Gravitational waves: Opening a new window on the universe

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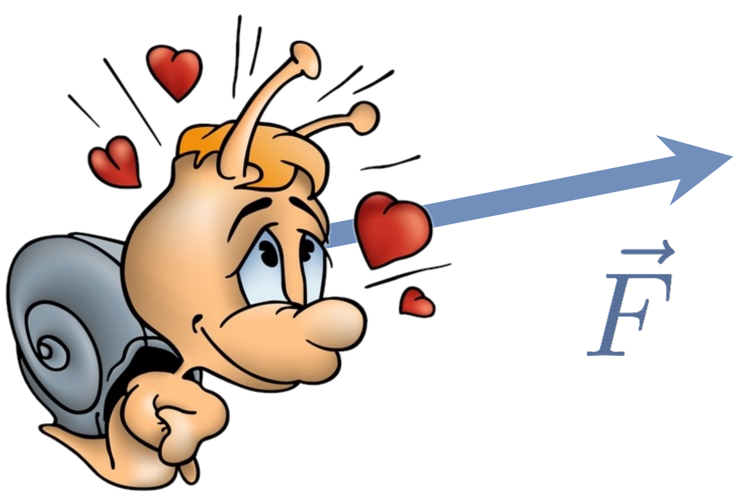
GraSPA summer school 2023

Contents

Gravitation: the classical theory

- Flat space, absolute time
- Instantaneous interaction between distant masses

$$\vec{F} = G \cdot m_1 m_2 \cdot \frac{1}{r^2} \cdot \vec{u}$$

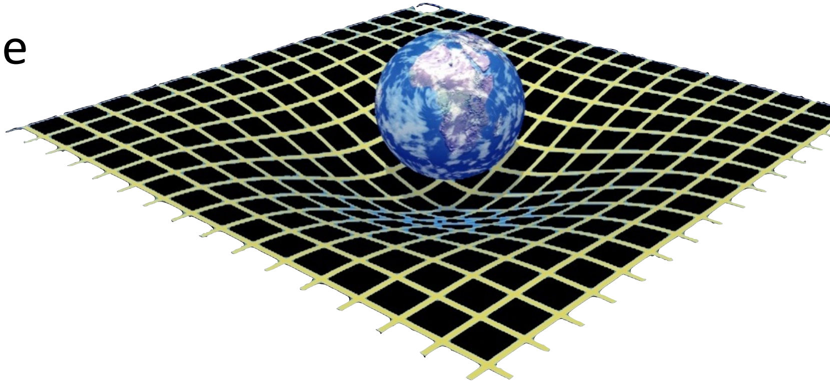


$\vec{F}$

$-\vec{F}$

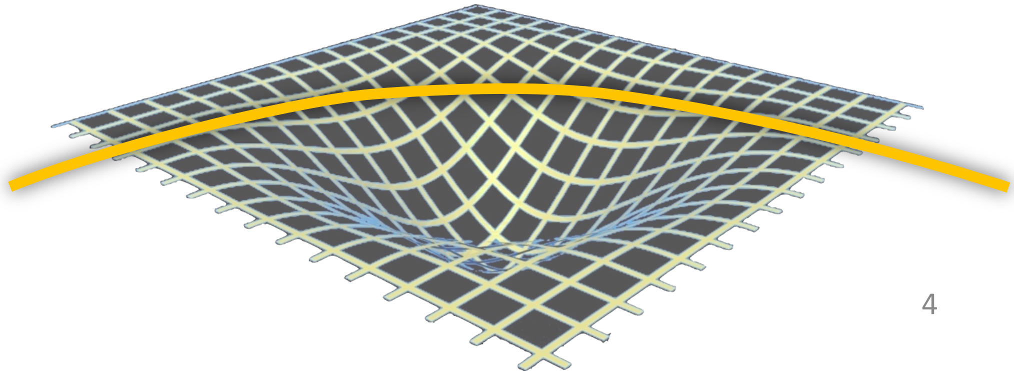
Gravitation: the modern theory

- **Theory of General Relativity (GR)**
- Einstein 1915-1918 : geometric theory of gravitation
- A mass "bends" and "deforms " space-time



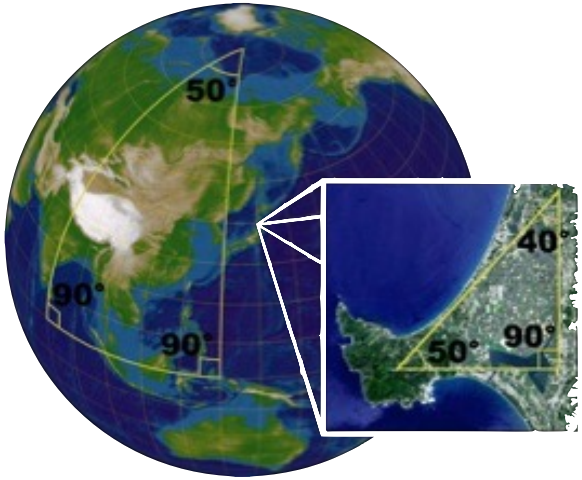
- The trajectory of a mass is influenced by the curvature of space-time

J. A. Wheeler : ***"Space tells matter how to move and matter tells space how to curve"***



Theoretical piece: curved space

- What is a curved space ? (= "manifold")
  - examples : sphere, saddle



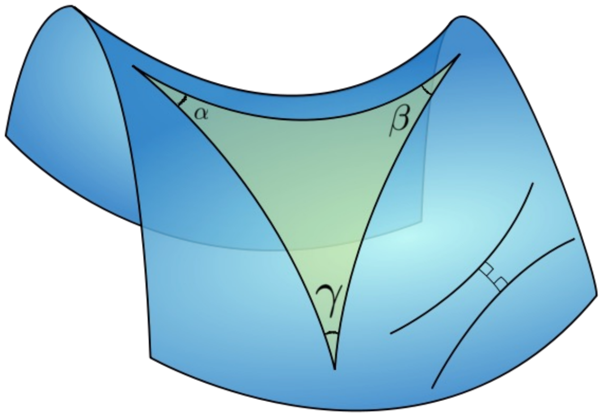
- Can we measure curvature ?
  - we cannot see our space from "outside"
  - but we can measure angles
  - the sum of the angles of a triangle is not always equal to π !

- positive curvature

$$\sum \text{angles} = \alpha + \beta + \gamma > \pi$$

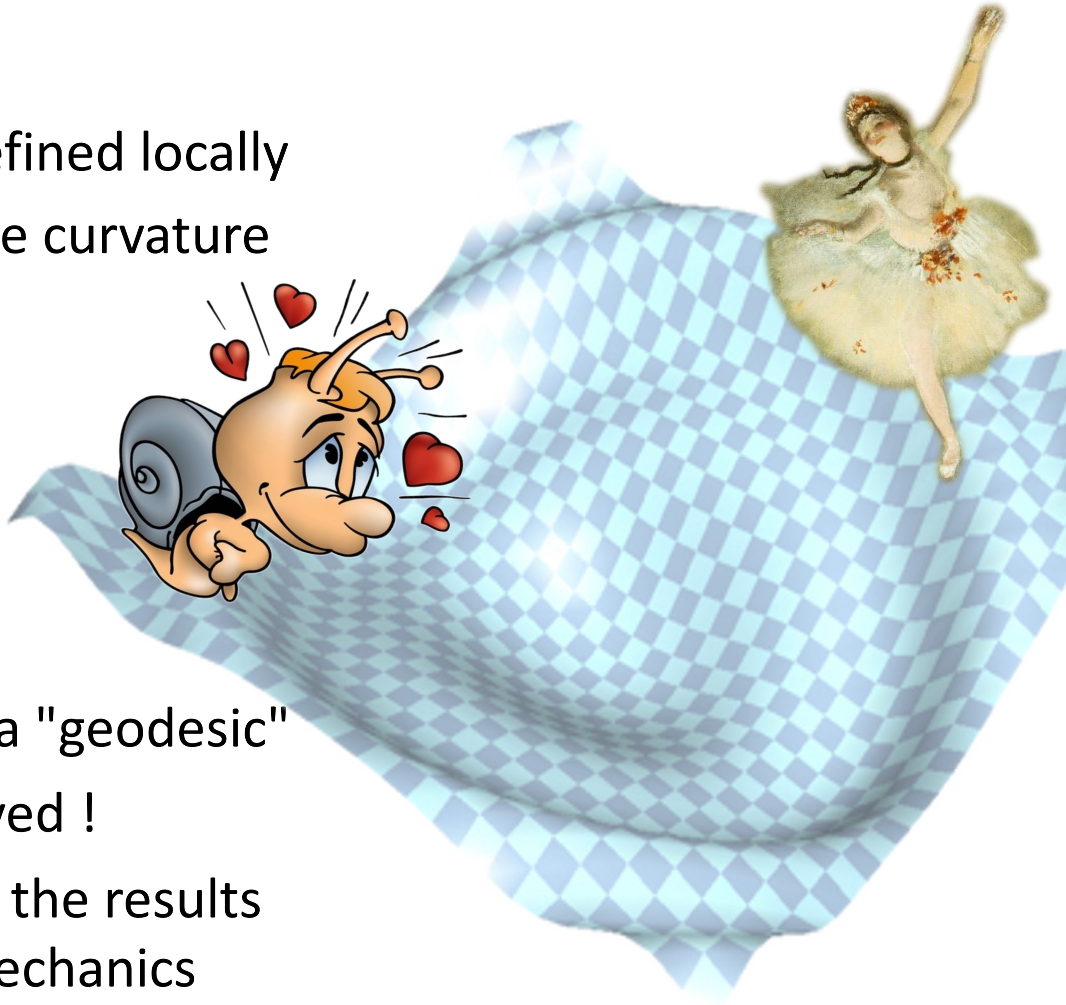
- negative curvature

$$\sum \text{angles} = \alpha + \beta + \gamma < \pi$$



Theoretical piece: curved space-time

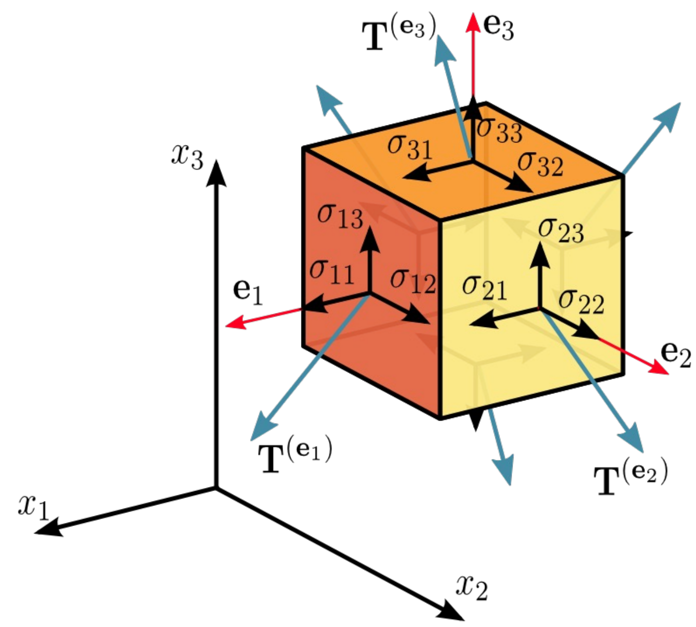
- In General Relativity
  - space is curved and time is defined locally
  - one cannot go "out" to see the curvature
    - "intrinsically" curved space
      => intrinsic curvature



  - go straight (free fall) = follow a "geodesic"
  - note that the time is also curved !
  - as a first approximation, finds the results (trajectories) of newtonian mechanics

Theoretical piece: tensors

- Tensor = mathematical object
- Does not depend on the coordinate system
- Extends the notion of vector
- In a specific coordinate system, multidimensional array
- Example:
  electrical conductivity of an anisotropic cristal



$$j^i = \sigma^i_j E^j$$

- Note : summation is implicit over repeated indices (Einstein convention)

$$\sigma^i_j E^j \equiv \sum_j \sigma^i_j E^j$$

Theoretical piece: the metric tensor

- In space-time, need to measure
  - the distance between two points
  - the angle between two vectors
- Measure of the distance between two infinitesimally close events in spacetime
- Need a "metric", start from the "line element" seen in special relativity :

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

with c = 1 !

- Which can be written $ds^2 = \eta_{\alpha\beta} dx^\alpha dx^\beta$

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad dx^0 = dt, \quad dx^1 = dx, \quad dx^2 = dy, \quad dx^3 = dz$$

- $\eta_{\mu\nu}$ is the metric of a flat spacetime, the Minkowski spacetime, used in special relativity

Theoretical piece: the metric tensor

- What if space is not flat ?

- The metric can be general : $g_{\mu\nu}$

- It contains all information about spacetime curvature
- It is a rank 2 tensor
- The curvature is also defined by another tensor, which depends on $g_{\mu\nu}$ and its derivatives: the Ricci tensor $R_{\mu\nu}$

- But what relates

  deformation of space-time and energy-momentum ?

The Einstein Field Equations

- Answer : the Einstein Field Equations (EFE)

still c = 1 !
would be $\frac{8\pi G}{c^4}$.

$$\underbrace{\left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R\right)}_{\text{curvature term}} = 8\pi G \underbrace{\left(T_{\mu\nu}\right)}_{\text{energy-momentum term}}$$

- Energy-momentum bends spacetime
- Spacetime tells mass (energy momentum) how to move
- These equations are non-linear

From Einstein Field Equations to Gravitational Waves

- Start from a flat space-time = Minkowski metric
  - Add a perturbation $h_{\mu\nu}$ to the metric : $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
  - Linearize Einstein Field Equations $(h_{\mu\nu} \ll 1)$
  - Choose a suitable coordinate system

    (« Transverse Traceless » or TT gauge)
- Obtain a wave equation

$$(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2})h_{\mu\nu} = 0$$

(in vacuum, no $T_{\mu\nu}$)

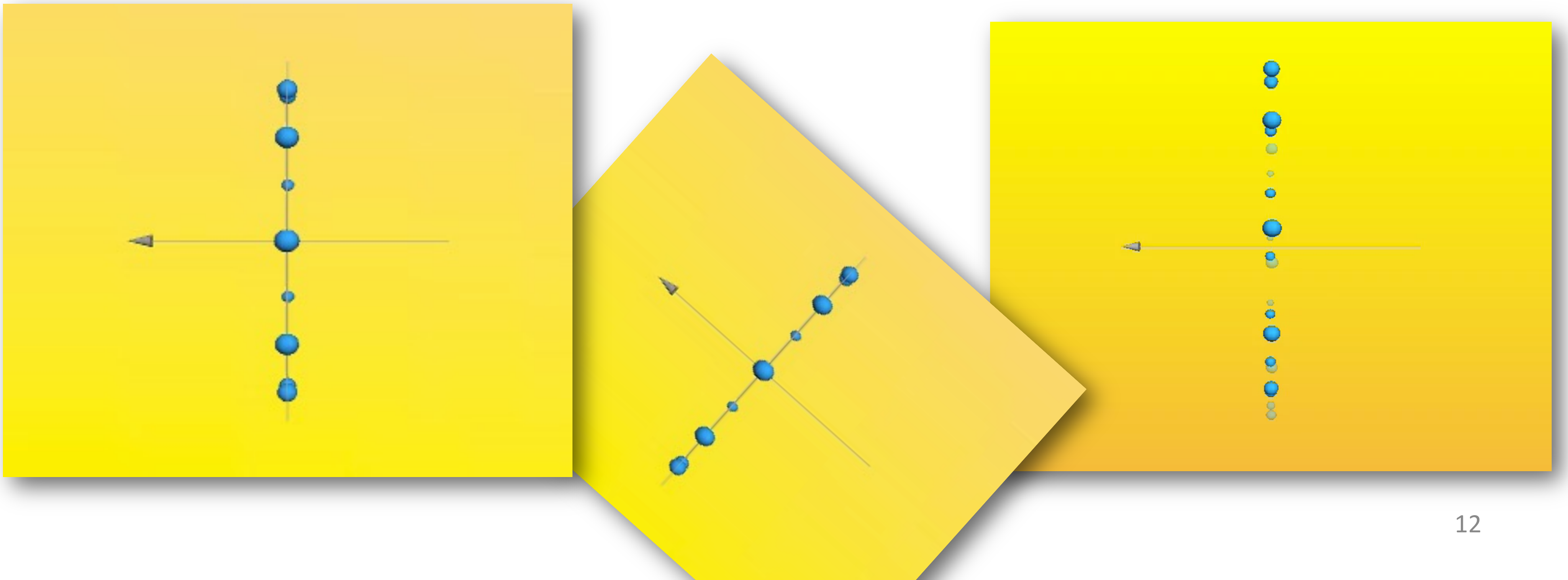
- Which solution is

$$h_{\mu\nu} = A_{\mu\nu} \cdot e^{-i(\vec{k}\cdot\vec{x}-\omega\cdot t)}$$

Gravitational waves: effect on matter

$$h_{\mu\nu} = A_{\mu\nu} \cdot e^{-i(\vec{k}\cdot\vec{x}-\omega\cdot t)}$$

- Transverse plane wave
- Propagating at the speed of light
- Two states of polarization: + and x
- Effect on free falling masses (test masses) in circle:

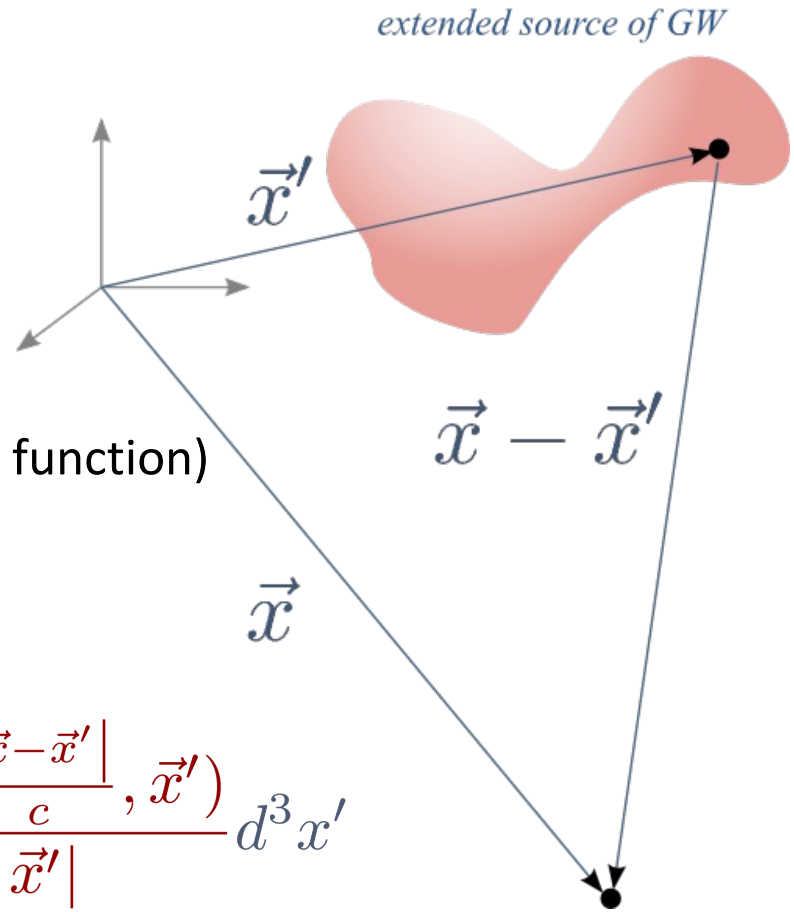


Gravitational waves: generation

- Linearized Einstein equations with a stress-energy tensor (source term)

now c = c

$$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}$$



- Use Green functions
  - Solutions of the wave equation in the presence of a point source (delta function)

- Retarded potential

$$\bar{h}_{\mu\nu}(t,\vec{x}) = -\frac{4G}{c^4}\int_{source} \frac{T_{\mu\nu}\left(t - \frac{|\vec{x}-\vec{x}'|}{c}, \vec{x}'\right)}{|\vec{x}-\vec{x}'|} d^3x'$$

Gravitational waves: generation

- Approximations :
  - isolated source
  - compact source
  - observer far from the source
    ($R = |\vec{x} - \vec{x}'|$ >> typical size of the source)
- Amplitude of the wave written as a function of

$$\bar{h}_{ij}(t) = \frac{2G}{Rc^4} \frac{d^2 I_{ij}}{d\,t^2}\left(t - \frac{R}{c}\right)$$

I_{ij} = reduced quadrupolar moment of the source

$$= \int_{source} d\vec{x}\; x_i x_j \; T_{00}(t, \vec{x})$$

$$\frac{G}{c^4} \approx 8.24 \times 10^{-45}\ \mathrm{s}^2 \cdot \mathrm{m}^{-1} \cdot \mathrm{kg}^{-1}$$

- Remark :
  Need a quadrupolar moment to generate a GW, the dipolar case is impossible (because of momentum conservation).

