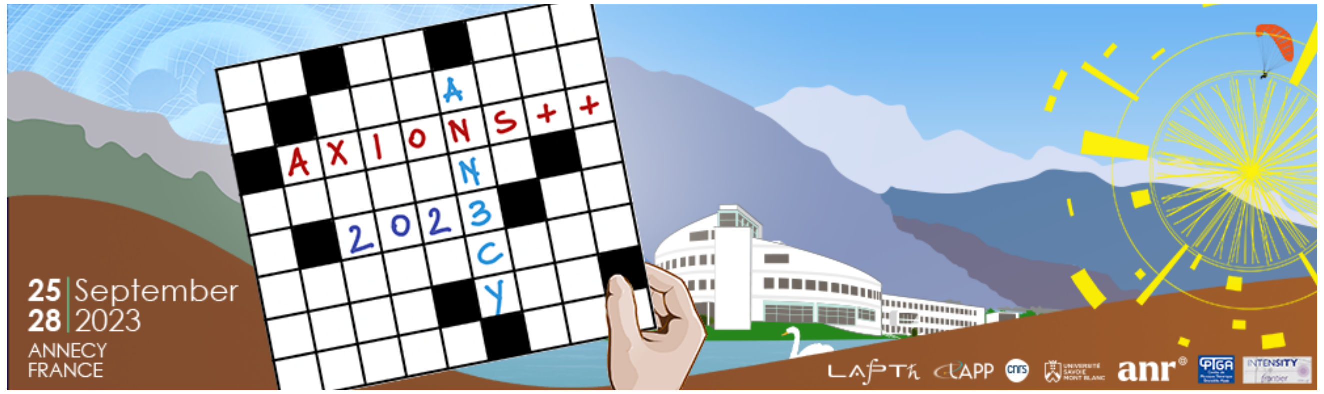
— Outlook —

Behind and Beyond the Standard Model

Axions++ 2023

LAPTH, Annecy, 28.09.2023



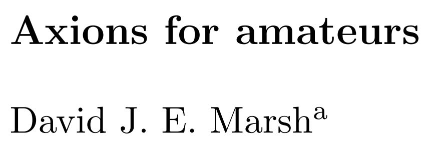
Christophe Grojean
DESY (Hamburg)
Humboldt University (Berlin)

(christophe.grojean@desy.de)

Disclaimer

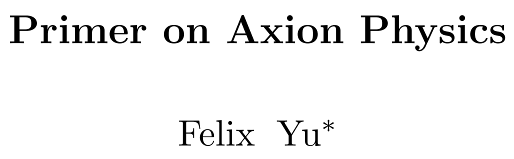
- **Organisers**: would you be available to deliver the keynote talk for the axions++ workshop?
- **Me**: I feel a bit embraced since I'm certainly not an expert on axion physics, despite my interest in the subject. But if you are happy with a broader outlook look, I would certainly be happy to prepare one. Furthermore I won't be able to come to Annecy on the first days of workshop, so I might not be the ideal speaker for the closing talk.

— Starting from the beginning during the summer—



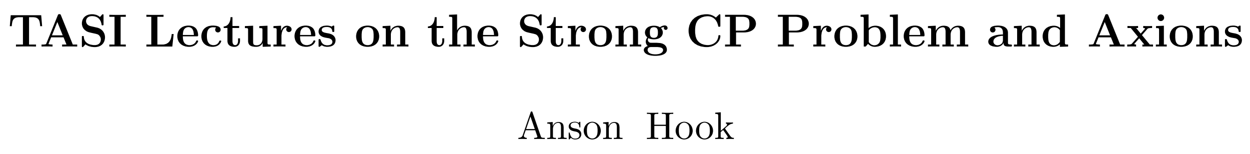
Axions for amateurs

David J. E. Marsh[a]



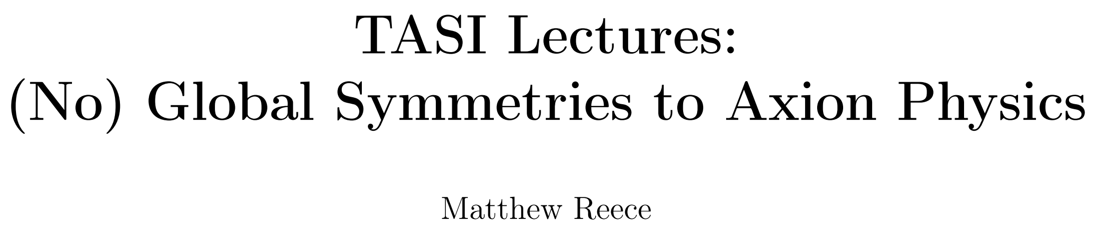
Primer on Axion Physics

Felix Yu*



TASI Lectures on the Strong CP Problem and Axions

Anson Hook



TASI Lectures:
(No) Global Symmetries to Axion Physics

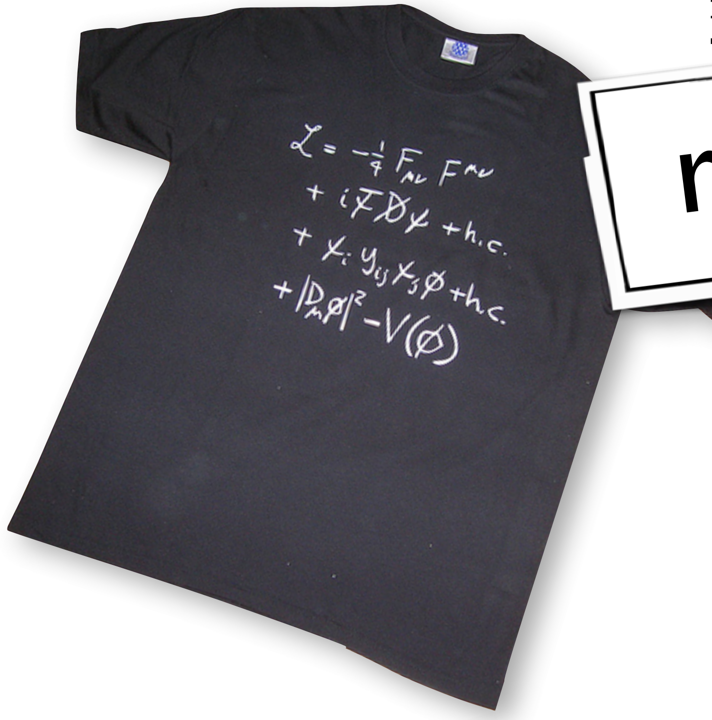
Matthew Reece

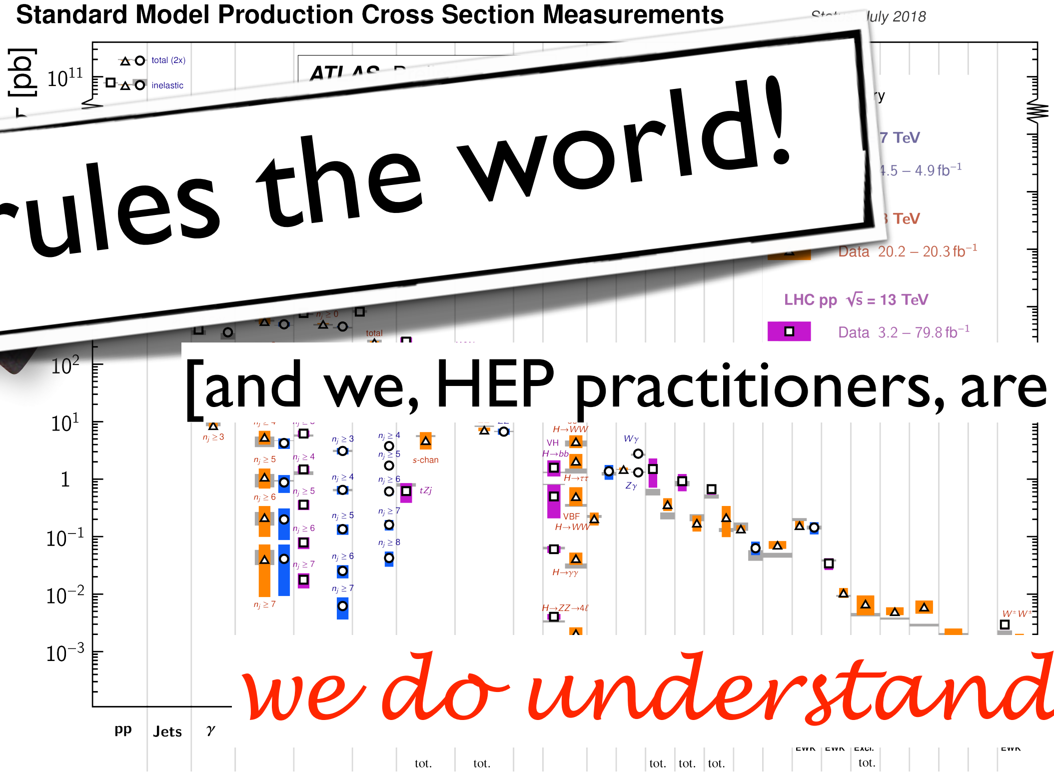
Standard Model …

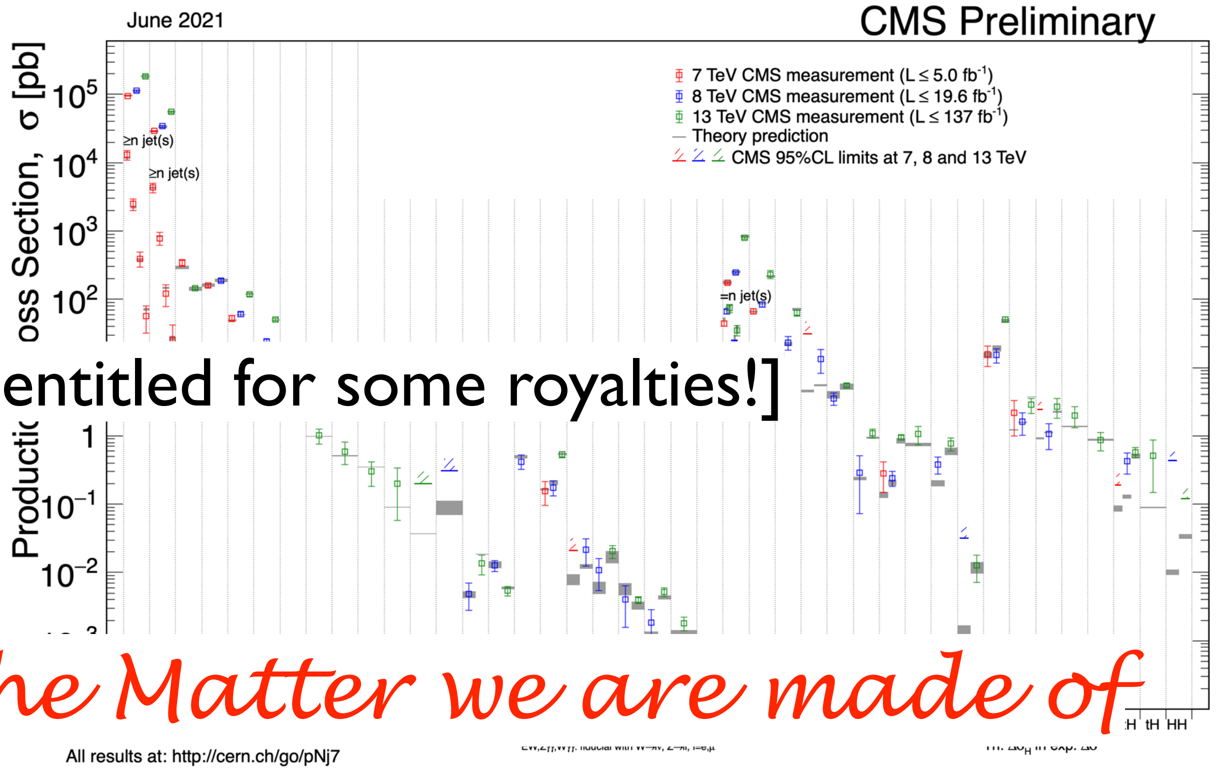
A. Bettini @ Erice'23

- 1961 S. Glashow $SU(2)\otimes U(1)$ weak mixing
- 1967-68 S. Weinberg, A. Salam spontaneous symmetry breaking
- 1971. G. 't Hooft how to renormalise weak interaction
- 1973. Gargamelle Discovery of neutral weak currents
- 1973 Quantum Chromodynamics

…J/Psi, b, W, Z, top, Higgs discoveries







rules the world!

