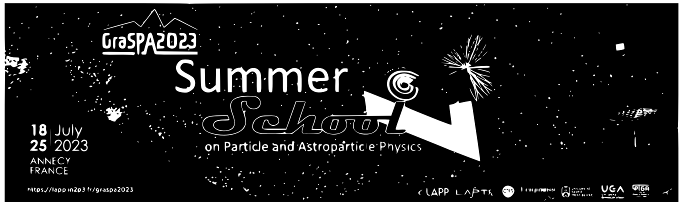
Neutrino Physics: theory

GraSPA Summer School 2023



Asmaa Abada

LPT - IJCLAB

two lectures

- Basics: brief history and basic concepts
- Oscillation phenomena and searches from many fronts
- Properties and Nature
- Theoretical frameworks and (Minimal) New Physics Models

Some references

- C. Giunti, C.W. Kim, “Fundamentals of Neutrino Physics and Astrophysics, Oxford University Press.
- R. N. Mohapatra and P. Pal, “Massive Neutrinos in Physics and Astrophysics, World Scientific
- M. Fukugita, T. Yanagida, “Physics of Neutrinos: and Application to Astrophysics (Theoretical and Mathematical Physics) ", Springer

Part 2

- Why the Standard Model cannot accommodate neutrino data
- Neutrino mass generation mechanisms
- Seesaw mechanisms
- How to distinguish between several possibilities? Effective Approach

$m_\nu \neq 0 \Rightarrow$ New Physics

Standard Model

- ν_L and no ν_R $\Longrightarrow$ No Dirac mass term: $\mathcal{L}_{m_D} = m_D\,(\overline{\nu_L}\nu_R + \overline{\nu_R}\nu_L)$

- No Higgs triplet $\Longrightarrow$ No Majorana mass term: $\mathcal{L}_{m_M} = \frac{1}{2} M \overline{\nu_L^c} \nu_L + h.c.$

Majorana field: $\Psi_\nu = \nu_L + \nu_L^c$ ➞ $\Psi_\nu = \Psi_\nu^c$ ➞ $\overline{\nu_L^c}\nu_L = \nu_L^T C \nu_L, \quad C = i\gamma^2\gamma^0$

- Lepton number symmetry is accidental $\Longrightarrow$ Non-renormalisable operators dim 5, 6 ...

SM $\equiv$ Effective theory of a larger one valid at a scale Λ

⬇

$$\delta\mathcal{L}^{d=5} = c^{d=5}\mathcal{O}^{d=5}, \quad \mathcal{O}^{d=5} = \frac{1}{\Lambda}\left\{(\phi\ell)^{\rm T}(\phi\ell) + h.c.\right\} \xrightarrow{\langle\phi\rangle = v} m_\nu \sim v^2/\Lambda$$

$m_\nu \sim \sqrt{\Delta m^2_{\rm atm}} \sim \sqrt{2\times 10^{-3}{\rm eV}^2} \Rightarrow \Lambda \sim 10^{15}$ GeV (Remarkably near $\Lambda_{\rm GUT}$!)