Orders of magnitude

- Amplitude:

$$h \approx \frac{G}{c^4} \cdot \frac{\ddot{I}}{R}$$

- Example with two orbiting objects : a binary system
  - M = total system mass, r = distance between the components
  - R = observer – system distance
  - $I \approx M.r^2$ hence $\ddot{I} \approx M \cdot v_{NS}^2 \approx E_c^{NS}$
    - where NS is the part of the source motion without spherical symmetry

- Hence

$$h \approx \frac{G}{c^4} \cdot \frac{E_c^{NS}}{R}$$

Orders of magnitude

- Luminosity: $L_{GW} \approx \frac{G}{c^5} \cdot \dddot{I}^2$
- Reminder: $\ddot{I} \approx E_c^{NS}$ hence $\dddot{I} \approx E_c^{NS}/T$
  - T = characteristic time of energy-momentum (or mass) motion from one side of the system to the other
- In case of a transient, violent event

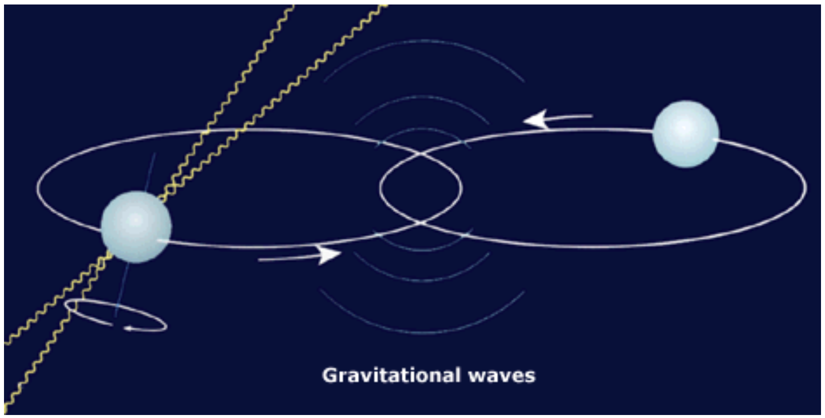
$$L_{GW} \approx \frac{G}{c^5} \cdot \dddot{I}^2 \approx \frac{G}{c^5} \cdot \left(\frac{E_c^{NS}}{T}\right)^2$$

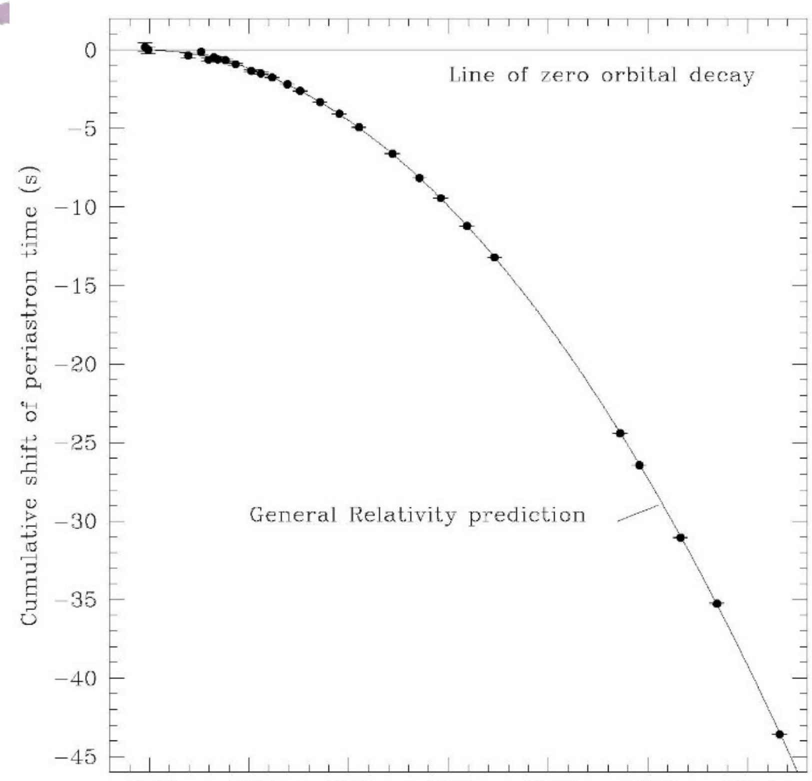
- For a quasi-stationary dynamics

$$L_{GW} \approx \frac{G}{c^5} \cdot \dddot{I}^2 \approx \frac{c^5}{G} \cdot \left(\frac{GM}{c^2 R}\right)^2 \cdot \left(\frac{v_{NS}}{c}\right)^6$$

where one introduces the Schwarzschild radius $R_S = \frac{2GM}{c^2}$

Indirect evidence: PSR 1913+16



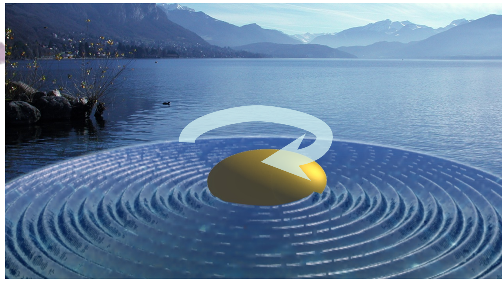


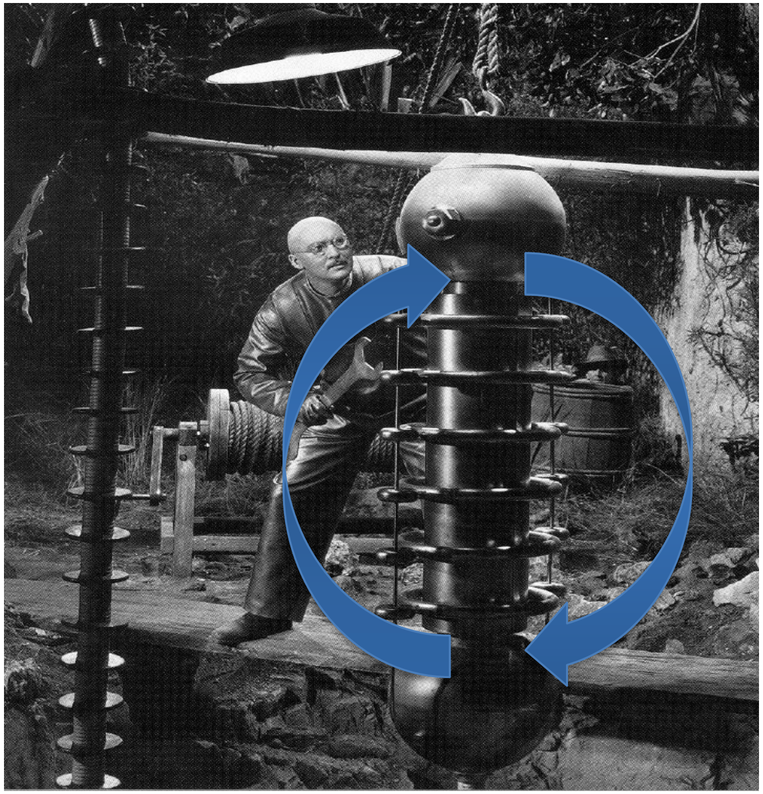
$$\dot{P}_{observe} / \dot{P}_{predit} = 1.0013 \pm 0.0021$$

- Binary system of neutron stars
- One neutron star is a radio pulsar
- Discovered in 1975 by Hulse and Taylor
- Studied by Taylor, Weisberg and co.
- Decay of the orbital period compatible with GW emission
- Frequency of GW emitted by PSR 1913+16: ~ 0.07 mHz
  - Undetectable by ground-based detectors (bandwidth 10 Hz- 10 kHz)

Orders of magnitude

- Mass distribution : needs a quadrupolar moment





- Examples for a binary system

$$h \approx 32\pi^2 \cdot \frac{G}{c^4} \cdot \frac{1}{R} \cdot M \cdot r^2 \cdot f_{orb}^2$$

- M = 1000 kg, r = 1 m, f = 1 kHz, R = 300 m

  $h \sim 10^{-35}$

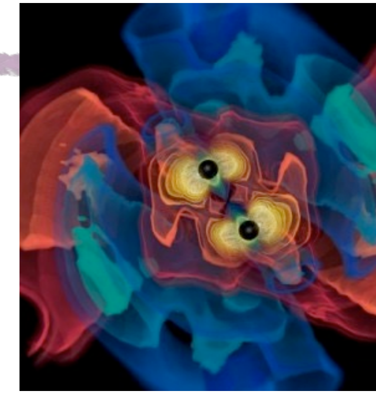
- $M = 1.4 \ M_\odot$, r = 20 km, f = 400 Hz, $R = 10^{23}$ m (15 Mpc = 48,9 Mlyr)

  $h \sim 10^{-21}$

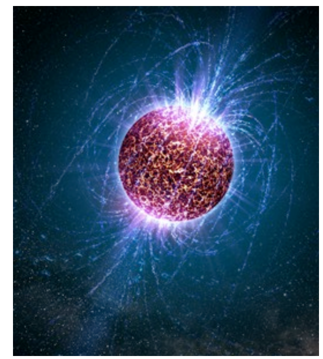
Doing it in a lab ? No way !

Astrophysical sources

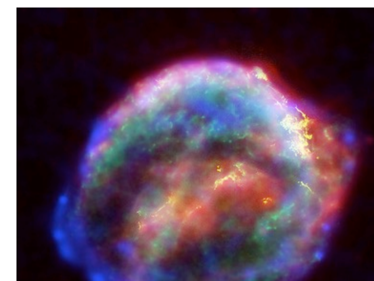
- Need high masses and velocities : astrophysical sources
- **Binary system**
  - Need to be compact to be observed by ground based detectors
    → Neutron stars, black holes
  - Signal well modeled but rates not well known… yet
- **Spinning neutron stars**
  - Nearly monotonic signals
  - Long duration
  - Strength not well known
- **Asymmetric explosion**
  - Ex: core collapse supernovae
  - « burst » transient
  - Not well modeled
- **Gravitational wave background**
  - First type : superposition of many faint sources
  - Second type : Residue of the Big Bang or Inflation
  - Stochastic in nature



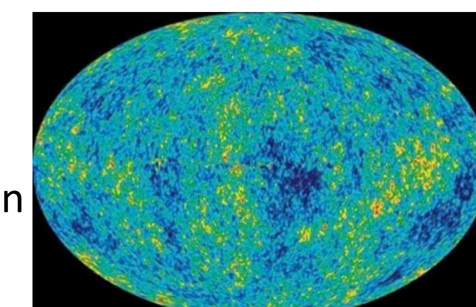
Credit: AEI, CCT, LSU



Casey Reed, Penn State



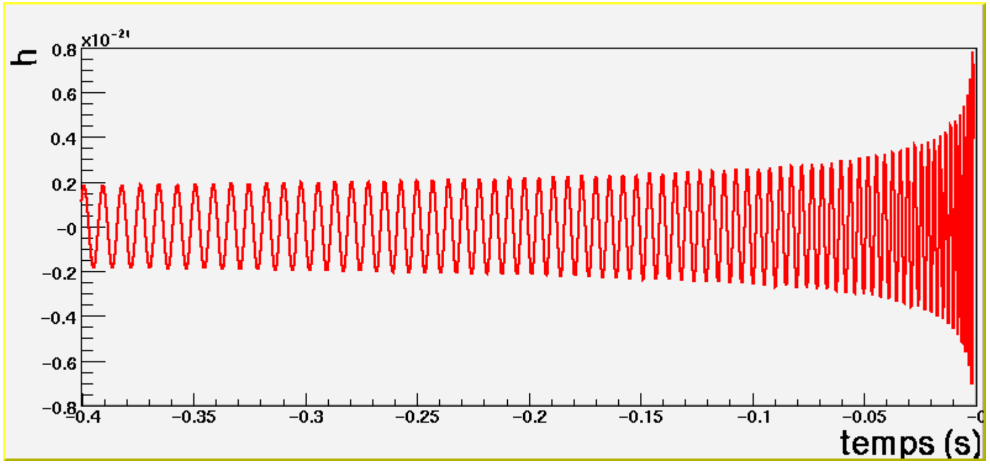
Credit: Chandra X-ray Observatory



NASA/WMAP Science Team

Coalescing binaries

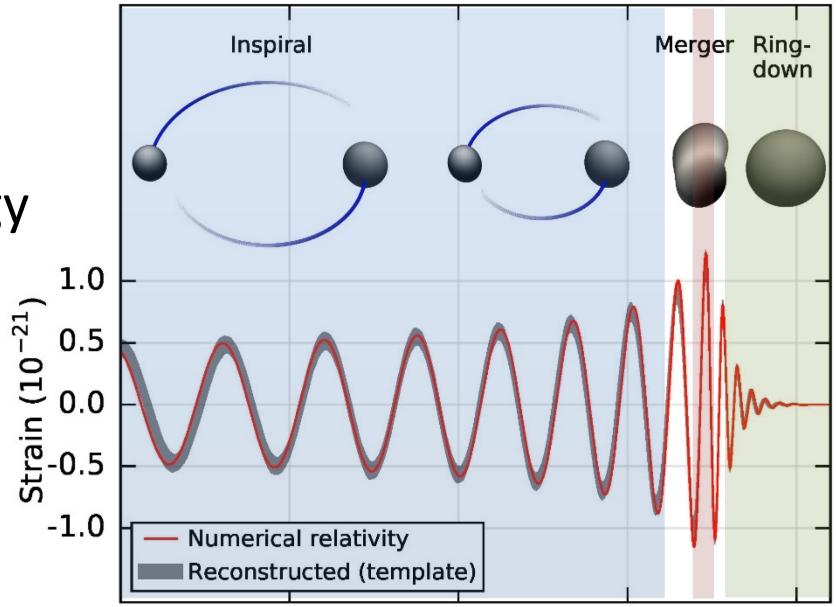
- Binary systems of compact stars at the end of their evolution
  - Neutron stars (NS) and/or black holes (BH)
- Very rare : a few events per million year per galaxy
- Typical amplitude at the detectors:
  - $h \approx 10^{-22}$ at 20 Mpc
- Very distinctive waveform



Coalescing binaries

- System may be binary neutron stars (BNS), binary black holes (BBH) or NS-BH
- Phases of the coalescence
- Inspiral
  - Masses m1 and m2 orbit each other
  - GW emission -> system looses energy
  - => Frequency ↗, amplitude ↗
  - Waveform characterised by a « chirp mass »

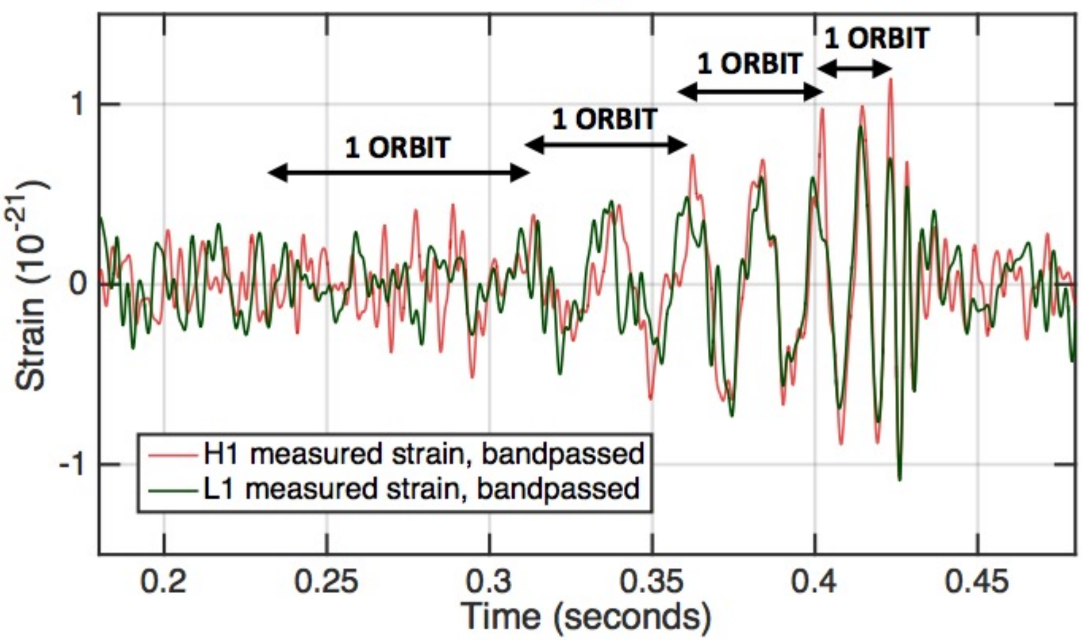
$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} = \frac{c^3}{G}\left[\frac{5}{96}\pi^{-8/3} f^{-11/3} \dot{f}\right]^{3/5}$$



- Merger: computed numerically (numerical GR)
- Ringdown: quasi-normal modes decomposition

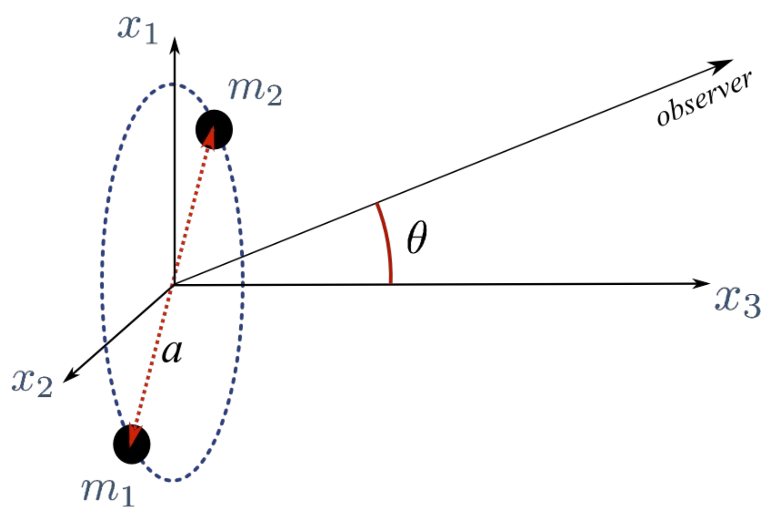
- First detection : GW150914

ΔL/L



one orbit
=
two GW cycles

- For the sake of simplicity, let's take a simple system :



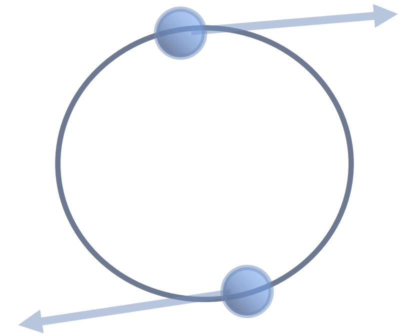
- Masses m_1 and m_2, total mass $M = m_1 + m_2$, reduced mass $\mu = \frac{m_1 m_2}{M}$
- Distance between stars: a, take circular orbits
- Compute h_+ and $h_\times$, the amplitude of the two modes of the emitted wave seen by an observer situated at a distance $R \gg a$

Coalescing binaries

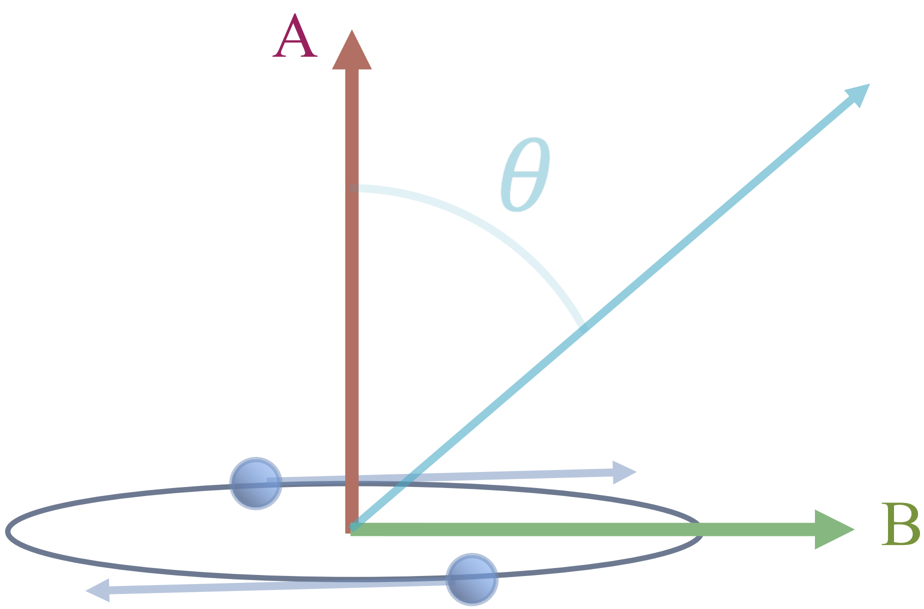
- Understanding the two polarization amplitudes

$$h_+(t) = \frac{4G\mu a^2\omega^2}{Rc^4}\frac{1+\cos^2\theta}{2}\cos 2\omega t$$

$$h_\times(t) = \frac{4G\mu a^2\omega^2}{Rc^4}\cos\theta\sin 2\omega t$$



Observer A : $\cos\theta = 1$
sees the two polarizations



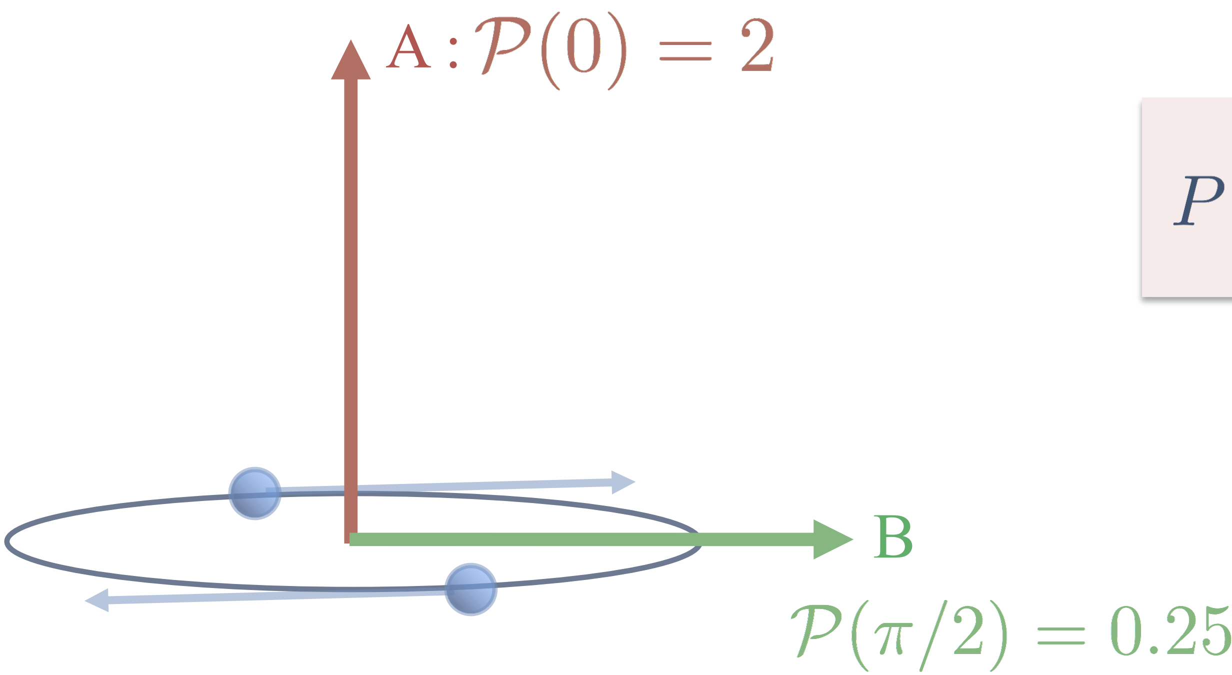
Observer B : $\cos\theta = 0$
sees a linear polarization

Coalescing binaries

- Radiated power per unit solid angle $$\frac{dP}{d\Omega} = \frac{2G\mu^2 a^4 \omega^6}{\pi c^5}\mathcal{P}(\theta)$$

$$\mathcal{P}(\theta) = \frac{1}{4}(1 + 6\cos^2\theta + \cos^4\theta)$$

- Radiated power non zero whatever the direction of emission



$$P = \frac{32G\mu^2 a^4 \omega^6}{5c^5}$$

Coalescing binaries

- Some examples
- Sun-Jupiter system

$$m_J = 1.9 \times 10^{27}\text{kg}, \quad a = 7.8 \times 10^{11}\text{m}, \quad \omega = 1.68 \times 10^{-7}\text{s}^{-1}$$

$$\Rightarrow \quad P = 5 \times 10^3 \quad \text{J/s}$$

  - Very small, compared to the light power emitted by the sun:

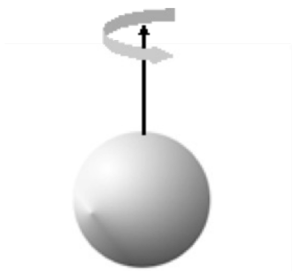
$$L_\odot \approx 3.8 \times 10^{26}\,\text{J/s}$$

- Binary pulsar PSR1913+16 (Hulse and Taylor)

$$P = 7.35 \times 10^{24}\ \text{J/s}$$

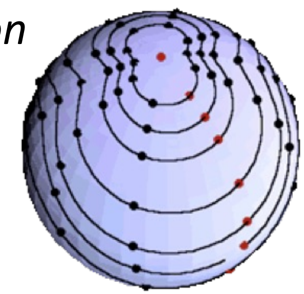
Continuous waves

- Rotating neutron stars $\nu \sim 1 - 10^3$ Hz $h \sim 10^{-25}$ at 3 kpc
- Not perfectly spherical

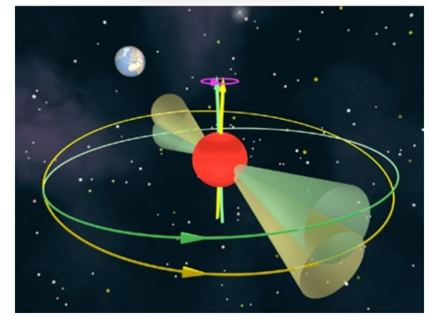


« mountains » or assymetry

Oscillation modes



precession



$$h_0 = \frac{4\pi^2 G}{c^4} \frac{I_{zz}\, \varepsilon\, f_{gw}^2}{d}$$

I_{zz} Moment of inertia along the rotation axis

$$\varepsilon = \frac{I_{xx} - I_{yy}}{I_{zz}}$$ Ellipticity in the equatorial plane

- I_{zz} and ε very poorly known
- Motion and orientation of the detector around the sun
  - Doppler modulation of the signal

The End of episode I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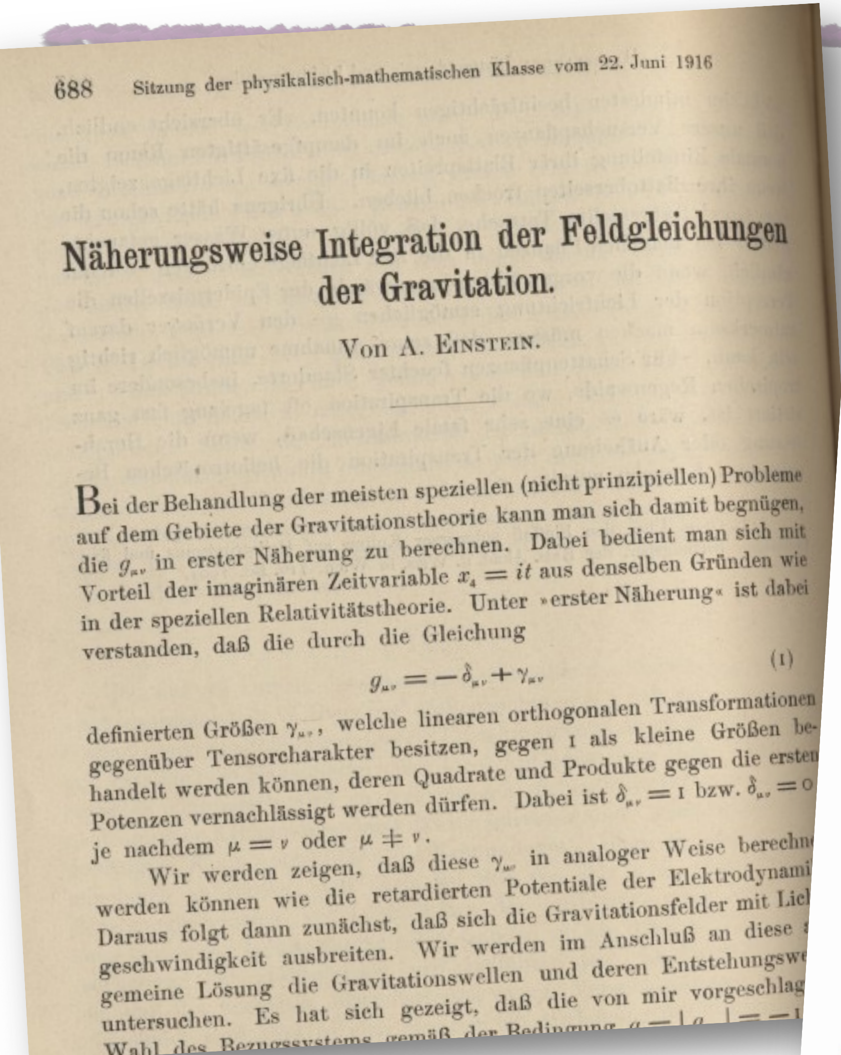
Gravitational waves


688 Sitzung der physikalisch-mathematischen Klasse vom 22. Juni 1916

Näherungsweise Integration der Feldgleichungen der Gravitation.

