PhD Days
Systematic investigations for the presence of quark matter in neutron star cores

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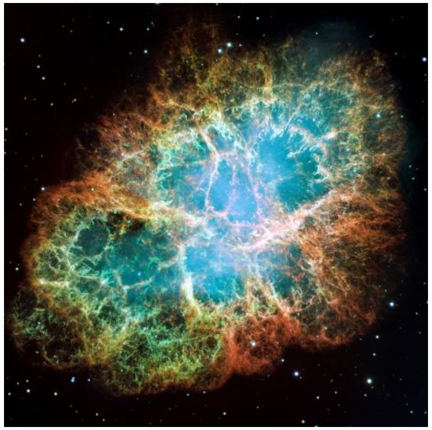
Supervised by Hubert Hansen and Joerg Aichelin

25/04/2023

Introduction: What is a neutron star?



Crab nebula from SN 1054

Remnants born in core-collapse supernovae from stars with $8\ M_\odot \leq M \leq 25\ M_\odot$

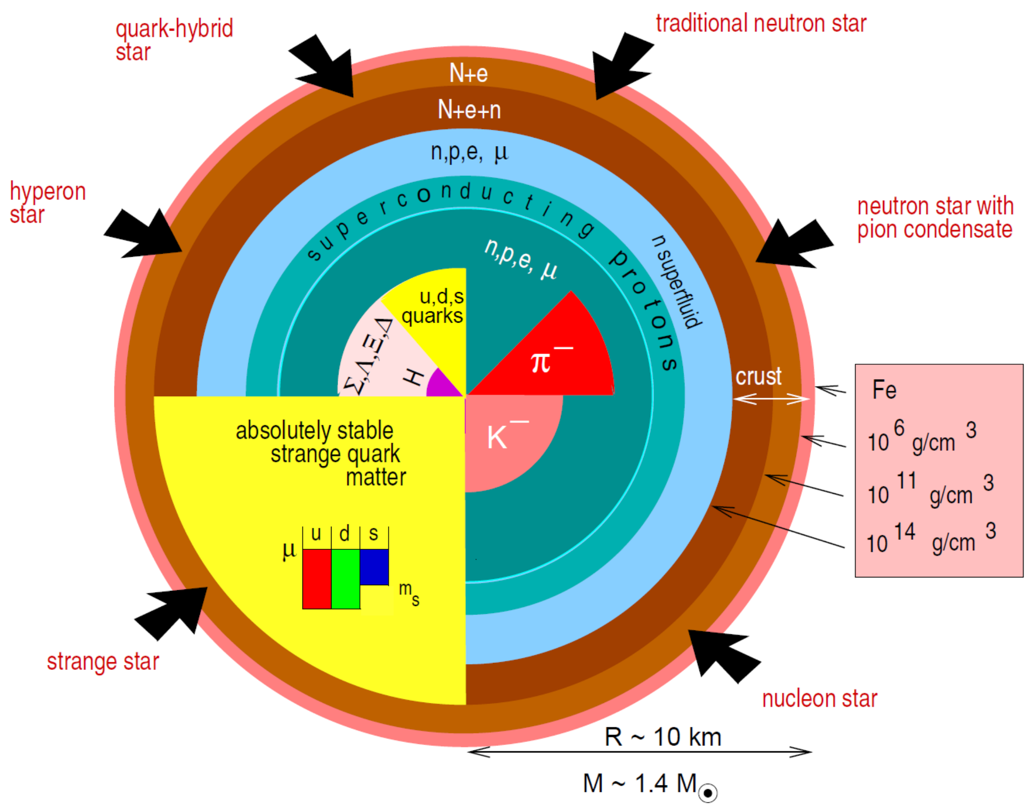
Some order of magnitudes :

- Very small star: $R \sim 15$ km ($\ll R_\odot = 696\,340$ km)
- But still relatively massive: $M \sim 1 - 2\ M_\odot$
- Schwarzschild radius: $R_S = \frac{2GM}{c^2} \sim 1$ km $\sim 0{,}1\ R$ (Sun: 0,000004 R)

- Average density: $\bar{\rho} \sim 5 \times 10^{14}$ g cm^{-3} ($\bar{n} \approx 0{,}3$ nucleon/fm^3)
- For an ordinary nucleus: $\rho_n \approx \rho_{sat} \approx 2{,}4 \times 10^{14}$ g cm^{-3} ($n_{sat} \approx 0{,}16$ nucleon/fm^3)
- Number of nucleons: $A \sim 10^{57}$

- Small temperatures: $T \sim 10^8$ K ~ 10 keV (at equilibrium)
- Fermi temperature (or Fermi energy) : $T_F \sim 100$ MeV – 1 GeV $\gg T$

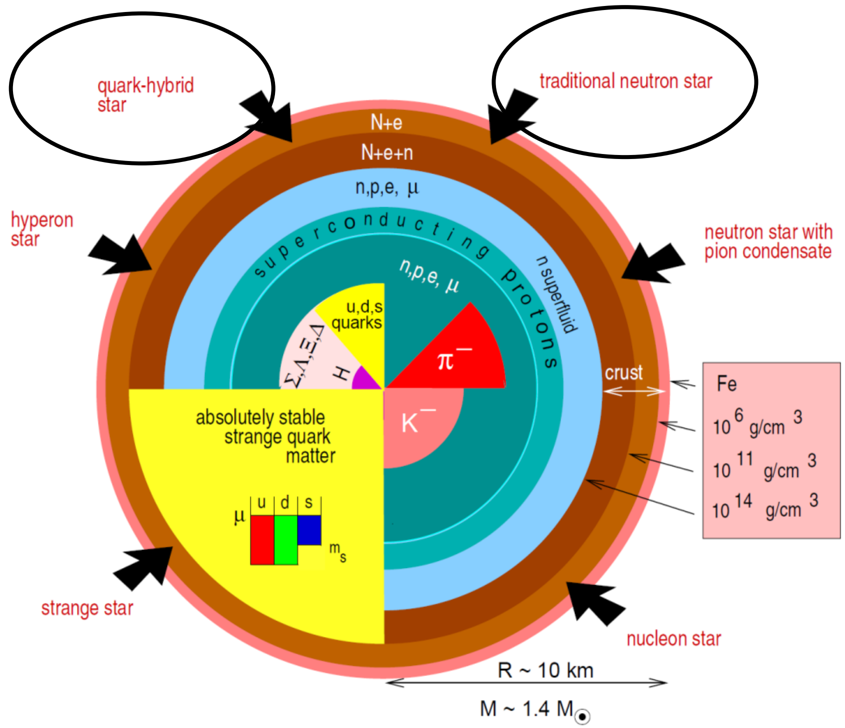
What is the state of neutron star matter?

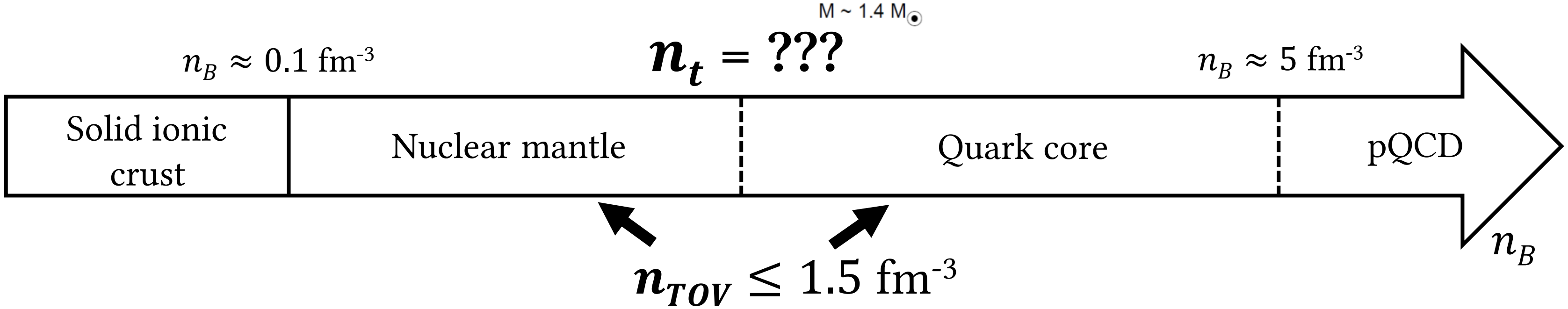


- Solid ionic crust with thickness ~ a few hundred meters
- Mantle of neutron rich nuclear matter
- "Exotic" composition could appear in the core

From Weber J.Phys.G27:465-474,2001

What is the state of neutron star matter?





