HS Symmetry at Infinite Distance and its Stringy Origin

José Calderón Infante

CERN

Based on 2305.05693 with Florent Baume and ongoing work with Irene Valenzuela

Landscapia, IPhT, CEA/Saclay, 29/11/2023

The Swampland Distance Conjecture

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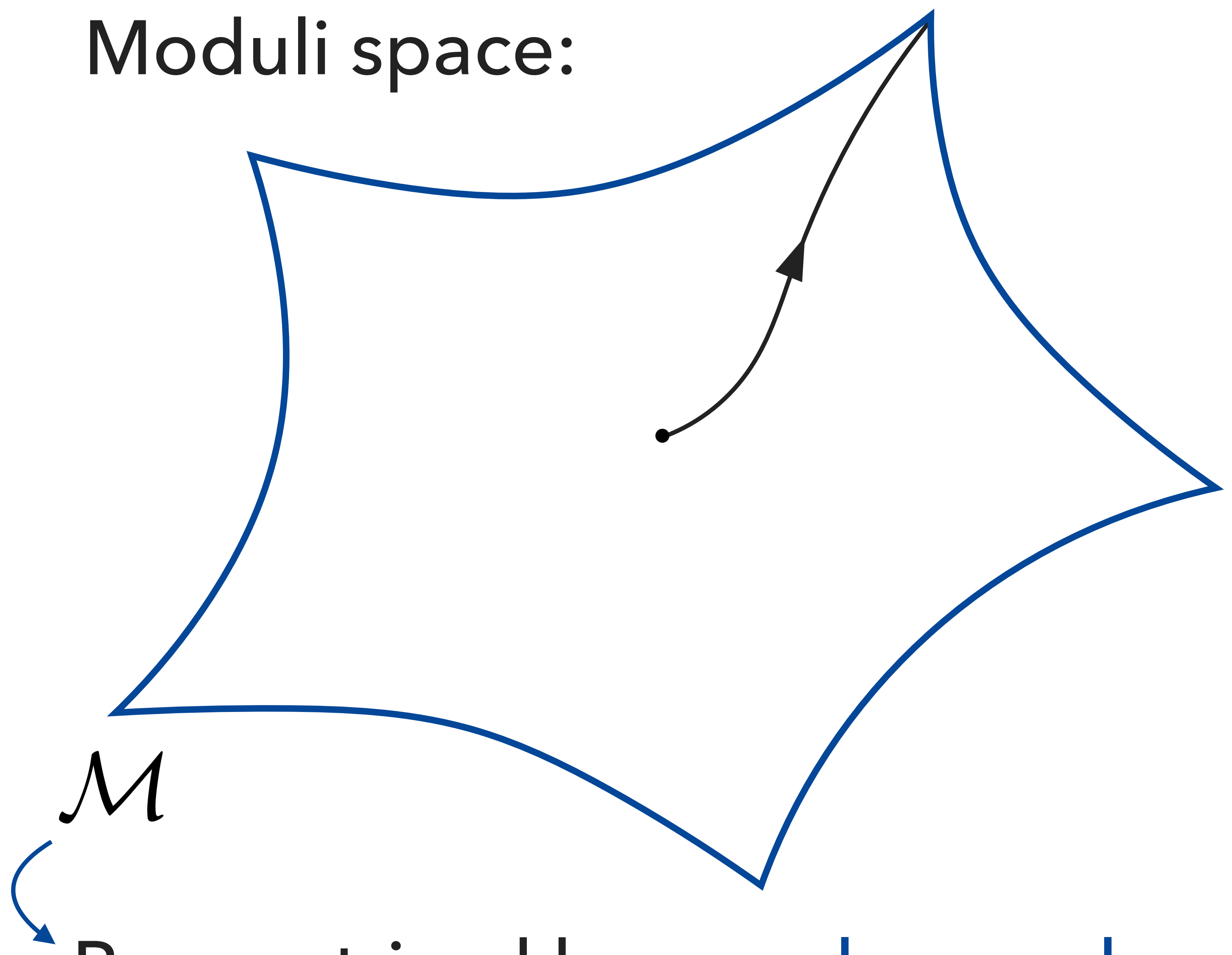
Moduli space:



Parametrized by massless scalars

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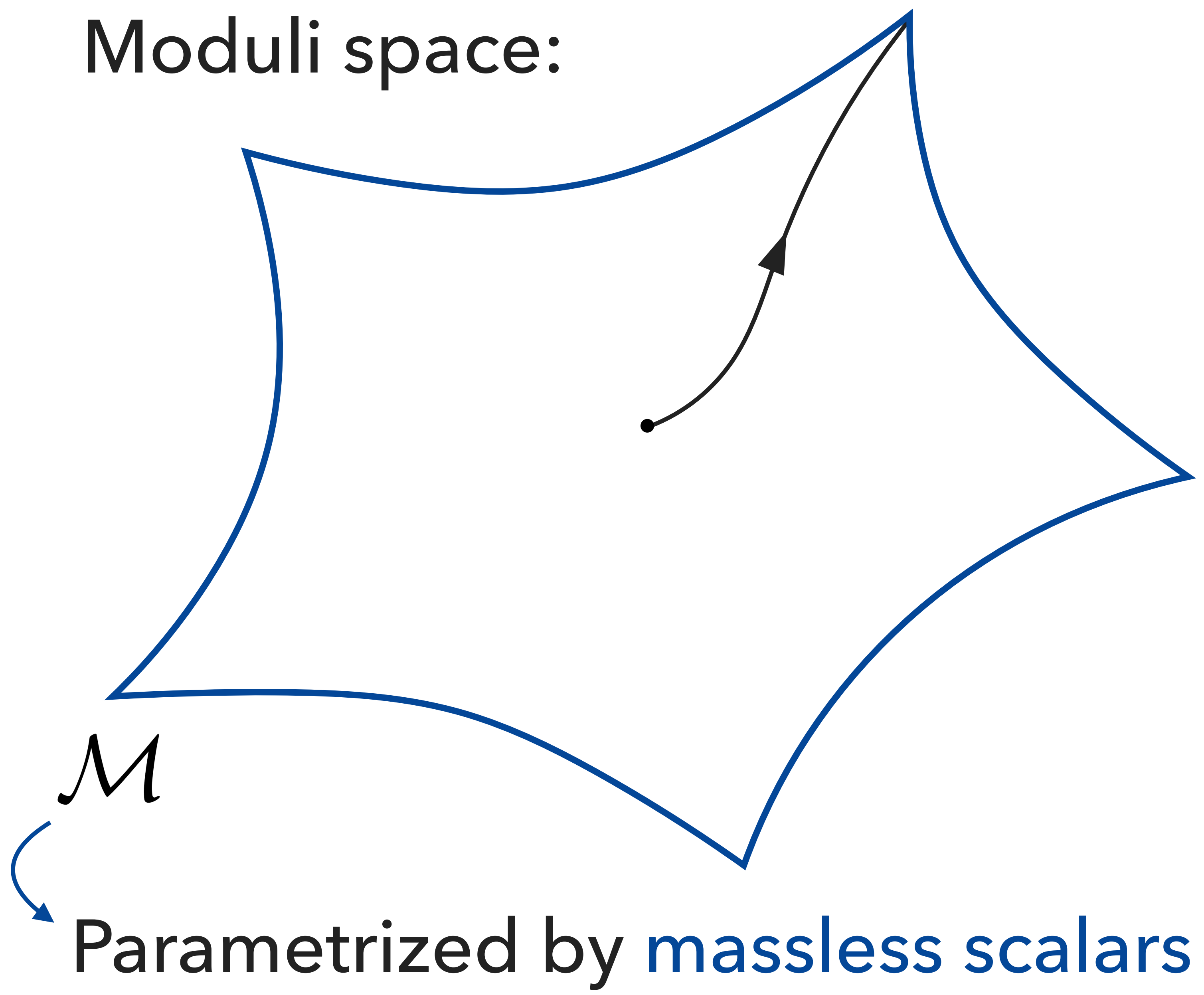

Swampland Distance Conjecture (SDC) [Ooguri, Vafa '06]

There is an infinite tower of states becoming light at infinite-distance points in moduli space:

$$M_{tower} \sim e^{-\alpha\,\Delta\phi} \quad \text{as} \quad \Delta\phi \to \infty \qquad (M_{Pl} = 1)$$

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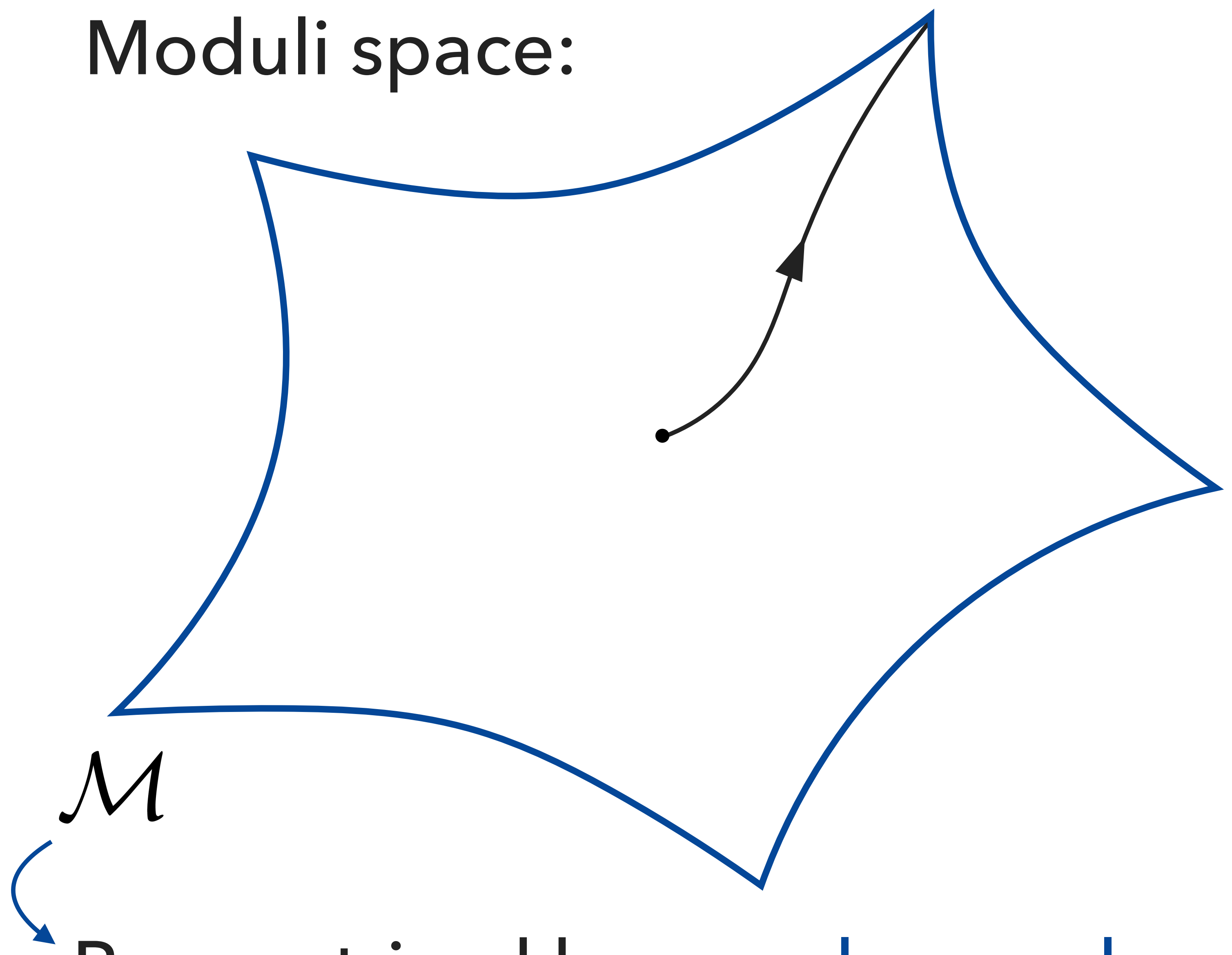

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Precise order one bounds on the exponential rates

- Lightest tower: [Etheredge, Heidenreich, Kaya, Qiu, Rudelius '22]
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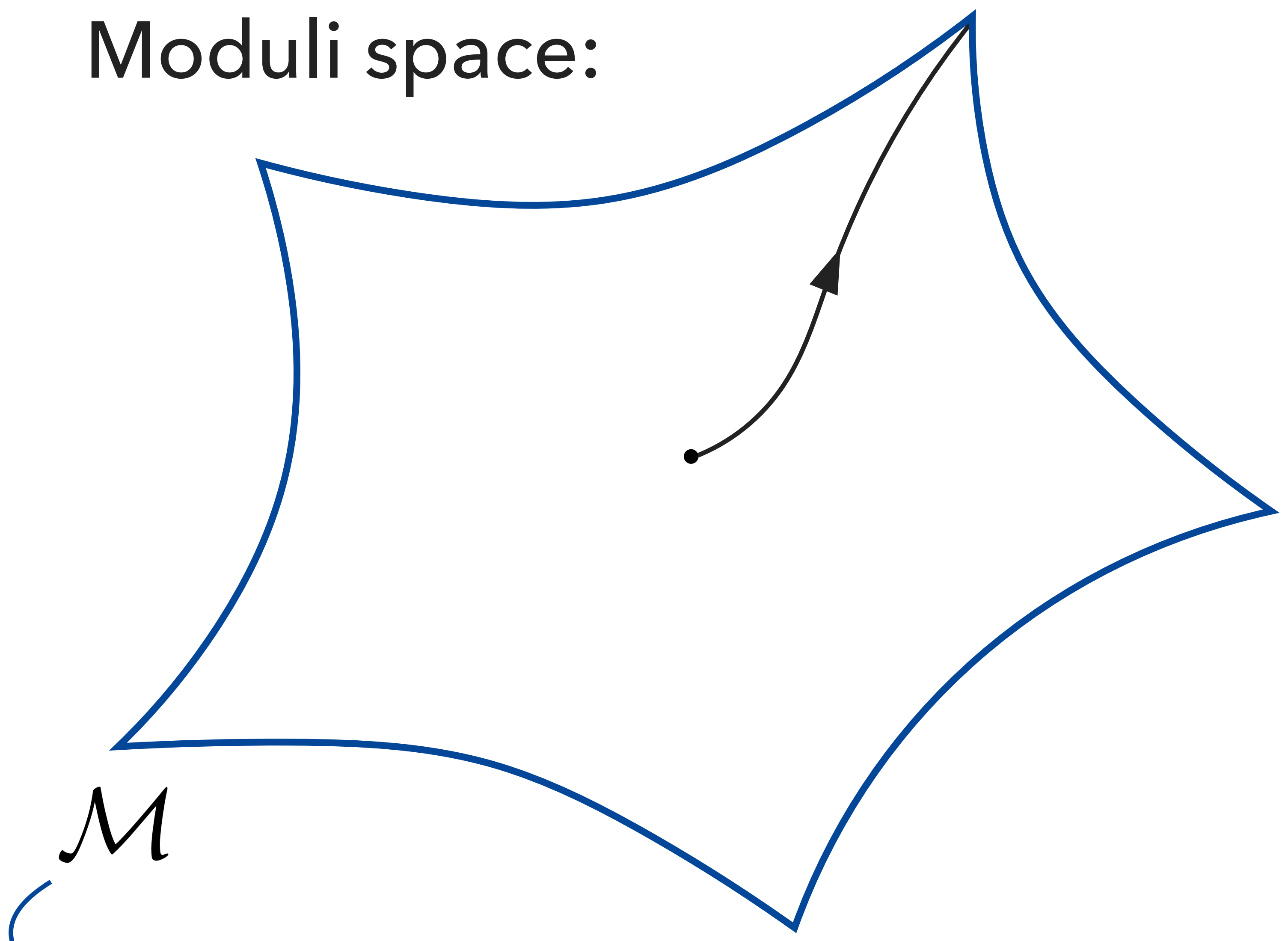

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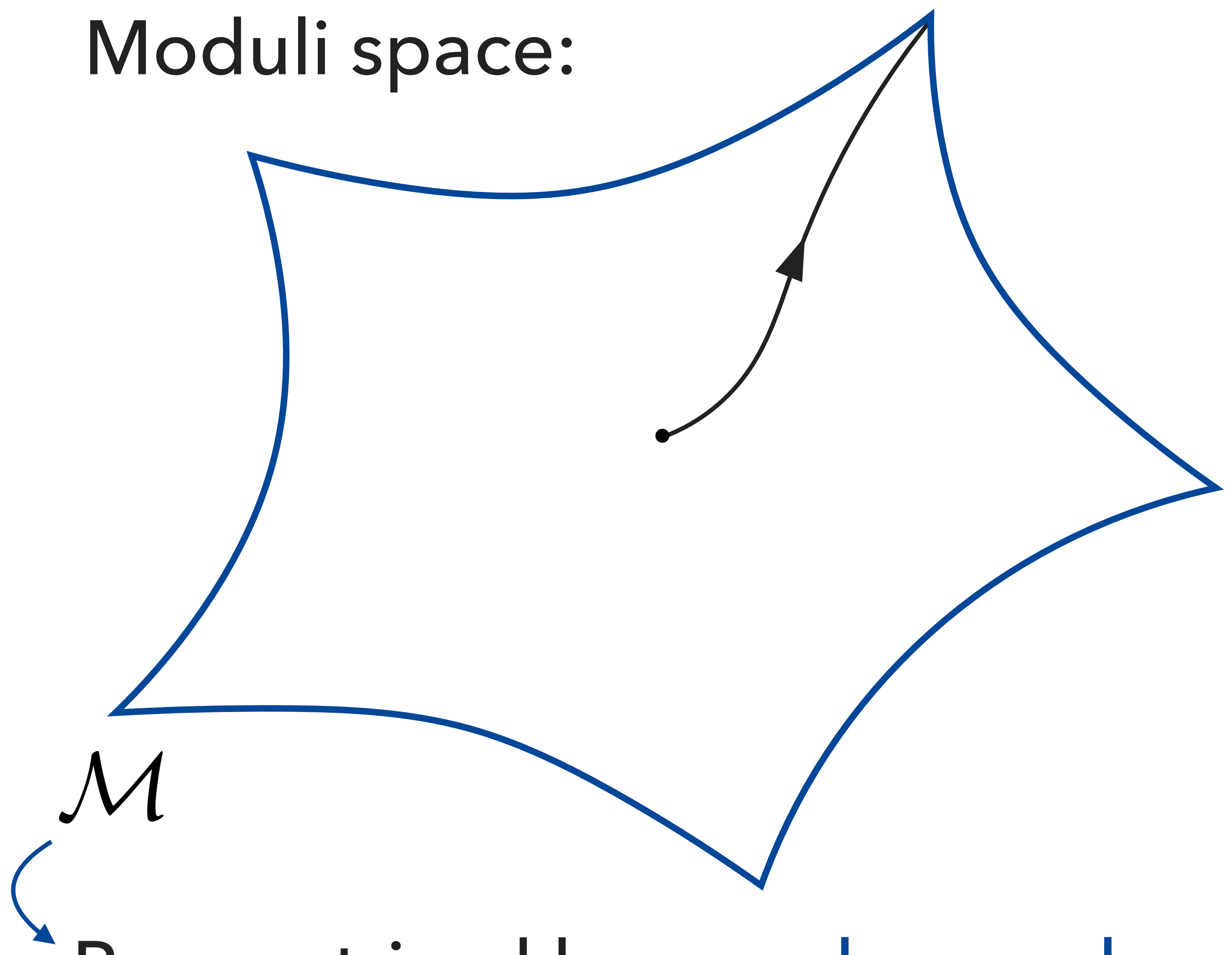

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Lots of top-down evidence!

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  + many many more!
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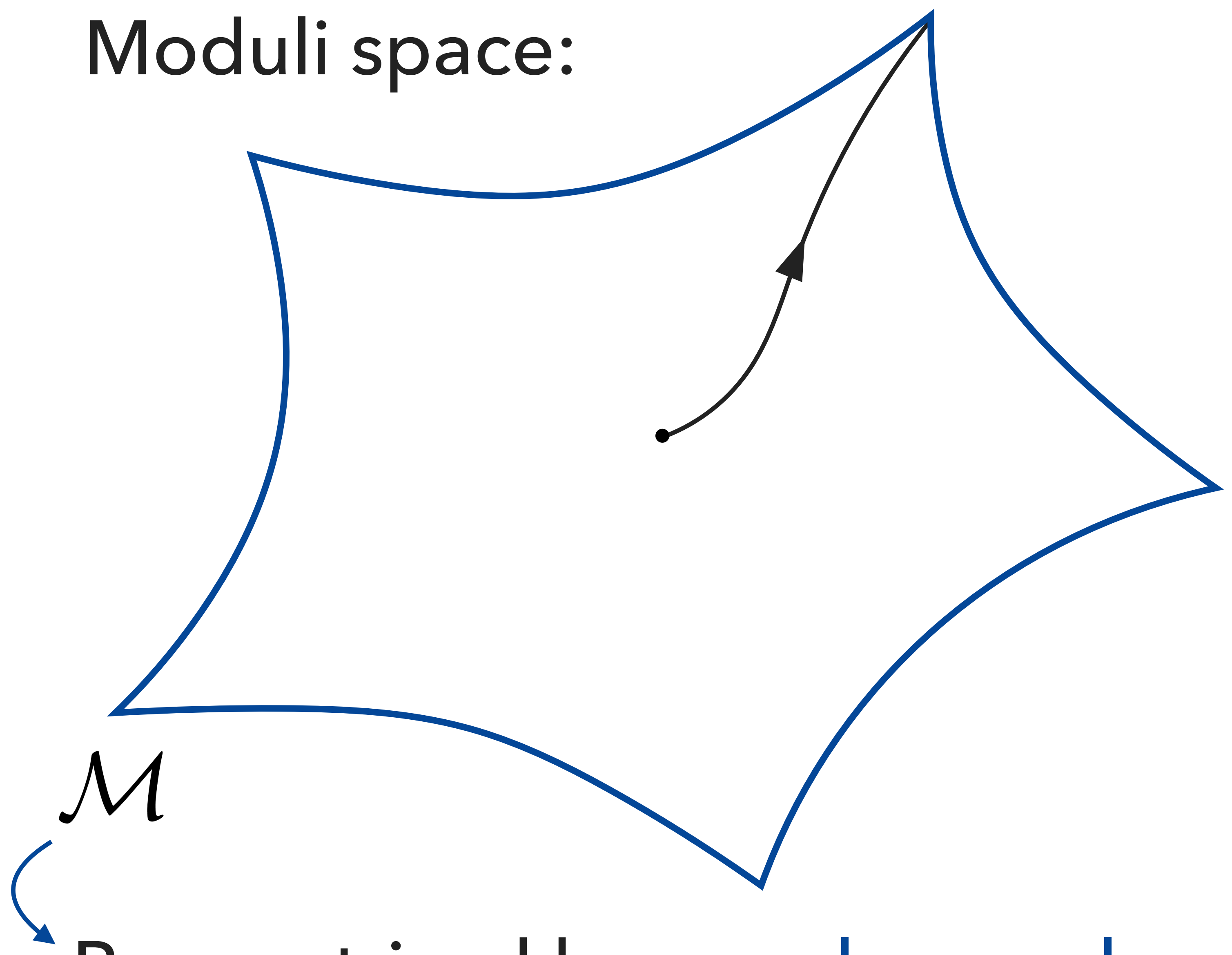

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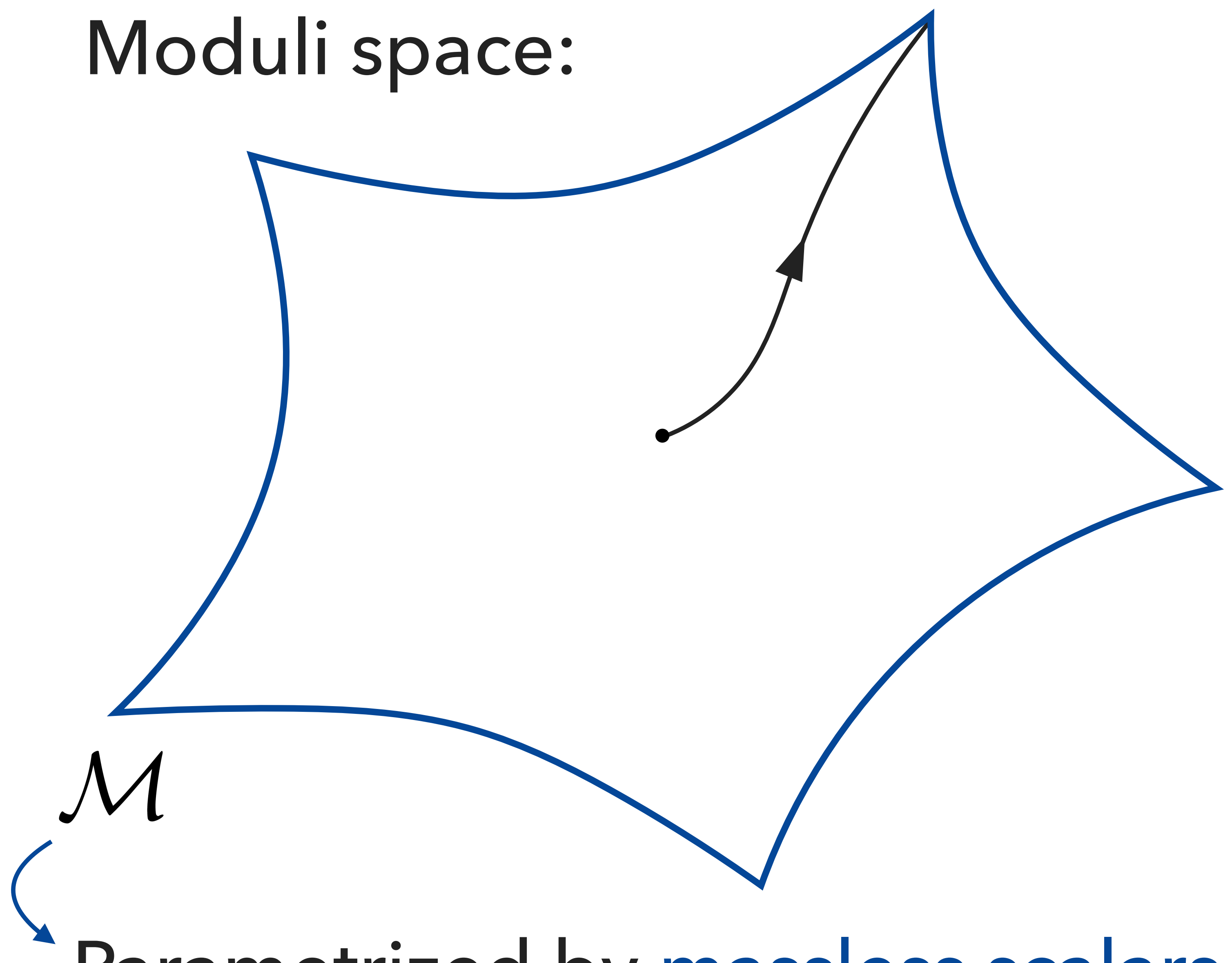

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+ connections, pheno, ...

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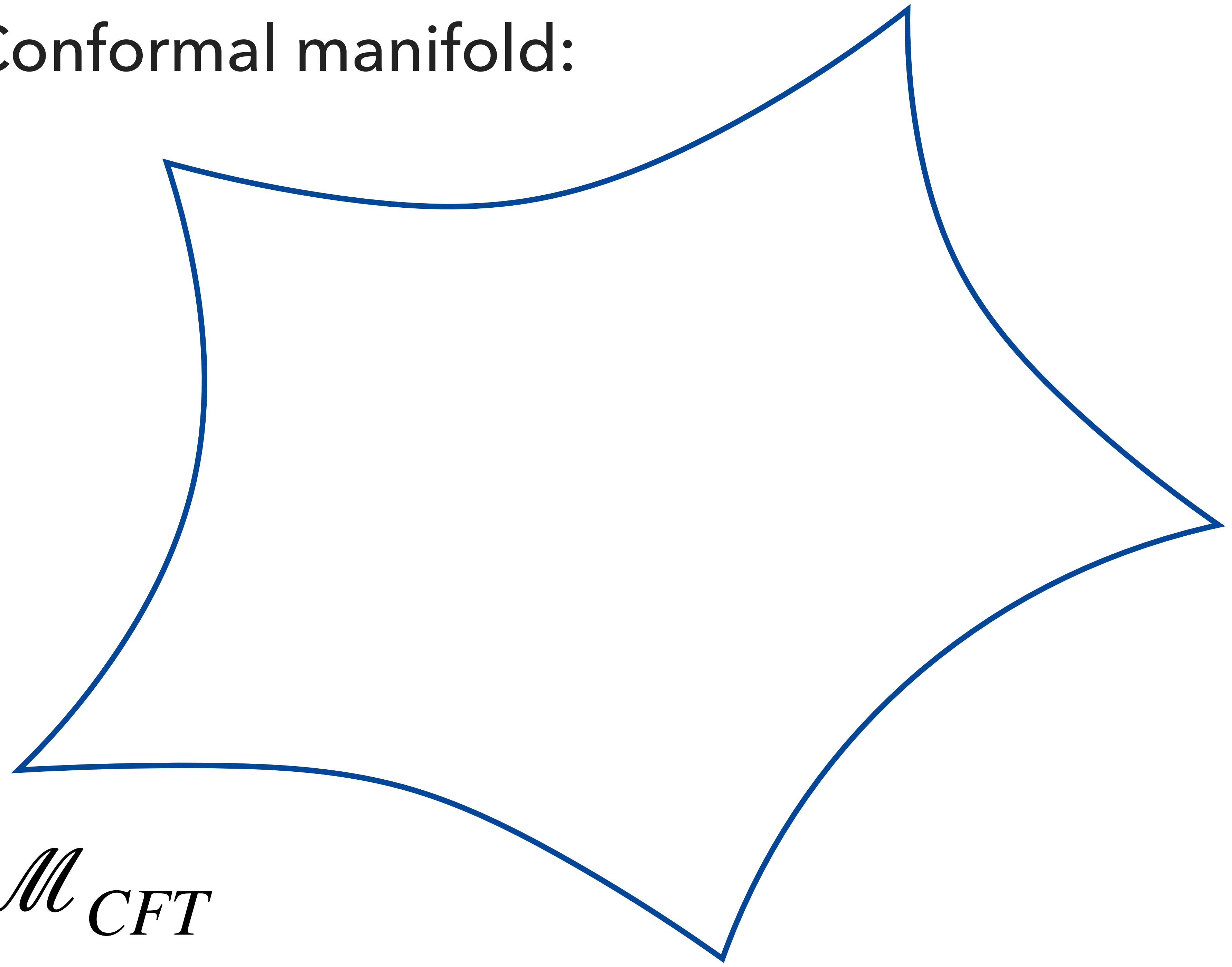

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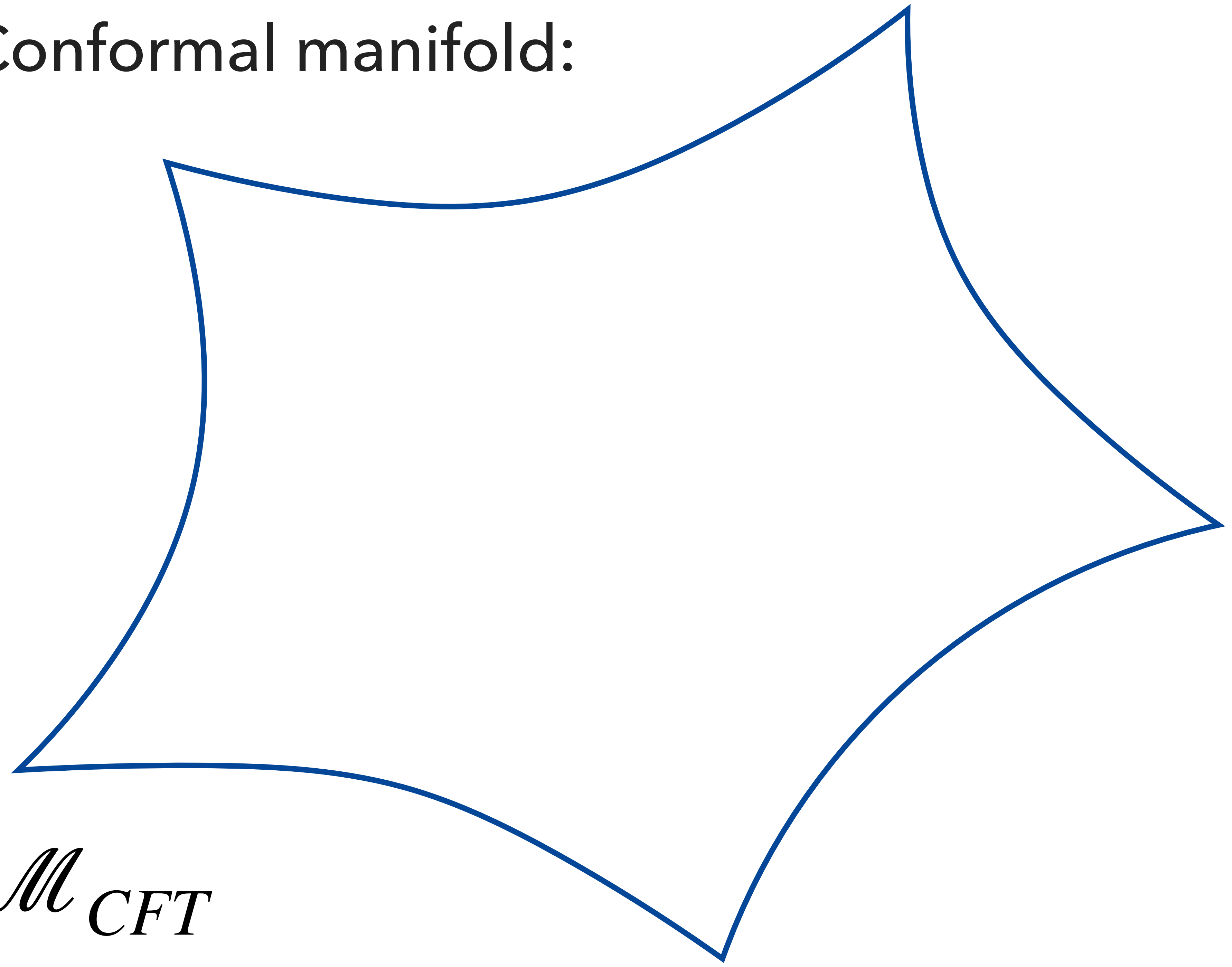

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→ Dynamical gravity in the bulk!

