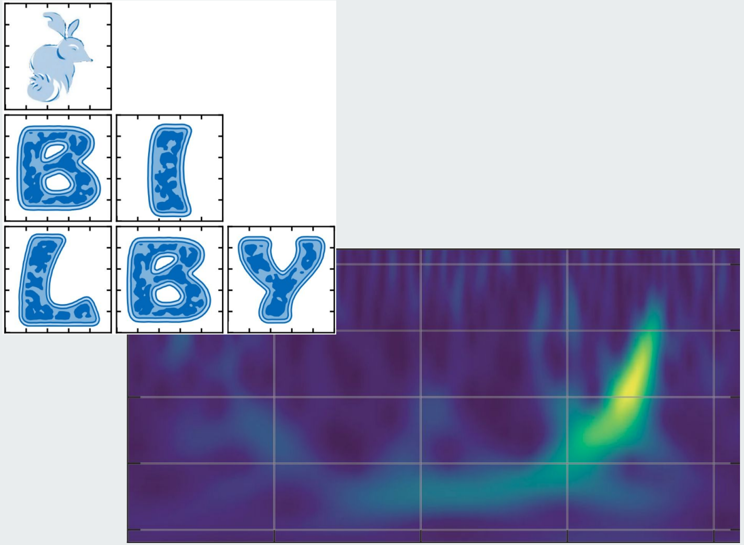
Measuring Neutron Star Equation of State with LVK

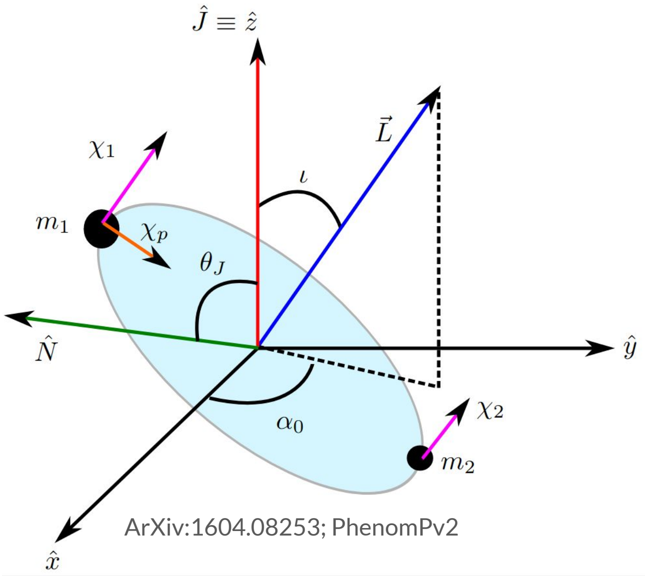
Jean-François Coupechoux
April 24, 2023

Introduction



- The detection of gravitational waves from the coalescence of a neutron star binary system, offers new opportunities to directly probe the properties of matter at the extreme conditions.
- Part I
  - Bayesian inference with fixed EOS
- Part II
  - Parameter estimation with ROQ

Parameter estimation



ArXiv:1604.08253; PhenomPv2

- **Bilby:** a user-friendly Bayesian inference library
- Bayes' theorem:

$$\overbrace{p(\theta|d,\mathcal{M}_A)}^{\text{Posterior distribution}} = \frac{\overbrace{\pi(\theta|\mathcal{M}_A)}^{\text{Prior}}\,\overbrace{\mathcal{L}(d|\theta,\mathcal{M}_A)}^{\text{Likelihood}}}{\underbrace{\mathcal{Z}(d|\mathcal{M}_A)}_{\text{Evidence}}}$$

GW170817 event

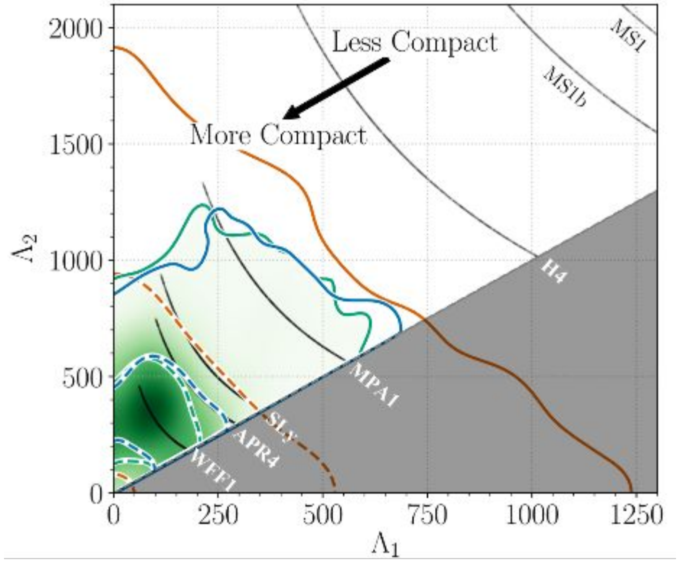
- Orange shadow: No relation between the two tidal deformabilities

