Deconstructing the landscape of heterotic flux compactifications

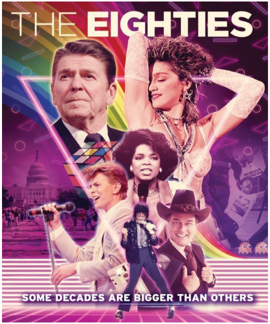
Dan Israël

Landscapia conference, Saclay, December 1 2023

★ D.I., Ilarion Melnikov, Ruben Minasian and Yann Proto, to appear

Introduction

Compactifications of the Heterotic string



- Heterotic strings : good old route to string pheno (minimal supersymmetry and GUTs easily). Assume a 10d space-time of the form :

$$\mathcal{M}_{10} = \mathbb{R}^{1,3} \times \mathcal{M}_6$$

where $\mathcal{M}_6$ is a compact manifold.

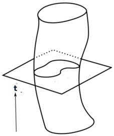
- Well-understood class: Calabi-Yau manifold with Hermitian-Yang-Mills gauge bundle $\mathcal{V} \subset E_8 \times E_8$.

➥ complex structure, complexified Kähler and bundle deformations: well-known moduli problem.

★This problem is alleviated in the case of compactifications with 3-form flux that I will discuss in this talk.

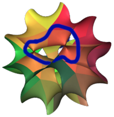
Compactification with 3-forum flux : pecularities

- Generic type of geometric heterotic compactifications preserving minimal SUSY in 4d.
- Internal manifold $\mathcal{M}_6$ non-Kähler hence existence theorems no longer apply:
  - for the hermitian metric on $\mathcal{M}_6$ (Yau theorem)
  - for the HYM connection on $\mathcal{V}$ (Donaldson-Uhlenbeck-Yau theorem)
- Because of the 3-form flux, size of some internal cycles is fixed around the string scale
  ➥ validity of the low-energy supergravity is dubious
- Fortunately, at least in principle, heterotic compactification with fluxes accessible to 2d worldsheet methods (unlike type II flux compactifications)



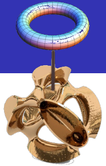
$(0,2)$ SCFT on the string worldsheet

$\longleftrightarrow$



Non-linear sigma model with $\mathcal{M}_6$ target space

Principal torus bundles



- A single class of heterotic flux compactifications is rather well-understood: principal T^2 bundles over $K3$ surfaces
  - ★ Originally motivated from M-theory duals *(Dasgupta, Rajesh, Sethi 99)*
  - ★ Preserve at least 4 supercharges
  - ★ The T^2 fiber is typically string-size, while the $K3$ volume can be large
- For 8 supercharges they may exhaust all heterotic vacua with an underlying non-linear sigma-model description *(Melnikov, Minasian, Theisen 12)*
  - ★ Worldsheet model: IR limit of a *torsional GLSM* *(Adams, Ernebjerg, Lapan 06)*
  - ★ New supersymmetric index and threshold corrections from localization *(DI Sarkis 15/16, Angelantonj, DI, Sarkis 16)*
- Large base limit ➥ eight-dimensional theories compactified on T^2. Suggests an important role played by $O(2,18;\mathbb{Z})$ dualities.

Goals

- Original goal: explore the topological features of heterotic flux compactifications in order to refine and broaden the landscape of such string vacua

- In keeping with the title of the conference, exactly the opposite happened: we realized that a big part of this flux landscape may be trivialized after all.

Outline

1. Geometry and topology of heterotic flux vacua
2. Narain lattices, rational CFTs and dualities
3. Dualities of five-dimensional flux vacua
4. Four-dimensional vacua: towards a general story
5. Conclusions

Geometry and topology of heterotic flux vacua

Supersymmetric heterotic compactifications on $\mathbb{R}^{3,1} \times \mathcal{M}_6$

SUSY conditions : Hull–Strominger system *(Hull 86, Strominger 86)*

- Gravitino variation ➥ covariantly constant spinor on $\mathcal{M}_6$: $(\nabla - \frac{1}{2}H)\epsilon = 0$
  - ★ $SU(3)$ structure (J, Ω) with $J \wedge \Omega = 0$, $J^3 = \frac{3i}{4}\Omega \wedge \bar{\Omega}$
  - ★ integrable complex structure such that J is a $(1,1)$ form and Ω a $(3,0)$-form
- Dilatino variation ➥ $\mathrm{d}(||\Omega|| J^2) = 0$, *i.e.* $\mathcal{M}_6$ is conformally balanced
- Gaugino variation ➥ holomorphic vector bundle V with primitive curvature $\mathcal{F}$, *i.e.* $\mathcal{F} \wedge J^2 = 0$ (zero-slope Hermitian-Yang-Mills)