Beyond the Standard Model

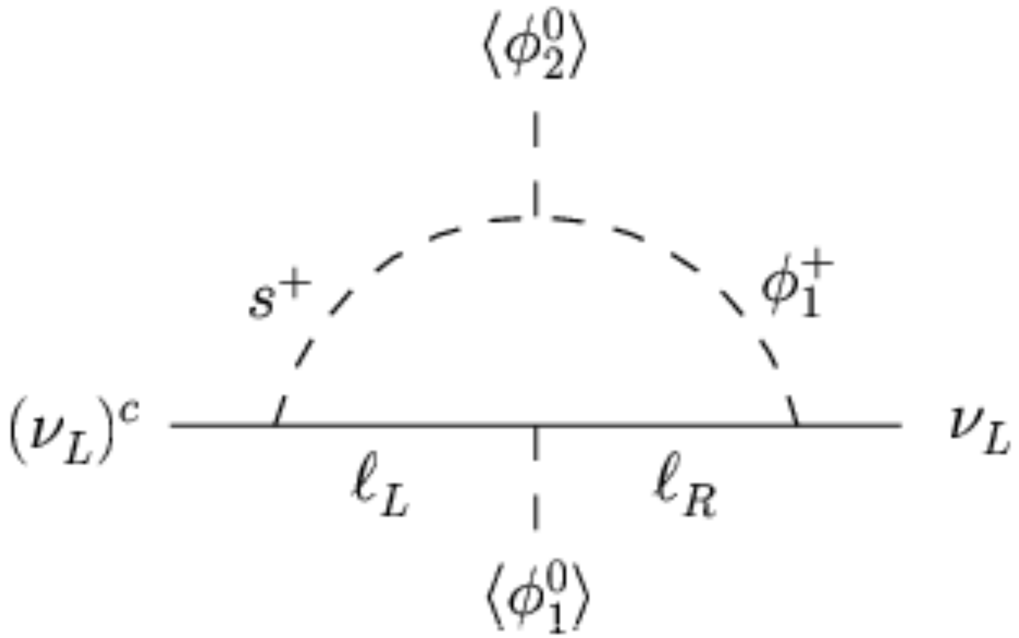
Typically 3 possible ways to generate $m_\nu \neq 0$:

- Seesaw mechanism can be achieved via
  1. type I with RH neutrino exchange
  2. type II with scalar triplet exchange
  3. type III with fermionic triplet exchange
- Radiative corrections ➛ MSSM extended $+\not{R}_p$, Zee model, $\cdots$
- Extra dimensions ➛ alternative to the seesaw

Radiative masses

Models at low energy ➔ radiative masses $\rightarrow m_\nu$, e.g.,

- SUSY with $\not{R}_p$ $(R_p = (-1)^{\mathsf{L+3B+2S}} = +1$ particles or -1 superparticles)
- Extended Higgs sector, like in the Zee Model:



➔ $$m_\nu = \begin{pmatrix} 0 & \alpha & \beta \\ \alpha & 0 & \epsilon \\ \beta & \epsilon & 0 \end{pmatrix},$$

$\epsilon \ll \alpha \sim \beta$

➔ Zee model predictions already in conflict with solar neutrino data ➔ excluded!!!

Neutrino masses at Tree-Level

Example: SM $+\nu_R$

ν_R (massive and Majorana) : gauge singlet ➜ does not break gauge invariance!!

$$L = L_{SM} + Y^\nu \bar{L} \nu_R \phi + M_R \nu_R \nu_R$$

General mass term Dirac+Majorana[a]

($+\nu_L$ with or without Majorana mass[b])

$$\mathcal{L}_M^{(\nu)} = \frac{1}{2} (\bar{\nu}_L \ \overline{\nu^c}_R) \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} + h.c.$$

[a] Minkowski, Ramond, Yanagida, Mohapatra, Senjanovic, Gell Mann, ...

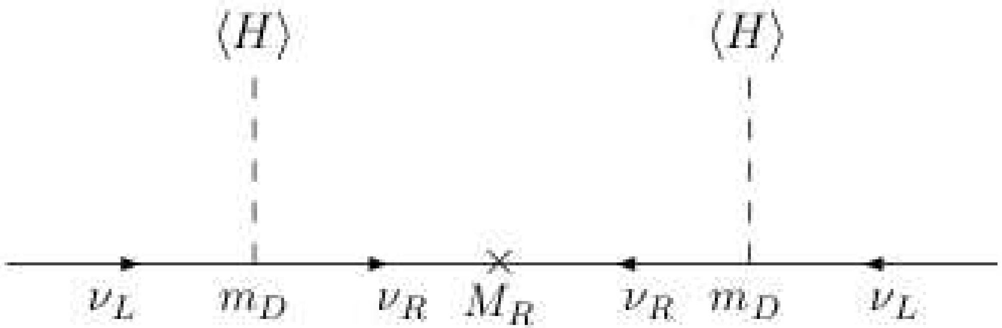
[b] such a term $\overline{\nu_L^c} m_L \nu_L$ ➜ Isospin Triplet