$$\texttt{Uniform}\ \Lambda_2 \in [0, 5000]\ \ \texttt{Uniform}\ \Lambda_1 \in [0, 5000]$$

- Green shadow: EOS-insensitive relation to impose a common EOS for the two bodies

$$\texttt{Uniform}\ \Lambda_s \in [0, 5000]\quad \Lambda_a(\Lambda_s, q)$$

- Blue shadow: Parametrized EOS expresses the logarithm of the adiabatic index of the EOS Γ(p; γi), as a polynomial of the pressure p, where γi = (γ0, γ1, γ2, γ3) are the free EOS parameters.



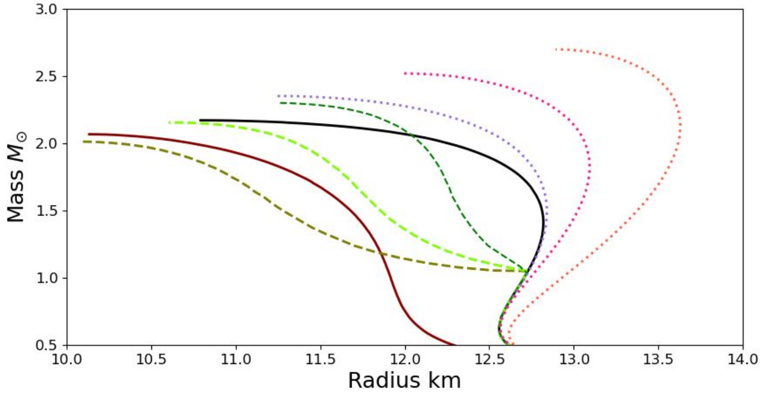
Marginalized posterior for the tidal deformabilities of the two binary components of GW170817 (arXiv:1805.11581)

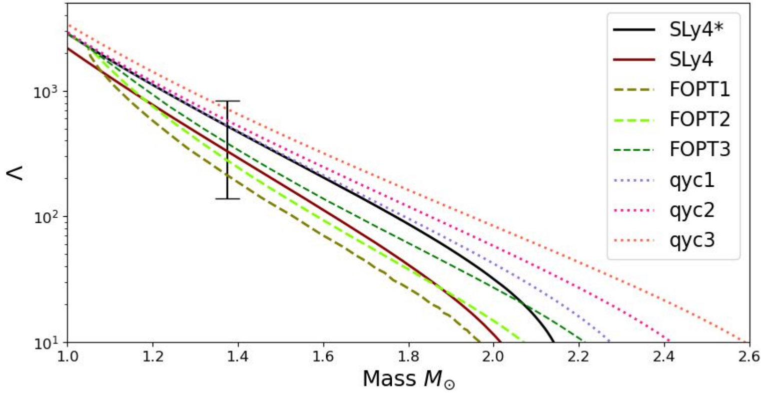
Bayesian inference with fixed equations of state

arXiv:2302.04147
J-F Coupechoux, R Chierici, H. Hansen,
J. Margueron, R. Somasundaram and V. Sordini

