Exploring the Interior of N=1 Field Spaces

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Deconstructing the Landscape a.k.a. “Landscapia” — Saclay
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Introduction

String Theory (and its compactifications) come with a number of **scalar fields** whose vacuum expectation values determine the **properties of the effective theory**

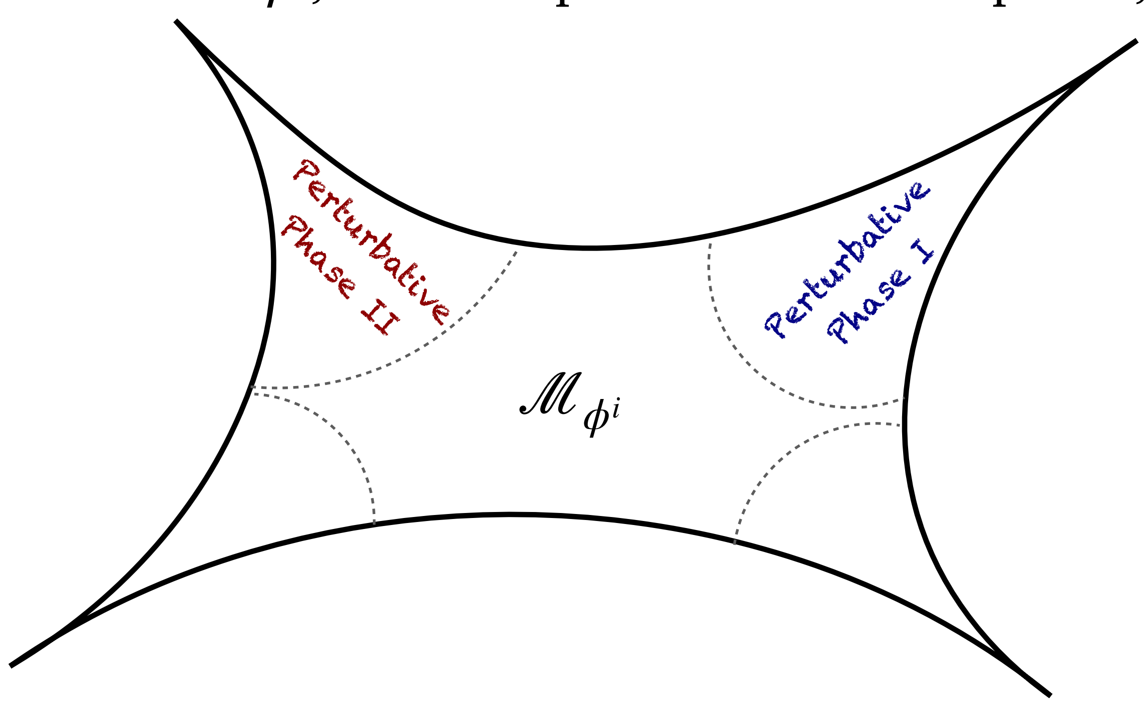
→ values of couplings, masses of states, value of the EFT cut-off …

Families of EFTs from string theory parametrized by the values of the scalar fields

→ scalar field space $\mathscr{M}_{\phi^i}$

Structure of $\mathscr{M}_{\phi^i}$ gives information about general properties of the theory

→ allowed values for ϕ^i, different perturbative descriptions, dualities … .



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→ comes equipped with a metric which can be computed in a perturbative limit of the theory
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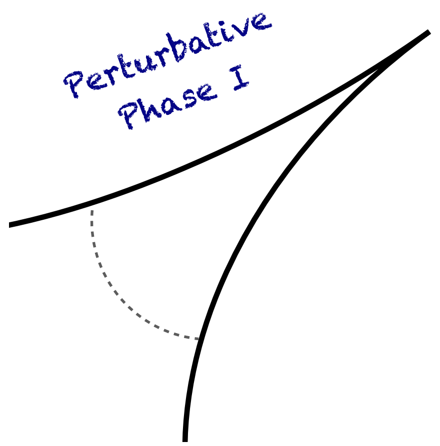