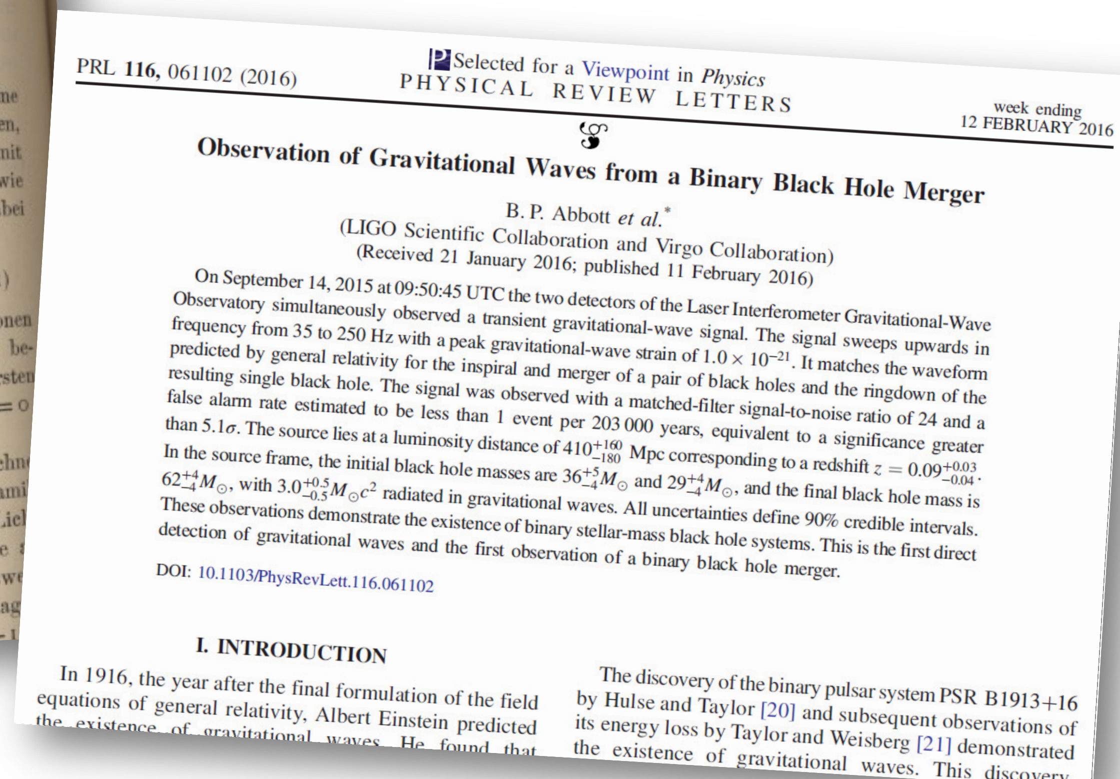
Von A. Einstein.

Bei der Behandlung der meisten speziellen (nicht prinzipiellen) Probleme auf dem Gebiete der Gravitationstheorie kann man sich damit begnügen, die $g_{\mu\nu}$ in erster Näherung zu berechnen. Dabei bedient man sich mit Vorteil der imaginären Zeitvariable $x_4 = it$ aus denselben Gründen wie in der speziellen Relativitätstheorie. Unter »erster Näherung« ist dabei verstanden, daß die durch die Gleichung

$$g_{\mu\nu} = -\delta_{\mu\nu} + \gamma_{\mu\nu} \quad (1)$$

definierten Größen $\gamma_{\mu\nu}$, welche linearen orthogonalen Transformationen gegenüber Tensorcharakter besitzen, gegen 1 als kleine Größen behandelt werden können, deren Quadrate und Produkte gegen die ersten Potenzen vernachlässigt werden dürfen. Dabei ist $\delta_{\mu\nu} = 1$ bzw. $\delta_{\mu\nu} = 0$ je nachdem $\mu = \nu$ oder $\mu \neq \nu$.

Wir werden zeigen, daß diese $\gamma_{\mu\nu}$ in analoger Weise berechn… werden können wie die retardierten Potentiale der Elektrodynami… Daraus folgt dann zunächst, daß sich die Gravitationsfelder mit Lic… geschwindigkeit ausbreiten. Wir werden im Anschluß an diese … gemeine Lösung die Gravitationswellen und deren Entstehungsw… untersuchen. Es hat sich gezeigt, daß die von mir vorgeschlag…


PRL 116, 061102 (2016) Selected for a Viewpoint in Physics PHYSICAL REVIEW LETTERS week ending 12 FEBRUARY 2016

Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott *et al.**
(LIGO Scientific Collaboration and Virgo Collaboration)
(Received 21 January 2016; published 11 February 2016)

On September 14, 2015 at 09:50:45 UTC the two detectors of the Laser Interferometer Gravitational-Wave Observatory simultaneously observed a transient gravitational-wave signal. The signal sweeps upwards in frequency from 35 to 250 Hz with a peak gravitational-wave strain of 1.0×10^{-21}. It matches the waveform predicted by general relativity for the inspiral and merger of a pair of black holes and the ringdown of the resulting single black hole. The signal was observed with a matched-filter signal-to-noise ratio of 24 and a false alarm rate estimated to be less than 1 event per 203 000 years, equivalent to a significance greater than 5.1σ. The source lies at a luminosity distance of 410^{+160}_{-180} Mpc corresponding to a redshift $z = 0.09^{+0.03}_{-0.04}$. In the source frame, the initial black hole masses are $36^{+5}_{-4}M_{\odot}$ and $29^{+4}_{-4}M_{\odot}$, and the final black hole mass is $62^{+4}_{-4}M_{\odot}$, with $3.0^{+0.5}_{-0.5}M_{\odot}c^2$ radiated in gravitational waves. All uncertainties define 90% credible intervals. These observations demonstrate the existence of binary stellar-mass black hole systems. This is the first direct detection of gravitational waves and the first observation of a binary black hole merger.

DOI: 10.1103/PhysRevLett.116.061102

I. INTRODUCTION

In 1916, the year after the final formulation of the field equations of general relativity, Albert Einstein predicted the existence of gravitational waves. He found that …

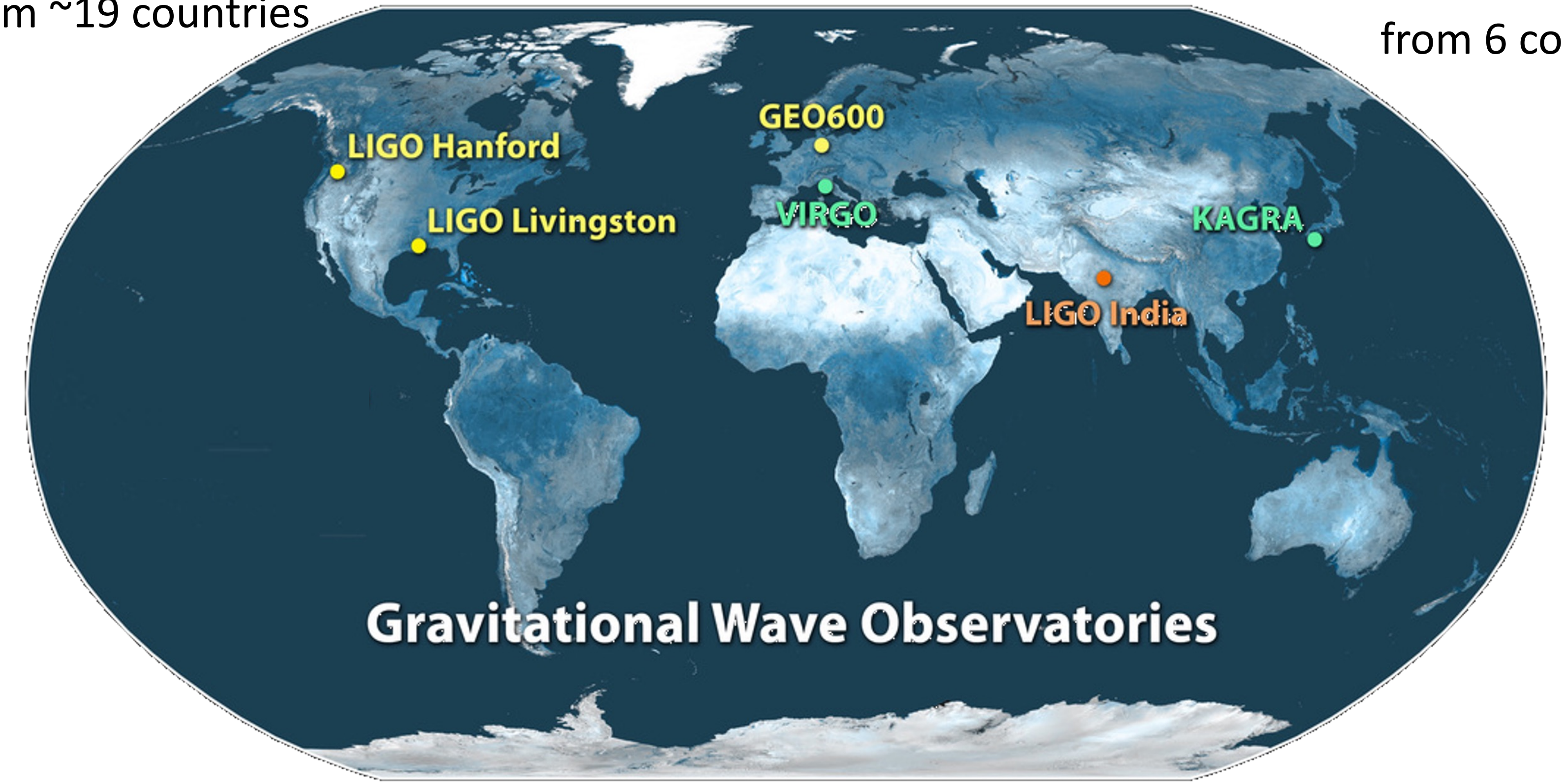
The discovery of the binary pulsar system PSR B1913+16 by Hulse and Taylor [20] and subsequent observations of its energy loss by Taylor and Weisberg [21] demonstrated the existence of gravitational waves. This discovery, …

Gravitational wave observatories

Network of interferometric detectors
Advanced LIGO – Advanced Virgo

LSC : ~1400 members
~127 institutions
from ~19 countries

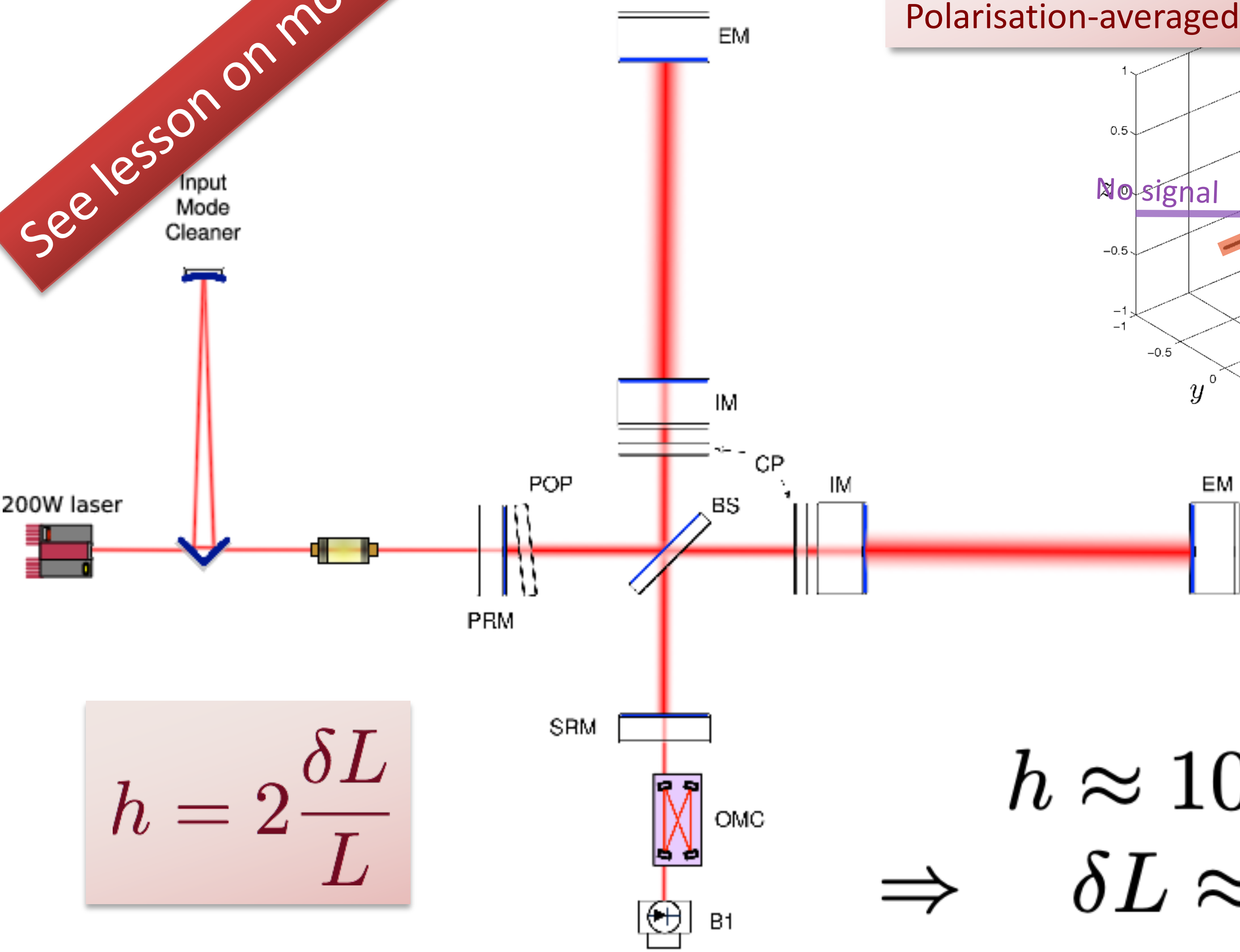
Virgo : ~400 membres
27 laboratories
from 6 countries



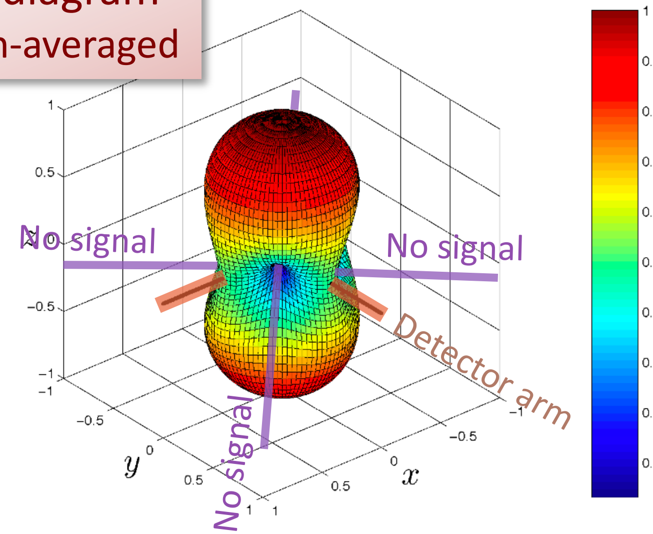
LVC = LIGO-Virgo Collaboration

Michelson interferometer : a “sensor” of gravitational waves

See lesson on monday



Antenna diagram
Polarisation-averaged

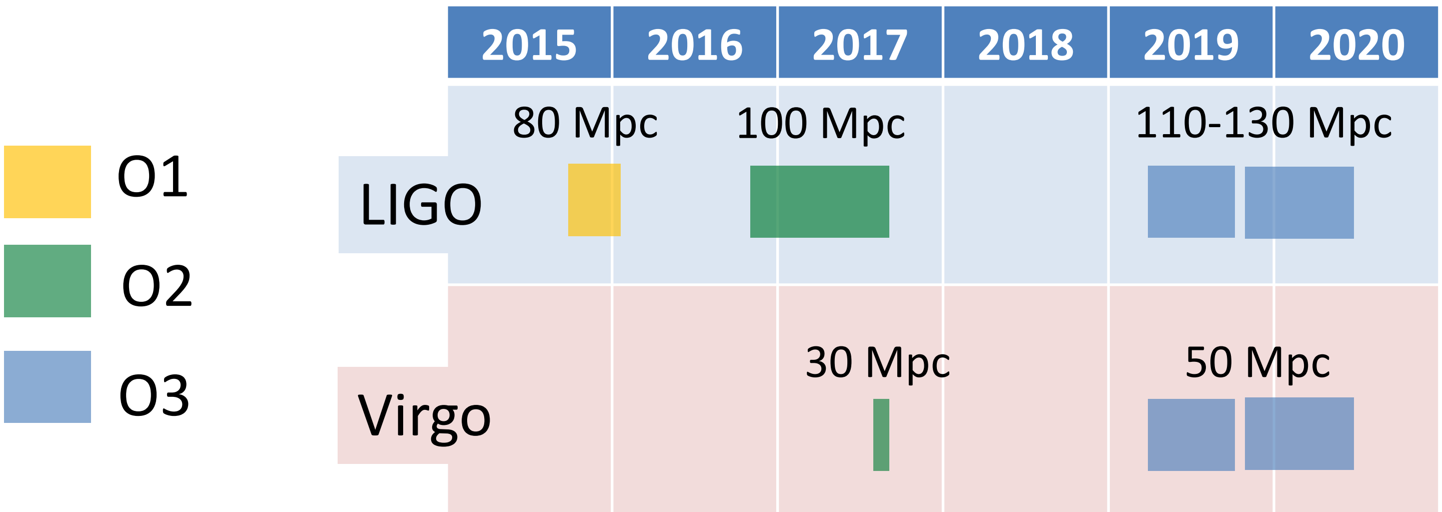


$$h = 2\frac{\delta L}{L}$$

$$h \approx 10^{-23}$$

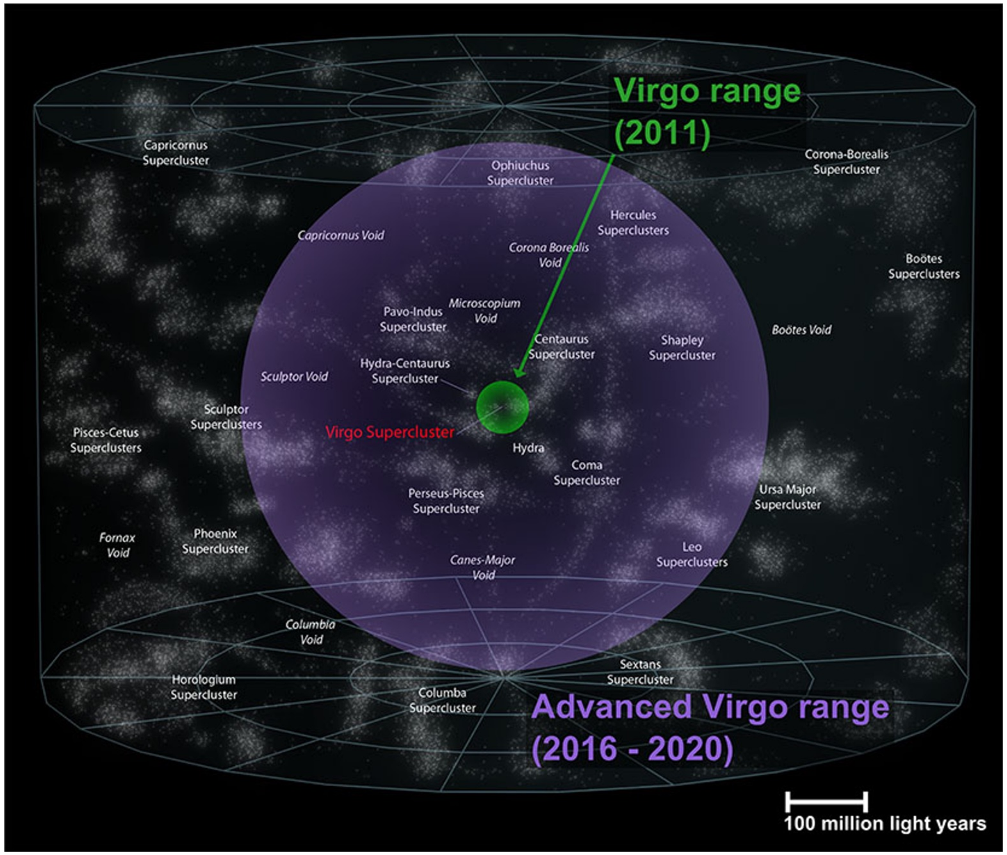
$$\Rightarrow \quad \delta L \approx 10^{-20}\ \mathrm{m}$$

LIGO-Virgo past runs



Horizon distance

- « Horizon » distance :
  - Distance at which a particular reference event emitted a signal which can be detected with Signal over Noise Ratio (SNR) = 8
- Reference event = binary neutron star coalescence with 1.4 $M_{\odot}$ for each component