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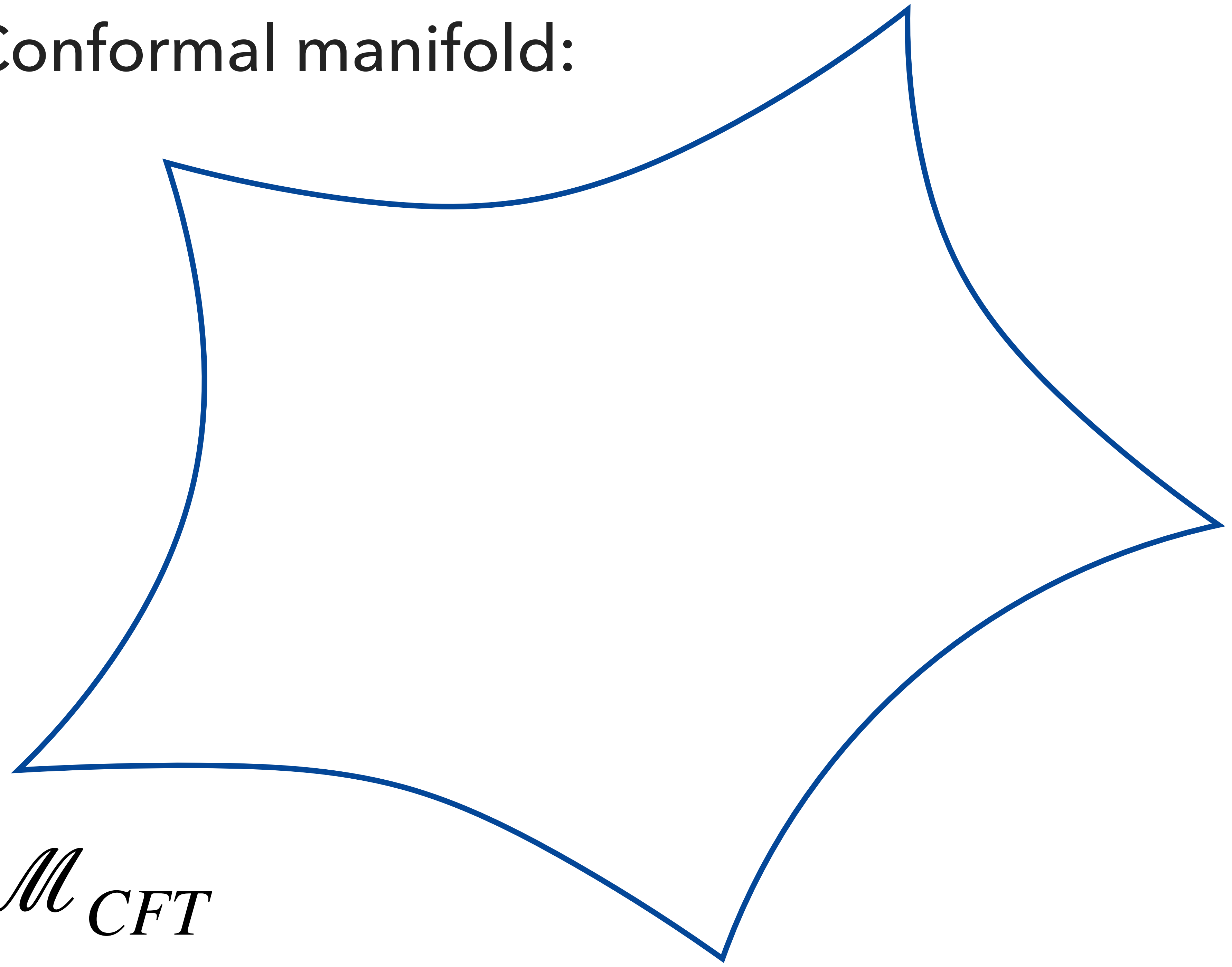

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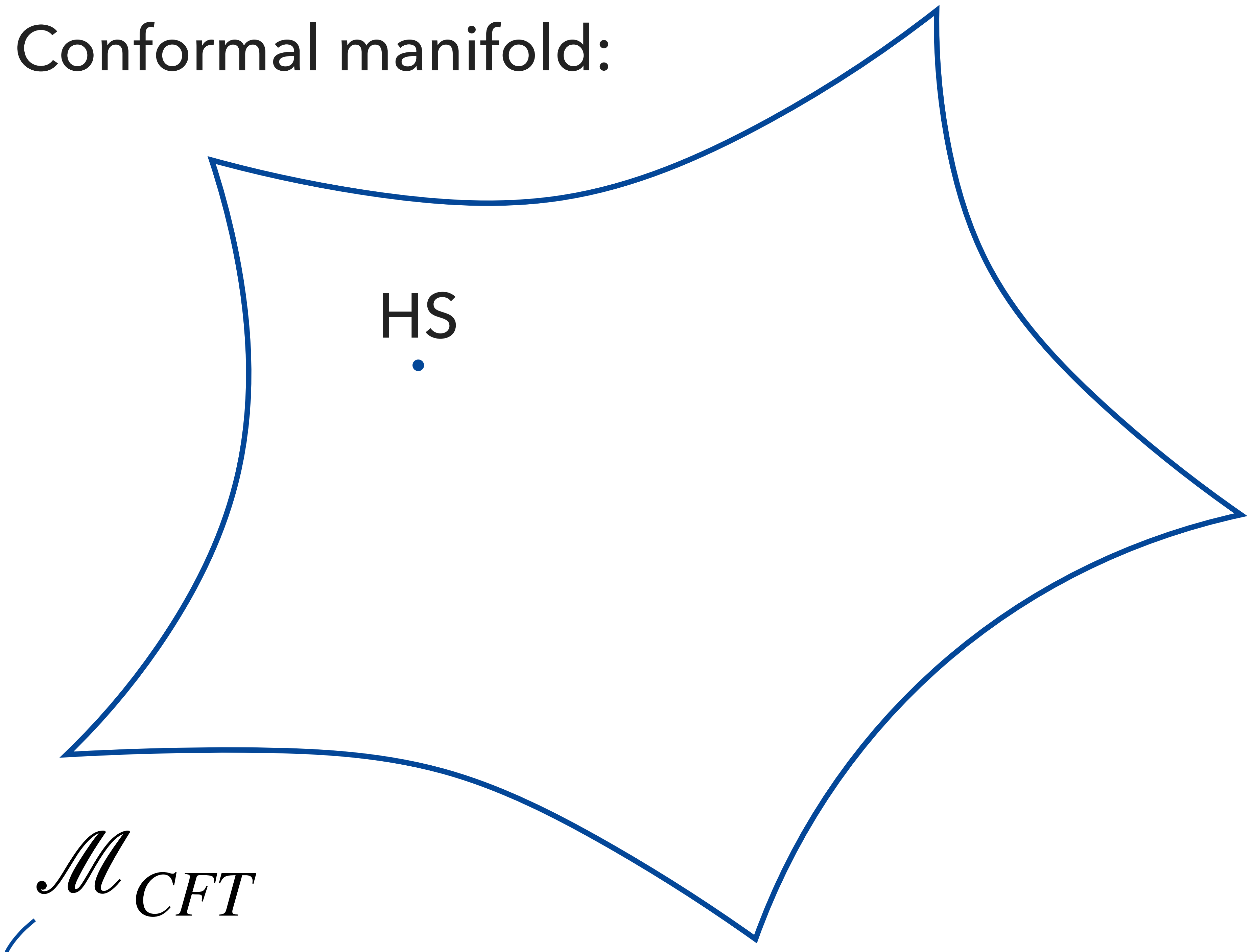

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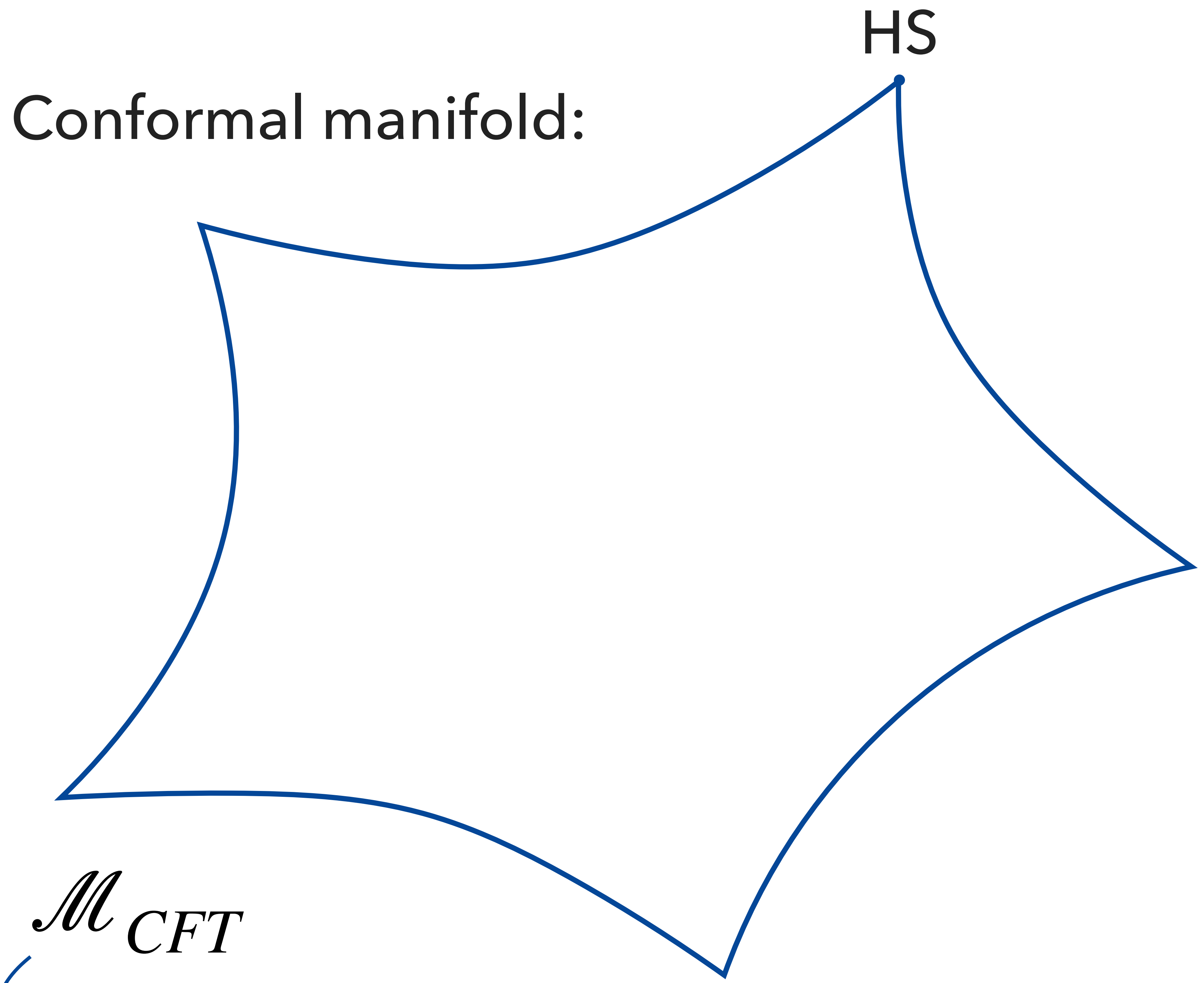

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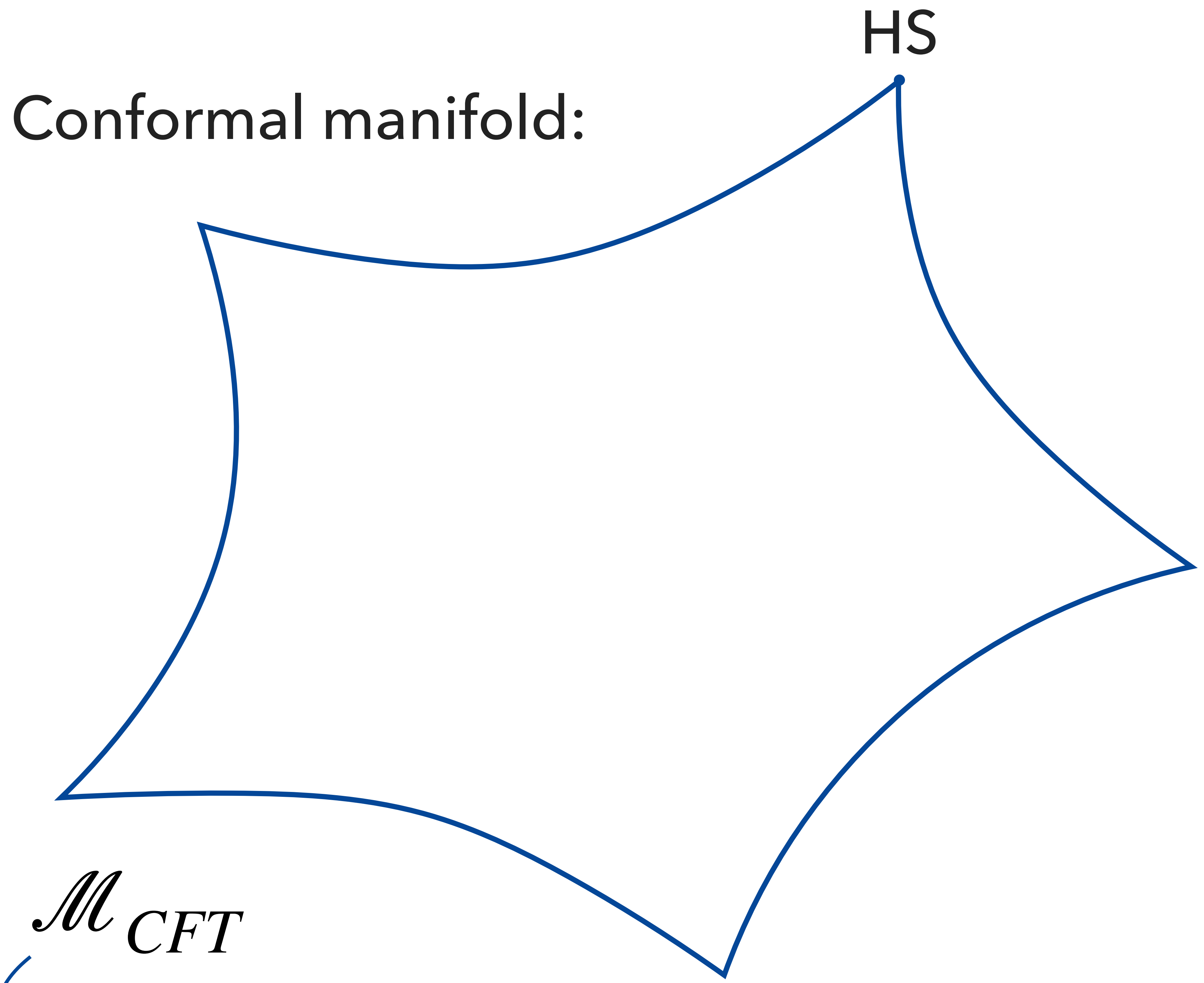

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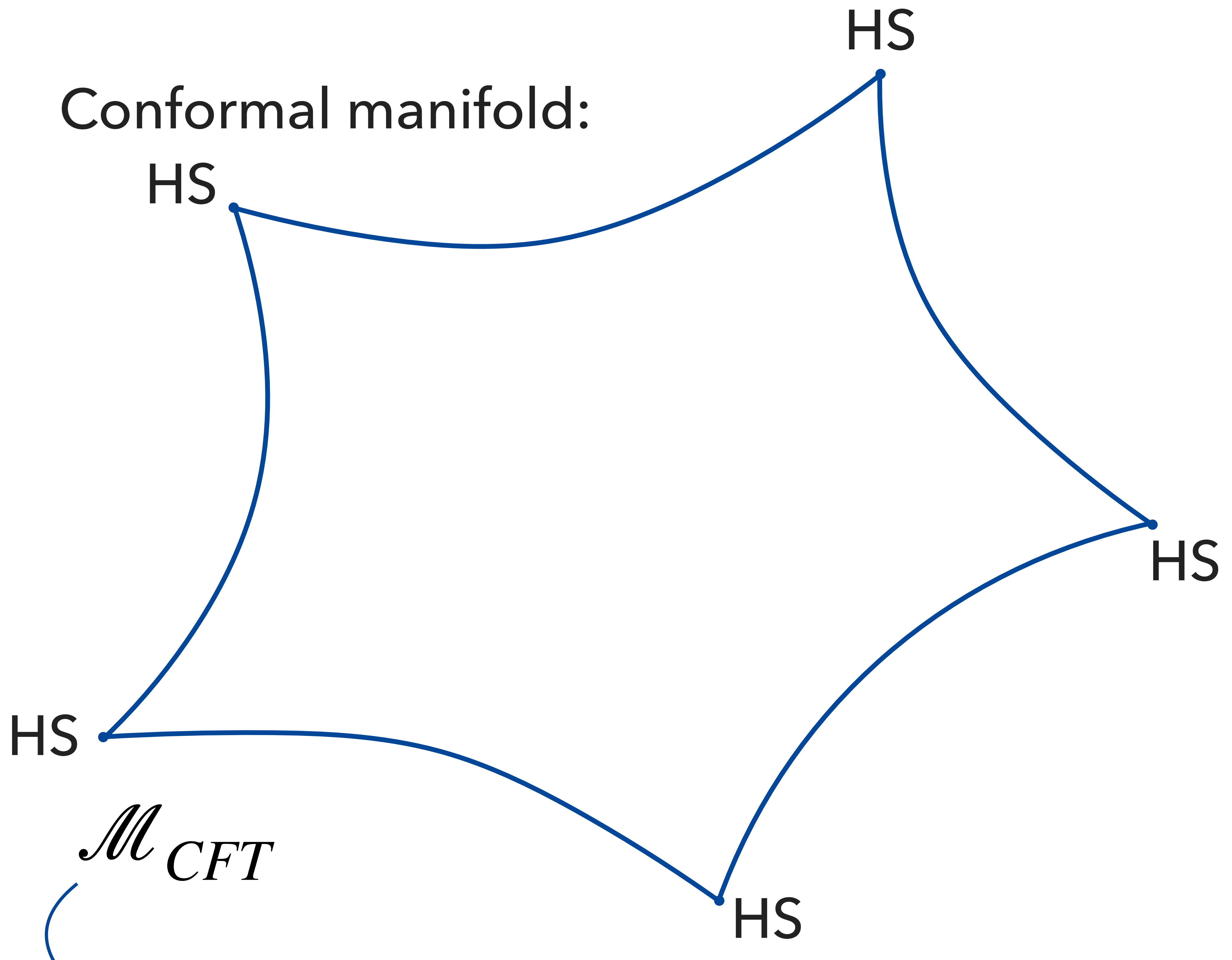

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Zamolodchikov distance

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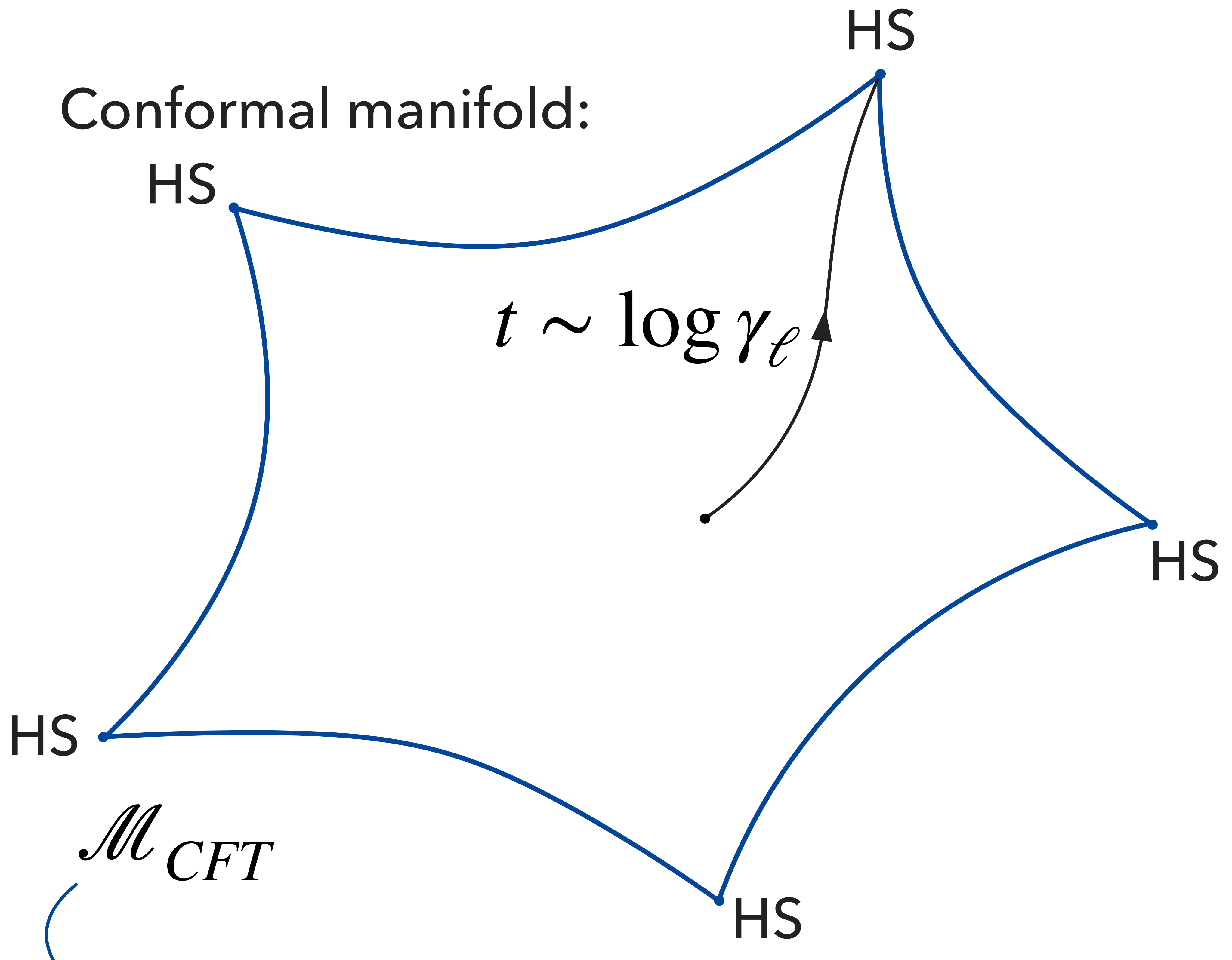

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Towards a CFT Distance Theorem

Can we **prove** this using CFT techniques?

[Perlmutter, Rastelli, Vafa, Valenzuela '20]

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Conformal perturbation theory + HS symmetry

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Evaluated at HS point!

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! No extra assumption, e.g., no supersymmetry + existence of energy-momentum is crucial!

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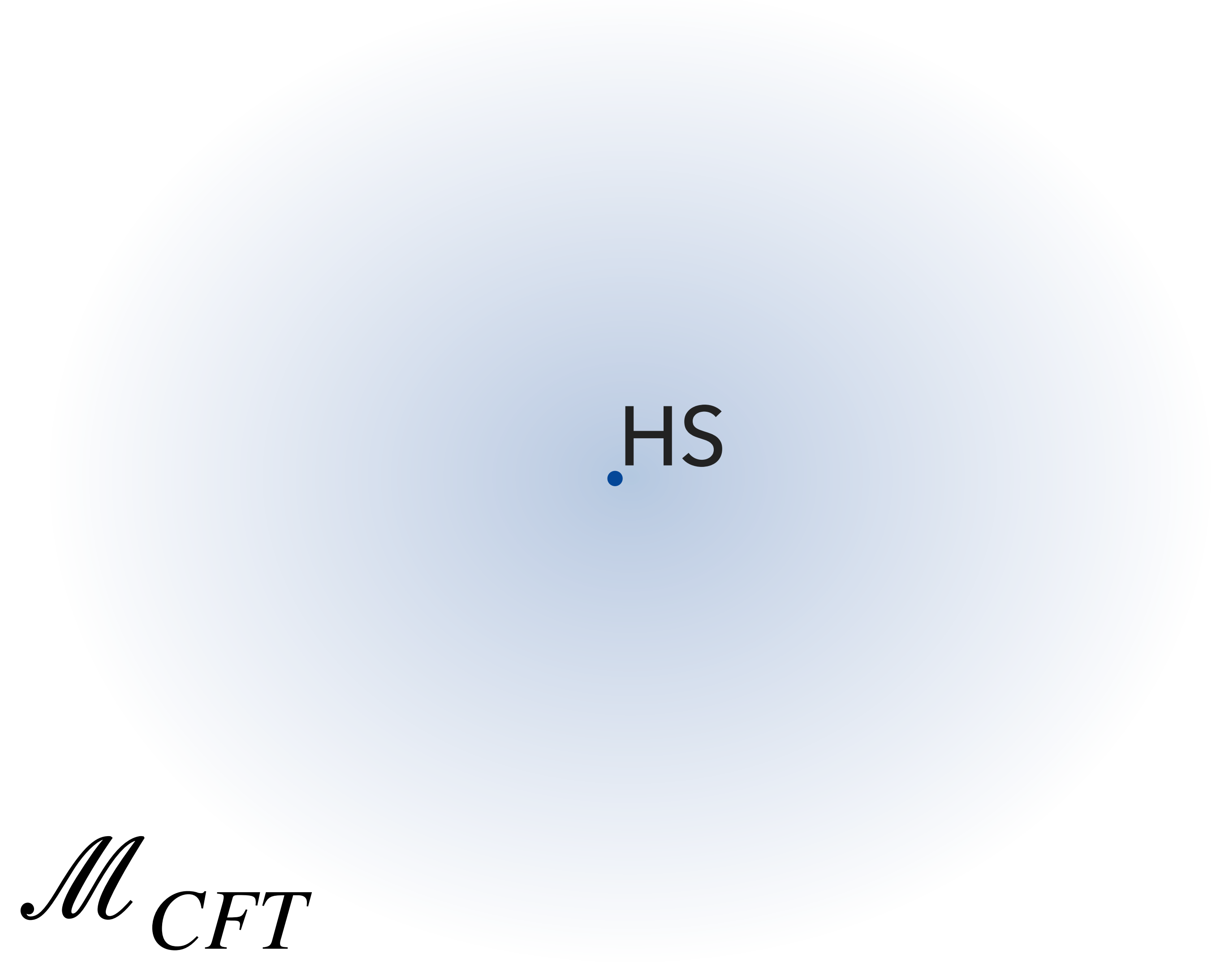

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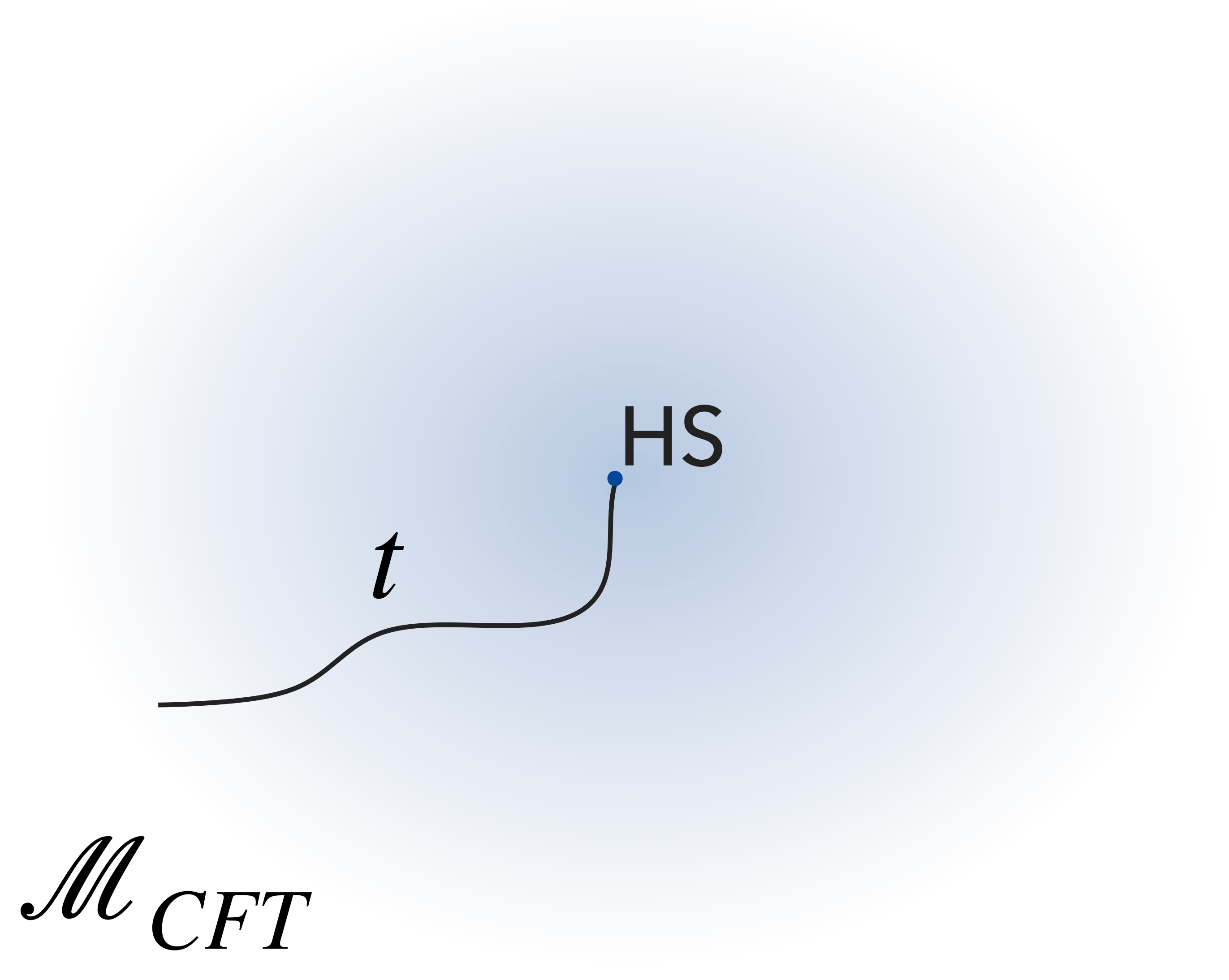