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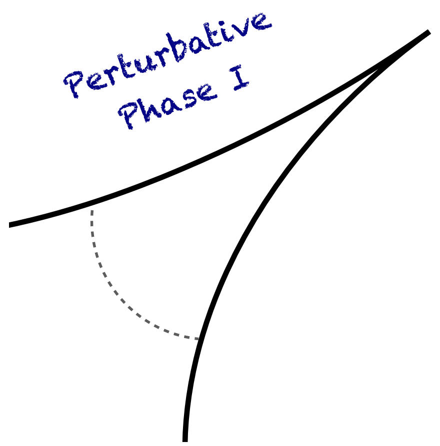

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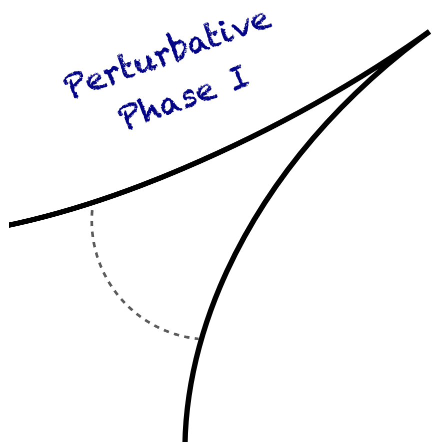

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With less supersymmetry can sometimes rely on non-renormalization theorems to describe moduli space away from perturbative limits:

Example 4d N=2: Moduli space factorizes into vector- and hypermultiplet sector and only one factor contains the string coupling $\rightarrow$ tree-level exact.

Question: What about the more realistic cases in 4d with minimal (or no) supersymmetry?

Warm-up: Type IIA Calabi—Yau compactifications

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Gather some intuition from 4d N=2 first — Specifically Type IIA Compactifications on CY 3-fold X_3

- Moduli space spanned by:
  - *Type II dilaton + axionic partner*
  - *Complex structure moduli of X_3 + axionic partners*

  hypermultiplets

  - *(complexified) Kähler moduli of X_3*

  vector multiplets

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- N=2 supersymmetry ensures factorization $\mathscr{M} = \mathscr{M}_{\text{HM}} \times \mathscr{M}_{\text{VM}}$.

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  → can trust the structure derived from string CFT