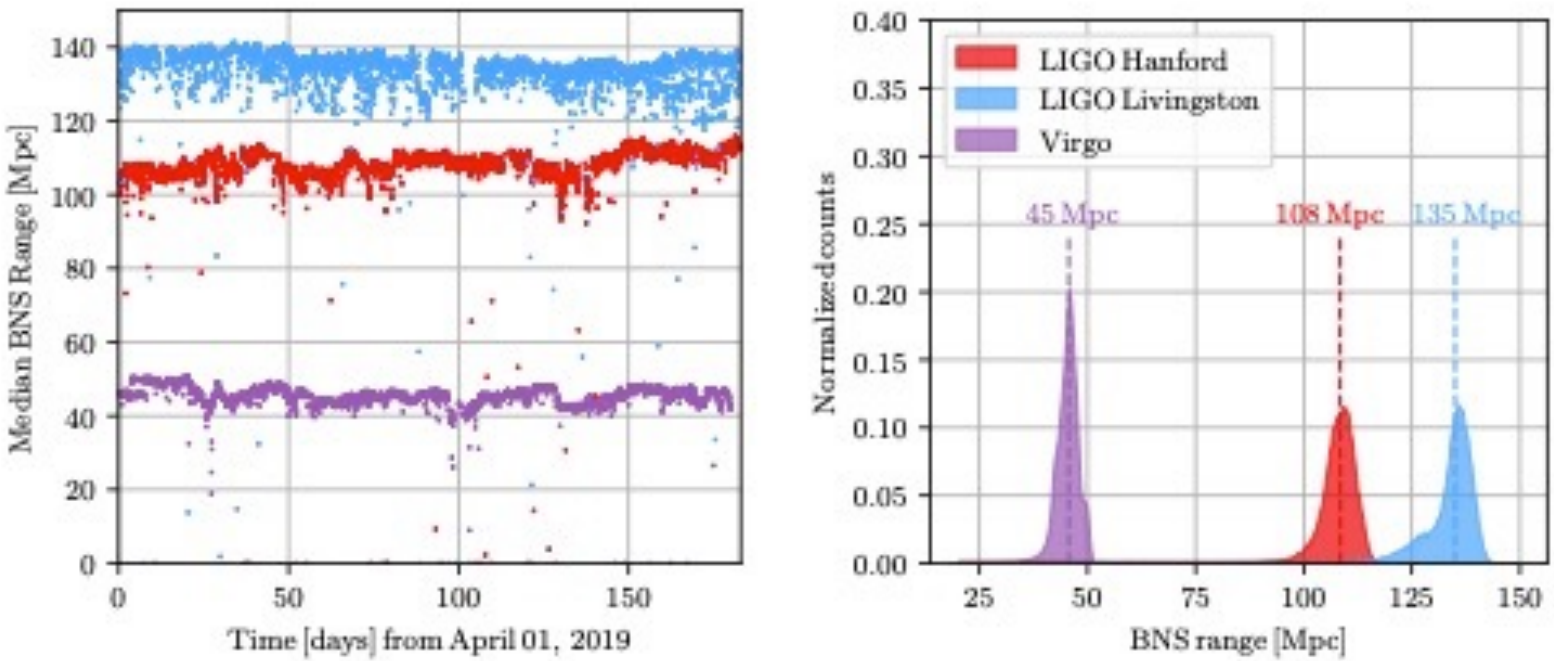


Improving the sensitivity (or horizon) by a factor 10

Increase the volume (or event rate) by 10^3

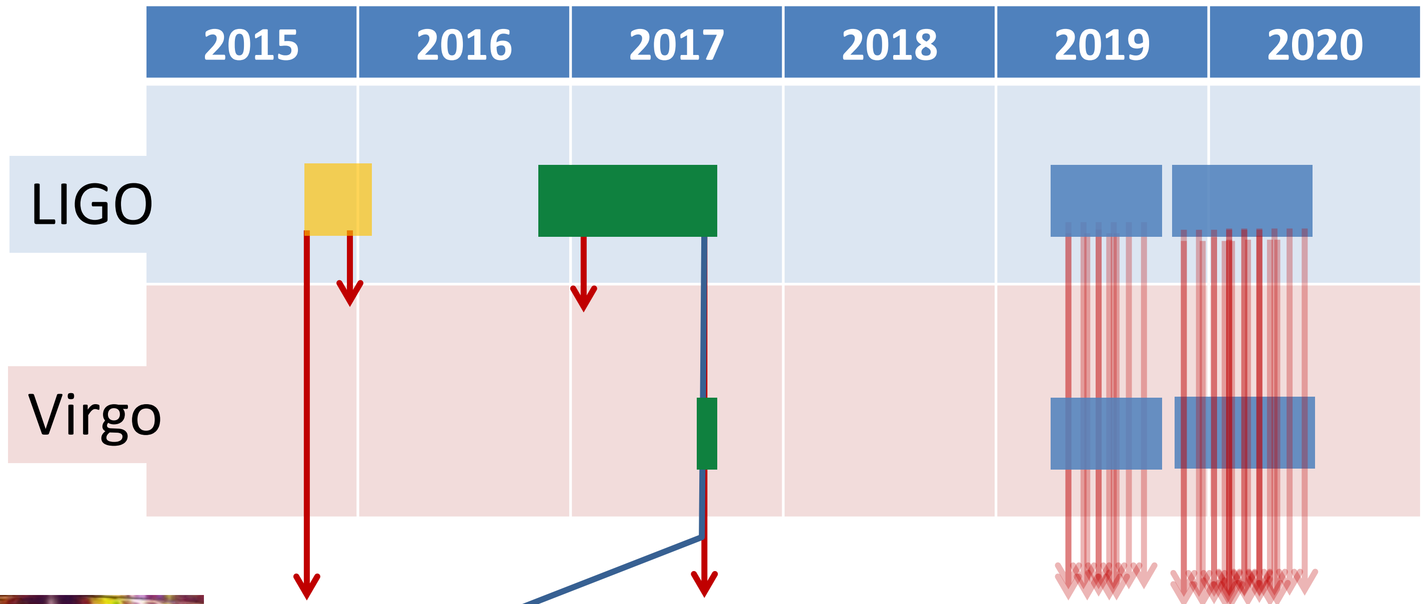
Horizon distance

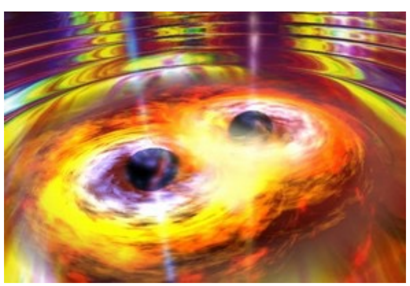
- « Horizon » distance :
  - Distance at which a particular reference event emitted a signal which can be detected with Signal over Noise Ratio (SNR) = 8
- Reference event = binary neutron star coalescence with 1.4 $M_\odot$ for each component



- Can define a horizon distance for BBH or any event type

Events and alerts





GW150914

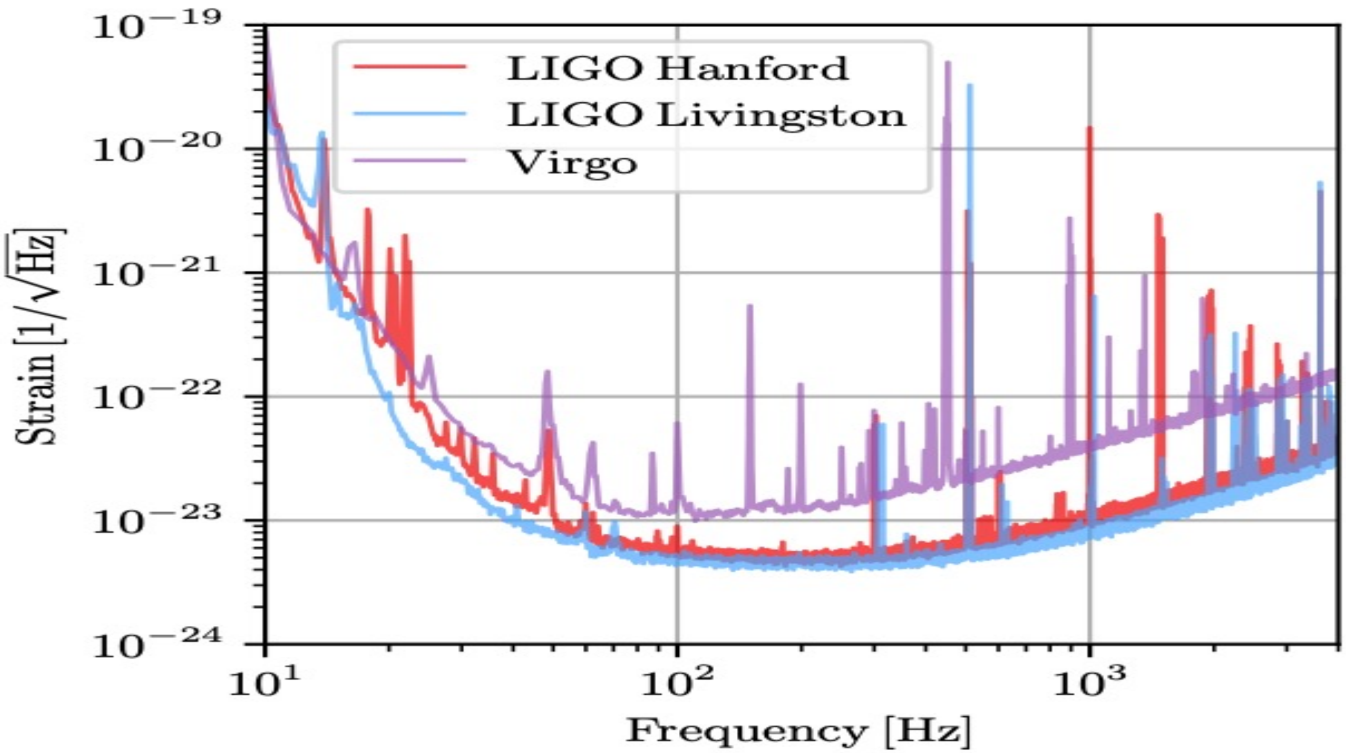
GW170814
First event with
Virgo!

During O3, 90 events of which
binary black hole (BH) coalescences,
2 coalesc. of two neutron stars (NS),
2 coalescences of NS + BH

GW170817
+ multi-messenger detection!

O3a run

- April 1, 2019 – October 1, 2019 (O3 = April 1, 2019 – March 27, 2020)
  - 3 detectors simultaneously observing : 44.5 % (81.4 days)
    - H1 = LIGO Hanford, L1 = LIGO Livingston



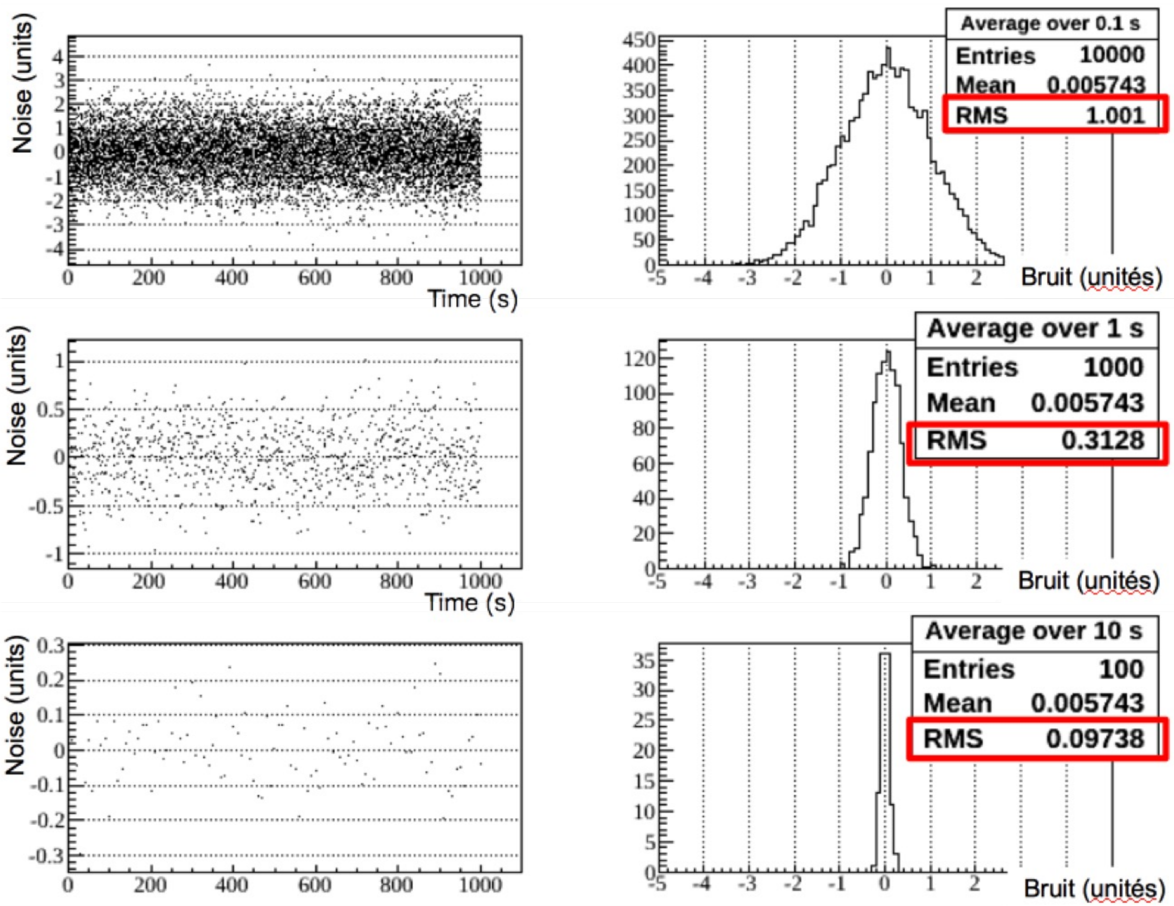
- Strain sensitivities for H1 and L1 : similar
  - ~ $5.10^{-24}/\sqrt{Hz}$ @ 100 Hz

but what is this unit "$1/\sqrt{Hz}$" ?

Characterizing noise level

- Hypothesis : constant signal S_0 in gaussian noise $\mathrm{N\,e}^{-\frac{1}{2}\frac{(x-<x>)^2}{\sigma_x^2}}$

Noise alone values **Distributions**



If T is the averaging time, the noise variance goes as

$$\sigma_{noise} \propto \frac{1}{\sqrt{T}}$$

Characterizing noise level

- Variance can be expressed as

$$\sigma_{noise} = \frac{D}{\sqrt{T}}$$

- Where D characterizes the level of noise
- *D* is written in terms of

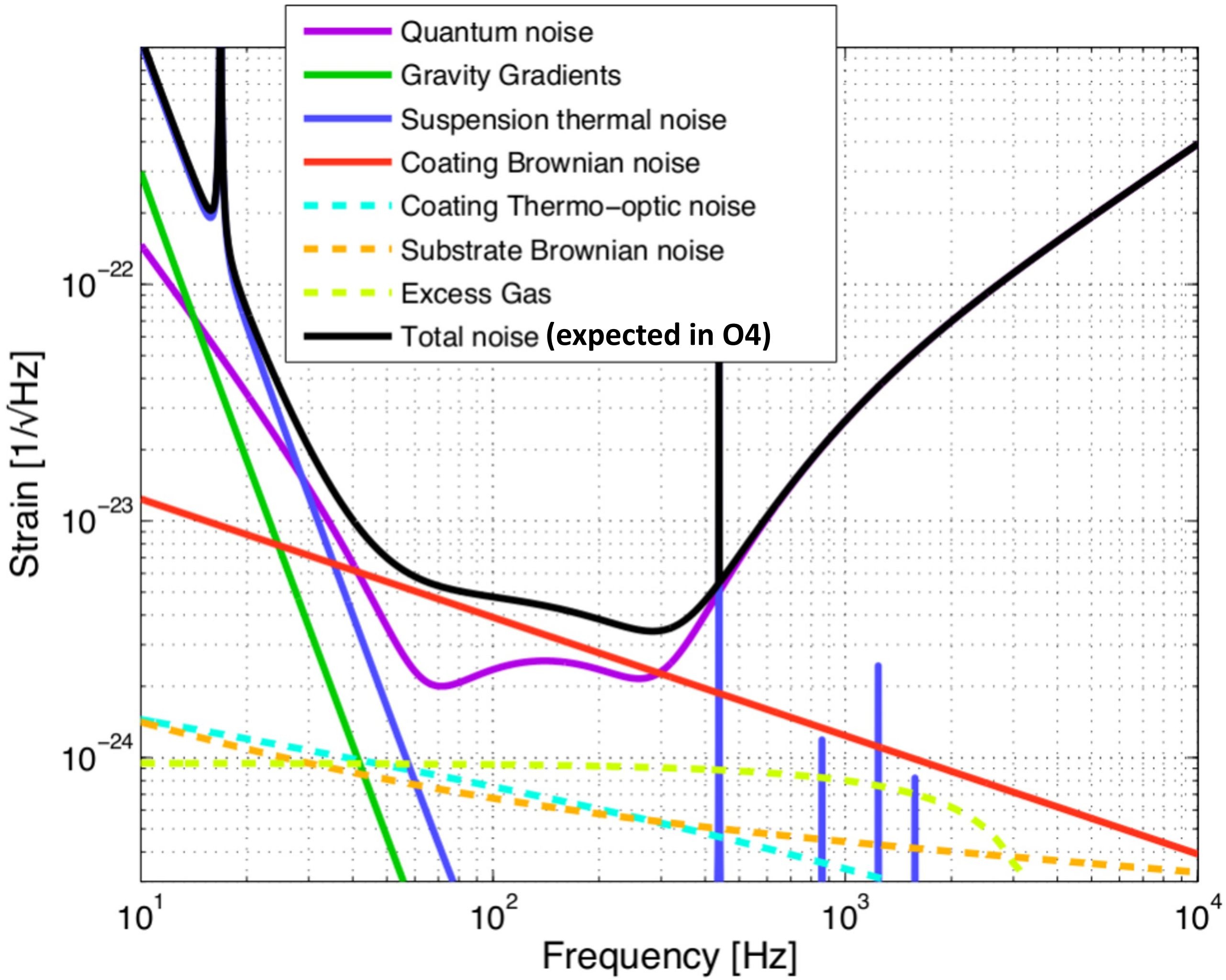
$$\frac{\text{data units}}{\sqrt{\text{Hz}}}$$

- Its value is the value of the noise variance when averaging over 1 s of signal
- Doing a Fourier transform, $D(f)$ is also expressed in terms of

$$\frac{\text{data units}}{\sqrt{\text{Hz}}}$$

Nominal sensitivity of Advanced Virgo

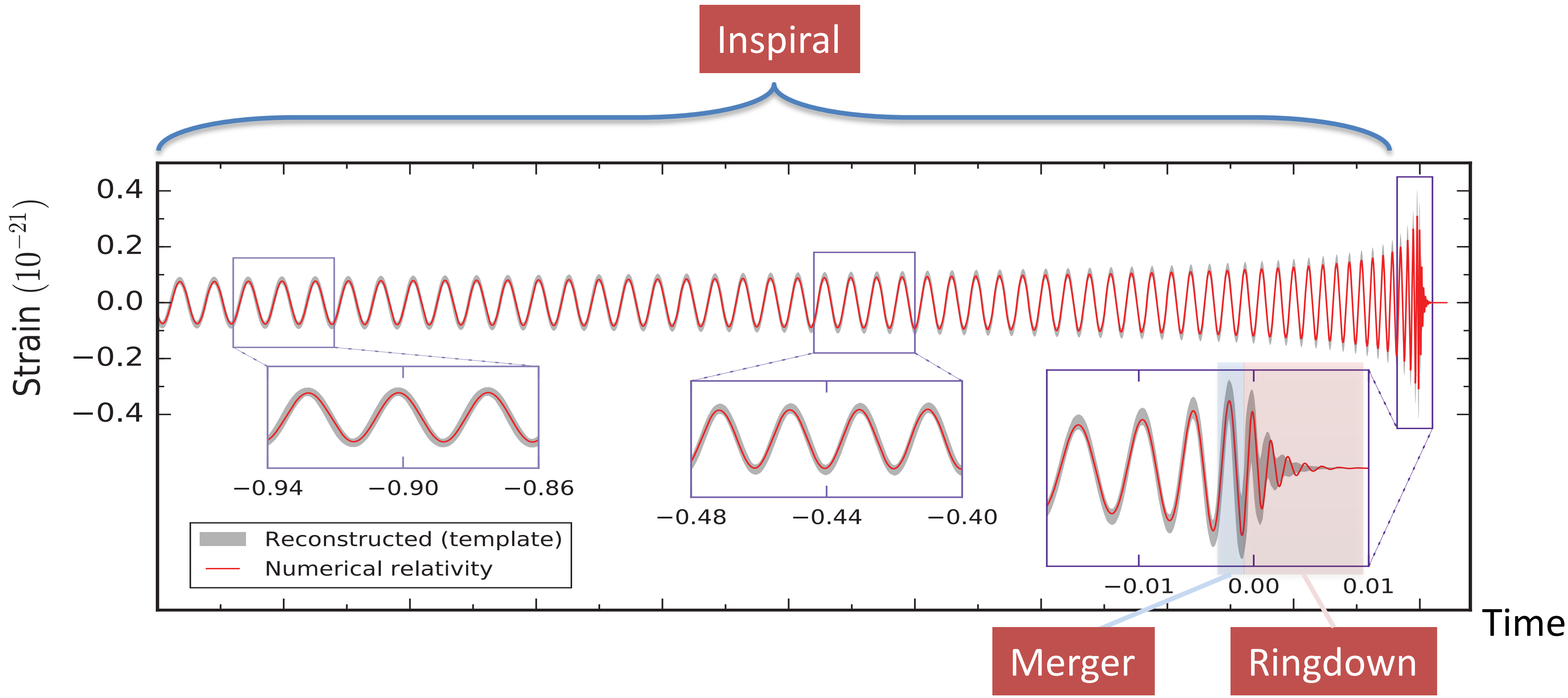
Fundamental noise only
Possible technical noises not shown



Sum of all the noises

Searching for the coalescence of a binary system of compact objects (CBC)

- **Target:** Signals from the coalescence of a binary system of compact objects
- Phases of the coalescence:



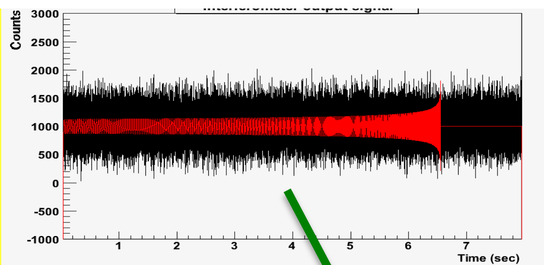
- Waveform characterized by « chirp mass »

$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} = \frac{c^3}{G}\left[\frac{5}{96}\pi^{-8/3} f^{-11/3} \dot{f}\right]^{3/5}$$

Searching for the coalescence of a binary system of compact objects (CBC)

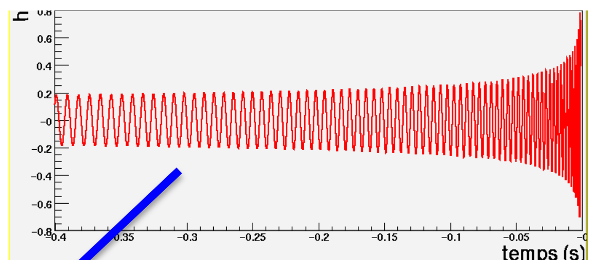
- Template based search
  - Production of a bank of templates (theoretical waveforms)
  - Optimal filtering = weighted inter-correlation btw signal and template

Signal buried into noise



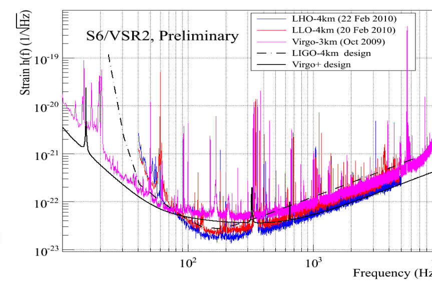
$\otimes$

Template



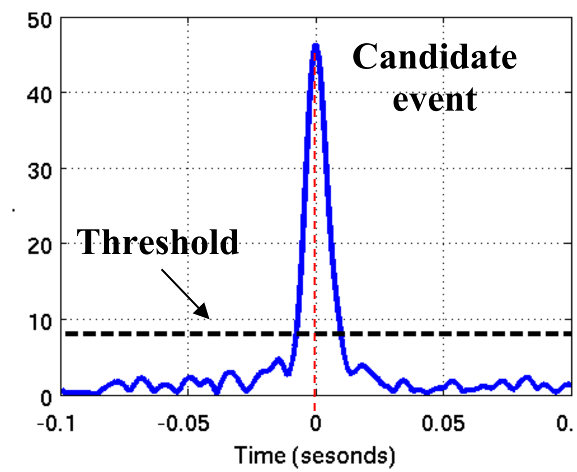
/

Detector noise spectral density



$$\langle \tilde{a}, \widetilde{T} \rangle = 2\left[\int_0^\infty \frac{\tilde{a}(f).\widetilde{T}^*(f)}{S_n(f)}\mathrm{d}f + \mathrm{c.c.}\right]$$