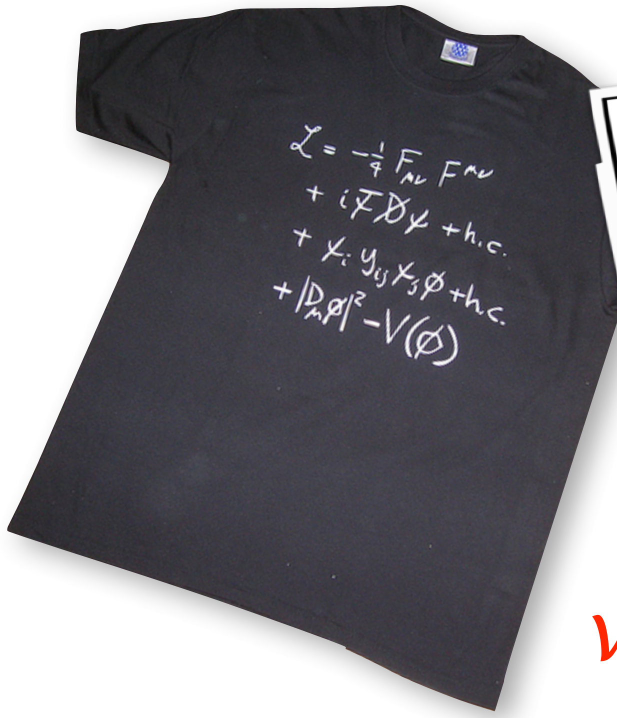
[and we, HEP practitioners, are all entitled for some royalties!]

we do understand the Matter we are made of

Standard Model and Beyond

The standard model is a wonderful achievement, but obviously incomplete

- Matter-antimatter asymmetry
- Dark Matter
- Dark Energy
- Flavour, CP, Hierarchy, Quantum gravity



is not enough

[and we all have to return our royalties!]

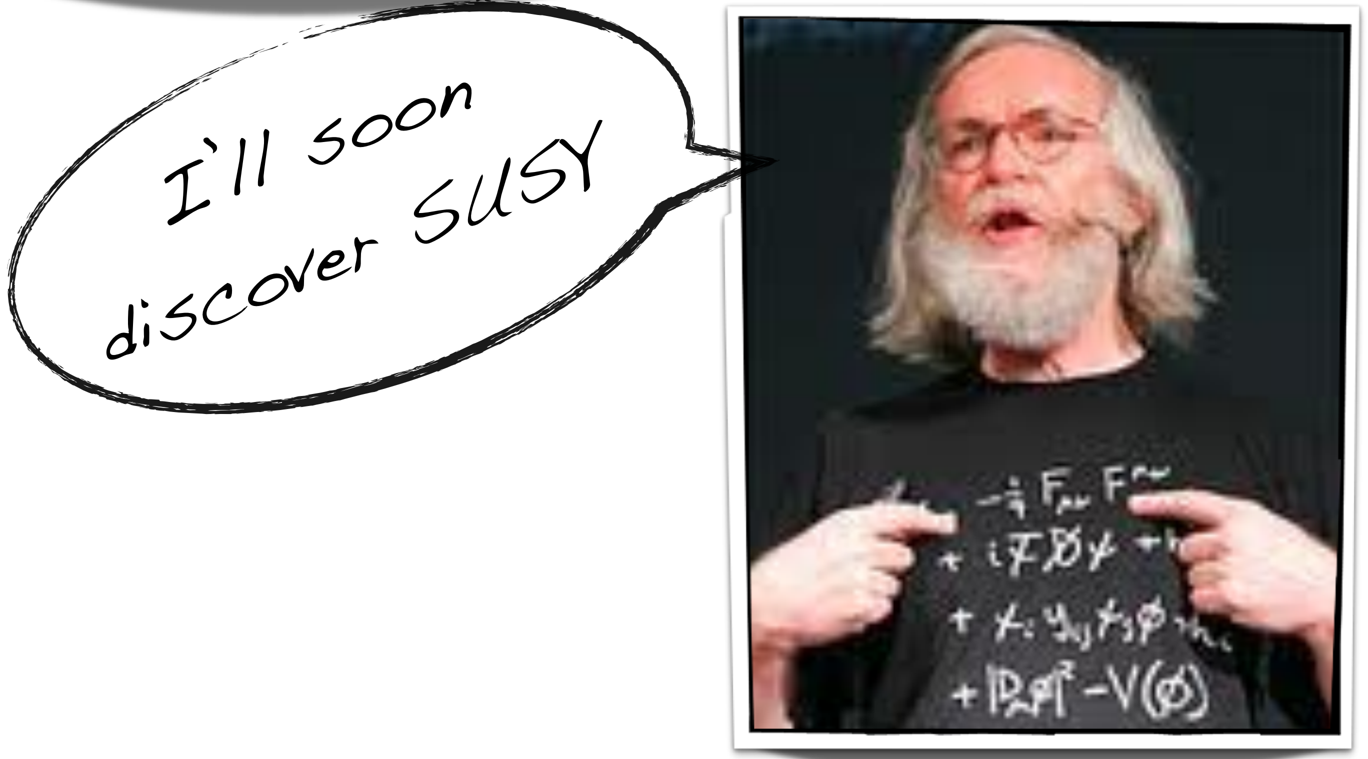
we do not understand the Matter the Universe is made of

BSM is turning 50

Not a new story… many (failed?) attempts

- GUT (Pati-Salam '73, Georgi-Glashow '74)
- SUSY ('71) / MSSM ('77-'81)
- Technicolor ('79)
- Large extra dimension, aka ADD ('98)
- Warped extra dimension, aka RS ('99)
- Composite Higgs ('84-'03)
- Little Higgs ('01)
- Higgsless ('03)
- Relaxion ('15)
- …

**If they had been successful,
they would be part of the SM…**



The LHC Legacy (so far)

- **SM confirmed to high accuracy up to energies of several TeV**
- **Higgs boson discovered**
- **Absence of new physics**

Naturalness agents might not explain DM/baryogenesis
Traditional New Physics models are under siege

New approaches: relaxion, Nnaturalness, clockwork…
Dark Sector/Cosmology might set the weak scale

The remarkable and successful operation of the LHC

(made possible thanks to technological advancements, accelerator performance, detector resolution, high-performance computing and data handling, as well as higher-order theoretical calculations)

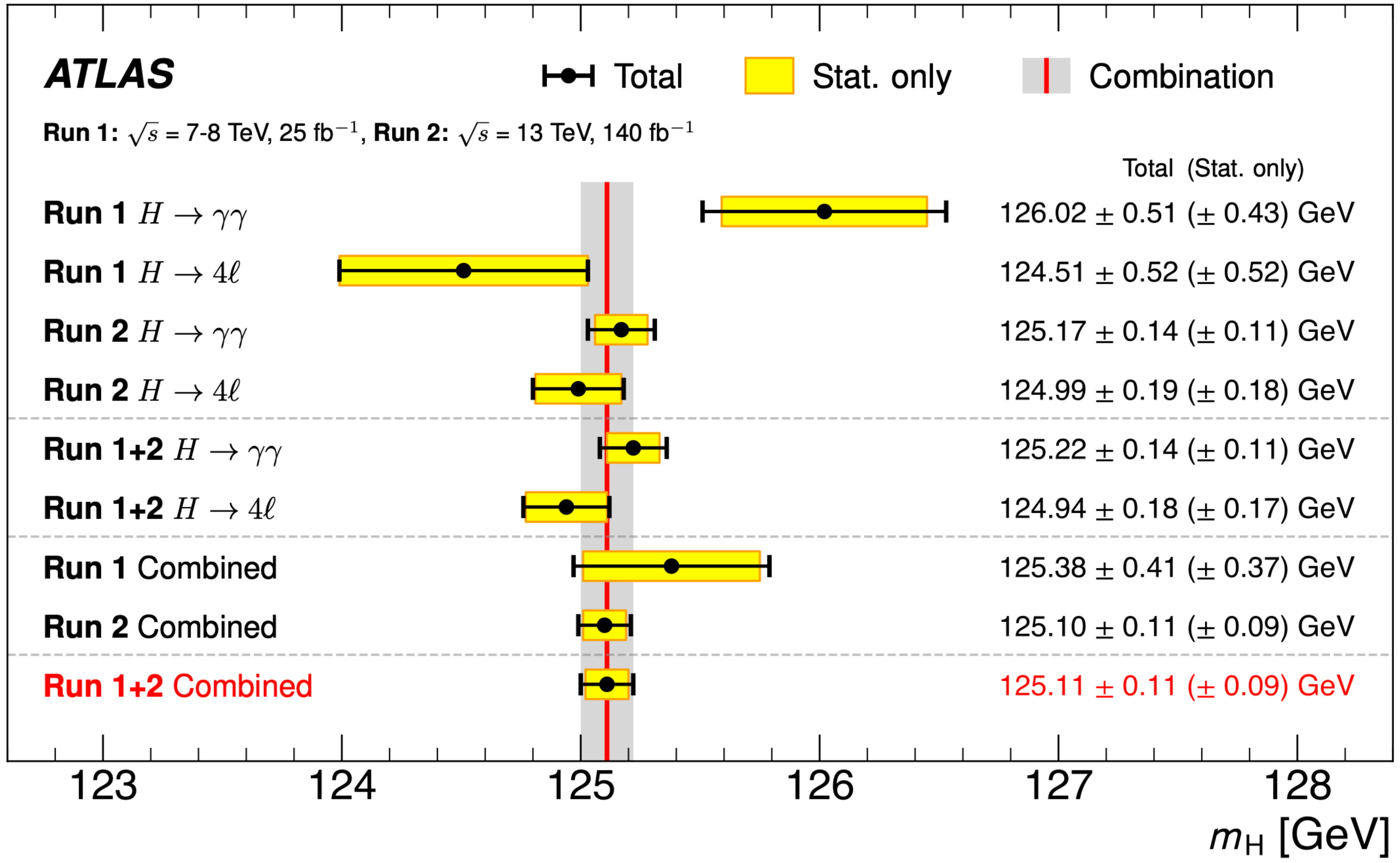
… also changed the nature of the LHC itself:

☺ it is not only an exploration machine but it also performs legacy precision measurements,

☺ a multi-messenger experiment on its own.

The LHC as a Precision Machine

The LHC is regularly surprising the community with its ability to deliver **precise measurements**



0.09% precision in Higgs mass determination

The LHC as a Precision Machine

The LHC is regularly surprising the community with its ability to deliver **precise measurements**

This potential for precision measurements relies on a firm **control of experimental systematic uncertainties**.

The systematic uncertainties for a hadron collider experiment depend on a careful evaluation of the **detector performance**, in the challenging pileup environment.

Detector simulations at future hadron collider like FCC-hh in presence of the O(1000) pileup are not reliable enough to make robust statements in the general context of precision studies.

We proved the ability to do precision measurements of the Higgs boson (for rare decays, ttH and self coupling) on the basis of conservative and believable assumptions. But to go beyond this needs a level of sophistication in the simulations, and in the understanding of physics backgrounds, that is not available today.

We need at least the **HL-LHC** to validate assumptions on the control of backgrounds and systematic errors at **FCC-hh**.

What is the scale of New Physics?

“IR Simplicity”

High Scale Wishes

small FCNC: $\frac{gF_{\mu\nu}\bar{\psi}H\sigma^{\mu\nu}\psi}{M_{\mathrm{NP}}^2}$ 10^3TeV?

tiny neutrino masses: $\frac{(LH)^2}{M_{\mathrm{NP}}}$ 10^{14}GeV?

slow proton decay: $\frac{UUDE}{M_{\mathrm{NP}}^2}$ 10^{16}GeV?

SM features some “accidental” selection rules that generic new physics doesn’t share.

Low Scale Wishes

small EDMs: $\mathrm{arg det Y} \leq 10^{-10}$

↪ axion?

tiny vacuum energy: $\Lambda \approx M_{\mathrm{NP}}^4 \gg (10^{-3}\mathrm{eV})^4$

↪ ?

light Higgs boson: $m_H^2 \approx M_{\mathrm{NP}}^2 \gg (125\mathrm{GeV})^2$

↪ light susy?

SM has some structural deficiencies that call for new physics at low scale.

