Quadratic Coupling of Axions to Photons



Based on arXiv: 2307.10362





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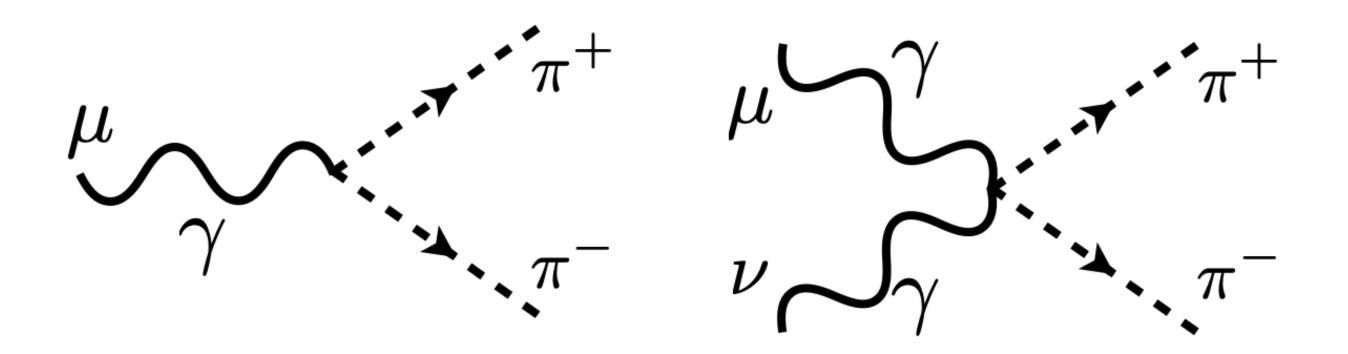
$$\mathcal{L}_{a^2F^2} \supset c_{F^2} \frac{\alpha}{16\pi^2} \left(\frac{a}{f_a}\right)^2 F_{\mu\nu} F^{\mu\nu} \qquad ?$$