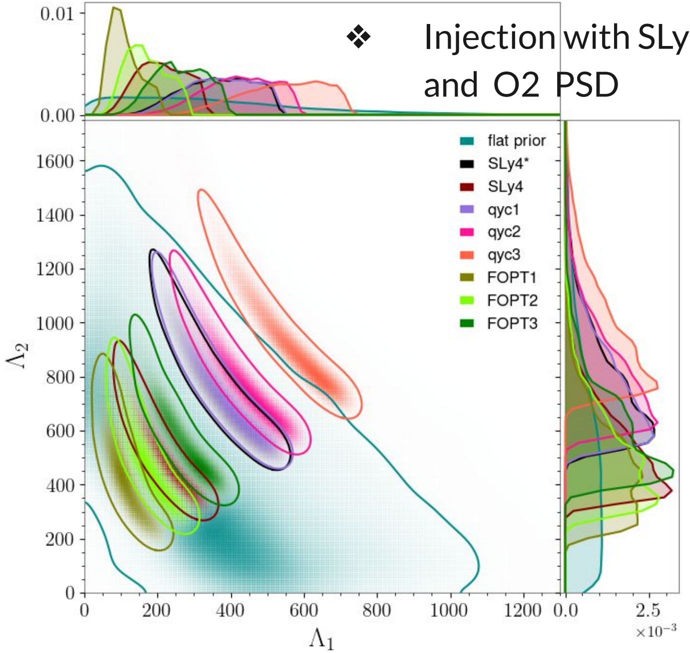
Equation of state

- Two nucleonic EOS SLy4 and SLy4*
- FOPT first order phase transition occurring at $n_t = 2\, n_{sat}$ with the sound speed $c^2 = 2/3$.
- qyc quarkyonic models adapted to beta-equilibrated matter in compact stars.

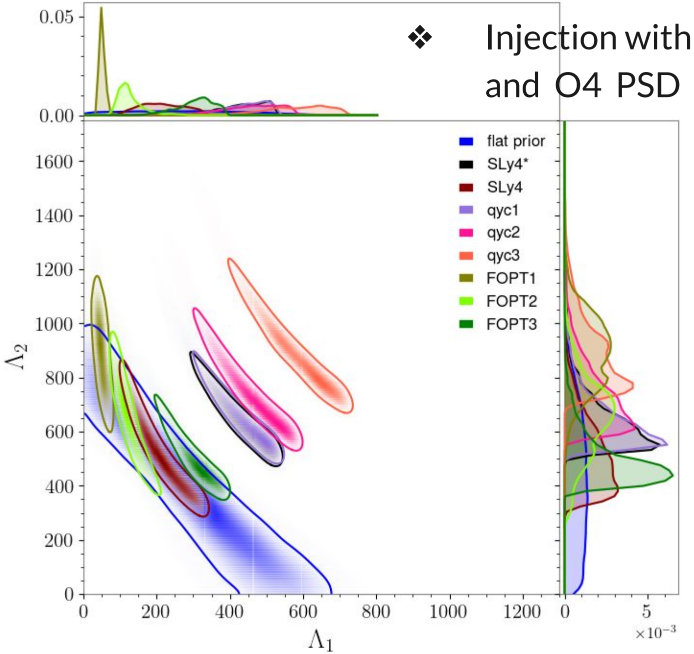




Simulated data like GW170817



- ❖ Injection with SLy4 and O2 PSD



- ❖ Injection with SLy4 and O4 PSD

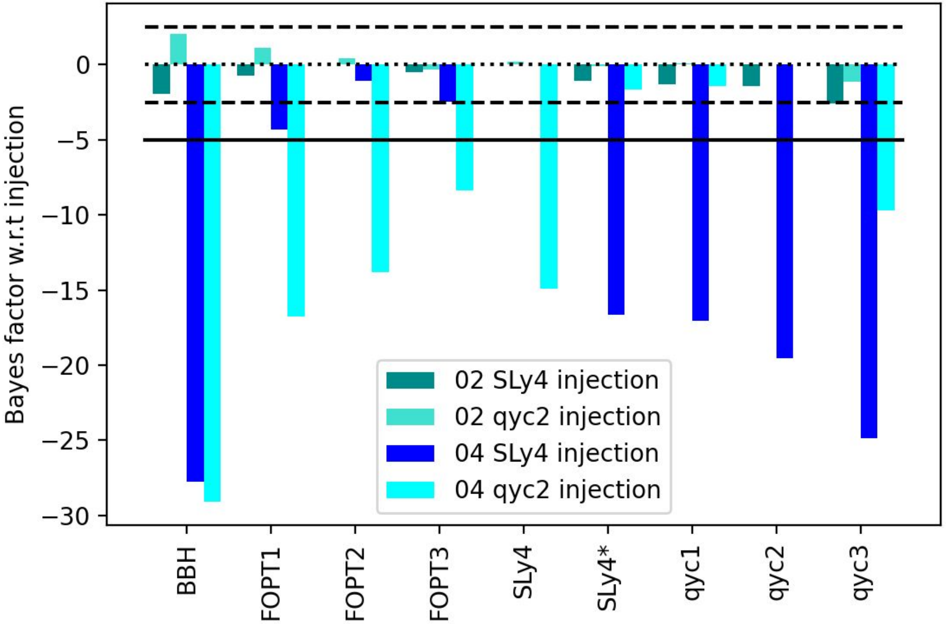
Bayes factor

The Bayes factor defined by

$$\mathcal{B}_{AB} = \frac{\mathcal{Z}_A}{\mathcal{Z}_B}$$

compare the ability of two models to describe the same data via the Jeffrey's scale.