Optimal filtering result: ρ(t)



- Very sensitive to the phase evolution

Searching for the coalescence of a binary system of compact objects (CBC)

- Intrinsic parameters
  - masses, spins (aligned)
    drive
    - the system dynamics
    - the waveform evolution

m_1, m_2, s_1, s_2

- Extrinsic parameters
  - Orientation of the binary, initial phase,…
    impact :
    - Arrival time of the signal
    - Global amplitude and phase
  - Maximized over
    (no need of templates)

$\iota, \Psi, \phi_0, t_0, \ldots$

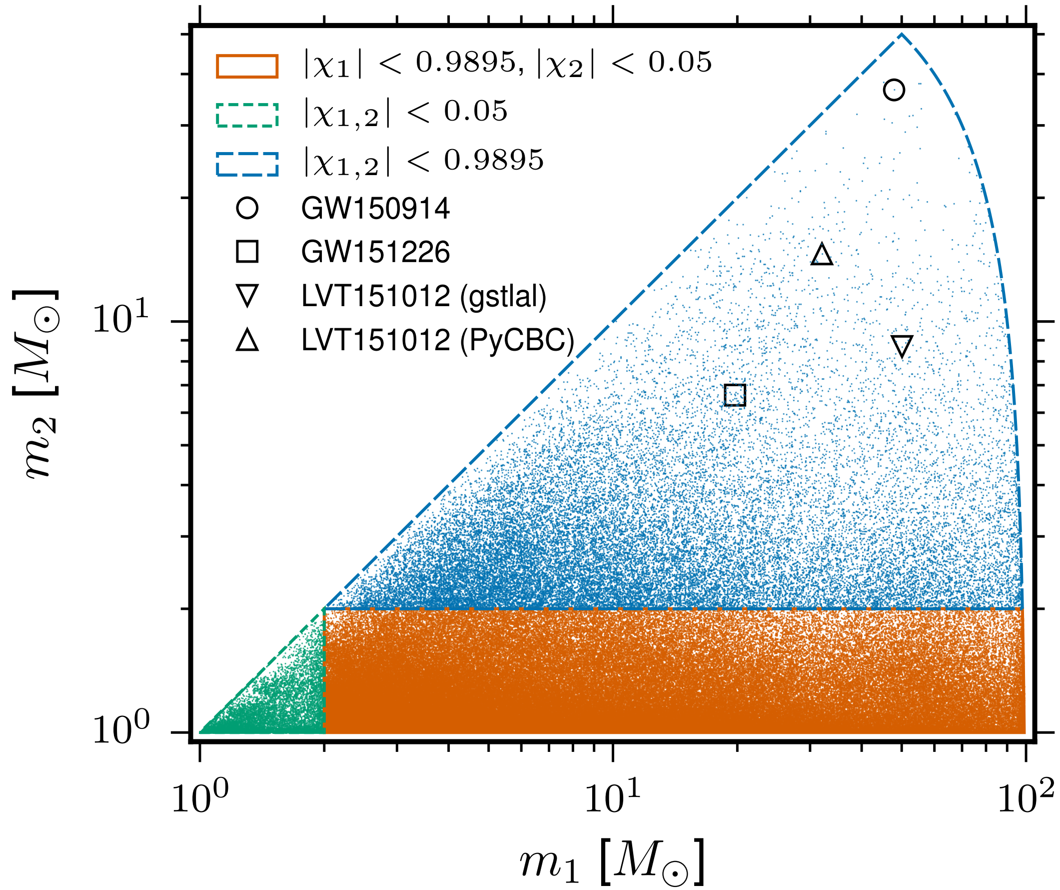
Searching for the coalescence of a binary system of compact objects (CBC)

- For each template
  - Extract the maximum in the signal-to-noise time series

$$\rho(t)$$

- Refinements : χ^2, coincidence, data quality, …

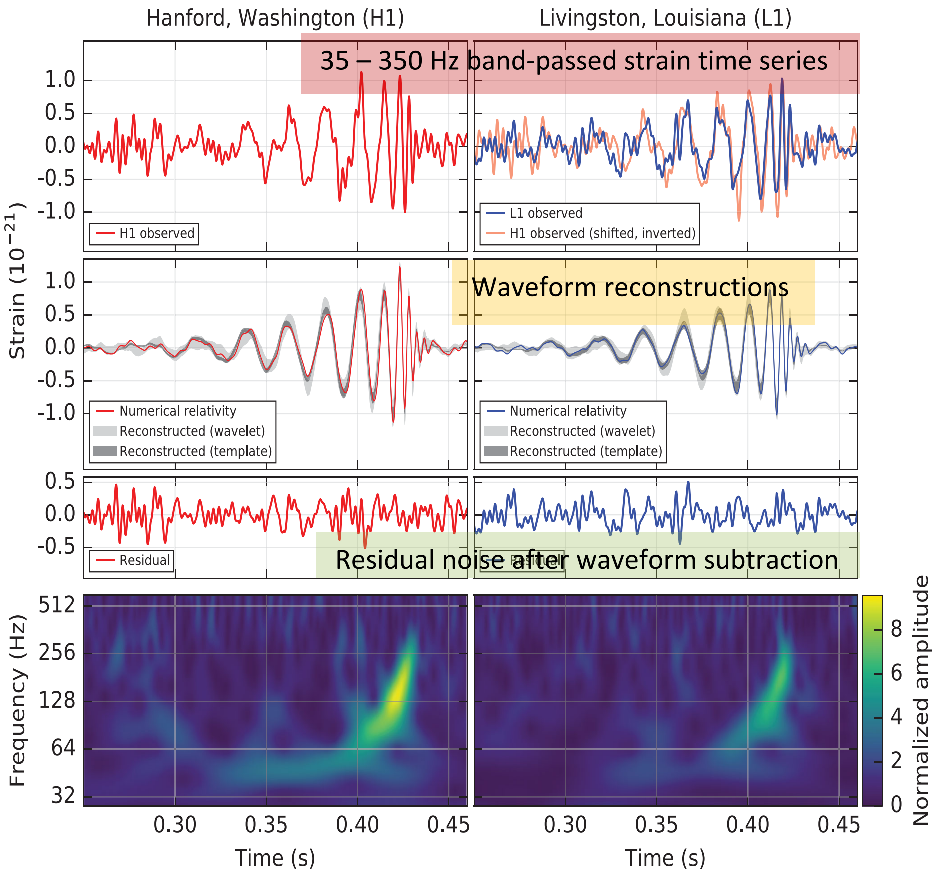
- This is only for the detection…
  - Going further needs parameter estimation



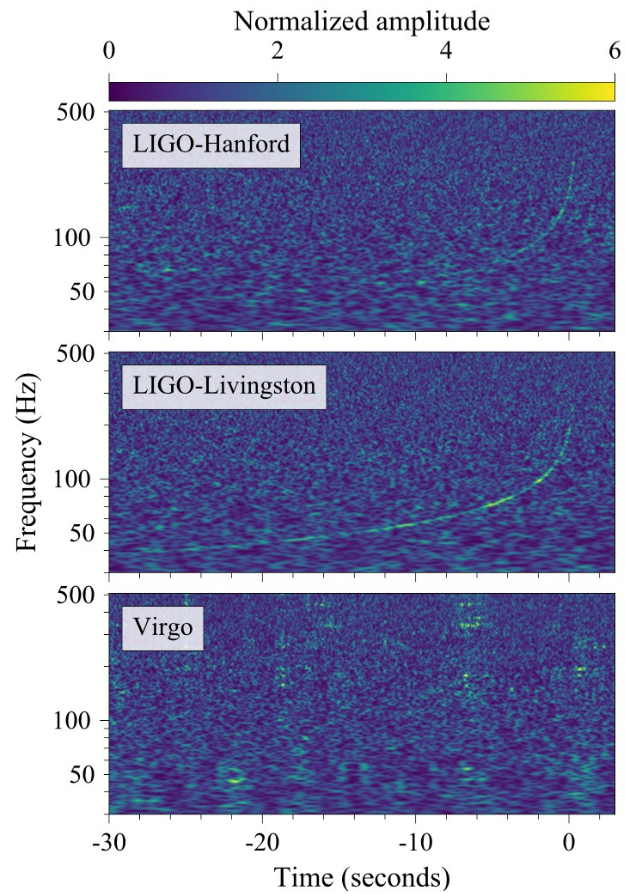
- Each point represents a template (test waveform)
- 4-D parameter space scanned with ~250,000 templates

Naked eye view of GW150914 and GW170817

GW150914



GW170817



- Waveform reconstructed
  - Coherent signal in both detectors
  - Agreement with best-fit theoretical waveforms
    (waveforms from perturbative theory + NR = Numerical Relativity)
- Residual noise consistent with instrumental noise

Parameter Estimation

- Intrinsic parameters (8)
  - Masses (2) + Spins (6)

- Extrinsic parameters (9)
  - Location : luminosity distance, right ascension, declination (3)
  - Orientation: inclination, polarization (2)
  - Time and phase of coalescence (2)
  - Eccentricity (2)

Parameter space of 17 dimensions !

+ 10 to acount for various systematic uncertainties.

- Estimation of the parameters of the source

- Reconstruct the Probability Density Function = "PDF"= $p(\vec{\theta}|\vec{d})$
  that a waveform of parameters $\vec{\theta}$
  is present in the data $\vec{d}$

Parameter Estimation

- Estimation of the source parameters

PDF = Probability Density Function

PDF to have a waveform of parameters $\vec{\theta}$ given the data $\vec{d}$

Prior PDF to have a waveform of parameters $\vec{\theta}$.
(before considering the data)

$$p(\vec{\theta}|\vec{d}) \propto \mathcal{L}(\vec{d}|\vec{\theta}) \cdot p(\vec{\theta})$$

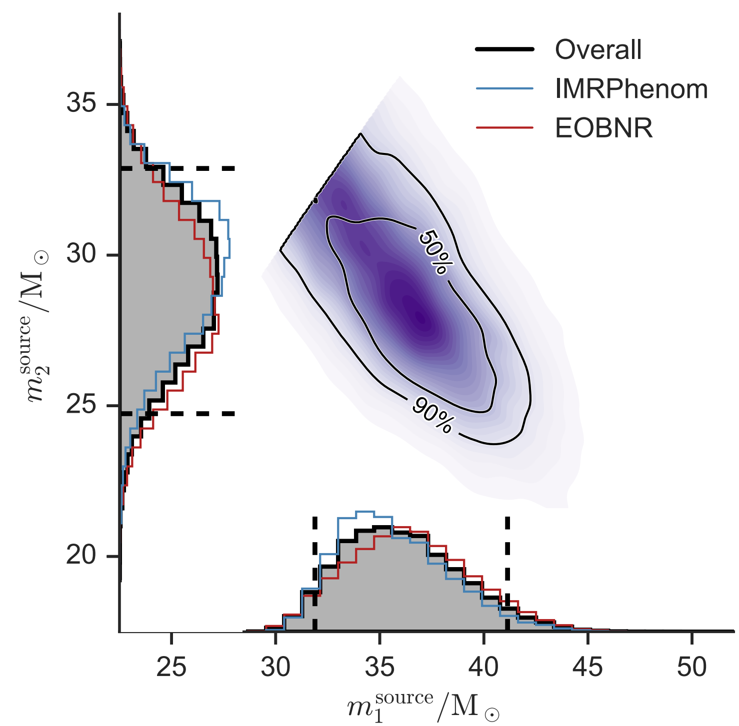
Likelihood that the data $\vec{d}$ contains a signal, or waveform, of given parameters $\vec{\theta}$.

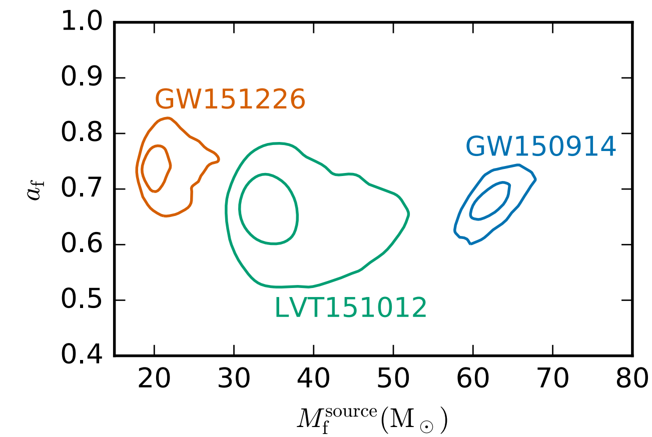
Based on the optimal filtering described in lesson II

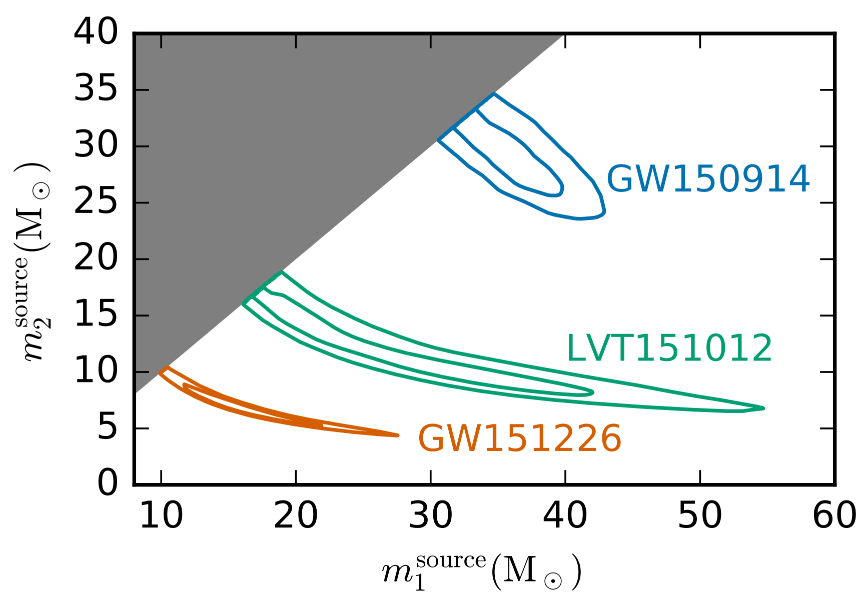
$\Leftrightarrow$ use the Bayes Theorem

Parameter Estimation

- Bayesian framework
- Various methods to sample the parameter space :
  - MCMC = Markov Chain Monte-Carlo
  - Nested sampling
- Example for some intrinsic parameters

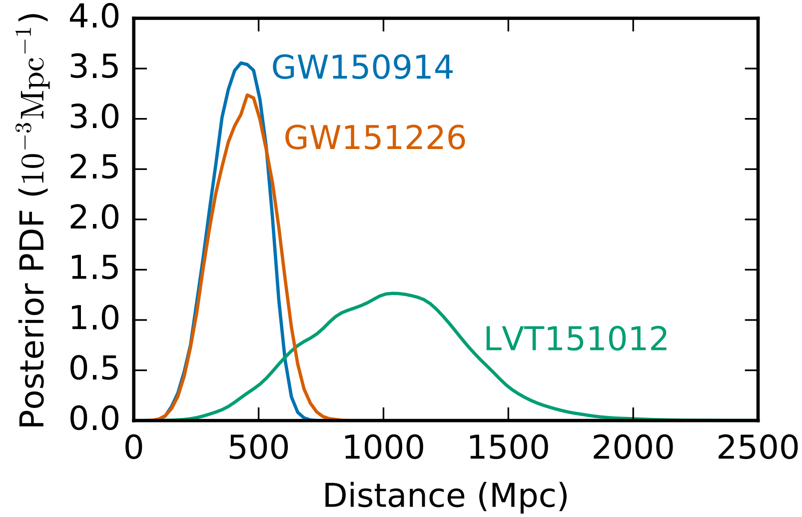


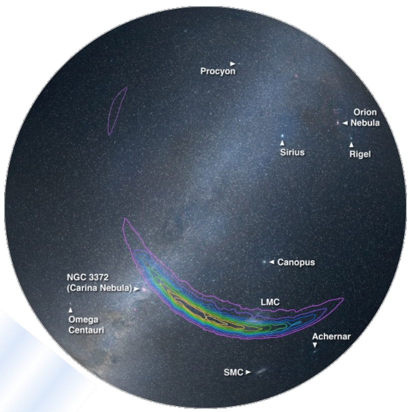


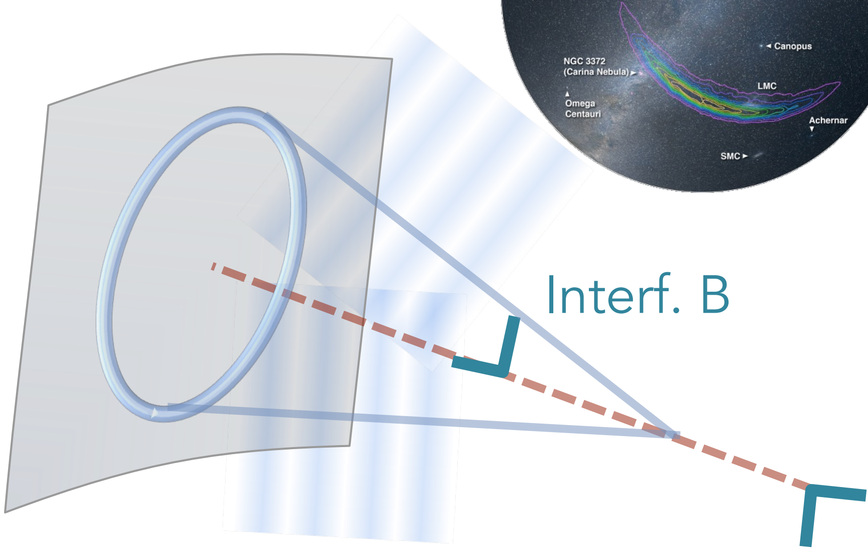


Extrinsic Parameters : examples

- Amplitude depends on masses, distance, and geometrical factors
  - Distance – inclination degeneracy
  - Distant sources with favorable orientations are preferred







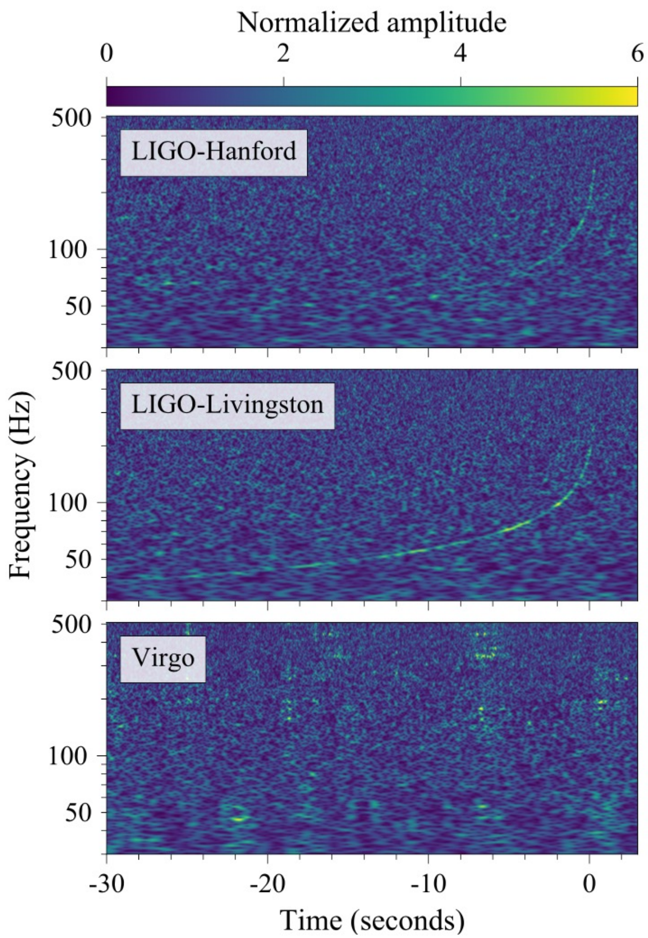
- Source location
  - inferred primarily from
    - time of flight $6.9^{+0.5}_{-0.4}$ ms for GW150914
    - amplitude and phase consistency
  - Limited accuracy with two detector network
  - Sky locations with good detector response are preferred

Most important events

- First detection : GW150914
- First binary neutron star coalescence : GW170817
- Coalescences of a neutron star and a black hole : GW200105 and GW 200115
- Most massive final black hole : GW190521

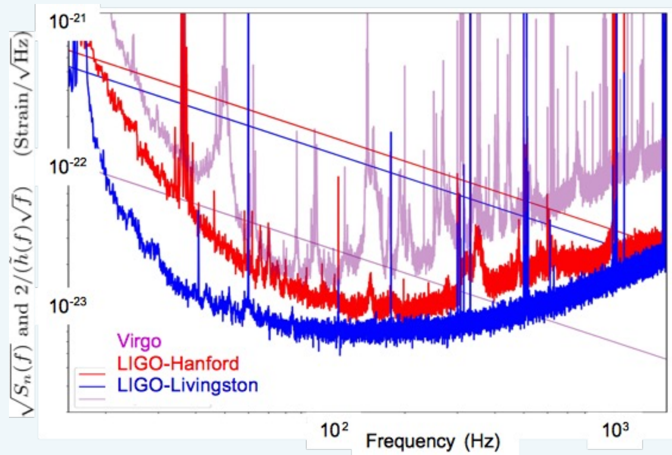
GW170817 : the merger of two neutron stars

- Detected on August 17, 2017 at 12:41:04.4 UTC
- Combined SNR = 32.4
- False alarm rate f < 1 over 80000 years

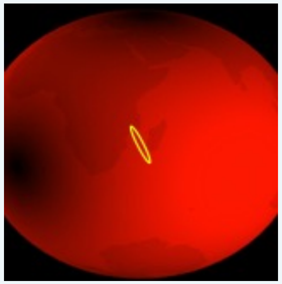


Abbott et al., PRL, 119, 161101 (2017)

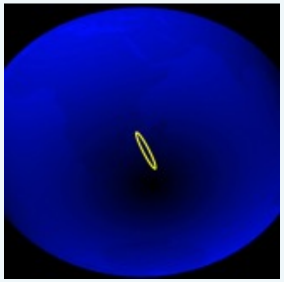
- Weak signal in Virgo
  - Lower sensitivity + unfavorable orientation
  - Does not participate to the detection
  - Significant effect on parameter estimation
  - Particularly sky localization



Antenna pattern projected on Earth
(darker = less sensitive)

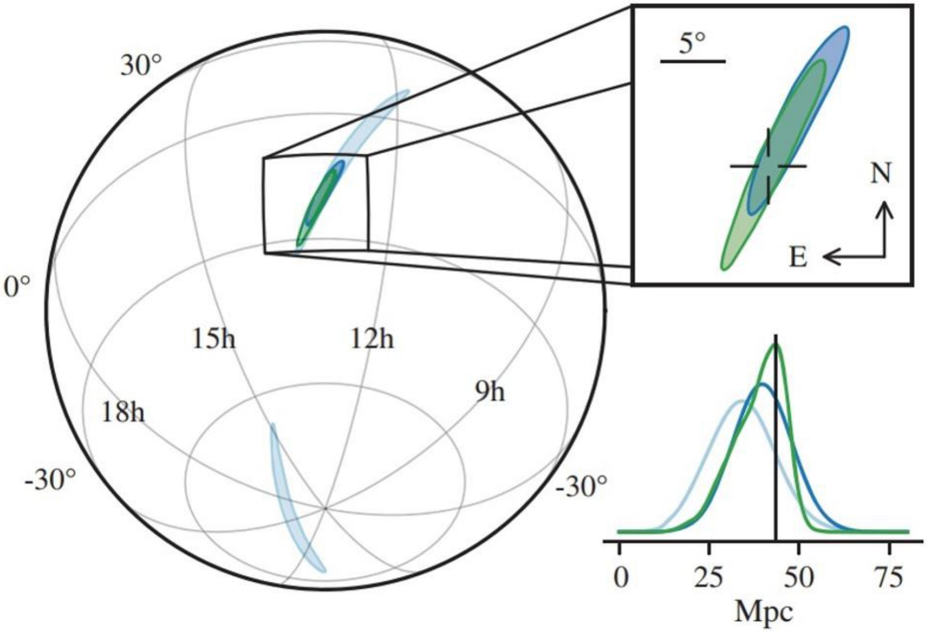


LIGO (Livingston)