  ↔ mirror symmetry to complex structure moduli of $\tilde{X}_3$

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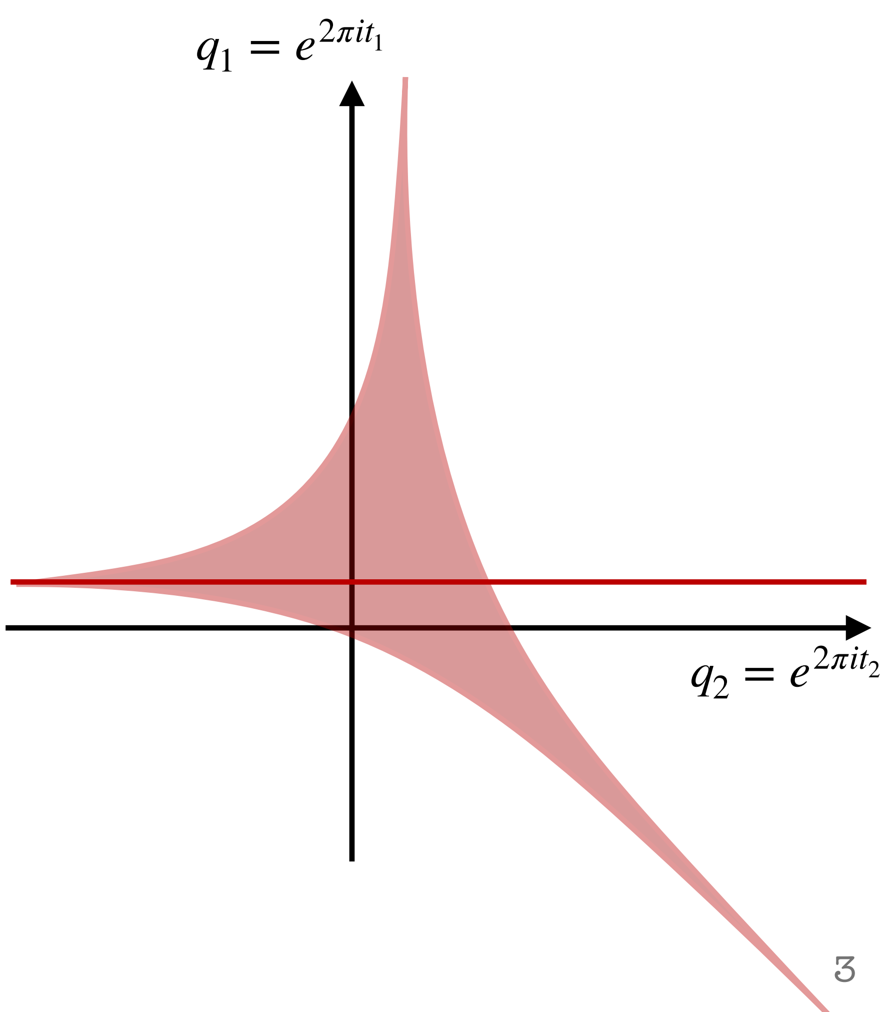

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- Thanks to factorization can describe small volume regime of $\mathscr{M}_{\rm VM}$
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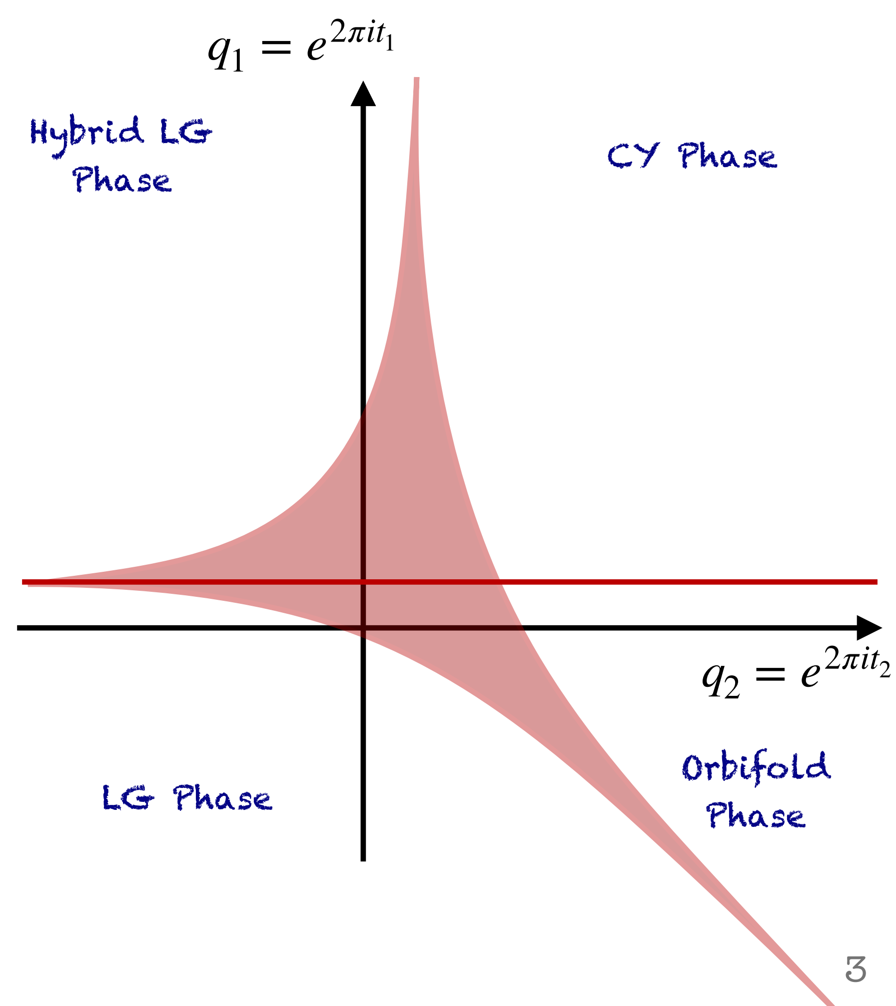
  - *at small volume get phases different from CY phase, e.g. orbifold phases, Landau-Ginzburg or hybrid phases.*