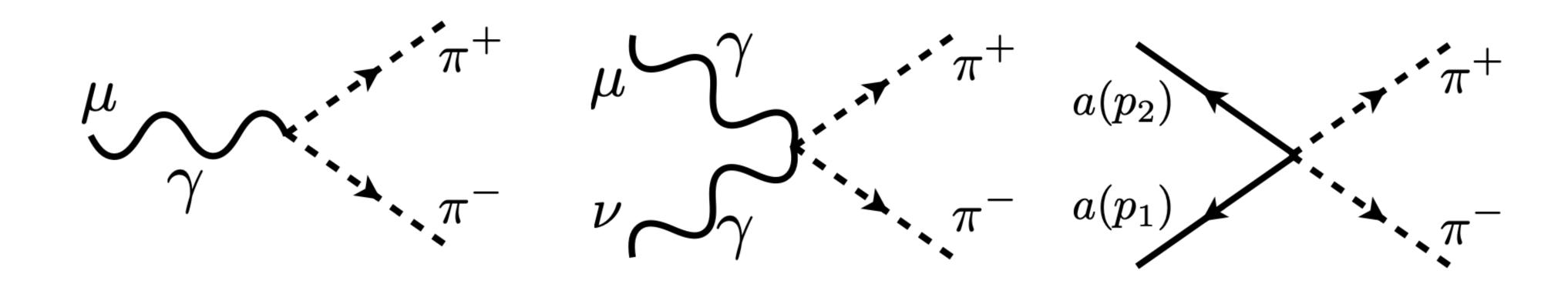


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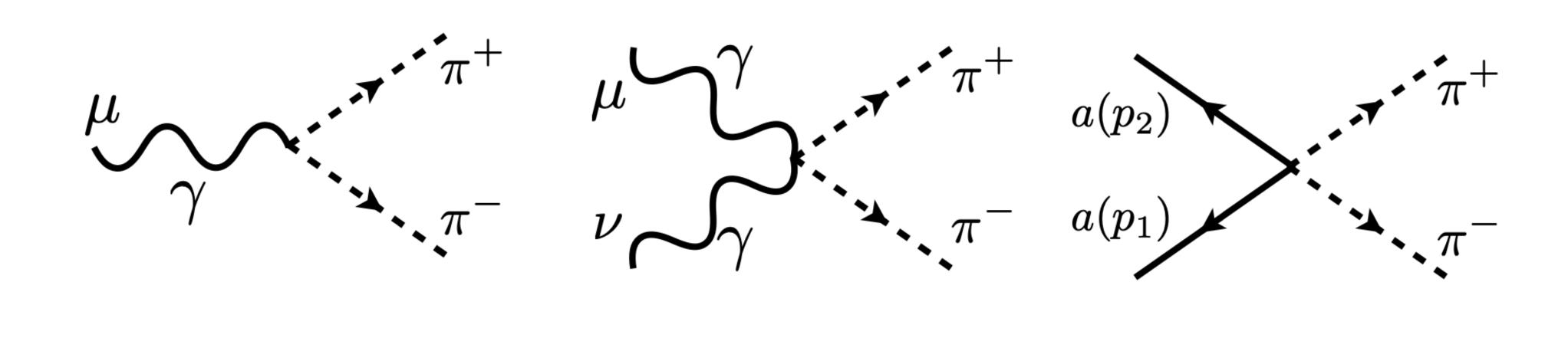
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$$\mu \qquad \pi^+ \qquad \mu \qquad \pi^+ \qquad a(p_2) \qquad \pi^+ \qquad \pi^+ \qquad \pi^- \qquad \nu \qquad \gamma \qquad \pi^- \qquad a(p_1) \qquad \pi^- \qquad a(p_1) \qquad \pi^- \qquad a(p_2) \qquad a(p_2)$$

$$iM^{\mu\nu} = \left(\frac{-i}{8\pi^2}\right) \frac{e^2}{f_a^2 (m_u + m_d)^2} \left(m_u m_d m_\pi^2 + 2(m_d - m_u)^2 (k_1 \cdot k_2)\right) \left(\frac{g^{\mu\nu} (p_1 \cdot p_2) - p_2^{\mu} p_1^{\nu}}{p_1 \cdot p_2}\right) \times \left[1 - \frac{2m_\pi^2}{s} \left(\text{Li}_2\left(\frac{2\sqrt{s}}{\sqrt{s} - \sqrt{s - 4m_\pi^2}}\right) + \text{Li}_2\left(\frac{2\sqrt{s}}{\sqrt{s} + \sqrt{s - 4m_\pi^2}}\right)\right)\right].$$

• This results in a modified value for α :

$$\alpha \simeq \alpha_0 \left(1 + \frac{\alpha_0 m_u m_d a^2}{12 \pi f_a^2 (m_u + m_d)^2} \right)$$

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• Note the same effect can be achieved by inclusion of operator:

$$\mathcal{L}_{a^2F^2} \supset c_{F^2} \frac{\alpha}{16\pi^2} \left(\frac{a}{f_a}\right)^2 F_{\mu\nu} F^{\mu\nu}$$

$$c_{F2} = \pi m_u m_d / 3 (m_u + m_d)^2 \sim 0.2$$

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$$\langle a(t)^2 \rangle \equiv \rho_{\rm DM}/m_a^2$$