Diagonalisation : $\mathcal{L}_M^{(\nu)} = \frac{1}{2}\left(\bar{L}\ \bar{R}^c\right) \begin{pmatrix} m_L & 0 \\ 0 & M_R \end{pmatrix} \begin{pmatrix} L^c \\ R \end{pmatrix} + h.c.$

mass eigenstates: [in the limit $m_D \ll M_R$]

$$\nu \simeq L + L^c = \nu^c\ , \quad \rightarrow \quad \tilde{m}_L \sim -m_D \frac{1}{M_R} m_D^T$$

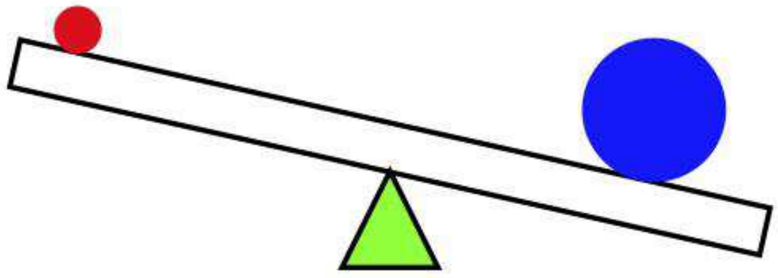
$$N \simeq R + R^c = N^c\ , \quad \rightarrow \quad \tilde{M}_R \sim M_R$$



$$M_R \sim \Lambda$$

One generation case:

$$\tilde{m}_L \sim \frac{m_D^2}{M_R} \ll M_R$$
$$\tilde{M}_R \sim M_R$$



$$\begin{cases} m_D \sim 200\mathrm{GeV} \rightarrow \geq \text{heaviest fermion} \\ M_R \sim 10^{15}\mathrm{GeV} \rightarrow \text{close to GUT} \end{cases}$$

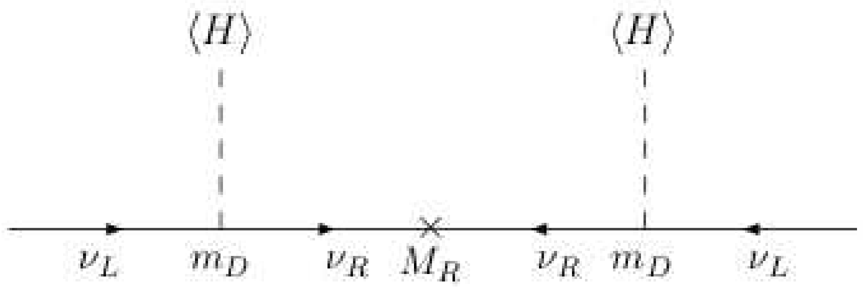
⬇

$$m_\nu \propto \sqrt{\Delta m^2_{\mathrm{atm}}} \sim (10^{-2} - 10^{-1})\mathrm{eV}$$

See-Saw mechanism, SM + ν_R

$$\mathcal{L} = \mathcal{L}_{SM} + \lambda^{\nu}_{Jk} \bar{L}_k \nu_{R_J} H - \frac{1}{2} \bar{\nu}_{R_J} M_{R_J} \nu^c_{R_J} + \lambda_\alpha H^c \bar{e}_{R_\alpha} \ell_\alpha \ , \quad m_D = \lambda^\nu \ v$$