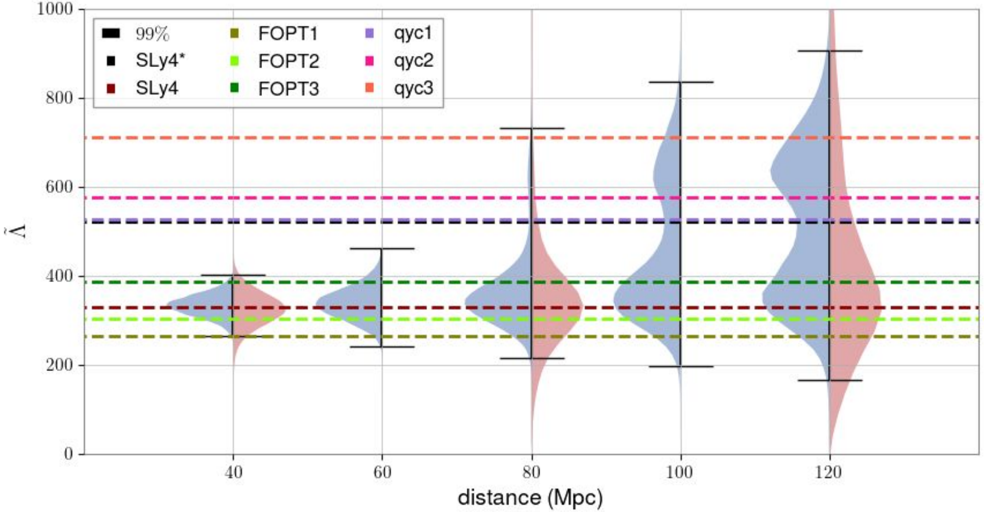
$\lvert \ln \mathcal{B}_{AB} \rvert$	Probability	
< 1	< 0.731	Inconclusive
2.5	0.924	Moderate evidence
5	0.993	Strong evidence



Impact of the distance of the source on O4 signal

Impact of the distance on the shape of the reconstructed signal assuming the O4 PSD and SLy4 EoS.

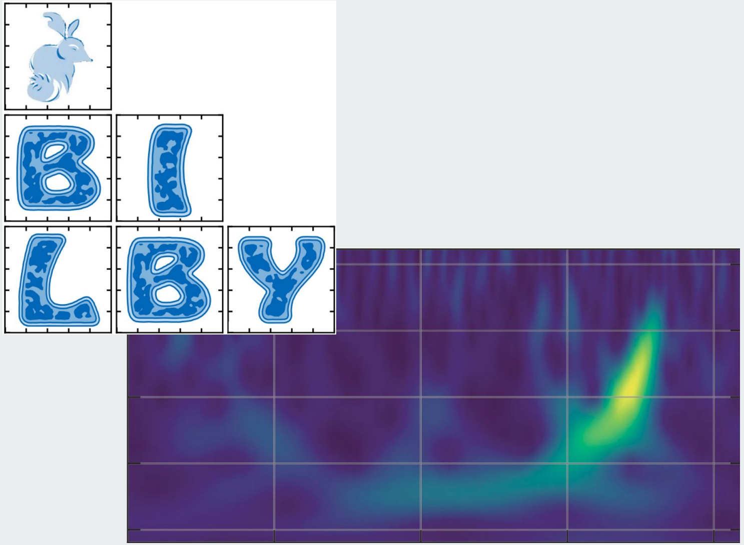
- The dimensionless effective tidal deformability within an uncertainty 7 times better than the one obtained in O2.
- Any GW170817-like-single-event within a distance of 100 Mpc would imply significantly improved constraints.
- The average BNS event rate of having a BNS merger as close as GW170817 is 5-48 years.



