A dS from higher dimensions

G. Bruno De Luca - Stanford University

Based on
2104.13380 with Silverstein and Torroba
2104.12773 with Tomasiello
2109.11560, 2212.02511, 2306.05456 with De Ponti, Mondino, Tomasiello
\+ work in progress

Deconstructing the String Landscape, IPhT CEA/Saclay
Nov 30, 2023

Two challenging problems

- Top down prescribes extra dimensions

$$S = m_D^{D-2} \int \sqrt{-g_D} R_D + \dots$$

1. How to describe 4-dimensional physics? [separation of scales]

   - Swampland conjectures
   - Proposed compactifications
   - For AdS, CFT constraints

[Ooguri, Vafa, '07,
Lüst,Palti, Vafa, '19, ...]

[Kachru, Kallosh, Linde, Trivedi '03, DeWolfe, Giryates, Kachru, Taylor, '05,
Polchinksi, Silverstein '09,
Petrini, Solard, Van Riet '13,
Cribiori, Junghans, Van Hemelryck, Van Riet, Wrase '21, …,
Carrasco, Coudarchet, Marchesano, Prieto, '23
Farakos, Morittu, '23]

[Polchinksi, Silverstein '09,
Conlon, Quevedo '18, Alday, Perlmutter '19,
Apers, Montero, Van Riet, Wrase '22,...]

2. How to describe *realistic* 4-dimensional physics? [dS or more general acc. expansion]

   - Conjectures, explicit constructions, consistency conditions, …

- This talk: how to get constraints from the equations of motion and a way to evade them

Physics of gravity compactifications

- At low energies $S = m_D^{D-2} \int \sqrt{-g_D} R_D +$ matter

$$ds_D^2 = e^{\frac{2}{D-2} f(y)} (g_d^\Lambda(x) + g_n(y))$$

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$$ds_D^2 = e^{\frac{2}{D-2}f(y)}(g_d^\Lambda(x) + g_n(y))$$

$$\left[\hat{T}^{(d)} \equiv m_D^{2-D}g_d^{\mu\nu}\left(T_{\mu\nu} - \frac{T}{D-2}g_{\mu\nu}\right), \tilde{T}_{mn} \equiv m_D^{2-D}\left(T_{mn} - \frac{T^{(d)}}{d}g_{mn}\right)\right]$$

- Equations of motion

$$\frac{1}{D-2}e^{-f}\Delta(e^f) = \frac{1}{d}\hat{T}^{(d)} - \Lambda$$

$$R_{mn} - \nabla_m\nabla_n f + \frac{1}{D-2}\nabla_m f\nabla_n f = \Lambda g_{mn} + \tilde{T}_{mn}$$

- Spectrum of spin 2 fluctuations given by

$$g_{4\,\mu\nu}(x) = g_{\mu\nu}^{(\Lambda)}(x) + \sum_i h_{\mu\nu}^i(x)\psi_i(y)$$

$$\Delta_f\psi_i \equiv \Delta\psi_i - \nabla f\cdot\nabla\psi_i = m_i^2\psi_i$$

[Csaki, Erich, Hollowood, Shirman, ‘00, Bachas,Estes, ‘11]

- What can we prove in general, that applies to any solution?

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smooth internal space, no boundaries:

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$\leqslant 0$ for classical sources and no O-planes

[Gibbons '84, de Wit, Smit, Hari Dass '87 , Maldacena-Nuñez, '00]

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- Synthetic Ricci curvature in effective dimension $N = 2-d$
- Studied in the Optimal Transport literature, controls the spectrum of Δ_f

[Sturm '06, Lott, Villani '07, Villani '09, Ambrosio, Gigli, Savaré 14 , …]

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- Useful to prove theorems on the spectrum of $\Delta_f \equiv \Delta - \nabla f \cdot \nabla$ and bound $m^2_{KK}/|\Lambda|$

$$\text{Ric}^{2-d,f}_{mn} = \Lambda g_{mn} + \tilde{T}_{mn} \quad \geqslant -|\Lambda|$$

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[GBDL, Tomasiello, ‘21]

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For fluxes, scalar fields, scalar potentials, D-dim cosmological constants and localized sources with positive tension

Reduced Energy Condition

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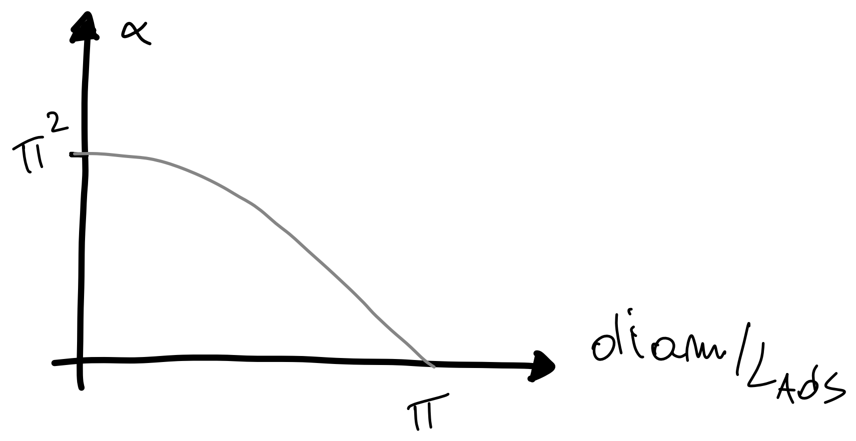
- Useful to prove theorems on the spectrum of $\Delta_f \equiv \Delta - \nabla f \cdot \nabla$ and bound $m^2_{KK}/|\Lambda|$
- If $m_0 = 0$ $\left[\Longrightarrow \;\; \psi_0 = \text{const.} \right.$ [GBDL, De Ponti, Mondino, Tomasiello, ‘23] $\left. \right]$

[GBDL, De Ponti, Mondino, Tomasiello, ‘22]

$$\frac{m_1^2}{|\Lambda|} \geqslant \alpha(\mathrm{diam}/L_{AdS}) \frac{L^2_{AdS}}{\mathrm{diam}^2}$$

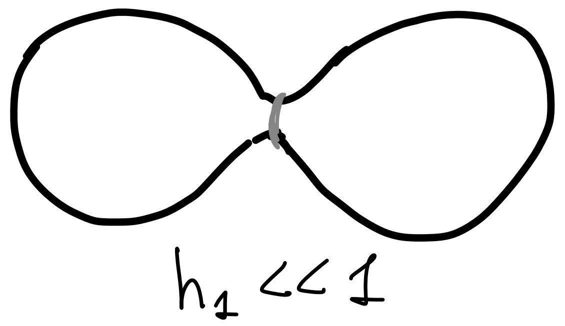


$\Longrightarrow$ Separation of scales achieved if diam $\ll L_{AdS}$

Intuitive, but now rigorous even with D-brane singularities and warping

$$m_1^2 \geqslant \frac{1}{4} h_1^2$$

Cheeger constant [Cheeger '69]



$\Longrightarrow$ Rigorous even in presence of O-planes

Can be used to check sep. of scale in explicit proposed examples. e.g. in

[**D**eWolfe, **G**iryavets, **K**achru, **T**aylor, ‘05 Acharya, Benini, Valandro ‘06, Junghans ‘20, Marchesano, Palti, Quirant, Tomasiello ‘20]

$$h_1^2 \sim N^{-1/2} \,, |\Lambda| \sim N^{-3/2}$$

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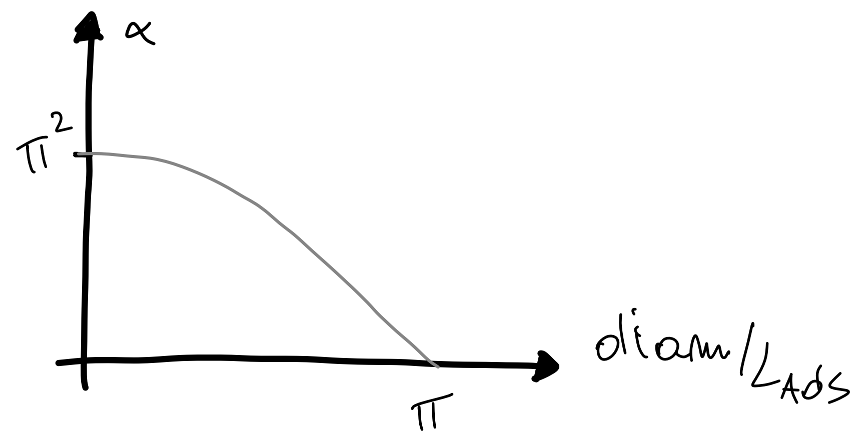

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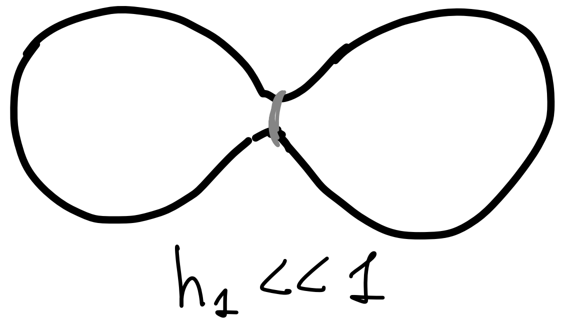