What is the state of neutron star matter?

- Strongly interacting matter (nuclei, nucleons)
- Total charge would destabilize the star ⟶ charge neutral
- Weak equilibirum :

$$\mathrm{n} \leftrightarrow p + e^- + \bar{\nu}_e \qquad\qquad e^- \leftrightarrow \mu^- + \nu_e + \bar{\nu}_\mu$$

$$\mu_\mathrm{n} = \mu_p + \mu_e \qquad\qquad \mu_\mu = \mu_e$$

⇨ β-equilibrium drives composition for each density n_B

Introduction : The dense matter equation of state

- Hydrostatics equilibrium in compact stars is ruled by the TOV equation: static (non-rotating) and spherical hypothesis

$$\frac{dP}{dr} = -\frac{Gm}{r^2}\rho\left(1+\frac{P}{\rho}\right)\left(1-\frac{2Gm}{r}\right)^{-1}\left(1+\frac{4\pi r^2 P}{m}\right)$$

- The entire star structure is determined by the EoS $P(\rho)$

  ⟶ encodes all the model dependence

- Measurements of neutron star properties (M, R, Λ) ⟶ model constraints
- Objectives:
  - Confront the purely nuclear vs hybrid hypotheses
  - Assuming a phase transition occurs, what can the transition density be?

The nuclear EoS

- Meta-model of Margueron *et al* (2018)

$$e_N(x,\delta) = t_N(x,\delta) + \sum_\alpha \frac{1}{\alpha!}({v_\alpha}^{sat} + \delta^2 {v_\alpha}^{sym}) x^\alpha u_\alpha(x) \quad \text{with } x = \frac{n_B - n_{sat}}{3n_{sat}}, \delta = \frac{n_n - n_p}{n_B}$$

- Simple, flexible ⟶ explores all possible nuclear EoS
- Directly parametrized by the nuclear empirical parameters:

X	E_{sat}	E_{sym}	n_{sat}	L_{sym}	K_{sat}	K_{sym}	Q_{sat}	Q_{sym}	Z_{sat}	Z_{sym}	$m^\star$	$\Delta m^\star$
Order	0	0	1	1	2	2	3	3	4	4		
Unit	MeV	MeV	fm^{-3}	MeV	MeV	MeV	MeV	MeV	MeV	MeV		
X_{min}	-17.5	27	0.15	20	190	-400	-1200	-2000	-4000	-5000	0.6	-0.1
X_{max}	-14.5	37	0.17	80	300	300	1000	5000	5000	5000	0.8	0.2

- Unified extension in the crust, using a compressible liquid drop model (CLDM) for the inhomogeneous phases
- Finite size parameters are fitted to the masses of known nuclei

*J. Margueron, R. Homann Casali, and F. Gulminelli, Physical Review C **97**, 025805 (2018)*
T. Carreau, F. Gulminelli, and J. Margueron, The European Physical Journal A, vol. 55, (2019)
Grams, G. , Somasundaram, R. , Margueron, J. ,Reddy, S, Physical Review C , Vol. 105, No. 3 (2022)

The effective Nambu—Jona-Lasinio model

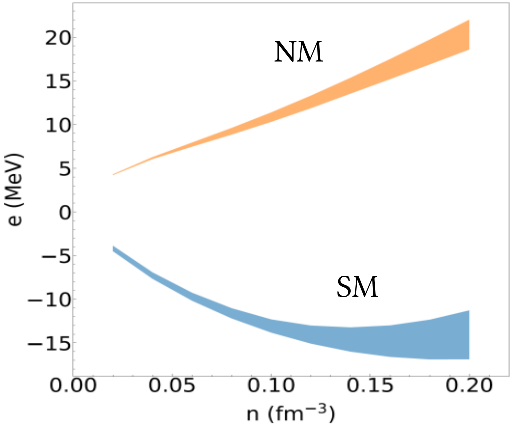
- Effective quantum field theory (relativistic)
- Quark degrees of freedom (3 flavors, no gluons)
- Reproduces the flavor symmetries of QCD:
  - Spontaneous breaking of $SU(N_f)_A$ ⟶ Goldstone mechanism
  - Symmetry restoration at finite μ
- Model parameters (coupling constants, bare masses) fitted to hadronic data in the vacuum (quantification of uncertainties → Pfaff et. al. (2023))
- Repulsive vector couplings ξ_ω and ξ_ρ kept as free parameters
- Connection to a nuclear model via the Maxwell construction

A Pfaff, H Hansen, J Aichelin, and J M. Torres-Rincon, Phys. Rev. C ***107****, 045204 (2023)*

Bayesian method

- Generate a large number (~ 10^8) of hybrid models:
- Flat prior on:
  - The nuclear empirical parameters on the nuclear side
  - The 2 free parameter of our NJL parametrization on the quark side: $\xi_\omega, \ \xi_\rho$

- Impose the model to be "reasonable":



  - Thermodynamic consistency (nuclear model):
    - $0 < c_s < 1$
    - $\frac{dP}{dn_B} > 0$
    - $e_{sym} = \frac{1}{n_B}\frac{\partial^2 \rho}{\partial \delta^2} > 0$
  - Compatibility with the ab initio χEFT energy bands (NM + SM) of Drischler *et al* (2016)
  - Possibility of a PT to quark matter before reaching the TOV mass

C. Drischler, K. Hebeler, and A. Schwenk, Phys. Rev. C, vol. 93, p. 054314, May 2016.

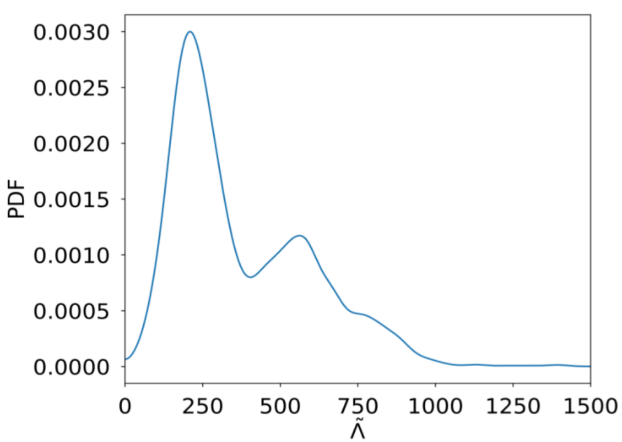
Bayesian method

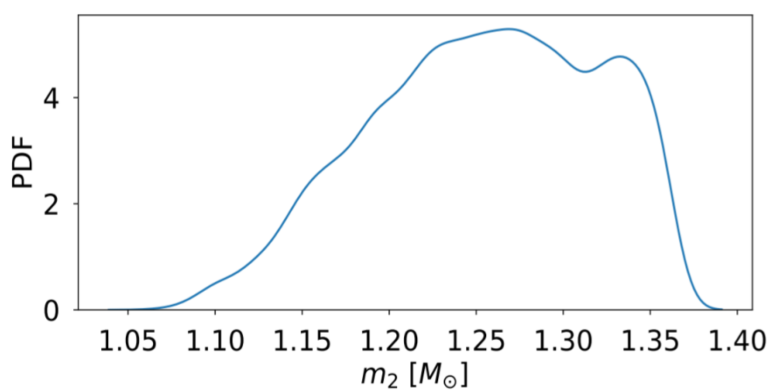
- Attribute weights to models based on their reproduction of experimental results:
  - Reaching a large enough TOV mass (J0740+6620 : $M = 2.08 \pm 0.07 M_\odot$)
  - Goodness of the fit to the experimental masses of nuclei:

$$w_{AME} = \frac{1}{N}\exp\left(-\frac{\chi^2}{2}\right) \quad \text{with } \chi^2 = \frac{1}{\nu}\sum_i \left(\frac{M^i - M^i{}_{AME}}{\sigma^i}\right)^2$$