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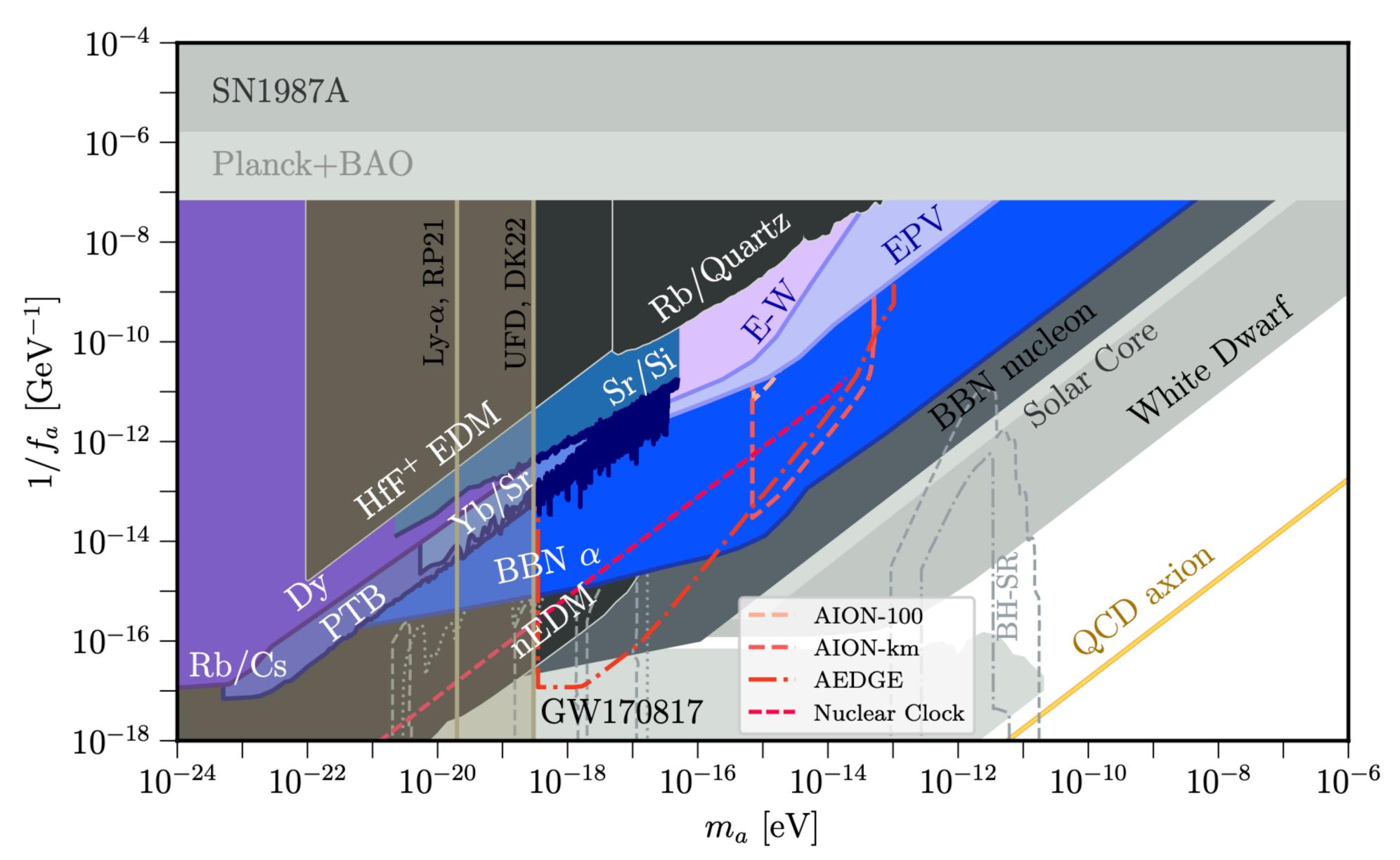
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• One sees the same result noticing the threshold correction to α 's running.

See [2307.14962] Kim, Lenoci, Perez, Ratzinger

Bounds in axion parameter space



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• In the case for ALPs, we may build models with these features:

We use dynamics which break the shift-symmetry which also couple to photons!

A little ALP model building

Consider the ALP model:

$$\mathcal{L}_{\text{UV}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}_L i \not\!\!D \psi_L + \bar{\psi}_R i \not\!\!D \psi_R$$
$$+ y \phi \bar{\psi}_L \psi_R + y \phi^{\dagger} \bar{\psi}_R \psi_L + \partial_{\mu} \phi \partial^{\mu} \phi^{\dagger} - V(\phi, \phi^{\dagger})$$

with a potential given by:

$$V(\phi, \phi^{\dagger}) = \lambda \left(\phi^{\dagger} \phi - \frac{f_a^2}{2} \right)^2 + g^2 \left(\phi^{\dagger} \phi - \frac{f_a^2}{2} \right) \left(1 - \cos \left(\frac{a}{f_a} \right) \right)$$

A little ALP model building

• Integrating out the fermions and then the radial modes leads to:

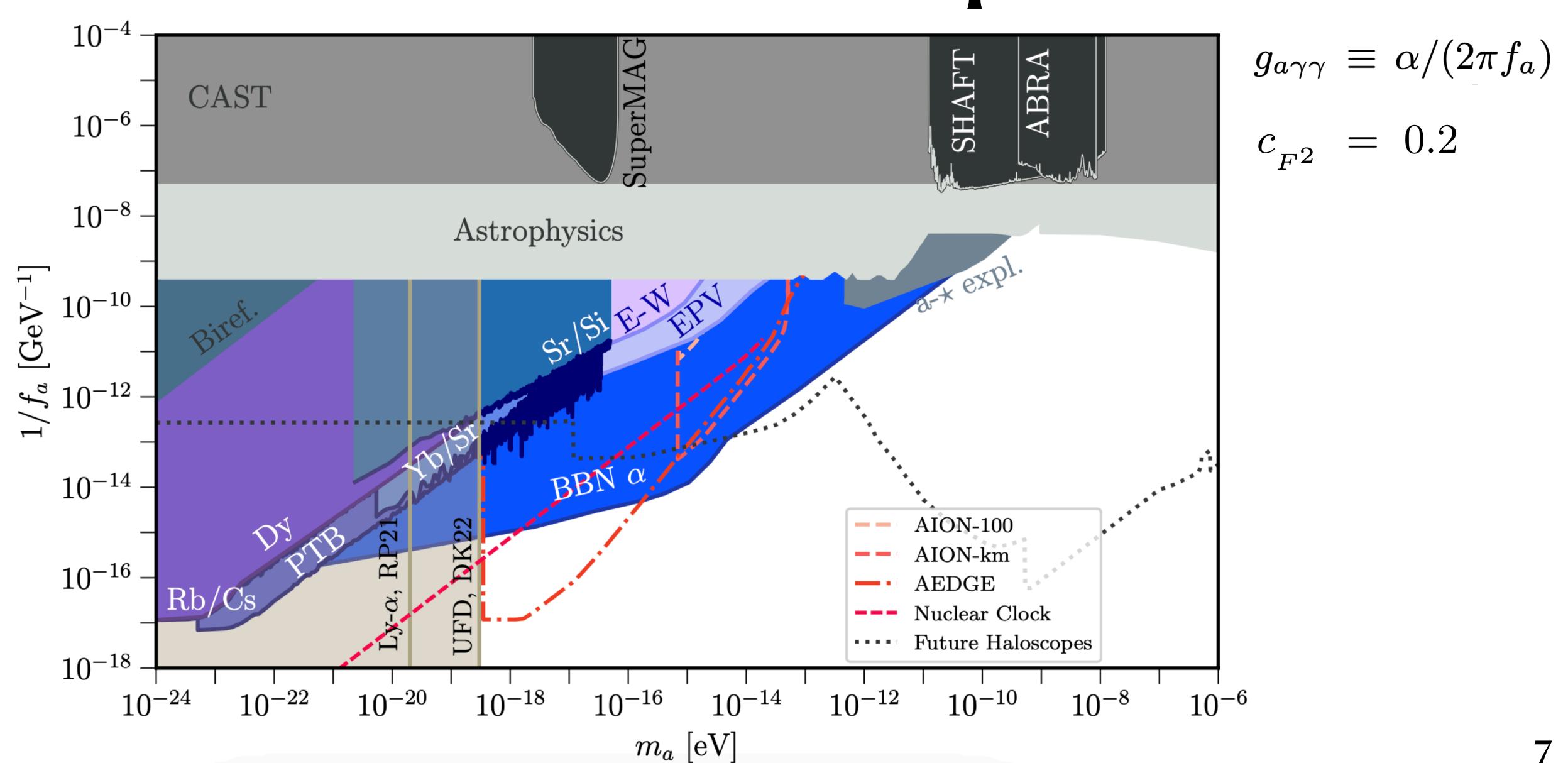
$$\mathcal{L}_{a^2 F^2}^{1-\mathrm{loop}} \supset \frac{i^2}{16\pi^2} \frac{2}{3M_{\psi}} \left[M_{\psi} \frac{\rho}{f_a} \right] (iQ_{\psi} e)^2 F_{\mu\nu} F^{\mu\nu}$$

$$\Rightarrow \frac{1}{48\pi^2} (Q_{\psi}e)^2 \frac{g^2}{f_a^2 M_{\rho}^2} a^2 F_{\mu\nu} F^{\mu\nu}$$