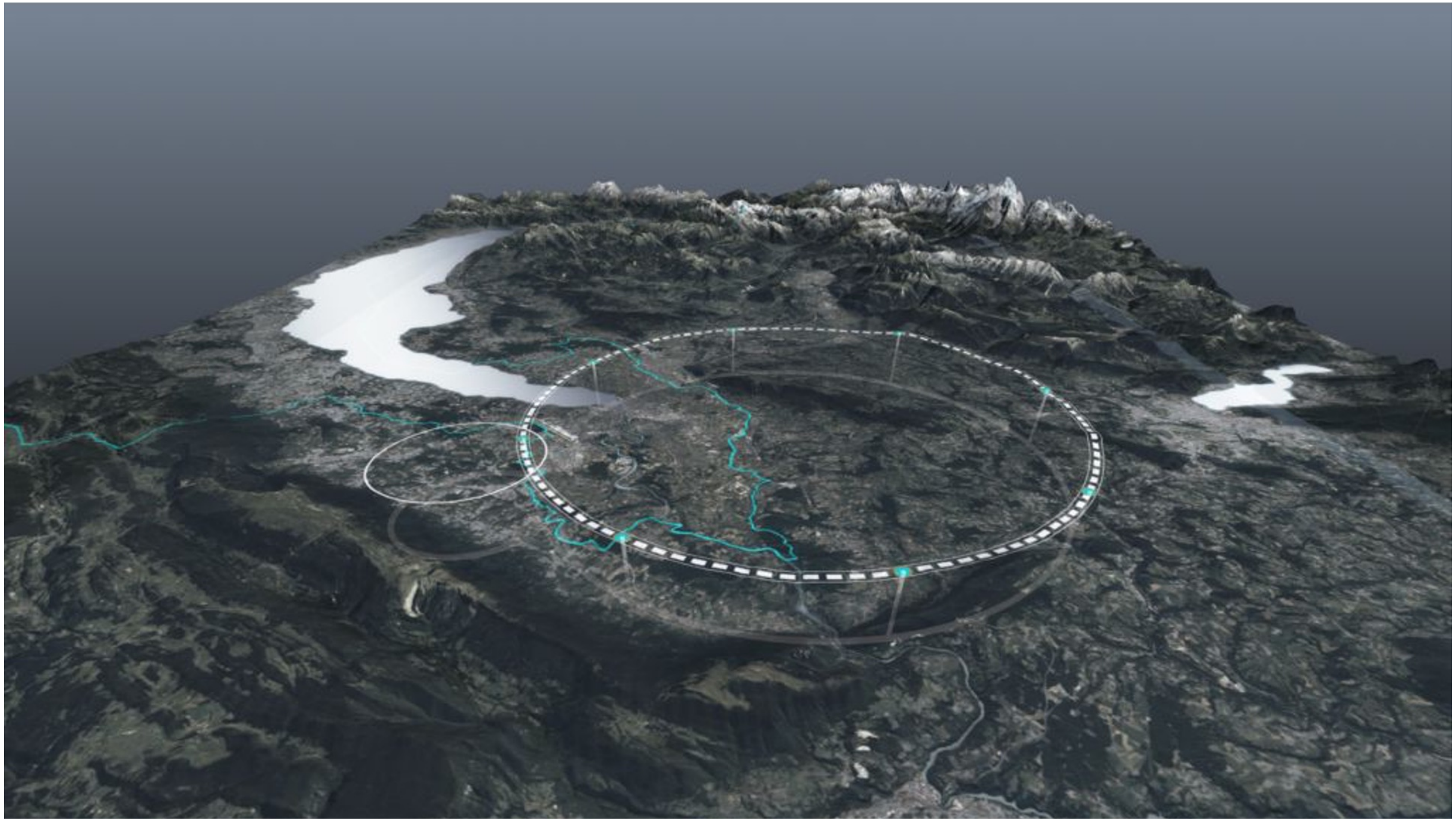
“Naturalness”

We know for sure that New Physics exists.
But no clear indication of the energy scale to probe.
We need a broad, versatile and ambitious programme that
1. will achieve legacy precision measurements,
2. can push the frontiers of the unknown.

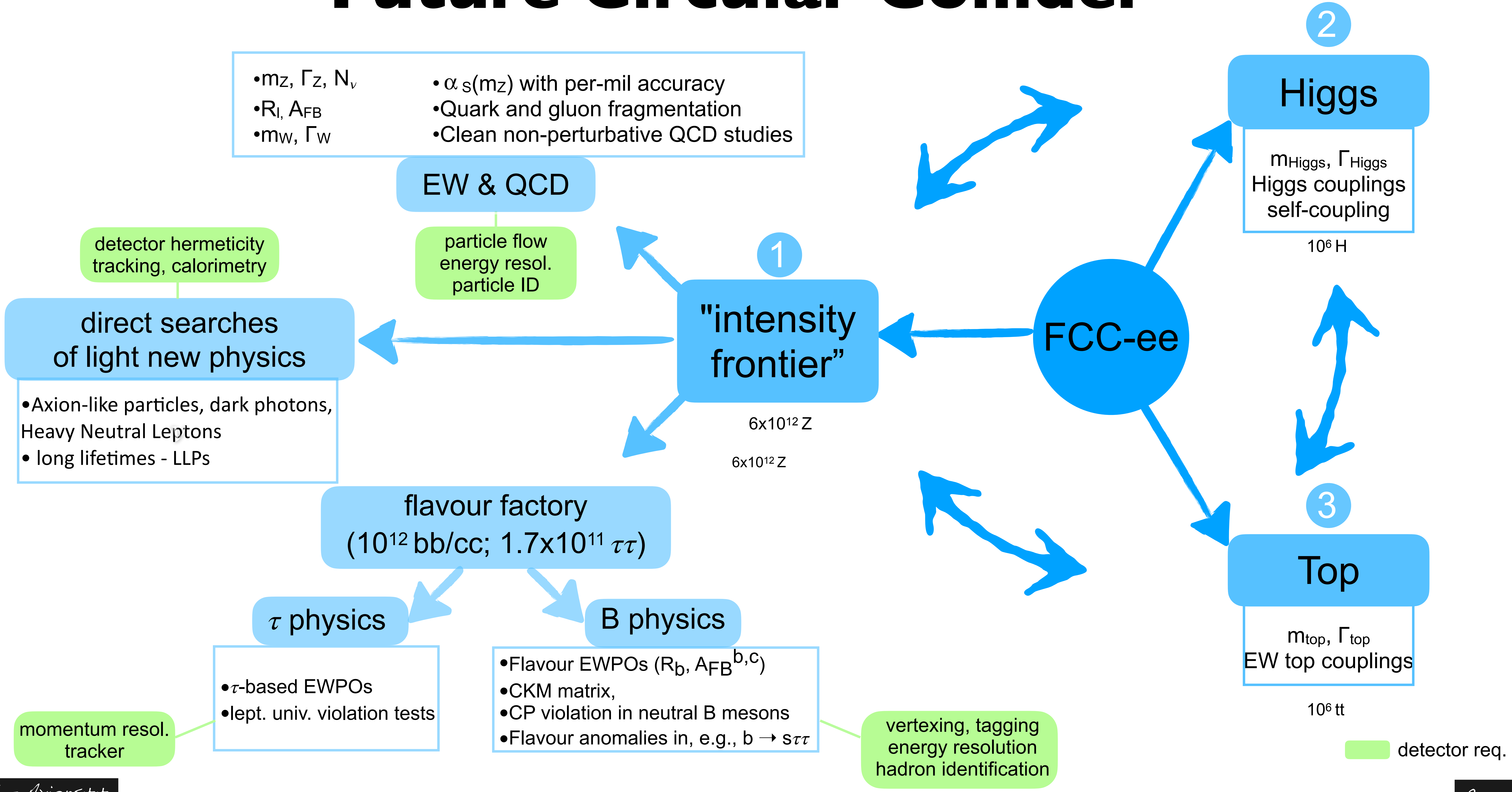
TWO FRONTIERS TO EXPLORE

Future Circular Collider

a versatile machine able to perform exquisite measurements at the EW/Higgs/top threshold
and to probe both the intensity and energy frontiers



Future Circular Collider



- m_Z, Γ_Z, N_ν
- R_l, A_{FB}
- m_W, Γ_W
- $\alpha_S(m_Z)$ with per-mil accuracy
- Quark and gluon fragmentation
- Clean non-perturbative QCD studies

EW & QCD

particle flow
energy resol.
particle ID

detector hermeticity
tracking, calorimetry

direct searches
of light new physics

- Axion-like particles, dark photons, Heavy Neutral Leptons
- long lifetimes - LLPs

①

"intensity frontier"

6×10^{12} Z

6×10^{12} Z

FCC-ee

②

Higgs

m_{Higgs}, Γ_{Higgs}
Higgs couplings
self-coupling

10^6 H

③

Top

m_{top}, Γ_{top}
EW top couplings

10^6 tt

flavour factory
(10^{12} bb/cc; 1.7×10^{11} $\tau\tau$)

τ physics

- τ-based EWPOs
- lept. univ. violation tests

momentum resol.
tracker

B physics

- Flavour EWPOs (R_b, $A_{FB}^{b,c}$)
- CKM matrix,
- CP violation in neutral B mesons
- Flavour anomalies in, e.g., $b \to s\tau\tau$

vertexing, tagging
energy resolution
hadron identification

🟩 detector req.

Intensity Frontier: Why not → What for?

— multi-vacuua relaxion —

Graham, Kaplan, Rajendran ’15

ϕ slowly rolling field that scans the Higgs mass during the cosmological evolution of the Universe

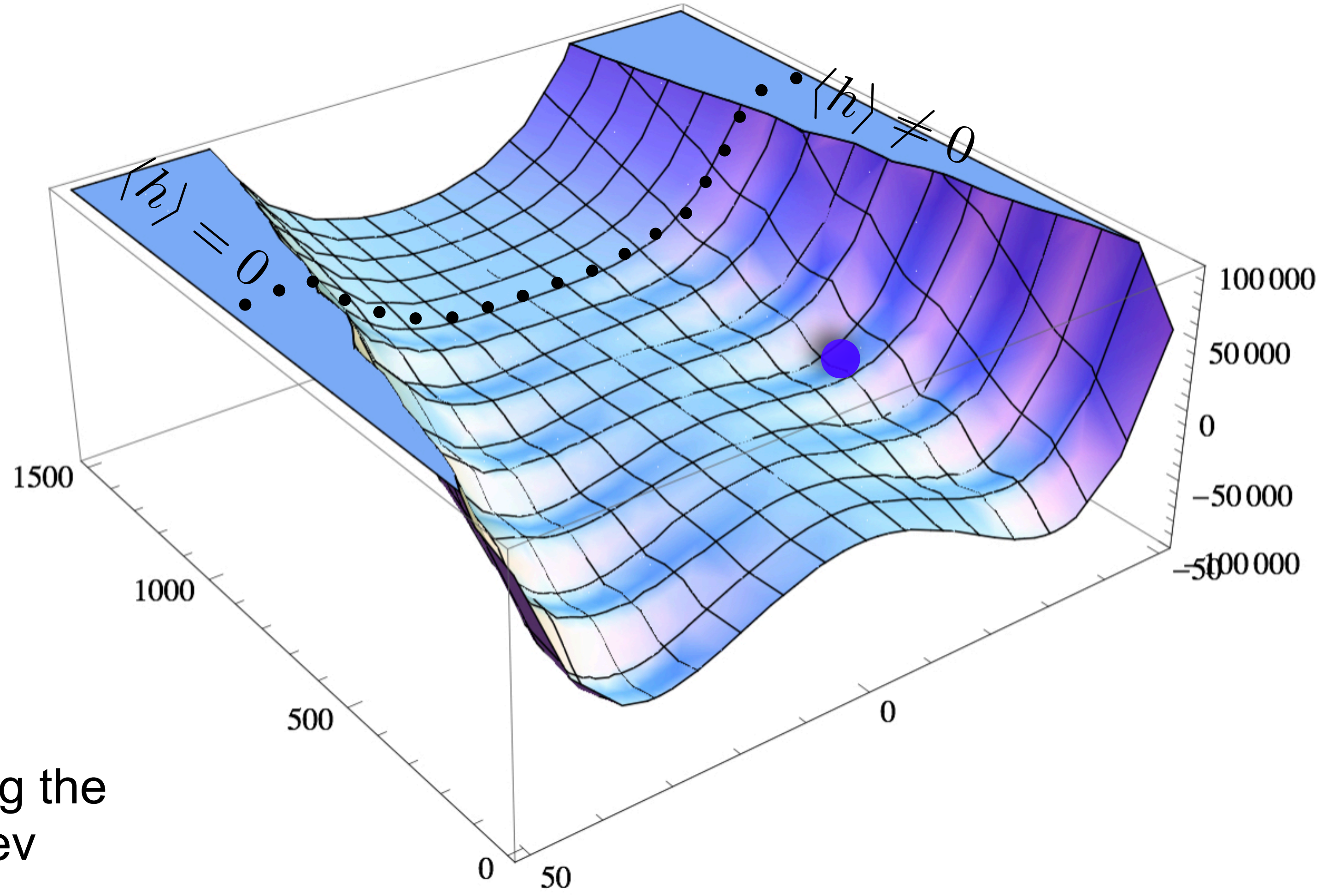
$$\Lambda^2\left(-1+f\left(\frac{g\phi}{\Lambda}\right)\right)|H|^2+\Lambda^4 V\left(\frac{g\phi}{\Lambda}\right)+\frac{1}{32\pi^2}\frac{\phi}{f}\tilde{G}^{\mu\nu}G_{\mu\nu}$$

Higgs mass depends on ϕ

potential needed to force ϕ to roll-down in time (during inflation)

axion-like coupling that will seed the potential barrier stopping the rolling onces the Higgs develops its vev

$$\Lambda_{\mathrm{QCD}}^3\, h\, \cos\frac{\phi}{f}$$



Intensity Frontier: Why not → What for?

