(De)Constructing Scale Separation with Weak Gravity and Anisotropies

Niccolò Cribiori

Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

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Based on work with: G. Dall'Agata, F. Farakos, C. Montella, D. Junghans, G. Tringas, V. Van Hemelryck, T. Van Riet, T. Wrase

- Swampland conjectures are a window into quantum gravity.
- They encode consistency conditions that low energy EFTs must obey and that are not obvious from IR perspective.
- Explicit realization of UV/IR mixing in quantum gravity.
- Typically tested in **top-down** constructions in string theory. Needed to gain confidence.

In the first part

I will explore a *complementary* approach:

- Assume swampland conjectures to be principles of QG
- Apply them to 4D and 5D supergravity (**bottom-up**)

The strategy is *coarse*, but it can lead to results on

- Scale-separated anti-de Sitter vacua
- De Sitter vacua

For this approach, need to use robust conjectures:

Weak Gravity Conjecture (WGC) $\Lambda_{UV} \lesssim g\, M_p$

[Arkani-Hamed, Motl, Nicolis, Vafa '06]

In the second part

I will discuss a **top-down** construction of scale-separated AdS_4 vacua in type IIA/M-theory.

Anisotropies in the internal manifold crucial for scale separation.

Is this lesson more general?

First Part:
Deconstructing Scale Separation with Weak Gravity

Warm-up

Consider 4d N=1 SUGRA with 1 vector. Turn on FI term

$$V_{FI} = \frac{1}{2} g^2 \xi^2 > 0$$

If ξ quantized [Seiberg '10; Distler, Sharpe '10], two related facts:

- Limit $V_{FI} \to 0$ leads to global $U(1)_R$
  However, there cannot be global symmetries in QG.
- Cosmological constant of order WGC cutoff

$$|V_{FI}| \simeq g^2 \overset{WGC}{\gtrsim} \Lambda_{UV}^2$$

Model not protected against corrections [NC, Farakos, Tringas '21]

Pure FI terms are in the swampland

In agreement with [Komargodski, Seiberg '09]

Strategy can be applied to models with $Q \geq 8$ supercharges.

- **All** gSUGRA with SUSY AdS: constraints on scale separation [Tsimpis '12]

$$\frac{L_{KK}}{L_H} \overset{?}{\ll} 1$$

- Certain gSUGRA with dS vacua

Note: WGC formulated in flat space; extension to (A)dS unclear. Recent proposal: *Positive Binding Conjecture*. [Aharony, Palti '21; Palti, Sharon '22; Andriolo, Michel, Palti '22]

Here, I will *assume* that curvature corrections to

$$\Lambda_{UV} < g\, M_P$$

are small in the SUGRA regime. See e.g. [Huang, Li, Song '06].

Weak Gravity vs Scale Separation(1/2)

[NC, Dall'Agata '22; NC, Montella '23]

- Consider gSUGRA with SUSY AdS vacua in 4D and 5D. The SUSY AdS vacuum energy is the gravitino mass

$$V_{AdS} = -m_{3/2}^2$$

- With at least 8 supercharges, SUSY relates

$$m_{3/2} \longleftrightarrow g$$

e.g. due to (very) special geometry.

- We can rewrite the vacuum energy as ($q = 1$ since quantized)

$$V_{AdS} = -g^2$$

- What is the gauge group?

Weak Gravity vs Scale Separation (2/2)

[NC, Dall'Agata '22; NC, Montella '23]

- According to [Louis, Lüst, Ruter '17] on SUSY AdS vacua

$$G \to H_R \times H_{mat}, \qquad H_R = \begin{cases} SO(N) & d=4 \\ [S]\,U(N/2) & d=5, \quad [N=8] \end{cases}$$

  - H_R gauged by graviphoton $\sim X^\Lambda A_\Lambda$
  - H_{mat} gauged by matter vectors $\sim \partial_i X^\Lambda A_\Lambda$

- We can split the contributions to the vacuum energy

$$V_{AdS} = -g^2 = -(g_R^2 + g_{mat}^2) < -g_R^2$$

$$i.e. \qquad L_H^{-2} = |V_{AdS}| \geq g_R^2 \overset{WGC}{\gtrsim} \Lambda_{UV}^2$$

- If $\Lambda_{UV} \sim \Lambda_{KK} \Rightarrow$ **no scale separation**.