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$$h_1^2 \sim N^{-1/2}\,, |\Lambda| \sim N^{-3/2}$$

- Also upper bounds, e.g:

$$m_k^2 \leqslant a(n)\max\left\{\text{sup}(\partial f)^2, \frac{1}{n-1}\left(|\Lambda| + \frac{1}{D-2}\text{sup}(\partial f)^2\right)\right\} + b(n)k^{2/n}\text{Vol}_f^{-2/n}$$

[GBDL, Tomasiello, ‘21 using Hassannezhad, ‘13]

- Even assuming the REC, these do not exclude separation of scales

[cf. Collins, Jafferis, Vafa, Xu, Yau, ‘22,
Cribiori, Junghans, Van Hemelryck, Van Riet, Wrase ‘21,]

Violating the REC

- Generically, with only “positive energy” ($T^{(d)} < 0$), it is easy to stabilize positive internal curvature
  - A simple understanding is through the effective potential
    - Equivalent to the D-dimensional eoms after the warp-factor constraint is enforced

[Douglas, ‘09]

$$V_{\text{eff}}[g_n, \phi; u] \propto m_D^{D-2} \int_{M_n} \sqrt{g_n} u^2 \left(-R_n - 3\frac{(\nabla u)^2}{u^2} - T_\phi^{(d)} \right)$$

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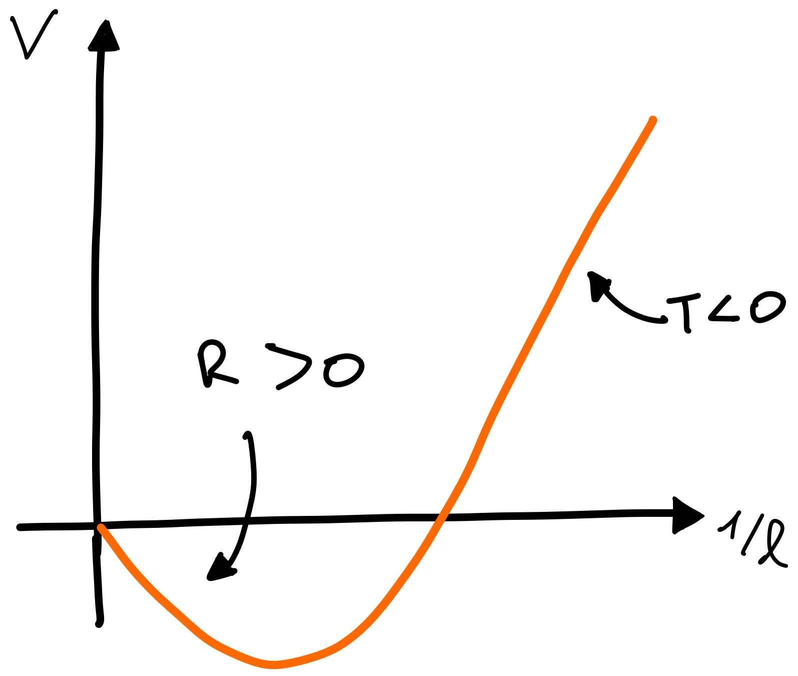

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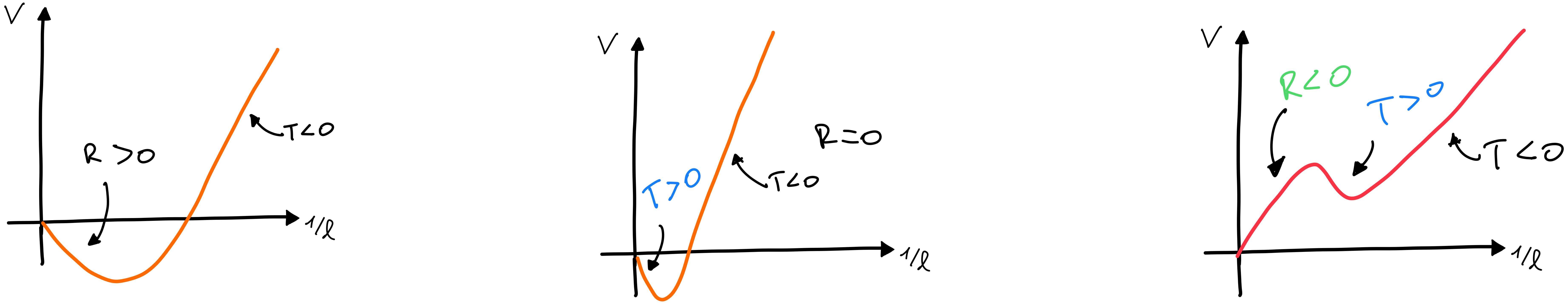

- But if $T_\phi^{(d)}$ includes also negative contributions, one can stabilize zero and negative curvature [cf. Douglas, Kallosh, ‘10]
  - Much richer structure: the length scales (e.g. diameter) and KK modes are not tied to the curvature
  - Negative curvature in particular has no moduli (rigidity)

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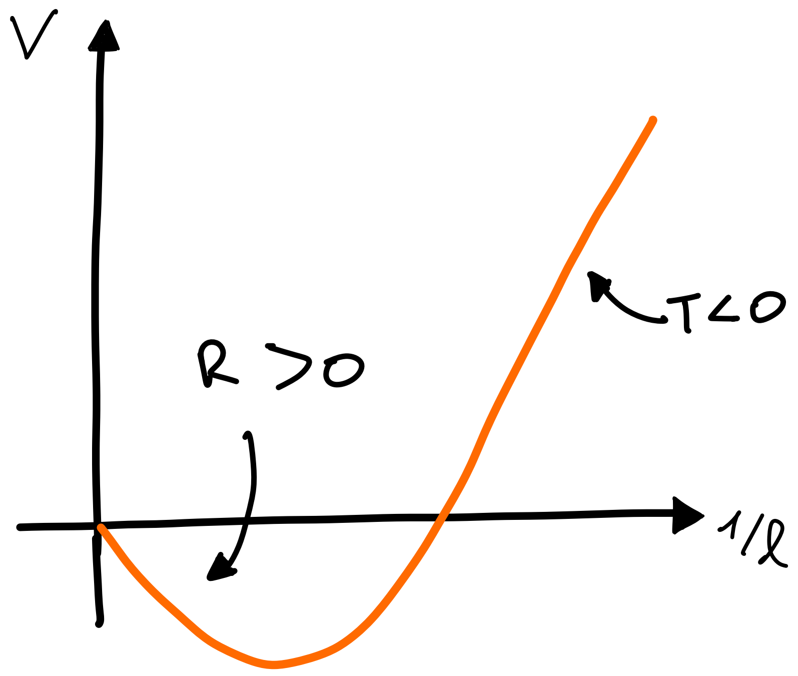