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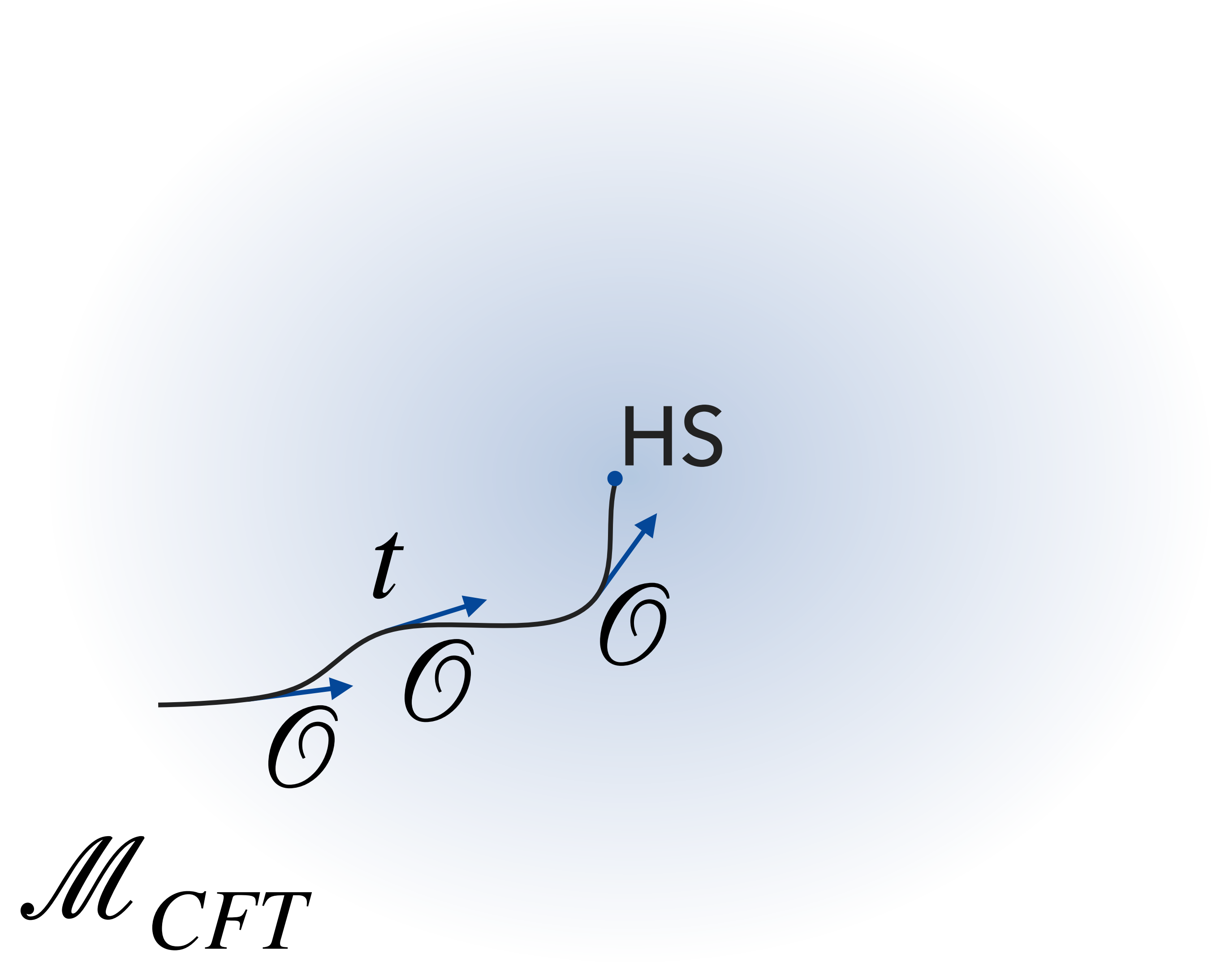

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Conformal perturbation theory

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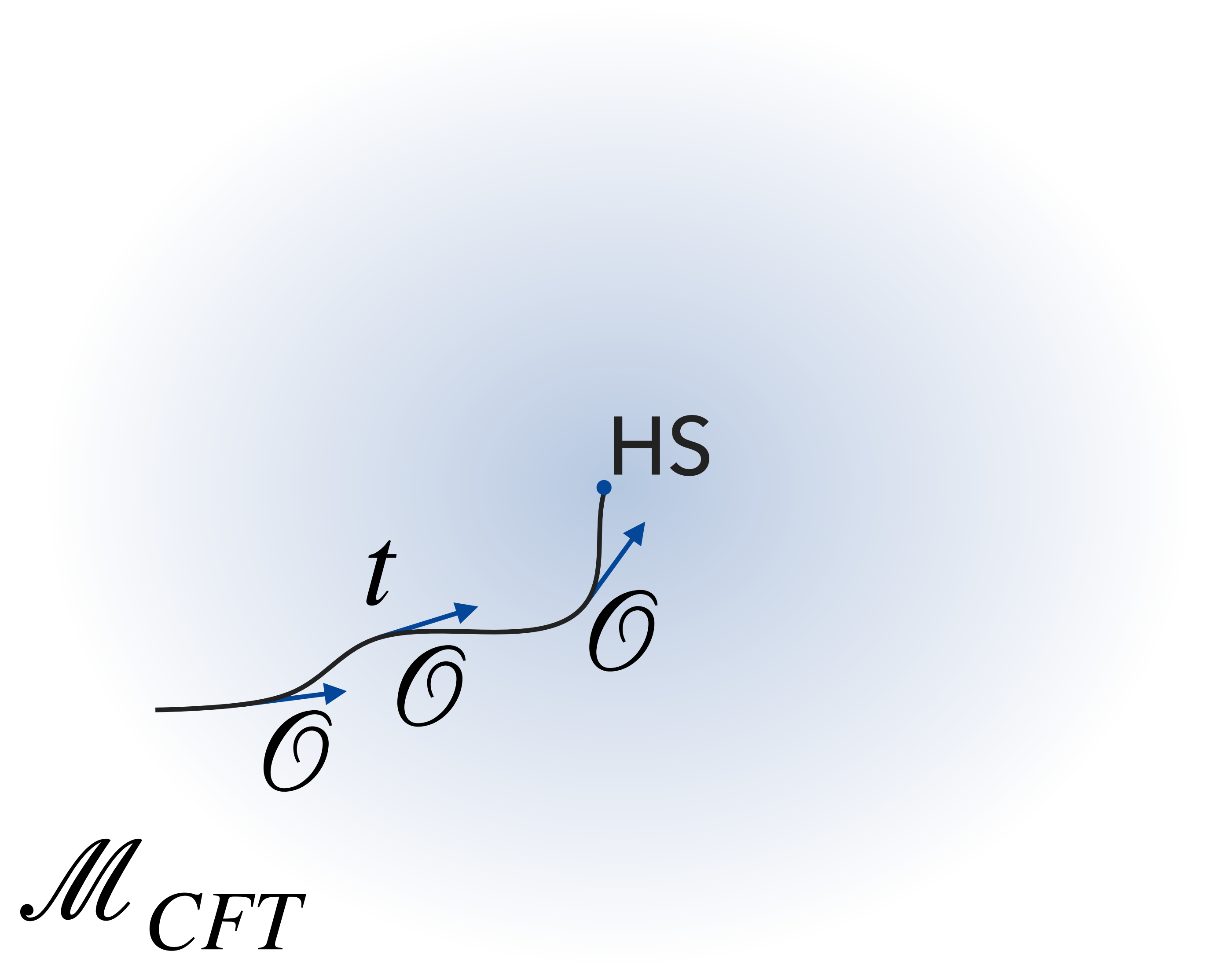

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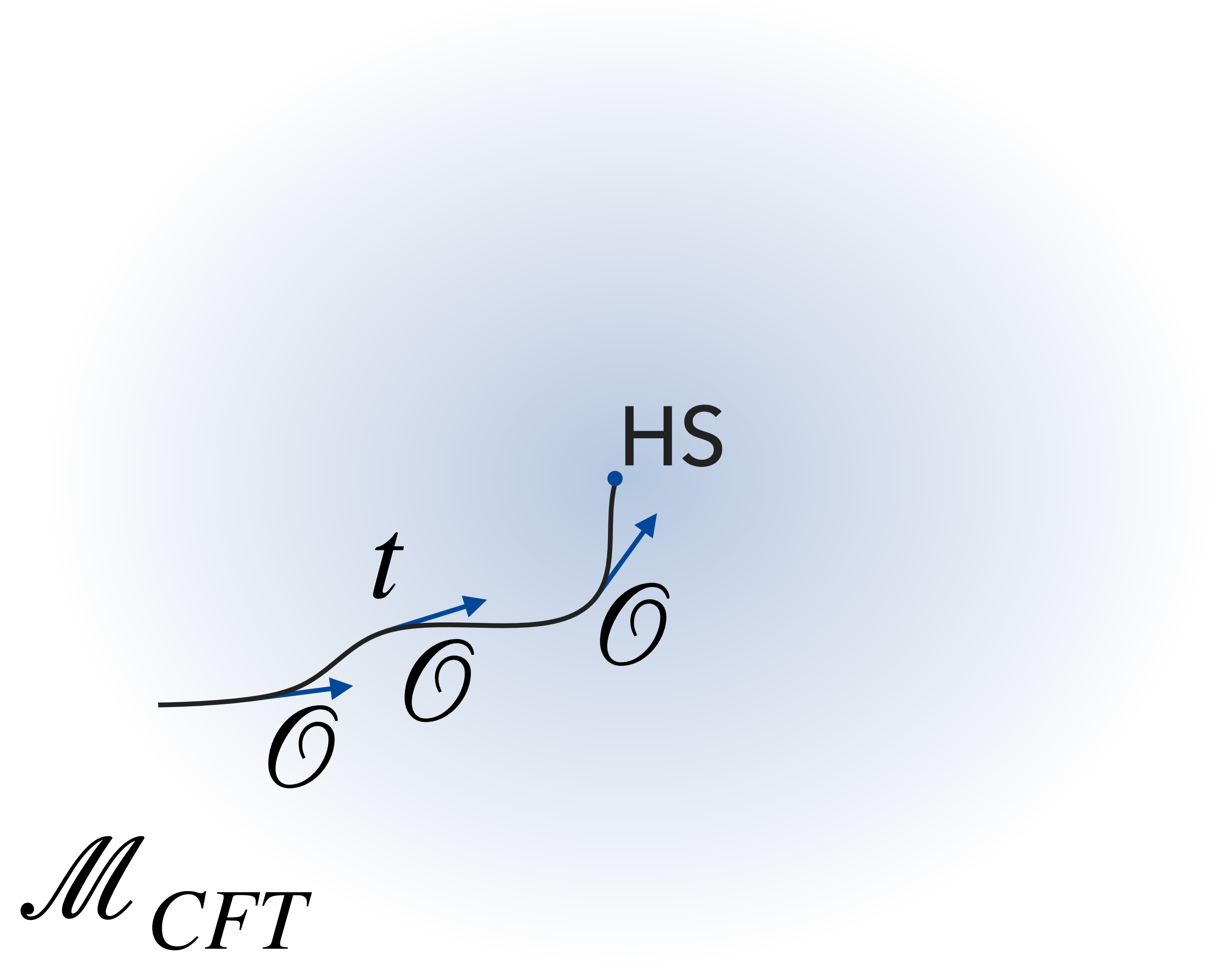

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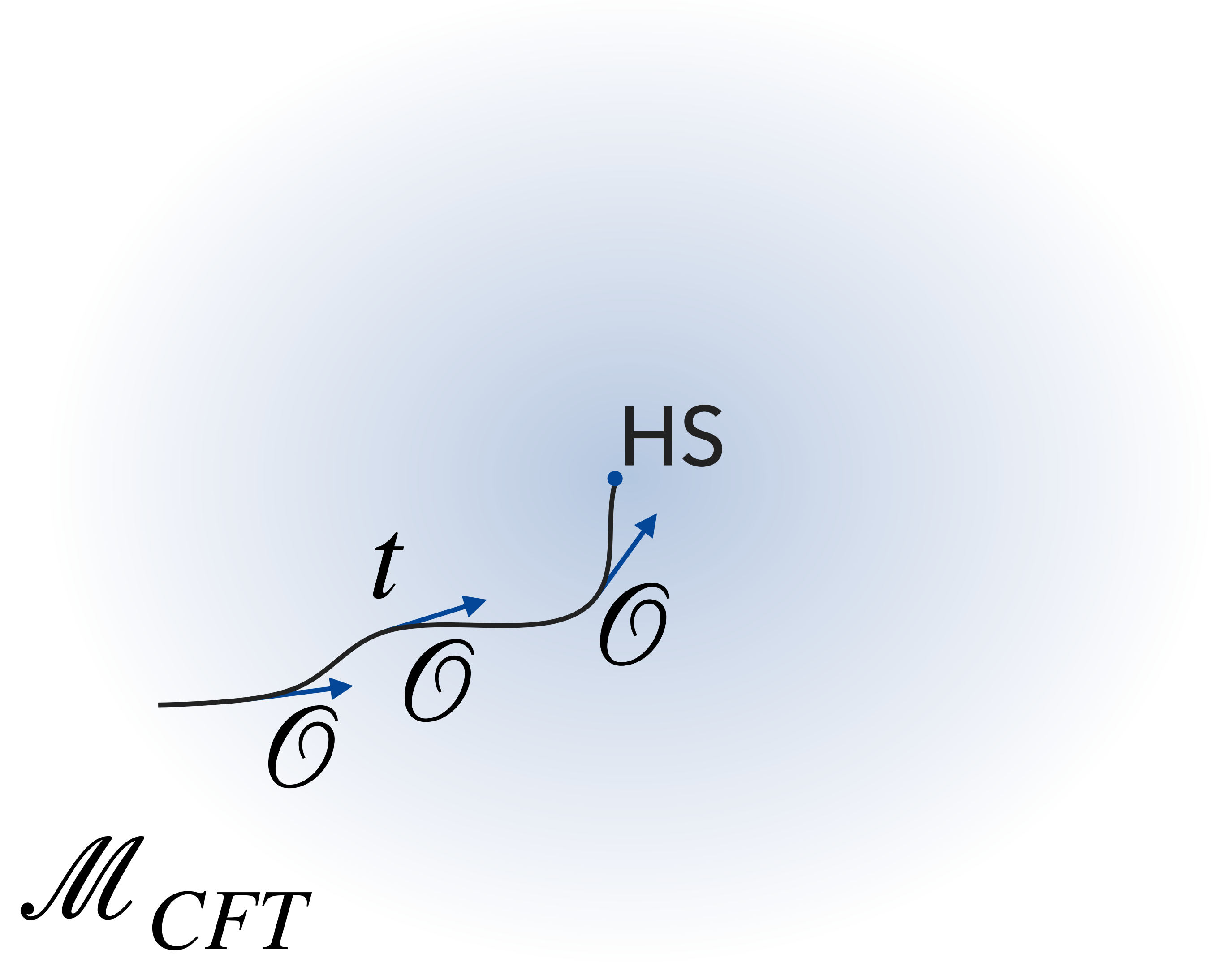

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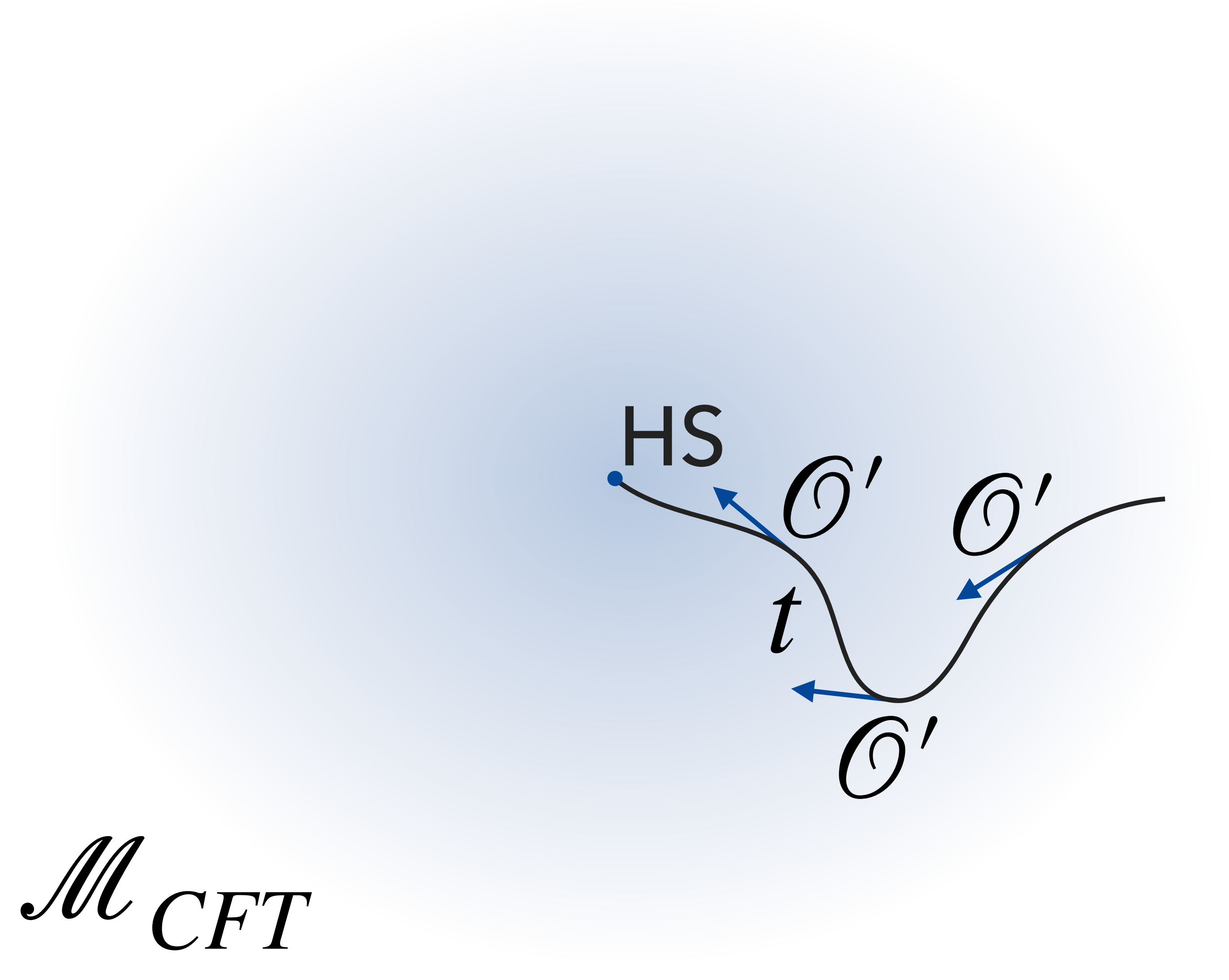

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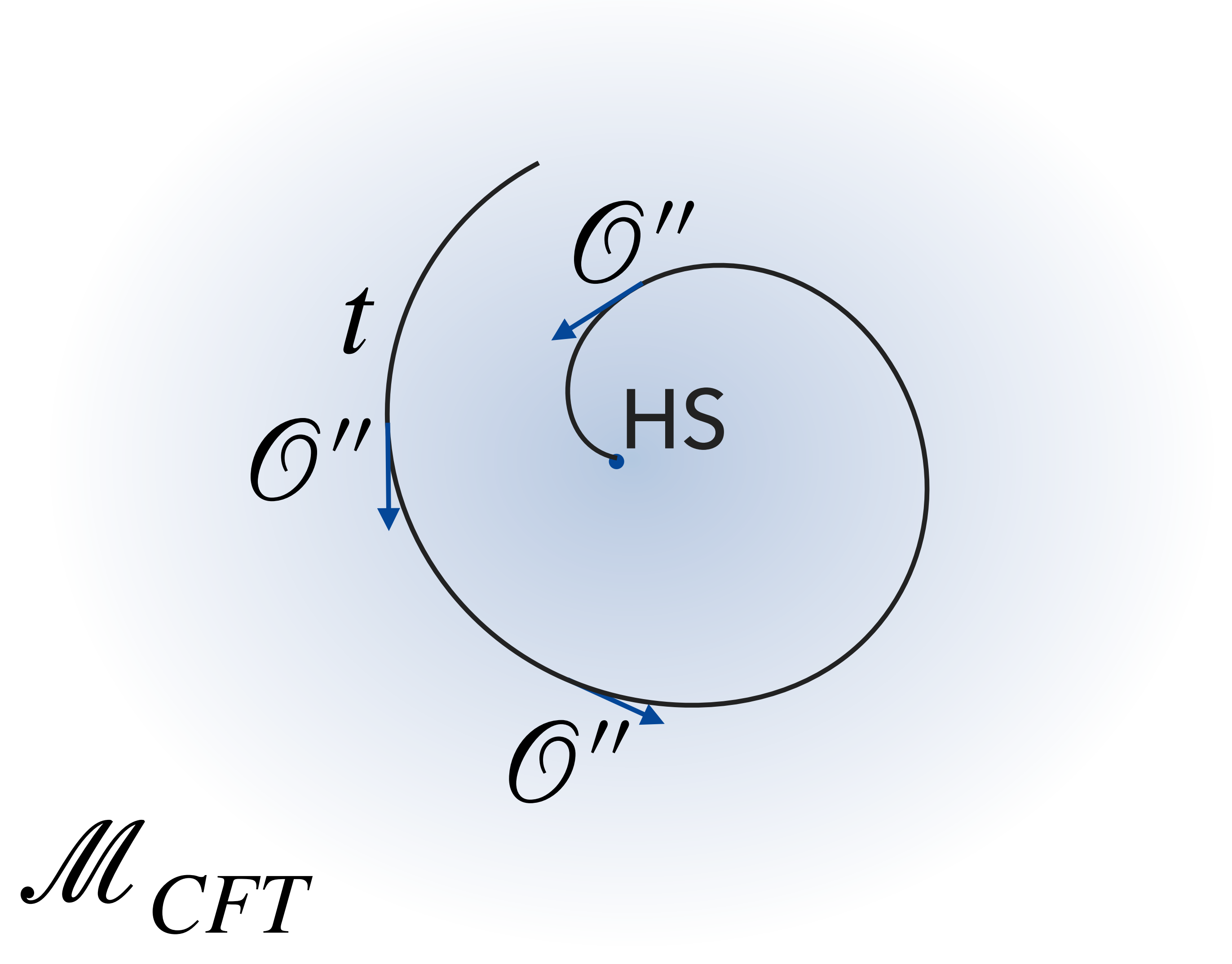

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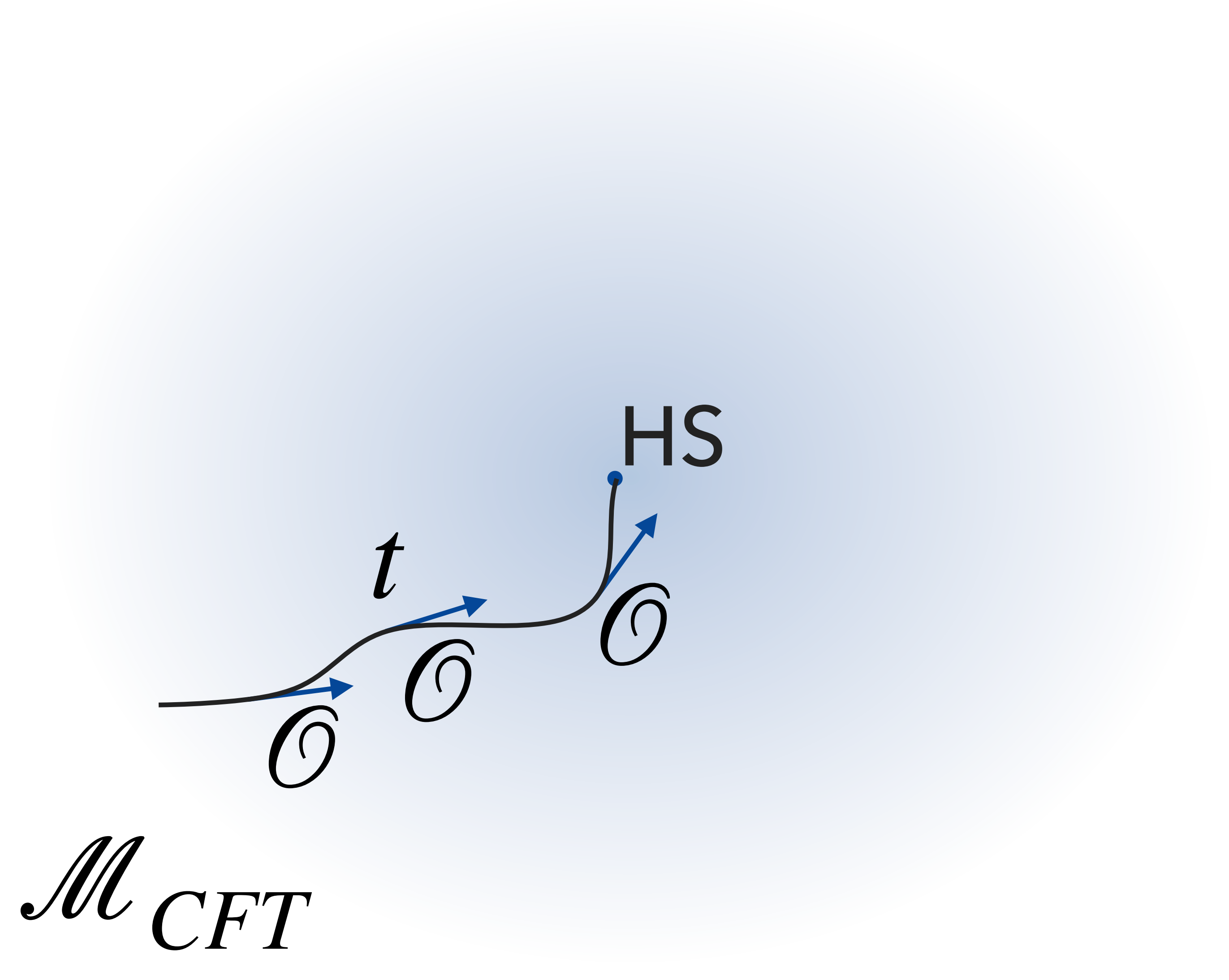

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Zamolodchikov distance!
(given $\langle \mathcal{O}\mathcal{O} \rangle \sim 1$)

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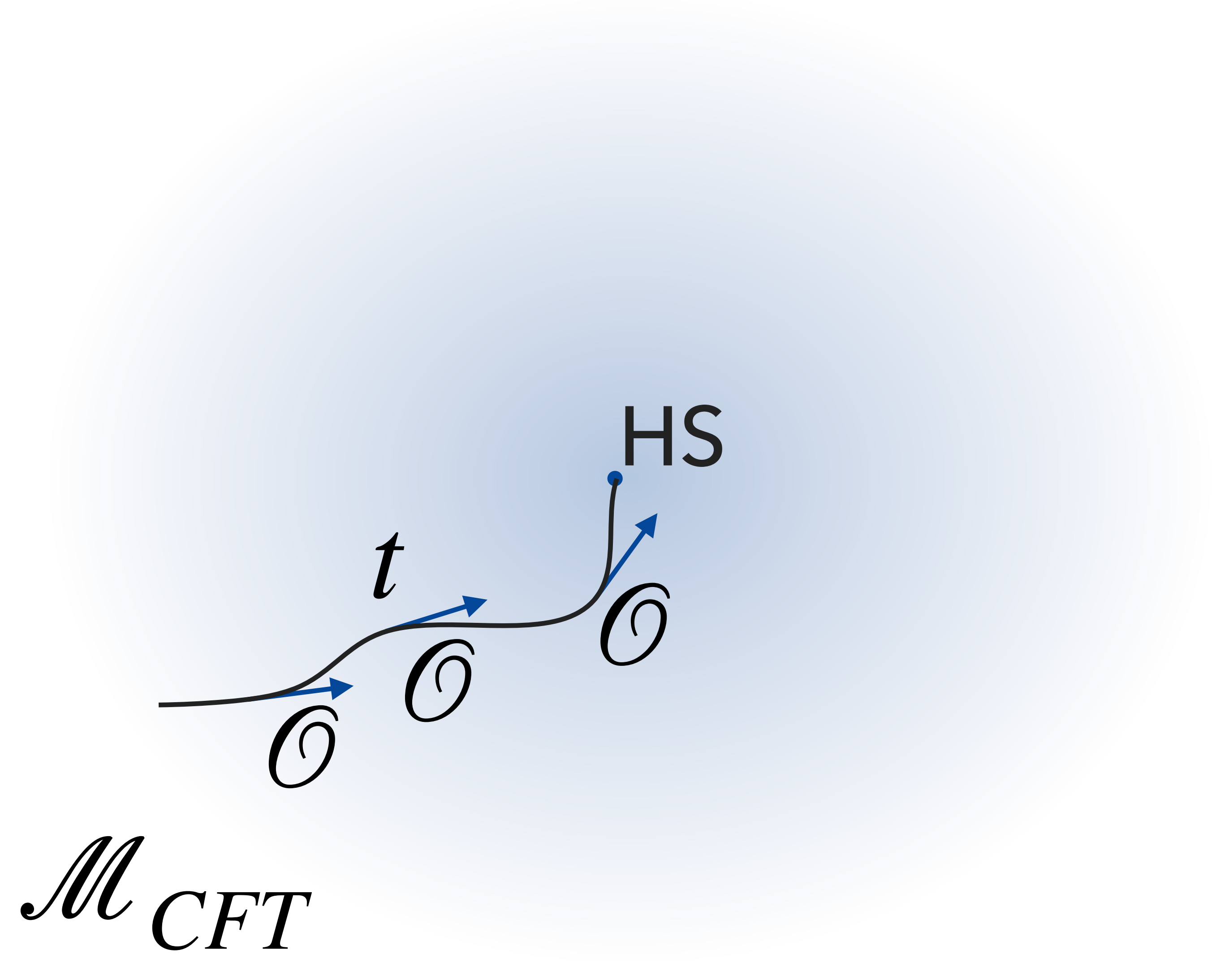

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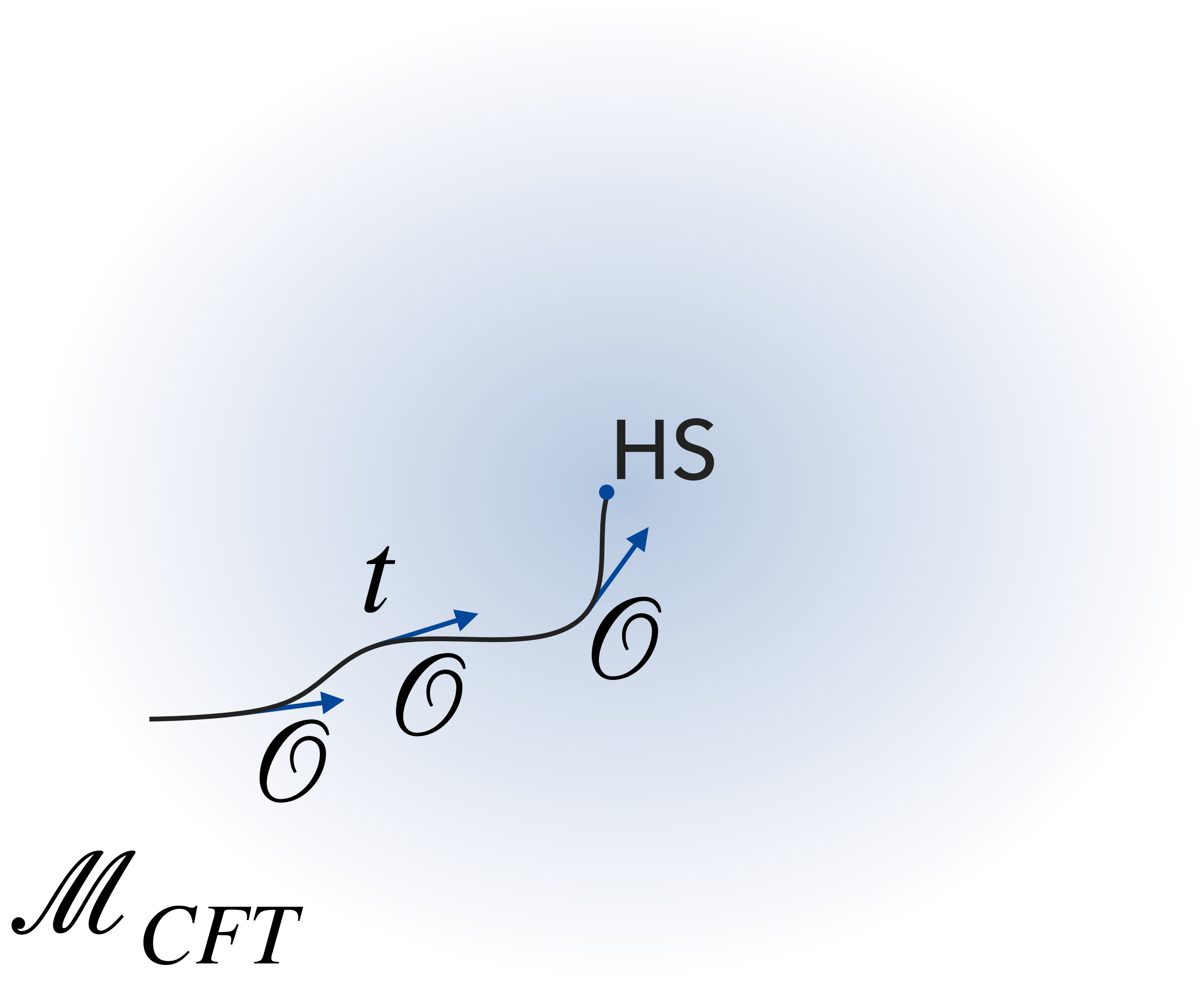