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- Scalar field space spanned by [Grimm '10]
  - complex structure moduli of X_4
  - complexified volumes of divisors of B_3

$$T_i = \frac{1}{2}\int_{D_a} J \wedge J + i \int_{D_a} C_4$$

J: Kähler form on B_3
D_a: Generators of $\mathrm{Eff}^1(B_3)$
C_4: Type IIB RR four-form

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- What happens away from the overall large volume limit?

  1. *small curve limit for* ***some*** *curves in* B_3
  2. *Mixing between c.s. and Kähler sector*

Structure of Kähler field space

- Consider first small curve limits in B_3.

- Naively might expect a similar pattern as in Type IIA → shrinking genus-0 curves also fall in three classes??

IIA on CY3:
[Witten '96]

- $\mathcal{N}_{C|B_3} = \mathcal{O}(-1) \oplus \mathcal{O}(-1)$ — *only curve shrinks → can trust classical geometry*
- $\mathcal{N}_{C|B_3} = \mathcal{O}(-2) \oplus \mathcal{O}(0)$ — *divisor shrinks to curve → classical geometry trustable due to enhanced supersymmetry*
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- For F-theory on $T^2 \to B_3$ can at best be true for curves not intersecting the anti-canonical divisor $\bar{K}_{B_3}$ (curves that do not 'see' the breaking of supersymmetry "$\mathcal{N} = 2$ curves")

- Interesting case: what happens if we shrink a curve C such that $\bar{K} . C > 0$?
→ can we trust the classical geometric picture and the Kähler potential derived from it?

Genuine N=1 effects — Case I

Focus on curves with $\bar{K}\,.\,C > 0$:

- Possibilities genus 0 curve with $\bar{K}\,.\,C = 1$

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- *Consistency check: for curve with $\mathscr{N} = \mathcal{O}(-2) \oplus \mathcal{O}(0)$ correction vanish and we can still trust the geometric picture.*

Genuine N=1 effects — Case II

[MW '22]

Consider now $\bar{K}_{B_3} \cdot C = 2$ and $\mathcal{N}_{C|B_3} = \mathcal{O}(0) \oplus \mathcal{O}(0)$.

→ C is fiber of rationally-fibered $B_3 : C \to B_2$ ↔ theory dual to heterotic string on CY3.
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Diverges in the limit [Klaewer, Lee, Weigand, MW '20]

- Via duality can argue that (at least in simple cases) a strong coupling singularity is reached for gauge theory on $D = B_2$.

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- All other (vertical) divisors have minimal quantum volume:

$$\frac{1}{\alpha^2} \text{Re}\, T_a \Big|_{\text{sing.}} = - \frac{\text{Re}\, T_a^{(0)}}{\mathcal{V}_{B_2}^{(0)}} \left(\frac{b}{8\pi} \log \xi + \text{const} . \right) + \text{Re}\, T_a^*$$

ζ : Complex structure parameter of X_4

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Shrinking of curve with $\mathcal{N} = \mathcal{O}(0) \oplus \mathcal{O}(0)$ is even worse than for $\bar{K} \cdot_{B_3} C = 1$.