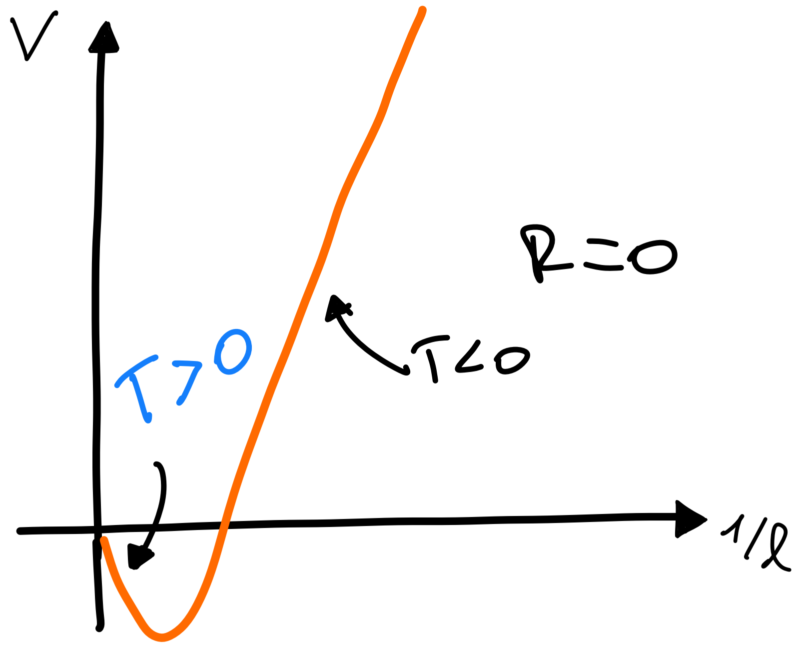

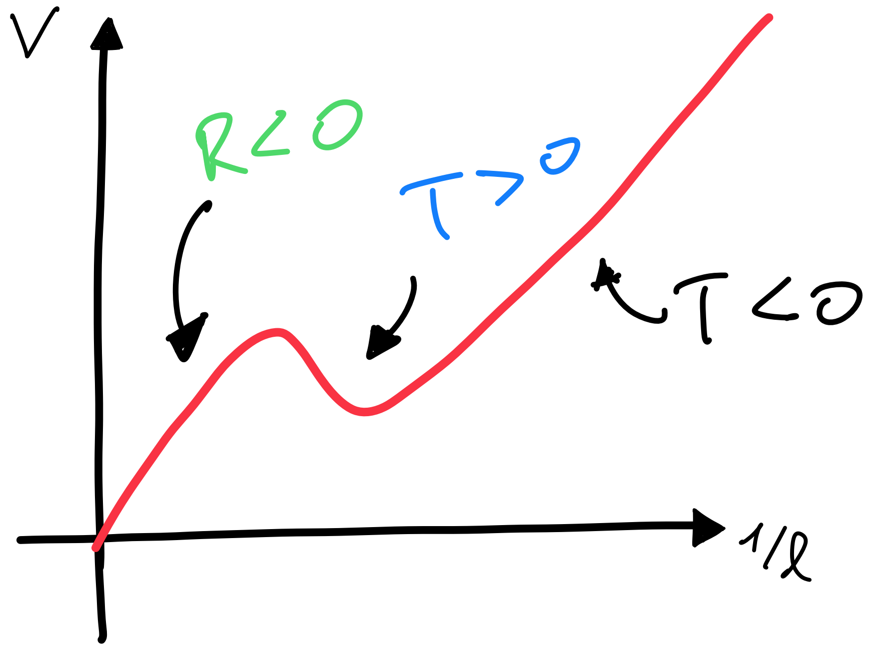

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  - Much richer structure: the length scales (e.g. diameter) and KK modes are not tied to the curvature
  - Negative curvature in particular has no moduli (rigidity)
- Many other possibilities for negative energy and uplift, (KKLT, LVS, supercritical,…)
  - Another simple possibility: O-planes and large gradients

[Silverstein, Torroba, Dodelson, Dong ‘13;
Córdova, GBDL, Tomasiello, ‘18 ‘19]

$R_n = 0 +$ Casimir $\to \Lambda_4 < 0$

[GBDL, De Ponti, Mondino, Tomasiello, ‘22]

- With a compact internal space, Casimir energy density can be automatically generated
- If the space has small circles, with antiperiodic BCs for fermions, Casimir energies are of the form

[Arkani-Hamed, Dubovsky, Nicolis, Villadoro ‘07]

[cf. Maldacena, Milekhin, Popov ‘18]

$$T_{ij} \sim R_c(y)^{-D} g_{ij} \qquad T_{ab} \sim -\frac{D-k}{k}\underline{R_c(y)}^{-D} g_{ab}$$

other directions

circle directions

small circle size

- Then solve the semi-classical equations:

$$-\frac{2}{\sqrt{-g_D}}\frac{S_D^{\text{(class.)}}}{\delta g^D_{MN}} = \langle T_{MN}^{\text{(Cas.)}} \rangle$$

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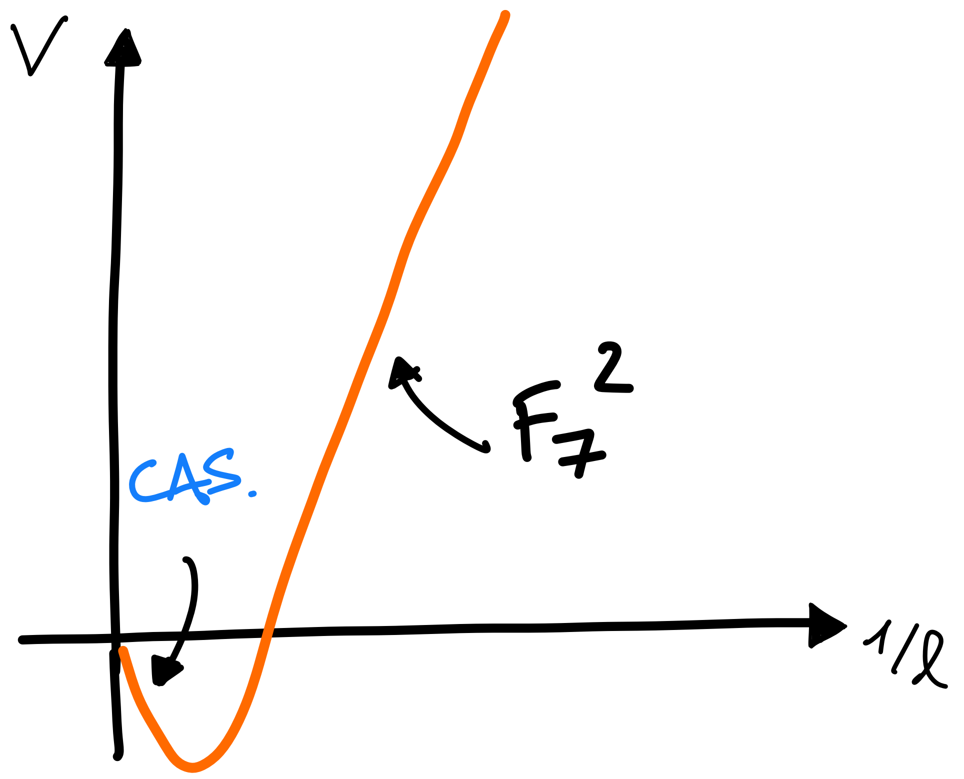
$$ds_{11}^2 = L_4^2 ds^2_{AdS_4} + R_c^2 ds^2_{T^7}$$

$$T^{Cas}_{\mu\nu} = |\rho_c| \ell_{11}^9 R_c^{-11} g_{\mu\nu} \qquad T^{Cas}_{ij} = -\frac{4}{7} |\rho_c| \ell_{11}^9 R_c^{-11} g_{ij}$$

$$F_7 = f_7 vol_{T^7} \qquad \frac{1}{\ell_{11}^6} \int F_7 = N_7$$

$$\Longrightarrow \quad \frac{L_4^2}{R_c^2} = \frac{2401}{4608} \frac{N_7^6}{\rho_c^4} \gg 1$$



$$\frac{R_c^{11}}{\ell_{11}^{11}} \sim N_7^{22/3} \gg 1$$

QG effects under control

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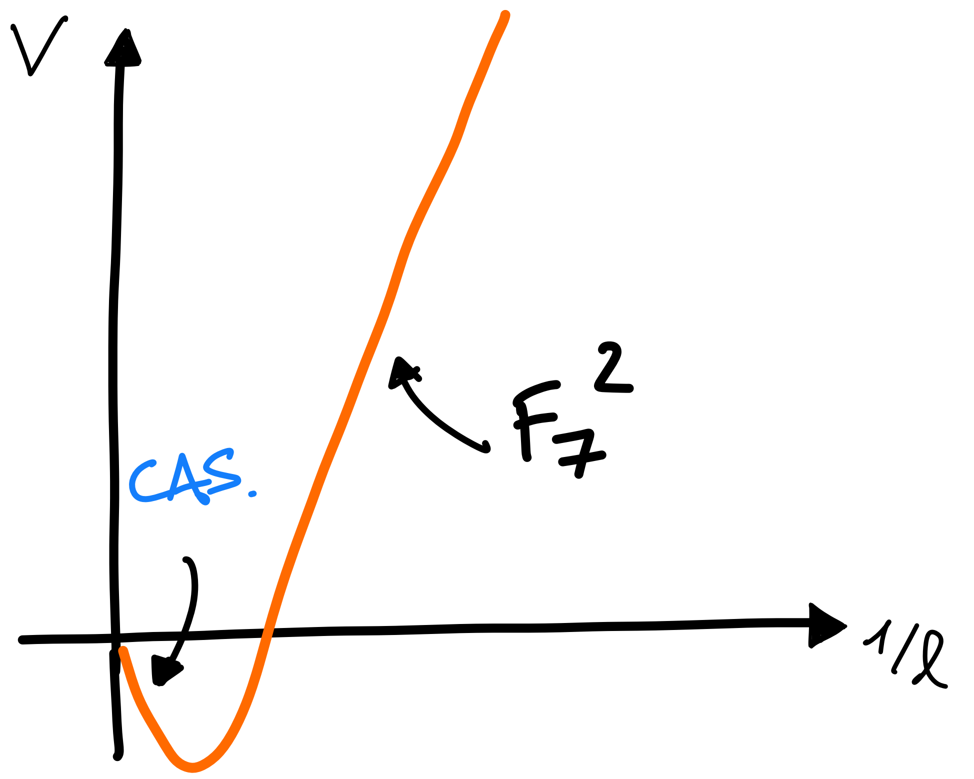

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QG effects under control

parametric separation of scales!