(Weakly-broken) HS symmetry

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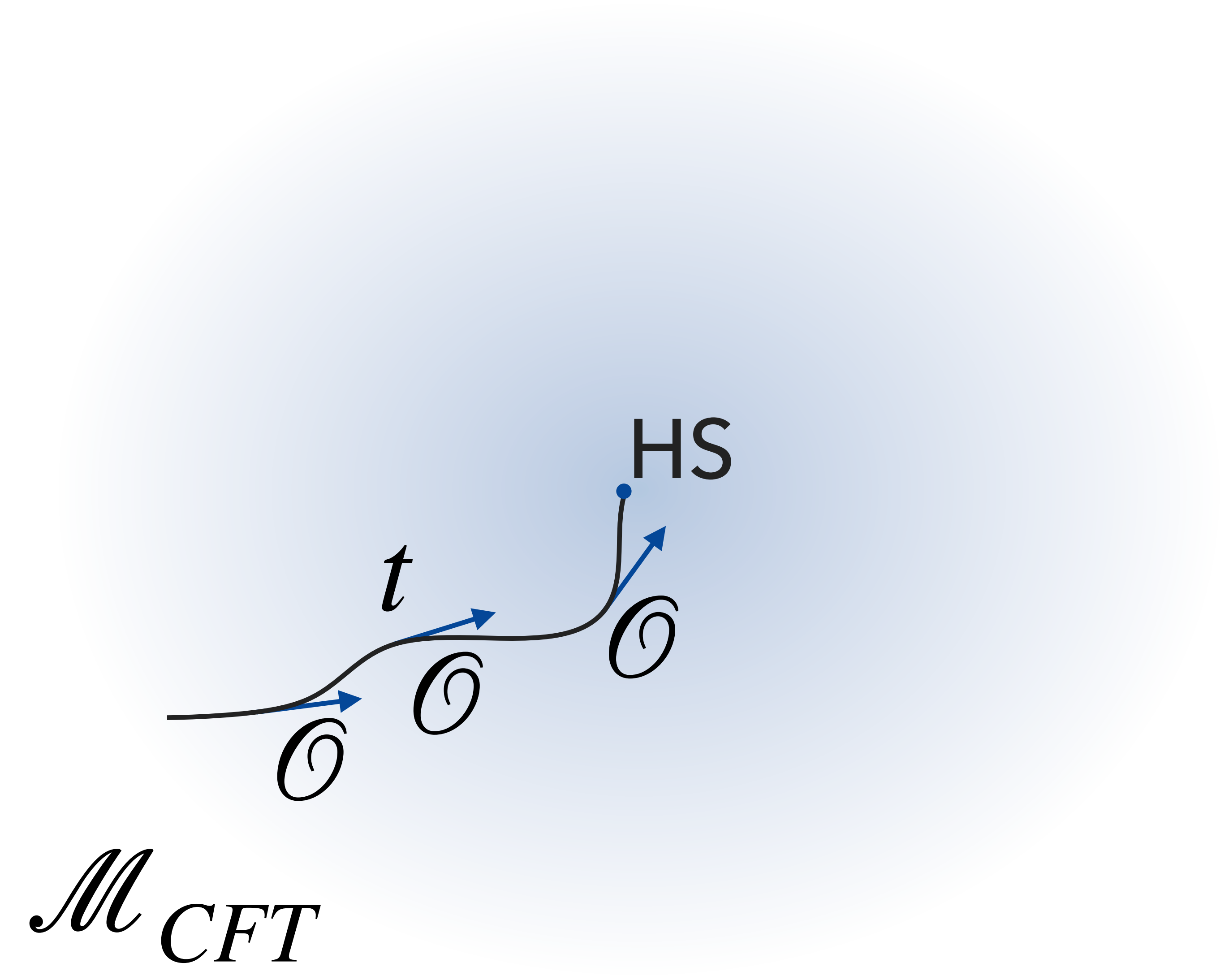
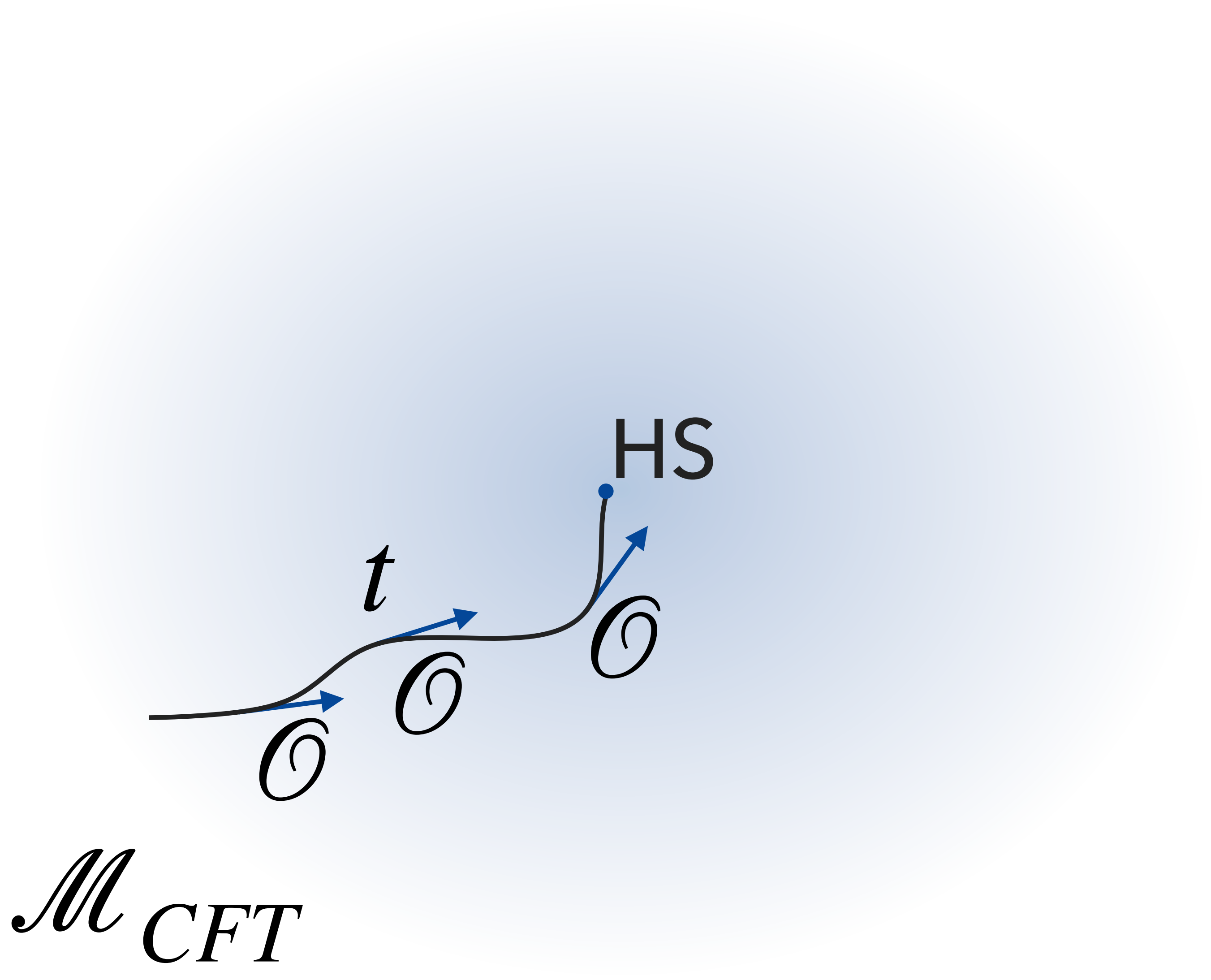


(Weakly-broken) HS symmetry

$$\partial \cdot J_\ell = g(t)\, K_{\ell-1}$$

Sketch of the Proof



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$$\partial \cdot J_\ell = g(t) K_{\ell-1}$$

$g(t) \to 0$ as $t \to$ HS

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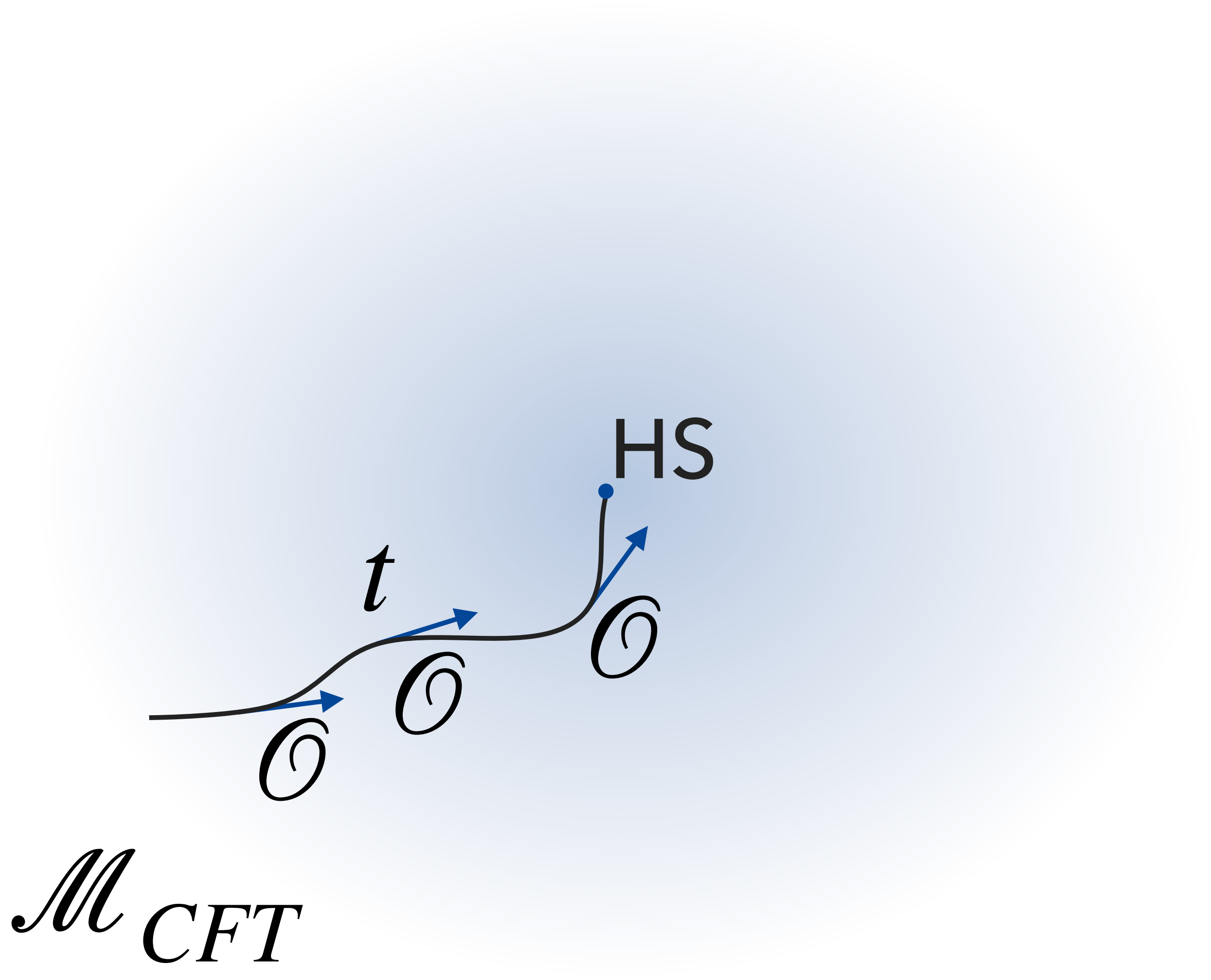

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$$C_{JJ\mathcal{O}} \simeq C^{HS}_{JJ\mathcal{O}} + C^{HS}_{JK\mathcal{O}}\, g + C^{HS}_{KK\mathcal{O}}\, g^2 + \cdots$$

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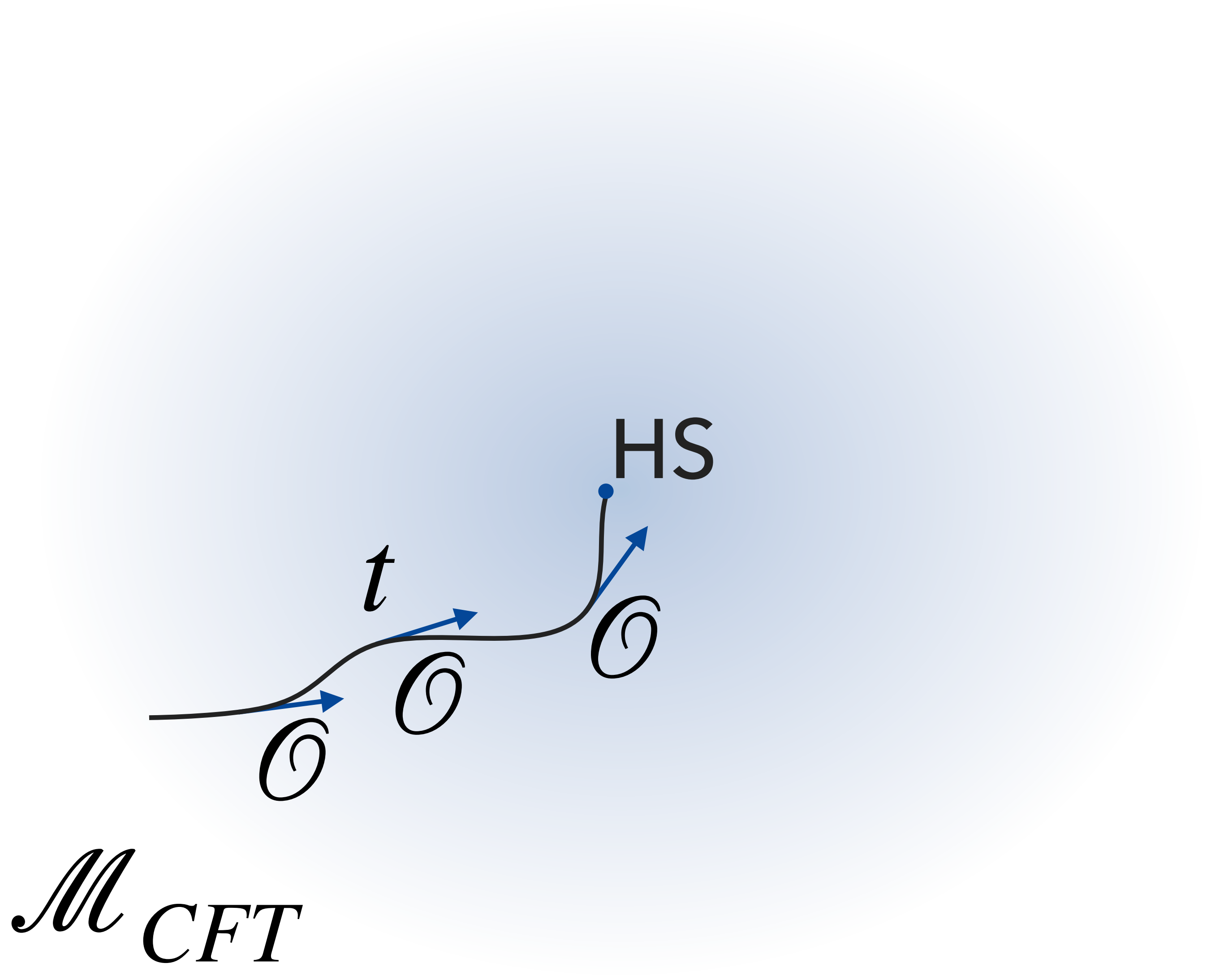

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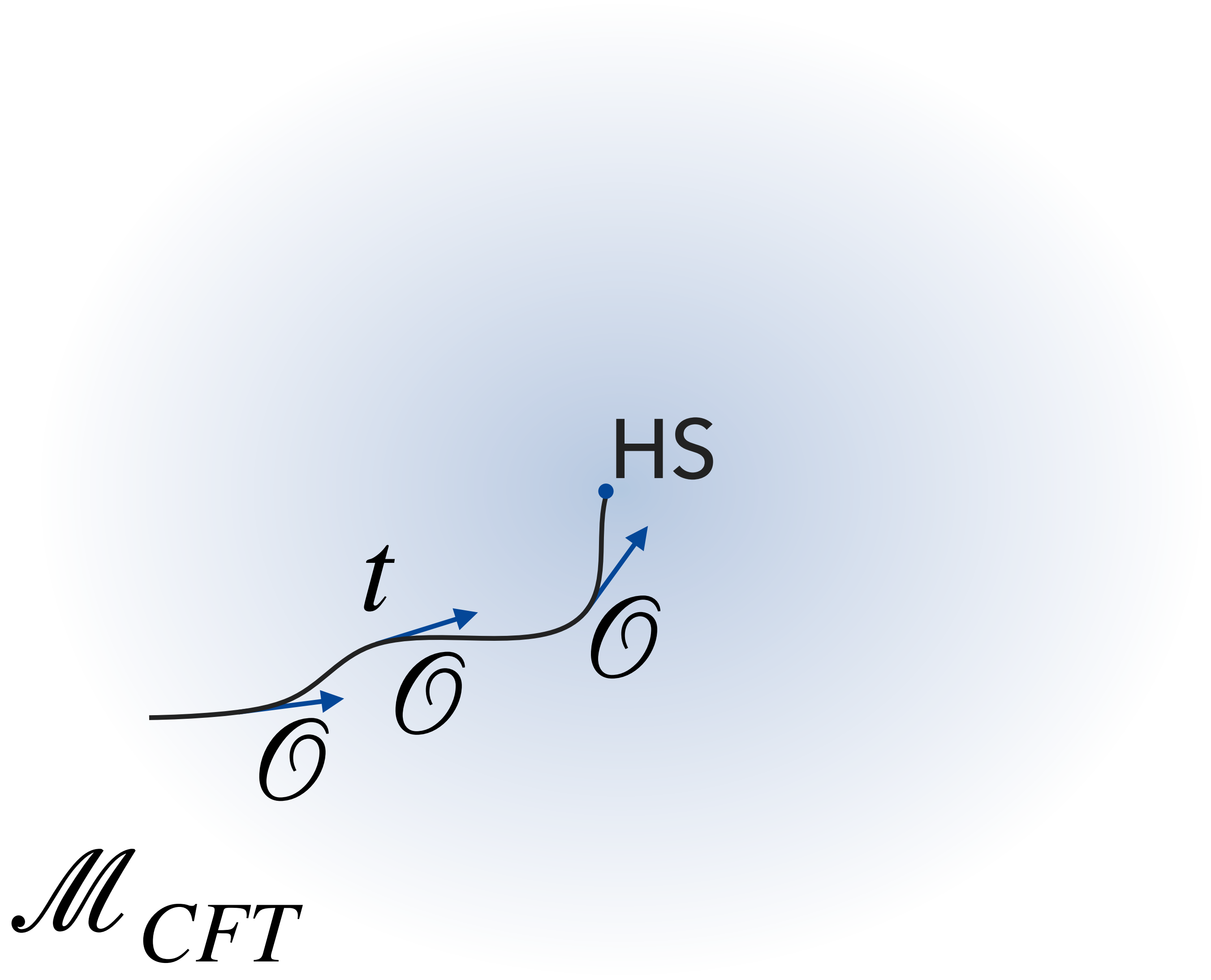

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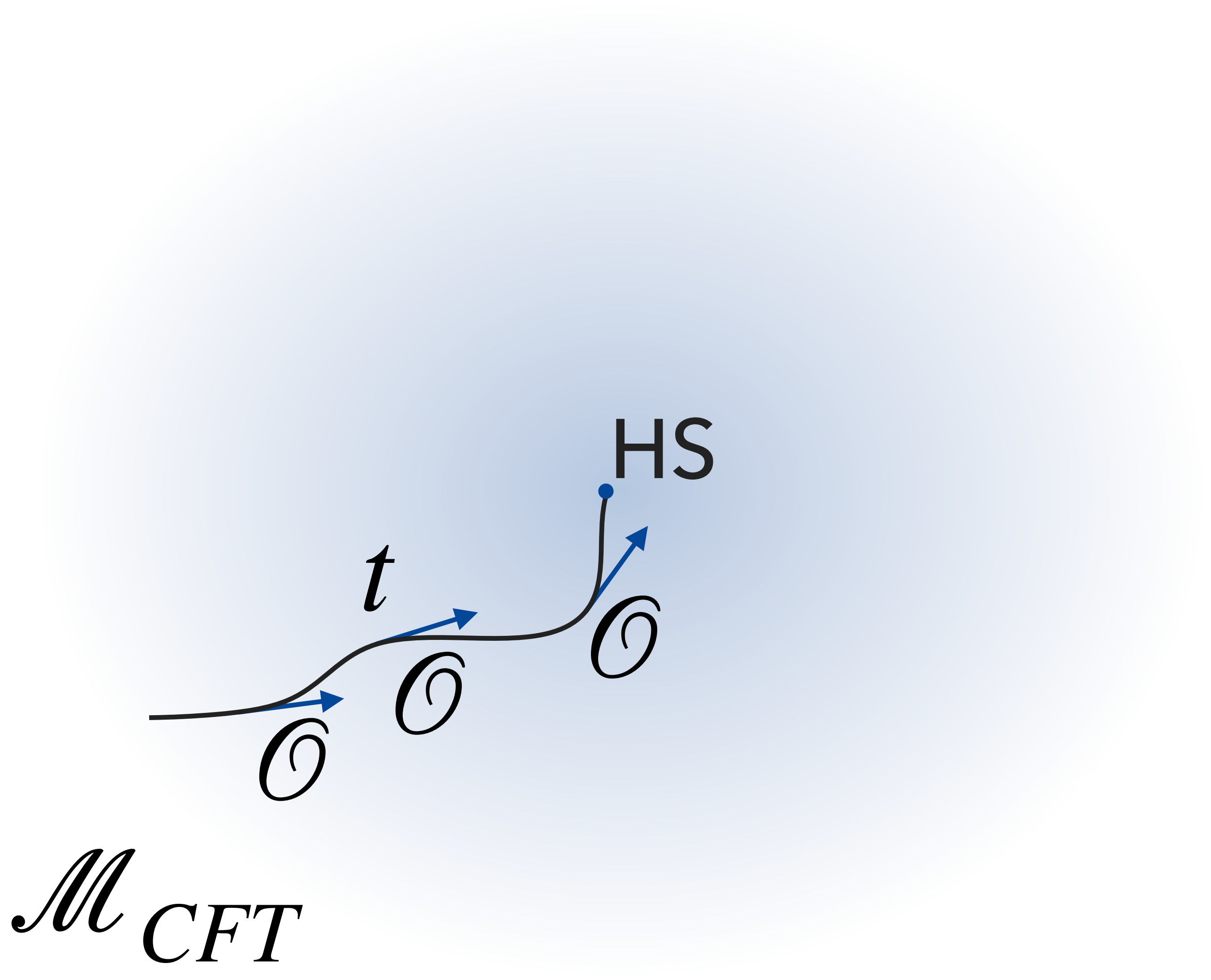

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A bit more complicated than this...

→ More details in [Baume, JCI '23]

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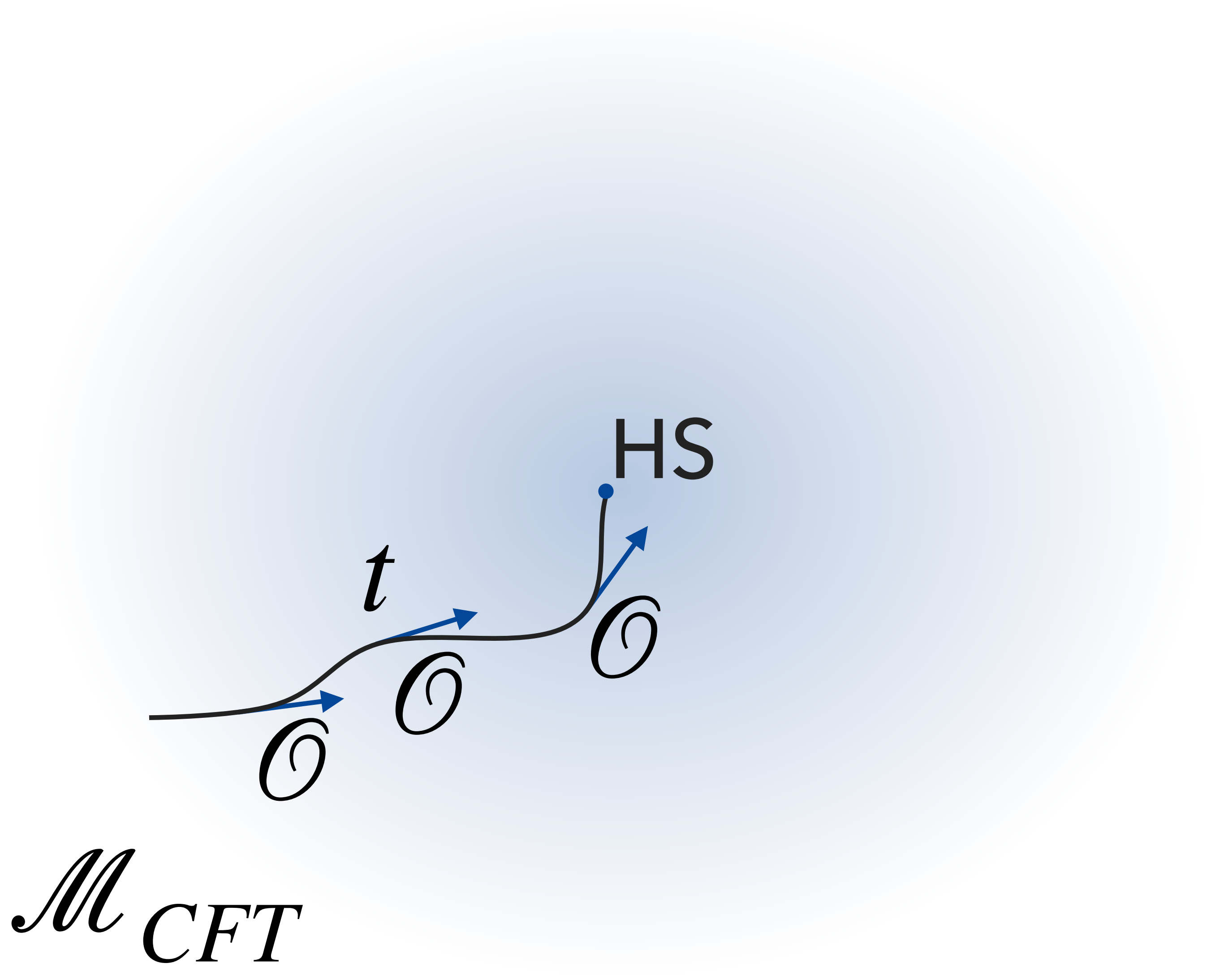

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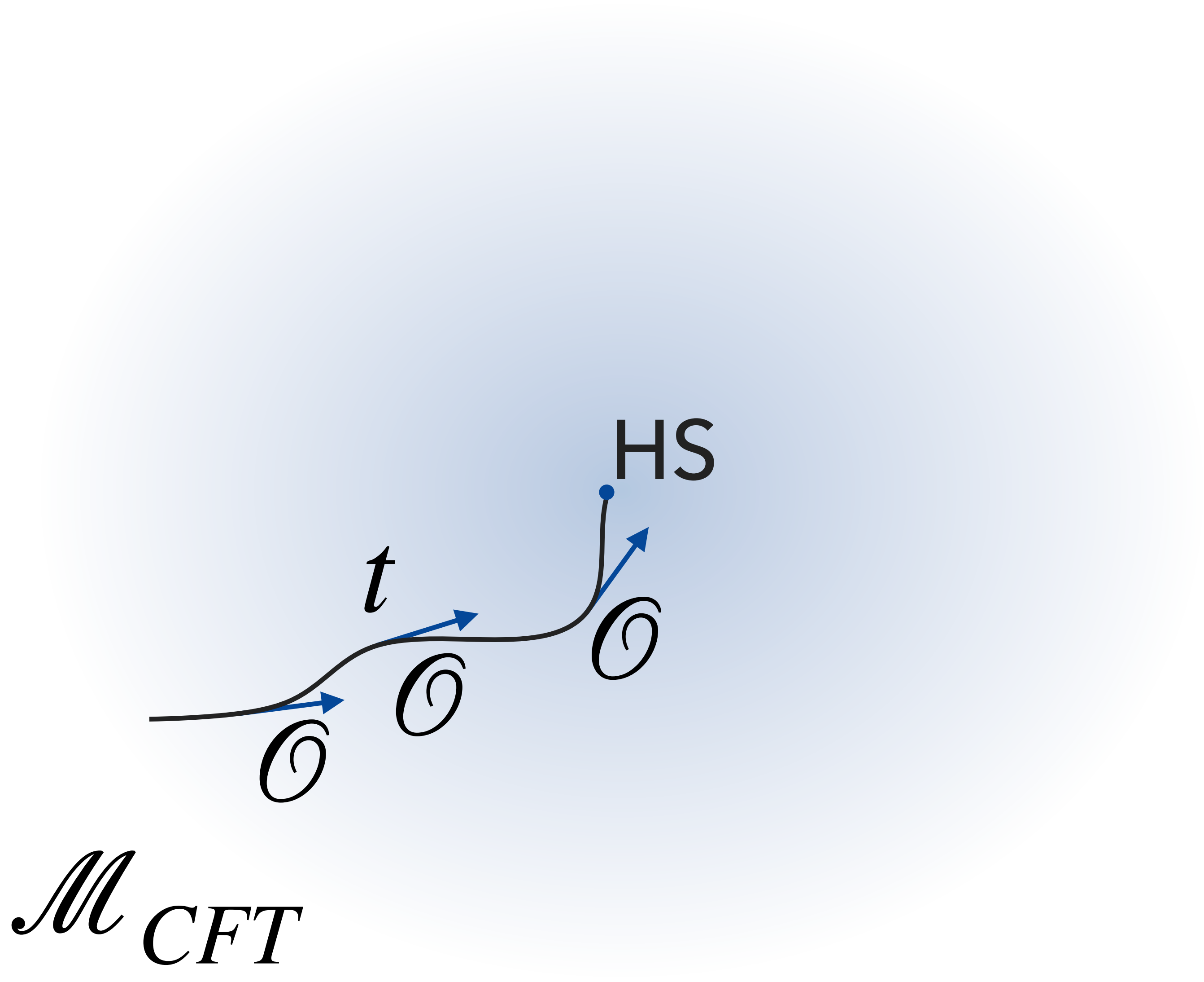

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HS symmetry constraints

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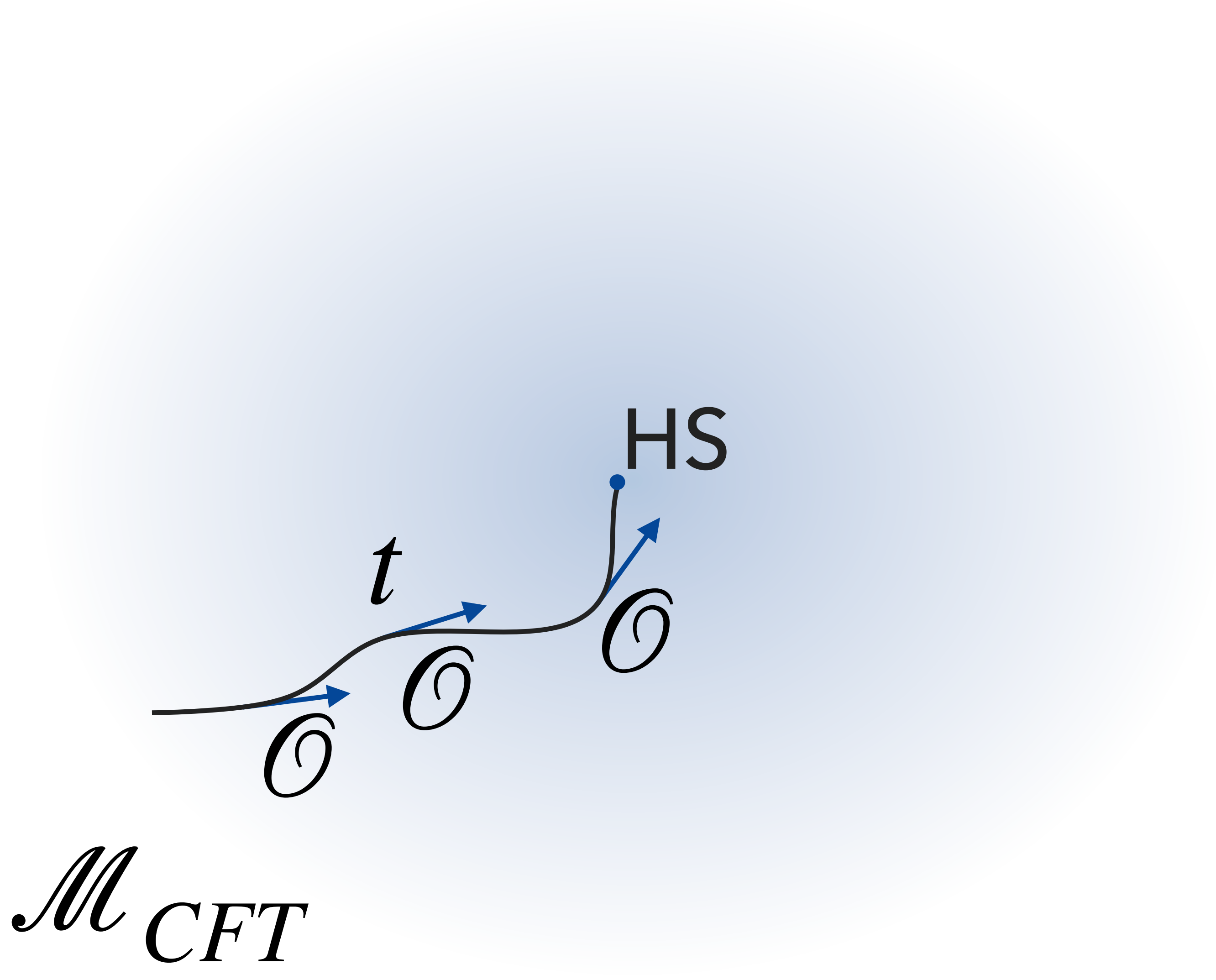