Bianchi identity

- Non-Kähler-ness of $\mathcal{M}_6$ measured by 3-form flux: $H = i(\bar{\partial} - \partial)J$.
- H obeys a non-trivial Bianchi identity

$$\mathrm{d}H = \frac{\alpha'}{4}\Big(\mathrm{tr}\, R(\nabla^+) \wedge R(\nabla^+) - \mathrm{tr}\mathcal{F} \wedge \mathcal{F}\Big) + \mathcal{O}\big((\alpha')^2\big)$$

➥ Hull connection $\nabla^+ = \nabla + \frac{H}{2}$ ensures that e.o.m. satisfied at order α' *(Ivanov 09)*

- Non-linear in the flux ➥ hard to solve!

Principal T^2 bundles over $K3$ surfaces $\pi : X \to S$

SU(3) structure

(Becker[2], Fu, Tseng, Yau 06)

- Pair of globally defined one-forms Θ^I on X ($\Theta^I = d\theta^I + \mathcal{A}^I$ locally) such that

$$d\Theta^I = \pi^*(F^I), \quad \omega^I = \left[\frac{F^I}{2\pi}\right] \in H^2(S, \mathbb{Z}) \ , \quad I = 1, 2$$

- Moduli of the T^2 fiber $(\tau, \rho = b + i\mathfrak{A})$ ➥ $\Theta = \Theta_1 + \tau\Theta_2$
- $SU(3)$ structure on X, provided $\Omega_S \wedge F = 0$, *i.e.* F has no (0,2) component:

$$\Omega_X = e^{2\Phi}\sqrt{\frac{\alpha'\mathfrak{A}}{\tau_2}}\,\Omega_S \wedge \Theta \ , \quad J_X = e^{2\phi} J_S + i\frac{\alpha'\mathfrak{A}}{2\tau_2}\,\Theta \wedge \bar{\Theta}$$

- Preserve eight supercharges if $F^{(2,0)} = 0$ ➥ *assume from now*

Gauge bundle

- Pullback of a connection $\hat{\mathcal{A}}$ of a HYM bundle V on S and "Wilson lines" $\mathbf{a}_I$

$$\mathcal{A} = \pi^*(\hat{\mathcal{A}}) + \mathbf{a}_I \Theta^I$$

- Consider in the following Abelian bundles (structure group G_V in the Cartan torus of $E_8 \times E_8$)

➥ first Chern class defines a lattice vector $\mathbf{W} \in (\Gamma_8 + \Gamma_8) \otimes H^2(S, \mathbb{Z})$

H-flux, Bianchi identity and flux quantization

- H-flux follows from supersymmetry

$$H = i(\bar\partial - \partial)J_X = \underbrace{iJ_S \wedge (\bar\partial - \partial)e^{2\Phi}}_{H_h\ (horizontal)} - \underbrace{\mathcal{G}_{IJ}F^I}_{H_{v,I}\ (vertical)} \wedge \Theta^J \ , \quad \mathcal{G}_{IJ} = \frac{\alpha' \mathfrak{A}}{\tau_2}\begin{pmatrix} 1 & \tau_1 \\ \tau_1 & |\tau|^2 \end{pmatrix}$$

- The Bianchi identity at order α' gives *(Melnikov, Minasian, Sethi 14)*

$$-\mathcal{G}_{IJ}F^I \wedge F^J = \frac{\alpha'}{4}\left(\mathrm{tr}\, R_+^2 - \mathrm{tr}\, \mathcal{F}^2\right) + \mathcal{O}(\alpha'^2)$$

Flux quantization conditions *(Melnikov, Minasian, Theisen 12)*

- Decomposition of the B-field $B = B_h + B_{v,I} \wedge \Theta^I + \frac{\alpha'}{2} b\epsilon_{IJ}\Theta^I \wedge \Theta^J$