- Non-susy and unstable for M2 bubble nucleation
- Compatible with AdS distance conjecture, $m_{KK}^2 \sim |\Lambda|^{1/d}$

[Lust, Palti, Vafa, ‘19
Gonzalo, Ibáñez, Valenzuela , ‘21]

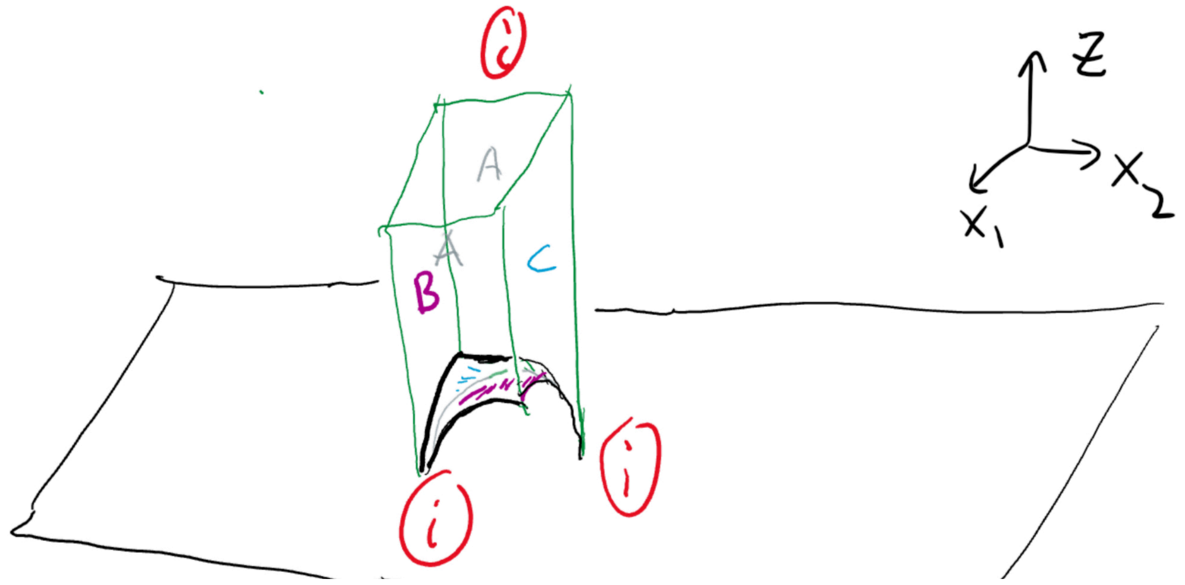
- [Also non stable dS possible in this way but not under parametric control]

Hyperbolic manifolds $\rightarrow \Lambda_4 > 0$

[e.g. Vinberg ‘ 93, Ratcliffe ‘06]

- Negative curvature and explicit metric, smooth manifolds. Quotients of hyperbolic space by subgroups Γ of its isometries
  - Recent explicit constructions by gluing right-angled polytopes

[Italiano, Martelli, Migliorini , ‘20]



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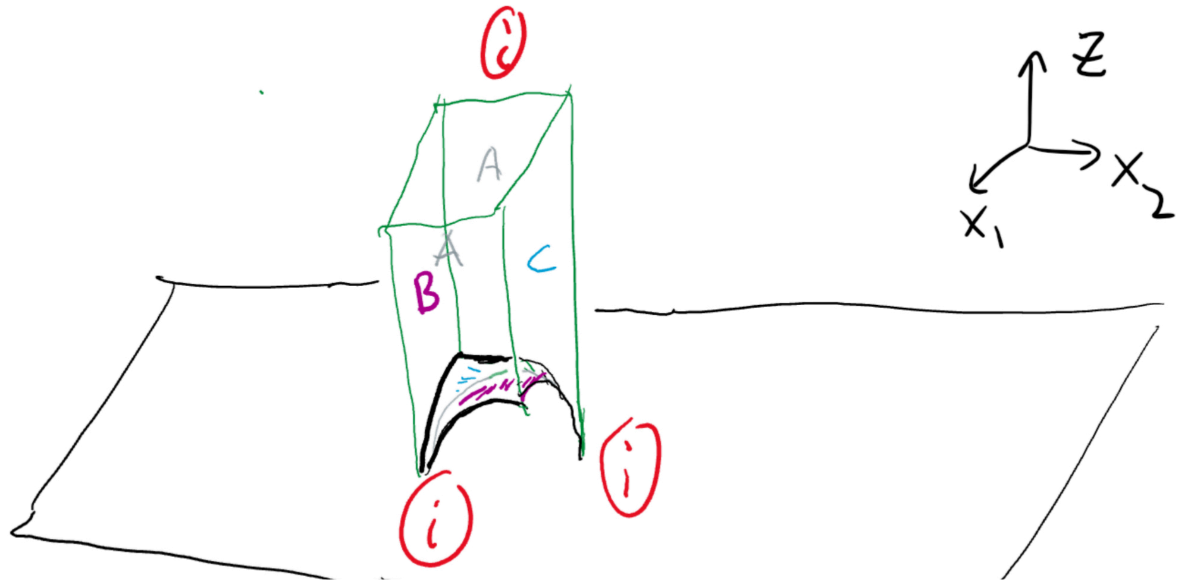

- Have one or more cusps: regions with small slowly varying circles

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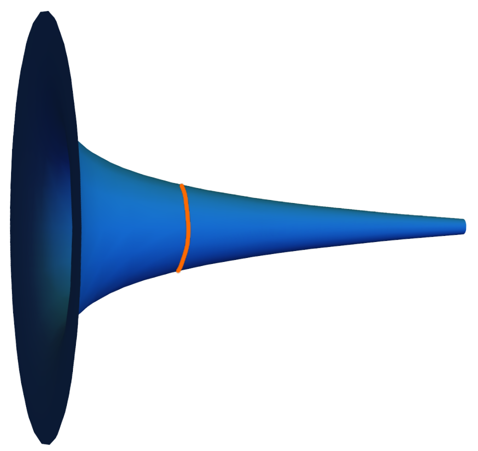
R_c

$$0 \leqslant y \leqslant y_c$$



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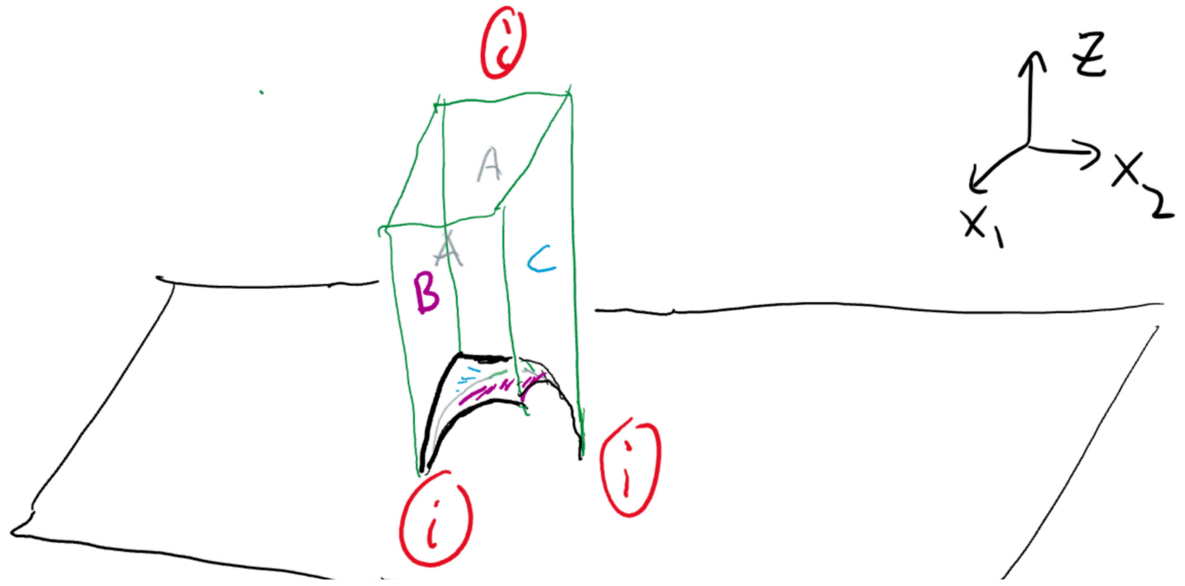

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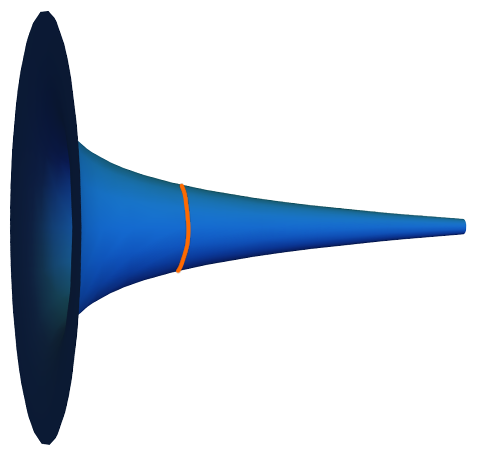

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- Cusps can be capped off in a smooth way: Anderson-Dehn filling to compact Einstein spaces

[Anderson ‘06]

