Tadpole & Moduli in Type IIB on the 1^9 LG Model

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(Based on 2310.00770, with Katrin Becker)

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Outline

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  - Internal SCFT (with appropriate $c = 9$) given by Landau-Ginzburg models – no Kähler moduli to sabilize, and *all* complex structure moduli stabilized by a flux-induced superpotential.
- Becker, Gonzalo, Walcher, Wrase (2022): All supersymmetric flux vacua found in the paper above leave a large number of moduli massless!

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- Our main interest is in studying the competition between the flux tadpole and the number of stabilized moduli in these models. An exhaustive test of the tadpole conjecture may be possible.

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  $S = \int d^2 z d^4\theta K(\Phi_1, \bar{\Phi}_1, \ldots, \Phi_n \bar{\Phi}_n) + (\int d^2 z d^2\theta \mathcal{W}(\Phi_1, \ldots, \Phi_n) + \text{ h.c.})$

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- Tensor N of these together $\rightsquigarrow c = \frac{3Nk}{k+2}$, which equals 9 if $(k, N) = (1, 9), (2, 6), (3, 5), (6, 4)$. We will focus on the first model.

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  $g_1:(\Phi_1,\Phi_2,\Phi_3,\Phi_4,\Phi_5,\Phi_6,\Phi_7,\Phi_8,\Phi_9)\mapsto-(\Phi_2,\Phi_1,\Phi_3,\Phi_4,\Phi_5,\Phi_6,\Phi_7,\Phi_8,\Phi_9)$.

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- $\Rightarrow h^{2,1}(M)^+ = 63$.

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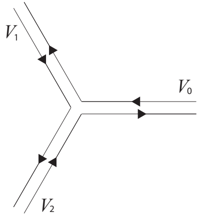

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- $1^{\otimes 9}/\mathbf{Z}_3$: The orbifold projects RR ground states to $H^{(p,q)}(M) \ni \Omega_{\vec{\ell}} \leftrightarrow |\ell^1,\ldots,\ell^9\rangle$ with $p+q=3$, $\ell^i=1,2$, $\sum_{i=1}^9 \ell^i = 0 \mod 3$.

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- The "3-forms" are classified as:

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- Tadpole cancellation: $N_{\text{flux}} + N_{\text{D3}} = 12$, where $N_{\text{flux}} = \int_M F_{RR} \wedge H_{NS}$.

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- To ensure $N_{\rm flux} \leq 12$, sufficient to restrict to up to 12-Ω solutions.
- Already in BBVW and BGWW, solutions with $N_{\rm flux} \leq 12$ were found. The minimum value of $N_{\rm flux}$ among these solutions is 8. Qn: Is there a "shorter" solution?

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- Dependence of W on complex structure moduli $t^{\vec{\ell}}$: $\int_\Gamma \Omega = \int_\Gamma \exp -\mathcal{W}(t^{\vec{\ell}})$, where $\mathcal{W}(t^{\vec{\ell}}) = \sum_{i=1}^{9} \Phi_i^3 - \sum_{\vec{\ell}} t^{\vec{\ell}} \vec{\Phi}^{\vec{\ell}-\vec{1}}$.

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- The Hessian DDW has the same rank r as the mass matrix (BGWW). $r =$ the number of massive moduli. Using a "homogeneous" basis of 3-cycles yields simpler formulas for the matrix elements of DDW.

Systematic Exploration of Solutions

- Tadpole cancellation $\Rightarrow N_{\mathrm{flux}} + N_{\mathrm{D3}} = 12$.
- It suffices to restrict search for fluxes with $N_{\mathrm{flux}} \leq 12$.
- For each Ω_I turned on, N_{flux} increases by at least 1.
- Hence, restrict search to ≤ 12-Ω_I's turned on.
- Searching through all possible 12-Ω configurations computationally not affordable – we organize solutions with fewer Ω_I's turned on.
- n is a crude lower bound for N_{flux} for the set of n-Ω flux backgrounds. We find the best lower bound, and the full set of fluxes that saturate it, for $n = 1, 2, 3, 4$.

1, 2, 3-Ω Solutions

$G = \sum B\Omega$	$T_{\min}$	Index Range	# Massive Scalars
$G = \frac{1}{\sqrt{3}}\Omega_I$	27	$\alpha = 1, \ldots, 35$ $\alpha = 57, \ldots, 63$	16 22
	54	$A = 36, \ldots, 63$	22
$G = \frac{i}{3}(\Omega_1 - \Omega_\alpha)$	18	$\alpha = 2, \ldots, 35$ $\alpha = 57, \ldots, 63$	16, 24, or 26 28 or 32
$G = \frac{i}{3}(\Omega_1 + \Omega_\alpha + \Omega_\beta)$ or $G = \frac{i}{3}(\Omega_{57} + \Omega_\alpha + \Omega_\beta)$	27	$\alpha, \beta = 2, \ldots, 35,$ $57, \ldots, 63$ $\alpha, \beta = 58, \ldots, 63$	$\{16, 20, 22, 24, 28, 29, 30,$ $32, 34, 36, 38, 40, 42, 46\}$

4-Ω Solutions

Minimum Flux Tadpole Contribution: $\mathrm{T}_{\mathrm{min}} = 12$

First example within the physical bound.

$$G = \frac{1}{3\sqrt{3}}\left(-\Omega_1 + \Omega_{\vec{\ell}_\alpha} + \Omega_{\vec{\ell}_\beta} - \Omega_{\vec{\ell}_\gamma}\right)$$

Family of Solutions:

$$G = \pm\omega^r \frac{1}{3\sqrt{3}}\left(-\Omega_1 + \omega^p\Omega_\alpha + \omega^{q-p}\Omega_\beta - \omega^q\Omega_\gamma\right)$$