➥ $B'_{v,I} = B_{v,I} + \frac{\alpha'}{4}\mathrm{Tr}(a_I \mathcal{A})$ transforms as the connection of a line bundle

$$\nu_I = \left[\frac{dB'_{v,I}}{2i\pi}\right] = \left(\frac{1}{\alpha'}\mathcal{G}_{IJ} - b\epsilon_{IJ} + \frac{1}{2}\mathbf{a}_I \cdot \mathbf{a}_J\right)\omega^J + \mathbf{a}_I \cdot \mathbf{W} \in H^2(S,\mathbb{Z})$$

- Topology of the bundle and quantization condition summarized in the vector

$$v = \begin{pmatrix} \omega^I \\ \nu_I \\ \mathbf{W} \end{pmatrix} \in \Gamma_{2,2+16} \otimes_{\mathbb{Z}} H^2(S,\mathbb{Z}) \quad , \qquad -\tfrac{1}{2} v \bullet v = 24 \quad (Bianchi)$$

Narain lattices, rational CFTs and dualities

Moduli quantization and rational CFTs

- There is a deep (although mysterious) relationship between the quantization conditions and rational CFTS. Consider the Narain lattice

$$p \in \Gamma_{2,2+16} = \Gamma_{2,2} + \Gamma_8 + \Gamma_8 \,, \quad p = w^I e_I + n_I e^{*I} + \mathbf{\Lambda} \,, \quad e_I \cdot e^{*J} = \delta_I{}^J \,, \quad \eta = \begin{pmatrix} 0 & \mathbb{I}_2 & 0 \\ \mathbb{I}_2 & 0 & 0 \\ 0 & 0 & \mathbb{I}_{16} \end{pmatrix}$$

- In terms of the moduli, the right T^2 momenta p_R given by

$$p_{R,I} = \Big(e_I - (\tfrac{1}{\alpha'}\mathcal{G}_{IJ} - b\epsilon_{IJ} + \tfrac{1}{2}\mathbf{a}_I \cdot \mathbf{a}_J) e^{*J} + \mathbf{a}_I \Big) \cdot p = n_I - (\tfrac{1}{\alpha'}\mathcal{G}_{IJ} - b\epsilon_{IJ} + \tfrac{1}{2}\mathbf{a}_I \cdot \mathbf{a}_J) w^J + \mathbf{a}_I \cdot \mathbf{\Lambda}$$

- Generators of left-moving chiral algebra obtained from non-trivial solutions of

$$n_I = (\tfrac{1}{\alpha'}\mathcal{G}_{IJ} - b\epsilon_{IJ} + \tfrac{1}{2}\mathbf{a}_I \cdot \mathbf{a}_J) w^J + \mathbf{a}_I \cdot \mathbf{\Lambda} \in \mathbb{Z}$$

- No Wilson lines ($\mathbf{a}_I = 0$): the T^2 is a rational CFT *(Gukov, Vafa 02)*
- Correspond precisely to the quantization conditions from the fluxes! *(DI 13)*
- The quantized vector v giving the topology of the flux compactification could be understood as an element of the left lattice of a (partially)-rational CFT.

Narain dualities

- Lattices automorphisms of the Narain lattice $\Gamma_{2,18}$ are associated with dualities mapping toroidal CFTs to isomorphic ones, among which
  - ★ Factorized dualities: $\varphi_{f,1}: \ p \mapsto n^1 e_1 + w_1 e^{*1} + w^2 e_2 + n_2 e^{*2} + \mathbf{\Lambda}$
  - ★ B-field shifts $\varphi_b[\alpha]: \ p \mapsto w^I e_I + (n_I + \alpha \epsilon_{IJ} w^J) e^{*I} + \mathbf{\Lambda}$, $\alpha \in \mathbb{Z}$
  - ★ Wilson line shifts $\varphi_w[\mathbf{k}_I]: \ p \mapsto w^I e_I + \left(n_I + \mathbf{k}_I \cdot \mathbf{\Lambda} - \frac{1}{2}\mathbf{k}_I \cdot \mathbf{k}_J w^J\right) e^{*I} + \mathbf{\Lambda} - \mathbf{k}_I w^I$ with $\mathbf{k}_I \in \Gamma_8 + \Gamma_8$
- Accompanied by transformations of moduli and phase factors in their action on vertex operators

➥ for example factorized duality for an S^1 gives

$$(R, \mathbf{a}) \mapsto \left(\frac{R}{R + \mathbf{a}^2/2}, \frac{-\mathbf{a}}{R + \mathbf{a}^2/2}\right) , \quad \mathcal{V}_{nw} \mapsto e^{inw} \mathcal{V}_{wn}$$