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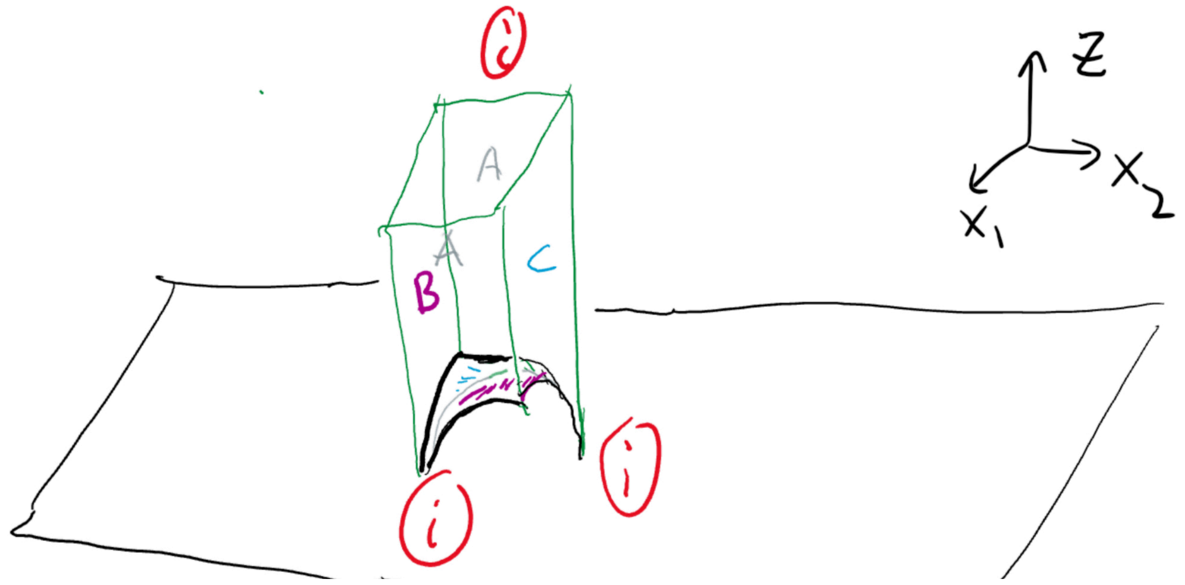

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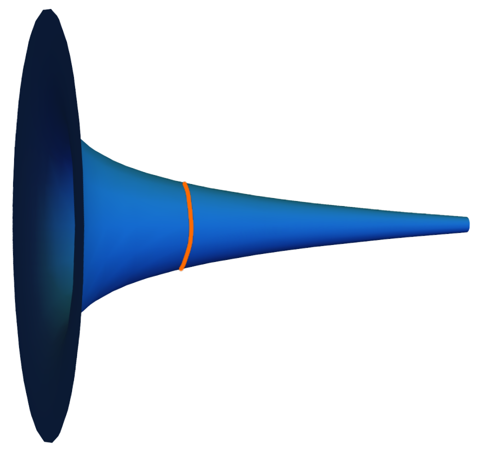

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[Anderson '06]

- They are rigid in $d > 2$: the hyperbolic structure is completely determined by the topology (no moduli space)
  - Also the filled Einstein manifolds are rigid
  - $-R_7$ is gapped at second order in h_{ij}

[Anderson '06]

[e.g. Besse '87]

- Rigidity: essentially, we only have to stabilize the volume modulus

[cf. Kaloper, March-Russell, Starkman, Trodden, '00]

$$ds_7^2 = \ell_7^2 \hat{ds_7}^2$$

$$m_4^2 = m_D^{D-2} \int_{M_7} \sqrt{g} u$$

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- Important: the negative contribution sits in the middle!

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- If $0 < a \ll 1$:

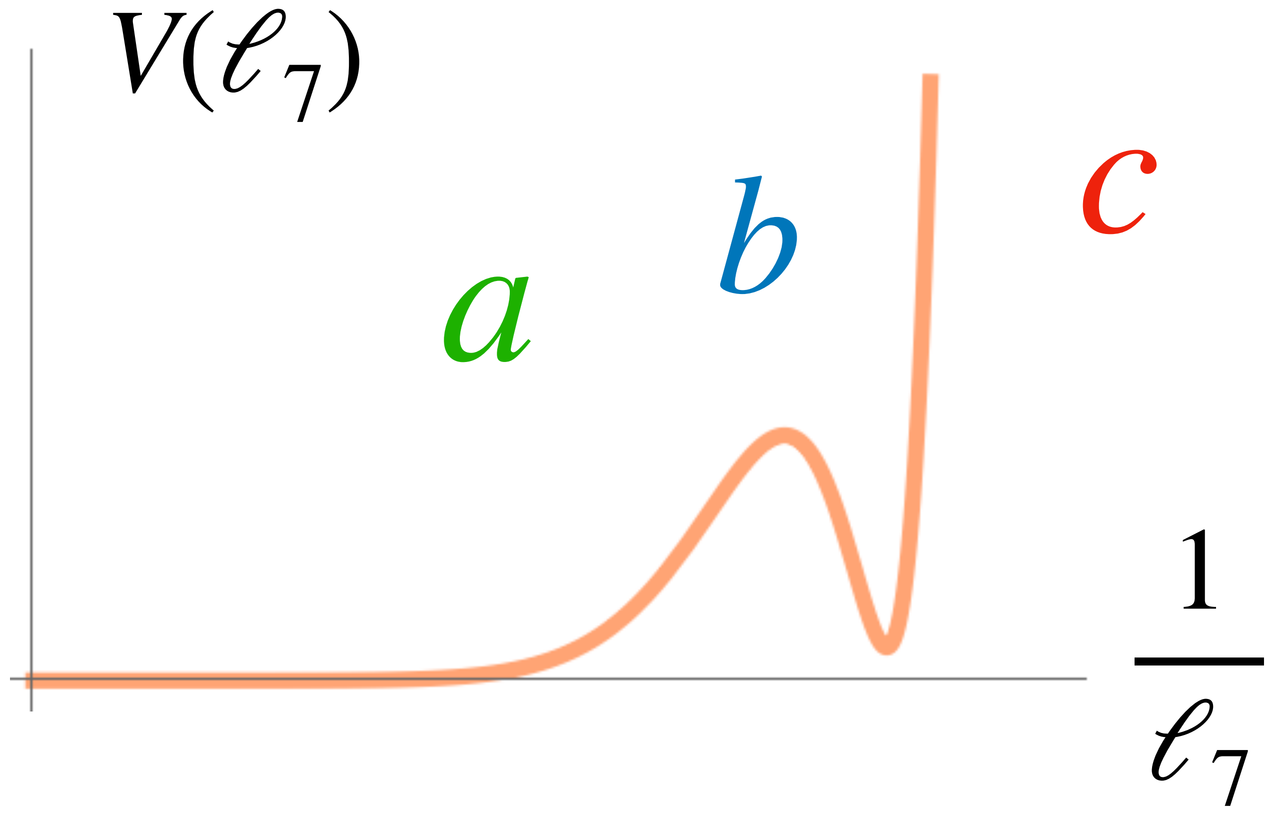
a c b

$$\int\sqrt{g}u^2\alpha > 0\text{, and}\int\sqrt{g}u^2\alpha + \int\sqrt{g}u^2\gamma \sim -\int\sqrt{g}u^2\beta \Rightarrow$$

Positive first term

Competition of classical and quantum effects

3 terms power law stabilization:



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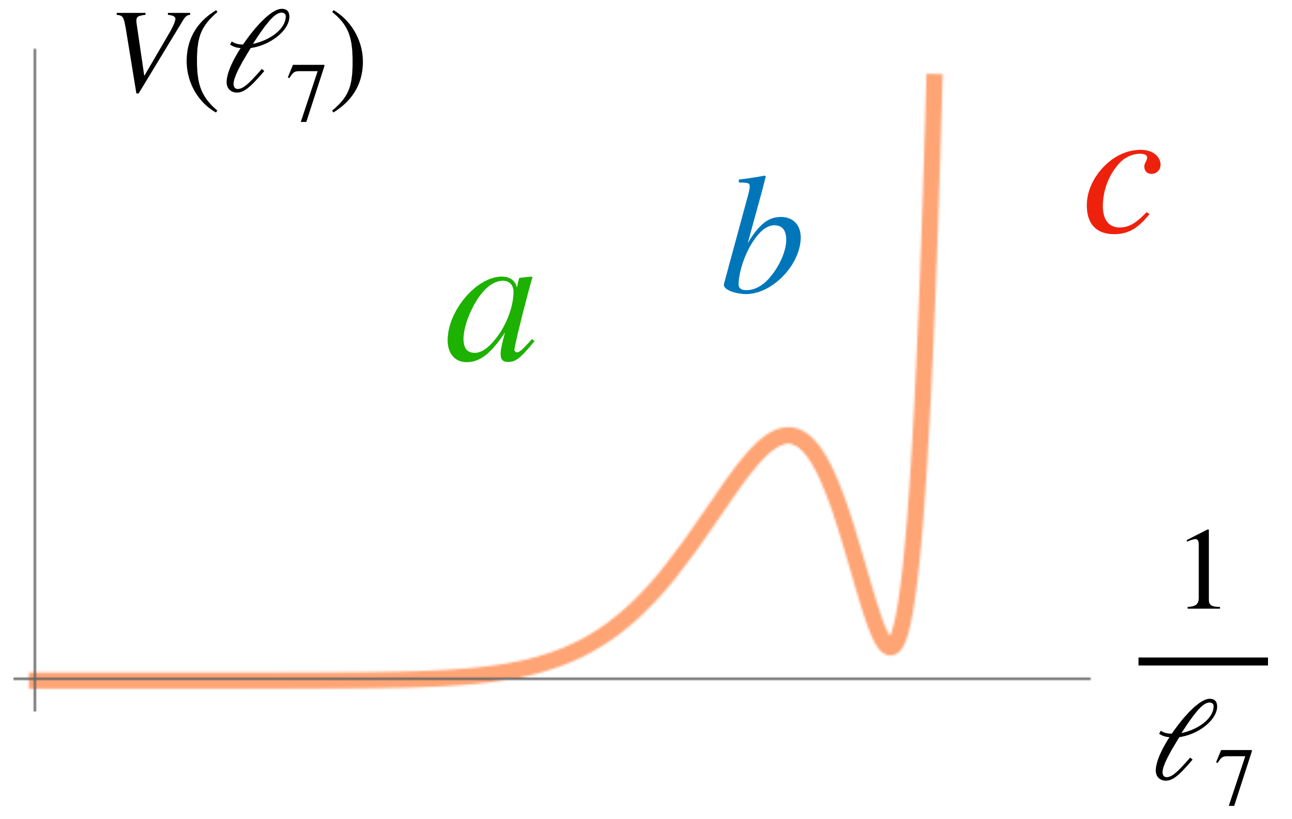