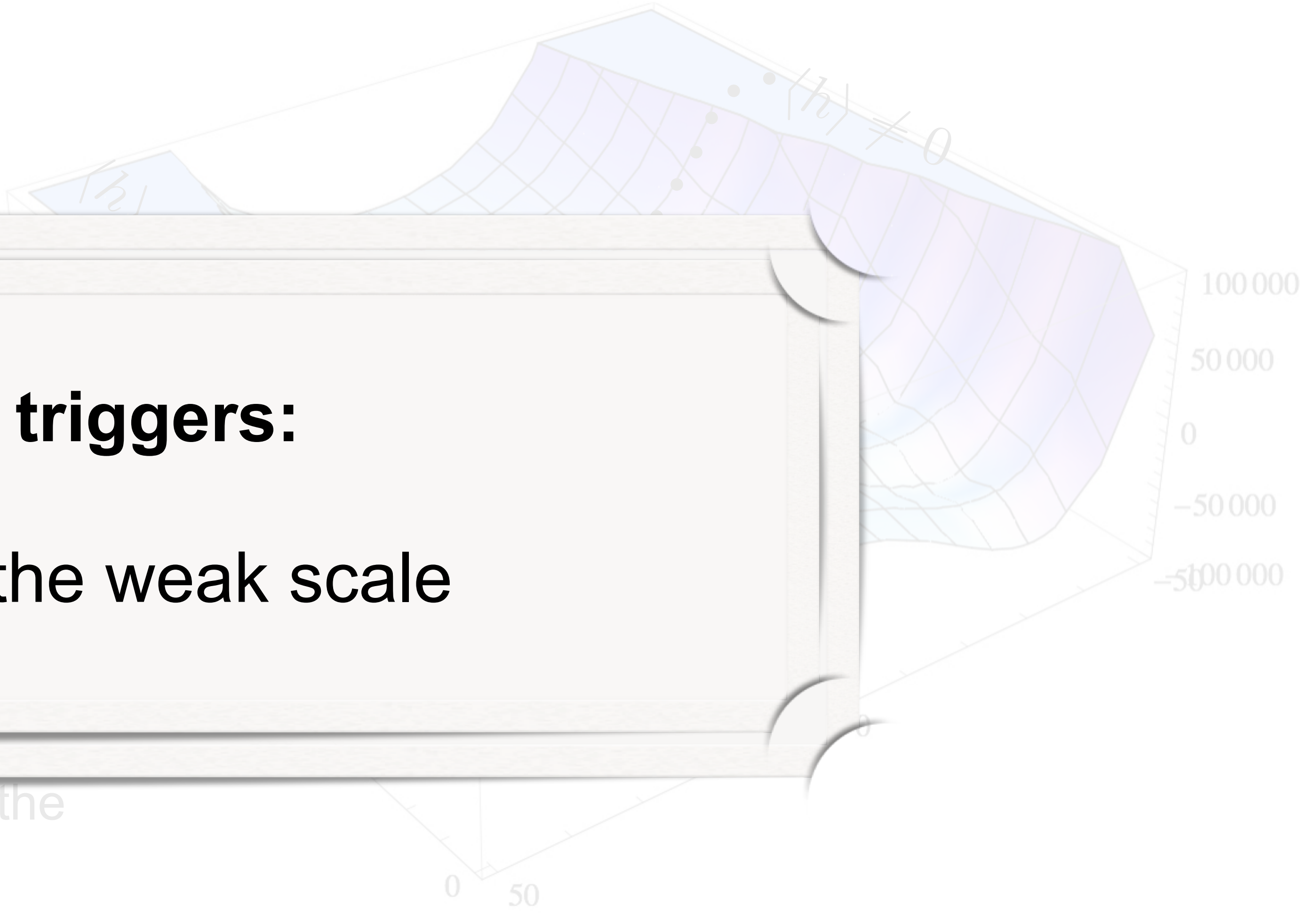
— multi-vacuua relaxion —

Graham, Kaplan, Rajendran ’15

ϕ slowly rolling field that scans the Higgs mass during the cosmological evolution of the Universe

$\Lambda^2 \left(-1 + f \right.$

Higgs mass depends on ϕ



Cosmological weak scale triggers:

the cosmological evolution sets the weak scale

that will seed the potential barrier stopping the rolling onces the Higgs develops its vev

$$\Lambda_{\text{QCD}}^3 \, h \cos \frac{\phi}{f}$$

Phenomenological Signatures

Energy Frontier might not be the best place to look into

only BSM physics below Λ~$10^{7÷9}$GeV could be in the form of

(very) light and very weakly coupled axion-like scalar fields

$$m_\phi \sim \left(\frac{g\,\Lambda^5}{f\,v^2}\right)^{1/2} \sim (10^{-20} - 10^{2})\,\mathrm{GeV}$$

Phenomenological Signatures

A QFT rationale for light and weakly coupled degrees of freedom

with spectacular signatures across different scales

Espinosa et al '15

Flacke et al '16

Choi and Im '16

—interesting cosmology signatures—

- ◎ BBN constraints
- ◎ decaying DM signs in γ-rays background
- ◎ ALPs
- ◎ superradiance

—interesting signatures @ Flavour—

- ◎ production of light scalars by B and K decays

—interesting atomic physics—

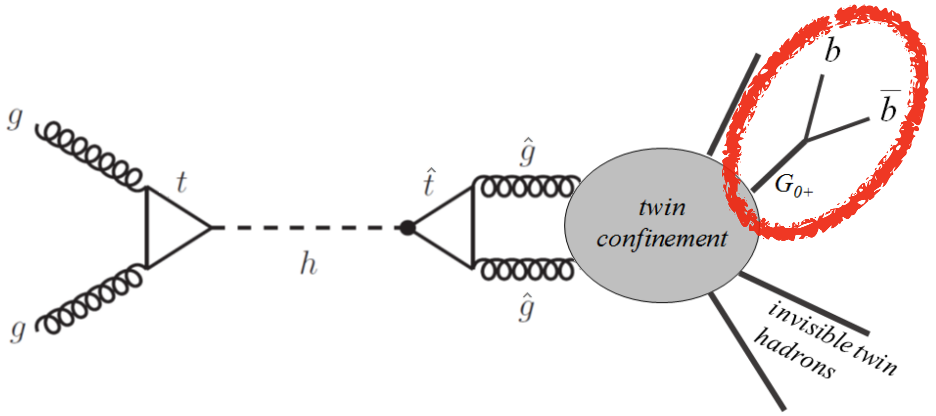
- ◎ change of atom sizes
- ◎ relaxion halo around earth/sun which induce $\delta m_e/m_e$ and $\delta\alpha/\alpha$

Banerjee et al '19

Collider Searches for Light New Physics

- **LLP searches with displaced vertices**
  e.g. in twin Higgs models glueballs that mix with the Higgs and decay back to b-quarks

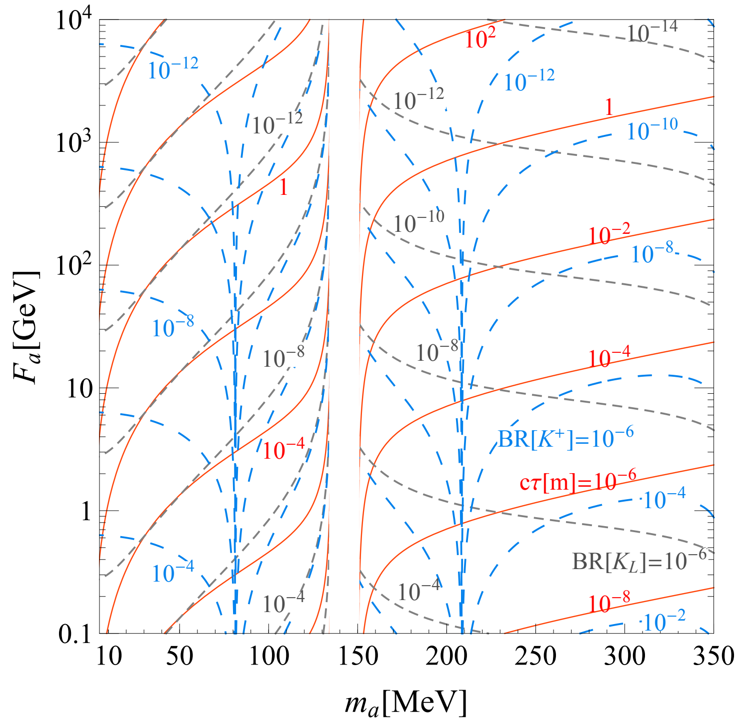
Craig et al, arXiv:1501.05310



- **Rare decays** Gori et al arXiv:2005.05170
  e.g. ALP mixing w/ SM mesons:

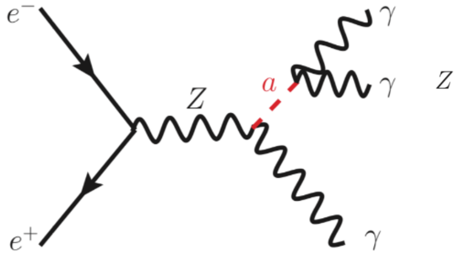
$$K_L \to \pi^0 a \to \pi^0 \gamma\gamma \text{ (KOTO)}$$
$$K^+ \to \pi^+ a \to \pi^+ \gamma\gamma \text{ (NA62)}$$

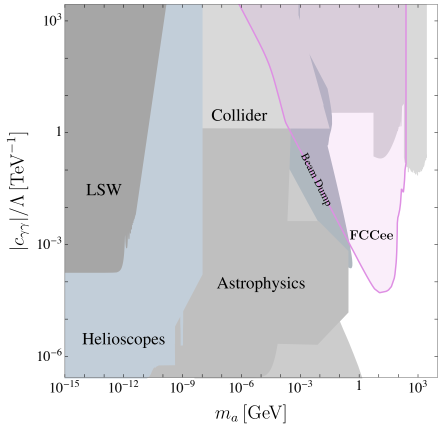
$$\mathcal{L} = \frac{\alpha_s}{8\pi F_a} a G_{\mu\nu} \tilde{G}^{\mu\nu}$$

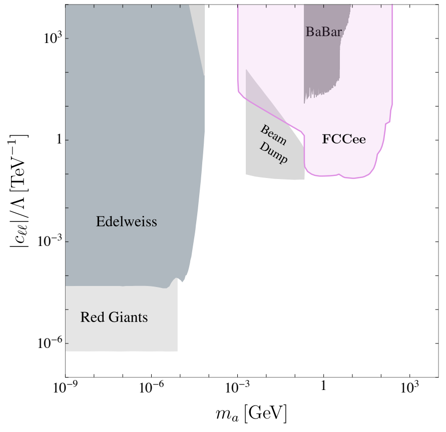


- **ALPs@ colliders**
  e.g. $e^+e^- \to \gamma a$

  $e^+e^- \to ha$







Knapen, Thamm arXiv:2108.08949

Astro/Cosmo → long-lived ALPs
colliders → short-lived ALPs MeV+

Physics of Light Degrees of Freedom

light scalars are un-natural in QFT unless they are

pseudo Nambu-Goldstone bosons

PNGB	approximate symmetry	symmetry breaking	origin
$\pi^\pm$, π^0	SU(2)xSU(2)/SU(2)	m_u, m_d	non-perturbative
$W_L^\pm$, Z_L^0	SU(2)xSU(2)/SU(2)	m_t-m_b & g’	perturbative
h(125)	SO(5)/SO(4)?	vacuum misalignment?	non-perturbative?
a?	$U(1)_{PQ}$?	QCD instantons	?

— rich and diverse pNGB phenomenology —

How serious is the strong CP problem ? (sorry if trivial)

G. Perez @ Ascona '23

- 3 levels of formulating the strong CP problem, *assuming CP is respected by the UV*:

$(i)\ \bar{\theta} = \theta - \arg\left[\det\left(Y_u Y_d\right)\right] \lesssim 10^{-10}$, is it a problem?