From Narain to torus bundles

- Large base limit ➥ decompactification to a 8d theory which has manifestly $O(2,18;\mathbb{Z})$ duality symmetry
- Topology change under Buscher T-duality with non-trivial fibration and H-flux *(Bouwkneg, Evslin and Mathai 2004)*
- Generalization to Narain dualities of heterotic flux compactifications that may change the topology *(Evslin, Minasian 08)*
- Worldsheet derivation using the torsional GLSM approach *(DI 13)*

  ➥ not only a symmetry of the infrared theory (NLSM on flux backgrounds) but of its UV completion as a $K3$ gauged linear sigma models coupled to axion fields.
- New supersymmetric index of flux backgrounds can be expressed in a manifestly duality-invariant form.

Dualities of five-dimensional flux vacua

A case of study: trivial gauge bundle

- Consider a single circle bundle over a $K3$ surface
  ➥ unique class $\omega = h\omega_p$, with $h \in \mathbb{Z}$ and ω_p primitive in $H^2(S,\mathbb{Z}) = \Gamma_{3,19}$.
- We have the unique quantization condition

$$\nu = (R^2 + \mathbf{a}^2/2)\omega \in H^2(S,\mathbb{Z})$$

- Solved for some $\ell \in \mathbb{Z}$

$$\nu = \ell\omega_p \ , \quad h(r^2 + \mathbf{a}^2/2) = \ell \ \leftrightarrow \ v = \begin{pmatrix} h\omega_p \\ \ell\omega_p \\ 0_{16} \end{pmatrix}$$

- The Bianchi identity takes the form $-\frac{1}{2} v \bullet v = h\ell(-\omega_p \cdot \omega_p) = 24$
- Radius is quantized as $(r^2 + \mathbf{a}^2/2) = \ell/h$
- This space is not simply connected: by a direct computation, $\pi_1(X) = \mathbb{Z}_h$

Dualities

Factorized duality g_f (ordinary T-duality)

$$v = \begin{pmatrix} h\omega_p \\ \ell\omega_p \\ \mathbb{O}_{16} \end{pmatrix} \mapsto v' = g_f v = \begin{pmatrix} \ell\omega_p \\ h\omega_p \\ \mathbb{O}_{16} \end{pmatrix} , \quad (R, \mathbf{a}) \mapsto \left(\frac{kR}{\ell}, -\frac{k\mathbf{a}}{\ell}\right)$$

★This space Y has $\pi_1(Y) = \mathbb{Z}_\ell$ ➥ topology change under the T-duality action

Unwinding duality (simply-connected)

$$v = \begin{pmatrix} \omega_p \\ \ell\omega_p \\ \mathbb{O}_{16} \end{pmatrix} \mapsto v' = g_f g_w[\mathbf{k}] v = \begin{pmatrix} (\ell - \frac{\mathbf{k}^2}{2})\omega_p \\ \omega_p \\ -\mathbf{k}\omega_p \end{pmatrix} , \ (R, \mathbf{a}) \mapsto \left(\frac{R}{2\ell + \mathbf{a}\cdot\mathbf{k}}, -\frac{\mathbf{a}+\mathbf{k}}{2\ell + \mathbf{a}\cdot\mathbf{k}}\right)$$

- By choosing $\mathbf{k} \in \Gamma_8 + \Gamma_8$ such that $\mathbf{k}^2 = 2\ell$ (always possible): dual description of the non-Kähler heterotic flux geometry as a direct product $K3 \times S^1$
- The S^1 principal bundle is replaced by a line bundle (together with a shift of the Wilson line).

Remarks on non-simply connected models

- Simply connected model with $\tilde{\ell} = \ell h$: consider the $\mathbb{Z}_h$ shift symmetry along S^1 acting on the vertex operators as

$$g_h \circ \mathcal{V}_{nw\mathbf{\Lambda}} = \mathcal{V}_{nw\mathbf{\Lambda}} e^{2i\pi n/h}$$

➥ The corresponding orbifold gives the background

$$v = \begin{pmatrix} h\omega_p \\ \ell\omega_p \\ \mathbb{O}_{16} \end{pmatrix}$$

- In the "unwound" description of the simply connected background, this would correspond to $g'_h = g_f g_w[\mathbf{k}] g_h (g_f g_w[\mathbf{k}])^{-1}$, that acts on the vertex operators in a non-geometric fashion:

$$g'_h \circ \mathcal{V}_{nw\mathbf{\Lambda}} = e^{2i\pi w/h} e^{-2i\pi \mathbf{k}\cdot\mathbf{\Lambda}/h} \mathcal{V}_{nw\mathbf{\Lambda}}$$

- Likewise, one can construct CHL-like models from non-simply connected flux backgrounds. Their unwound dual would be non-geometric, having a non-trivial winding action

➥ the flux background is sometimes the simpler description!