An example

M-theory on SE_7 manifolds gives 4D N=2 gSUGRA specified by [Gauntlett, Kim, Varela, Waldram '09; Hristov, Looyestjin, Vandoren '09]

$$F = \sqrt{X^0(X^1)^3}$$

and quaternionic metric $ds^2 = \frac{1}{4\rho^2}\left(d\rho^2 + (d\sigma - i(\xi d\bar{\xi} - \bar{\xi} d\xi))^2\right) + \frac{1}{\rho} d\xi d\bar{\xi}$.

On the AdS vacuum a U(1) ⊂ U(1)×U(1) factor survives

$$\mathcal{P}^x_\Lambda = e_\Lambda \delta^{x3}, \qquad e_\Lambda = (1, -3).$$

The vacuum energy can be rewritten as

$$V_{AdS} = -6 g_R^2 q^2$$

These vacua are not scale separated and thus not really 4D.

Result and implications

- In [NC, Dall'Agata '22, NC, Montella '23] explicit argument given for N=2 and N=8 gSUGRA in 4D and 5D. Compatible with [Montero, Rocek, Vafa '22].

- No clear obstruction in going beyond 5D, or in specializing to $8 < Q < 32$. (Indeed [Apruzzi, De Luca, Gnecchi, Lo Monaco, Tomasiello '19] proved no scale separation in SUSY AdS_7.)

- When combined with [Ooguri, Vafa '16], no $d > 4$ EFT in AdS, regardless of SUSY?

- $Q \leq 4$ still not covered (3D $N \leq 2$ and 4D $N \leq 1$). Known examples: [DeWolfe, Giryavets, Kachru, Taylor '05; NC, Junghans, Van Hemelryck, Van Riet, Wrase '22; Farakos, Van Riet, Tringas '22; De Luca, De Ponti, Mondino, Tomasiello '22; Carrasco, Coudarchet, Marchesano, Prieto '23; Tringas '23; Farakos, Morittu '23]

Weak gravity vs de Sitter

[NC, Dall'Agata, Farakos '20; Dall'Agata, Emelin, Farakos, Morittu '21; NC, Montella '23]

- The scalar potential of gSUGRA is schematically

$$V = g^2 - m_{3/2}^2$$

- Assuming vanishing gravitino mass on the vacuum, we can repeat a similar analysis as for AdS

$$V_{dS} = g^2 = g_{WGC}^2 + g_{rest}^2$$
$$\geq g_{WGC}^2 \overset{WGC}{\gtrsim} \Lambda_{UV}^2$$

- In dS, natural IR cutoff $L_H^{-1} \sim H \sim \Lambda_{IR}$. Then

$$\Lambda_{IR}^2 \sim V_{dS} \gtrsim g_{WGC}^2 \sim \Lambda_{UV}^2$$

For good EFT $\Lambda_{UV} \gg \Lambda_{IR}$, hence these EFTs are in the swampland

Results and implications

- Vacua with massless charged gravitini in tension with WGC. Independent from stability.
- All known stable dS_4 vacua in $N \geq 2$ gSUGRA [Fre, Trigiante, Van Proeyen '02] are in the swampland.
- However, stable dS_5 vacua [Cosemans, Smet '05; Ogetbil '08] evade the argument. To be investigated further, also in light of [Hebecker, Schreyer, Venken '23].
- $Q \leq 4$ at the Lagrangian level (e.g. 4D $N = 1$) still not covered. Compatible with [Andriot, Horer, Marconnet '22].

A general lesson

Certain SUGRA models know about swampland conjectures, in some sense. More arguments and examples in [NC, Farakos '23].

This points towards the following general lesson

$$\text{SUGRA} + \text{conjecture A} \iff \text{SUGRA} + \text{conjecture B}$$

$$\text{SUGRA} + \text{known conjecture(s)} \iff \text{SUGRA} + \text{new conjecture(s)}$$

Second Part:
Constructing Scale Separation with Anisotropies

DGKT: reasons of concern

Setup: mIIA supergravity on SU(3)-structure manifolds with O6 planes (and possibly D6). Solution governed by free parameter $n \sim \int F_4$, such that for $n \to \infty$ one gets large volume, weak coupling and parametric scale separation.

[Behrndt, Cvetic '04; Deredinger, Kounnas, Petropoulos, Zwirner '04; Lüst, Tsimpis '04; DeWolfe, Giryavets, Kachru, Taylor '05]

1. Non-vanishing Romans mass: no smooth M-theory uplift in which sources become geometry à la [Atiyah, Hitchin '85]

2. "Smeared" sources: trick to solve Bianchi identities and EOMs

$$dF = \delta \quad \to \quad dF = \rho$$

However, unclear physical meaning if sources are orientifolds.