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In general: Field space geometry for small genuine $\mathcal{N} = 1$ curves not describable by classical geometry

→ corrections are big and field space does not necessarily factorize anymore.

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- $\mathcal{N} = 1$ theory behaves significantly different from $\mathcal{N} = 2$ counterpart

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In general: Field space geometry for small genuine $\mathcal{N} = 1$ curves not describable by classical geometry

→ corrections are big and field space does not necessarily factorize anymore.

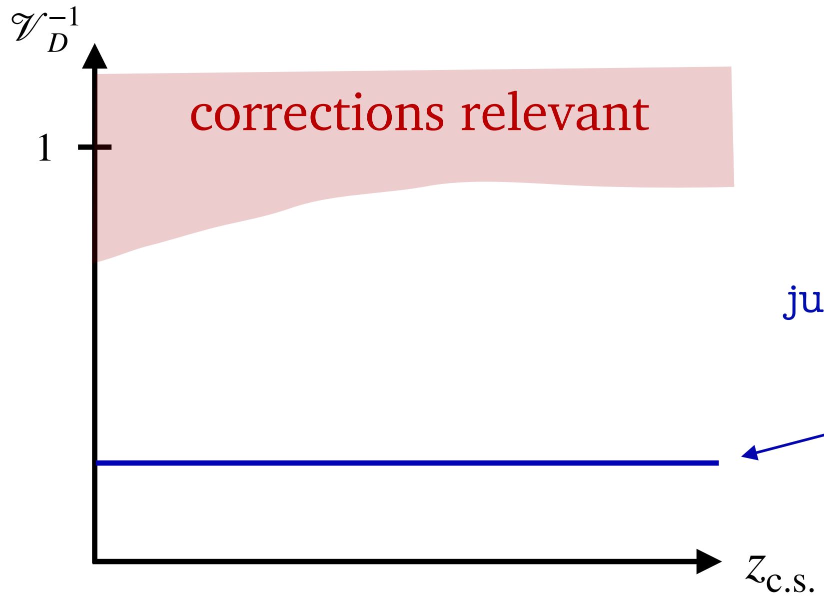
Question: Away from small curve limits can I still trust the classical field space structure?

→ does $\mathcal{M} \simeq \mathcal{M}_{\text{c.s.}} \times \mathcal{M}_{\text{Kahler}}$ only break down for very small volumes?

→ or corrections important for large complex structure?

Mixing in the Complex Structure Sector

Might expect that the mixing between Kähler and complex structure sectors is sufficiently suppressed as long as divisor volumes $\mathcal{V}_D \gg 1$:



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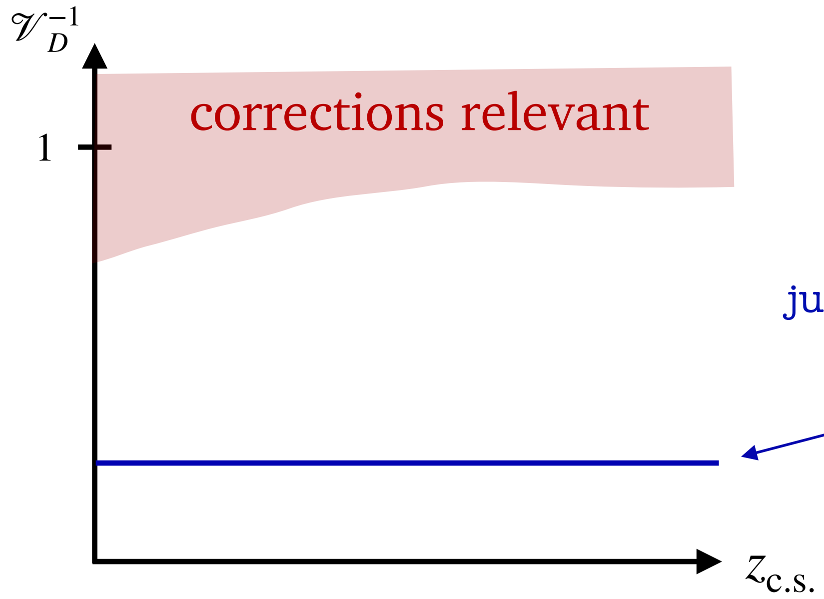

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Motivated by viewing F-theory via IIB orientifolds:

→ For Type IIB CY compactifications the complex structure is classically exact.