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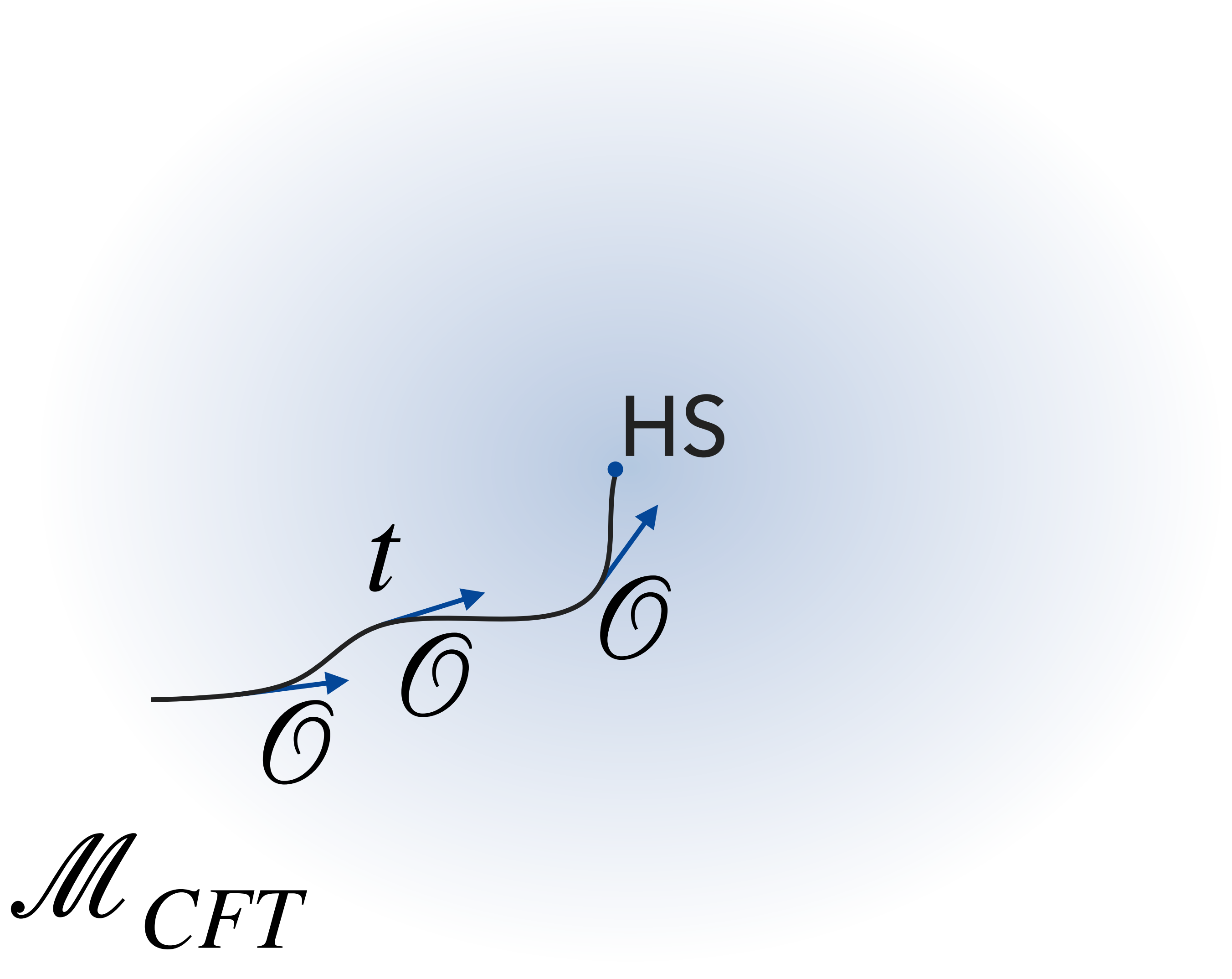
HS symmetry constraints

$$\Big\downarrow\ \gamma_\ell \sim g^2$$

$$\boxed{|C_{JJ\mathcal{O}}| \lesssim \gamma_\ell \text{ as } \gamma_\ell \to 0}$$

Sketch of the Proof



Conformal perturbation theory + (Weakly-broken) HS symmetry

$$\boxed{\left|\frac{d\gamma_\ell}{dt}\right| = \left|C_{JJ\mathcal{O}}\right| \lesssim \gamma_\ell \text{ as } \gamma_\ell \to 0}$$

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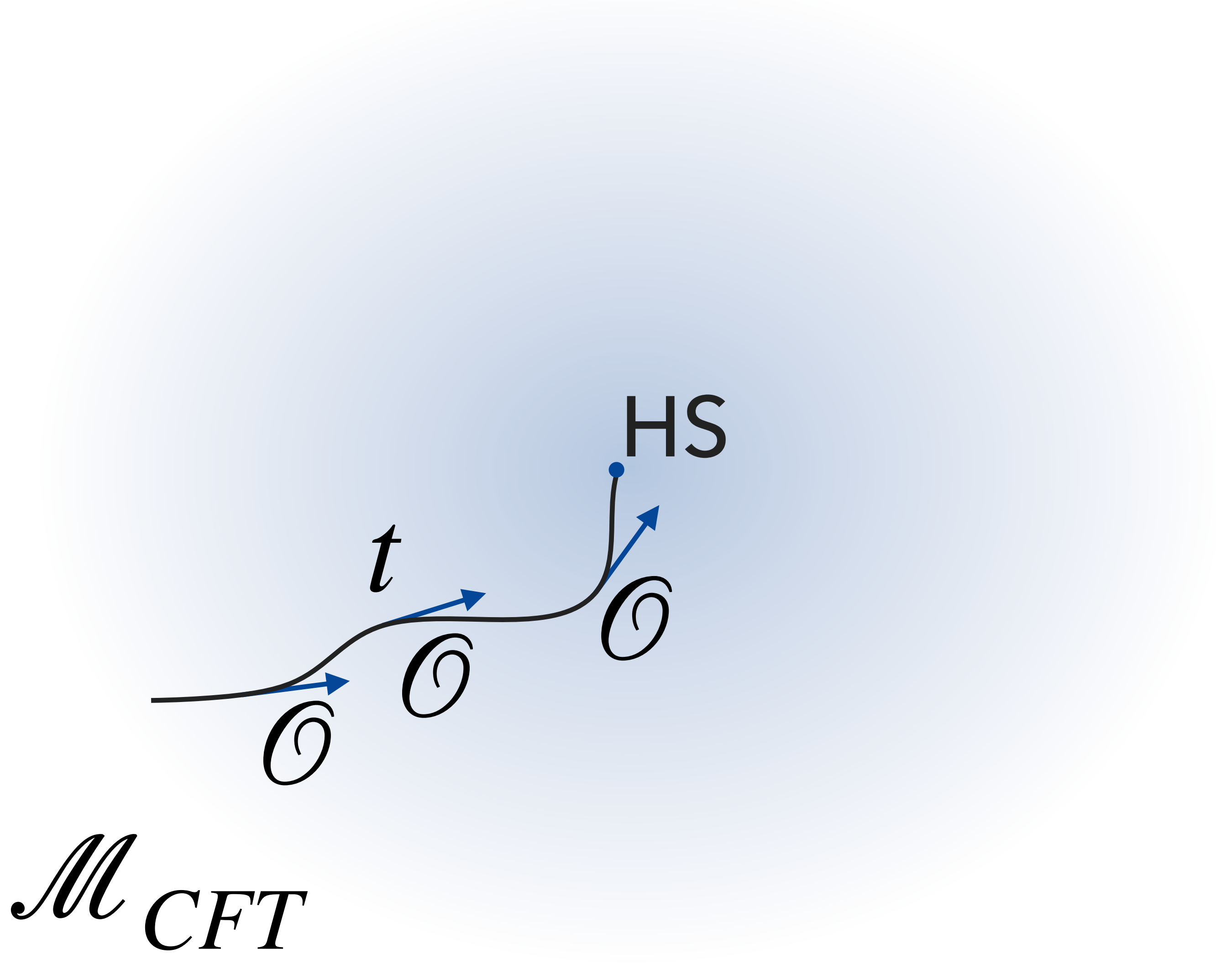

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Asymptotic version of bound in [van de Heisteeg, Vafa, Wiesner '20] !

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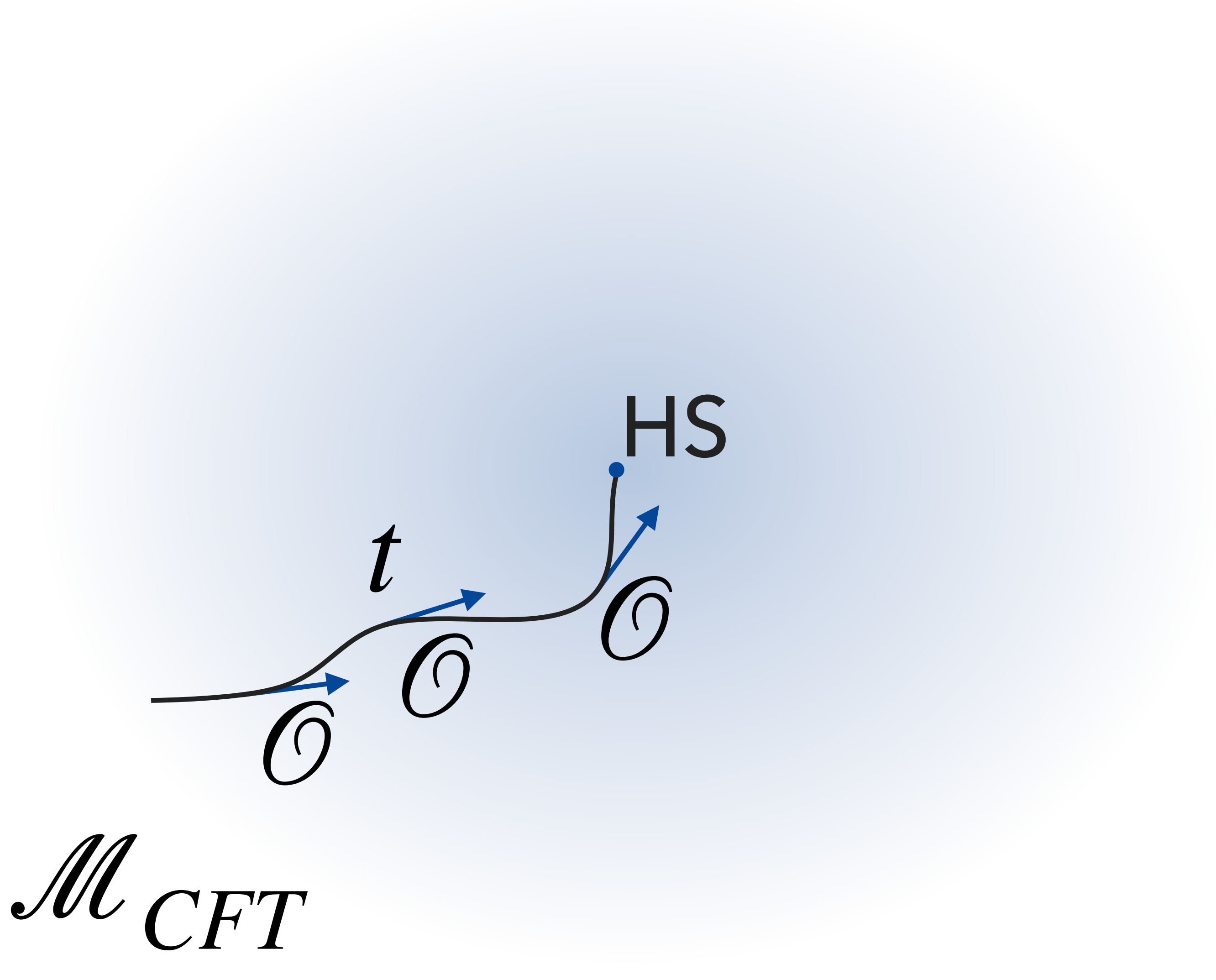

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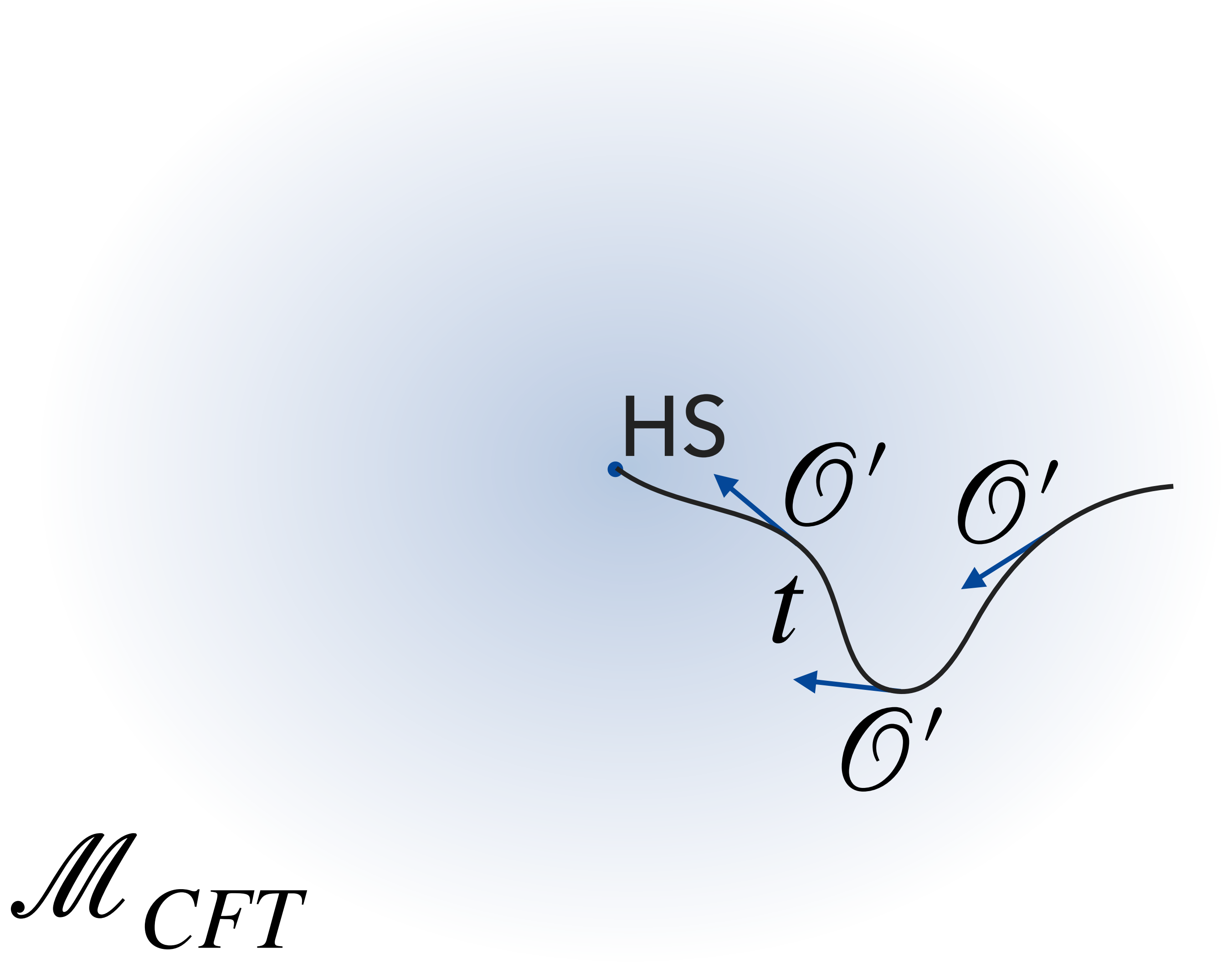

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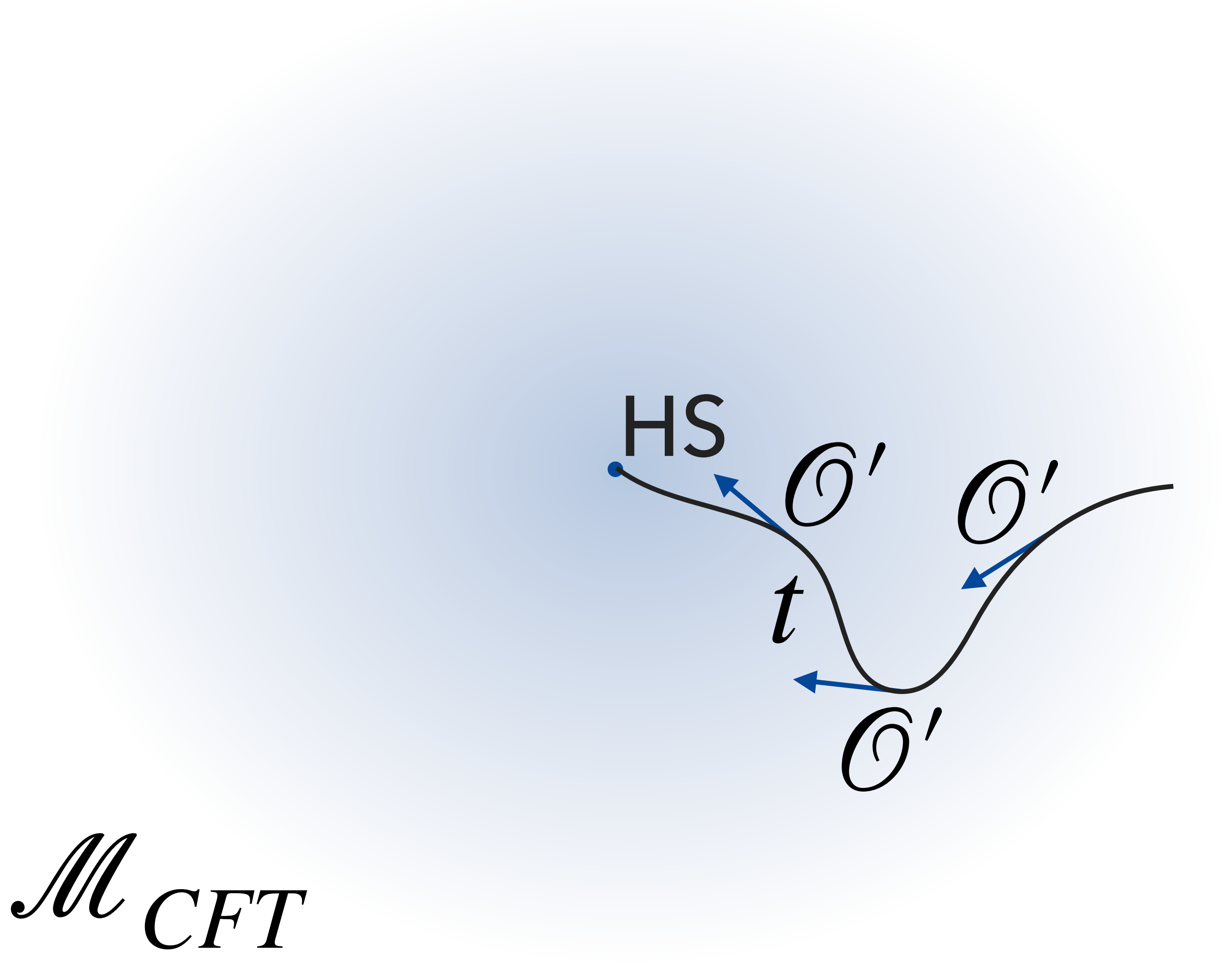

Conformal perturbation theory + (Weakly-broken) HS symmetry

$$\left|\frac{d\gamma_\ell}{dt}\right| = \left|C_{JJ\mathcal{O}'}\right| \lesssim \gamma_\ell \text{ as } \gamma_\ell \to 0$$

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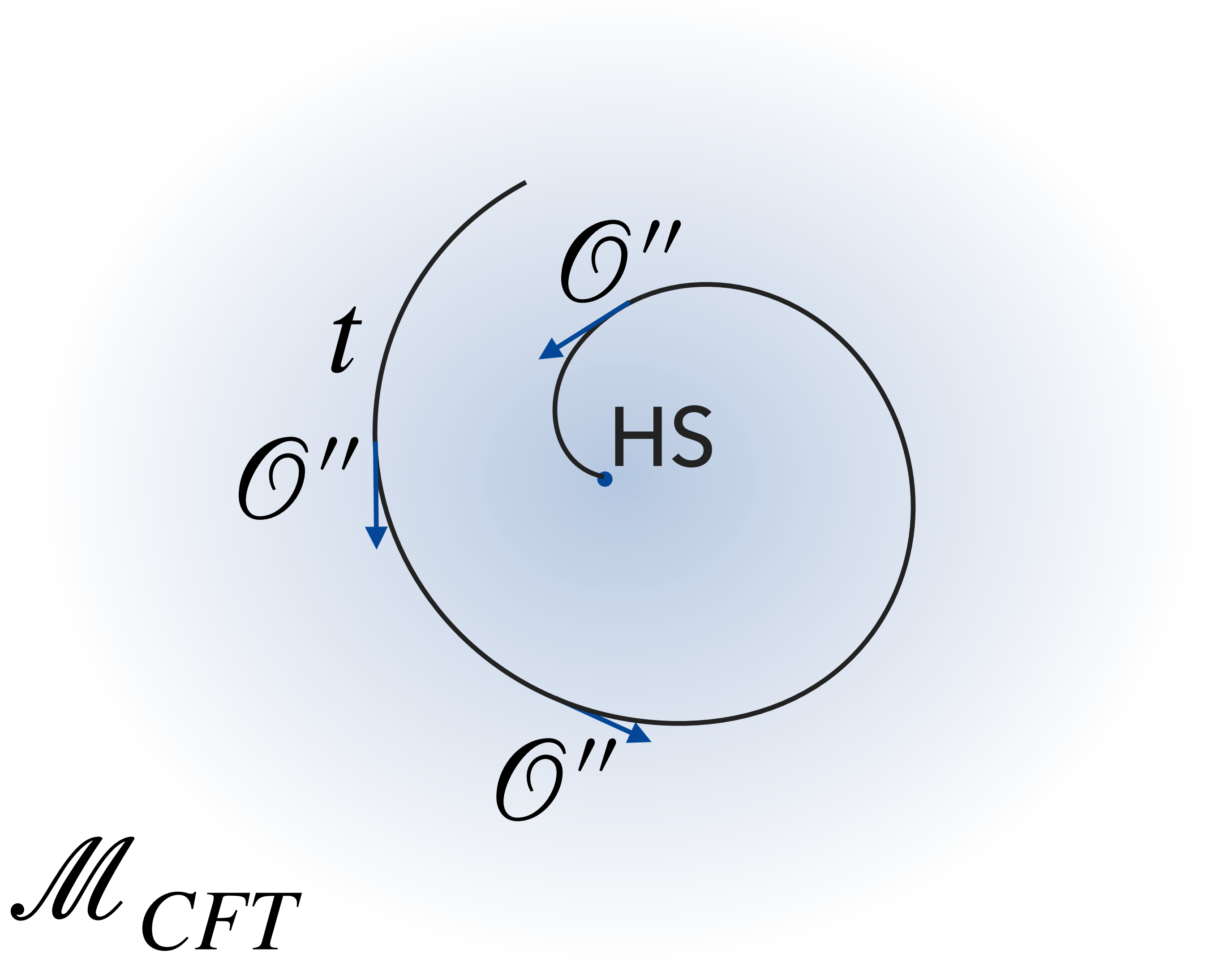
Sketch of the Proof



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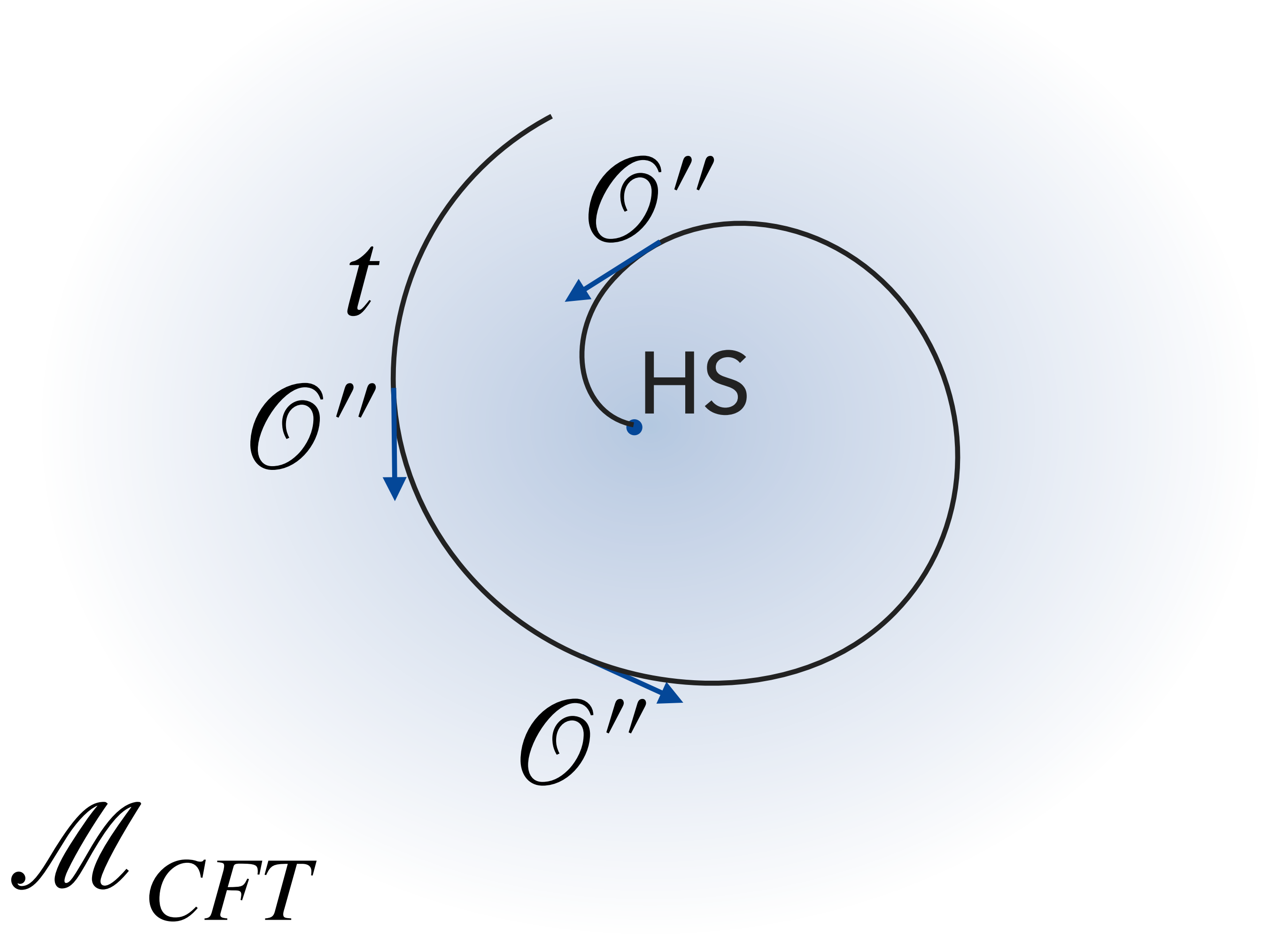

Conformal perturbation theory + (Weakly-broken) HS symmetry

$$\left|\frac{d\gamma_\ell}{dt}\right| = \left|C_{JJ\mathcal{O}''}\right| \lesssim \gamma_\ell \text{ as } \gamma_\ell \to 0$$

$\gamma_\ell \to 0$ as $t \to \infty$: HS point reached at infinite distance along trajectory

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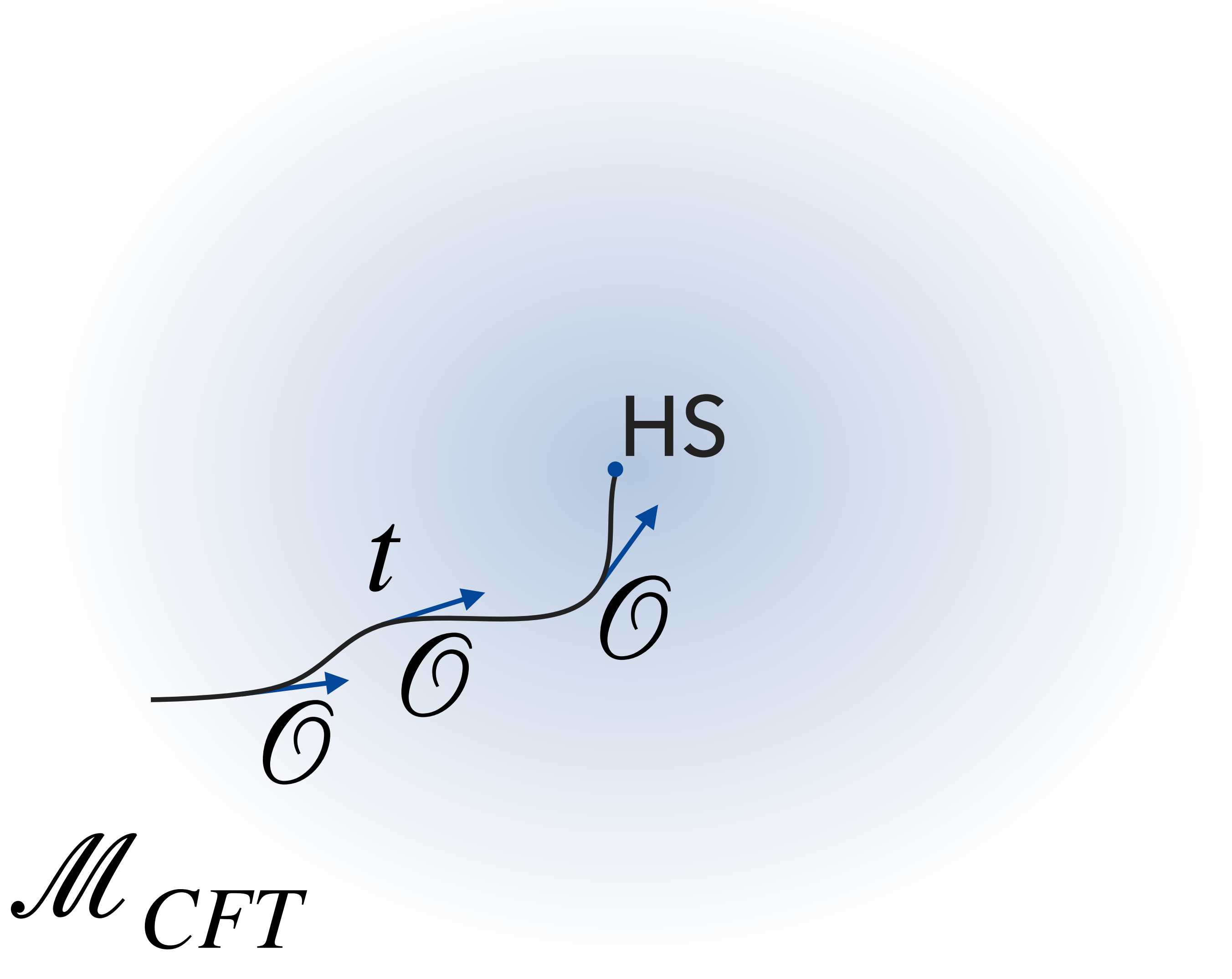
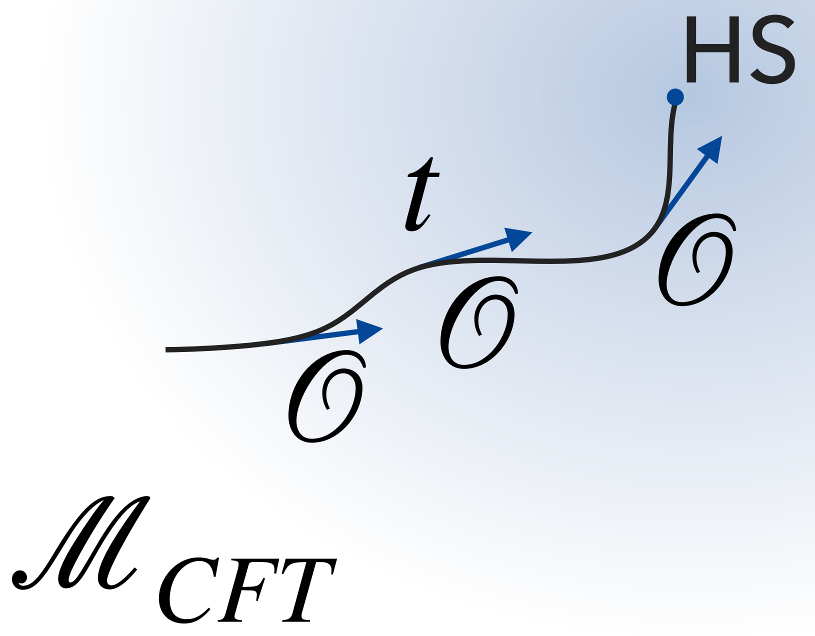


Conformal perturbation theory + (Weakly-broken) HS symmetry

$$\boxed{\left|\frac{d\gamma_\ell}{dt}\right| = \left|C_{JJ\mathcal{O}}\right| \lesssim \gamma_\ell \text{ as } \gamma_\ell \to 0 \quad \forall\, \mathcal{O}}$$

$\gamma_\ell \to 0$ as $t \to \infty$ $\forall\, \mathcal{O}$: HS point reached at infinite distance along any trajectory

Sketch of the Proof



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$\boxed{\gamma_\ell \to 0 \text{ as } t \to \infty \quad \forall J_\ell, \mathcal{O} : \text{All HS points are at infinite distance}}$ ✓

Towards a Proof of the CFT Distance Conjecture

CFT Distance Conjecture

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CFT Distance Conjecture

I. HS point $\longrightarrow$ Infinite distance ✓

Towards a Proof of the CFT Distance Conjecture

CFT Distance Conjecture

✓

I. HS point $\longrightarrow$ Infinite distance

Conformal perturbation theory

$$\frac{d\gamma_\ell}{dt} = -\,C_{JJ\mathcal{O}}$$

$\oplus$

$C_{JJ\mathcal{O}} \lesssim \gamma_\ell$ as $\gamma_\ell \to 0$

(Weakly-broken) HS symmetry

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II. Infinite distance $\longrightarrow$ HS point

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II. Infinite distance $\longrightarrow$ HS point

$$\exists \mathcal{O} :\ C_{JJ\mathcal{O}} \neq 0$$

CFT Distance Criterion $\Big\downarrow$ Sufficient but not necessary

Finite distance

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CFT Distance Conjecture

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No HS symmetry

$\Big\downarrow$?