  - Reproduction of the tidal deformability $\tilde{\Lambda}$ from GW170817 (Abbott *et al*, 2019)

$$w_{GW170817} = \frac{1}{N}\sum_j p_{LVC}\left(m_2{}^j\right) \times p_{LVC}\left(\tilde{\Lambda}^j\right) \qquad \tilde{\Lambda} = \frac{16}{13}\frac{(m_1 + 12m_2)m_1{}^4\Lambda_1 + (m_2 + 12m_1)m_2{}^4\Lambda_2}{(m_1 + m_2)^5}$$





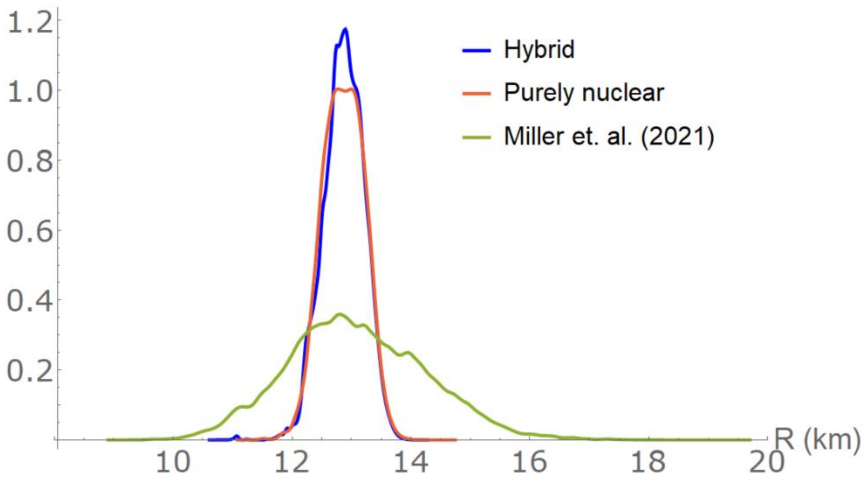
$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} \approx 1.188\ M_\odot$$

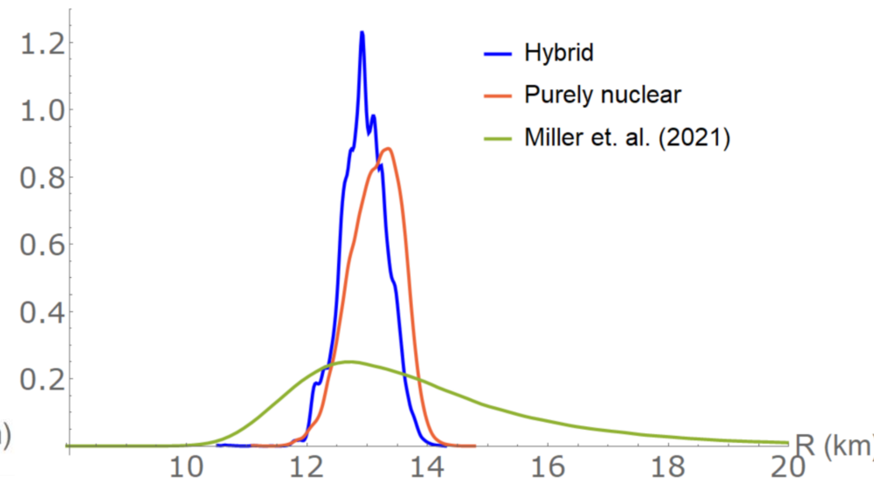
Abbott et al., Physical Review X 9, 011001 (2019)

Neutron star radii

Comparison with estimations of the radius from the NICER mission

J0030+0451 : $M = 1.44 \pm 0.14 M_{\odot}$ J0740+6620 : $M = 2.08 \pm 0.07 M_{\odot}$





→ Both model assumptions are compatible with NICER data

Inclusion of a quark PT has weak influence on the radii

M. C. Miller et al 2021 ApJL ***918*** *L28*

Neutron star radii

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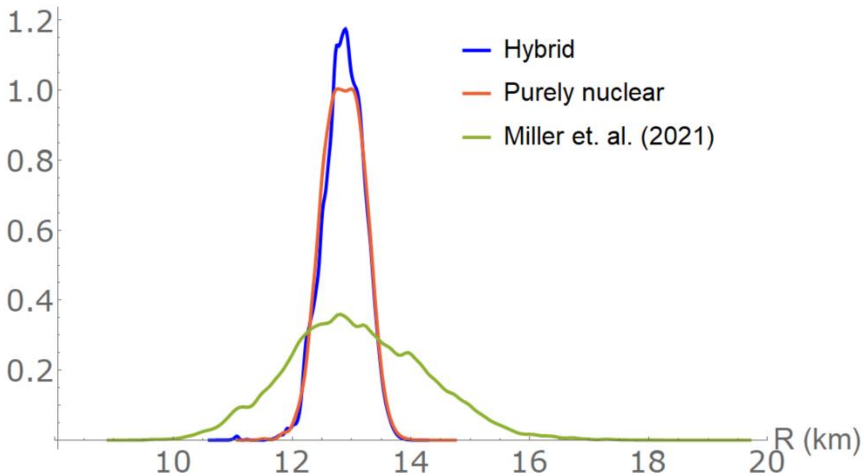
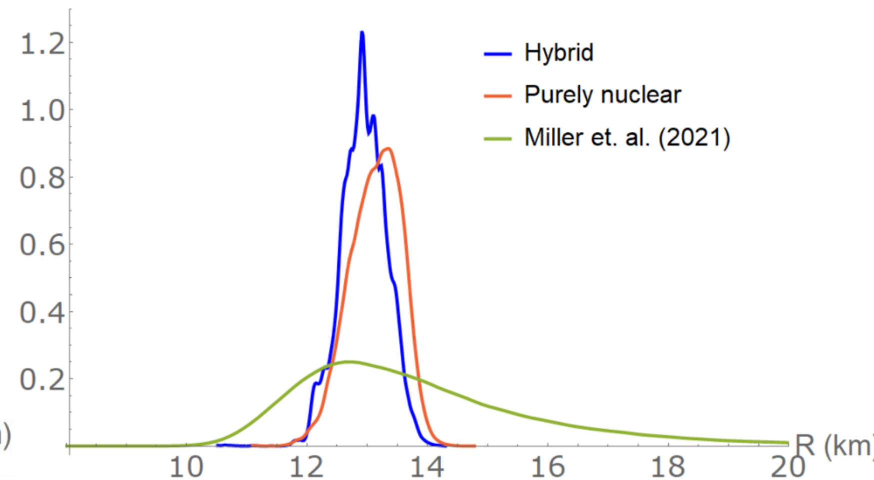
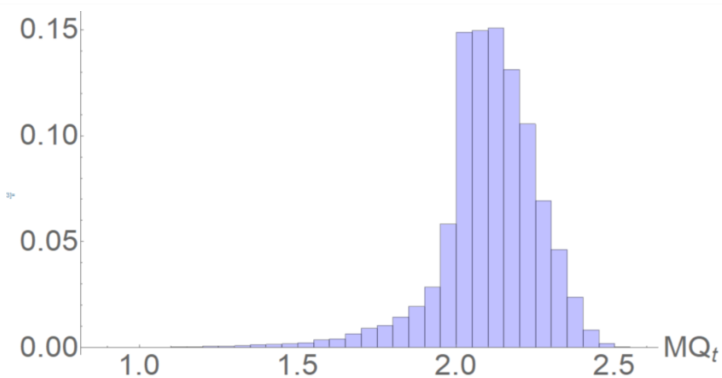