Parameter estimation with Reduced Order Quadrature

Article in preparation
J-F Coupechoux and H. Qi

State of the art



- Bayesian inference for a low mass system takes weeks or even months to run with traditional methods.

- One way to significantly reduce offline parameter estimation run time is using Reduced Order Quadrature (ROQ) rule, see references 1, 2, and 3. This method has been implemented in Bilby package.

- We expect to have the ROQ method widely used in O4.

Gains obtained with ROQ

- The usual technique takes about two days using 160 CPUs with parallel_bilby
- The Bayesian analysis with ROQ takes about two days using only one CPU with bilby_pipe

With ROQ, Bayesian analysis of a neutron star merger can be performed during the day.

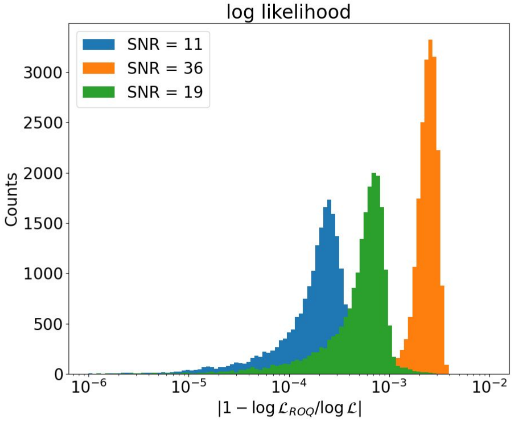
ROQ likelihood vs Standard likelihood

- $L \simeq [f_{high} - f_{low}]T$ terms for the standard likelihood calculation:

$$
\begin{aligned}
&2\log\mathcal{L}(d|\vec{\Lambda}) + (d|d) \\
&\quad = 2\Re\sum_{l=1}^{L}\frac{4\Delta f \tilde{d}^{*}(f_l)}{S_n(f_l)}\tilde{h}(f_l;\vec{\Lambda}) - \sum_{l=1}^{L}\frac{4\Delta f}{S_n(f_l)}\tilde{h}(f_l;\vec{\Lambda})\tilde{h}^{*}(f_l;\vec{\Lambda})
\end{aligned}
$$

- $N_L + N_Q$ terms for the ROQ likelihood calculation:

$$
\begin{aligned}
&2\log\mathcal{L}(d|\vec{\Lambda})_{ROQ} + (d|d) \\
&\quad = 2\Re\sum_{j=1}^{N_L}\omega_j(t_c)\tilde{h}(F_i;\vec{\Lambda}) - \sum_{k=1}^{N_Q}\Psi_k\tilde{h}(\mathcal{F}_k;\vec{\Lambda})\tilde{h}^{*}(\mathcal{F}_k;\vec{\Lambda})
\end{aligned}
$$



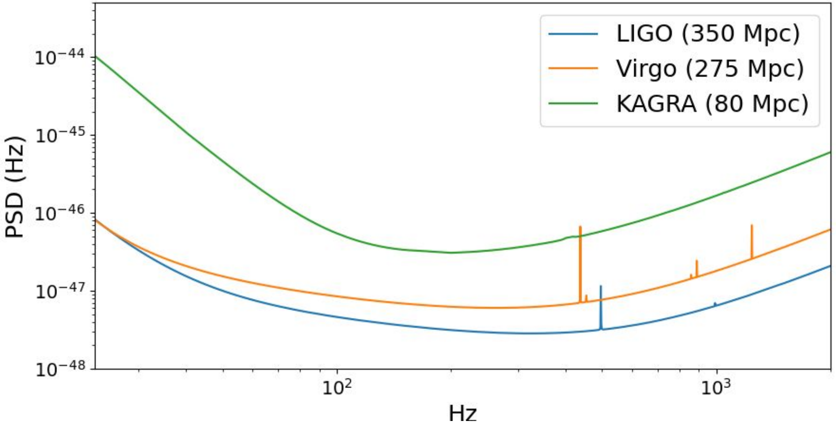
The likelihoods from the two methods are indistinguishable, up to a tiny fractional difference.