Four-dimensional vacua: towards a general story

General unwinding

- 4d flux vacua with Abelian gauge bundle characterized by a vector $v = \begin{pmatrix} \omega^I \\ \nu_I \\ \mathbf{W} \end{pmatrix}$
- Quantization conditions (assuming $\mathbf{a}_I \cdot \mathbf{W} = 0$)

$$\nu_I = \Big(\tfrac{1}{\alpha'} \mathcal{G}_{IJ} + \tfrac{1}{2} \mathbf{a}_I \cdot \mathbf{a}_J - b\epsilon_{IJ} \Big) \omega^J \in H^2(S, \mathbb{Z})$$

- Simply connected geometries obtained from a pair of primitive 2-forms $\omega_p^{1,2}$

$$v = \begin{pmatrix} \omega_p^1 \\ \omega_p^2 \\ \ell_{11}\omega_p^1 + \ell_{12}\omega_p^2 \\ \ell_{21}\omega_p^1 + \ell_{22}\omega_p^2 \\ \mathbf{W} \end{pmatrix} \mapsto v' = \begin{pmatrix} (\ell_{11} - \frac{1}{2}\mathbf{k}_1 \cdot \mathbf{k}_1)\omega_{\mathrm{p}}^1 + (\ell_{12} + \alpha - \mathbf{k}_1 \cdot \mathbf{k}_2)\omega_{\mathrm{p}}^2 \\ (\ell_{21} - \alpha)\omega_{\mathrm{p}}^1 + (\ell_{22} - \frac{1}{2}\mathbf{k}_2 \cdot \mathbf{k}_2)\omega_{\mathrm{p}}^2 \\ \omega_{\mathrm{p}}^1 \\ \omega_{\mathrm{p}}^2 \\ \mathbf{W} - \mathbf{k}_1\omega_{\mathrm{p}}^1 - \mathbf{k}_2\omega_{\mathrm{p}}^2 \end{pmatrix}$$

unwinding is possible by choosing $\mathbf{k}_1 \cdot \mathbf{k}_1 = 2\ell_{1,1}$, *etc.*

- Remark: for general gauge bundle, reach a point in the bundle/K3 moduli space allowing to (smoothly) abelianize?

Non-simply connected 4d models

- More general T^2 bundles with classes

$$\omega^1 = m_1 \omega_p^1\,,\ \omega^2 = m_1 m_2 \omega_p^2\,,\ m_{1,2} \in \mathbb{N}$$

- One has then $\pi_1(X) = \mathbb{Z}_{m_1} \times \mathbb{Z}_{m_1 m_2}$
- These non-simply connected flux backgrounds can be obtained from freely-acting shift orbifolds of the simply connected models
- Likewise, the shift orbifold on in the unwound dual description involves winding shifts ➥ non-geometric.
- A large class of CHL-like flux vacua and their unwound duals also exists in the T^2 case

Conclusions

Summary

- We have found compelling evidence that $\mathcal{N} = 2$ heterotic flux backgrounds are T-dual to ordinary $K3 \times T^2$ with line bundles and certain Wilson lines configurations
- An important caveat is that, for non-simply connected flux geometries, their unwound duals involve a non-geometric orbifold action on the Narain lattice
- Along the way we have found a rich landscape of CHL-like backgrounds, taking advantage of the non-simply-connected nature of most of these flux vacua
- The case of $\mathcal{N} = 1$, not discussed by lack of time, is very different as the $(2, 0)$ component of the curvature cannot be removed by T-duality
- Other $\mathcal{N} = 1$ construction from quotients à la Borcea-Voisin of $\mathcal{N} = 2$ models *(Becker Tseng Yau 09, DI Proto 23)*
  ➥ the unwinding map may shed light on non-Kähler resolutions in flux vacua (in progress)

Thank you for your attention.