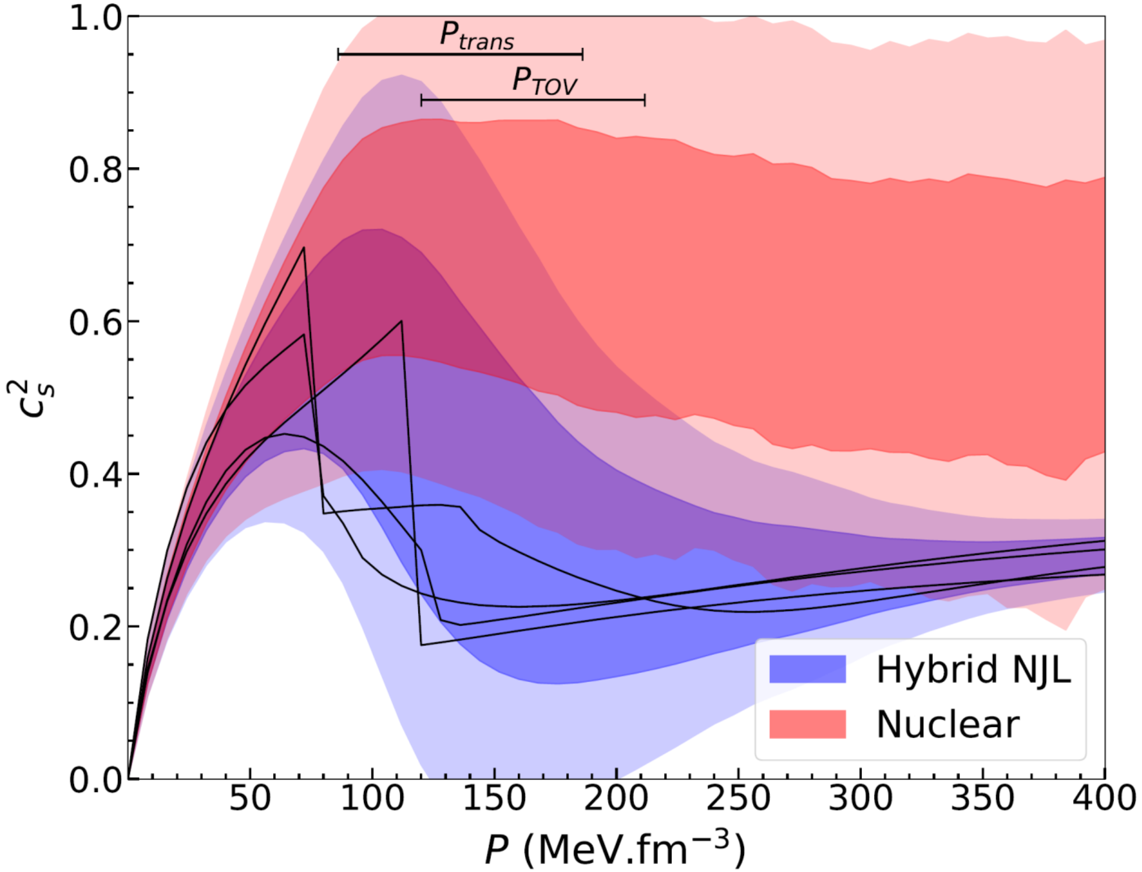




Quark cores typically appear only in very massive stars

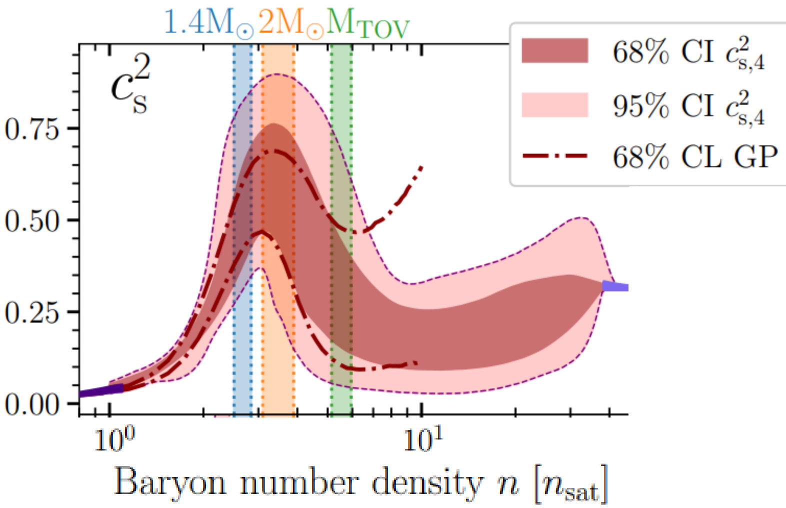
Global EoS properties

Sound speed $c_s{}^2 = \frac{dP}{d\rho}$

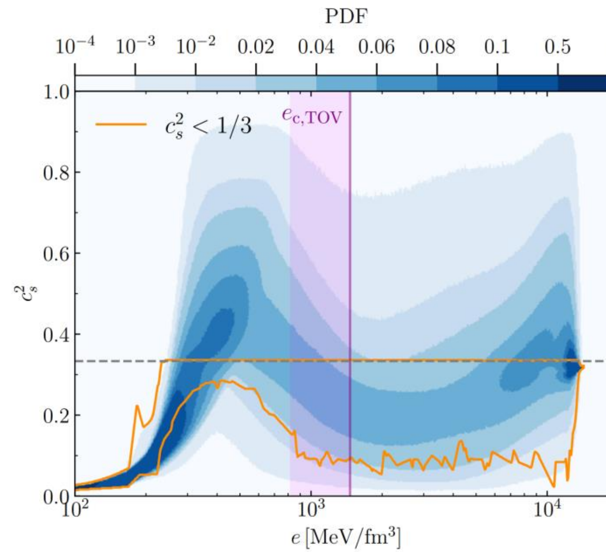


Global EoS properties

https://arxiv.org/abs/2303.11356

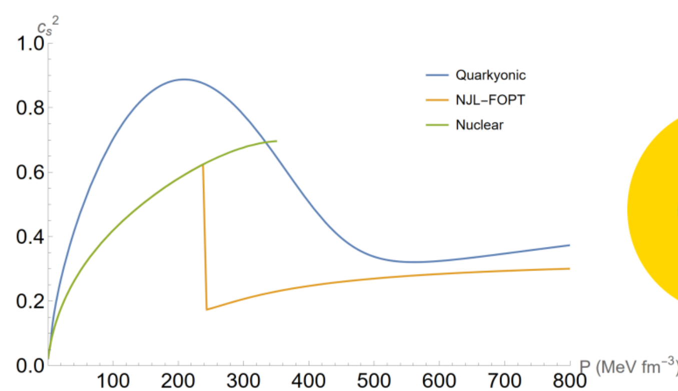


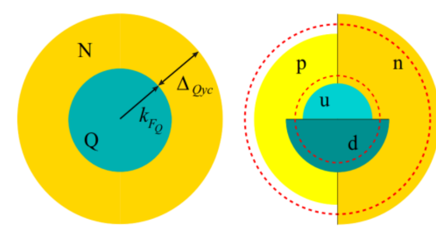
S Altiparmak, C Ecker, and L Rezzolla
The Astrophysical Journal Letters (2022)



Quarkyonic
phase transition ?