(who knows?)

$(ii)\ \bar{\theta} = \lesssim 10^{-10} \ll \theta_{\mathrm{KM}} = \arg\left\{\det\left[Y_u Y_u^{\dagger}, Y_d Y_d^{\dagger}\right]\right\}$, is it a problem?

(not if these are natural/protected and sequestered)

$(iii)\ \bar{\theta} = \lesssim 10^{-10} \ll \theta_{\mathrm{KM}}$, but $\bar{\theta} = \bar{\theta}_{\mathrm{bare}} + \epsilon\, \theta_{\mathrm{KM}} \ln\left(\Lambda_{\mathrm{UV}}/M_W\right)$, is it a problem?

(ϵ appears in 7 loops and contains several other suppression factor)

but in the MSSM, θ has 1-loop RG running from the gluino mass phase

- Should we be more cautious / more generic? [at least till we reach $\mathcal{O}\left(10^{-16}\right)$ precision]

Axion Cosmology

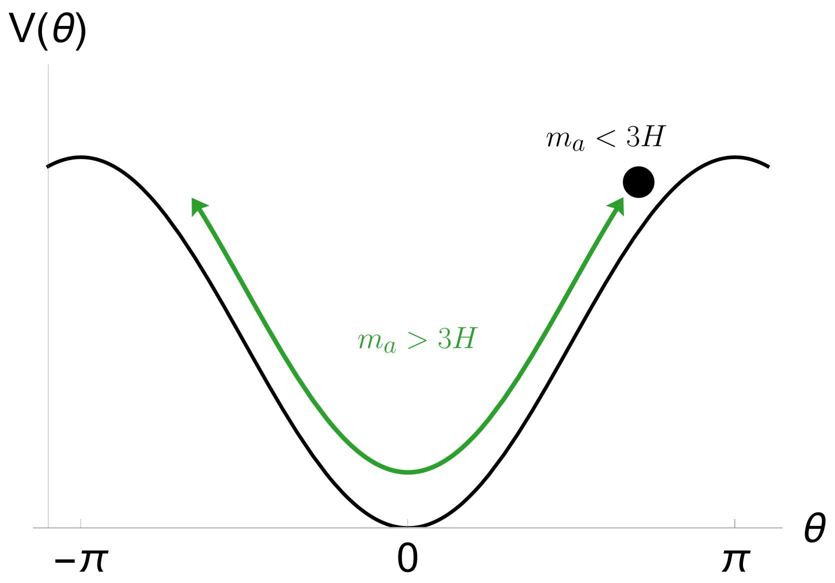
G. Servant @ Planck'22

Start with ALP lagrangian $$\mathcal{L} = \frac{1}{2} g^{\mu\nu}\partial_\mu a \partial_\nu a - V(\phi) = \frac{1}{2} g^{\mu\nu}\partial_\mu a \partial_\nu a - m_a^2(T) f^2 \left[1 - \cos\left(\frac{a}{f}\right)\right]$$

Define homogeneous zero-mode $\bar{a}(t) \equiv f\Theta(t)$

Neglecting fluctuations, it satisfies $\ddot{\Theta} + 3H\dot{\Theta} + m_a^2(T)\sin(\Theta) = 0,$

With initial conditions: $\Theta(t_i) = \Theta_i, \; \dot{\Theta}(t_i) = 0.$ ***standard assumption***



- $m_a \ll 3H \iff \rho_a \propto a^0$ (Frozen)
- $m_a \gg 3H \iff \rho_a \propto a^{-3}$ (Oscillating)

-> standard misalignement mechanism

For $\Theta_i \sim 1$ $$\rho_{\rm DM} \sim \rho_{osc}\left(\frac{a_{osc}}{a_0}\right)^3 \sim m_a^2 f_a^2 \left(\frac{T_0}{T_{osc}}\right)^3$$

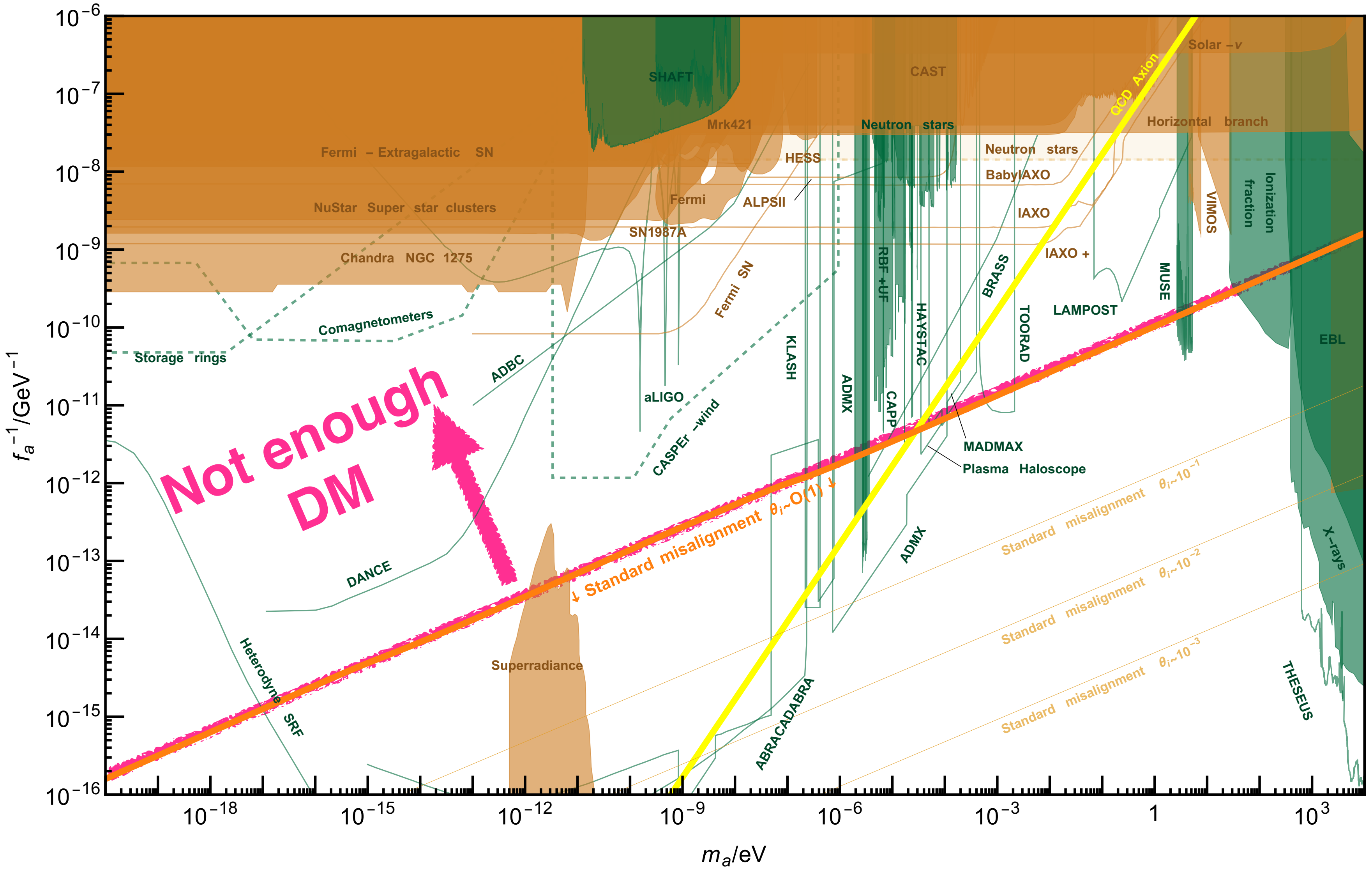
$$T_{osc} \sim \sqrt{m_a M_{Pl}}$$

1

Axion Cosmology

Conventional misalignement makes too little DM for low f_a .

G. Servant @ Planck’22



Not enough DM

Axion Cosmology

radial/axion interplay to enlarge DM parameter space

G. Servant @ Planck'22

If PQ symmetry is broken explicitly at high energies
—> mexican hat potential is tilted

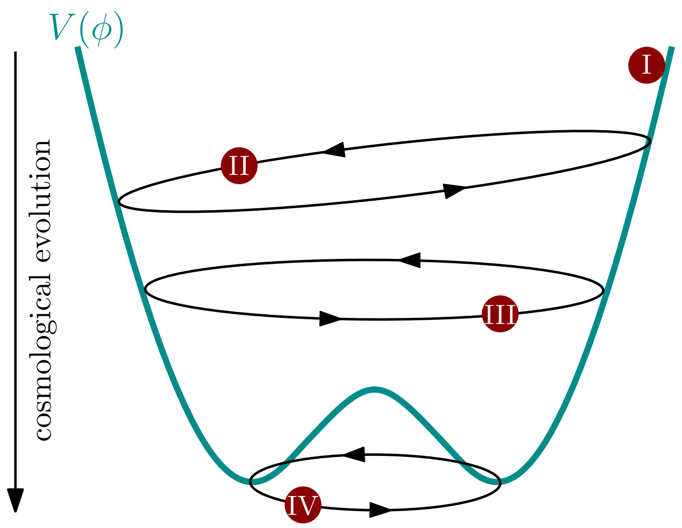


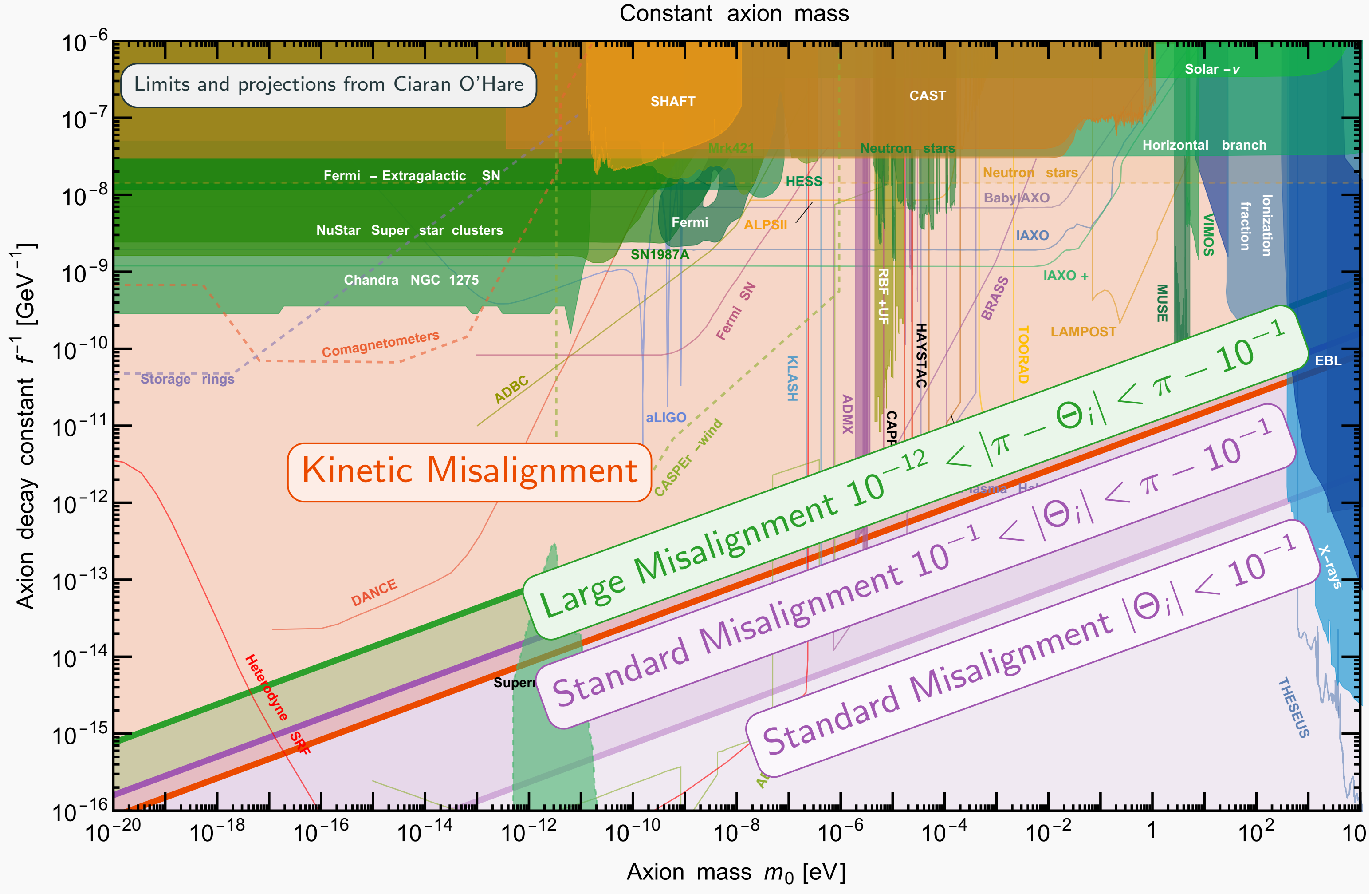
Figure by P. Simakachorn

If radial mode of PQ field starts at large VEV, the angular mode gets a large kick in the early universe