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- To reduce a, add bulk regions (or reduce cusps)

Backreacted smooth solution in a filled cusp

- At the end of the filled cusp, approximately only radial dependence
  - PDEs → ODEs

[Anderson '06]

$$ds_{11}^2 = u(y)ds_{4,\Lambda}^2 + dy^2 + R_c^2(y)ds_{\mathbb{T}^5}^2 + R^2(y)d\theta^2$$

$$0 = 4A'\left(\frac{5R_c'}{R_c} + \frac{R'}{R}\right) + 6(A')^2 - \frac{1}{4}e^{-8A}f_0^2 - \frac{1}{2}e^{-2A}C + \frac{5R'R_c'}{RR_c} - \frac{|\rho_c|}{2R_c^{11}} + \frac{10(R_c')^2}{R_c^2}$$

Full set of 11D EOMs

$$\begin{aligned} A'' &= -A'\left(4A' + \frac{5R_c'}{R_c} + \frac{R'}{R}\right) + \frac{1}{3}e^{-8A}\left(\frac{3}{4}e^{6A}C + f_0^2\right) - \frac{|\rho_c|}{2R_c^{11}} \\ \frac{R_c''}{R_c} &= -\frac{R_c'\left(4A' + \frac{5R_c'}{R_c} + \frac{R'}{R}\right)}{R_c} - \frac{1}{6}e^{-8A}f_0^2 + \frac{3|\rho_c|}{5R_c^{11}} + \frac{(R_c')^2}{R_c^2}\,, \\ \frac{R''}{R} &= -\frac{R'\left(4A' + \frac{5R_c'}{R_c} + \frac{R'}{R}\right)}{R} + \frac{1}{6}\left(-e^{-8A}f_0^2 - \frac{3|\rho_c|}{R_c^{11}}\right) + \frac{(R')^2}{R^2} \end{aligned}$$

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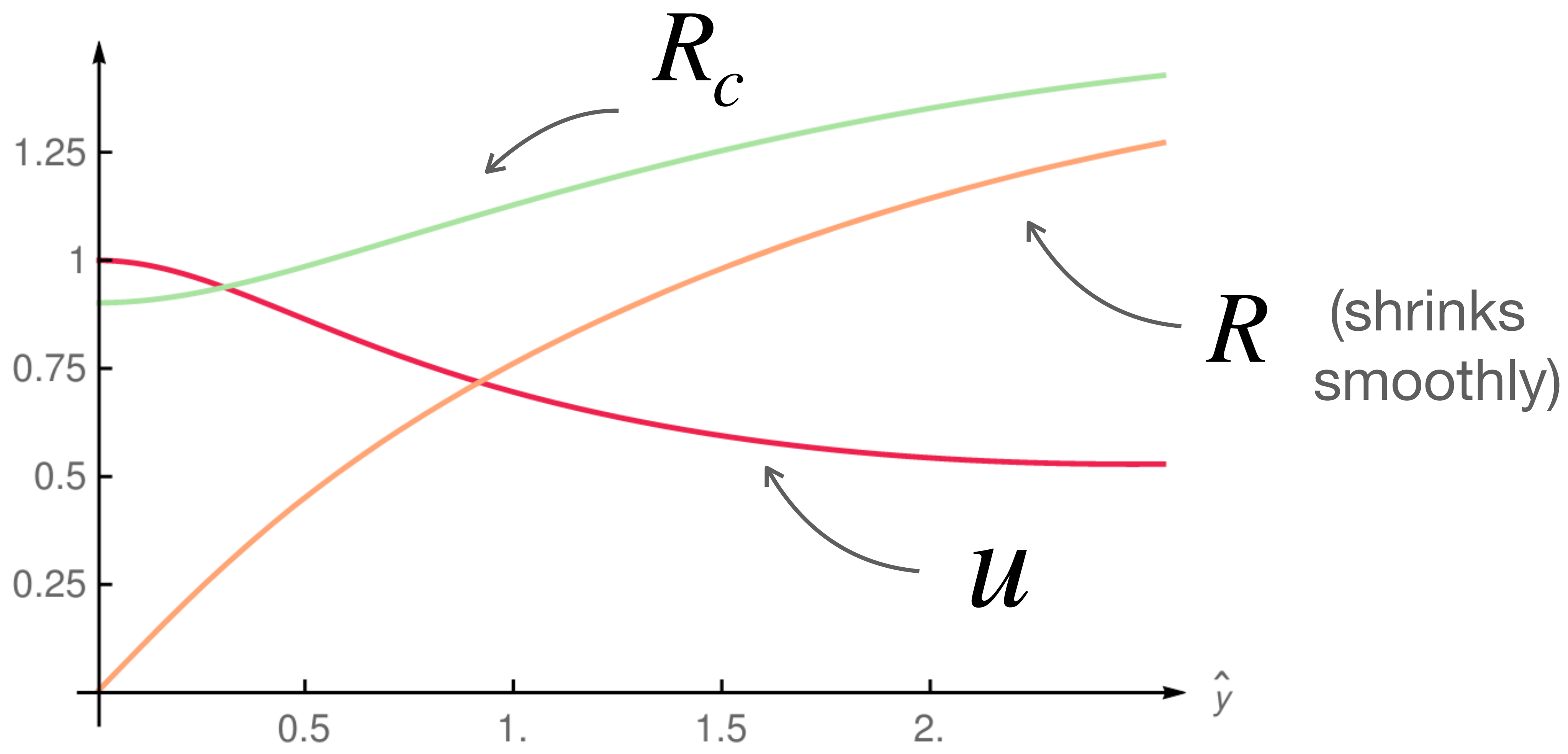

(functions rescaled for clarity, but can make $R \gg R_c \gg \ell_{11}$)

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[Italiano, Martelli, Migliorini , '20]

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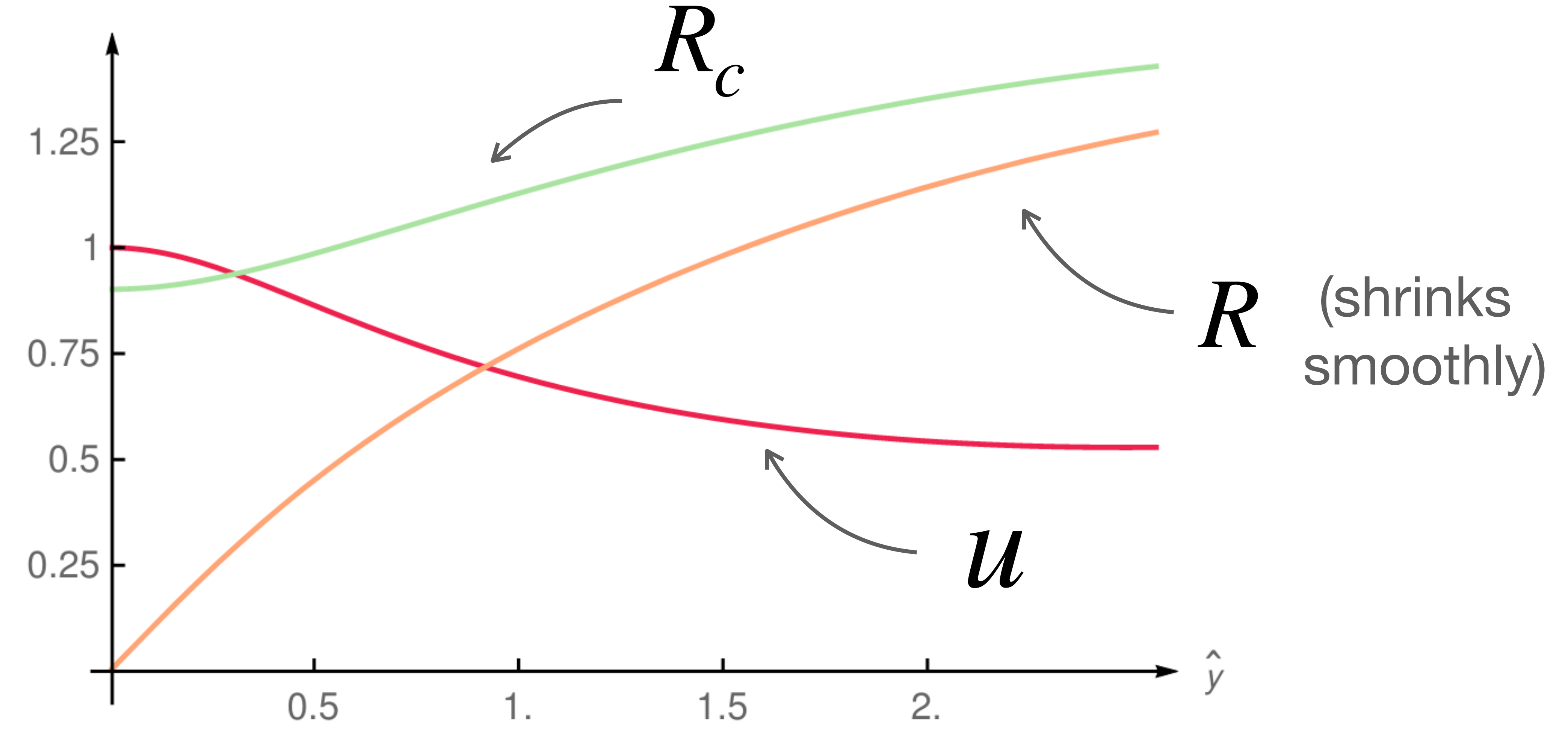