Microscopical interpretation
for the sound speed peak





Conclusions

- The bayesian framework allows us to consistently take into account all types of experimental knowledge.
- Hybrid NJL stars usually have a small QC that only appears in very heavy stars
- Relatively low impact on the radii

  ➡ Low observability of the PT with GW/X-ray measurements only

- Substantial effect on the susceptibilites (sound speed)
- Left for future works :
  - Influence of quark pairing (color superconductive phases)
  - Finite extent of the phase transition (crossover)

The NJL Lagrangian

Quark/gluon interaction

$$\mathcal{L}_{QCD} = \underbrace{\bar{\psi}(i\gamma\mu\partial_{\mu} - \hat{m} + \hat{\mu}\gamma_0)\psi}_{\text{Quarks}} + \overbrace{\bar{\psi}\gamma^{\mu}\frac{\lambda_a}{2}A_{\mu}{}^{a}\psi}^{\text{Quark/gluon interaction}} - \underbrace{\frac{1}{4}F_{\mu\nu}{}^{a}F^{\mu\nu}{}_{a}}_{\text{Gluons}}$$

Quarks

Gluons

$$\mathcal{L}_{NJL} = \bar{\psi}(i\gamma\mu\,\partial\mu - \hat{m} + \hat{\mu}\gamma_0)\psi + \sum_{C} G^{C}(\bar{\psi}\Gamma^{C}\psi)^2 + \mathcal{L}'_{t\ Hooft}$$

- Interaction parametrized by several coupling constants associated to the different channels considered: G_S, G_ρ, G_ω, K
- Total interaction must preserve the flavor symmetries
- $U(1)_A$ symmetry group broken by the ‘t Hooft term (anomaly)

The mean field approximation

- Assume small fluctuations of the fields around a mean value

$$\hat{\mathcal{O}} = \langle \hat{\mathcal{O}} \rangle + \delta \hat{\mathcal{O}}$$

- For scalar interactions : $G_S(\bar{\psi}\psi)^2 \approx \underbrace{2GS\langle\bar{\psi}\psi\rangle\bar{\psi}\psi}_{\text{Mass modification}} + GS\langle\bar{\psi}\psi\rangle^2$

- For vector interactions $G_V(\bar{\psi}\gamma_0\psi)^2 \approx \underbrace{2GV\langle\psi^\dagger\psi\rangle\psi^\dagger\psi}_{\text{Chemical potential modification}} + GV\langle\psi^\dagger\psi\rangle^2$

- With correct flavor factors following $SU(N_f = 3)$ algebra:

$$m_i = mi_{0\,\_\,4}GS\,\langle\bar{\psi}_i\psi_i\rangle + 2K\langle\bar{\psi}_j\psi_j\rangle\langle\bar{\psi}_k\psi_k\rangle$$

$$i, j, k = u, d, s$$

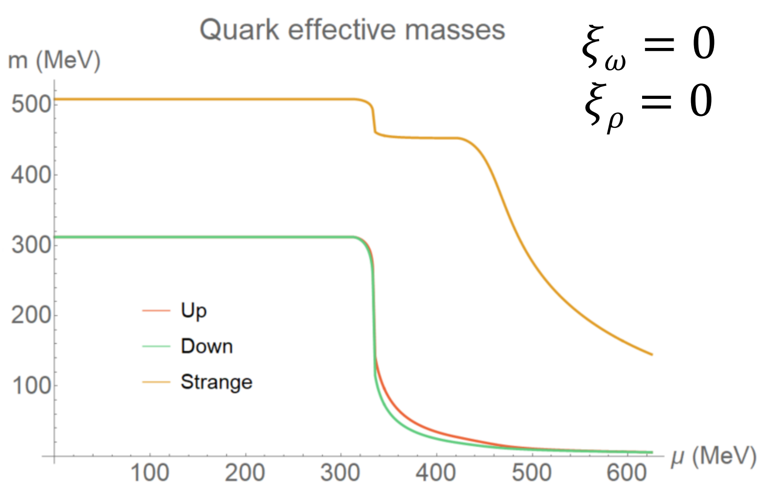
$$\widetilde{\mu}_i = \mu_i - \frac{4}{3}G_\omega(n_i + nj + nk) - \frac{4}{3}G_\rho(2ni - nj - nk)$$

The phase transition

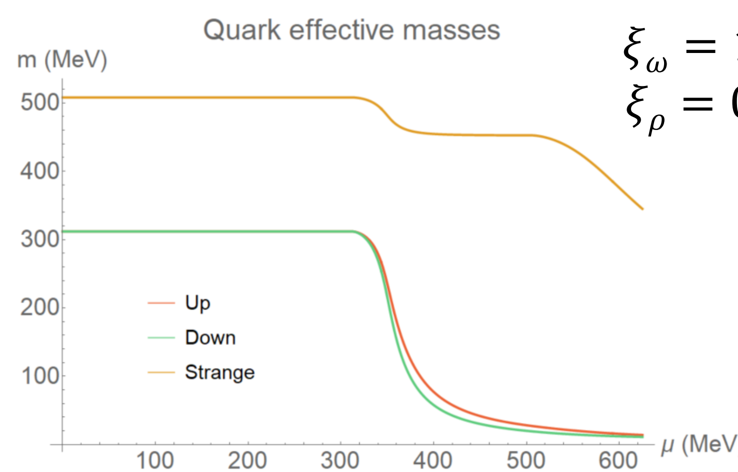
- Fix external conditions for NS at equilibrium:
  - Zero temperature
  - Charge neutrality
  - β-equilibrium

$$\Longrightarrow \begin{cases} \mu_u = \mu + \frac{2}{3}\mu_e \\ \mu_d = \mu_s = \mu - \frac{1}{3}\mu_e \end{cases}$$

- Chiral symmetry restoration



$\xi_\omega = 0$
$\xi_\rho = 0$



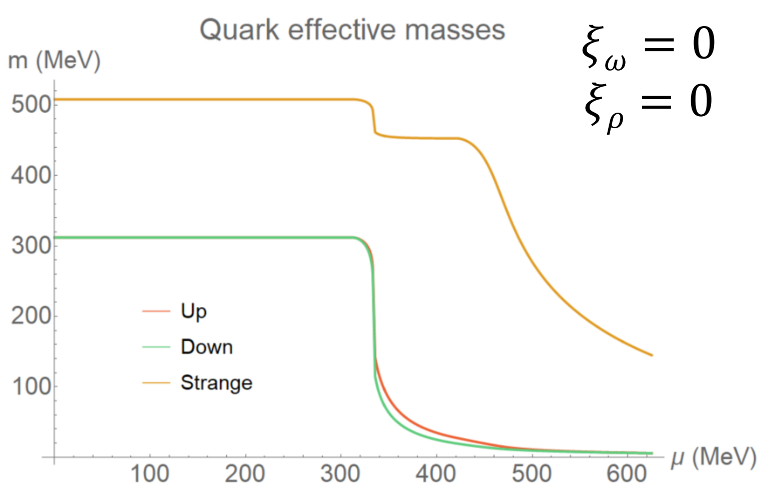
$\xi_\omega = 1$
$\xi_\rho = 0$

The phase transition

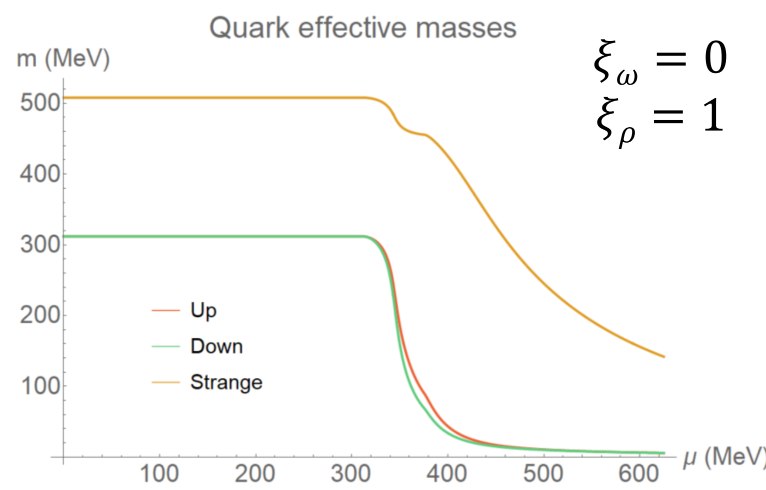
- Fix external conditions for NS at equilibrium:
  - Zero temperature
  - Charge neutrality
  - β-equilibrium

$$\Longrightarrow \begin{cases} \mu_u = \mu + \frac{2}{3}\mu_e \\ \mu_d = \mu_s = \mu - \frac{1}{3}\mu_e \end{cases}$$