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CFT Distance Conjecture

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Conformal perturbation theory

$\oplus$

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Towards a Proof of the CFT Distance Conjecture

CFT Distance Conjecture

I. HS point $\longrightarrow$ Infinite distance ✔

Conformal perturbation theory

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Conformal perturbation theory

$\oplus$

(Weakly-broken) HS symmetry

Interesting questions for the future!

… But let me ask something else

Stringy nature of HS points ?

Strings in the Conformal Manifold

Inspiration: Emergent String Conjecture [Lee, Lerche, Weigand '19]

Leading tower → KK modes → Decompactification

Leading tower → Excitations of weakly-coupled string

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Problem: $M_s \lesssim R_{AdS}^{-1}$ → String in a highly-curved background... **hard to study!**

→ Rely on CFT results and extract clues !

A Distance Conjecture Approach

In flat space: Value of $\alpha \rightarrow$ Nature of the tower

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$$\alpha = \sqrt{\frac{d-2+n}{n(d-2)}}$$ $\Rightarrow$ Decompactification of n extra dimensions

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→ Three different values: $\alpha = \left\{ \sqrt{\frac{2}{3}}, \sqrt{\frac{7}{12}}, \frac{1}{\sqrt{2}} \right\}$ [Perlmutter, Rastelli, Vafa, Valenzuela '20]

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So… What is going on**?!**
Irene's puzzle

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E.g. $\mathcal{N} = 2$ SCQCD

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E.g. $\mathcal{N} = 1$ SCQCD

A Distance Conjecture Approach

In flat space: Value of $\alpha \rightarrow$ Nature of the tower

$$\alpha = \sqrt{\frac{d-2+n}{n(d-2)}}$$ ➡ Decompactification of n extra dimensions

$$\alpha = \frac{1}{\sqrt{d-2}}$$ ➡ Emergent string limit

Caveat: Different values found for decompactification to running solution
[Etheredge, Heidenreich, McNamara, Rudelius, Ruiz, Valenzuela '23]

From the CFT: Restrict to zoo of 4d SCFTs with simple gauge group (Lagrangian) admitting large N

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Goal: Understand this case!

Convex Hull for AdS5xS5

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Type IIB on an 5-sphere

$$S = \frac{M_{Pl}^3}{2} \int d^5x \sqrt{-g} \left(R - (\partial\hat{\Phi})^2 - (\partial\hat{R})^2 - V(\hat{\Phi}, \hat{R}) \right)$$

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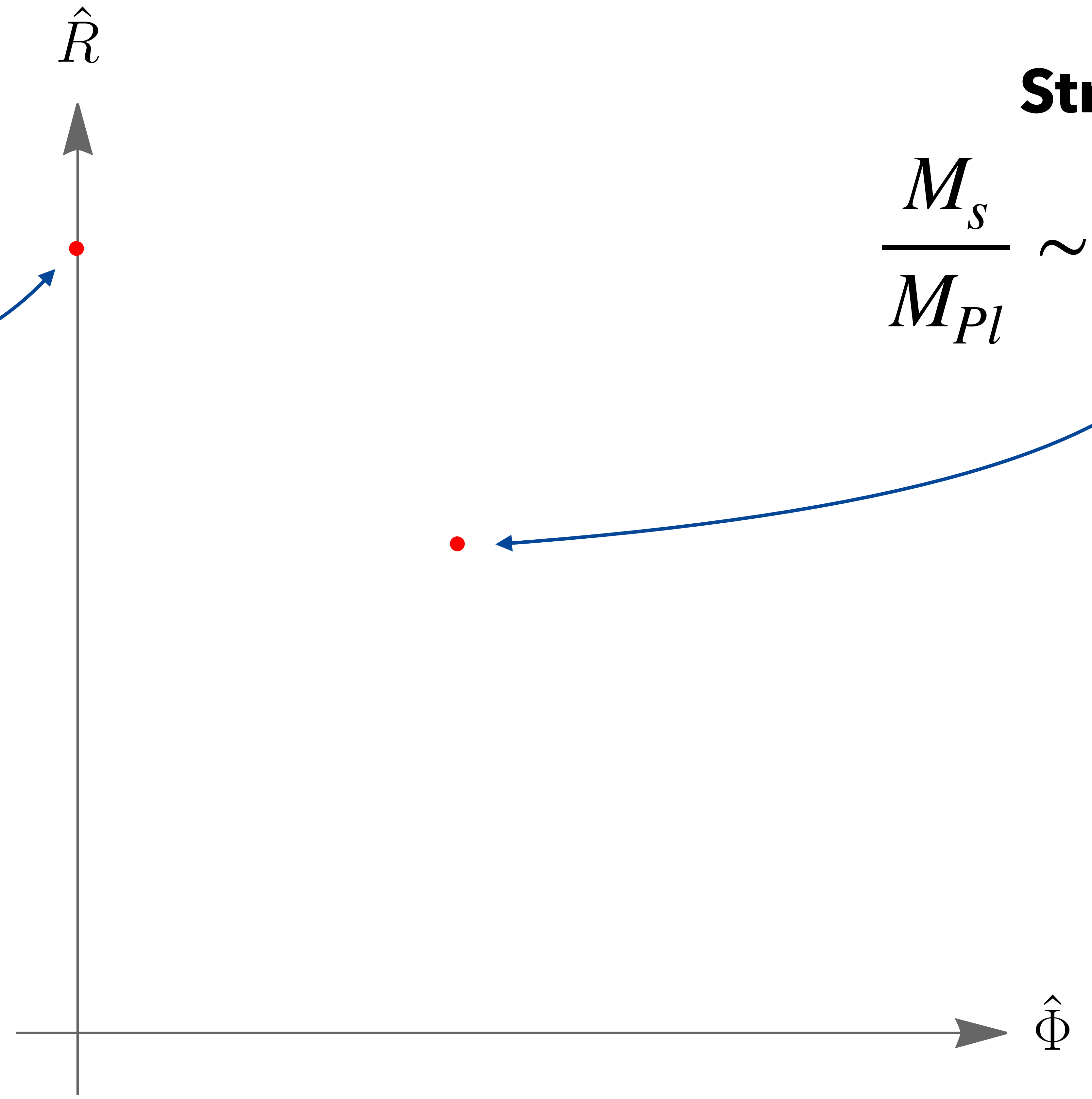

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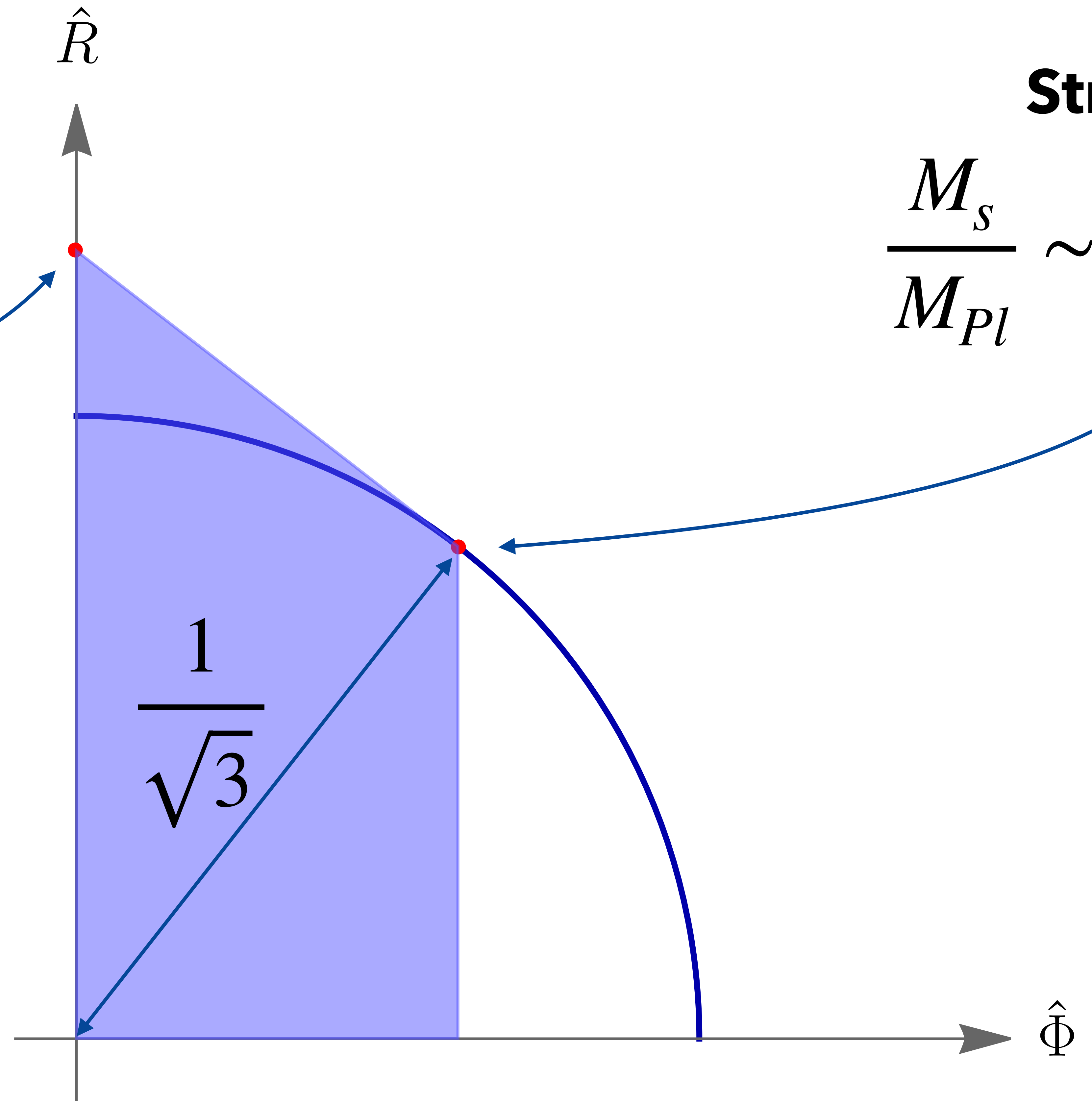

Convex hull condition

[JCI, Uranga, Valenzuela '20]

↓

Sharpened SDC

[Etheredge, Heidenreich, Kaya, Qiu, Rudelius '22]

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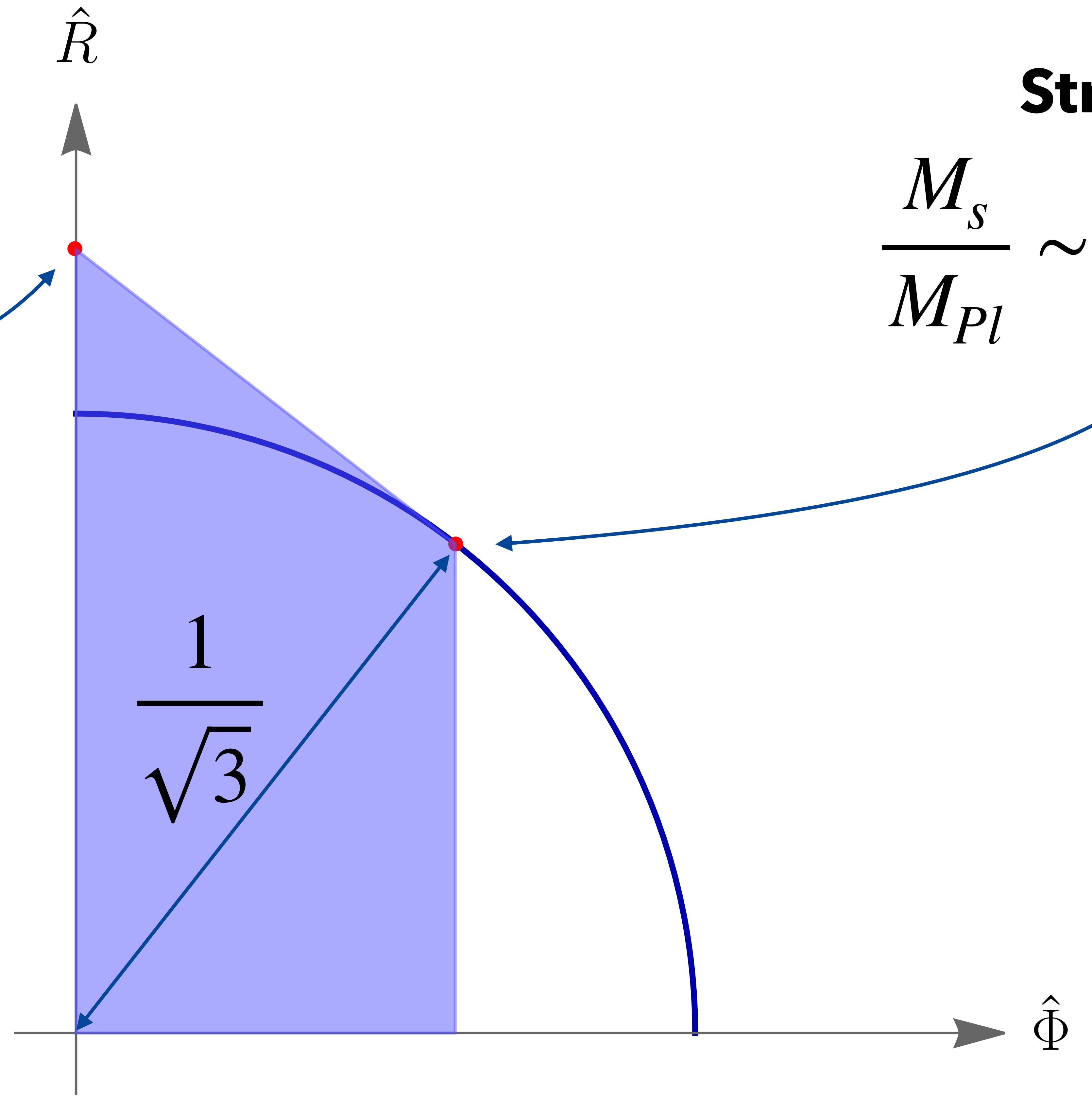

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Violated!?

$AdS_5 \times S^5$ moduli space

$$\alpha = \frac{1}{2\sqrt{2}} \neq \frac{1}{\sqrt{2}} \text{ (fix } \hat{R}\text{)}$$

CFT value not reproduced!

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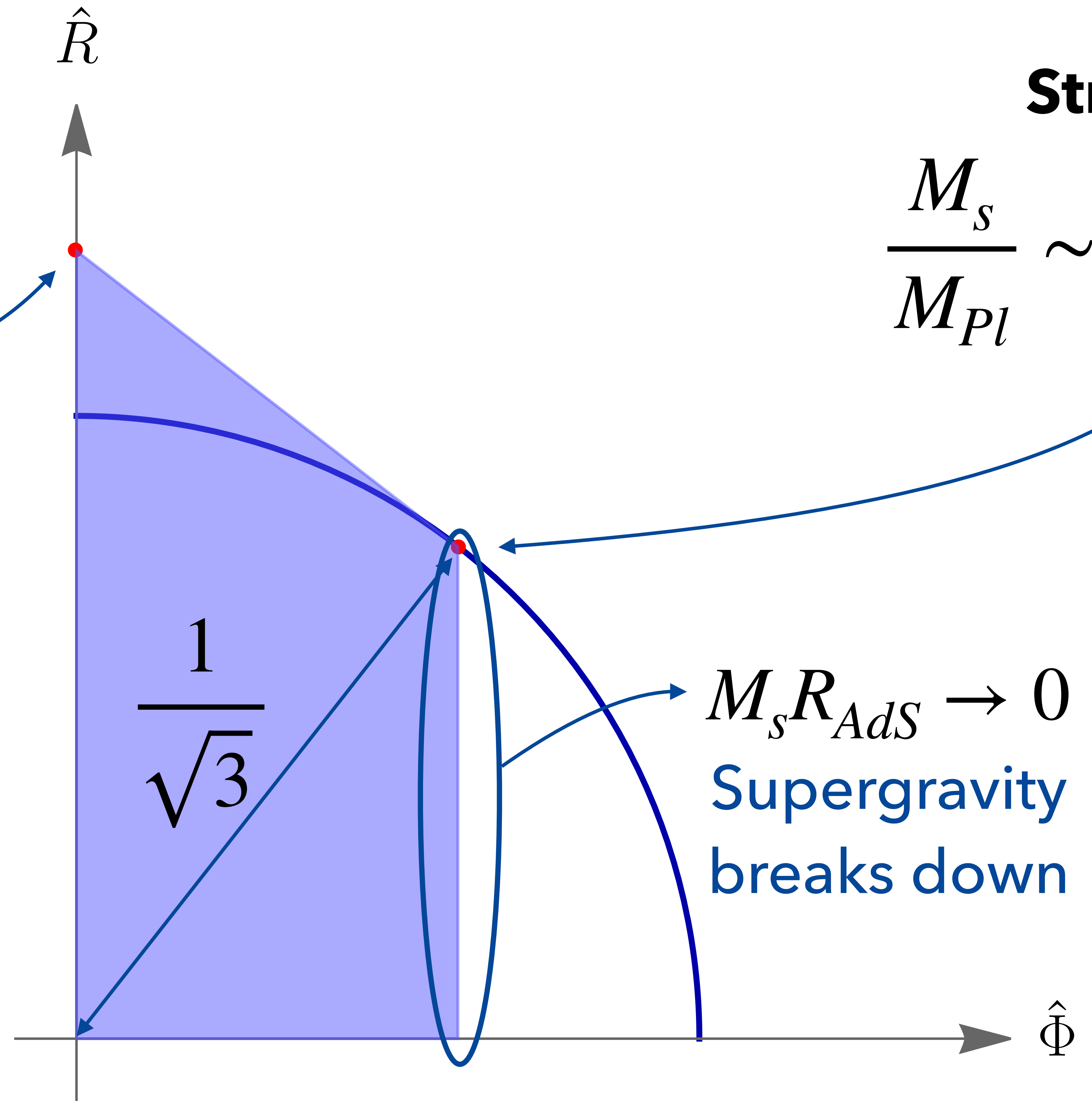

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Violated!?

$M_s R_{AdS} \to 0$

Supergravity breaks down

$AdS_5 \times S^5$ moduli space

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$\mathcal{N} = 4$ SU(N) gauge theory in 4d

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No CFT distance in the N-direction

$$N \sim e^{\alpha_N \hat{R}} \; ? \quad :($$

See [Basile, Montella '23] for progress

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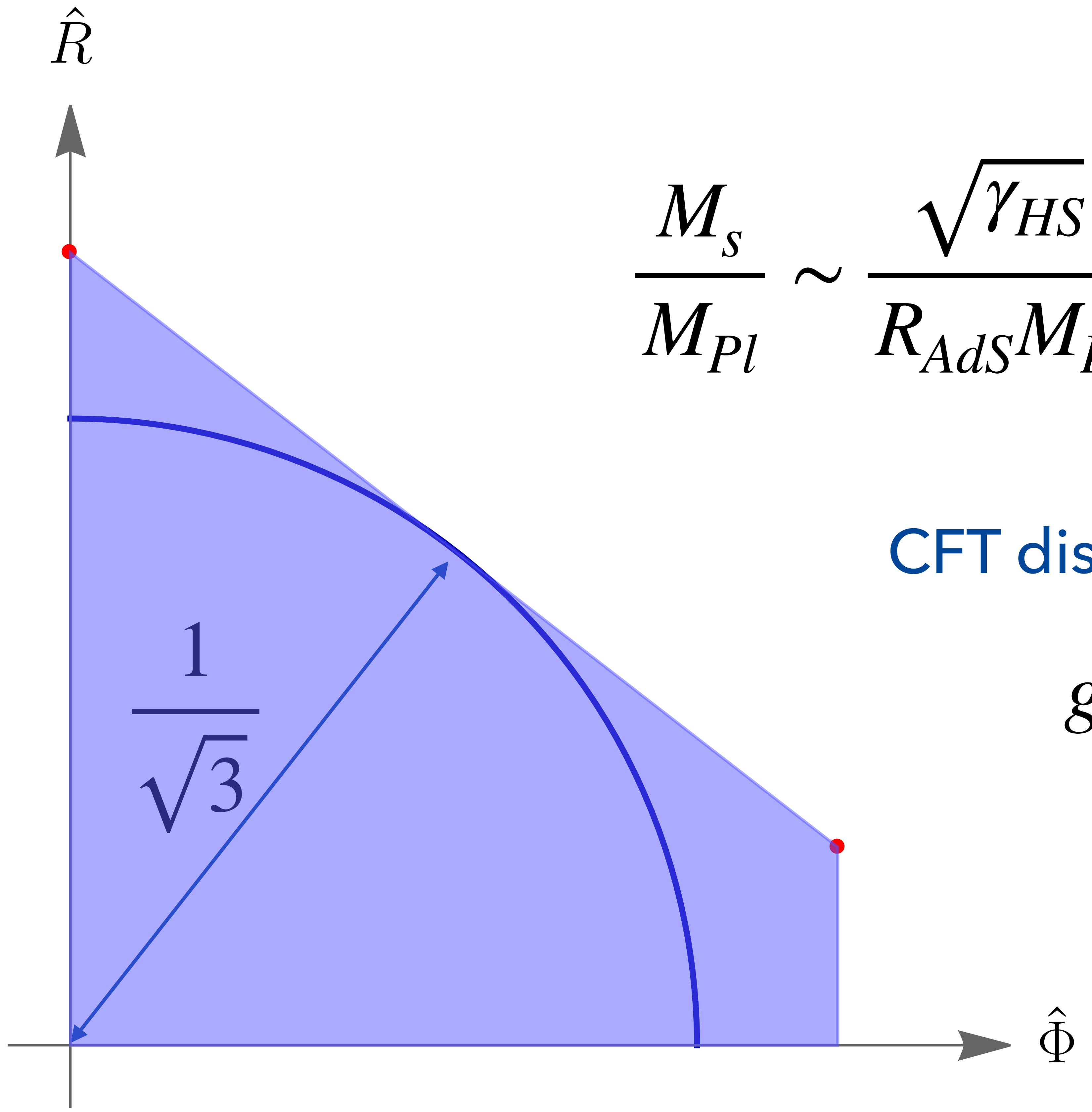

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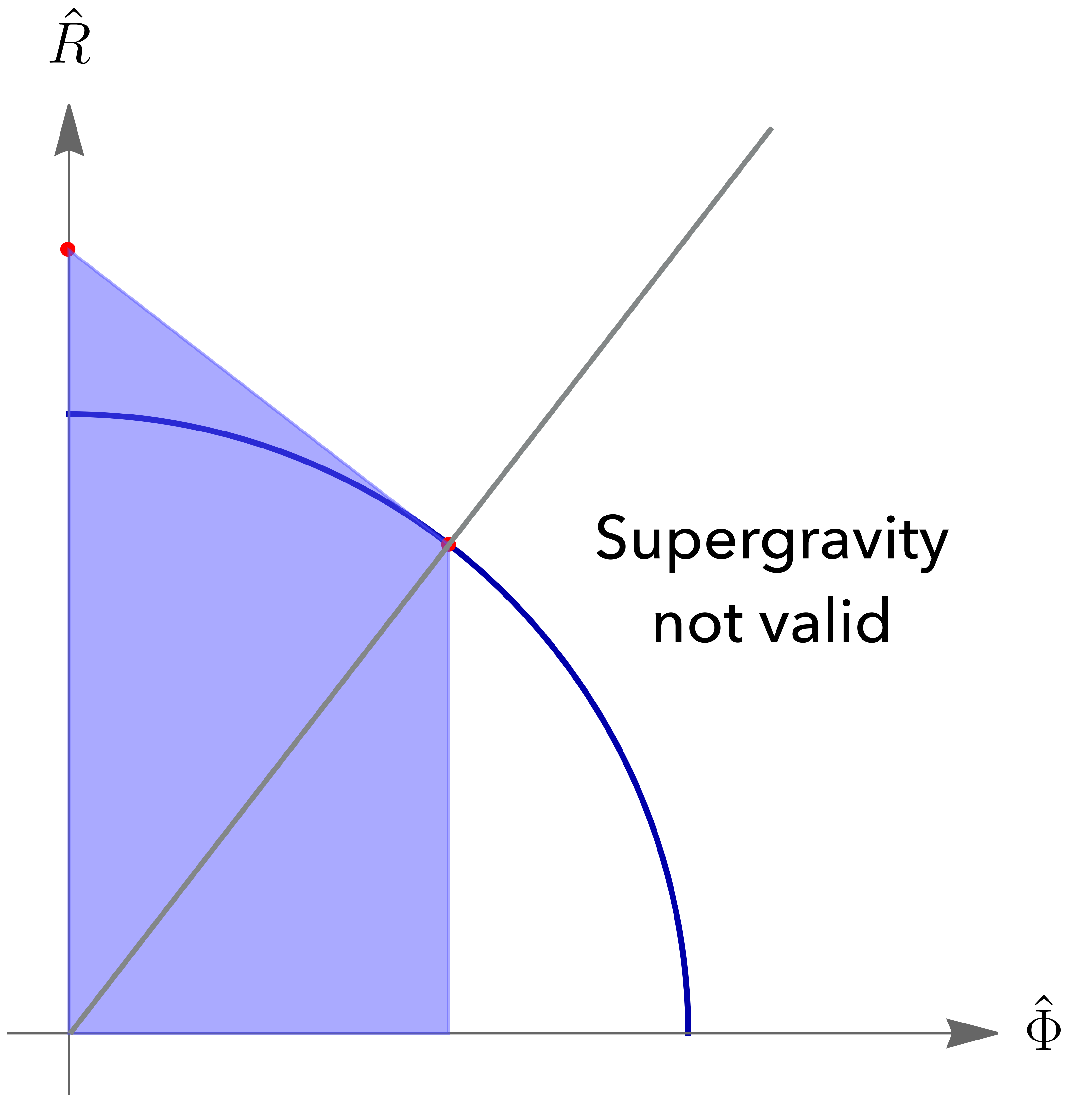
Convex Hulls Summary



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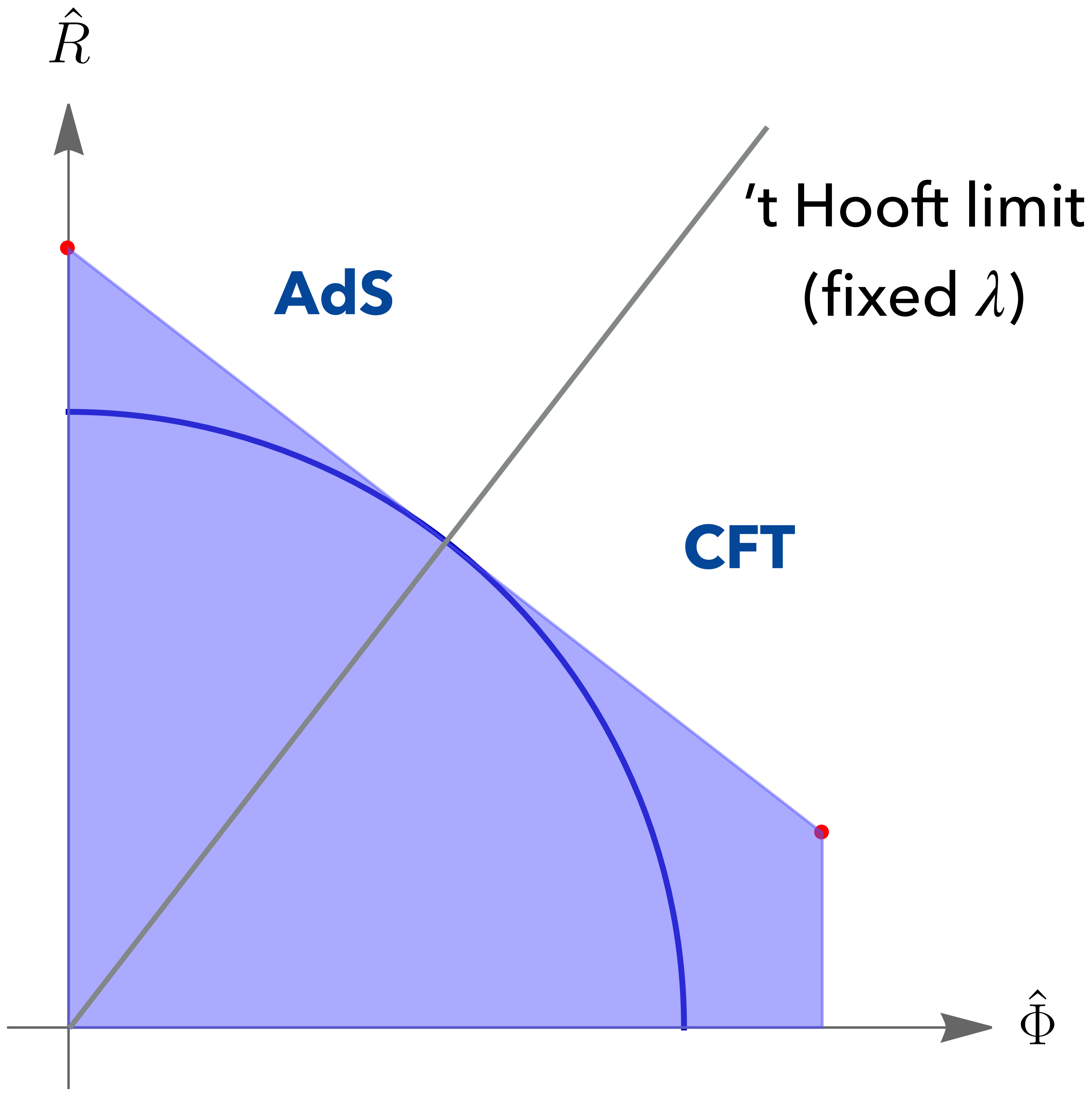
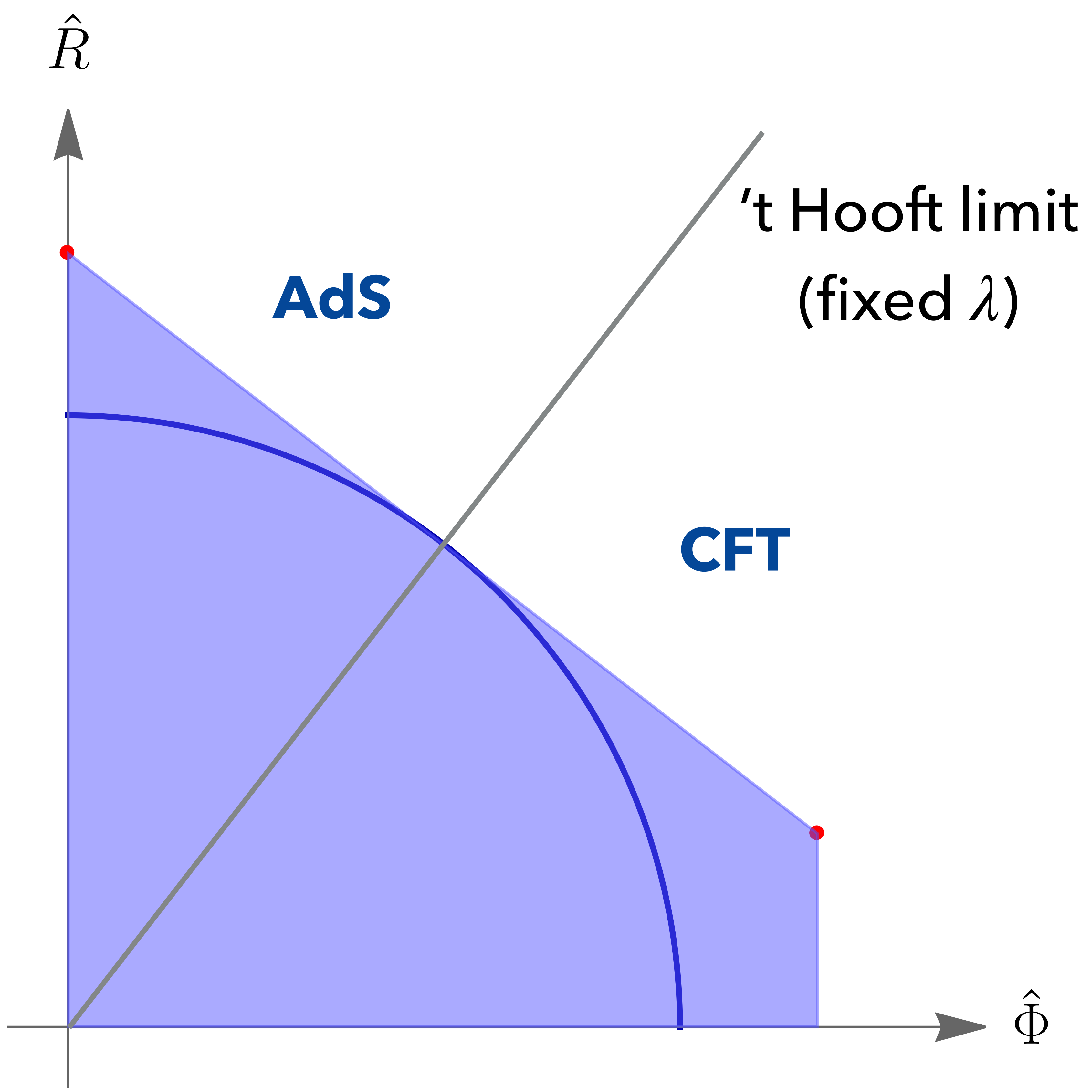


Convex Hulls Summary

Notice:

Convex hulls for AdS and CFT glue nicely together!