Virgo

GW170817 : source localization



- Sky location:
  - rapid loc. with HL: 190 deg^2
  - rapid loc. with HLV: 31 deg^2 - final loc. with HLV: 28 deg^2

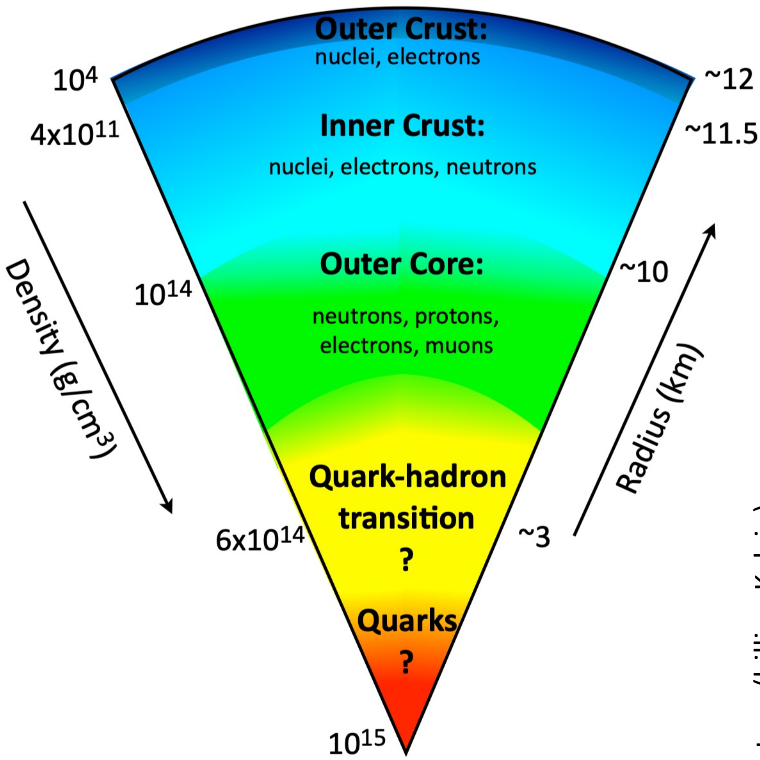
- Luminosity distance: 40 Mpc
  (~120 millions of light-years)

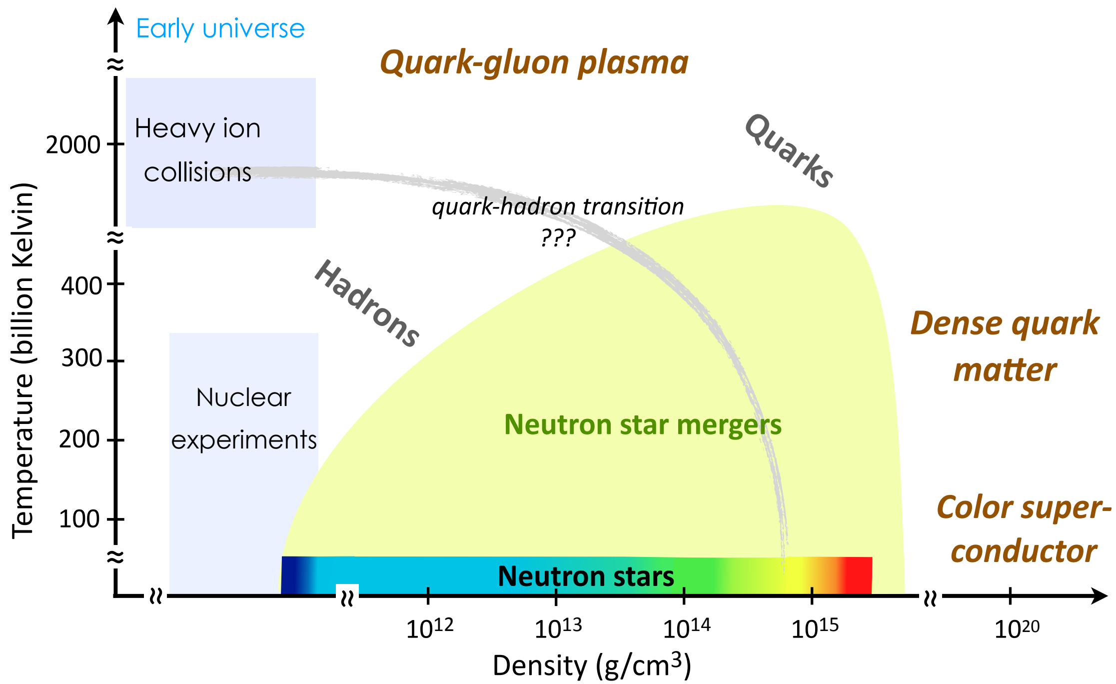
→ 3D position: 380 Mpc^3

- Source closest and best localized even today
- Triggered electromagnetic and neutrino followup observations
- Identified NGC4993 as the host galaxy

Neutron star: internal structure

- Density





Neutron star: internal structure

- Spherical symmetry body in GR
  - Isotropical material
  - Gravitational equilibrium, stationary
- => Tolman-Oppenheimer-Volkoff (TOV) equation:

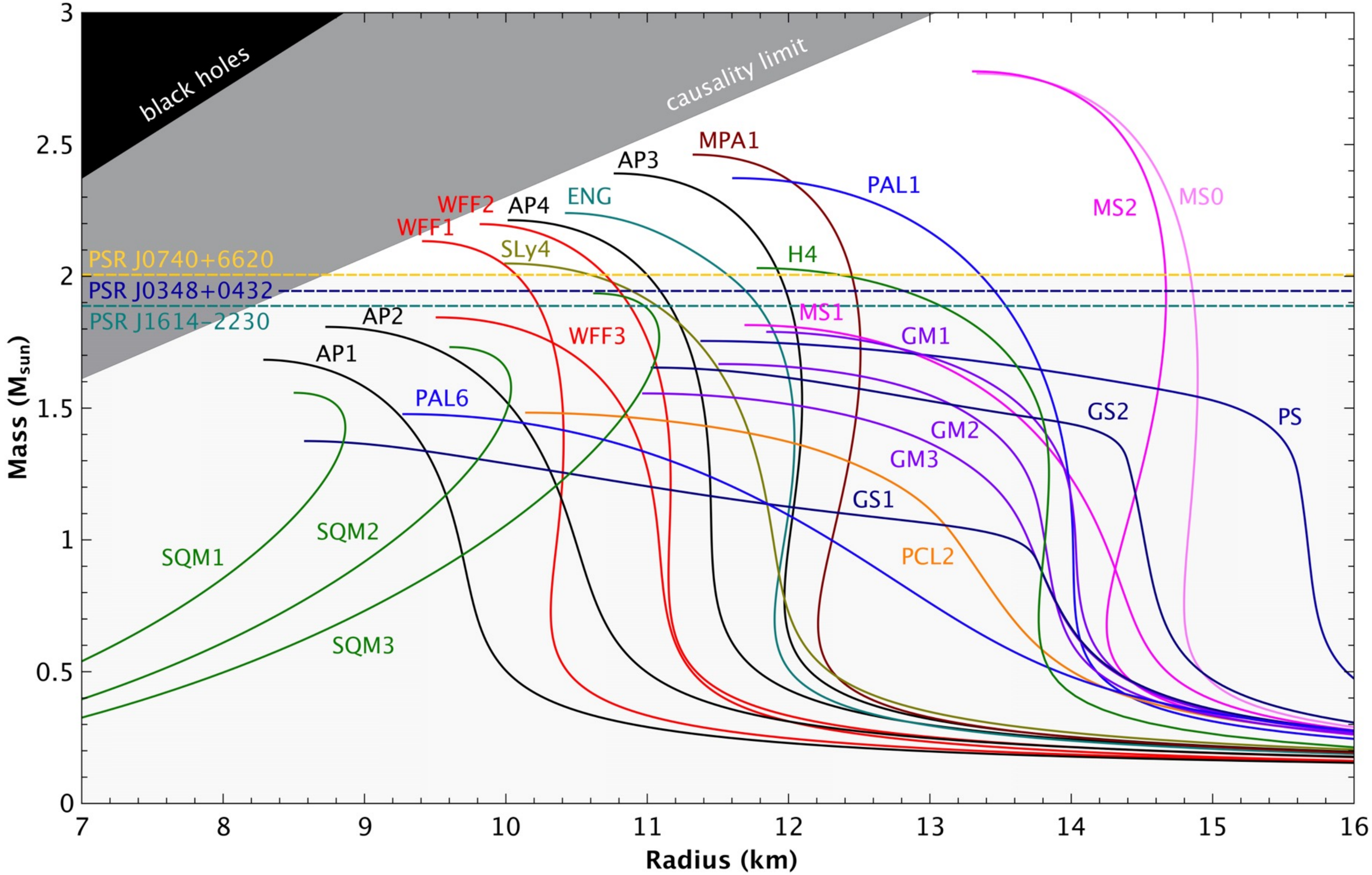
$$\frac{dP}{dr} = -\frac{Gm}{r^2}\rho\left(1+\frac{P}{\rho c^2}\right)\left(1+\frac{4\pi r^3 P}{mc^2}\right)\left(1-\frac{2Gm}{rc^2}\right)^{-1}$$

- r radial coordinate, $\rho(r)$ energy density, $P(r)$ pressure
- $m(r)$ total mass in a sphere of radius r

- If includes the equation of state (EOS) $F(P,\rho)=0$
  - => completely determines the internal structure
- But F is poorly known !

Neutron star: internal structure

- Equations of state



http://www3.mpifr-bonn.mpg.de/staff/pfreire/NS_masses.html arXiv:1603.02698 **[astro-ph.HE]**

GW170817 : intrinsic parameters

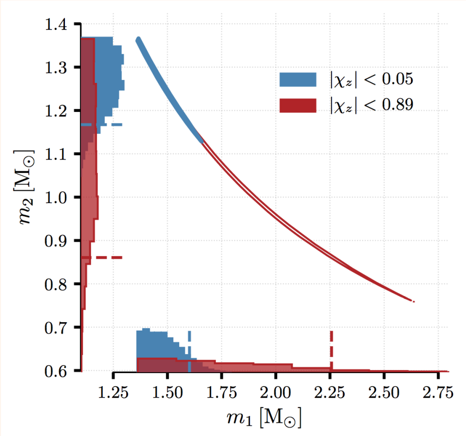
Abbott et al., PRL, 119, 161101 (2017)

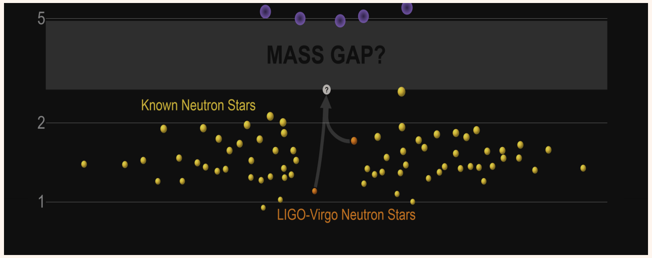
	low-spin ($\lvert\chi\rvert < 0.05$)	high-spin ($\lvert\chi\rvert < 0.89$)
$M_{chirp} (M_\odot)$	$1.188^{+0.004}_{-0.002}$	
m_1 $(M_\odot)$	1.36–1.60	1.36 – 2.26
m_2 $(M_\odot)$	1.17–1.36	0.86 – 1.36
m_{tot} $(M_\odot)$	$2.74^{+0.04}_{-0.01}$	$2.82^{+0.47}_{-0.09}$

Object masses

Degeneracy btw mass ratio and spin aligned components.

→ Masses < 2.3 $M_\odot$

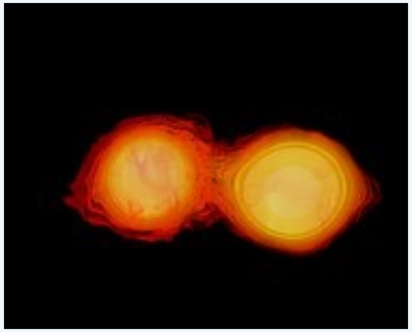




Masses consistent with neutron stars

Equation of state of neutron stars

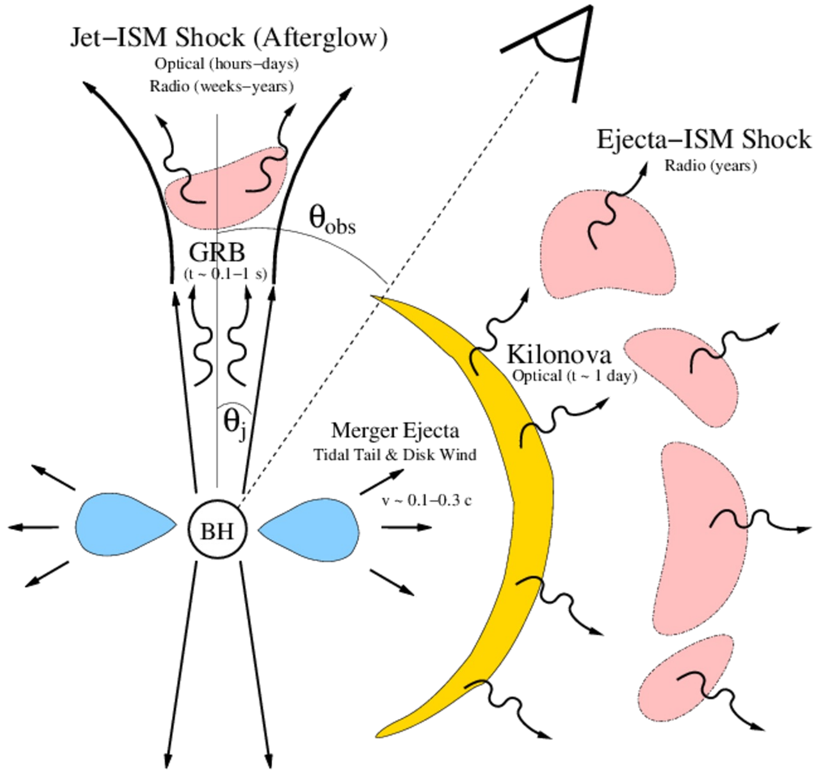
Tidal field of the companion ⇒ Deformation of the neutron star ⇒ Imprint on the shape of the GW for f>600 Hz



Merger happens earlier than w/o tidal effect, final spin modified

Result favors equations of state of neutron stars that predict more compact stars: radius < 15 km

Expected electro-magnetic counterparts?



Metzger & Berger, ApJ, 746, 48 (2012)

Short gamma-ray burst (sGRB):

Jet

→ prompt γ-ray emission

- few seconds after merger
- last for <2 s
- beamed

Interaction of jet with interstellar medium

→ afterglow emission

- few days after merger
- evolves from X-ray to radio

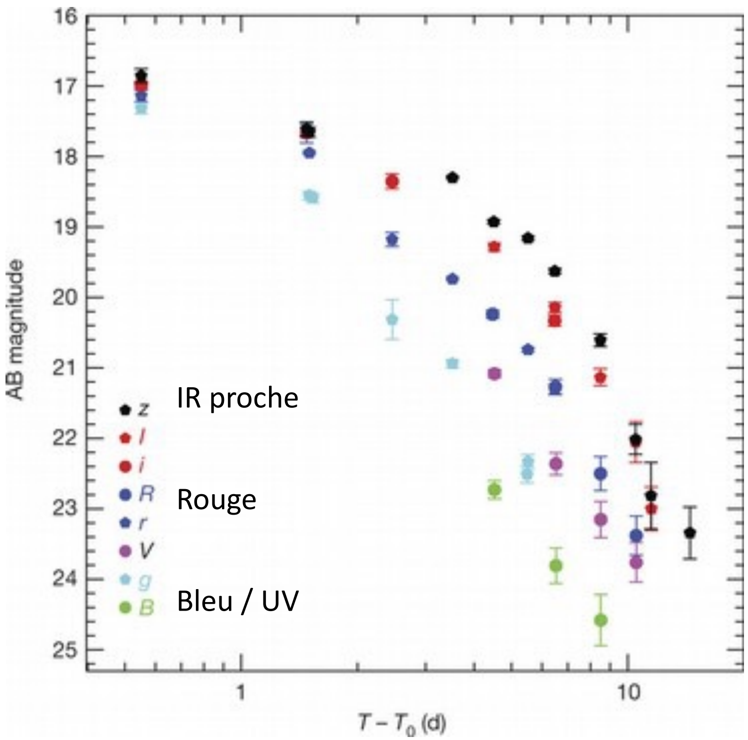
Kilonova (or macronova)

Conversion of hot ejected matter into r-processed elements, disintegration and thermal emission

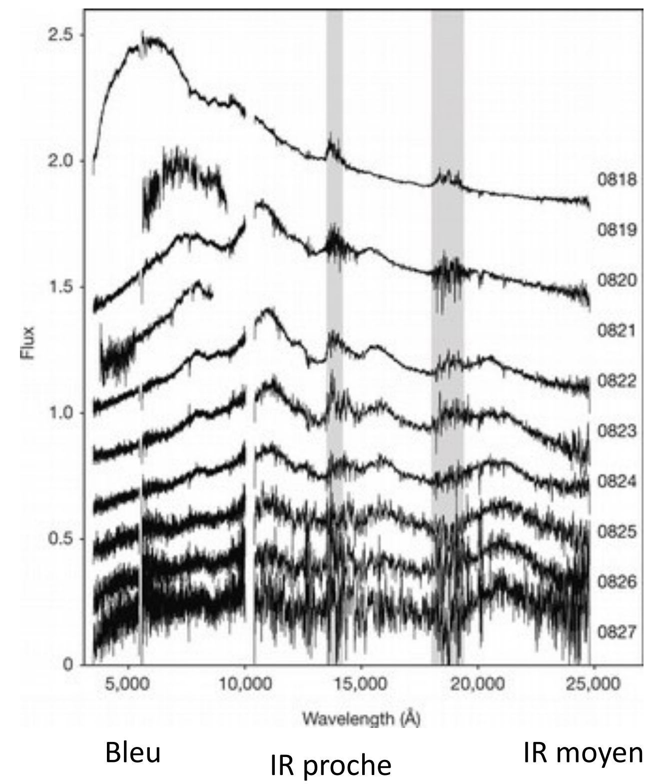
→ black body continuum + broad structures

- few hours-days after merger
- visible in UV, optical, IR
- rapid spectral evolution

Optical transient evolution



Light curves

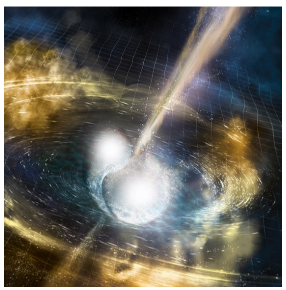


Spectrum evolution

- Consistent with kilonova (=macronova) models
- First spectroscopic identification of a kilonova
- Probably the main source of heavy elements in the universe

GW170817 : association btw GW/GRB, speed of gravitational waves

Emission during the merger
-> GW and γ-rays



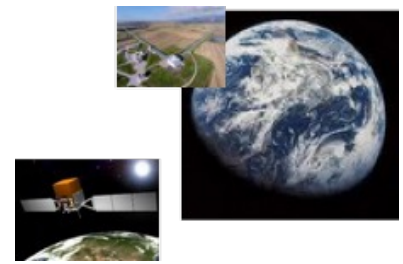
Hypothèse :
les γ sont émis entre
0 et 10 s après les OG

Propagation
over 40 Mpc

GW

γ-rays

Detection



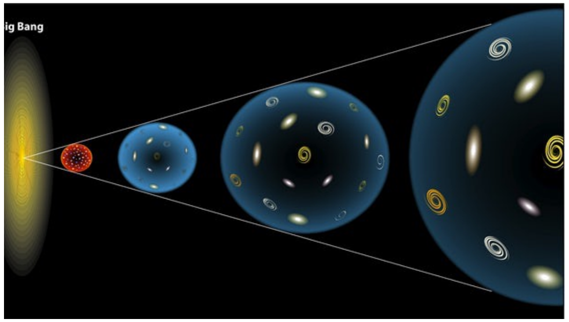
γ –rays detected
1.75 ± 0.05s after GW
from merger

Difference btw speed of light and speed of GW

$$[-3 \times 10^{-15}; +7 \times 10^{-16}] \times c$$

Hubble constant measurement

H_0 = today's expansion rate of the universe



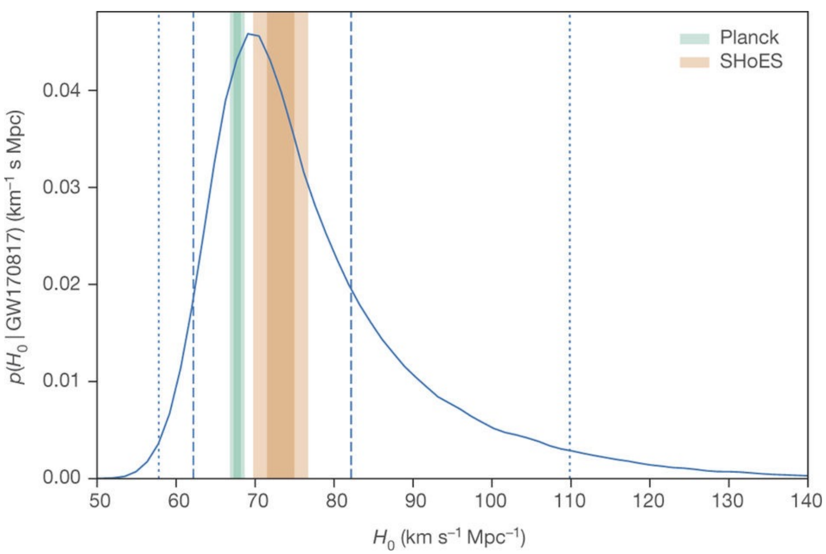
GW170817 may be used as a standard siren

$$D_{luminosity} = H_0 \times v_r$$

Estimated from the GW signal: $(43.8^{+2.9}_{-6.9}\,\mathrm{Mpc})$

Determined from the redshift of host galaxy $(3017 \pm 166\,\mathrm{km/s})$

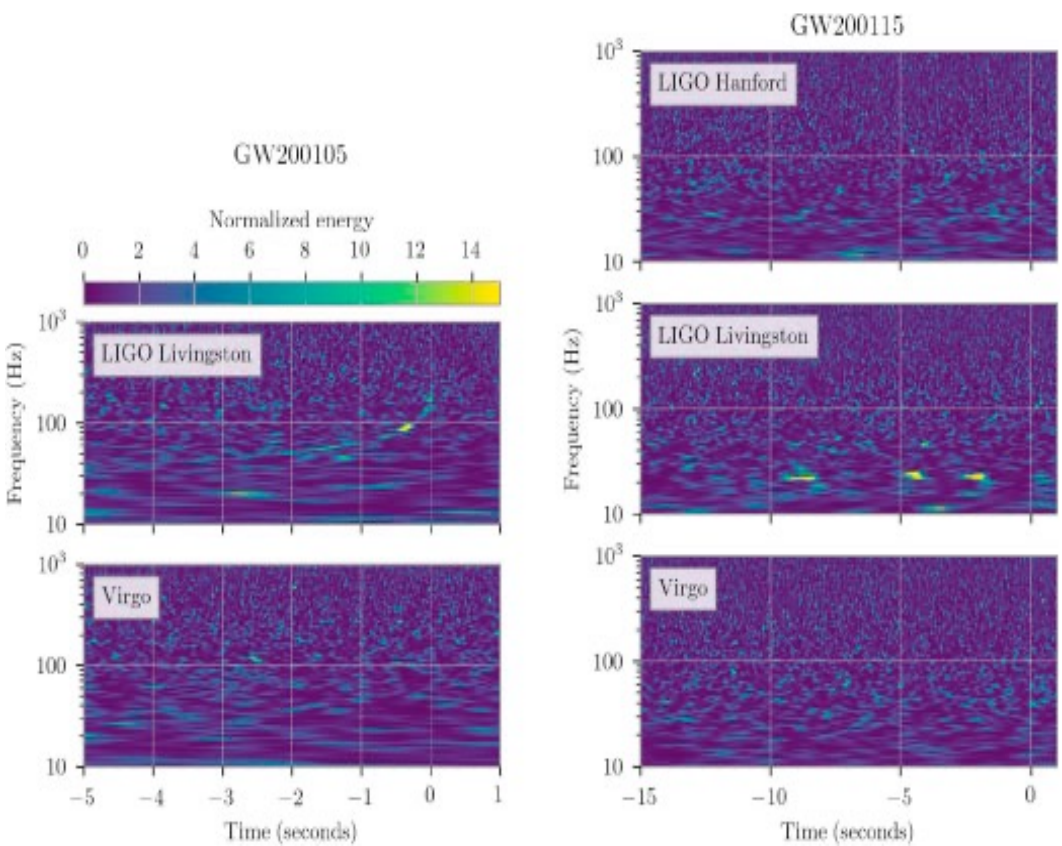
→ $H_0 = 70^{+12}_{-8}\,\mathrm{km/s\ Mpc^{-1}}$