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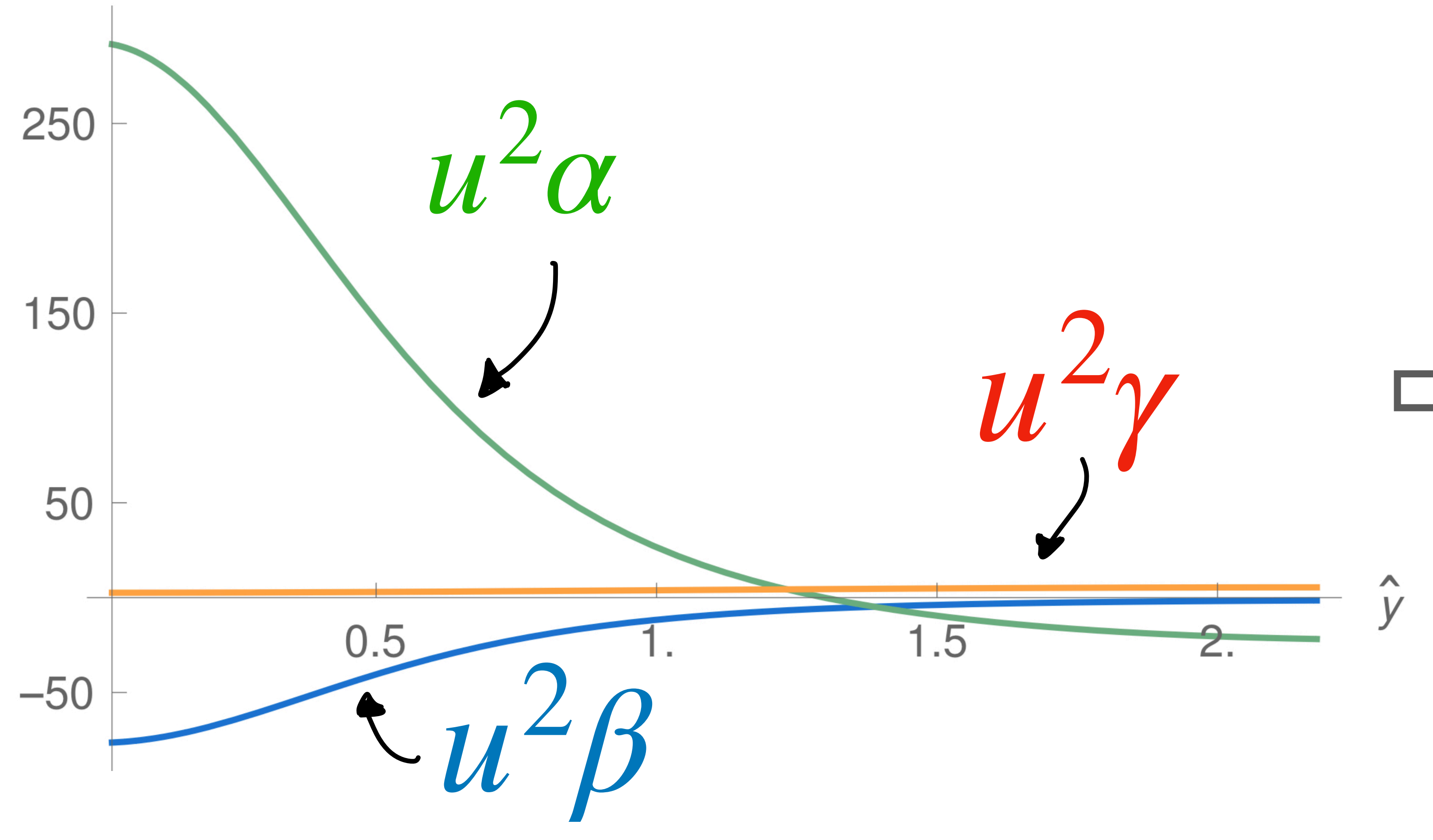
- We can check our general estimates explicitly:



$$a > 0$$ ✓

$$\frac{\int\sqrt{g}u^2(\alpha+\gamma)}{\int\sqrt{g}u^2\beta} \sim 1.06$$ ✓

$$\delta^2 V > 0$$ ✓

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Backreaction on the whole manifold?

- We have described explicit families of hyperbolic manifolds and constructed piece-wise de SItter solutions of the M-theory equations of motion.

- Compare this with Anderson's proof of the existence of the filled metric [Anderson '06]

$$ds^2_{\text{cusp}} = \frac{dr^2}{r^2} + \frac{r^2}{r_j^2} ds^2_{T^{n-1}}$$

$$r \geqslant 0$$

Glued at

$$r = r_j > 1$$

$$ds^2_{\text{BH}} = \left(\frac{dr^2}{V(r)} + V(r) d\theta^2 + r^2 ds^2_{\mathbb{R}^{n-2}} \right) / \mathbb{Z}^{n-2}$$

$$r \geqslant 1 \qquad V(r) = r^2 \left(1 - r^{1-n}\right)$$

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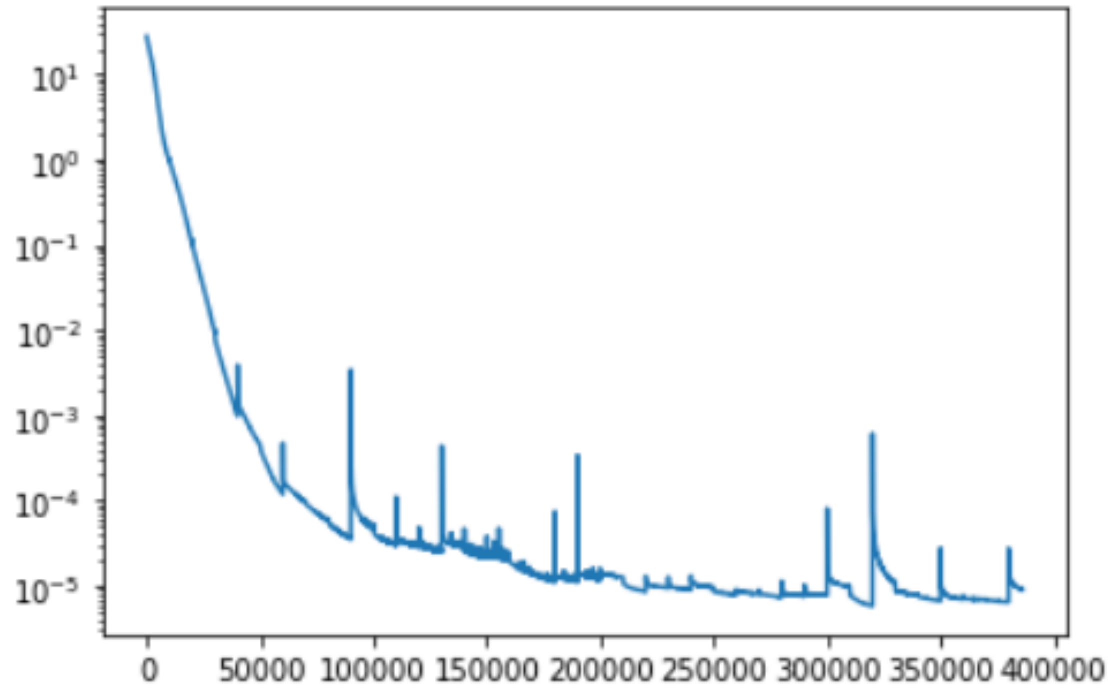
M = 3