- Chiral symmetry restoration



$\xi_\omega = 0$
$\xi_\rho = 0$



$\xi_\omega = 0$
$\xi_\rho = 1$

The equation of state

Fermi pressure of a free gas of quasi-quarks

Additional contribution from vector interactions

$$P_{tot} = P_{quarks} - B_{eff} + P_{vector} + P_{leptons}$$

Effective " bag " pressure

Fermi pressure of electrons (+ muons)

$$P_{vector} = \frac{2}{3} G_{\omega}(n_u + nd + ns)^2 + G_{\rho}(n_u - nd)^2 + \frac{1}{3} G_{\rho}(n_u + n_d - 2n_s)^2$$

- ω interactions couple to the total baryonic density of the system (symmetric in flavor)
- ρ interactions couple to the isospin and flavor hypercharge densities (asymmetric in flavor)

Hybrid EoS

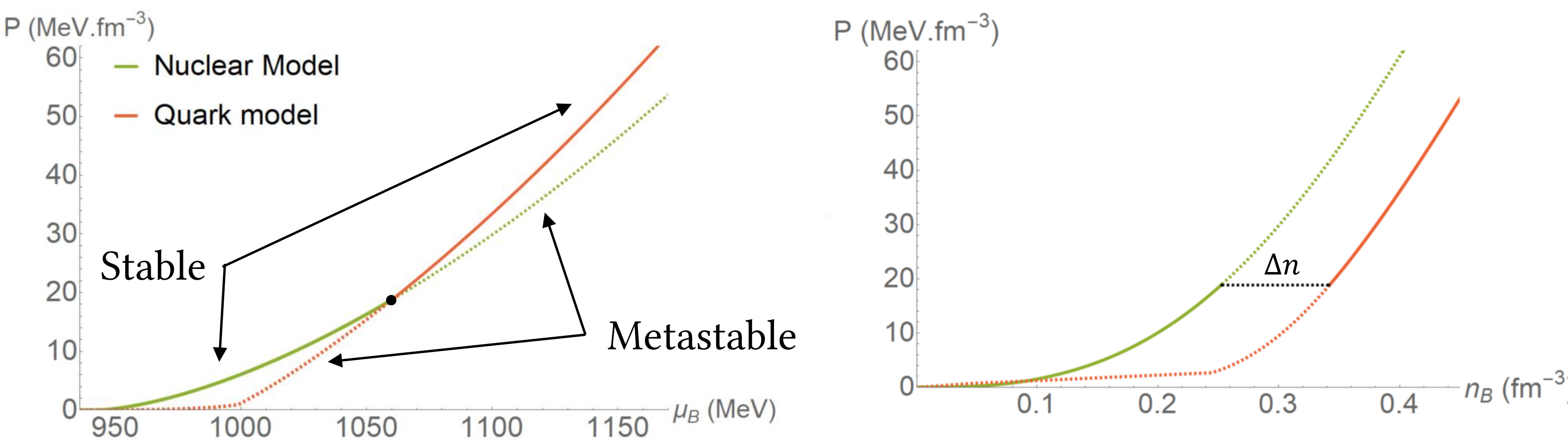
- Deconfinement phase transition:

  Nuclear matter (n, p, e, μ) ⟶ Quark matter (u, d, s, e)

- Gibbs thermodynamical equilibrium conditions :

  $P_N = P_Q$ $\mu_N = \mu_Q$ $T_N = T_Q$

- Grand canonical ensemble : $\Omega = -P$



Maxwell's construction (first order)

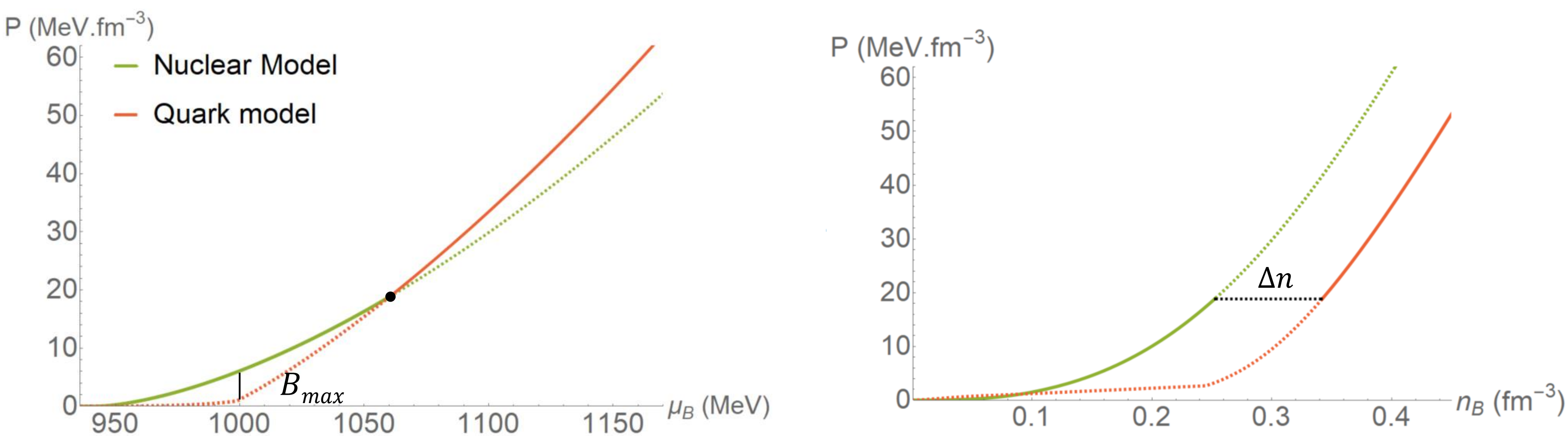
Hybrid EoS

- NJL pressure defined up to constant; we allow the freedom:

$$P_Q \to P_Q + B^* \qquad \rho_Q \to \rho_Q - B^*$$

where B^* is a constant free parameter satisfying $B^* < B_{max}$

- Maxwell's construction not totally consistent : μ_e discontinuity

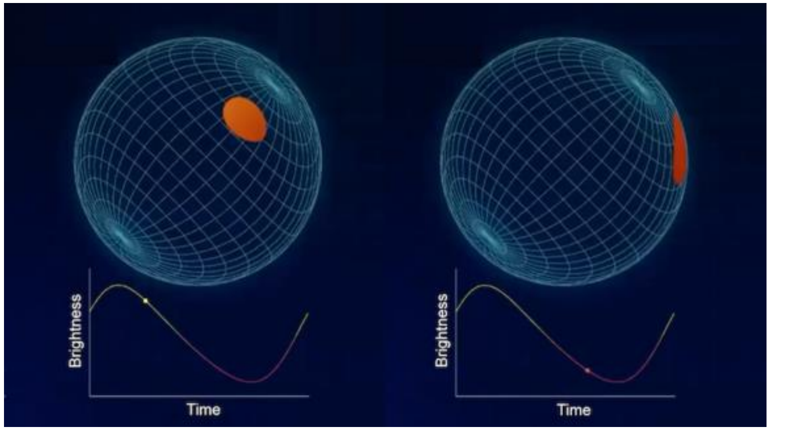


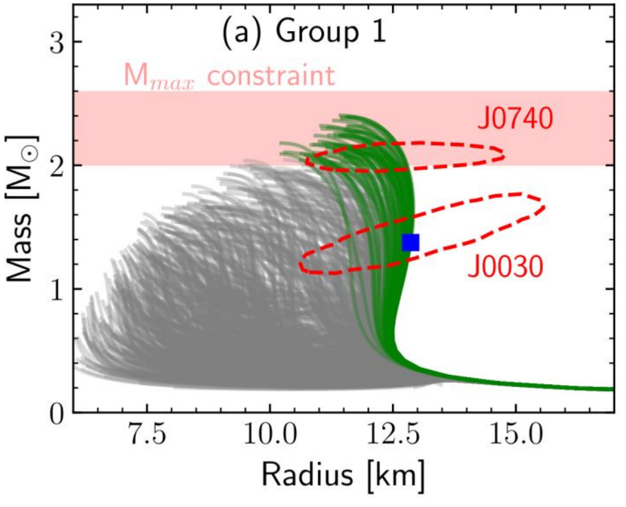
Maxwell's construction (first order)