Majorana Eigenstates(3×3) : $\begin{cases} \nu = L + L^c = \nu^c \ , & \rightarrow \ \tilde{\mathrm{m}}_{\mathrm{L}} \sim -m_D \frac{1}{M_R} m_D^T \\ N = R + R^c = N^c , & \rightarrow \ \tilde{M}_R \sim M_R \end{cases}$



$\begin{cases} m_D \sim 200 \ \ \text{GeV} \\ M_R \sim 10^{15} \text{GeV} \end{cases} \rightarrow m_\nu \propto \sqrt{\Delta m^2_{\mathrm{atm}}} \sim (10^{-2} \ - \ 10^{-1})\mathrm{eV} \leftarrow \ \lambda^\nu \sim \mathcal{O}(1)$

$\lambda^\nu \sim \lambda_e \ \leftarrow \ M_R \sim \text{few} \ \ \text{TeV}$

Testable ! low scale seesaw

Sources of CP violation?

General

- For N Dirac fermions, their (unitary) mixing matrix contains:
  ➔ $\frac{N(N-1)}{2}$ mixing angles and $\frac{(N-1)(N-2)}{2}$ CPV phases

- In the case of for N Majorana fermions, N phases reabsorbed in fields redefinition:
  ➔ $\frac{N(N-1)}{2}$ mixing angles and $\frac{N(N-1)}{2}$ CPV phases [N-1 Majorana CPV phases]

Dirac or Majorana

recall that Majorana field: $\Psi_\nu = \nu_L + \nu_L^c$ ➔ $\Psi_\nu = \Psi_\nu^c$ ➔ $\overline{\nu_L^c}\nu_L = \nu_L^T C \nu_L, \quad C = i\gamma^2\gamma^0$

♦ Dirac mass term: $\mathcal{L}_{m_D} = m_D\left(\overline{\nu_L}\nu_R + \bar{\nu}_R\nu_L\right)$

♦ Majorana mass term: $\mathcal{L}_{m_M} = \frac{1}{2}M_{ij}\overline{\nu_{L_i}^c}\nu_{L_j} + h.c. \qquad \overline{\nu_L^c}\nu_L = \nu_L^T C \nu_L, \quad C = i\gamma^2\gamma^0$

➔ How many independent physical states?

Additional CP phases!!

Example: Seesaw case ☞ **Yukawa couplings λ^ν** ← Complex!

$$\lambda^\nu = \frac{1}{v}\sqrt{M_N}\, R\, \sqrt{m_\nu}\, U^\dagger$$ [Casas-Ibarra parametrisation]

$R =$ orthogonal matrix, $U = U_{\rm PMNS}$

(Crucial: $R \neq 1$ in general)

Sources of CP violation both in R and $U_{\rm PMNS}$!

- with 3 RH neutrinos, R is orthogonal 3×3 matrix → 3 complex angles θ_i
- $U = U_{\rm PMNS}$ → one Dirac CPV phase δ + two Majorana CPV phases α_1, α_2

↓

Effect of these phases in EDMs, LNV processes, LFV observables ? ...

and in the leptonic CP asymmetry required for a sucessfull leptogenesis

↓

☞ **BAU from leptogenesis**: $\left(R(\theta_i),\ M_1,\ m_1,\ \theta_{ij},\ \delta,\ \alpha_1,\ \alpha_2\right)$

Observations : Neutrino masses and BAU

☞ **Need dynamical mechanism to generate** Baryon Asymmetry of the Universe η_B:

Primordial abundances of light elements $(D, {}^3He)$ + CMB Anisotropies sensitive to η_B

$$\eta_B^{\mathrm{BBN}} = \frac{n_B}{n_\gamma}\Big|_{\mathrm{BBN}} \approx (5.7 - 6.7) \times 10^{-10}, \quad \eta_B^{\mathrm{CMB}} = \frac{n_B}{n_\gamma}\Big|_{\mathrm{CMB}} \approx (6.10 \pm 0.04) \times 10^{-10}$$

