String Universality and (Mixed) Anomalies

Based on:
2008.09117 with F. Apruzzi and L. Lin
2008.10605 and 2106.07654 with M. Cvetic, L. Lin, and H. Y. Zhang (+ follow-ups)
and
2107.14227 and 2302.00007 with A. Debray, J. J. Heckman, and M. Montero
2212.04503 with P.-K. Oehlmann and T. Schimannek

Deconstructing the String Landscape - December 1, 2023

LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

Markus Dierigl

Higher-form symmetries

[Gaiotto, Kapustin, Seiberg, Willett ’14]

Consider a gravitational theory in **d dimensions** with a **U(1) p-form gauge field** C_p

(Abundant in string theory, e.g., RR-fields, B-field, …)

$$\frac{1}{2g^2}\int F_{p+1} \wedge {}^*F_{p+1}$$

Typically has **global higher-form symmetries:**

- **Electric p-form symmetry** $C_p \to C_p + \Lambda_p$

$q = d - p - 2$

- **Magnetic (d-p-2)-form symmetry** $\tilde{C}_q \to \tilde{C}_q + \tilde{\Lambda}_q$ with $\tilde{F}_{q+1} \sim {}^*F_{p+1}$

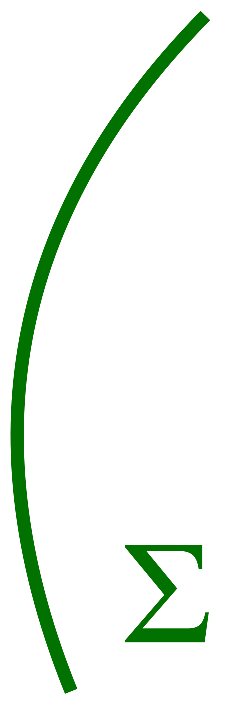
But **global symmetries are not allowed in quantum gravity**

See e.g. [Banks, Dixon ’88; Banks, Seiberg ’11; Harlow, Ooguri ’18]

➡ get rid of them!

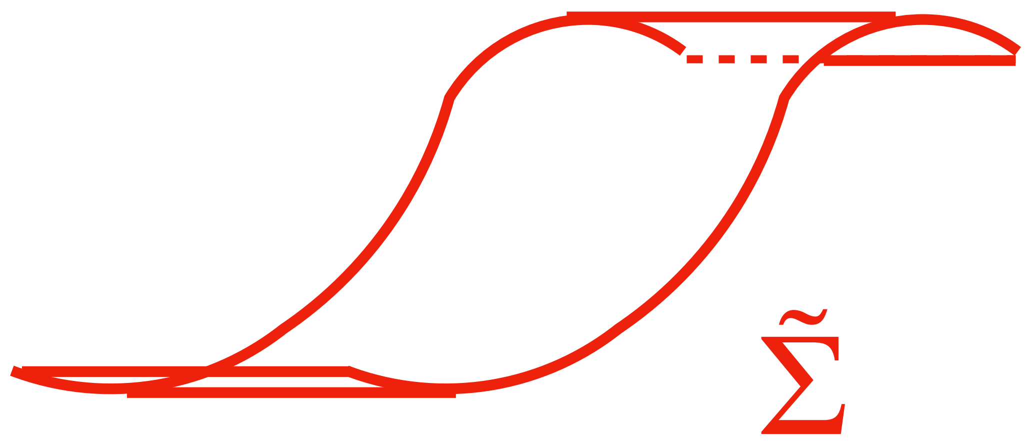
Breaking of higher-form symmetries

Include **dynamical charged objects** that break the higher-form symmetries:



$$\int_\Sigma C_p$$

p-dimensional



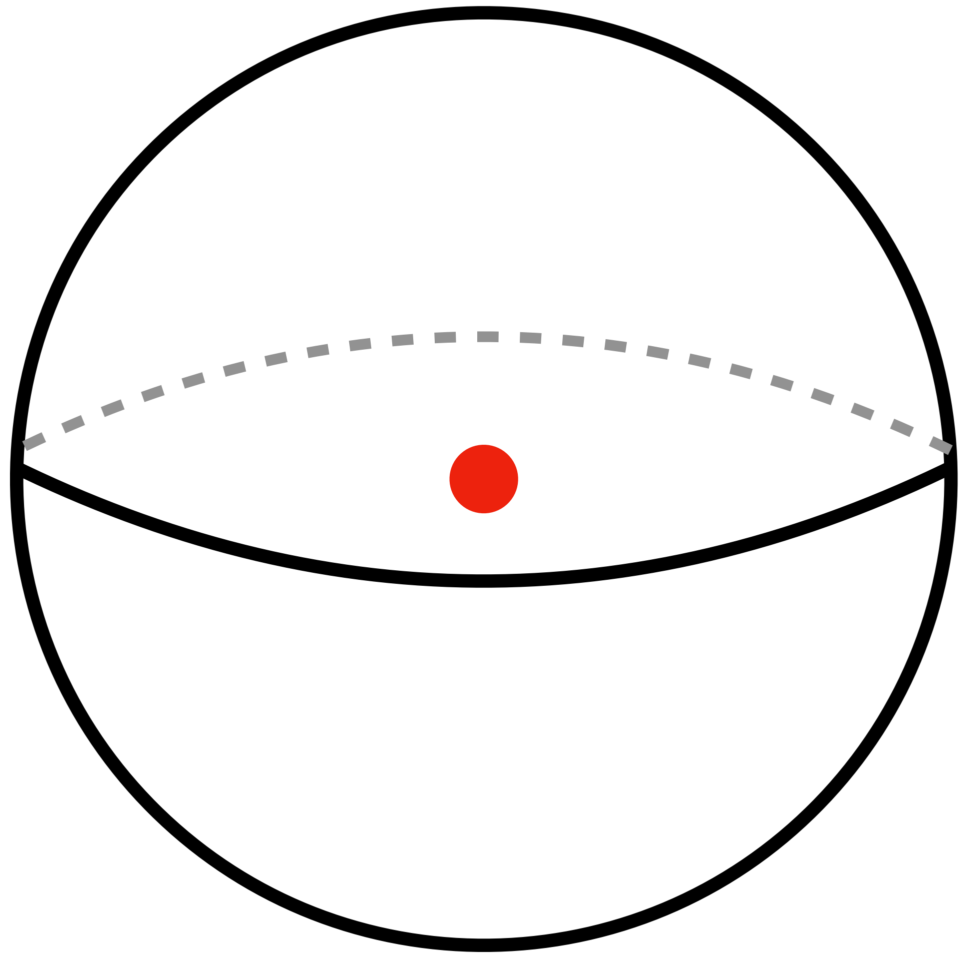
$$\int_{\tilde{\Sigma}} \tilde{C}_q$$

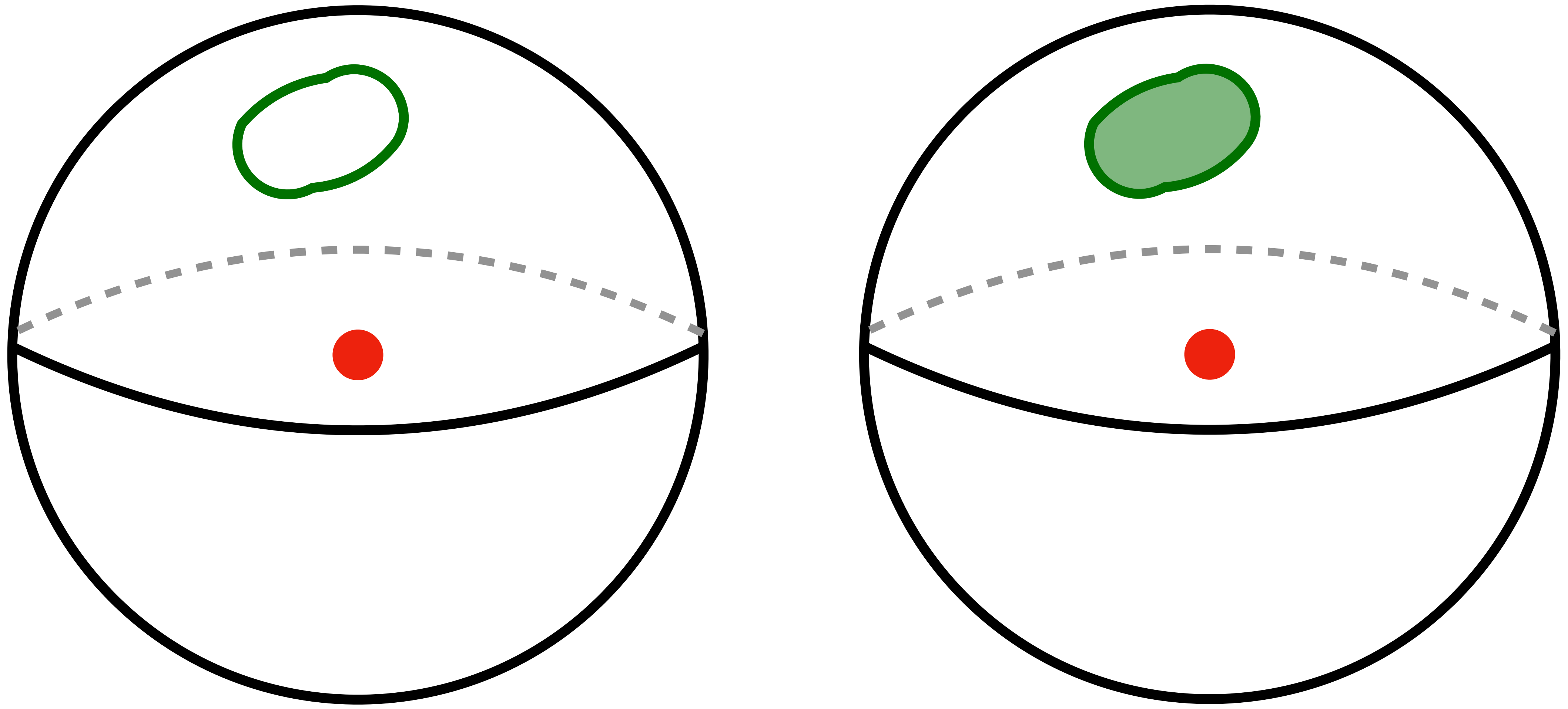
(d-p-2)-dimensional

➡ Break **everything**: no remnant discrete $\mathbb{Z}_N$ (if charges are in $N\mathbb{Z}$)