Independent measurement of H_0
→ may help to understand the current tension

GW200105 and GW200115
Neutron star and black hole

- Detected on January 5 and January 15 2020
- Combined SNR = 13.9 and 11.6



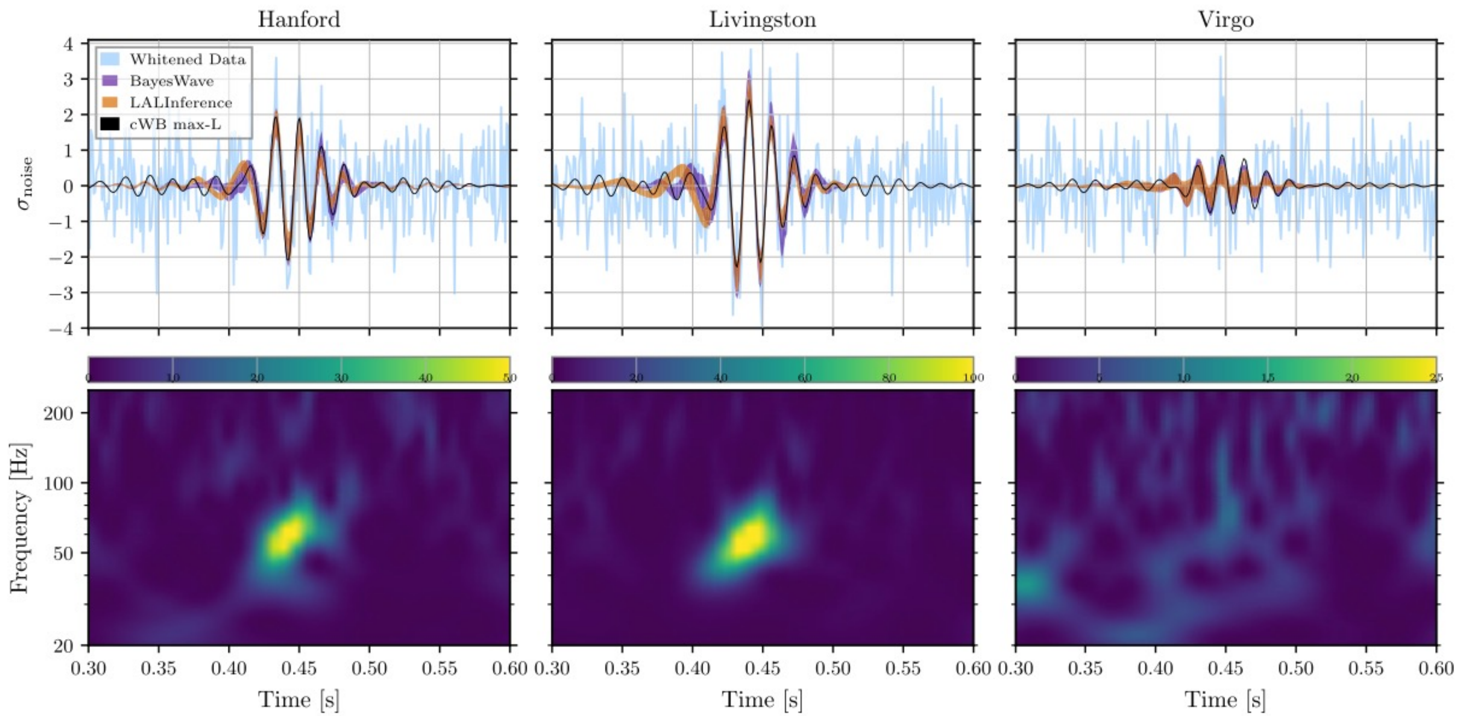
- Distance : $(280^{+110}_{-110} \text{ and } 300^{+150}_{-100})$ Mpc
- Masses : $(8.9^{+1.2}_{-1.5} \text{ and } 1.9^{+0.3}_{-0.2})\ M_\odot$
  and $(5.7^{+1.8}_{-2.1} \text{ and } 1.5^{+0.7}_{-0.3})\ M_\odot$
- Modeling the formation of such binaries is difficult
- No EM or neutrino counterparts

R. Abbott *et al* 2021 *ApJL* **915** L5

GW190521 : Big is big !

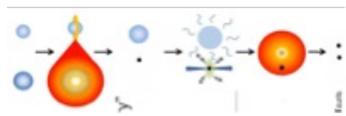
- Detected0 on May 21 2019 at 03:02:29 UTC
- Combined SNR = 14.7
- False alam rate f < 1 over 4900 years

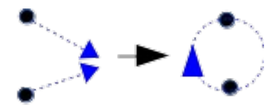
- Distance : $5.3^{+2.4}_{-2.6}$ Gpc (redshift of 0.82)
- Masses : $85^{+21}_{-14}\ M_\odot$ and $66^{+17}_{-18}\ M_\odot$
- Final black hole mass: $142^{+28}_{-16}\ M_\odot$
- First intermediate mass black hole



Non exhaustive list of current and future studies

- Astrophysical implications
  - Formation mechanism of NS or BH binaries





  - GRB origin, jet focusing / structure
  - Kilonovae modeling
  - Equation of state of neutron stars
  - Neutron star result of a merger: long or short-lived ?
  - Inference of binary neutron star population distribution and coalescence rate

$$R = 1540^{+3200}_{-1220} \quad \mathrm{Gpc}^{-3}.\mathrm{yr}^{-1}$$

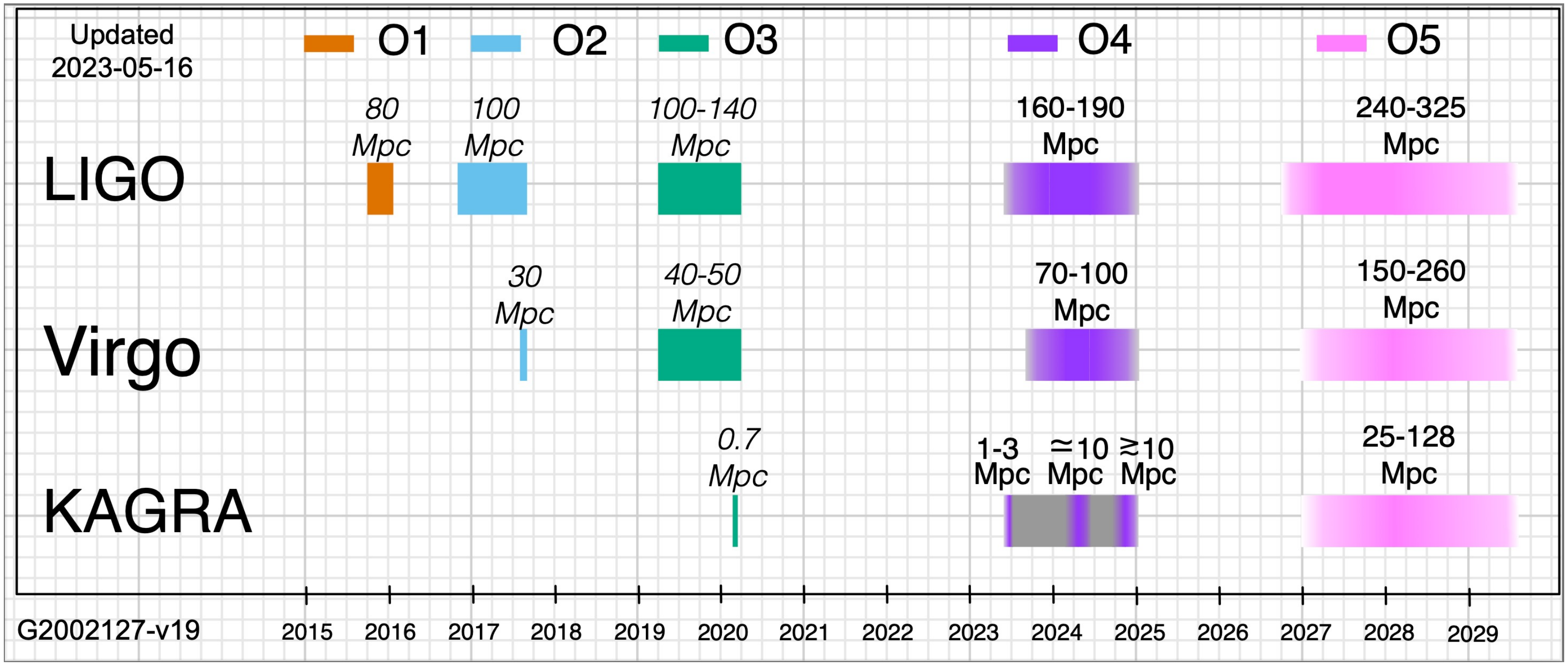
$$(R < 12600\,\mathrm{Gpc}^{-3}.\mathrm{yr}^{-1} \quad \mathrm{from} \quad 01)$$

  - GW stochastic background coming from BNS coalescences (astrophysical stochastic background)
    - To be detected in the coming years

- Tests of GR
  - Difference in speed between GW and light
  - Search for deviations from GR in GW waveforms
  - Study of the GW polarization
  - New limits on Lorentz invariance violation
  - New test of the equivalence principle

- Cosmologie
  - Independent measurement of the Hubble constant

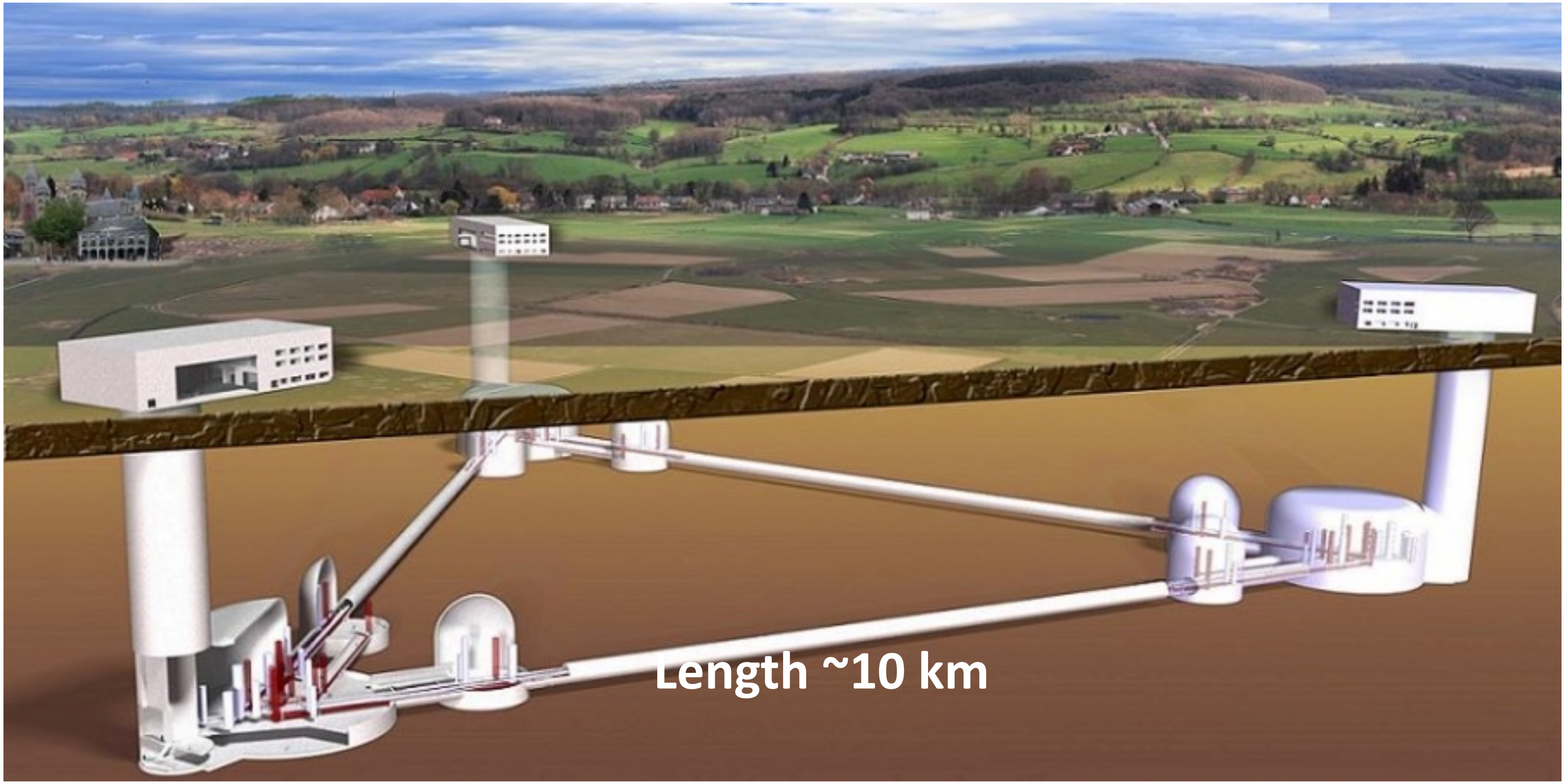
An eye on the future



https://observing.docs.ligo.org/plan/

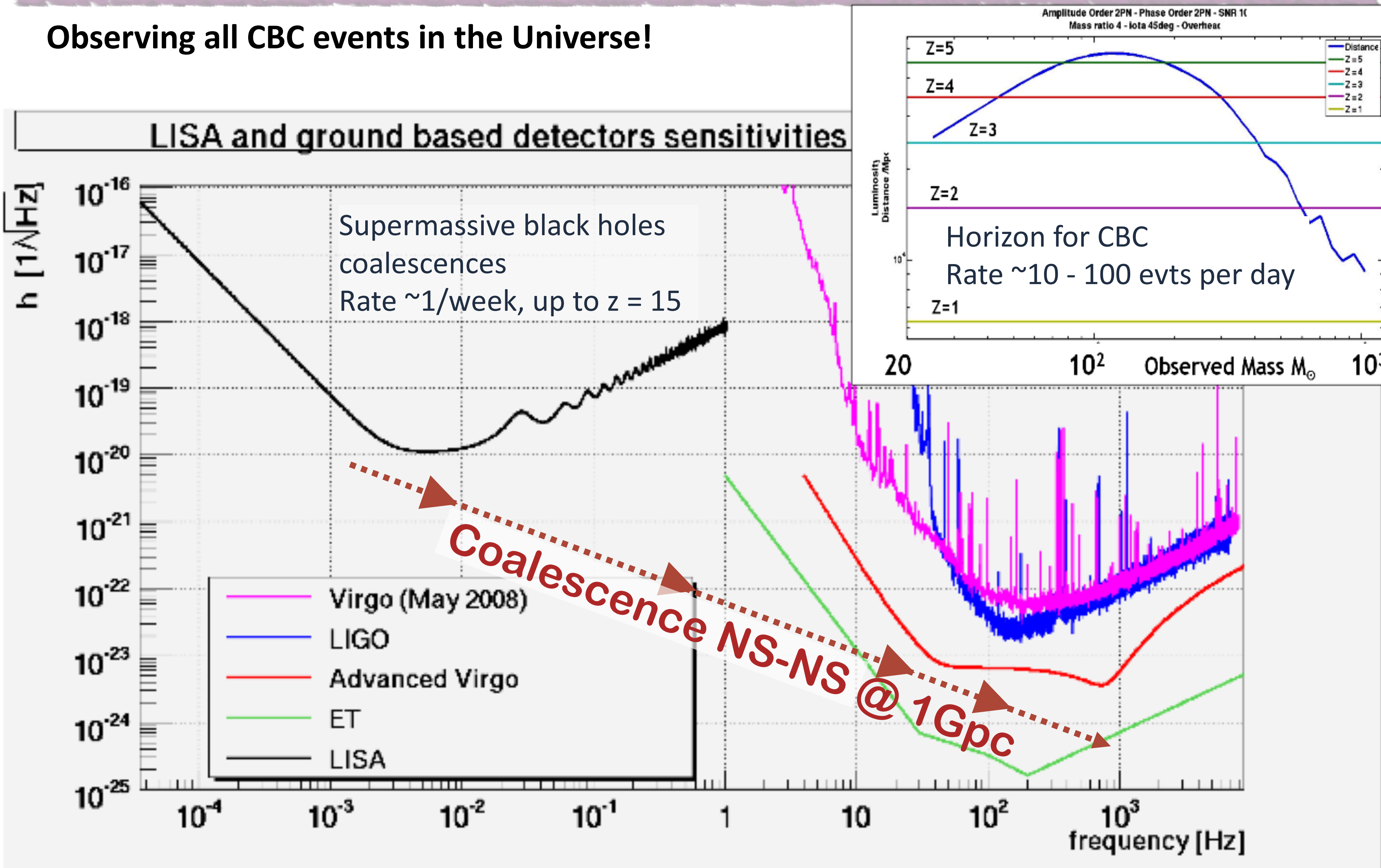
Einstein Telescope

- Third generation interferometer
- Located underground, ~10 km arms
- Technical design to be written in ~2024 -2025, detector operational after 2035?



ET and LISA performances

Observing all CBC events in the Universe!

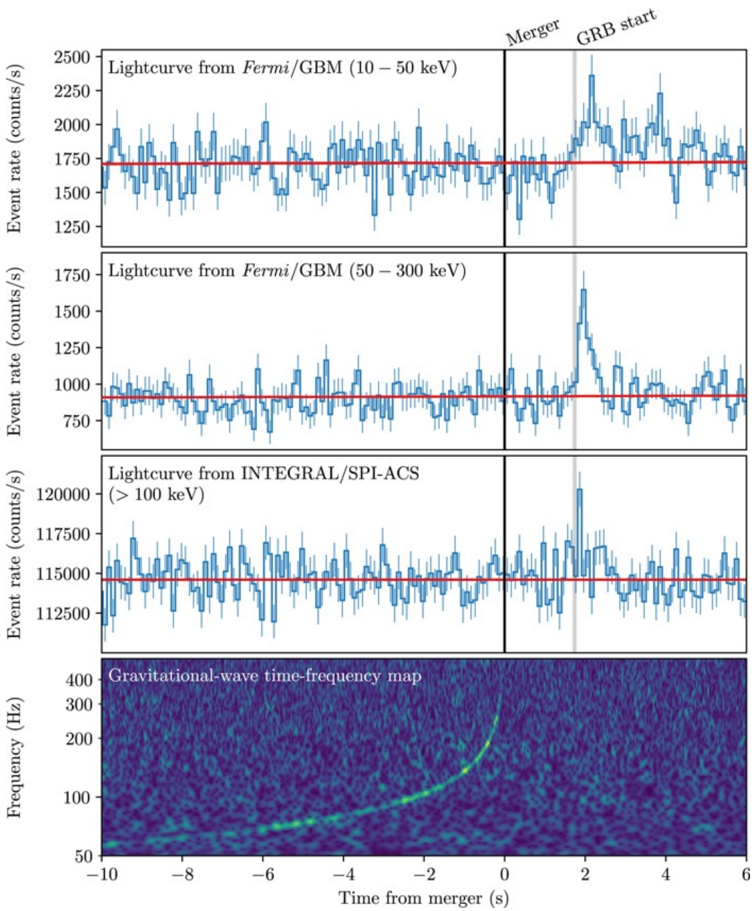


The End of episode II

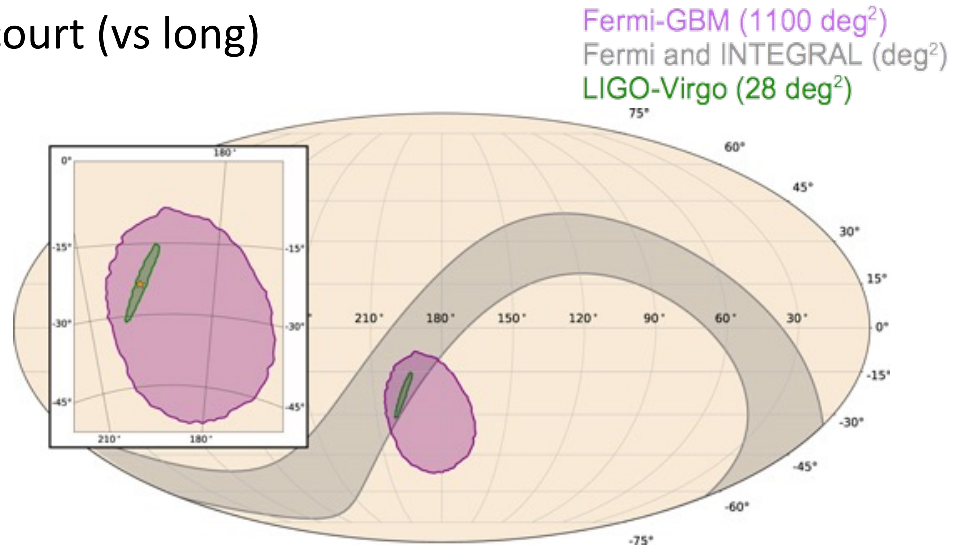
- Spares

A GRB seen by Fermi

- GRB170817A détecté par Fermi et INTEGRAL
  - Emission gamma ~ 1.7 s après la fusion
  - 3 fois plus probable d'être un GRB court (vs long)



Localisation sur le ciel (90% CL)



Probabilité d'une association aléatoire : 5.0×10^{-8}

-> association validée à 5.3 σ

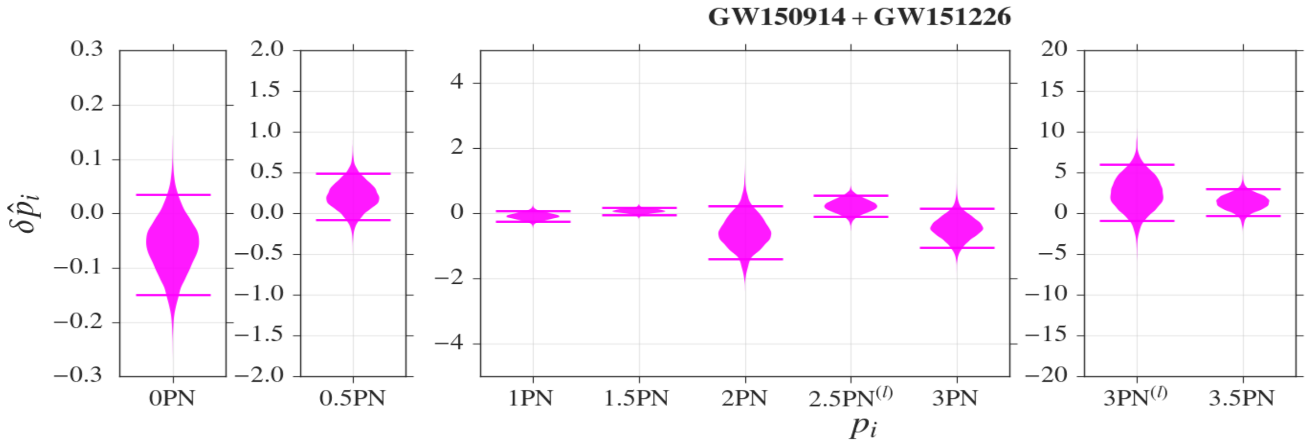
Première preuve que les fusions d'étoiles à neutrons sont les progéniteurs des GRB courts (au moins certains)