FILLING 1 CUSP



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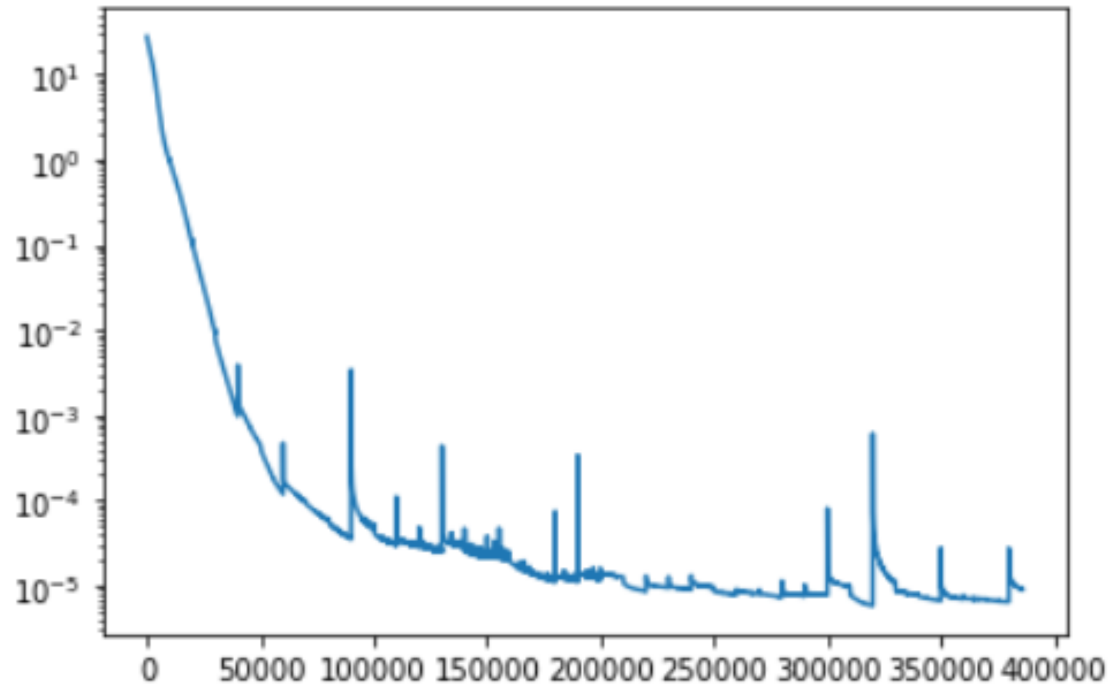
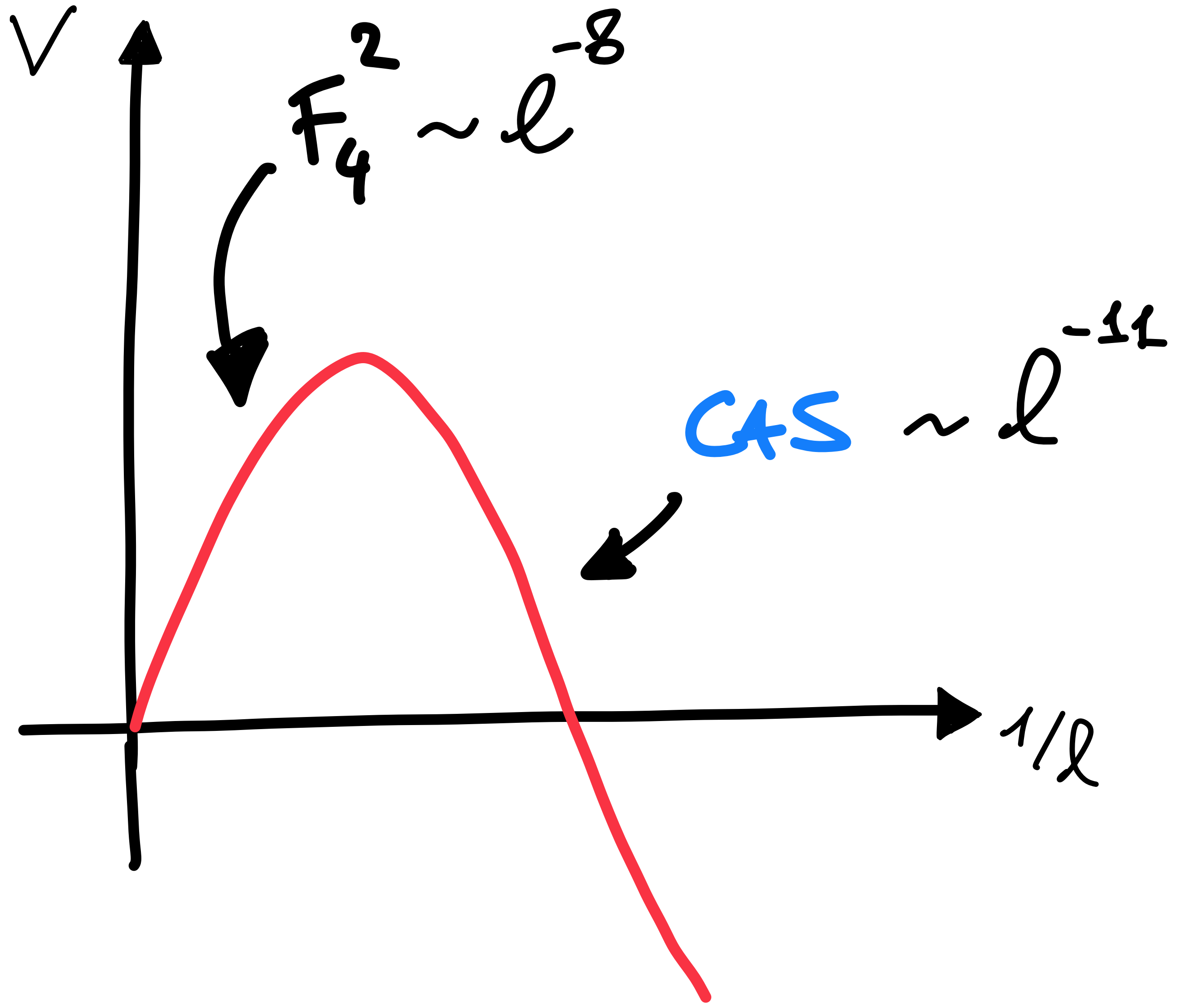


Thank

you!

An explicit uncontrolled dS with Casimir

- Consider M-theory on dS$_7 \times T^4$ (or dS$_4 \times S^3 \times T^4$), with magnetic F_4 on the torus



$$ds_{11}^2 = L_7^2 ds_{dS_7}^2 + R_c^2 ds_{T^4}^2$$

$$T_{\mu\nu}^{Cas} = |\rho_c| \ell_{11}^9 R_c^{-11} g_{\mu\nu} \qquad F_4 = f_4 vol_{T^4}$$

$$T_{ij}^{Cas} = -\frac{7}{4} |\rho_c| \ell_{11}^9 R_c^{-11} g_{ij} \qquad \frac{1}{\ell_{11}^3} \int F_4 = N_4$$

$$\Longrightarrow \quad \frac{R_c}{\ell_{11}} \sim N_4^{-2/3} \cancel{\gg} 1$$

$$\frac{L_7}{\ell_{11}} \sim N_4^{-11/3}$$

Recall for AdS$_4 \times T^7$

$$\left[\frac{R_c}{\ell_{11}} \sim N_7^{2/3} \gg 1\right]$$

Putting everything together

$$a \equiv \frac{\int_{M_7} \sqrt{g} u^2 \left(-R_7 - 3\frac{(\nabla u)^2}{u^2} \right)}{\int_{M_7} \sqrt{g} u^2 \frac{42}{\ell_7^2}}$$

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integrated Casimir

- And:
  - Tadpoles around the hyperbolic starting point are bounded and small
  - Full Hessian is likely to be positive, gapped:
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[extending Douglas, ‘09]

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[Douglas, Kallosh , '10]

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⇓

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- To increase a, reduce the flux

Available tuning
discrete topological
parameters.

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- Can we also solve all the equations of motion explicitly?

Organizing the equations of motion

- The 11D equations of motion can be obtained from the effective potential [Douglas, '09]

$$V_{\text{eff}}[u, g_7, C_6] \equiv \frac{1}{2\ell_{11}^9}\int_{M_7}\sqrt{g}\,u^2\left(-R_7 - 3\frac{(\nabla u)^2}{u^2} - \ell_{11}^9\rho_c R_c(y)^{-11} + \frac{1}{2}|F_7|^2\right)$$

$\delta g_{\mu\nu}^{11}$ ⇨ $\dfrac{\delta V_{\text{eff}}}{\delta u} = 0$ ⇨ $\Delta u = \dfrac{1}{3}\left(-R_7 + F_7^2 - \dfrac{\ell_{11}^9}{R_c^{11}}\right)u - \Lambda$ warp factor constraint

- When $\Lambda\ell_7^2 \ll 1$ is an analogue Schrodinger problem
- "Negative energy" → potential barriers for warping

δg_{ij}^{11} ⇨ $\dfrac{\delta V_{\text{eff}}}{\delta g_{7\,ij}} = 0$ ⇨ Set of 7d second order non-linear PDEs!

- Organized in terms of their geometrical origin:

$$\delta g_{7\,ij}(y) \equiv h_{ij}(y) + \frac{1}{7}g_{7\,ij}\delta\tilde{B}(y) \qquad \delta\tilde{B}(y) \equiv \delta V + \delta B(y)$$

anisotropies

volume

measure
inhomogeneinities

+ Flux equations and fixed $G_N = \int\sqrt{g_7}u^2$