With initial conditions: $$\frac{1}{2}\dot{\Theta}_i^2 \gg 2m^2(T_i)$$ **Delayed axion oscillations !**

-> kinetic misalignment mechanism

Co, Harigaya, Hall '19



QCD axion

QCD instantons generate the axion potential ($U(1)_{PQ}$x$SU(3)_C^2$ anomaly)
hence axion physics is fully IR determined

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a} + \Theta \frac{g_s^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a,\ \mu\nu}$$

➡

vacuum energy

$$\mathcal{E} = -2\cos\Theta \int \frac{d\rho}{\rho^5}\ C\ \frac{1}{g^8} \exp\left[-8\pi^2/g^2(\rho)\right]$$

[minium at θ=0 as expected]

QCD is asymptotically free: integral is dominated by IR (large instantons)

The tiny axion mass is due to mixing
with η’ and pion:

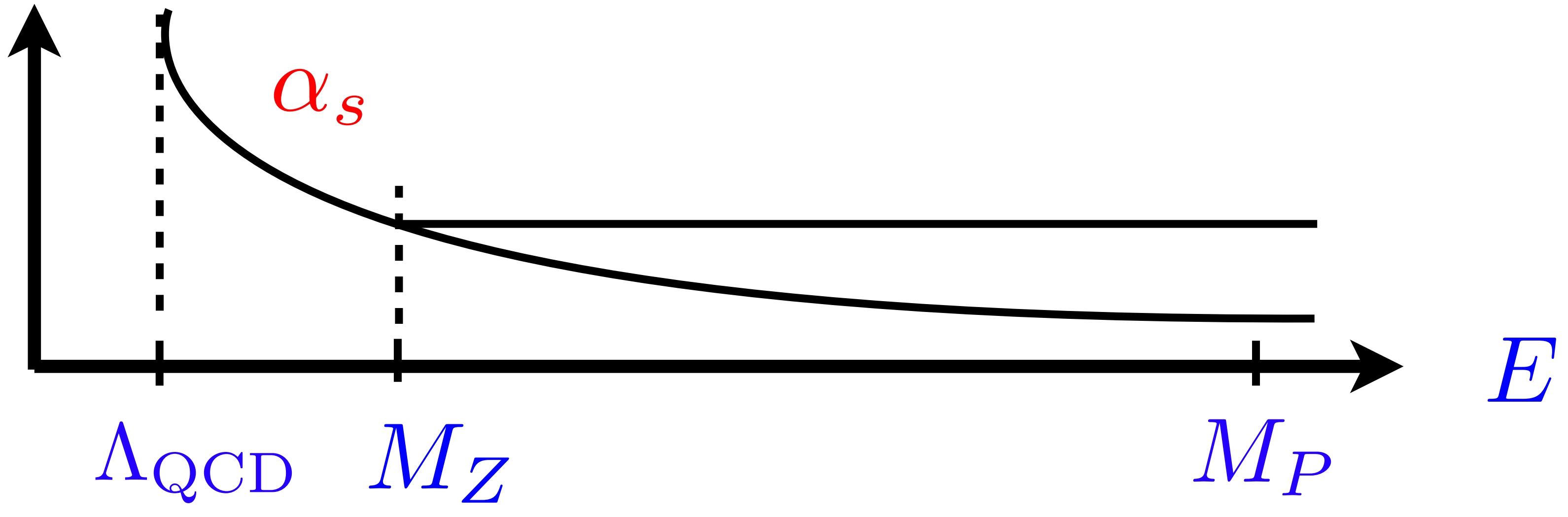
$$m_a^2 f_a^2 \sim m_\pi^2 f_\pi^2 \frac{m_u m_d}{(m_u + m_d)^2}$$

Once we know f_a, we know (almost) everything about axion physics (mass, couplings to SM…)

QCD axion model building

goal: introduce freedom in axion physics while still solving strong CP

- change QCD running such that small instantons can contribute



- color scalars
- extra-dimension

⇒ heavier axion (for fixed f_a)

- change confining gauge group
  - $SU(3)^n \to SU(3)_{\rm QCD}$ Agrawal+ '17 Hook '18
  - Mirror world

Dimopoulos++ '16

→ also alleviates the axion quality problem

$$m_a^2 f_a^2 = m_\pi^2 f_\pi^2 / n$$

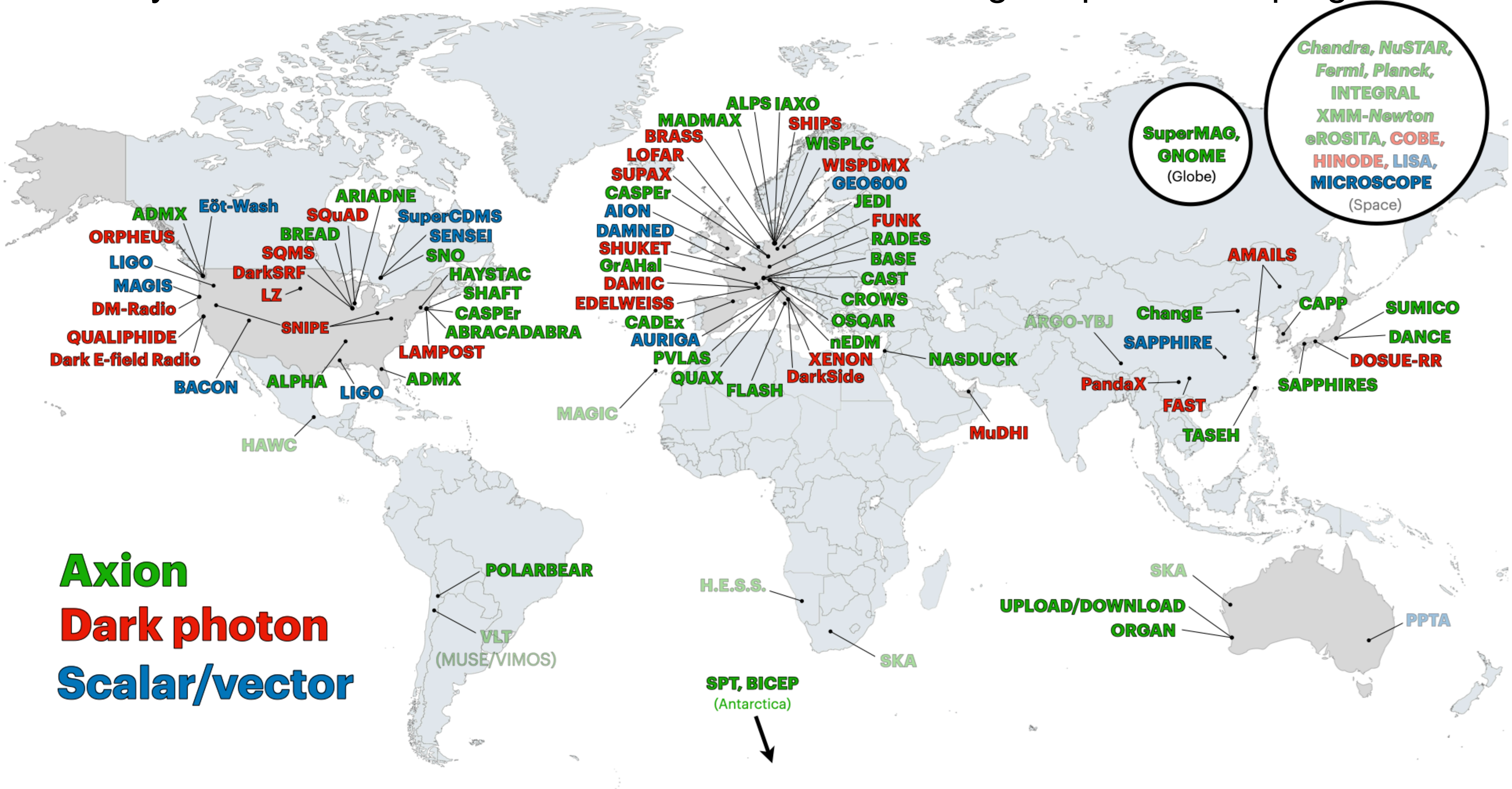
⇒ lighter axion (for fixed f_a)

- mixing among several axions

Gavela++ '23

QCD axion model building

many models with different characteristic scales ⇒ large experimental programme



Ciaran O'Hare

QCD axion model building

NEW EXPERIMENTAL FRONTIER

axion searches through GW observation

(see lecture by Valerie)

QCD axion model building

WARNING

all (but one if we are lucky) these experiments won't find anything!

Incarnation of Pascal's bet:

0 x ∞ can be finite.

QCD axion model building

Axion physics is a playground to learn about **non-perturbative** QFT physics (confinement, anomaly, instantons…).

But, relying on (approximate) global symmetries in the first place,

axions are also a laboratory of **Quantum Gravity**.

How global symmetries are broken by gravitational effects?

Axion quality problem!

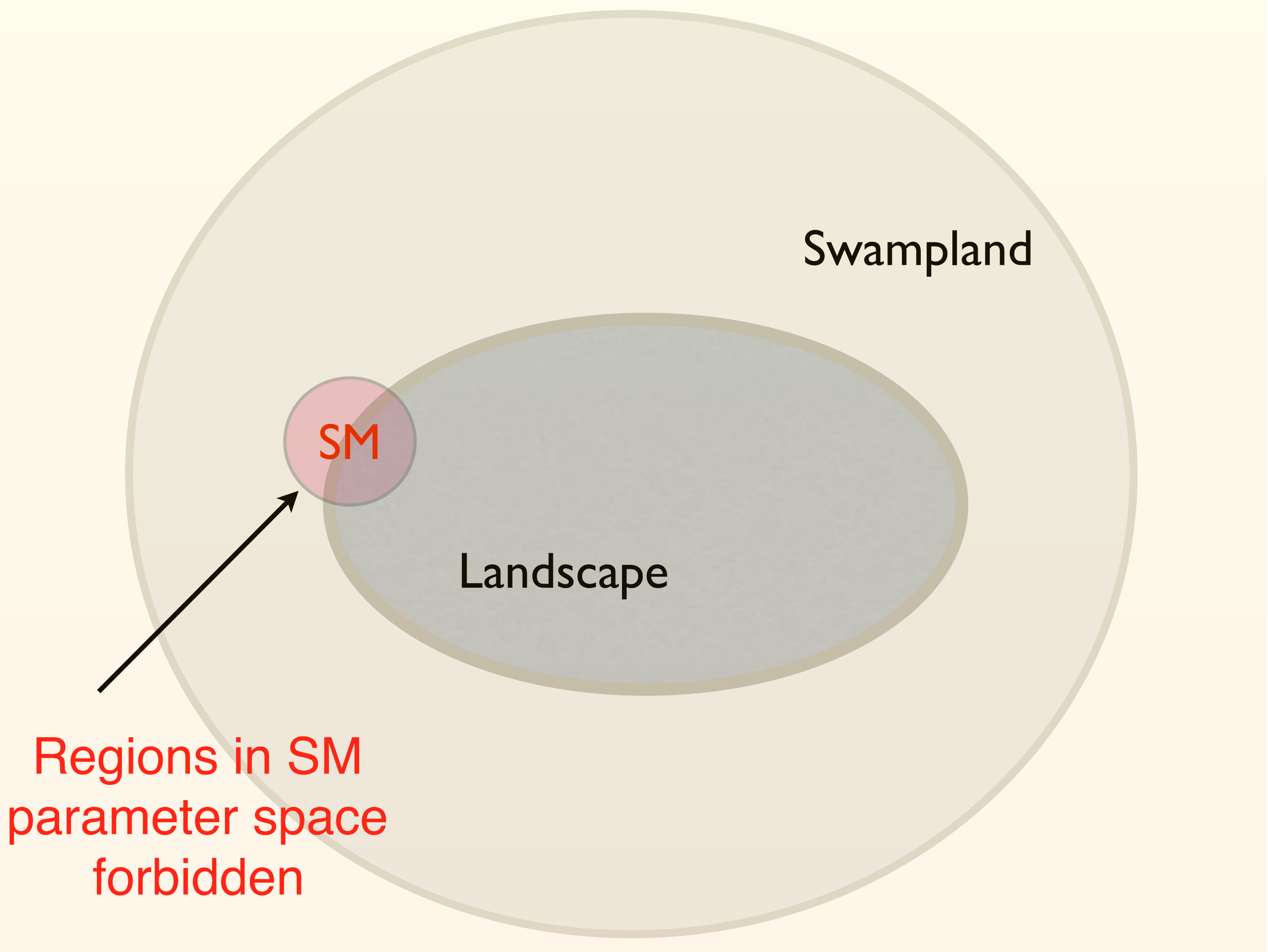
Superradiance is another fascinating effect connecting axion and gravity (but I won't talk about it today).