• One can read off the variation of $c_{_{F^2}}$ and α as:

$$c_{F^2} = \frac{4\pi}{3} Q_{\psi}^2 \frac{g^2}{M_{\rho}^2} \,, \quad \alpha(a) = \alpha \left(1 + \frac{Q_{\psi}^2 \alpha}{3\pi} \frac{g^2 a^2}{M_{\rho}^2 f_a^2} \right)$$

Bounds in ALP space



Summary

- Dynamics endowing axion with a mass can also lead to a quadratic coupling of axions to photons
- For QCD-axion other constraints stronger; but offers new ways to probe parts of parameter space
- For ALPs the quadratic coupling is easily generated & resulting constraints might be the strongest bounds in large regions of parameter space

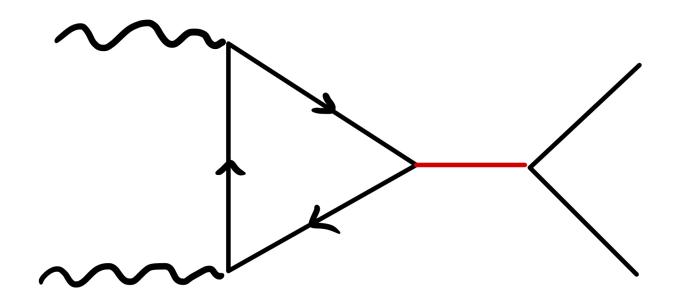
Thanks & Questions?

Additional slides

Calculation in ALP models

Shift symmetry breaking EFT

- Most of the diagrams cancel due to the shift symmetry of the axion, in the matching procedure, because they should not be able to generate shift symmetry breaking operators in the IR if they respect it in the UV.
- At lowest loop order the diagrams we are matching to in the high UV are of the form:



which gives the result in the main slides.

Calculation in ALP models

QCD-like model: SU(N)' x U(1)'

- A dark QCD-like sector with two equal mass quarks will generate a potential of form: $V(a) = -m_{\pi'}^2 f_{\pi'}^2 \cos\left(\frac{a}{2f_a}\right) \simeq -\frac{1}{2} m_a^2 a^2$
- One sees the fine structure in the new sector is modified in the same way as before

$$\alpha' \simeq \alpha'_0 \left(1 + \frac{\alpha'_0 a^2}{48 \pi f_a^2} \right)$$

• Through kinetic mixing of the photon and dark photon one finds that our fine structure constant is modified by:

(a^2)

$$\alpha \simeq \alpha_0 \left(1 + \chi^2 \alpha_0' \frac{a^2}{48 \pi f_a^2} \right)$$

BBN reminder & yield

• The proton-neutron mass difference is sensitive to variations in α ; the proton mass will get receive corrections from photons

• The equilibrium ratio of proton and neutron number densities is exponentially sensitive to this mass difference, this is why the constraint is so stringent

Why not more heavily suppressed?

• NDA expectation is that because of the shift-symmetry of the axion the operator should appear as: $\mathcal{O}((\partial_{\mu}a)^2\,f_a^{-4})$

• Since the operator we see is generated by the same dynamics as the potential the suppression is less severe than this

Why constraints not vanishing in massless 'a' limit?

• This is an artefact of treating axion as DM and holding the observed DM density fixed: $\langle a(t)^2\rangle\equiv \rho_{\rm DM}/m_a^2$

• For sufficiently light masses the amplitudes will become transplanckian

Why constraints not vanishing in massless pion limit?

- The simplest answer is that one treats the pion as massive throughout the calculation; although note that a vanishing up or down quark mass *does* prevent the variation in α !
- (There is also no need for an axion in this limit and eta prime will act as the axion to relax theta to zero.)

• One has thus assumed they are non-zero when writing in terms of a pion mass.

Threshold correction argument gives this physical intuition.

Region of interest in parameter space?

• The axion mass lies in a weird part of the parameter space, how did it get there?

• One can assume there exists a tuning / symmetry to allow the mass to naturally sit in this region, for example Zn "mirror" symmetry of Hook.