- The larger η_B is, the later ${}^4He$ producing processes stop and smaller the freeze-out of D and ${}^3He$ will be!
- $\frac{\Delta T}{T}$: analysed in Fourier k modes: at recombination time $T \simeq Z = 1000$ changes with baryon gravity $\rightarrow$ enhancement of odd-peaks disparity $\rightarrow$ odd/even peaks $\propto \eta_B$

Baryo(Lepto)genesis requirements

Sakharov '67 [a]

1. Baryon (Lepton) number violation
2. C & CP Violation
3. Out of equilibrium processes

$$n_{B(L)} \neq n_{\bar{B}(\bar{L})}$$

[a] 4th constraint : $B - L$ must be violated if baryogenesis occurs > EW scale

Leptogenesis

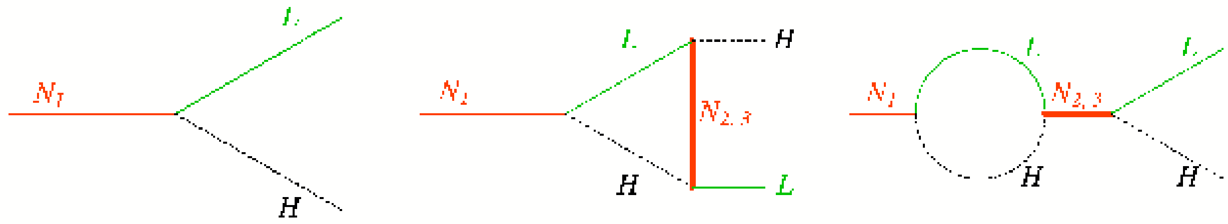
Basic Leptogenesis Mechanism (Seesaw type I)

Fukugita and Yanagida '86

$$\mathcal{L} = \mathcal{L}_{SM} + \lambda^{\nu}_{Jk} \bar{L}_k \nu_{R_J} H - \frac{1}{2} \bar{\nu}_{R_J} M_{R_J} \nu^c_{R_J} + h_\alpha H^c \bar{e}_{R_\alpha} \ell_\alpha$$

1. Out-of-equilibrium and CP violating decay of a heavy particle through an L-violating interaction can produce a lepton asymmetry
2. Lepton asymmetry transformed into *BAU* through sphaleron interactions:
   $B + L$-current is anomalous but $B - L$ and $1/3B - L_\alpha$ currents are not

$$\eta_B = -\left(\frac{24+4n_H}{42+9n_H}\right)\eta_L$$



1) L violation ← N_i Majorana

2) C & CP asymmetry: $\epsilon_1 = \sum_\alpha \epsilon_{\alpha\alpha} = \frac{\sum_\alpha \left(\Gamma(N_1 \to H \ell_\alpha) - \Gamma(\bar{N}_1 \to \bar{H} \bar{\ell_\alpha})\right)}{\sum_\alpha \left(\Gamma(N_1 \to H \ell_\alpha) + \Gamma(\bar{N}_1 \to \bar{H} \bar{\ell_\alpha})\right)}$

CP from the complexity of the λ^ν Yukawa matrix

3) Out-of-equilibrium decay of N_1 ← $\frac{\Gamma_{N_1}}{H(T=M_1)} \ll 1$

$\eta_L \propto K\, \epsilon_1$ is produced

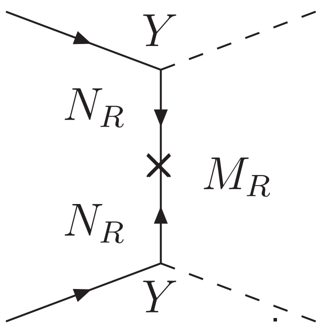
4) Sphalerons act: $\Delta L \to \Delta B$