Possible way out

1. Double T-dualize to massless IIA [Banks, Van den Broek '06]

2. Apply systematic strategy of [Junghans '20; Marchesano, Palti, Quirant, Tomasiello '20] to compute corrections to the smeared approximation

$$F_p = F_p^{(0)} + \epsilon F_p^{(1)} + \dots, \qquad \epsilon = n^{-\alpha}$$

where

$$dF_p^{(0)} = \rho, \qquad dF_p^{(1)} = (\rho - \delta)$$

and similarly for all fields.

Combining these two steps, we found possible evidence for scale separated AdS_4 vacua in massless IIA and M-theory [NC, Junghans, Van Hemelryck, Van Riet, Wrase '21]

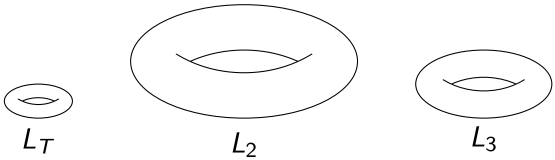
The setup

- Start from T^6 and formally perform 2 T-dualities (see [Banks, Van den Broek '06] for subtleties). Get Iwasawa manifold

$$ds^2 = (L_T e^1)^2 + (L_2 e^2)^2 + (L_3 e^3)^2 + (L_2 e^4)^2 + (L_3 e^5)^2 + (L_T e^6)^2,$$

$$de^1 = -e^{23} - e^{45}, \quad de^6 = -e^{34} - e^{25}$$

- 3 two-tori with sizes L_T, L_2 and L_3



- N=1 AdS_4 smeared solution with $F_2 \neq 0 \neq F_6$, while $F_0 = F_4 = H_3 = 0$ [Caviezel, Koerber, Kors, Lust, Tsimpis, Zagermann '08].

Scaling analysis

The solution can be scaled as

$$L_T \sim n^{(a-b-c)/4}, \quad L_2 \sim n^{(a-b+c)/4}, \quad L_3 \sim n^{(a+b-c)/4}, \quad L_H \sim n^{(a+b+c)/4}, \quad g_s \sim n^{(a-3b-3c)/4},$$

while preserving the orientifold charge.
Generalized in [Carrasco, Coudarchet, Marchesano, Prieto '23]

- Crucially, if $L_T \equiv L_2 \equiv L_3$, $(b = c = 0)$, **no scale separation**.
- Otherwise, choose $c \geq b$ (second torus bigger), while $b > 0$ needed for control. Hence, $L_2 \sim L_{KK}$ and scale separation governed by
$$\frac{L_{KK}}{L_H} \sim n^{-\frac{b}{2}}$$
- By tuning a, b, c we can find scale separated solutions at large volume and weak or strong coupling (M-theory)

Uplift to 11D

- Uplift of the smeared solution would not work [Banks, Van den Broek '06]. For example, Bianchi identities are not really solved, not even away from the sources ($dF_p = 0$), due to ρ.

- More in detail, the 11D EOMs imply $\hat{R}_7 > 0$.
  On the other hand, the smeared solution has $\hat{R}_7^{smeared} < 0$.

- Remarkably, backreaction corrections flip the sign!

$$\hat{R}_7 = \hat{R}_7^{smeared} + corrections > 0$$

Evidence for scale-separated, sourceless
(but classically singular) geometry in 11D SUGRA?

Conclusion

- Scale separation is a meaningful requirement for phenomenology in theories with extra dimensions.
- We gave evidence that $2 \leq N \leq 8$ $AdS_{4,5}$ vacua of gauged SUGRA are not scale separated if the WGC holds.
- $N = 0, 1$ supersymmetry in $d = 3, 4$ most promising chance to get scale separated AdS vacua. Indeed, all known examples are of this kind.
- Non-isotropic internal manifolds allow for large parameter space. [NC, Junghans, Van Hemelryck, Van Riet, Wrase '21; Carrasco, Coudarchet, Marchesano, Prieto '23; Tringas '23; Farakos, Morittu '23] New scale-separated solutions to be uncovered?

Thank you!

Extra slides

Parameteric scale separation

Consider a d-dimensional theory with scalar potential V.

- On maximally symmetric vacuum $L_H^{-1} \sim |V|^{\frac{1}{2}}$
- Scale of the extra dimension L_{KK}

Parametric scale separation is the requirement:

$$\frac{L_{KK}}{L_H} \ll 1$$

Estimating L_{KK} is non-trivial. Typically

$$L_{KK} \sim \mathrm{Vol}^{\frac{1}{10-d}},$$

but several effects (e.g. warping) can change it [Andriot, Tsimpis '18; De Luca, Tomasiello '21, De Luca, De Ponti, Mondino, Tomasiello '21, '23].