Particle Physics & Quantum Gravity

Can the SM be embedded in a theory of quantum gravity at the Planck scale?
Can QG be really decoupled at low energy?

Would certainly be true if any QFT can be consistently coupled to QG

Instead Vafa conjectured in 2005 that there exists a **swampland**



Ibanez @ SUSY'18

This conjecture has potentially far-reaching implications for phenomenology.

Landscape/Swampland Conjectures

1) No exact global symmetry

For a review, see Banks, Seiberg '10

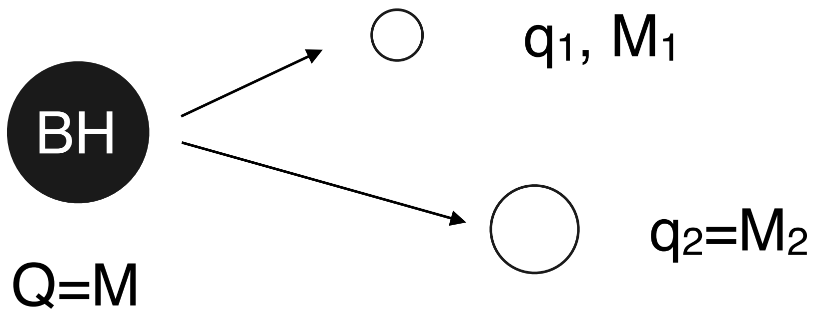
Black-holes decay without memory of global charges

2) Gravity is the weakest force

Arkani-Hamed, Motl, Nicolis, Vafa '06

In any UV complete U(1) gauge theory there must exist at least one charged particle with mass M such that: $M/M_P < g \, . \, q$

Why? otherwise extremal charged BH cannot decay!



BH can decay iff $M_1+M_2<M$, i.e. $M_1<M-M_2=Q-q_2=q_1$

Swampland Conjectures

3) $M_P \parallel \vec{\nabla}_{\phi_i} V(\phi_i) \parallel > c\, V(\phi_i)$ with c is O(1) for any field configuration

Obied, Ooguri, Spodyneiko, Vafa ’18

- Pure positive cosmological constant, i.e. vacuum energy, (dS vacuum) is forbidden
- Quintessence:

Agrawal, Obied, Steinhart, Rafa ‘18

$$\boxed{V(\phi) = \Lambda^4 e^{-\kappa\phi/M_P}}$$

$$0.6 > \kappa > c$$

Planck data ⤴ ⤴ swampland conjecture

- Quintessence + Higgs:

Denef, Hebecker, Wrase ‘18

$$\boxed{V(H,\phi) = \Lambda^4 e^{-\kappa\phi/M_P} + \lambda(|H|^2 - v^2)^2 + V_0} \Rightarrow V(\phi, H) = e^{-\kappa\phi/M_P}\left(V(H) + \Lambda^4_{cc}\right)$$

Higgs-quintessence coupling
⇒ 5th force signal

$$\frac{M_P \parallel \vec{\nabla}_{\phi_i} V(\phi_i) \parallel}{V(\phi_i)} = \begin{array}{ll} \dfrac{\kappa\Lambda^4}{\Lambda^4 + \lambda v^4 + V_0} & @(H=0, \phi=0) \\ \dfrac{\kappa\Lambda^4}{\Lambda^4 + V_0} & @(H=v, \phi=0) \end{array}$$

at least one of them is as small as

$$\mathcal{O}\left(\frac{\mathrm{cc}}{\mathrm{EW}^4}\right) \sim \frac{(10^{-3}\,\mathrm{eV})^4}{(100\,\mathrm{GeV})^4} \sim 10^{-56}$$

- Quintessence + axion:

Murayama, Yamazaki, Yanagida ‘18

$$\boxed{V(\theta,\phi) = \Lambda^4 e^{-\kappa\phi/M_P} + \Lambda^4_{QCD}(1 - \cos(\theta/f)) + V_0} \Rightarrow$$

quintessence-axion coupling needed

$$\frac{M_P \parallel \vec{\nabla}_{\phi_i} V(\phi_i) \parallel}{V(\phi_i)} = \begin{array}{ll} \dfrac{\kappa\Lambda^4}{\Lambda^4 + V_0} & @(\theta=0, \phi=0) \\ \dfrac{\kappa\Lambda^4}{\Lambda^4 + \Lambda^4_{QCD} + V_0} & @(\theta=\pi f, \phi=0) \end{array}$$

at least one of them is as small as

$$\mathcal{O}\left(\frac{\mathrm{cc}}{\mathrm{QCD}^4}\right) \sim \frac{(10^{-3}\,\mathrm{eV})^4}{(200\,\mathrm{MeV})^4} \sim 10^{-44}$$

Swampland Conjectures

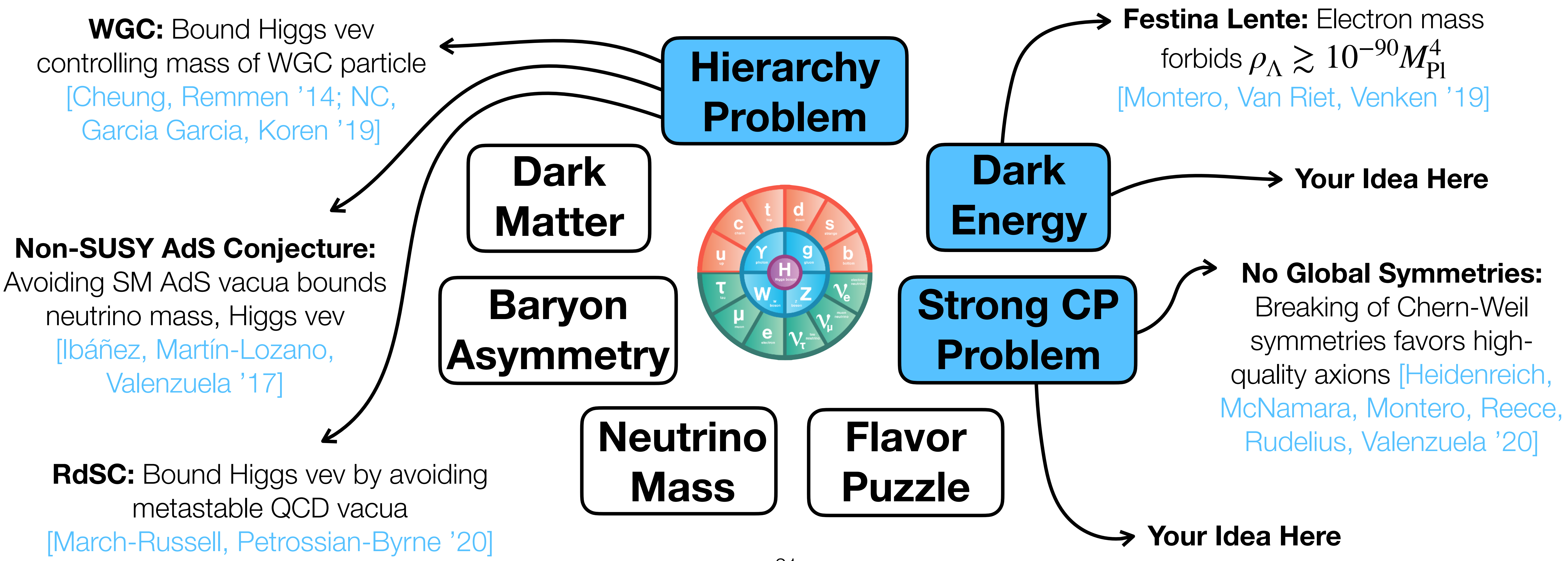
It is not that String Theory rules out the SM as we know it.
But non-trivial interactions among seemingly decoupled sectors must exist:
UV enforces interactions among IR degrees of freedom,
like anomaly conditions enforce constraints on IR physics.

New Perspectives on BSM

Two rules of thumb for applying SCs to BSM:

1. The more rigorous the conjecture, the less powerful for BSM.
2. Assumptions are required, so you need new predictions.

N. Craig @ Swamplandia '23



WGC: Bound Higgs vev controlling mass of WGC particle [Cheung, Remmen '14; NC, Garcia Garcia, Koren '19]

Non-SUSY AdS Conjecture: Avoiding SM AdS vacua bounds neutrino mass, Higgs vev [Ibáñez, Martín-Lozano, Valenzuela '17]

RdSC: Bound Higgs vev by avoiding metastable QCD vacua [March-Russell, Petrossian-Byrne '20]

Hierarchy Problem

Dark Matter

Baryon Asymmetry

Neutrino Mass

Flavor Puzzle

Dark Energy

Strong CP Problem

Festina Lente: Electron mass forbids $\rho_\Lambda \gtrsim 10^{-90} M_{\rm Pl}^4$ [Montero, Van Riet, Venken '19]

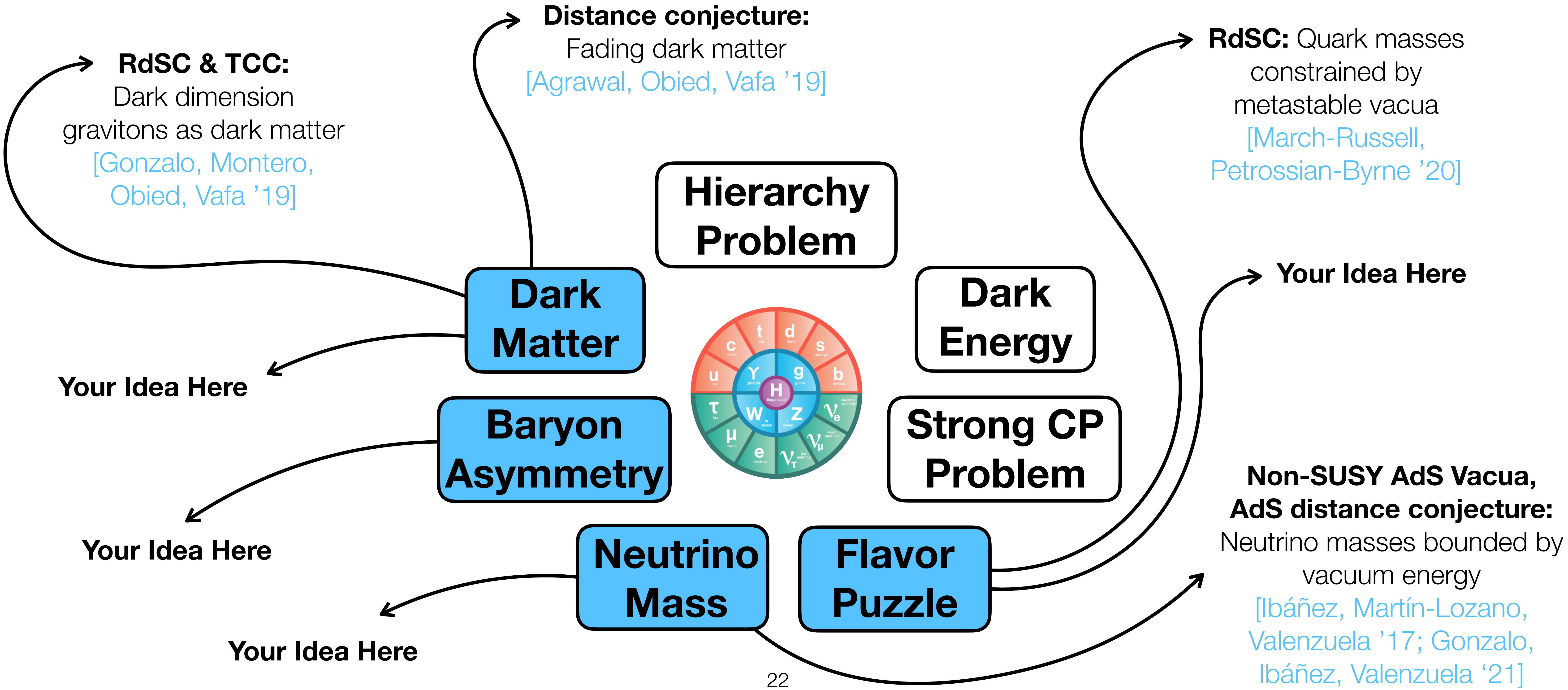
Your Idea Here

No Global Symmetries: Breaking of Chern-Weil symmetries favors high-quality axions [Heidenreich, McNamara, Montero, Reece, Rudelius, Valenzuela '20]