X-ray observations : the NICER mission

- Measurements of X-ray emitting hot spots
- Light curve heavily affected by gravity (Doppler effect)

⟶ Radius estimations

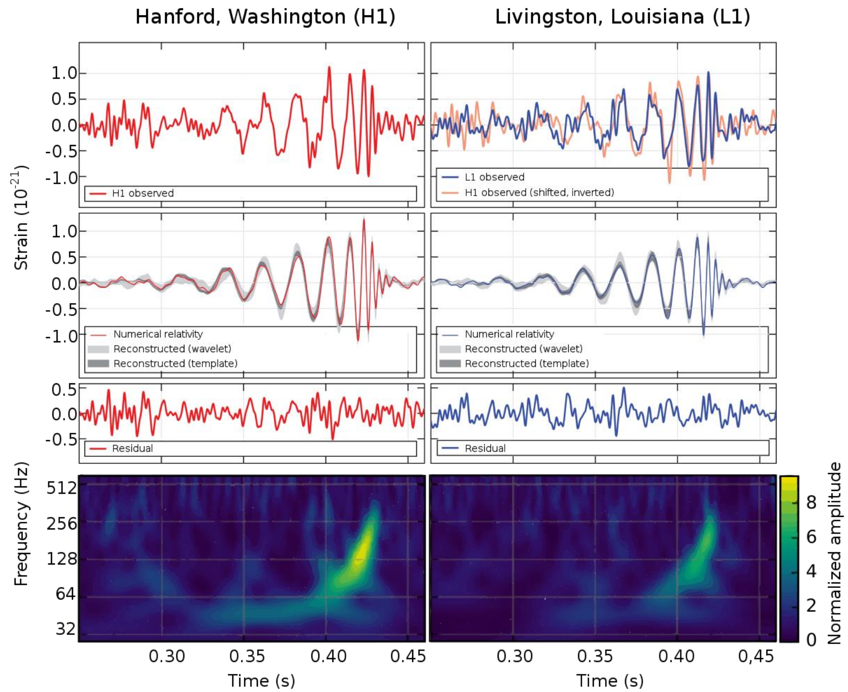


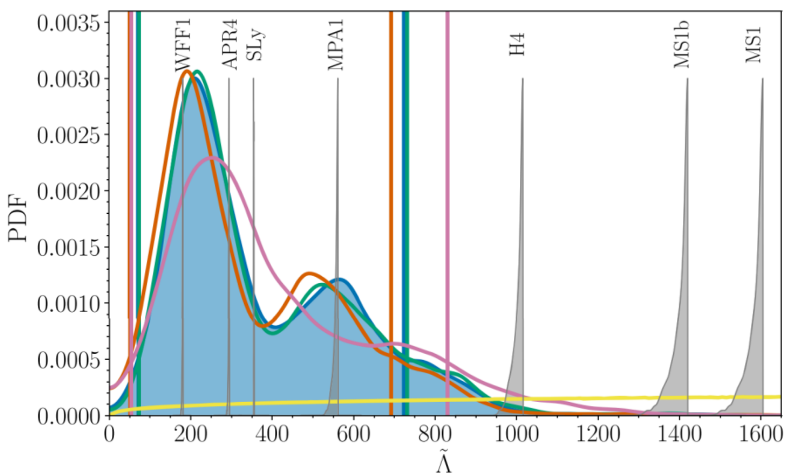


Somasundaram et al. arxiv:2112.08157

Gravitational observations : GW170817

- GW170817 : First observation with LIGO/VIRGO of the GW signal of a binary neutron star merger (BNS)
- EM counterpart confirmation
- $M_1 \approx M_2 \approx 1{,}4\ M_\odot$



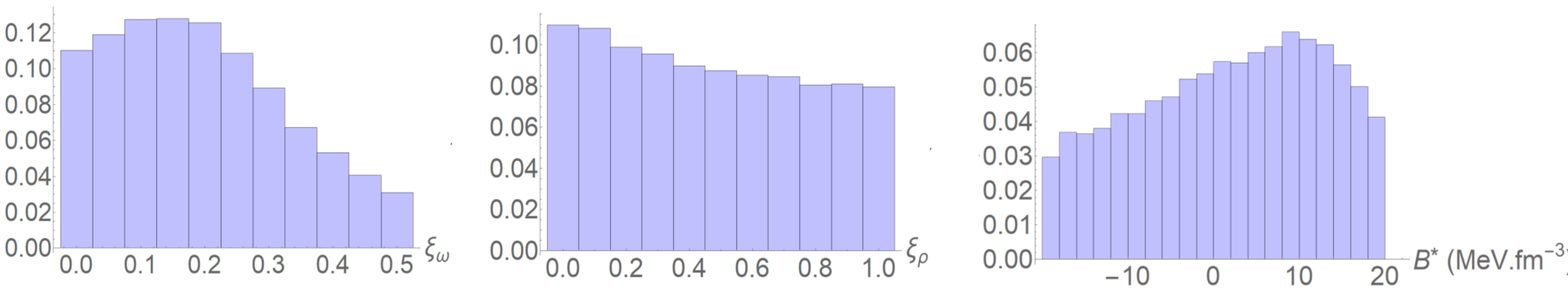


- Inspiral phase dependent on the tidal deformability Λ of the stars

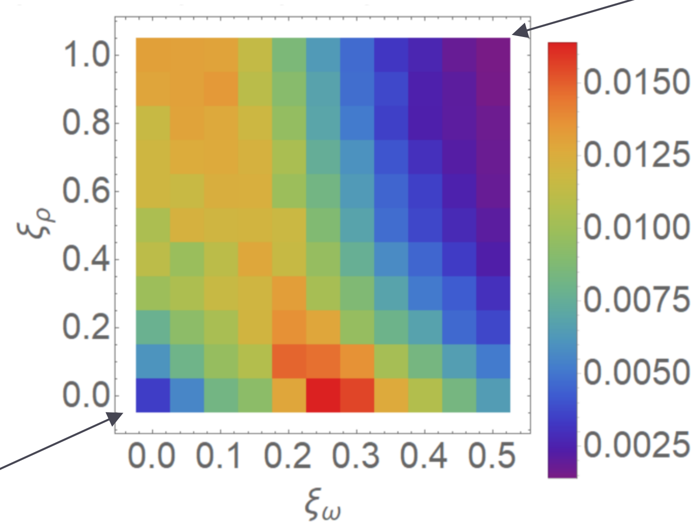
$$70 \leq \tilde{\Lambda} \leq 720 \text{ (90\% level)}$$