→ Can evaluate periods of X_4 reliably to infer structure of $\mathscr{M}_{c.s.}$.

→ Period integrals simplify close to boundaries of $\mathscr{M}_{c.s.}$ ⇒ good setting for e.g. searches for flux vacua.

Is this picture correct?

A simple Calabi—Yau fourfold

Consider a **very simple** elliptically-fibered Calabi-Yau fourfold

$$X_4 = (T^2 \to B_2) \times T^2 \qquad \Longrightarrow \qquad B_3 = B_2 \times T^2$$

Elliptically-fibered Calabi-Yau threefold

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Question: Can we already see in this theory what to expect got the mixing between complex structure sector and $\mathcal{V}_{B_3}$?

Therefore consider vector- and hypermultiplet sector of this F-theory comapctification:

- *complex structure moduli of $(T^2 \to B_2)$ and overall volume of B_2 + axionic partners* hypermultiplets

- *(complexified) Kähler moduli of B_2 + moduli of T^2* vector multiplets

Hypermultiplet Corrections to CY3 x T2

Focus on hypermultiplet sector of F-theory on $(T^2 \to B_2) \times T^2$

→ contains precisely the **volume modulus** and (part of) **the complex structure sector of** X_4.

F-theory on $(T^2 \to B_2) \times T^2$ dual to Type IIA on $T^2 \to B_2$.

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- Type IIA hypermultiplet sector receives corrections due to D2-brane instantons
- D2-brane instanton contributions to moduli space metric have been computed in

[Alexandrov, Banerjee '14]; see [Robes-Llana, M. Rocek, F. Saueressig, U. Theis, S. Vandoren, '06] for mirror dual Type IIB.

$$S_{4d}^{\text{corr.}} = S_{4d}^{(0)} + \sum \text{D2-instantons}$$

- effect on (mirror dual of) large complex structure limit moduli space has been investigated in [(Baume), Marchesano, MW '19]; see also [Alvarez-Garcia, Klaewer, Weigand '21]

→ effectively obstruct large complex structure limits!

Consequence for N=1 Theories

- Can break supersymmetry to N=1 e.g. through non-trivial fibration $X_4:\ X_3 \to \mathbb{P}^1 \qquad B_3 = B_2 \to \mathbb{P}^1$

  $\to$ classically $\mathscr{M}_{c.s.}(X_3) \subset \mathscr{M}_{c.s.}(X_4)$

- Expectation: corrections present in N=2 also correct N=1 theory

  $\to$ asymptotic regimes in $\mathscr{M}_{c.s.}(X_4)$ also receive corrections at finite $\mathscr{V}_{B_2}$ due to corrections to action of D3-brane instantons on $D = B_2 \subset B_3$