Dirac quantization

But also **don't break too much!** ➔ **Inconsistencies**





take a minimal magnetic object

$$\frac{1}{2\pi}\int_{S^{p+1}} F_{p+1} = 1$$

include electrically charged object

$$Q\int_{\Sigma_p \subset S^{p+1}} C_p = \int_{H_{p+1}} F_{p+1}$$

Dirac quantization

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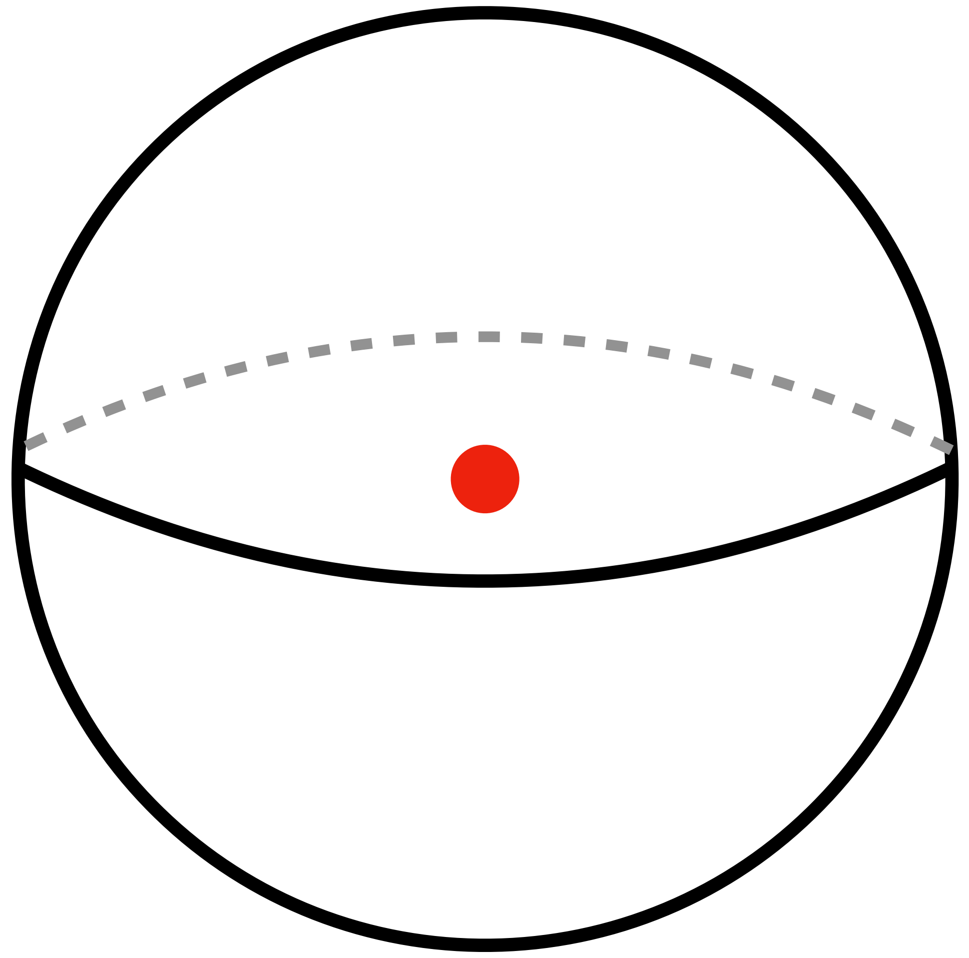
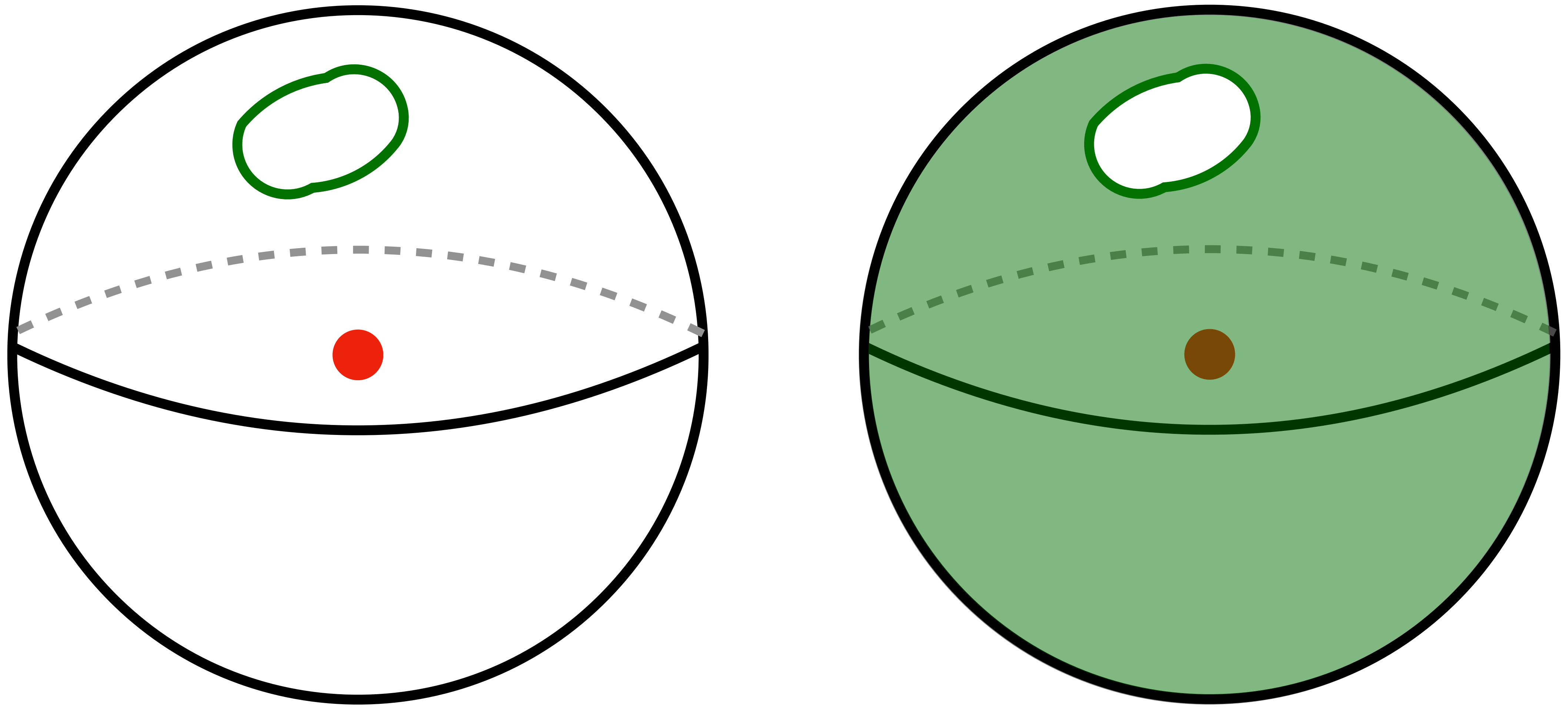


take a minimal magnetic object

$$\frac{1}{2\pi}\int_{S^{p+1}} F_{p+1} = 1$$

Such that $e^{iQ\int_\Sigma C_p}$ **single-valued**!