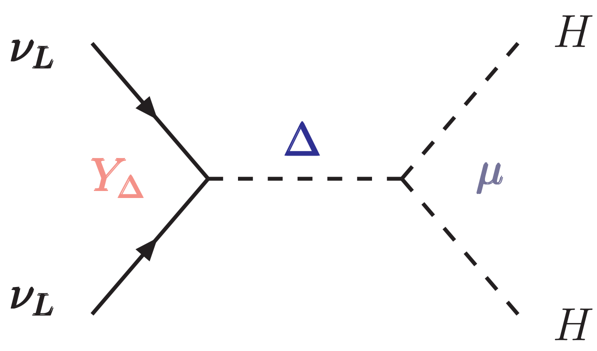
..., Solve Boltzmann Equations

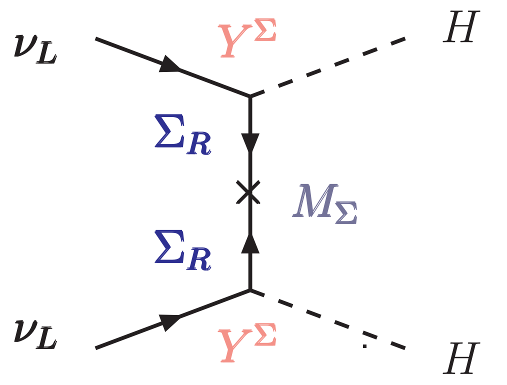
☞ **Neutrino mass generation mechanism at tree-level, other options?**

→ Other Seesaw Mechanisms

Seesaw I, II, III







type I (fermionic singlet)

$$m_\nu = -\frac{1}{2}v^2\, Y_N^T \frac{1}{M_N}\, Y_N$$

type II (scalar triplet)

$$m_\nu = -2v^2 Y_\Delta\, \frac{\mu_\Delta}{M_\Delta^2}$$

type III (fermionic triplet)

$$m_\nu = -\frac{v^2}{2}\, Y_\Sigma^T\, \frac{1}{M_\Sigma}\, Y_\Sigma$$

Minkowski, Gell-Man,

Ramond, Slansky

Yanagida, Glashow

Mohapatra, Senjanovic

Magg, Wetterich,

Nussinov

Mohapatra, Senjanovic

Schechter, Valle

Ma, Sarkar

Ma, Hambye et al.

Bajc, Senjanovic, Lin

A.A., Biggio, Bonnet, Gavela,

Notari, Strumia, Papucci, Dorsner

Fileviez-Perez, Foot, Lew...

☞ **How to disentangle among the different possibilities (BSM)?**

→ Use first the effective approach

☞ Neutrino masses require the addition of new fields

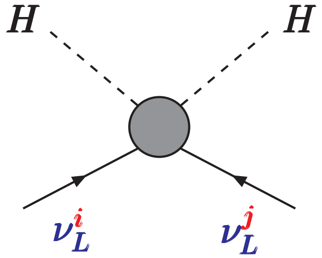
▶ Effects at low energy: effective theorie approach

☞ heavy fermion: $\frac{1}{\not{D} - M} \sim -\frac{1}{M} - \frac{1}{M}\not{D}\frac{1}{M} + \ldots$

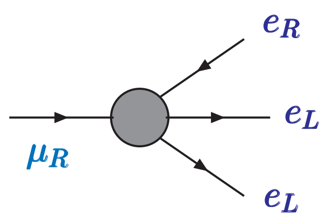
☞ heavy scalar : $\frac{1}{D^2 - M^2} \sim -\frac{1}{M^2} - \frac{D^2}{M^4} + \ldots$

➨ $$\mathcal{L}_{\mathrm{eff}} = \mathcal{L}_{\mathrm{SM}} + \frac{1}{M} c^{d=5} \mathcal{O}^{d=5} + \frac{1}{M^2} c^{d=6} \mathcal{O}^{d=6} + \cdots$$