PE ROQ Robustness Tests with BNS Mergers

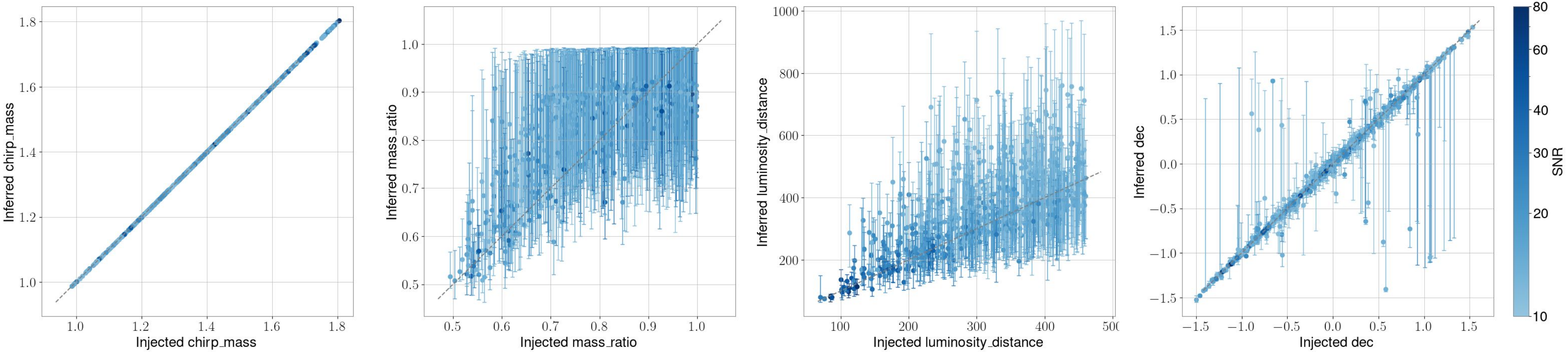
- 3000 BNS events are simulated using the IMRPhenomPv2_NRTidal approximant.
  - 1000 using mpa1 eos
  - 1000 using sly230a eos
  - 1000 using hqc18 eos

- The waveforms are injected in the LVK collaboration interferometer network.

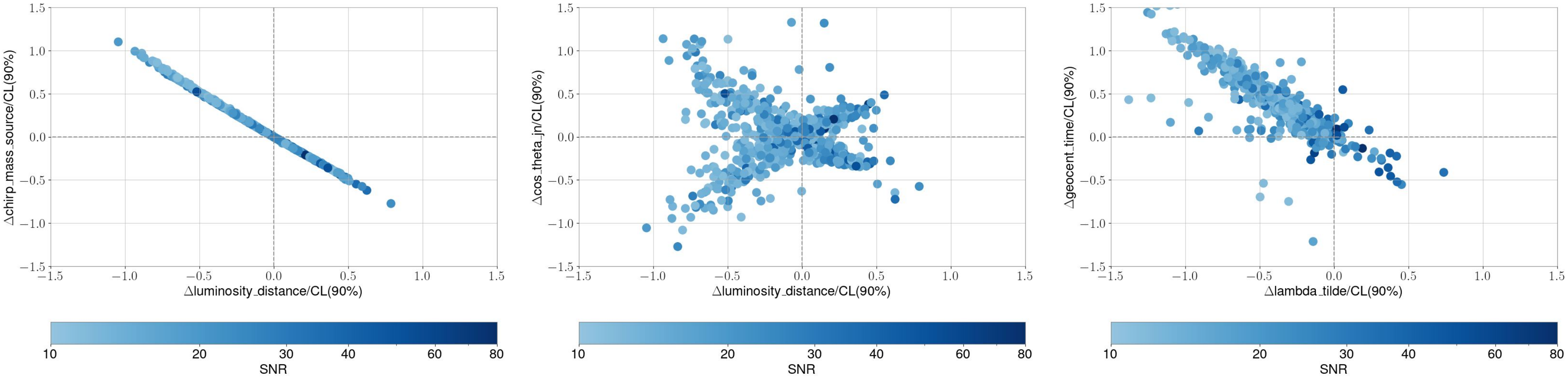
- The PSD modeling the detector noise is chosen equal to the expected value for the future run called O5