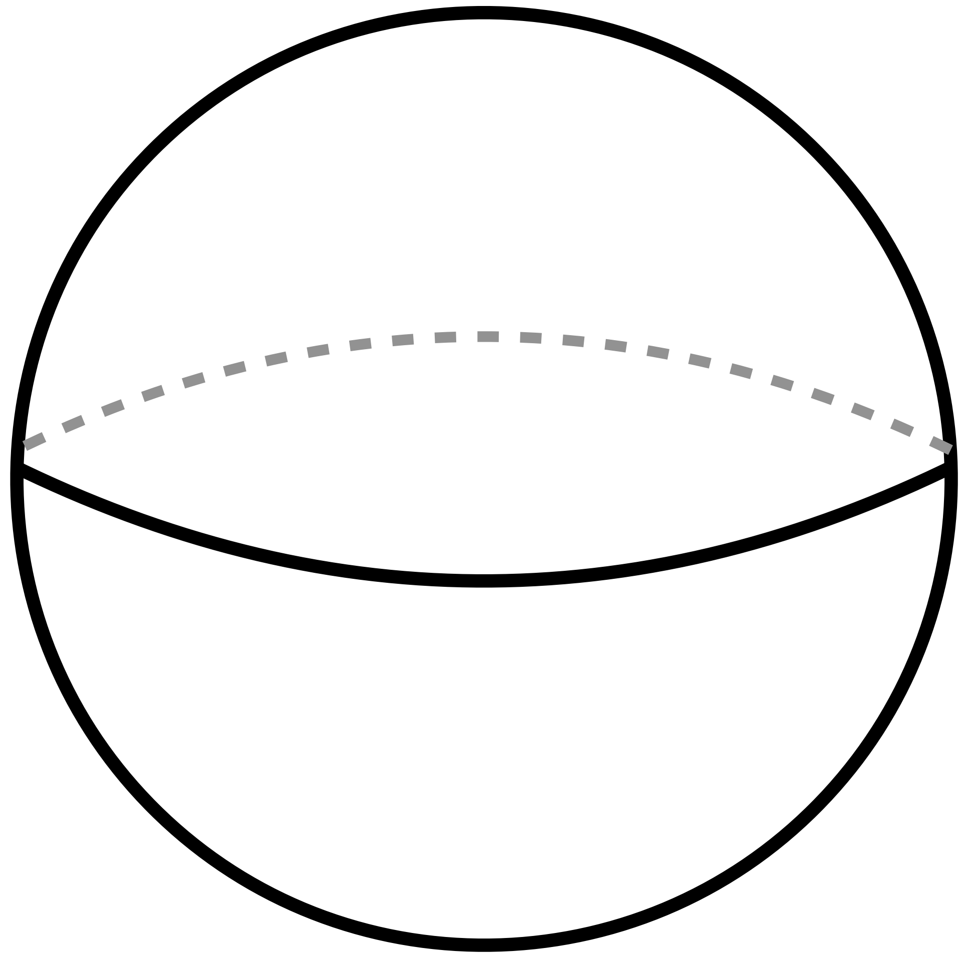
$$Q \in \mathbb{Z}$$



include electrically charged object

$$Q\int_{\Sigma_p \subset S^{p+1}} C_p = \int_{H'_{p+1}} F_{p+1}$$

Dirac quantization and large gauge trafos



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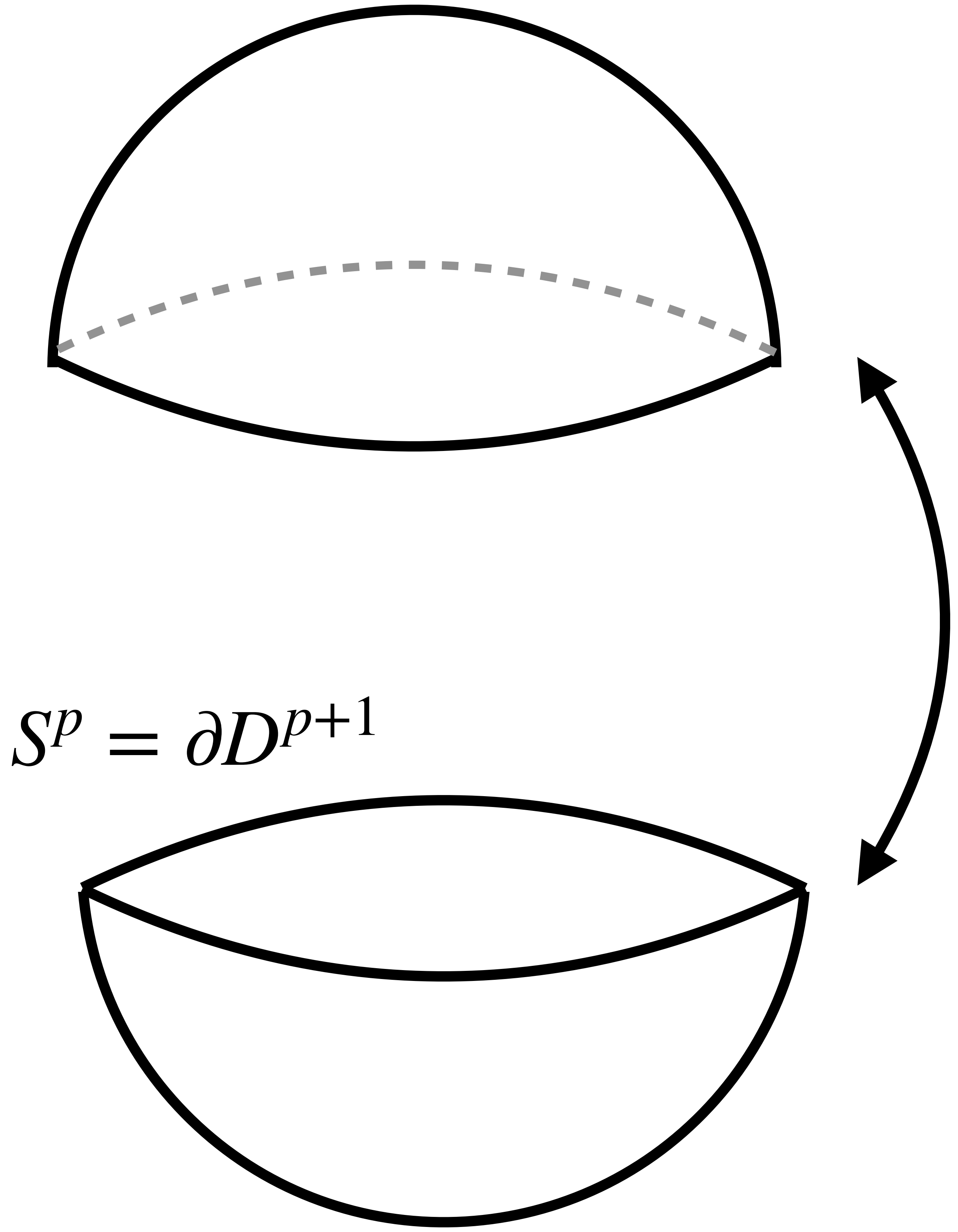
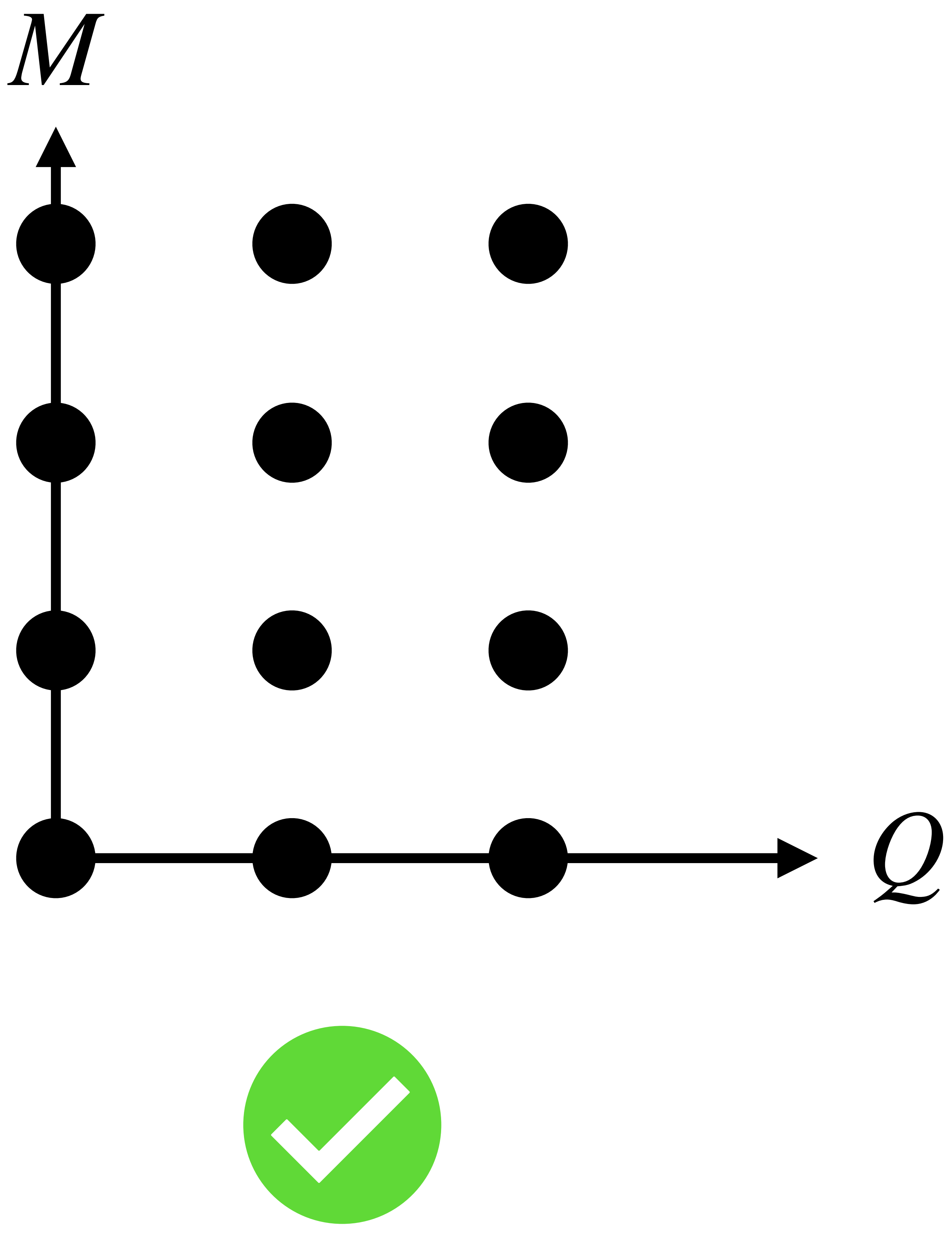