Testing GR with GW150914 (I)

- Most relativistic binary pulsar known today
  - J0737-3039, orbital velocity $v/c \sim 2 \times 10^{-3}$
- GW150914
  - Strong field, non linear, high velocity regime $v/c \sim 0.5$
- "Loud" SNR -> coarse tests
  - Waveform internal consistency check
  - Evidence for deviation from General Relativity in waveform ?
  - Bound on Compton wavelength (graviton mass)

Testing GR with GW150914 (II)

- No evidence for deviation from GR in waveform



- No evidence for dispersion in signal propagation
  - Bounds :

$$\lambda_g > 10^{13}\ \text{km}$$

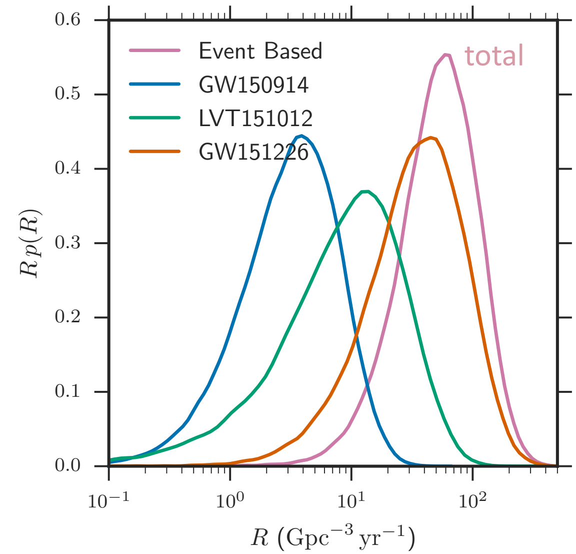
$$m_g \leq 1.2 \times 10^{-22}\ \text{eV/c}^2$$

$$\left(\frac{v}{c}\right)^2 = 1 - \left(\frac{hc}{\lambda_g E}\right)^2$$

  - More constraining than bounds from
    - Solar System observations
    - binary pulsar observations
  - Less constraining than model dependent bounds from
    - large scale dynamics of galactic clusters
    - weak gravitational lensing observations

Rate of BBH mergers

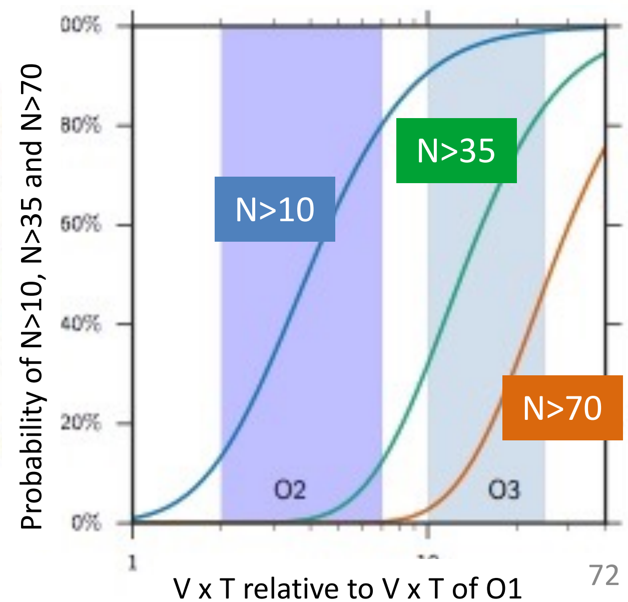
- Previously :
  - Estimations of the coalescence rate
    - Based on electromagnetic observations and population modeling
    - R ~ 0.1 – 300 $Gpc^{-3}\ yr^{-1}$
  - Previous LIGO-Virgo rate upper limits
    - R < 140 $Gpc^{-3}\ yr^{-1}$ for GW150914 parameters



- Astrophysical rate inference
  - Counting signals in experiment
  - Estimating sensitivity to population of sources
    - Depends on mass distribution (hardly known)

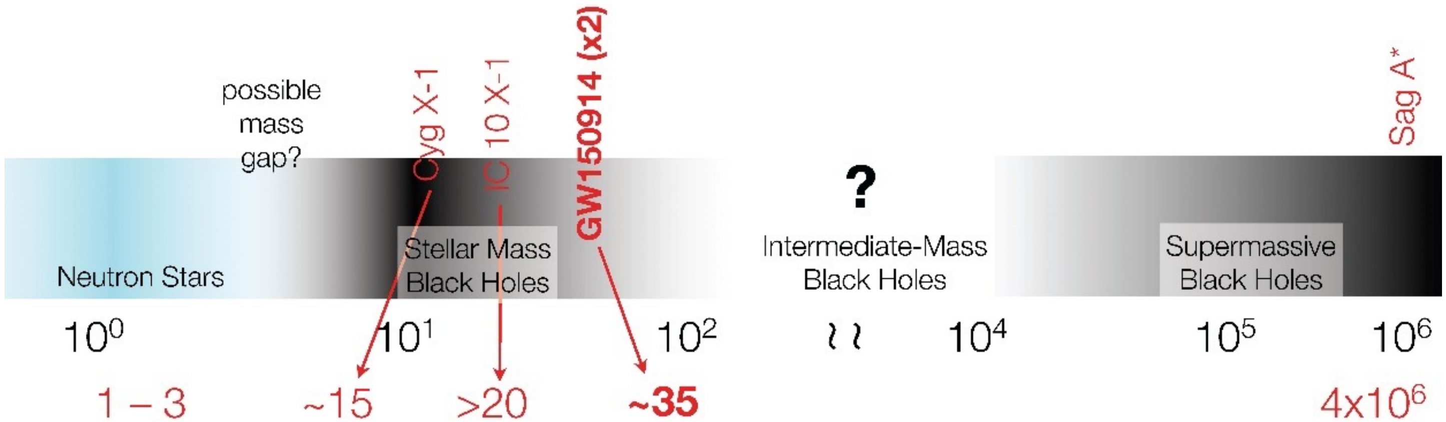
- Low statistics and variety of assumptions -> broad rate range
  - R ~ 9 – 240 $Gpc^{-3}\ yr^{-1}$

- Project expected number of highly significant events as a function of surveyed time x volume



Astrophysics implications

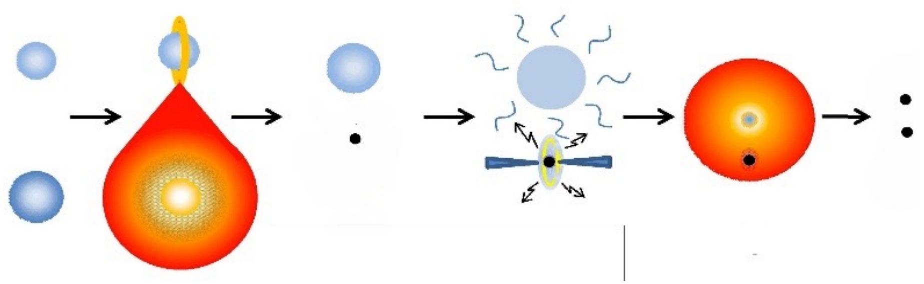
- Relatively massive black holes (> 25 $M_\odot$) exist in nature

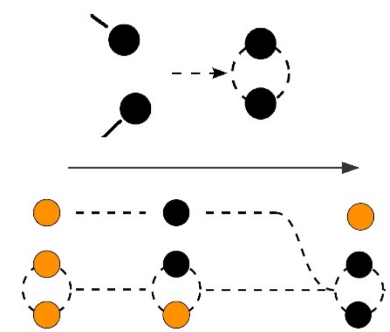


- Massive progenitor stars
  => low mass loss during its life
  => weak stellar wind
- Metallicity = proportion of elements heavier than He
  - High metallicity => strong stellar wind
- => formation of progenitors
  in a low metallicity environment

Astrophysics implications

- Binary black holes form in nature
  - Formation :
    - Isolated binaries
    - Dynamical capture (dense stellar regions)
  - GW150914 and GW151226 do not allow to identify formation channel
    - Future : information on the spins can help

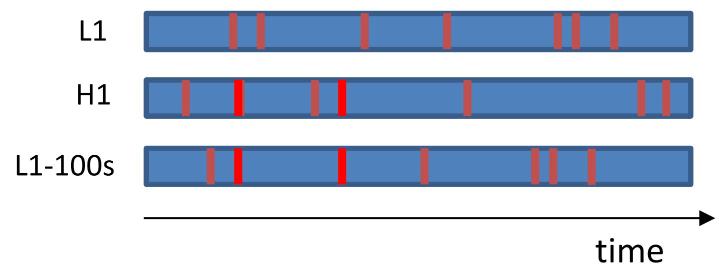




- Binary Black Holes merge within age of Universe at detectable rate
  - Inferred rate consistent with higher end of rate predictions (> 1 Gpc^{-3} yr^{-1})

False alarm rate

- False alarm rate
  - Measured from background estimated on data
  - Time shifts by N x 0.1 s between H1 and L1

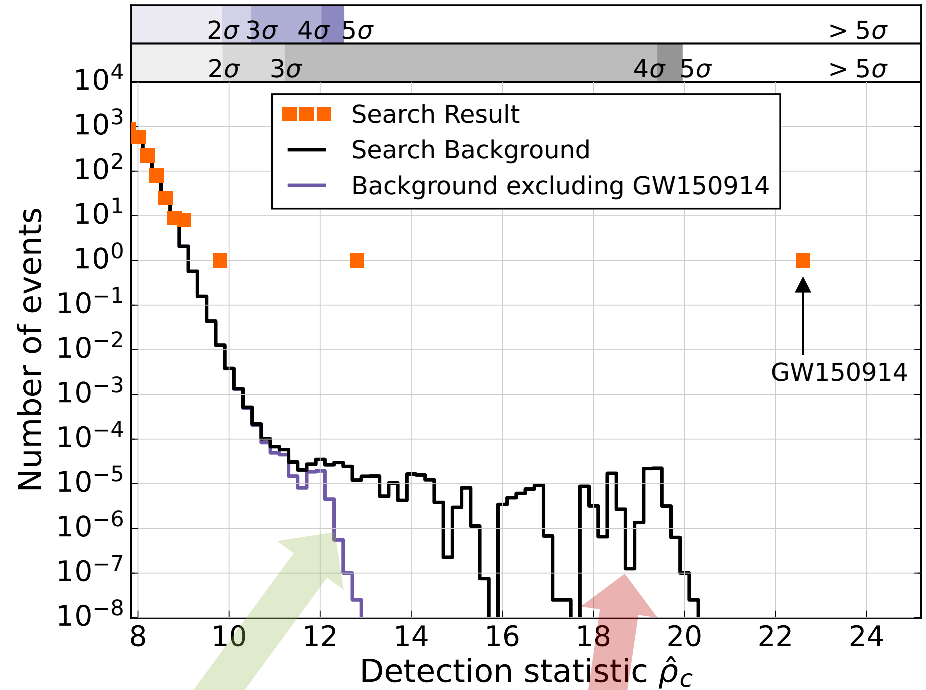


  - Case of GW150914, first analysis for February annoucement
    - N_{max} = 10^7 shifts, T_{bkgd} = 608,000 yrs
    - Account for trial factors
    - GW150914 louder than all background ➔ lower limit on significance

Event	Time (UTC)	FAR (yr^{-1})	$\mathscr{F}$	$\mathscr{M}$ ($M_\odot$)	m_1 ($M_\odot$)	m_2 ($M_\odot$)	χ_{eff}	D_L (Mpc)
GW150914	14 September 2015 09:50:45	$< 5 \times 10^{-6}$	$< 2 \times 10^{-7}$ ($> 5.1\,\sigma$)	28^{+2}_{-2}	36^{+5}_{-4}	29^{+4}_{-4}	$-0.06^{+0.17}_{-0.18}$	410^{+160}_{-180}
LVT151012	12 October 2015 09:54:43	0.44	0.02 ($2.1\,\sigma$)	15^{+1}_{-1}	23^{+18}_{-5}	13^{+4}_{-5}	$0.0^{+0.3}_{-0.2}$	1100^{+500}_{-500}

CBC BBH search result : GW150914

- Statistic
  - $\hat{\rho} = \rho / \{[1 + (\chi_r^2)^3]/2\}^{1/6}$
  - $\hat{\rho}_c = \sqrt{\hat{\rho}_{H1}^2 + \hat{\rho}_{L1}^2}$
- Significance
  - GW150914 is the loudest event in the search, $\hat{\rho}_c$ = 23.7
  - Individual triggers in L1 and H1 (forming GW150914): highest $\hat{\rho}$ in each detector
  - Significance $> 5.3\sigma$



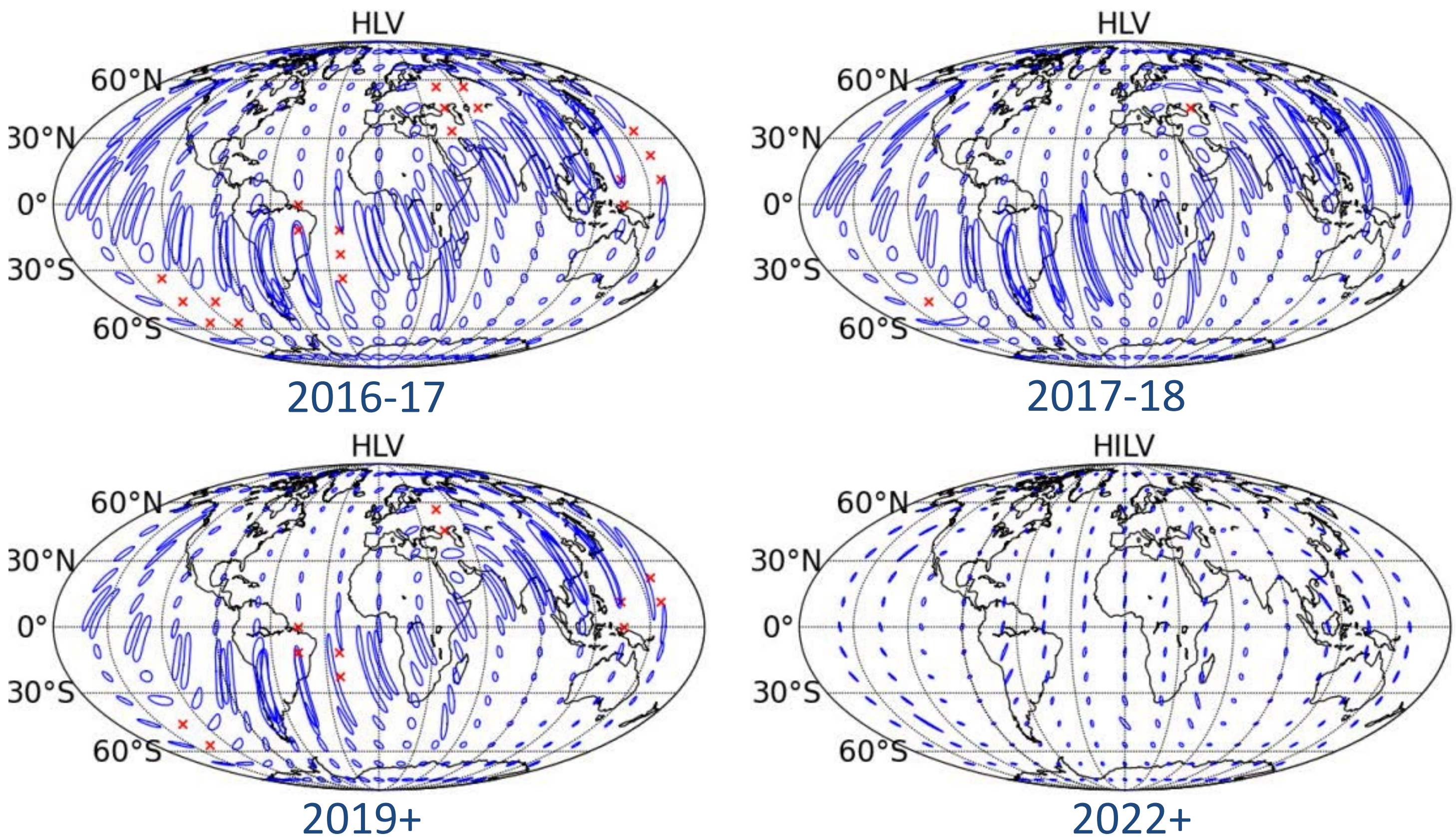
Background excluding contribution from GW150914 (gauge significance of other triggers)

Coincidences between single detector triggers from GW150914 and noise in other detector

Future Localization Prospects

Face-on BNS
@ 80 Mpc

Face-on BNS
@ 160 Mpc

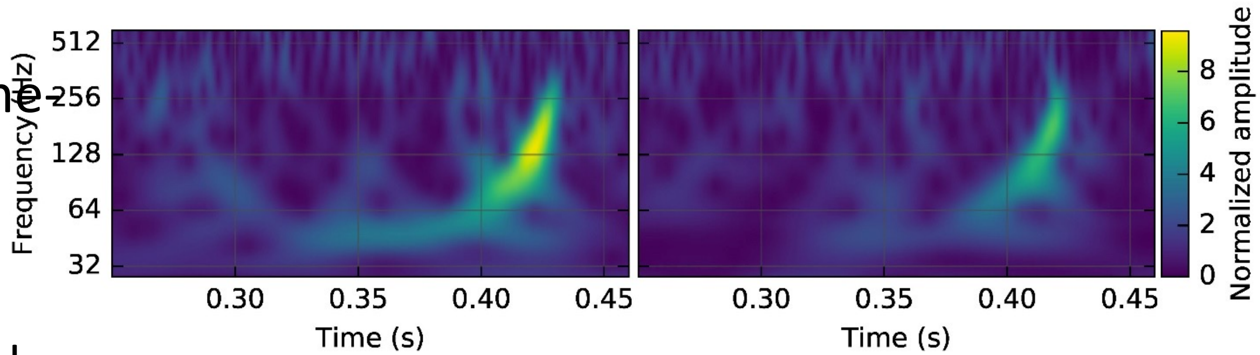


HLV = Hanford-Livingston-Virgo

HILV = Hanford-LIGO India-Livingston-Virgo

Generic Transient Search

- Operates without a specific search model
  - Identifies coincident excess power in time-frequency representations of h(t)
    - Frequency < 1 kHz
    - Duration < a few seconds



- Reconstructs signal waveforms consistent with common GW signal in both detectors using multi-detector maximum likelihood method
- Detection statistic
  - $$\eta_c = \sqrt{\frac{2E_c}{(1 + E_n/E_c)}}$$

E_c: dimensionless coherent signal energy obtained by cross-correlating the two reconstructed waveforms

E_n: dimensionless residual noise energy after reconstructed signal is subtracted from data

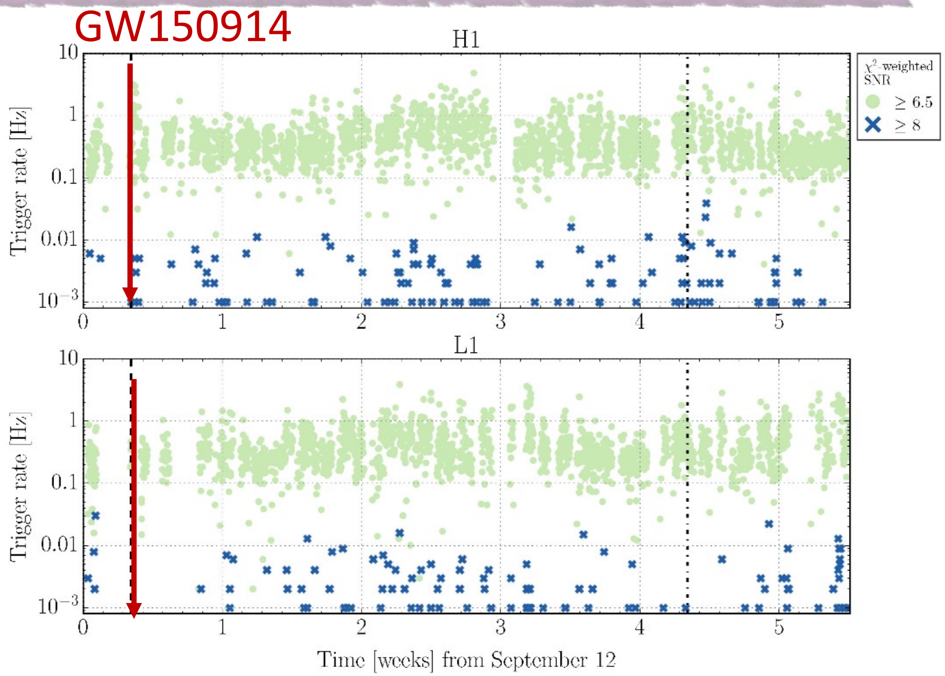
- Signals divided into 3 search classes based on their time-frequency morphology
  - C3 : Events with frequency increasing with time – CBC like

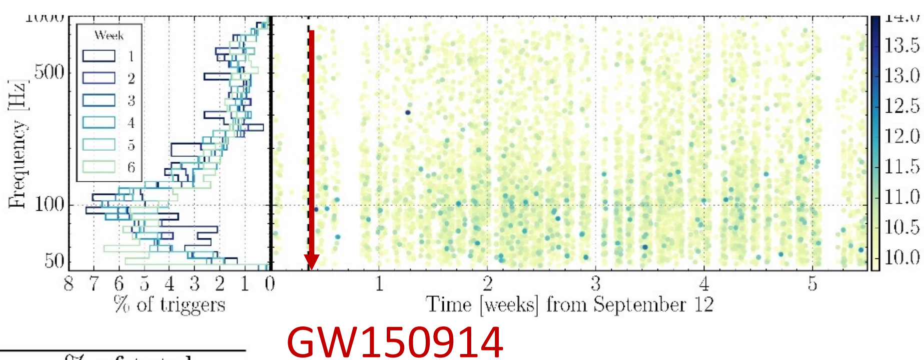
Data quality

- On analyzed period
  - Clean data set
  - Homogeneous background

- Data quality vetoes
  - Identify periods with intrumental or environmental problems
  - Veto those periods

  - GW150914 >> every background event even without DQ vetoes





Hanford			Livingston		
DQ veto category	Total deadtime (s)	% of total coincident time	DQ veto category	Total deadtime (s)	% of total coincident time
1	73446	4.62%	1	1066	0.07%
2	5522	0.35%	2	87	0.01%

Expected BBH Stochastic Background

- GW150914 suggests population of BBH with relatively high mass
- Stochastic GW background from BBH could be higher than expected
  - Incoherent superposition of all merging binaries in Universe
  - Dominated by inspiral phase
- Estimated energy density

$$\Omega_{\mathrm{GW}}(f = 25\,\mathrm{Hz}) = 1.1^{+2.7}_{-0.9} \times 10^{-9}$$

- Statistical uncertainty due to poorly constrained merger rate currently dominates model uncertainties
- Background potentially detectable by Advanced LIGO / Advanced Virgo at projected final sensitivity

$$\Omega_{\mathrm{GW}}(f) = \frac{f}{\rho_c}\frac{d\rho_{\mathrm{GW}}}{df}$$