(see later)



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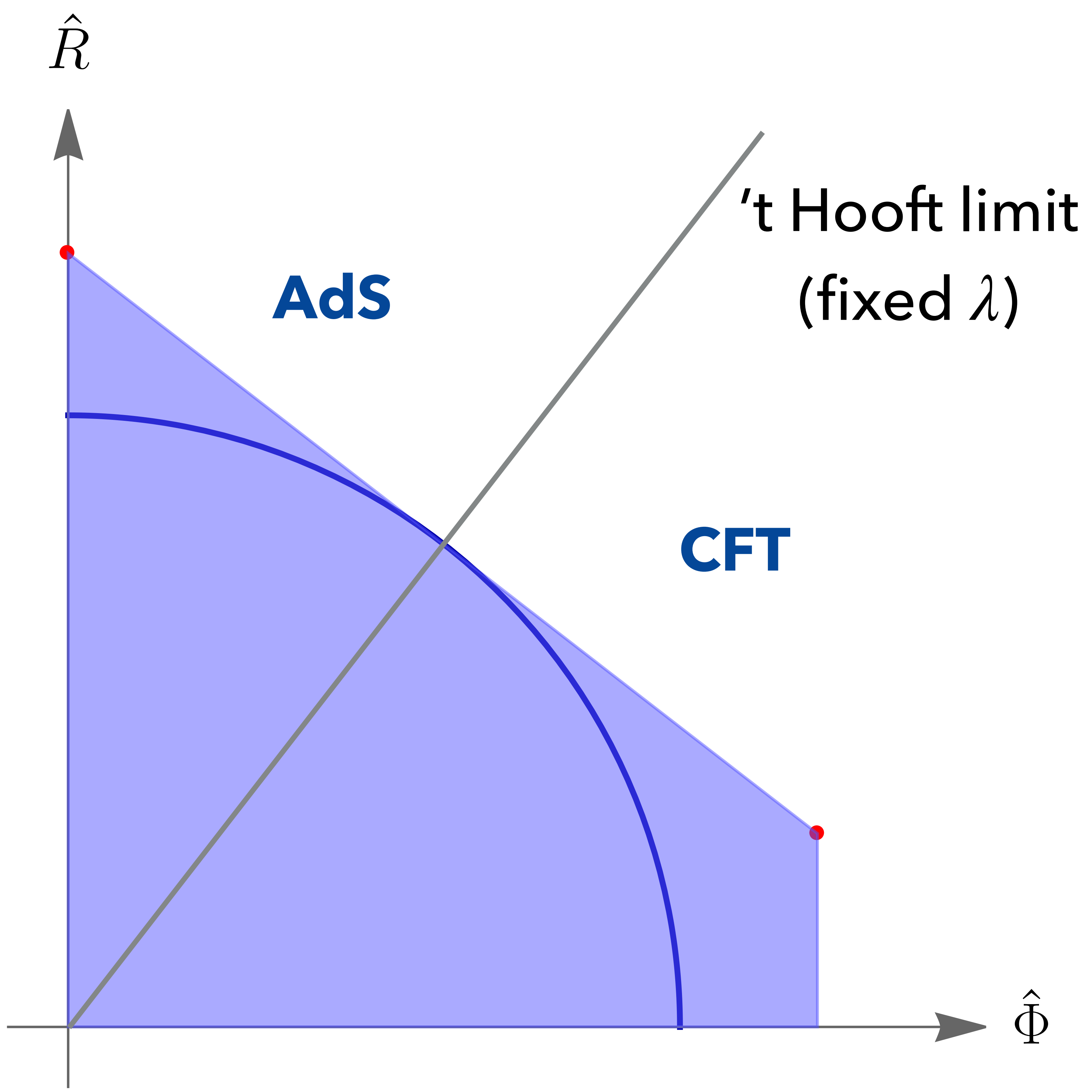

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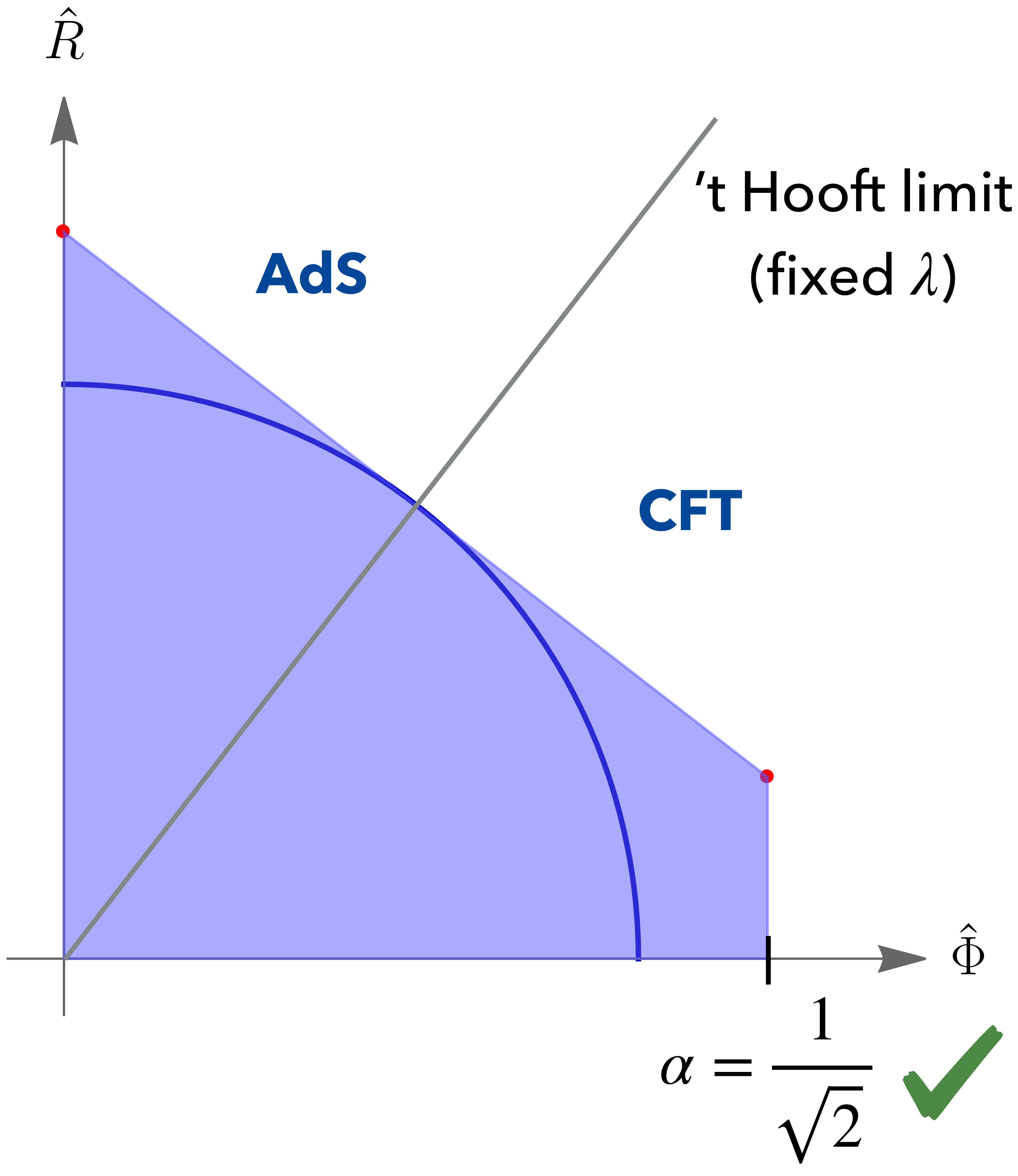

Fits with Type IIB string
but M_s/M_{Pl} corrected
in highly-curved regime!

Summary

A Detour: Scale Separation vs Sharpened SDC

BPS operators ↔ KK tower

$$\Delta_{BPS} \sim \mathcal{O}(1) \longleftrightarrow M_{KK} R_{AdS} \sim \mathcal{O}(1)$$

No scale separation from the CFT!

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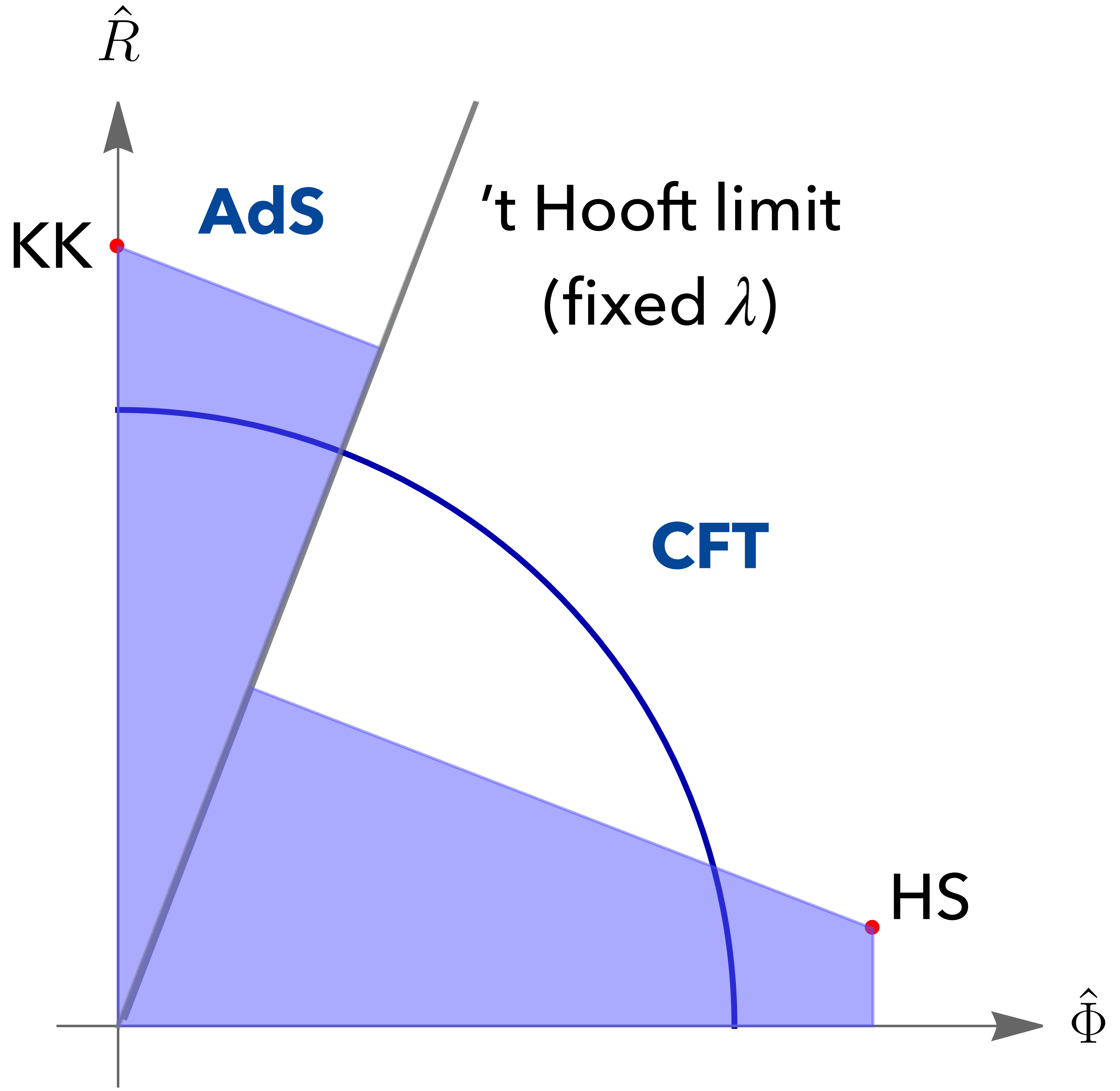
Anti-separation of scales ($\beta > 1/2$)



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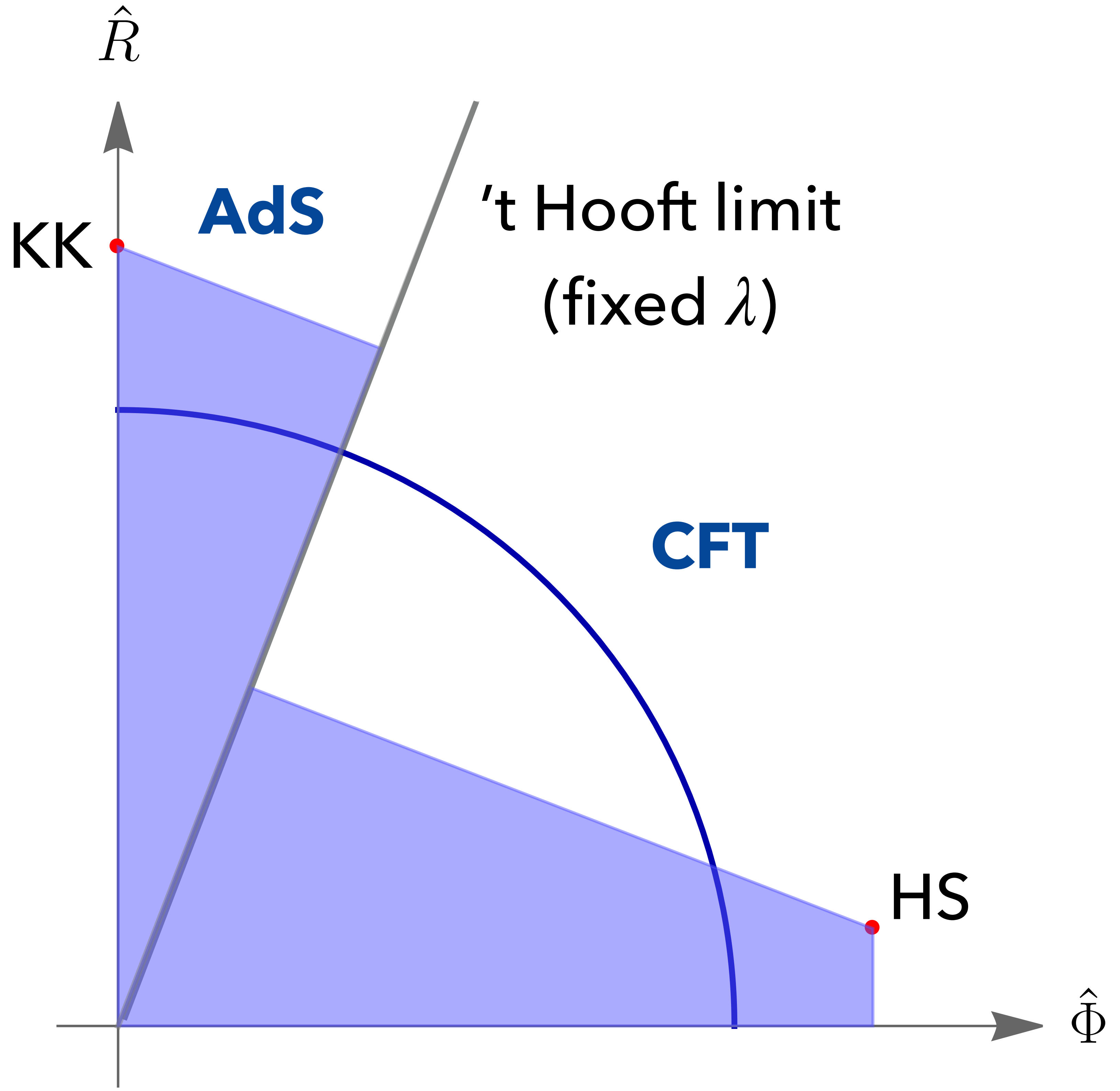

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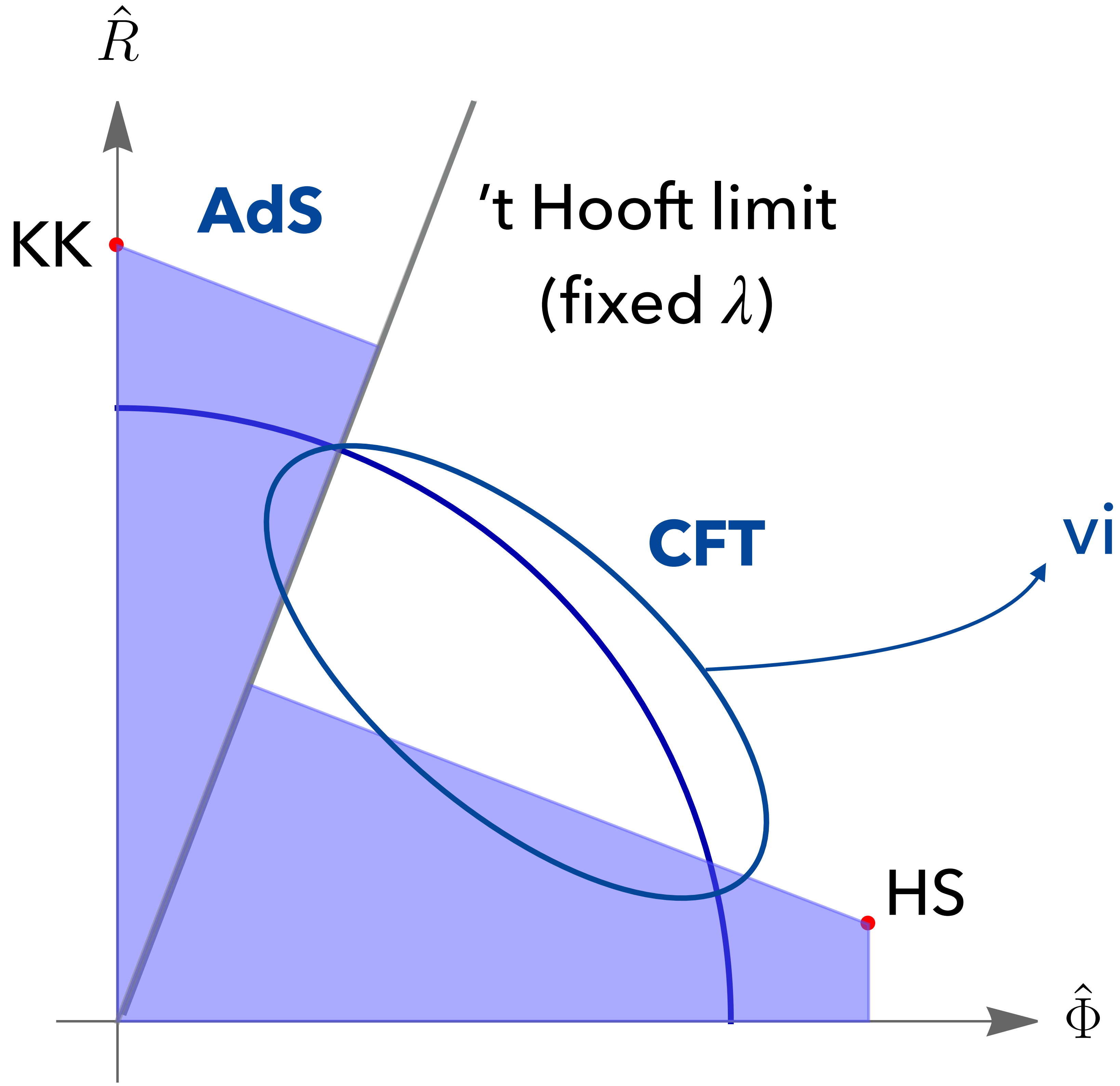

Sharpened SDC violation in the CFT !

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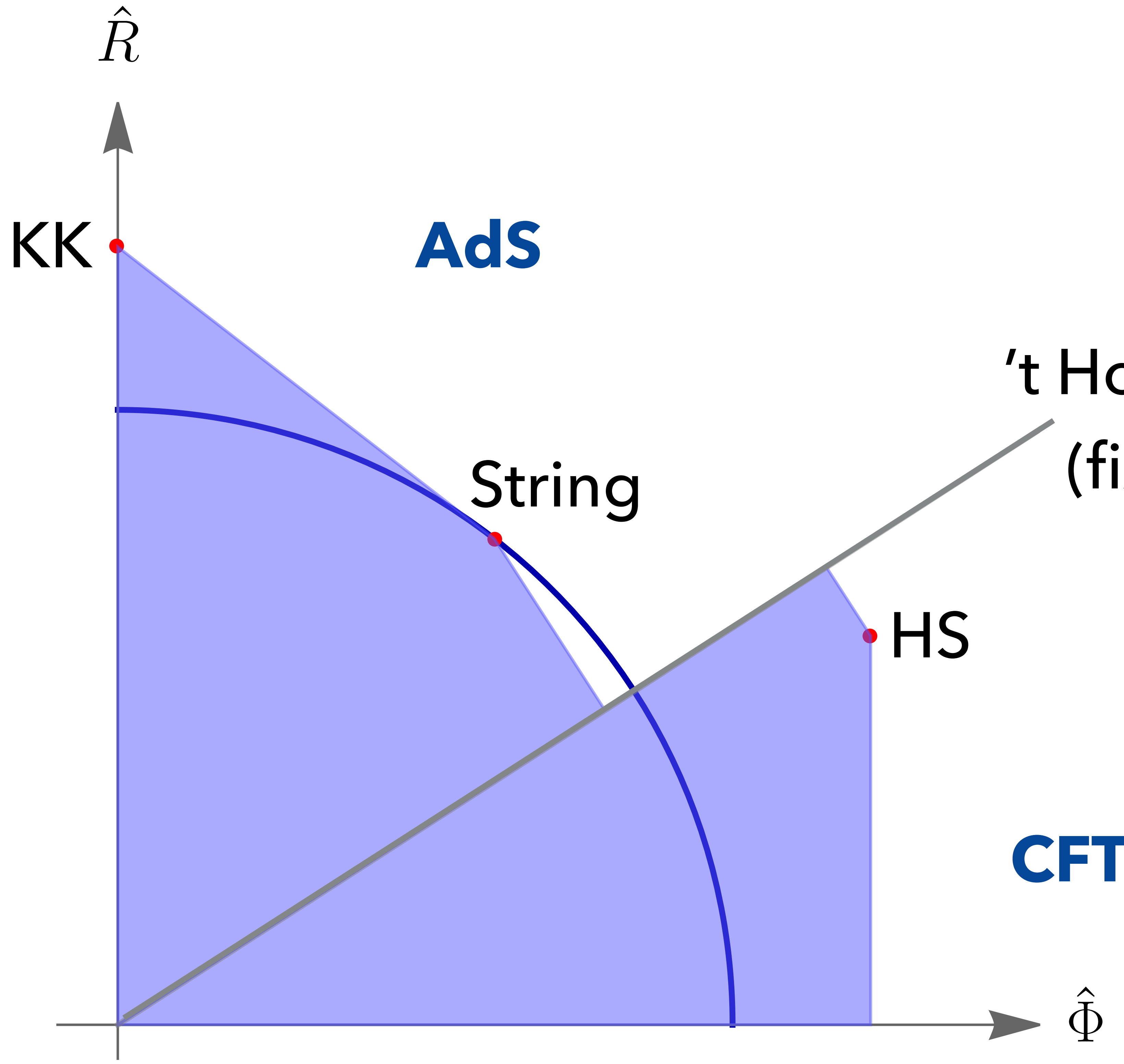

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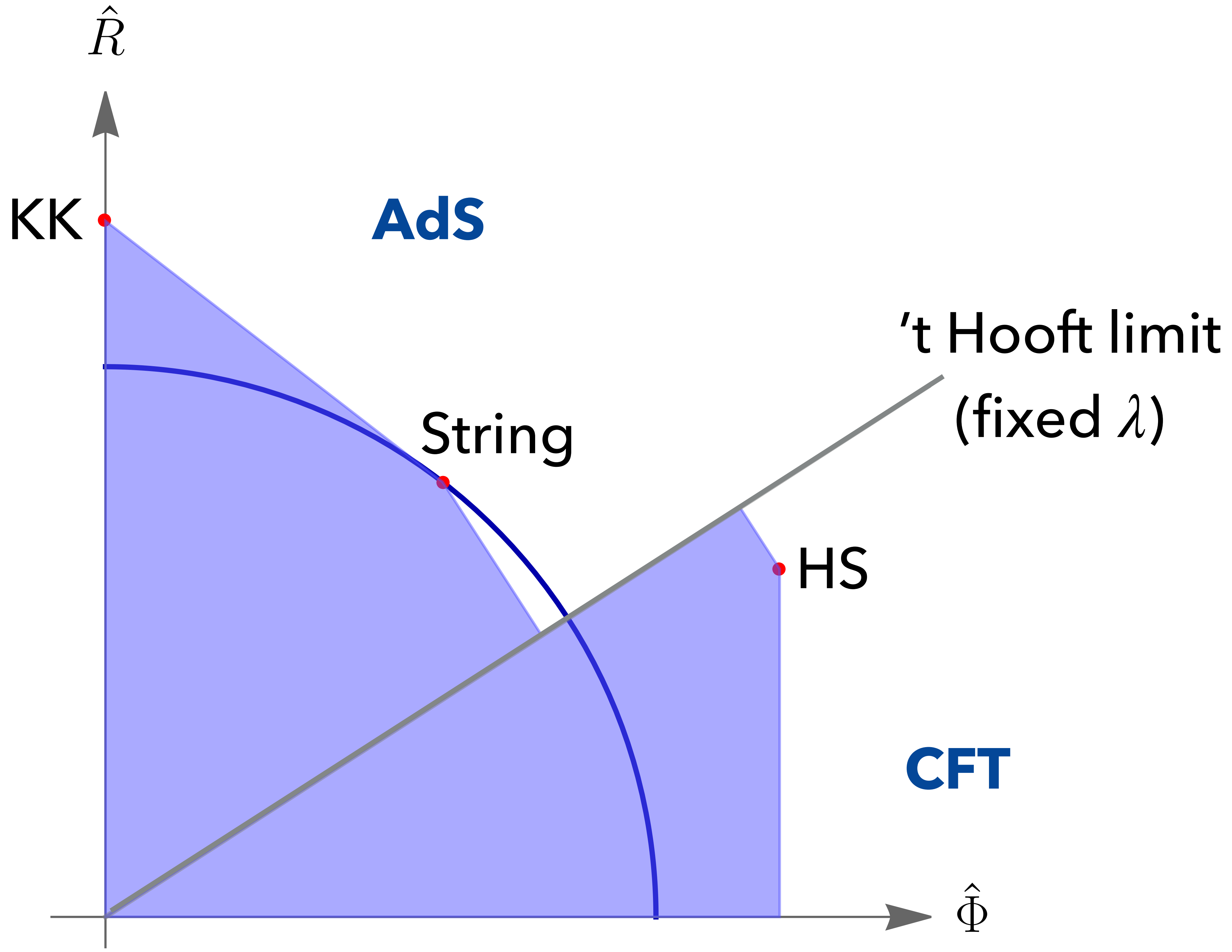

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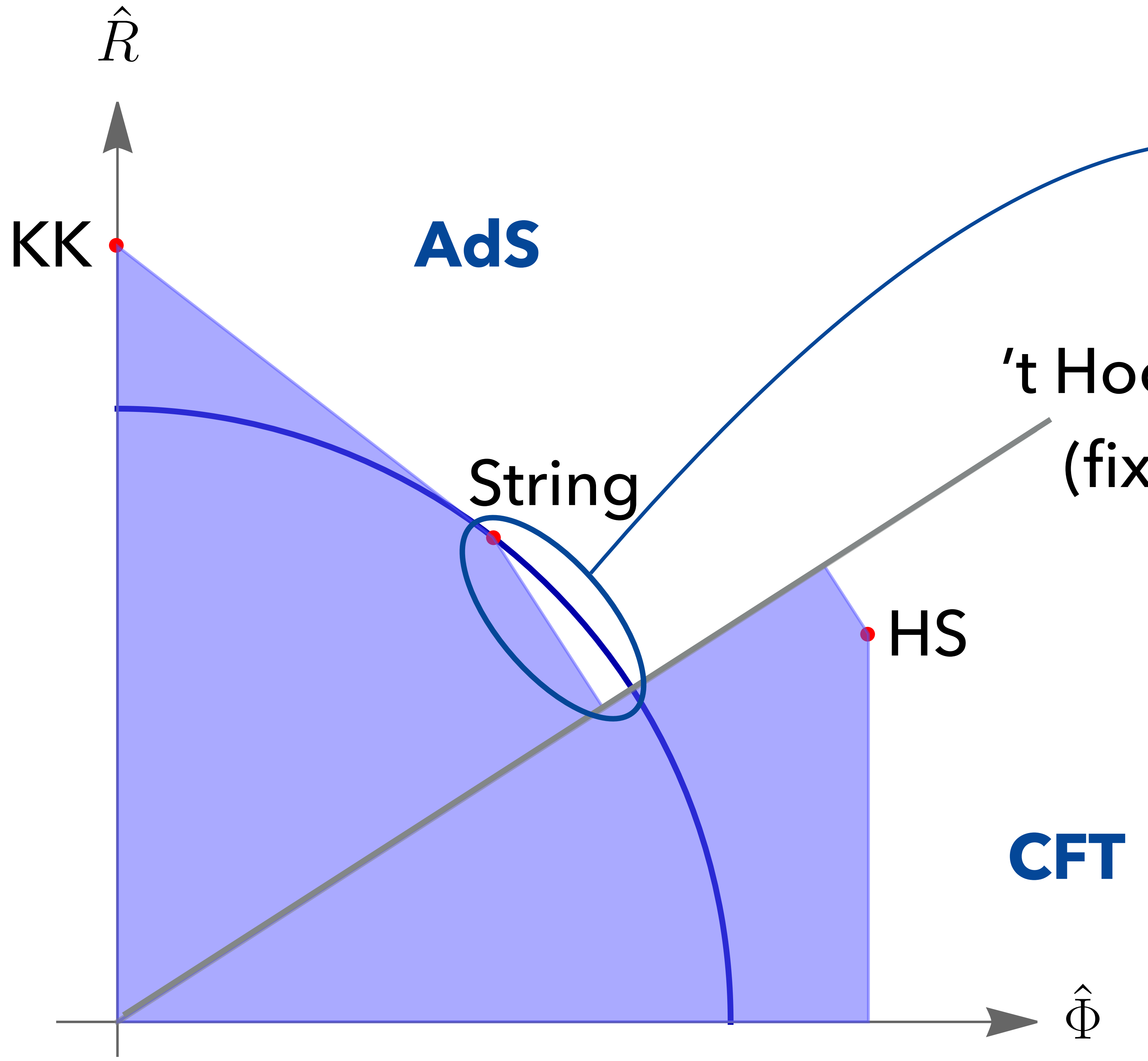