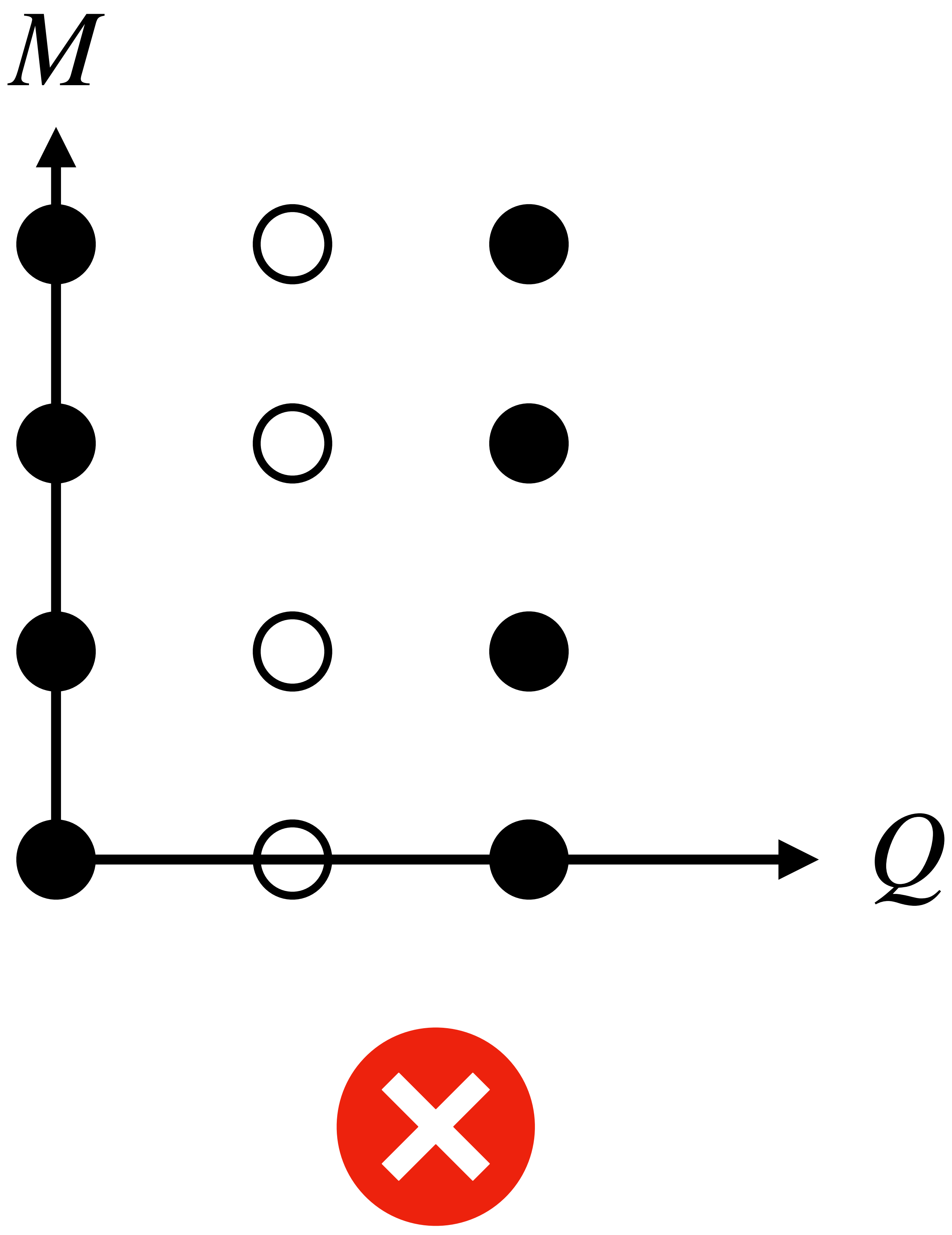
glue with large
gauge transformation:

$$\frac{1}{2\pi}\int_{S_p} \Lambda_p = 1$$

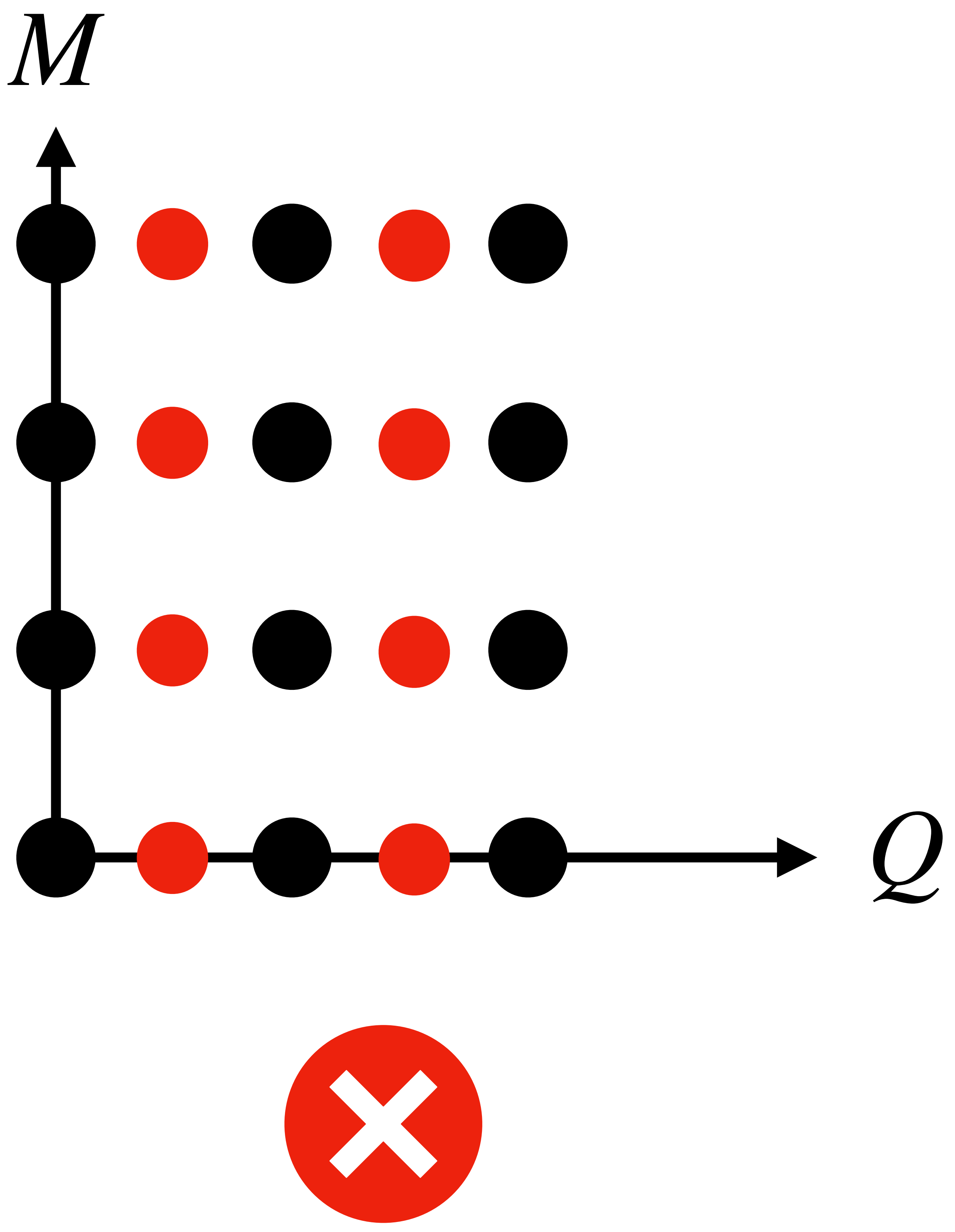
$$e^{iQ\int_{S^p} C_p} = e^{iQ\int_{S^p} C_p + \Lambda_p}$$ **invariant**