Quantization Condition:

$$\frac{1}{3}(-\omega^a + \omega^b + \omega^c - \omega^d) = N - \omega M, \qquad \text{iff} \qquad (-a - d + b + c) \mod 3 = 0$$

4-Ω Solutions

# Massive Scalars	(α, β, γ)			
12	(2, 6, 8) (2, 7, 9) (2, 12, 14) (2, 13, 15) (2, 23, 25)	(3, 5, 8) (3, 7, 10) (3, 11, 14) (3, 13, 16) (3, 21, 24) (3, 23, 26)	(4, 5, 9) (5, 12, 17) (5, 13, 18) (6, 13, 19) (5, 22, 27) (7, 22, 29)	
22	(2, 33, 34) (2, 29, 30) (2, 19, 20) (7, 31, 35) (11, 29, 35)	(3, 32, 34) (3, 28, 30) (3, 18, 20) (13, 27, 35) (17, 23, 35)	(4, 31, 34) (4, 27, 30) (13, 24, 34)	(5, 26, 30) (5, 16, 20) (6, 15, 20) (7, 14, 20) (8, 13, 20)
26	(2, 59, 60)	(4, 57, 60) (5, 59, 61)	(13, 57, 62) (21, 59, 63)	

8-Ω Solutions

$$G = \frac{1}{9}\left(-\Omega_1 + \Omega_{\vec{\ell}_{a_2}} - \Omega_{\vec{\ell}_{a_3}} + \Omega_{\vec{\ell}_{a_4}} - \Omega_{\vec{\ell}_{a_5}} + \Omega_{\vec{\ell}_{a_6}} - \Omega_{\vec{\ell}_{a_7}} + \Omega_{\vec{\ell}_{a_8}}\right)$$

Quantization Condition: $(-\vec{\ell}_1 + \vec{\ell}_{a_2} - \vec{\ell}_{a_3} + \vec{\ell}_{a_4} - \vec{\ell}_{a_5} + \vec{\ell}_{a_6} - \vec{\ell}_{a_7} + \vec{\ell}_{a_8}) \bmod 3 = 0$

Tadpole Contribution:

$T_{min} = 8$

 # Massive Scalars:

14

$(a_2, a_3, a_4, a_5, a_6, a_7, a_8)$	
(2, 8, 6, 15, 13, 19, 20)	(2, 8, 6, 25, 23, 29, 30)
(2, 9, 7, 14, 12, 19, 20)	(2, 9, 7, 24, 22, 29, 30)
(3, 8, 5, 16, 13, 18, 20)	(3, 8, 5, 26, 23, 28, 30)
(3, 10, 7, 14, 11, 18, 20)	(3, 10, 7, 24, 21, 28, 30)
(4, 9, 5, 16, 12, 17, 20)	(4, 9, 5, 26, 22, 27, 30)
(2, 14, 12, 25, 23, 33, 34)	(3, 14, 11, 26, 23, 32, 34)
(4, 15, 11, 26, 22, 31, 34)	(5, 17, 12, 28, 23, 33, 35)

The Shortest Vector Problem

- $\exists$ solutions with $N_{\rm flux} = 8$. Are their "shorter" fluxes? Answer: No.
- To rule out $N_{\rm flux} \leq T$, we proceed as follows. (In our code we implement this for $T = 7$).
- Choose T complex B_i's to be (possibly) non-zero and set the remaining to zero $\rightsquigarrow$ a set of $(126 - 2T)$ **R**-linear equations in the integer flux quanta y_i.
- The set $\{B_{i_1}, \ldots, B_{i_T}\} \neq 0$ gives a reduced expression for $\mathrm{N_{flux}}$,

$$\mathrm{N^{red}_{flux} = Q_{ij}\, c_i\, c_j} \quad \text{where} \quad \mathrm{c_i \in \mathbb{Z},\ i = 1, \ldots, 2T}$$

  and Q is symmetric, positive definite.
- Is $\mathrm{N^{red}_{flux}}(\vec{c}) \leq \mathrm{T}$ for a choice of integers c_i?

The Eigensieve Algorithm

- Define the level set in $\mathbb{R}^{2T}$: $L_T = \{(x_1, \ldots, x_{2T}) : \mathrm{N}^{\mathrm{red}}_{\mathrm{flux}}(\vec{x}) = T\} \subset \mathbb{R}^{2T}$.
- Points $\vec{x}$ outside of the ellipsoid L_T will have values, $\mathrm{N}^{\mathrm{red}}_{\mathrm{flux}} > T$.
- Are there integer lattice points on L_T or in its interior?
- Eigensieve: A combination of enumeration and sieving using the eigenvalues of Q, $\{\lambda_1, \ldots, \lambda_{2T}\}$ where $0 < \lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_{2T}$ and normalized eigenvectors, $\{\vec{v}_1, \ldots, \vec{v}_{2T}\}$.
  First carve out a finite region of $\mathbb{Z}^{2T}$ using the smallest eigenvalue of Q. Lattice points $\vec{p}$ in this region satisfying $|\vec{p}.\vec{v}_i| > \sqrt{\frac{T}{\lambda_i}}$, $i = 1(1)2T$, all lie outside of L_T are sieved off. Explicit evaluation of $\mathrm{N}^{\mathrm{red}}_{\mathrm{flux}}$ done on the remaining points.

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- All solutions found so far have number of massive moduli below the tadpole conjecture bound.
- Need to push exhaustive search up to 12-Ω solutions – moving away from symbolic computation in Mathematica necessary.
- Finally, we aim to extend all our analyses to other Gepner models.

Thank You!