$\Delta\mathcal{L}^{d\geq 5} = \frac{c^{d=5}}{M} \times$



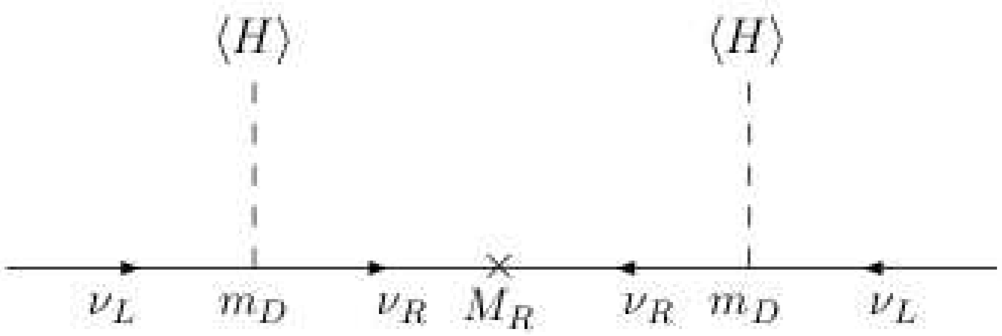
$+ \frac{c^{d=6}_{\mu eee}}{M^2} \times$

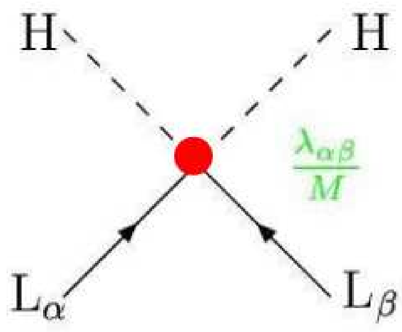


$+ \frac{c^{d=6}_{\ell_i \ell_j \gamma}}{M^2} \ldots$

Dimension 5

$$\delta\mathcal{L}^{d=5} = \frac{1}{2}\, c^{d=5}_{\alpha\beta} \left(\overline{\ell^c_{L\alpha}}\tilde{\phi}^*\right)\left(\tilde{\phi}^\dagger\,\ell_{L\beta}\right) + \text{h.c.}\,,$$





$$m_\nu = \frac{v^2 Y^\dagger Y}{M_R}$$

$$m_\nu = c^{d=5} v^2, \quad c^{d=5} = \frac{Y^\dagger Y}{M_R}$$

- ☞ $Y \sim 1$ ➞ $M_R \sim M_{\text{GUT}}$
- ☞ $Y \sim 10^{-6}$ ➞ $M_R \sim \text{TeV}$

$\mathcal{O}^{d=5}$ Operator violates lepton number L ➞ Majorana neutrinos

- ▶ **$\mathcal{O}^{d=5}$ is common to all models of Majorana neutrinos**

Higher order operators

☞ $\mathcal{O}^{d=5}$ operator: same for all SM extensions incorporating massive MAJORANA

☞ $\mathcal{O}^{d=6}$: 3 "types" of Dimension 6 operators relevant for **cLFV (dipole and 3-body)**

2 lepton-Higgs-photon: $\mathcal{O}^6_{\ell_i \ell_j \gamma} \sim L_i \sigma^{\mu\nu} e_j H F_{\mu\nu}$

$\mathcal{O}^6_{\ell_i \ell_i \gamma}$ ➜ anomalous magnetic or electric moments $(\propto$ Re or Im $\mathcal{C}^6_{\ell_i \ell_i \gamma}/\Lambda^2)$

$\mathcal{O}^6_{\ell_i \ell_j \gamma}$ ➜ **radiative decays** $\ell_i \to \ell_j \gamma$ $(\propto \mathcal{C}^6_{\ell_i \ell_j \gamma}/\Lambda^2)$

4 lepton: $\mathcal{O}^6_{\ell_i \ell_j \ell_k \ell_l} \sim (\ell_i \gamma_\mu P_{L,R} \ell_j)(\ell_k \gamma^\mu P_{L,R} \ell_l) \rightsquigarrow$ **3-body decays** $\ell_i \to \ell_j \ell_k \ell_l$, ...