Breaking correctly





left-over global symmetry



violation of Dirac quantization, gauge invariance

Other charges

Sometimes **other sectors** can **carry charges:**

- **Gauge sectors** with 2-form fieldstrength F $\qquad \int_{X_d} C_p \wedge \alpha \, \mathrm{tr}(F^k)$
- **Gravitational sectors** with curvature 2-form R $\qquad \int_{X_d} C_p \wedge \beta \, \mathrm{tr}(R^k)$
- **Other k-form symmetries** $\qquad \int_{X_d} C_p \wedge \gamma \, dC_k$

➡ All of them **need to satisfy quantization!**

Global data

The quantization of these background is sensitive to **'global data'** of the theory:

e.g. $\in \frac{1}{3}\mathbb{Z}$ for SU(3)/$\mathbb{Z}_3$

- **Gauge sector:** depends on **gauge group** $\int \mathrm{tr}(F^2) \in \mathbb{Z}$ **or** $\in \mathbb{Q}$

simply-connected e.g. SU(3)

- **Gravitational sector:** depends on **tangential structure** $\frac{1}{48}\int p_1 \in \mathbb{Z}$ **or** $\mathbb{Q}$

for spin manifolds

➞ **Restrictions from Dirac quantization**

Global data

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for spin manifolds

➔ **Restrictions from Dirac quantization**

Gauge group vs. gauge algebra

e.g. [Aharony, Seiberg, Tachikawa ’13]

Pure non-Abelian gauge theories have a center 1-form symmetry (e.g. $\mathbb{Z}_N$ for SU(N))

so in **quantum gravity** we need to:

- **Break it**; dynamical states transforming in the center (e.g. $\mathbf{N}$ of SU(N))
- **Gauge it**; forbidding representations, fractional instantons

Example: SU(3) x SU(3) has representation $(\mathbf{3},\mathbf{1})$

(SU(3) x SU(3))/$\mathbb{Z}_3$ does not but has $(\mathbf{3},\mathbf{3})$ or $(\mathbf{3},\overline{\mathbf{3}})$

↙ has fractional instantons

(After gauging include magnetic states)

Examples

- **Periodic scalars in 4d** $\int a\,\mathrm{tr}(F^2)$ [Cordova, Freed, Lam, Seiberg '18],…, [Reece '23], [Choi, Forslund, Lam, Shao '23] [Cordova, Hong, Wang '23] ← actual constraints
- **2-forms in 6d** $\int \alpha^i \Omega_{ij} B^j \wedge \mathrm{tr}(F_i^2)$ e.g. [Apruzzi, MD, Lin '20], [Braun, Larfors, Oehlmann '21]
- **4-forms in 8d** $\int C_4 \wedge \alpha_i\,\mathrm{tr}(F_i^2)$ [Cvetic, MD, Lin, Zhang '20]

Focus on this, since in $\mathcal{N} = 1$ SUGRA:

single 4-form coupling to **all** gauge sectors

8d Supergravity

[Cvetic, MD, Lin, Zhang '20]

$$\Delta \int C_4 \wedge \alpha_i \mathrm{tr}(F_i^2) = \int \Lambda_4 \wedge \alpha_i \mathrm{tr}(F_i^2)$$

$\rightarrow$ $\boxed{\sum_i \alpha_i \in \mathbb{Z}}$ otherwise **anomaly**

Include restrictions on gauge rank r :

- $r \leq 18$**:** from **anomaly inflow on dual strings**
  [Kim, Tarazi, Vafa '19], [Kim, Shiu, Vafa '19], [Lee, Weigand '19]
- $(r-2)$ divisible by 8: from **anomaly inflow on I-folds**
  [Montero, Vafa '20] see also [Lao, Minasian '23]

8d Supergravity

[Cvetic, MD, Lin, Zhang '20]

For non-Abelian gauge group:

$$G = \frac{\bigotimes_i G_i}{\mathbb{Z}_n}$$

The α_i are composed of:

$$\alpha_i = q_i k_i^2 m_i$$

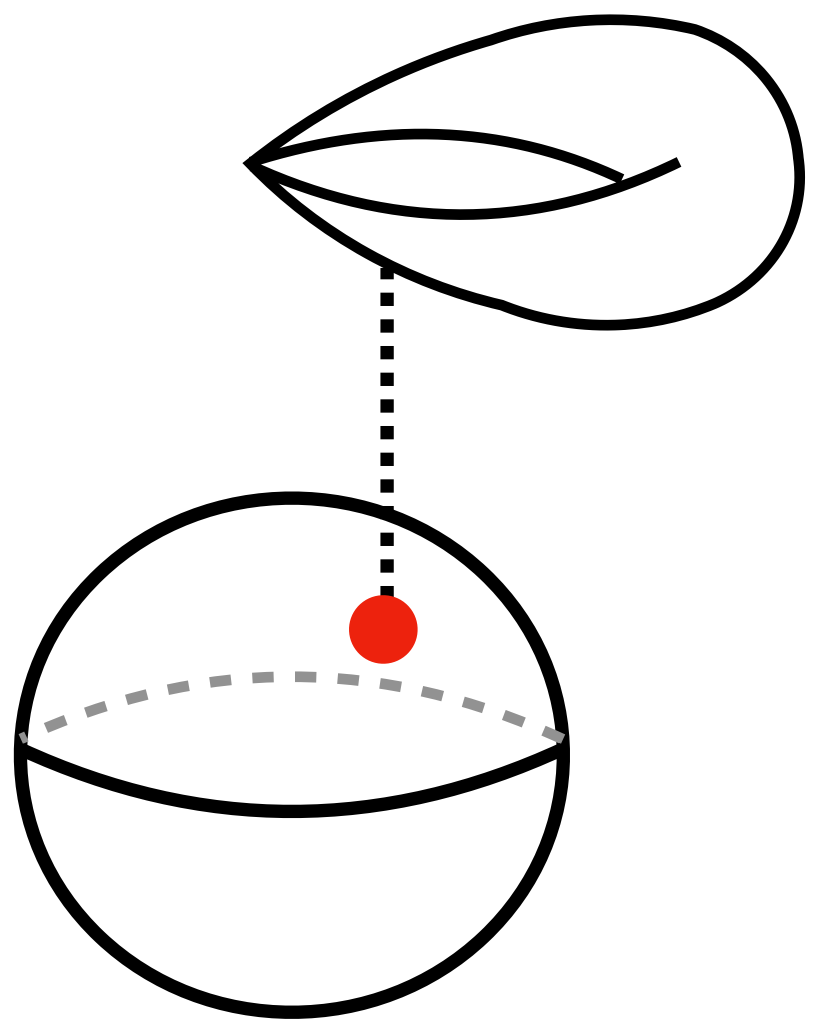
- q_i determined by fractional instantons
- k_i embedding coefficients
- m_i levels (has to be 1 for $r = 18$)