Your Idea Here

New Perspectives on BSM

N. Craig @ Swamplandia '23



RdSC & TCC:
Dark dimension
gravitons as dark matter
[Gonzalo, Montero,
Obied, Vafa '19]

Distance conjecture:
Fading dark matter
[Agrawal, Obied, Vafa '19]

RdSC: Quark masses
constrained by
metastable vacua
[March-Russell,
Petrossian-Byrne '20]

Hierarchy Problem

Dark Matter

Your Idea Here

Dark Energy

Your Idea Here

Baryon Asymmetry

Strong CP Problem

Your Idea Here

Neutrino Mass

Flavor Puzzle

Your Idea Here

Non-SUSY AdS Vacua,
AdS distance conjecture:
Neutrino masses bounded by
vacuum energy
[Ibáñez, Martín-Lozano,
Valenzuela '17; Gonzalo,
Ibáñez, Valenzuela '21]

Axion/ALP Power Counting

Axion/ALP=Goldstone boson → shift-symmetry

Bonnefoy, Grojean, Kley '22

$$a \to a + \epsilon f$$

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \frac{1}{2}\left(\partial_\mu a\right)\left(\partial^\mu a\right) + \frac{\partial_\mu a}{f}\sum_{\psi\in\mathrm{SM}} \bar{\psi} c_\psi \gamma^\mu \psi + \mathcal{O}\left(\frac{1}{f^2}\right)$$

hermitian matrices
(26 CP-even and 13 CP-odd couplings)

But shift-symmetry cannot be exact (PQ as approximate symmetry)

What are the allowed couplings of an ALP after (soft) breaking of shift-symmetry?

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \frac{1}{2}\left(\partial_\mu a\right)\left(\partial^\mu a\right) - \frac{a}{f}\left(\bar{Q}\tilde{Y}_u \tilde{H} u + \bar{Q}\tilde{Y}_d H d + \bar{L}\tilde{Y}_e H e + \mathrm{h.c.}\right)$$

generic matrices
(27 CP-even and 25 CP-odd couplings)

What is the power counting of these new couplings?

What are the conditions to recover a shift-symmetry?

ALP Shift-Invariance Conditions

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{2}\left(\partial_\mu a\right)\left(\partial^\mu a\right) + \frac{\partial_\mu a}{f}\sum_{\psi\in{\rm SM}} \bar{\psi} c_\psi \gamma^\mu \psi + \mathcal{O}\left(\frac{1}{f^2}\right) \xrightarrow{\psi \to e^{-ic_\psi a/f}\psi} \mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{2}\left(\partial_\mu a\right)\left(\partial^\mu a\right) - \frac{a}{f}\left(\bar{Q}\tilde{Y}_u \tilde{H} u + \bar{Q}\tilde{Y}_d H d + \bar{L}\tilde{Y}_e H e + {\rm h.c.}\right)$$

$$\tilde{Y}_{u,d} = i\left(Y_{u,d} c_{u,d} - c_Q Y_{u,d}\right)\ , \quad \tilde{Y}_e = i\left(Y_e c_e - c_L Y_e\right)$$

Conversely, what are the conditions on the couplings of a pseudo-scalar to recover shift-invariance?

Numbers of physical parameters

hermitian matrices (6 angles and 3 phases) generic matrices (9 angles and 9 phases)

	Shift-symmetric Wilson coefficients $c_{Q,u,d,L,e}$		Generic Wilson coefficients $\tilde{Y}_{u,d,e}$		Number of constraints	
	CP-even	CP-odd	CP-even	CP-odd	CP-even	CP-odd
Quark sector	17	9	18	18	1	9
Lepton sector	9	4	9	7	0	3

U(1)$_B$ and U(1)$_{L_i}$ conserved currents

$\partial_\mu a J^\mu$ added to Lagrangian

3*6-1=17
2*6-3=9

#L$_i$ remove 2 phases

#L$_i$ remove 2 phases

13 conditions on Ỹ's to recover a shift symmetry (1 CP-even and 12 CP-odd)

ALP Shift Invariance

The conditions for shift-symmetry can be written in an invariant way

$X_x = Y_x Y_x^\dagger$

- **Lepton sector**

$$\mathrm{Re}\,\mathrm{Tr}\left(X_e^{0,1,2}\tilde{Y}_e Y_e^\dagger\right) = 0$$

3 invariants

- **Quark sector**

$I_i=0$

$$I_u^{(1)} = \mathrm{Re}\,\mathrm{Tr}\left(\tilde{Y}_u Y_u^\dagger\right), \qquad I_u^{(2)} = \mathrm{Re}\,\mathrm{Tr}\left(X_u\tilde{Y}_u Y_u^\dagger\right), \qquad I_u^{(3)} = \mathrm{Re}\,\mathrm{Tr}\left(X_u^2\tilde{Y}_u Y_u^\dagger\right),$$

$$I_d^{(1)} = \mathrm{Re}\,\mathrm{Tr}\left(\tilde{Y}_d Y_d^\dagger\right), \qquad I_d^{(2)} = \mathrm{Re}\,\mathrm{Tr}\left(X_d\tilde{Y}_d Y_d^\dagger\right), \qquad I_d^{(3)} = \mathrm{Re}\,\mathrm{Tr}\left(X_d^2\tilde{Y}_d Y_d^\dagger\right),$$

$$I_{ud}^{(1)} = \mathrm{Re}\,\mathrm{Tr}\left(X_d\tilde{Y}_u Y_u^\dagger + X_u\tilde{Y}_d Y_d^\dagger\right),$$

$$I_{ud,u}^{(2)} = \mathrm{Re}\,\mathrm{Tr}\left(X_u^2\tilde{Y}_d Y_d^\dagger + \{X_u, X_d\}\tilde{Y}_u Y_u^\dagger\right),$$

$$I_{ud,d}^{(2)} = \mathrm{Re}\,\mathrm{Tr}\left(X_d^2\tilde{Y}_u Y_u^\dagger + \{X_u, X_d\}\tilde{Y}_d Y_d^\dagger\right),$$

$$I_{ud}^{(3)} = \mathrm{Re}\,\mathrm{Tr}\left(X_d X_u X_d\tilde{Y}_u Y_u^\dagger + X_u X_d X_u\tilde{Y}_d Y_d^\dagger\right)$$

$$I_{ud}^{(4)} = \mathrm{Im}\,\mathrm{Tr}\left(\left[X_u, X_d\right]^2\left(\left[X_d, \tilde{Y}_u Y_u^\dagger\right] - \left[X_u, \tilde{Y}_d Y_d^\dagger\right]\right)\right)$$

4 entangled conditions
between up and down sectors
⇒ collective nature

one algebraic relation ⇒ only **10 independent invariants**

13 flavour invariants all linear in $\tilde{Y}$'s (note that CP ensures that all but $I_{ud}^{(4)}$ vanish)

ALP Shift Invariance

These invariants define sum-rules
among ALP couplings to quarks and leptons.
They need to be tested to establish the ALP nature of a new light scalar.

RG Invariance

The set of invariants is closed under RG

$$\dot{I}_e^{(1)} = 2\gamma_e I_e^{(1)} + 6I_e^{(2)} + 2\,\mathrm{Tr}(X_e)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right),$$

$$\dot{I}_e^{(2)} = 4\gamma_e I_e^{(2)} + 9I_e^{(3)} + 2\,\mathrm{Tr}\left(X_e^2\right)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right),$$

$$\dot{I}_e^{(3)} = 6\gamma_e I_e^{(3)} + 12I_e^{(4)} + 2\,\mathrm{Tr}\left(X_e^3\right)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right)$$

$$\dot{I}_u^{(1)} = 2\gamma_u I_u^{(1)} + 6I_u^{(2)} - 3I_{ud}^{(1)} - 2\,\mathrm{Tr}(X_u)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right),$$

$$\dot{I}_u^{(2)} = 4\gamma_u I_u^{(2)} + 9I_u^{(3)} - 3I_{ud,u}^{(2)} - 2\,\mathrm{Tr}\left(X_u^2\right)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right),$$

$$\dot{I}_u^{(3)} = 6\gamma_u I_u^{(3)} + 12I_u^{(4)} - 3I_u^{'} - 2\,\mathrm{Tr}\left(X_u^3\right)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right),$$

$$\dot{I}_d^{(1)} = 2\gamma_d I_d^{(1)} + 6I_d^{(2)} - 3_{ud}^{(1)} + 2\,\mathrm{Tr}(X_d)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right),$$

$$\dot{I}_d^{(2)} = 4\gamma_d I_d^{(2)} + 9I_d^{(3)} - 3I_{ud,d}^{(2)} + 2\,\mathrm{Tr}\left(X_d^2\right)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right),$$

$$\dot{I}_d^{(3)} = 6\gamma_d I_d^{(3)} + 12I_d^{(4)} - 3I_d^{'} + 2\,\mathrm{Tr}\left(X_d^3\right)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right),$$

$$\dot{I}_{ud}^{(1)} = 2(\gamma_u + \gamma_d)I_{ud}^{(1)},$$

$$\dot{I}_{ud,u}^{(2)} = (4\gamma_u + 2\gamma_d)I_{ud,u}^{(2)} + 3I_u^{'} - 6I_{ud}^{(3)} - 2\,\mathrm{Tr}(X_u X_d X_u)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right),$$

$$\dot{I}_{ud,d}^{(2)} = (4\gamma_d + 2\gamma_u)I_{ud,d}^{(2)} + 3I_d^{'} - 6I_{ud}^{(3)} + 2\,\mathrm{Tr}(X_d X_u X_d)\left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)})\right),$$

$$\dot{I}_{ud}^{(3)} = 4(\gamma_u + \gamma_d)I_{ud}^{(3)},$$

$$\dot{I}_{ud}^{(4)} = 6\left(\gamma_u + \gamma_d + \frac{1}{2}\mathrm{Tr}(X_u + X_d)\right)I_{ud}^{(4)} - \mathrm{Im}\,\mathrm{Tr}\left([X_u, X_d]^3\right)(I_u^{(1)} + I_d^{(1)}).$$

$$\gamma_e = -\tfrac{15}{4}g_1^2 - \tfrac{9}{4}g_2^2 + \mathrm{Tr}\left(X_e + 3(X_u + X_d)\right)$$

$$\gamma_u \equiv -\tfrac{17}{12}g_1^2 - \tfrac{9}{4}g_2^2 - 8g_3^2 + \mathrm{Tr}(X_e + 3(X_u + X_d))$$

$$\gamma_d \equiv -\tfrac{5}{12}g_1^2 - \tfrac{9}{4}g_2^2 - 8g_3^2 + \mathrm{Tr}(X_e + 3(X_u + X_d))$$

closed set except for:

$$I_e^{(4)} = \mathrm{Re}\,\mathrm{Tr}\left(X_e^3 \tilde{Y}_e Y_e^\dagger\right)$$

$$I_u^{'} = \mathrm{Re}\,\mathrm{Tr}\left((X_u X_d X_u + \{X_d, X_u^2\})\tilde{Y}_u Y_u^\dagger + X_u^3 \tilde{Y}_d Y_d^\dagger\right)$$

$$I_d^{'} = I_u^{'}(u \leftrightarrow d)$$

but Cayley-Hamilton eq. tells us that these 3 invariants are actually linear combinations of our original set

↓

shift-invariance conditions are closed under RG

Shift-Invariance: Non-perturbative Condition

θ_{QCD} again

$$-\frac{C_g g_3^2}{16\pi^2}\frac{a}{f}\,\mathrm{Tr}\big(G_{\mu\nu}\tilde{G}^{\mu\nu}\big)$$

breaks shift-invariance non-perturbatively (instanton effects)

(in the operator basis where fermion couplings are derivative)

$$I_g \equiv C_g + \mathrm{Im}\,\mathrm{Tr}\left(Y_u^{-1}\tilde{Y}_u + Y_d^{-1}\tilde{Y}_d\right) = 0$$

is the basis independent condition for the shift-invariance to be maintained at the non-perturbative level

It can be shown again that this condition is **RG invariant**

$$\mu\frac{dI_g}{d\mu} = 0$$

whenever shift-symmetry holds (I_g=I_i=0 for i=1…13)

Executive summary on status of BSM

BAD NEWS

Experimentalists haven't found (yet)
what theorists told them they will find

GOOD NEWS

There are rich opportunities
for mind-boggling signatures
@ colliders and beyond