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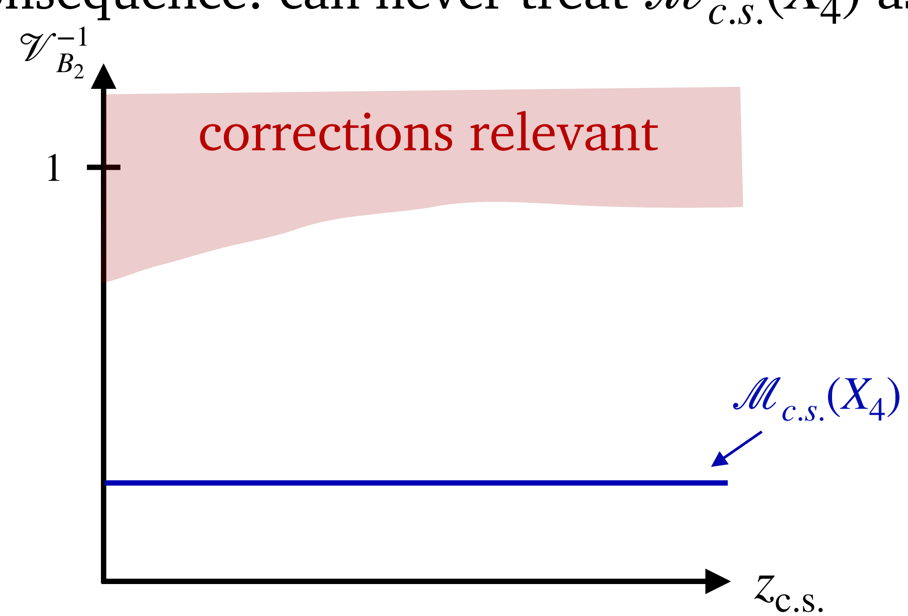

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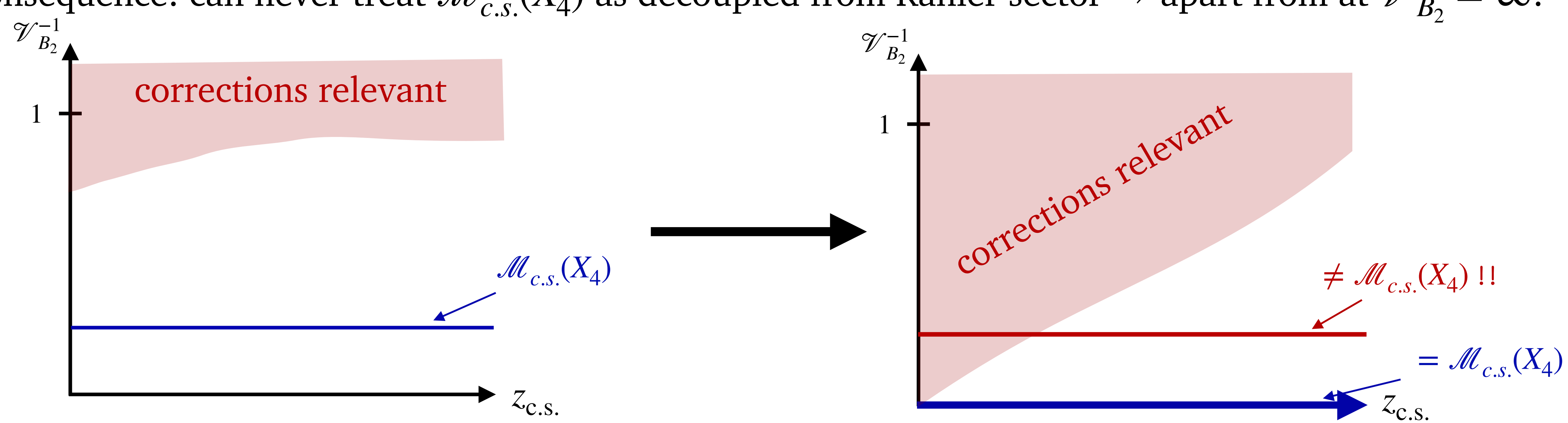

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- Can use N=2 intuition only if N=2 → N=1 breaking effects are infinitely diluted!
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- Explicitly considered F-theory compactifications on four-folds
  - genuine N=1 effects become large if curves intersected by anti-canonical divisor become small
    → N=2 breaking not diluted.
  - Mixing between complex structure and Kähler sector becomes important away from $\mathcal{V}_D = \infty$.
  - asymptotic regions in c.s. sector only describable through classical geometry in double-scaling limit
    (where N=2 supersymmetry is restored…)
    → similar effects to N=2 hypermultiplet sector at finite string coupling …

- **… what happens in general N=1 cases?**

Thank you!!