*LIGO/VIRGO (Abbott et al) : Phys. Rev. X **9**, 011001*

Posterior results – quark parameters

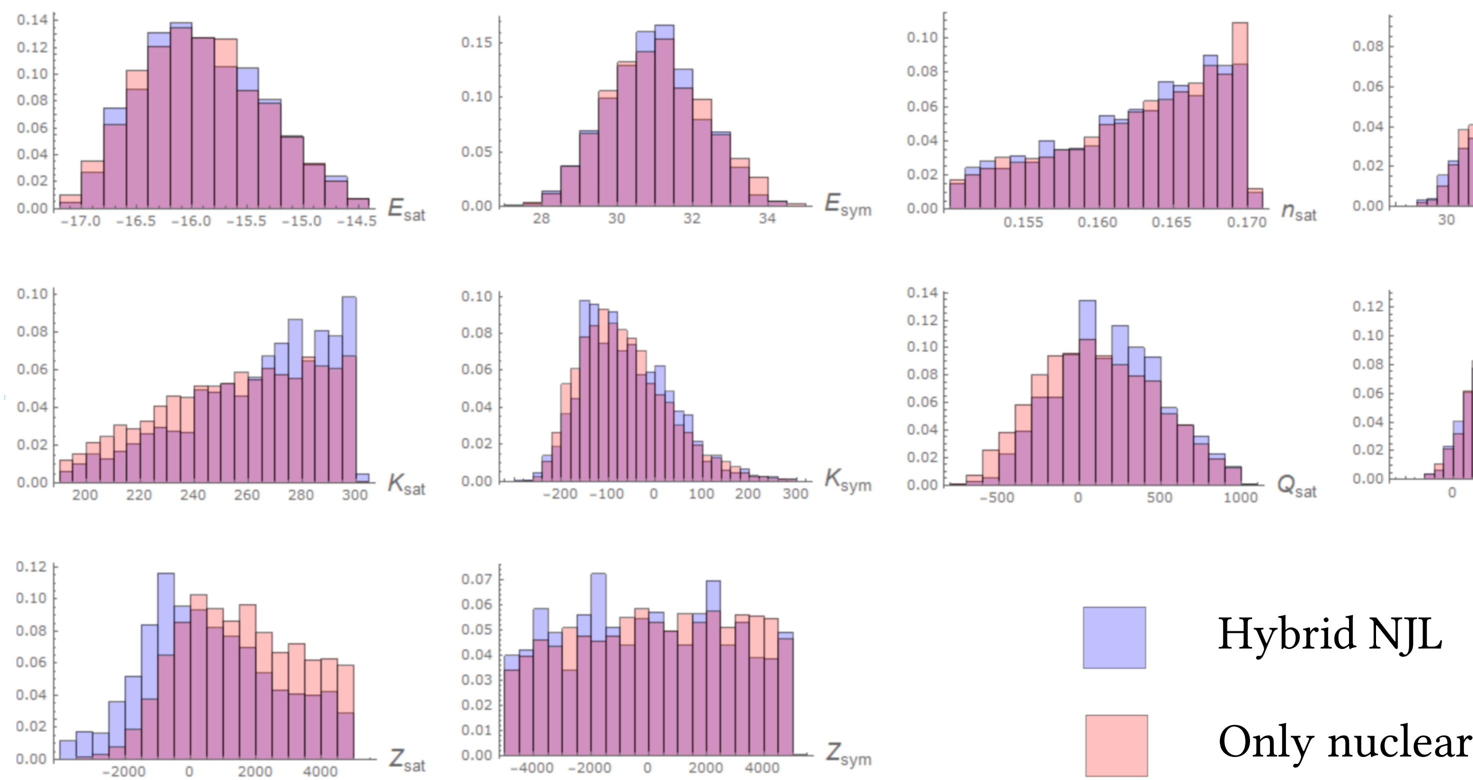


Quark EoS too stiff

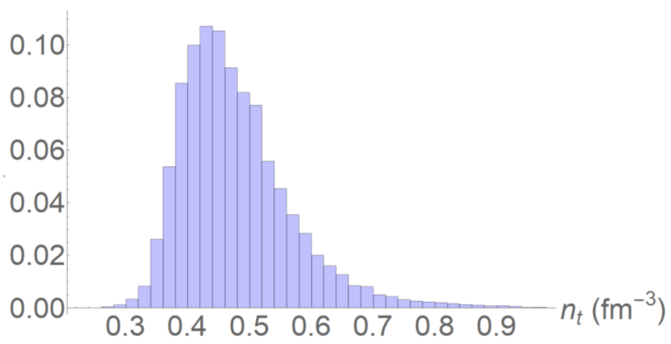


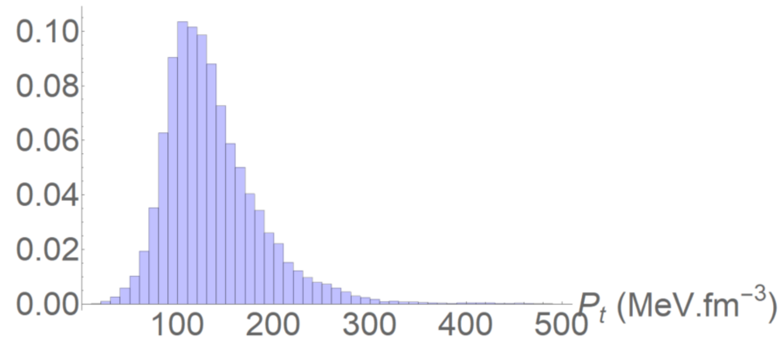
Quark EoS too soft

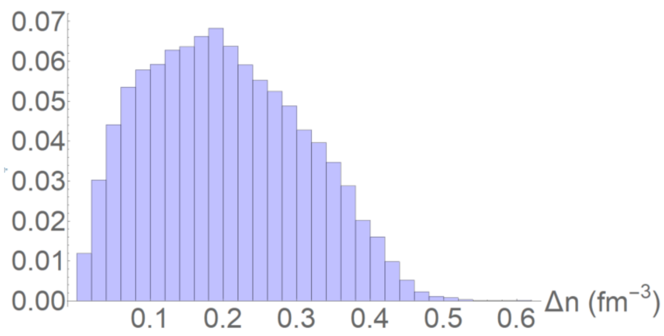
Posterior results – nuclear parameters



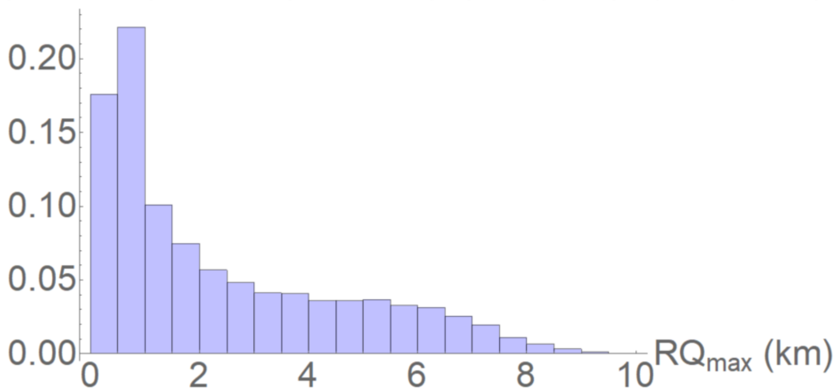
Posterior results – phase transition characteristics

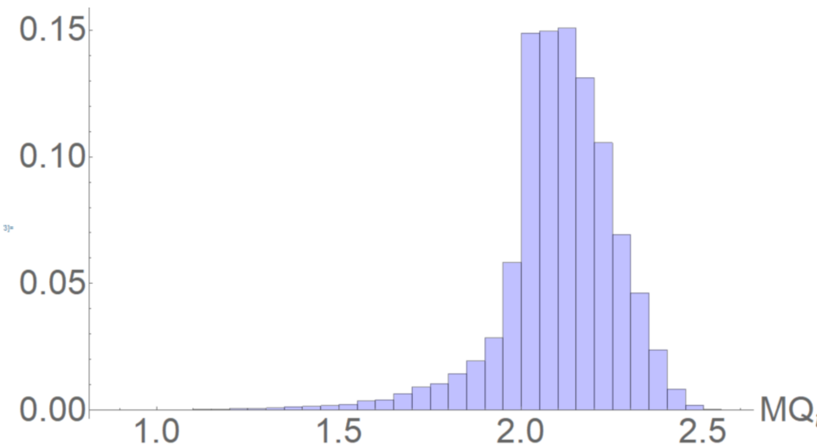






Posterior results – Hybrid star properties





$RQ_{max} \equiv$ Radius of the quark core in the maximum mass configuration
$MQ_t \equiv$ Total mass of the star as quarks start to appear