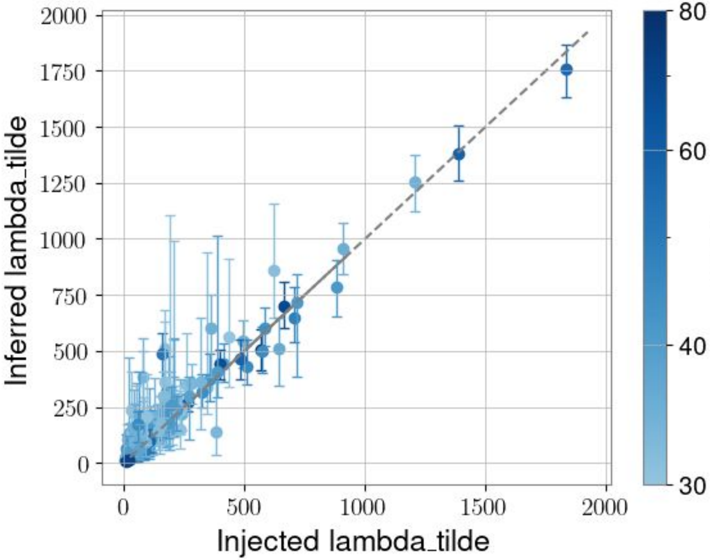
Inferred and injected parameters

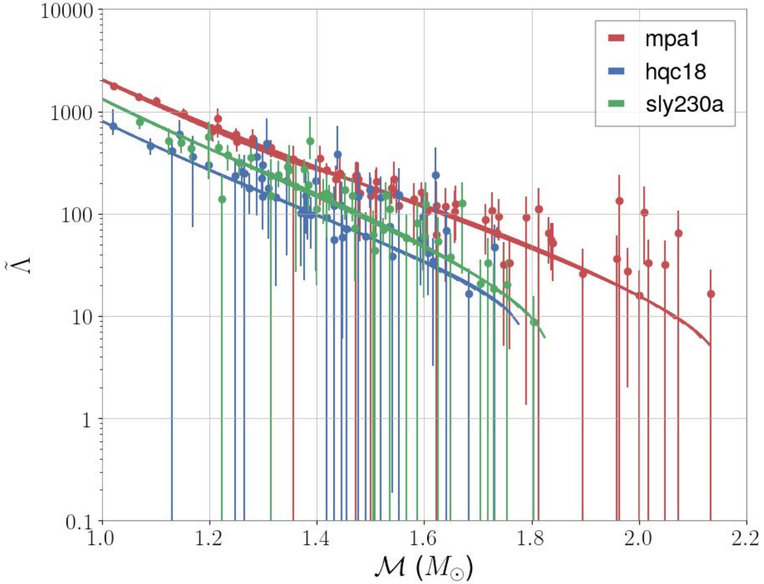


Bias plots


Δchirp_mass_source/CL(90%)
Δluminosity_distance/CL(90%)
Δcos_theta_jn/CL(90%)
Δluminosity_distance/CL(90%)
Δgeocent_time/CL(90%)
Δlambda_tilde/CL(90%)
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
SNR
10
20
30
40
60
80

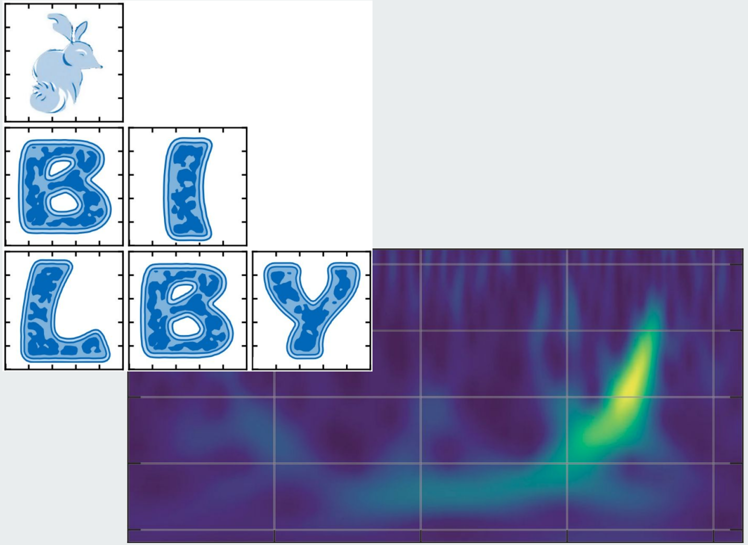
Effective tidal deformability





- ➔ Among the 3000 injections only 146 have a SNR higher than 30

Conclusion:



- For the moment the GW170817 event remains the best detection to constrain EOS.
- ROQ method is:
  - Accurate and reliable
  - Fast
  - Easy to use with Bilby
  - Already available
- Statistical studies possible thanks to ROQ.