Anomaly hints for (internal) geometry

Example: All G_i are SU(N_i) and $r = 18$: $$\sum_i \frac{N_i - 1}{2N_i} k_i^2 \in \mathbb{Z}$$

→ **strong bottom-up restrictions (without using string theory)**

Many such models can be derived **in string theory:**



F-theory on elliptically-fibered K3

Indeed there is a **geometrical condition:** on torsional sections

[Miranda, Persson '89]

$$\sum_i \frac{N_i - 1}{2N_i} k_i^2 \in \mathbb{Z}$$

see also [Hamada, Vafa '21]

Other string theory constructions

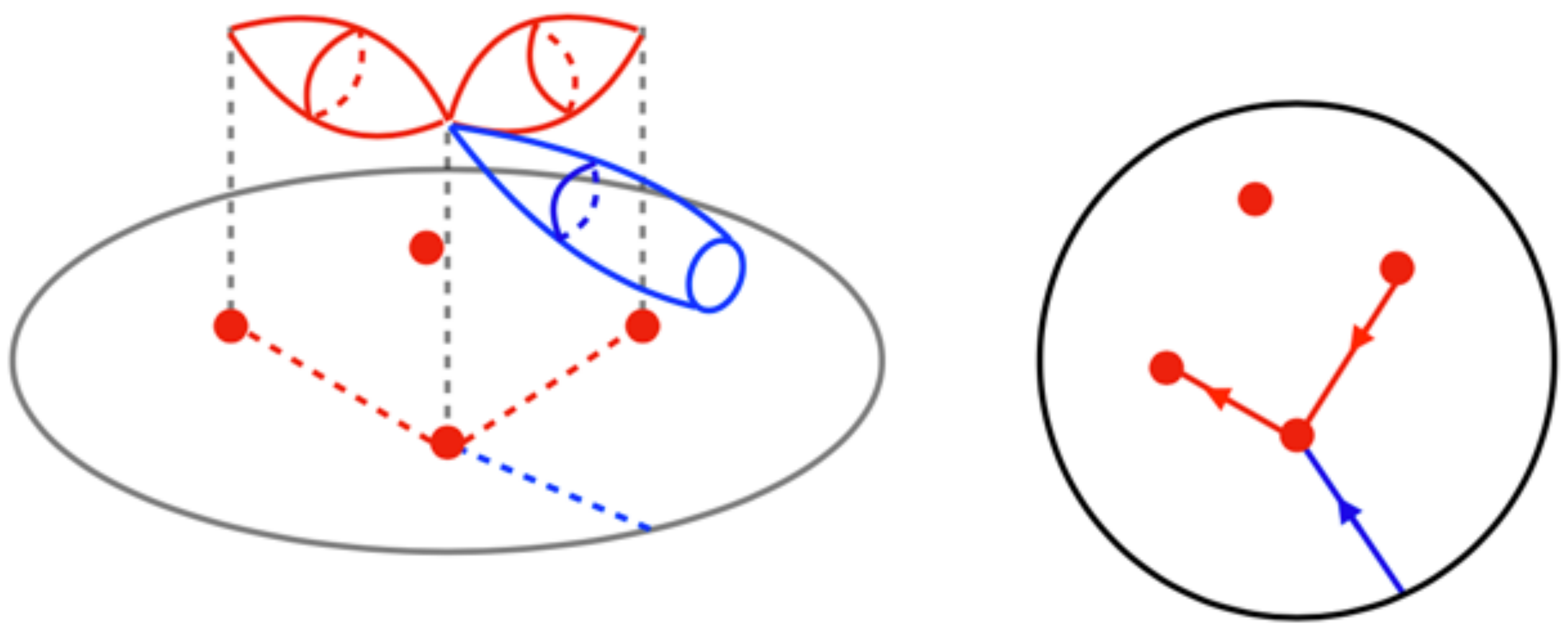
Same results from heterotic on T^2 with gauge fields

[Font, Fraiman, Grana, Nunez, Parra de Freitas ’20 + ’21]

$$\Lambda_{\text{root}} \hookrightarrow \Lambda_{\text{string}}$$

String junctions in type IIB and their M-theory analogs

[Cvetic, MD, Lin, Zhang ’21, ’22]



➜ **strong top-down restrictions (using string theory)**

Lesson 1:

➛ **Anomalies very powerful to constrain models and approach string universality**

- **Hints on geometry of string realization**
- **SUSY not needed for computations** (but powerful in restricting spectrum)
- Include **all (generalized) symmetries**

The flip side of the coin

Maybe string theory is not as unique as we think?!?

Think of a theory that has an anomaly parametrized by $\mathbb{Z}_k$
[Dai, Freed ’94], [Witten ’15], [Yonekura ’16], [Montero, Garcia-Etxebarria ’18]
which we want to cancel:

- **Embedding in larger (discrete) group**
  [Ibanez, Ross ’91], [Montero, Garcia-Etxebarria ’18]
- **Topological Green-Schwarz**
  [Garcia-Etxebarria, Hayashi, Ohmori, Tachikawa, Yonekura ’17]
- **Discrete modification of Bianchi identities**
  [Debray, MD, Heckman, Montero ’21], [MD, Oehlmann, Schimannek ’22]

No modification of local dynamics, hard to test, not unique!