2 lepton-2 quarks: $\mathcal{O}^6_{\ell_i \ell_j q_k q_l} \sim (\ell_i \gamma_\mu P_{L,R} \ell_j)(q_k \gamma^\mu P_{L,R} q_l)$ **$\mu - e$ in Nuclei, meson decays,**

(Higher order $\mathcal{O}^{d=7,8,..}$: ν (transitional) magnetic moments, NSI, unitarity violation, ...)

☞ **A specific example: Seesaw models**

Dimension 6 operators

Model	Effective Lagrangian $\mathcal{L}_{eff} = c_i \mathcal{O}_i$		
	$c^{d=5}$	$c_i^{d=6}$	$\mathcal{O}_i^{d=6}$
Fermionic Singlet	$Y_N^T \frac{1}{M_N} Y_N$	$\left(Y_N^\dagger \frac{1}{M_N^\dagger} \frac{1}{M_N} Y_N \right)_{\alpha\beta}$	$\left(\overline{\ell_{L\alpha}} \tilde{\phi}\right) i\partial\!\!\!/ \left(\tilde{\phi}^\dagger \ell_{L\beta}\right)$ LFV
Scalar Triplet	$4Y_\Delta \frac{\mu_\Delta}{M_\Delta^2}$	$\frac{1}{M_\Delta^2} Y_{\Delta\alpha\beta} Y^\dagger_{\Delta\gamma\delta}$	$\left(\overline{\widetilde{\ell_{L\alpha}}} \vec{\tau} \ell_{L\beta}\right)\left(\overline{\ell_{L\gamma}} \vec{\tau} \widetilde{\ell_{L\delta}}\right)$ LFV
		$\frac{\lvert\mu_\Delta\rvert^2}{M_\Delta^4}$	$\left(\phi^\dagger \vec{\tau} \tilde{\phi}\right)\left(\overleftarrow{D_\mu} \overrightarrow{D^\mu}\right)\left(\tilde{\phi}^\dagger \vec{\tau} \phi\right)$ Higgs-Gauge
		$-2\left(\lambda_3 + \lambda_5\right) \frac{\lvert\mu_\Delta\rvert^2}{M_\Delta^4}$	$\left(\phi^\dagger \phi\right)^3$ Higgs
Fermionic Triplet	$Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma$	$\left(Y_\Sigma^\dagger \frac{1}{M_\Sigma^\dagger} \frac{1}{M_\Sigma} Y_\Sigma \right)_{\alpha\beta}$	$\left(\overline{\ell_{L\alpha}} \vec{\tau} \tilde{\phi}\right) iD\!\!\!\!/ \left(\tilde{\phi}^\dagger \vec{\tau} \ell_{L\beta}\right)$ LFV

Fermions: if $Y \sim \mathcal{O}(1)$, $c^{d=6} \sim (c^{d=5})^2$ and the smallness m_ν would preclude observable effects from $\mathcal{O}_i^{d=6}$. Not the case for scalars!

Summary

Up to now, we have seen

- ☞ **Indisputable:** νs are massive and mix
- ☞ **SM$_{m_\nu}$:** strong potential for CP violation, for the Majorana nature
- ☞ **The SM must be extended:** extended Higgs sector, New particles, ...
- ☞ **The Seesaw mechanism:** fermionic and/or scalar new fields
  ➙ Seesaw type I, II, III
- ☞ **BAU and Neutrino problem:** common origin? ➙ Leptogenesis
- ☞ **The effective approach:** Dim 5 operator common to all NP extensions
- ☞ **The effective approach:** Dimension 6 and higher may help in disentangling among NP scenarios

Lepton Flavour Violation in the neutral sector :
➙ one may expect LFV in the charged sector too!