Comments

- If $L_{KK}^{d-8} \sim \int R_{10-d}$, scale separation requires negative tension sources [Gautason, Schillo, Van Riet, Williams '15]. The bottom-up argument I will present is agnostic about this assumption.

- Swampland conjectures suggest no scale separation in AdS [Gautason, Van Hemelryck, Van Riet '18; Lüst, Palti, Vafa '19; Blumenhagen, Brinkmann, Makridou '19...]

$$L_H \sim \sqrt{k}\,(L_{KK})^{\alpha} \qquad \text{e.g. } \alpha = 1 \text{ for } AdS_5 \times S^5$$

  ($\mathbb{Z}_k$ symmetry refinement [Buratti, Calderon, Mininno, Uranga '20])
  However there are counterexamples; see [Courdarchet '23] for up to date review.

- I will not use any of the above, but derive $L_{KK} \sim L_H$ via WGC.

The argument in 4D (1/2)

Idea: We want to show that the vacuum energy is completely fixed by the WGC gauge coupling with no free parameter.

The SUSY AdS vacuum energy is given by the gravitino mass

$$V_{AdS} = -3\bar{L}^{\Lambda} L^{\Sigma} \mathcal{P}^x_{\Lambda} \mathcal{P}^x_{\Sigma}$$

There is a relation between gravitino mass and gauge couplings [Hristof, Looyestijn, Vandoren '09]

$$\bar{L}^{\Lambda} L^{\Sigma} \mathcal{P}^x_{\Lambda} \mathcal{P}^x_{\Sigma} = -\frac{1}{2} \left(\mathrm{Im}\mathcal{N}^{-1}\right)^{\Lambda\Sigma} \mathcal{P}^x_{\Lambda} \mathcal{P}^x_{\Sigma}$$

Thus we can express V_{AdS} in terms of the gauge coupling

$$V_{AdS} = 3 \left(\mathrm{Im}\mathcal{N}^{-1}\right)^{\Lambda\Sigma} \mathrm{Tr}\, \mathrm{P}_{\Lambda} \mathrm{P}_{\Sigma},$$

where $2\mathrm{P}_{\Lambda} = \mathbb{I}\mathcal{P}^0_{\Lambda} + \sigma^x \mathcal{P}^x_{\Lambda}$.

The argument in 4D (2/2)

Identify and canonically normalise the WGC U(1) vector

$$A_\mu^{WGC} = \Theta_\Lambda A_\mu^\Lambda, \qquad g^2 = -\Theta_\Lambda \left(\mathrm{Im}\mathcal{N}^{-1}\right)^{\Lambda\Sigma} \Theta_\Sigma$$

Finally split $\mathrm{P}_\Lambda = \mathrm{P}_\Lambda^\perp + \mathrm{P}_\Lambda^\parallel$ (wrt A_μ^{WGC}) and find

$$V_{AdS} = 3\left(\mathrm{Im}\mathcal{N}^{-1}\right)^{\Lambda\Sigma}\left(\mathrm{Tr}\mathrm{P}_\Lambda^\parallel \mathrm{P}_\Sigma^\parallel + \mathrm{Tr}\mathrm{P}_\Lambda^\perp \mathrm{P}_\Sigma^\perp\right)$$

$$\leq 3\left(\mathrm{Im}\mathcal{N}^{-1}\right)^{\Lambda\Sigma}\mathrm{Tr}\mathrm{P}_\Lambda^\parallel \mathrm{P}_\Sigma^\parallel = -3g^2\mathrm{Tr}(q^2)$$

i.e.

$$|V_{AdS}| \geq 3g^2\,\mathrm{Tr}\left(q^2\right) \overset{WGC}{\gtrsim} \mathrm{Tr}(q^2)\,\Lambda_{UV}^2$$

Thus if $\Lambda_{UV} \sim \Lambda_{KK}$ there is **no scale separation** (assuming charge quantisation).

Backreaction corrections

- Identify the parameter governing the backreaction.
  In our case this is $\epsilon = n^{-b}$. Then, expand all fields in powers of ϵ.

- Solve EOMs and Bianchi identities perturbatively in ϵ.

- Remarkably, at first order **all** equations reduce to a single 1D Poisson equation for an ansatz function β

$$\nabla^2 \beta = \frac{e^{\phi}}{2}(\rho - \delta)$$

whose solution is

$$\beta(x) \sim \frac{(x)^2}{2} - |x|, \qquad x \in [-1, 1]$$

It controls the backreaction in the direction orthogonal to the sources.