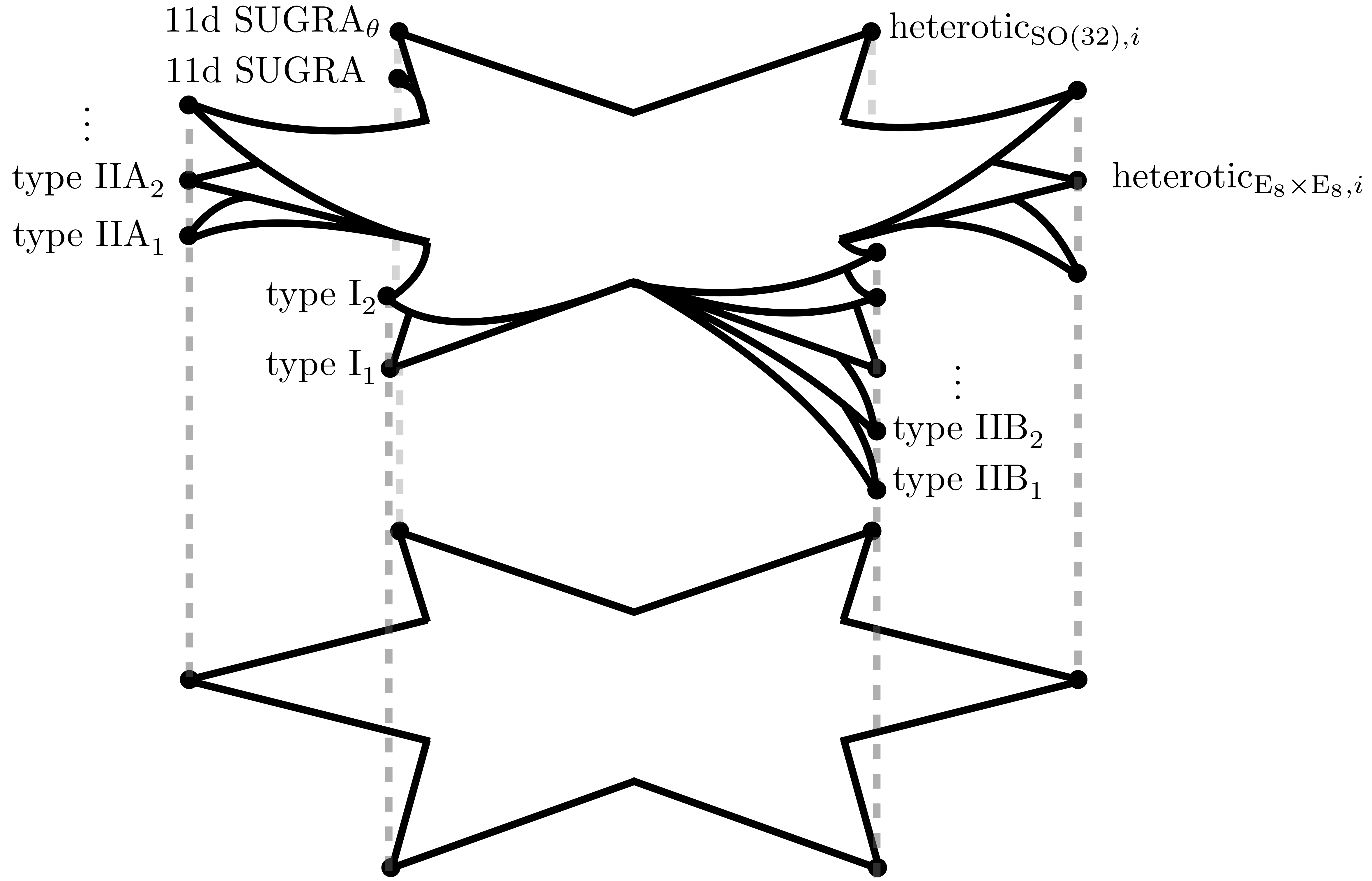
Example I: Duality in type IIB

[Debray, MD, Heckman, Montero '21 + '23]

Duality of type IIB is anomalous!

Can be cancelled by:

- Discrete modification of Bianchi identity of F_5
- Topological Green-Schwarz of BF type with $\mathbb{Z}_3$ higher-form fields



Example II: Discrete gauge theories in 6d

[MD, Oehlmann, Schimannek ’22]

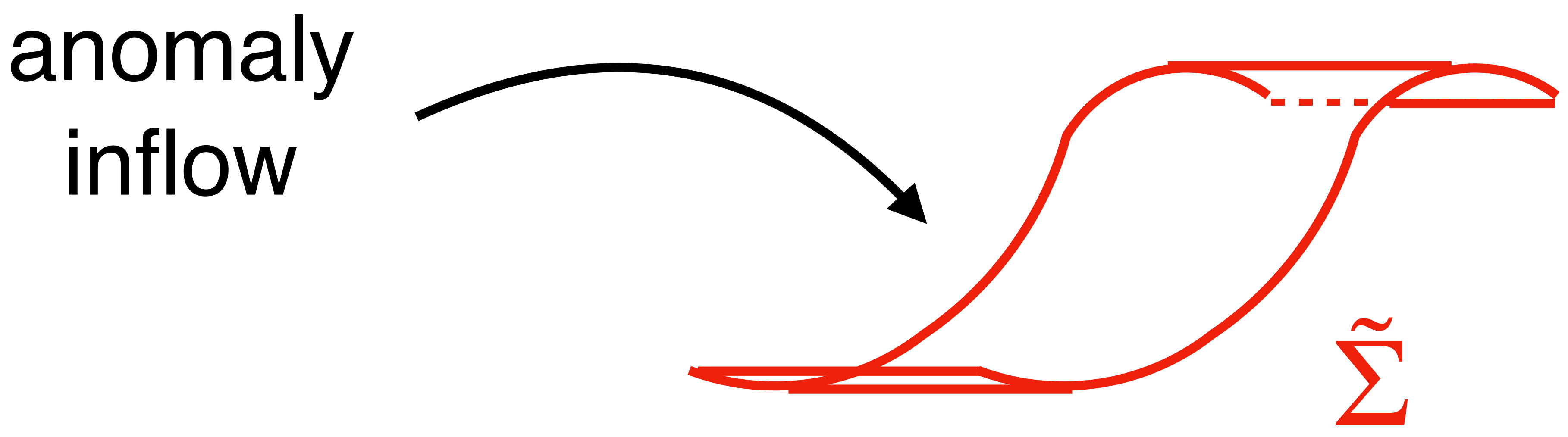
6d SUGRA with $\mathbb{Z}_3$ gauge group:

- Number of tensors
- Number of neutral hypers
- Number of charged hypers

	T	Neutral	Charged
Model 1	0	78	195
Model 2	0	78	195

different F-theory geometry

Same model? — No! — Anomaly cancellation differs!



can be tested by **twisted-twined genera of topological string**

see also [Basile, Leone ’23]

Lesson 2:

➛ **Anomalies and their cancellation can lead to discrete landscape?**

- **Forbidden? Why?**
- **Discrete Swampland constraints?**
- **Allowed? How many UV completions?**

Summary

➛ Anomalies very powerful to constrain models and approach string universality

➛ Anomalies and their cancellation can lead to discrete landscape?

Swamplandia in “Kloster Seeon” close to Munich: May 26 - May 30, 2024

(Back-to-back with “Strings & Geometry” in Hamburg, the week before)