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A Detour: Scale Separation vs Sharpened SDC

BPS operators ↔ KK tower

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ADC parameter [Lust, Palti, Vafa '19]

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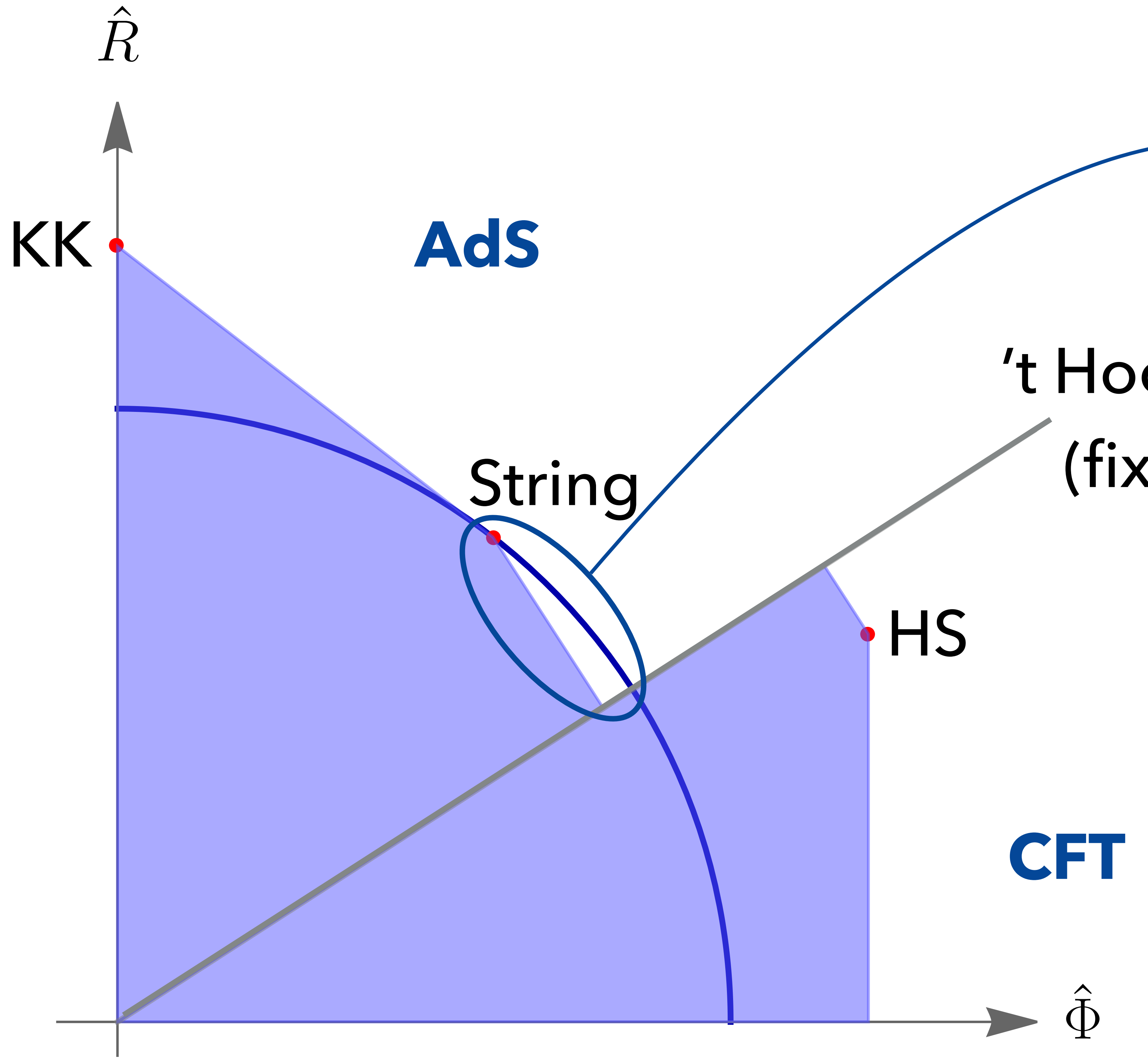

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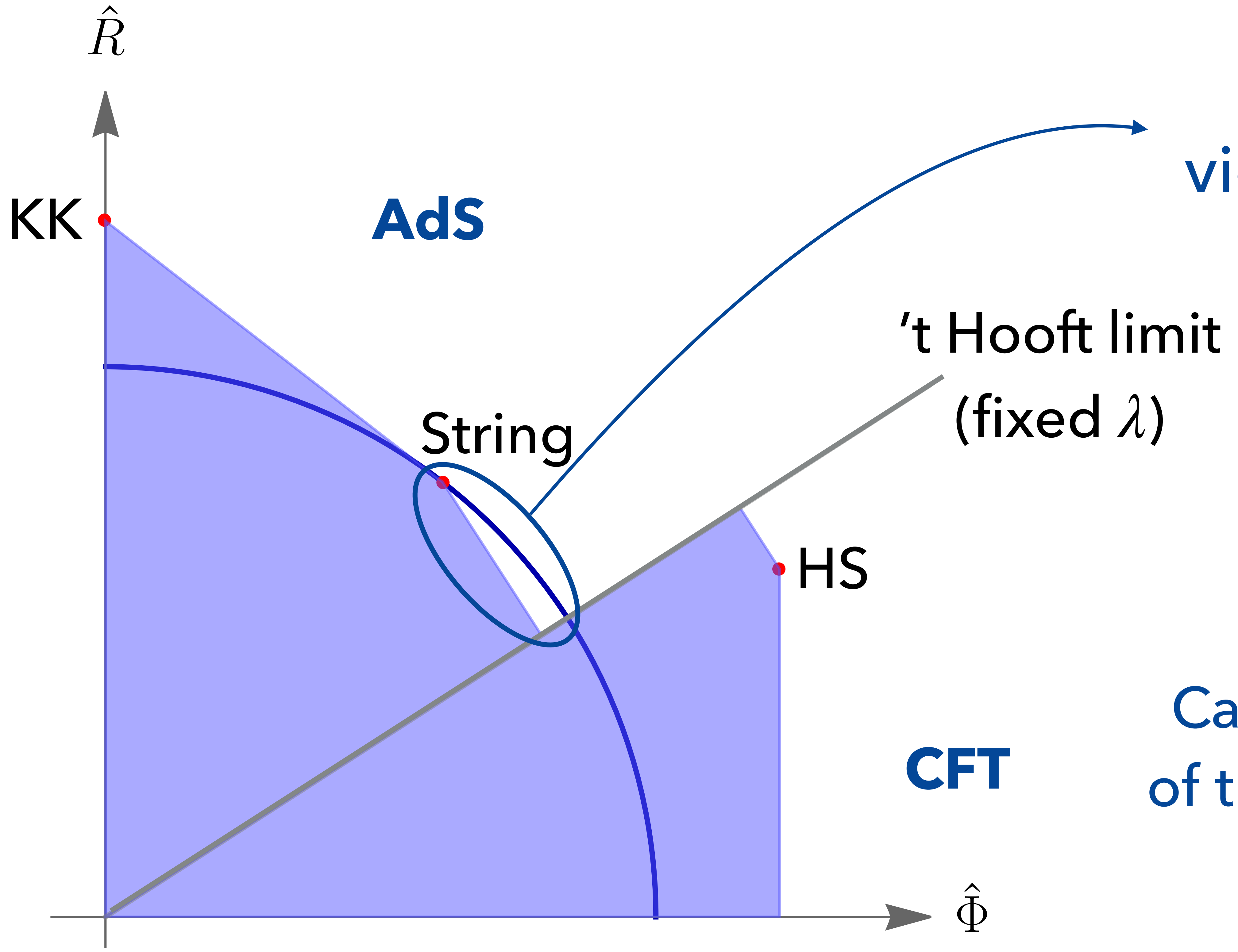

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Suggests link between Sharpened SDC and no scale separation

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Anything special with these three values **?**

Food for thought!

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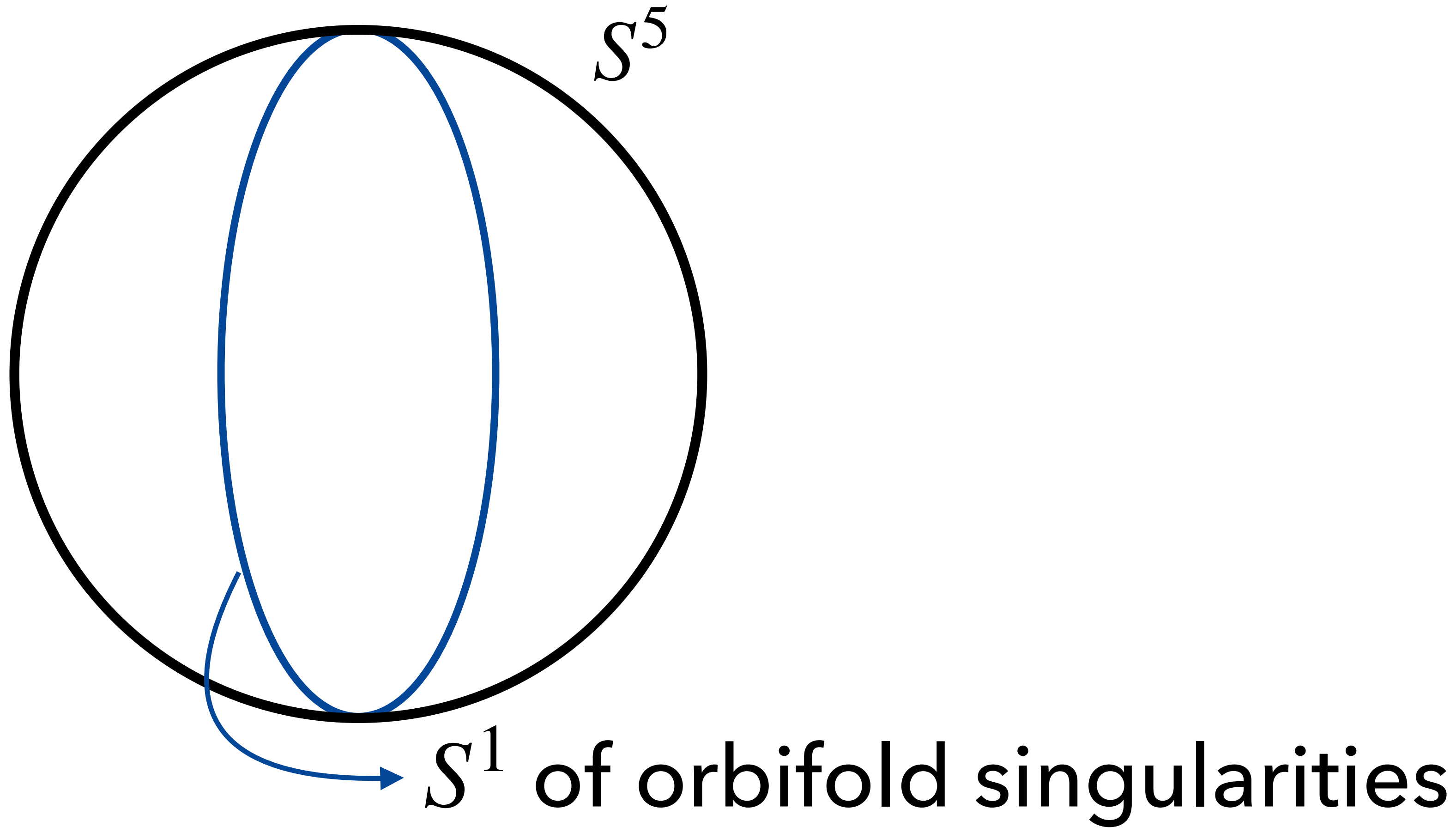

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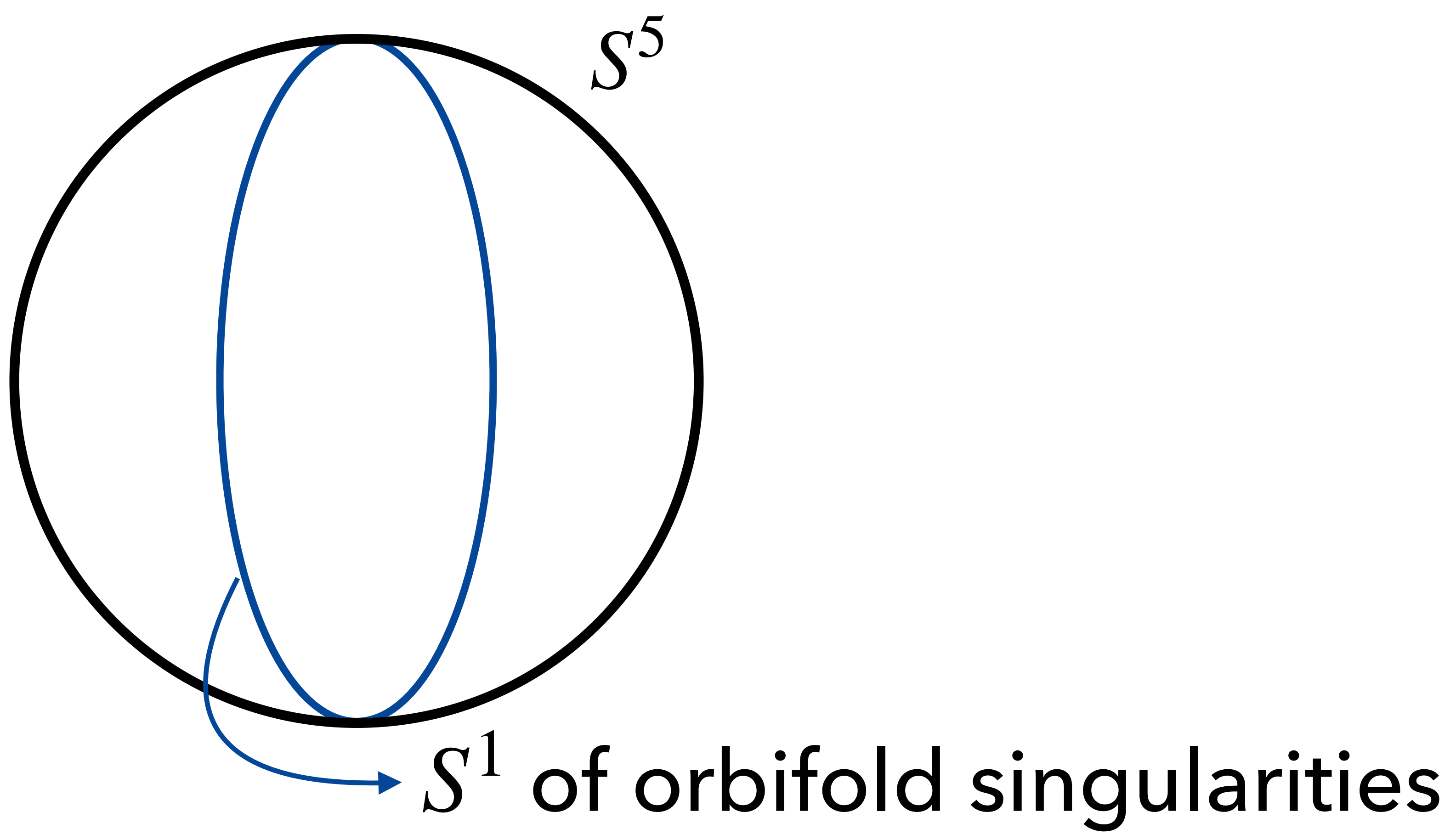

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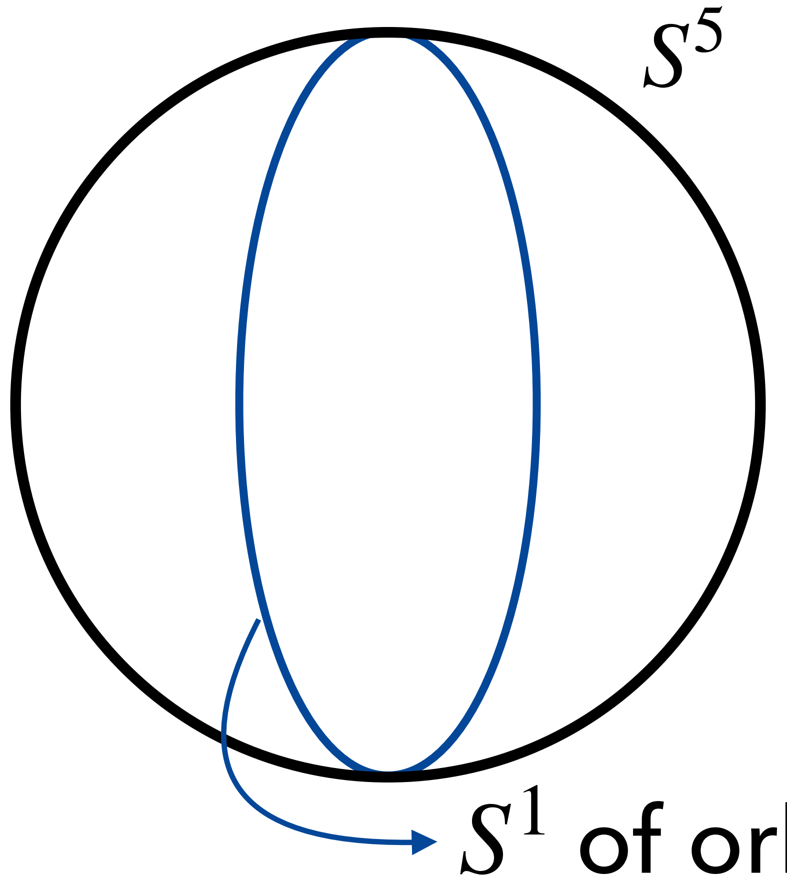

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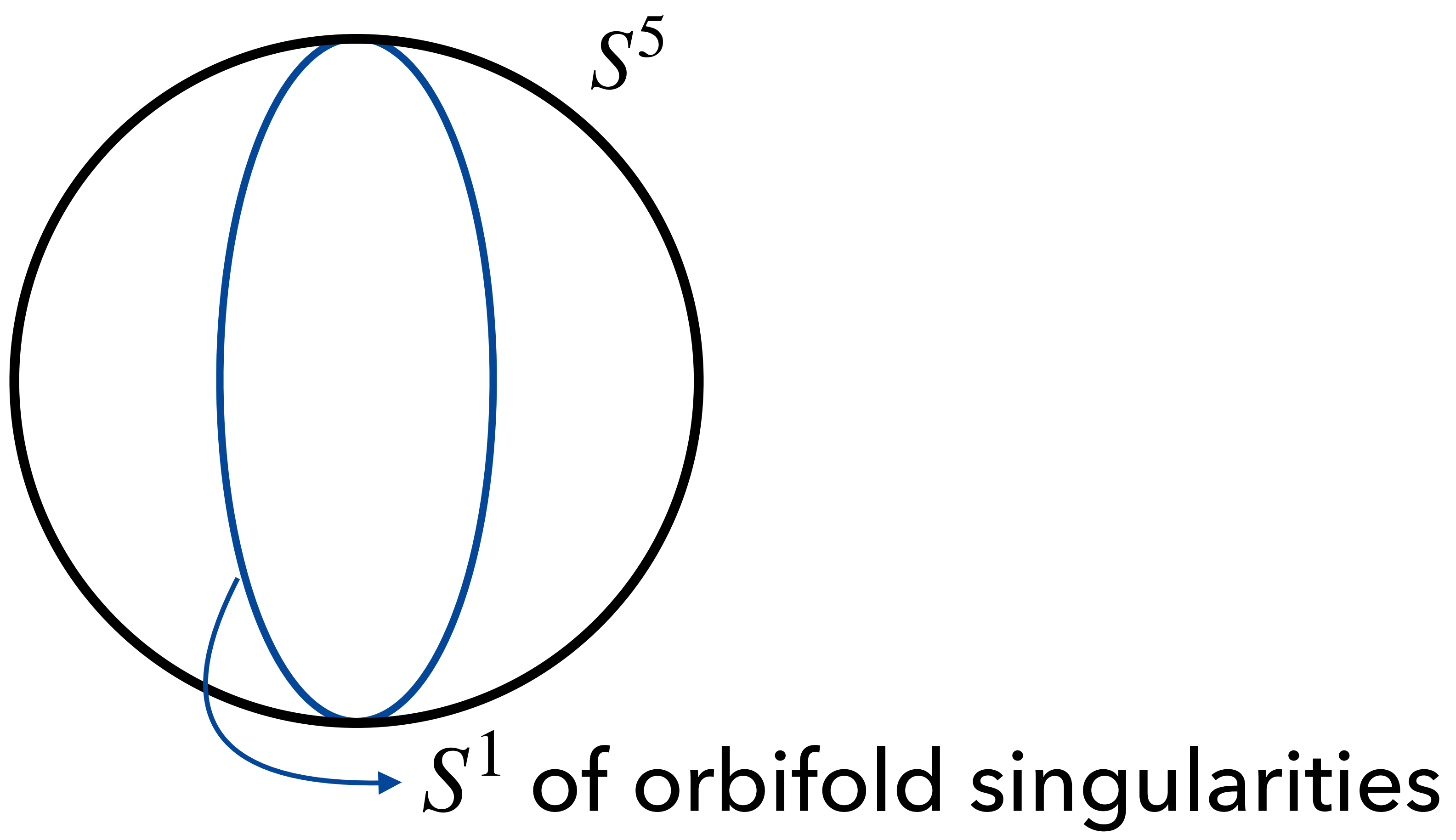

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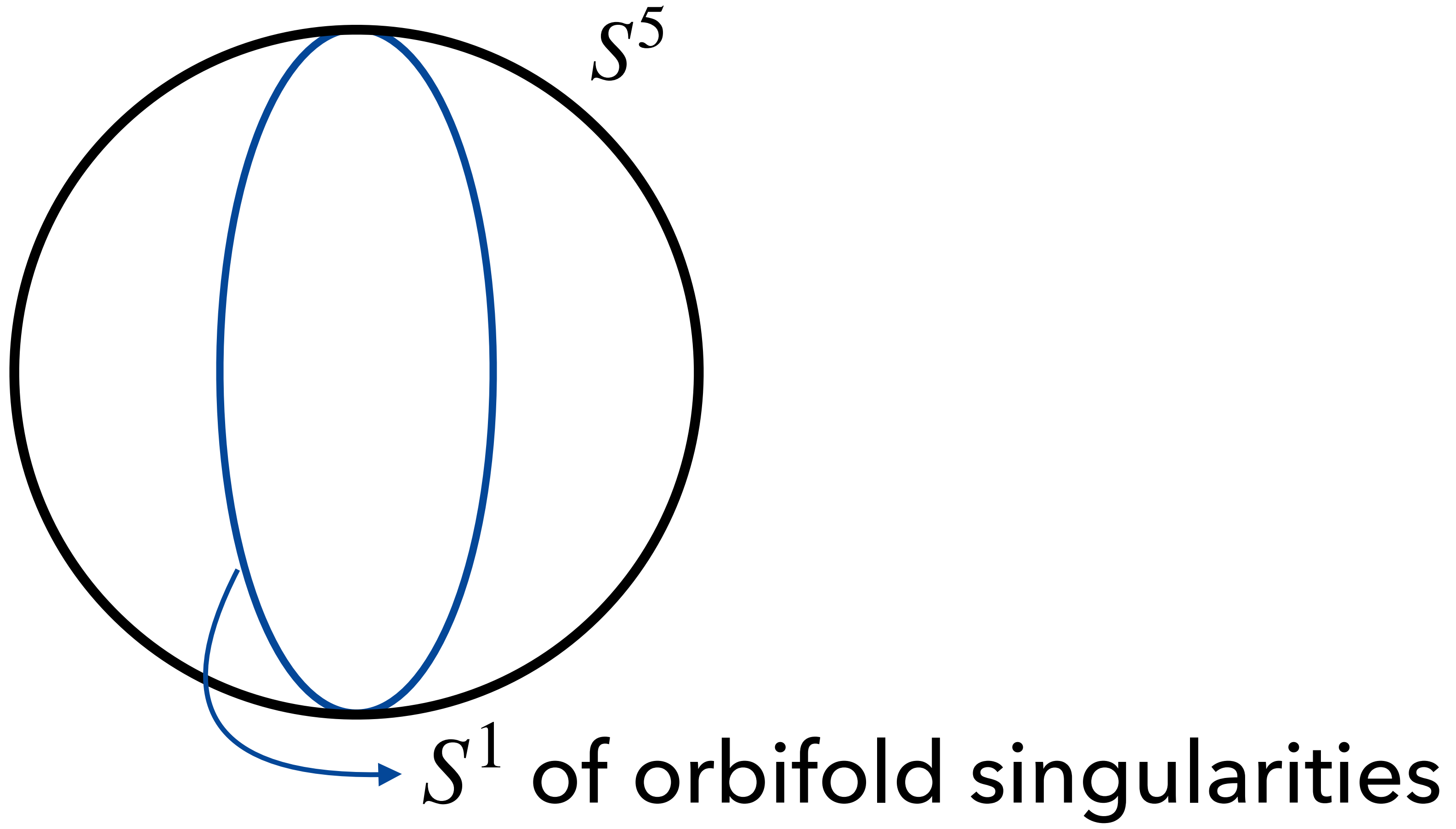

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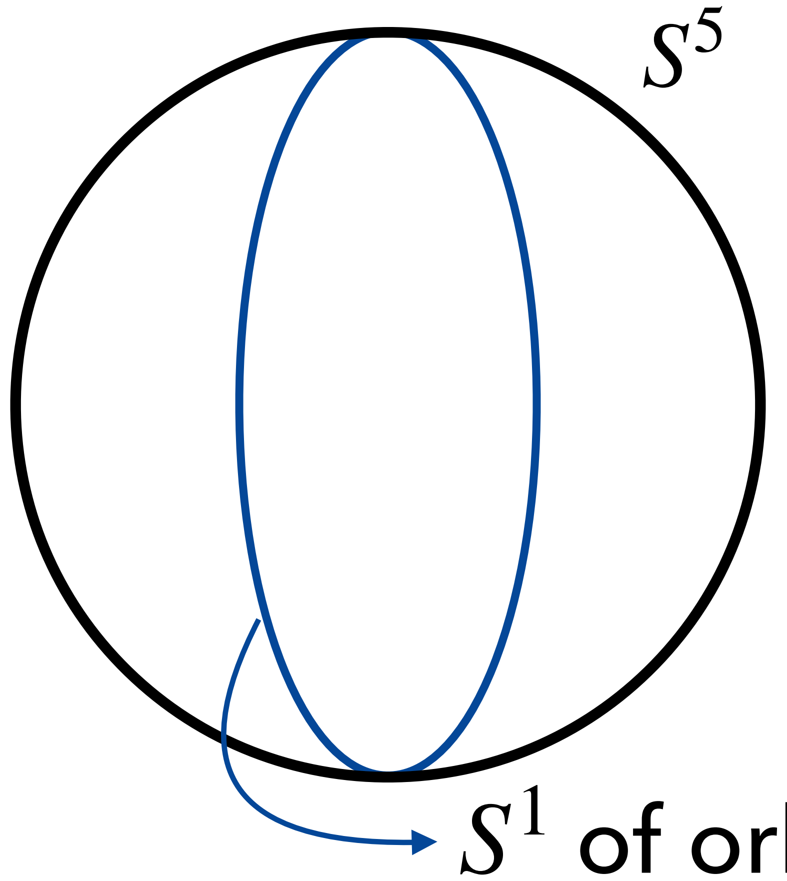

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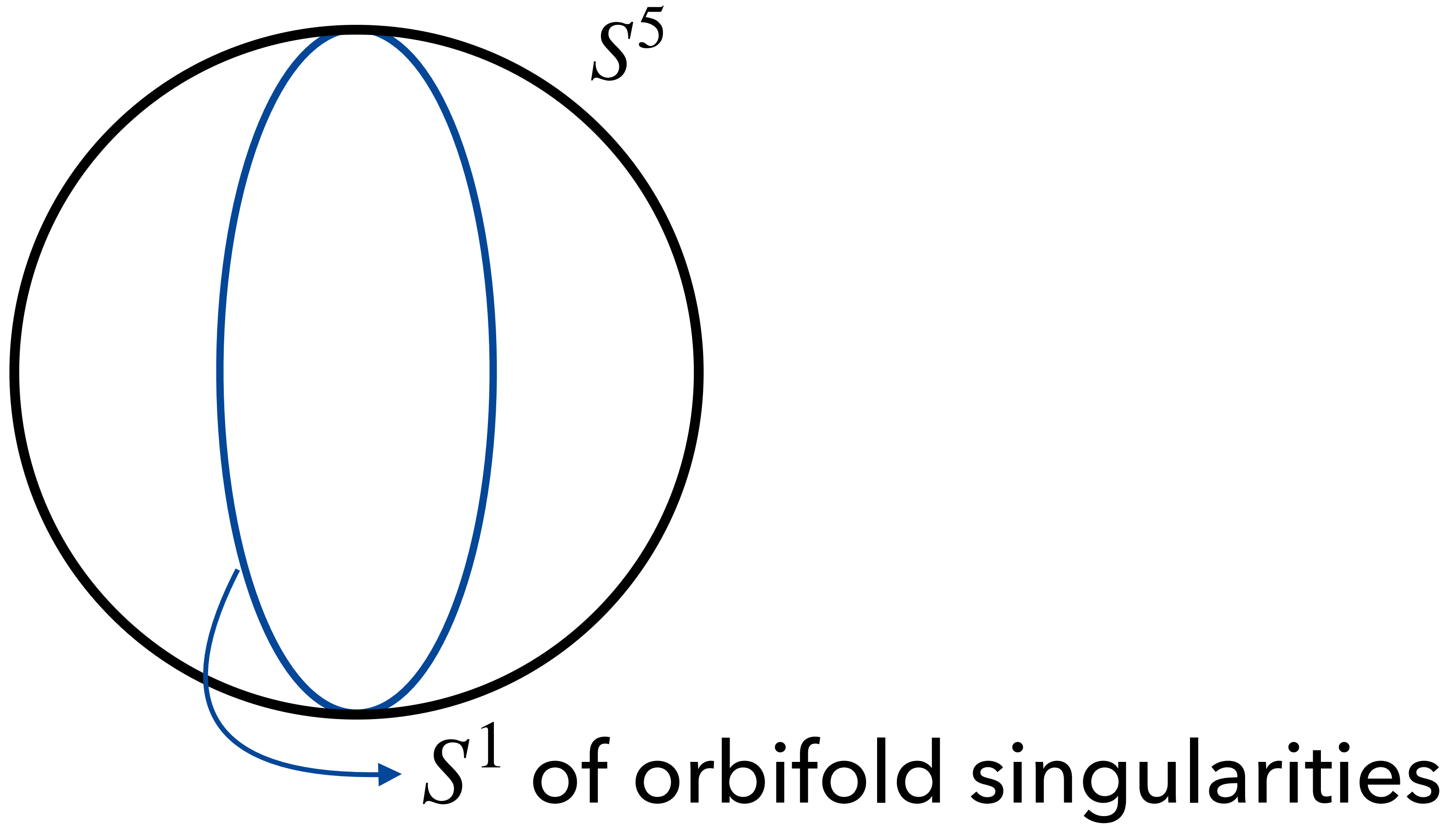

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