Novel directions in the ALP EFT

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HIDDeν
Hunting Invisibles: Dark sectors, Dark matter and Neutrinos

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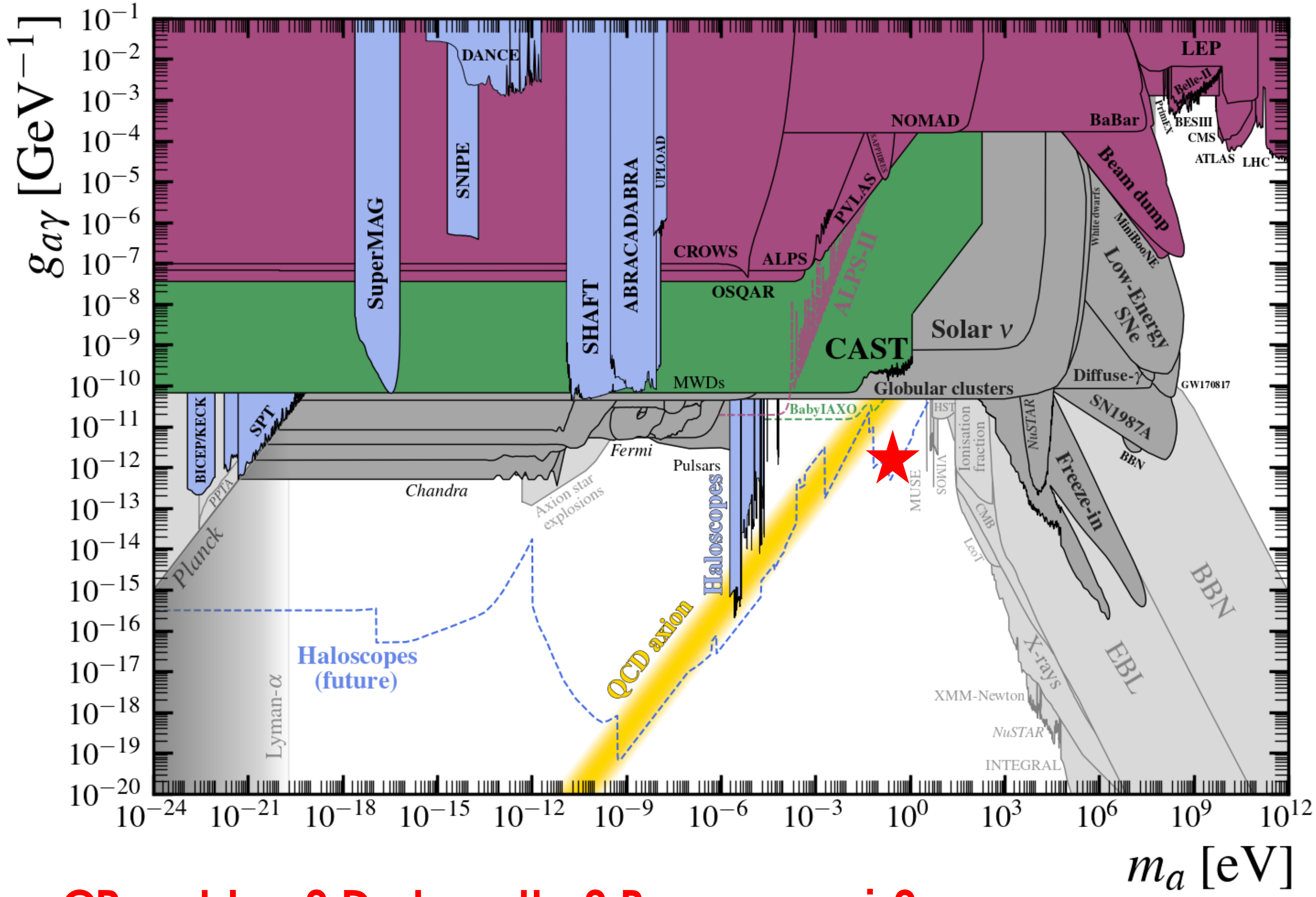
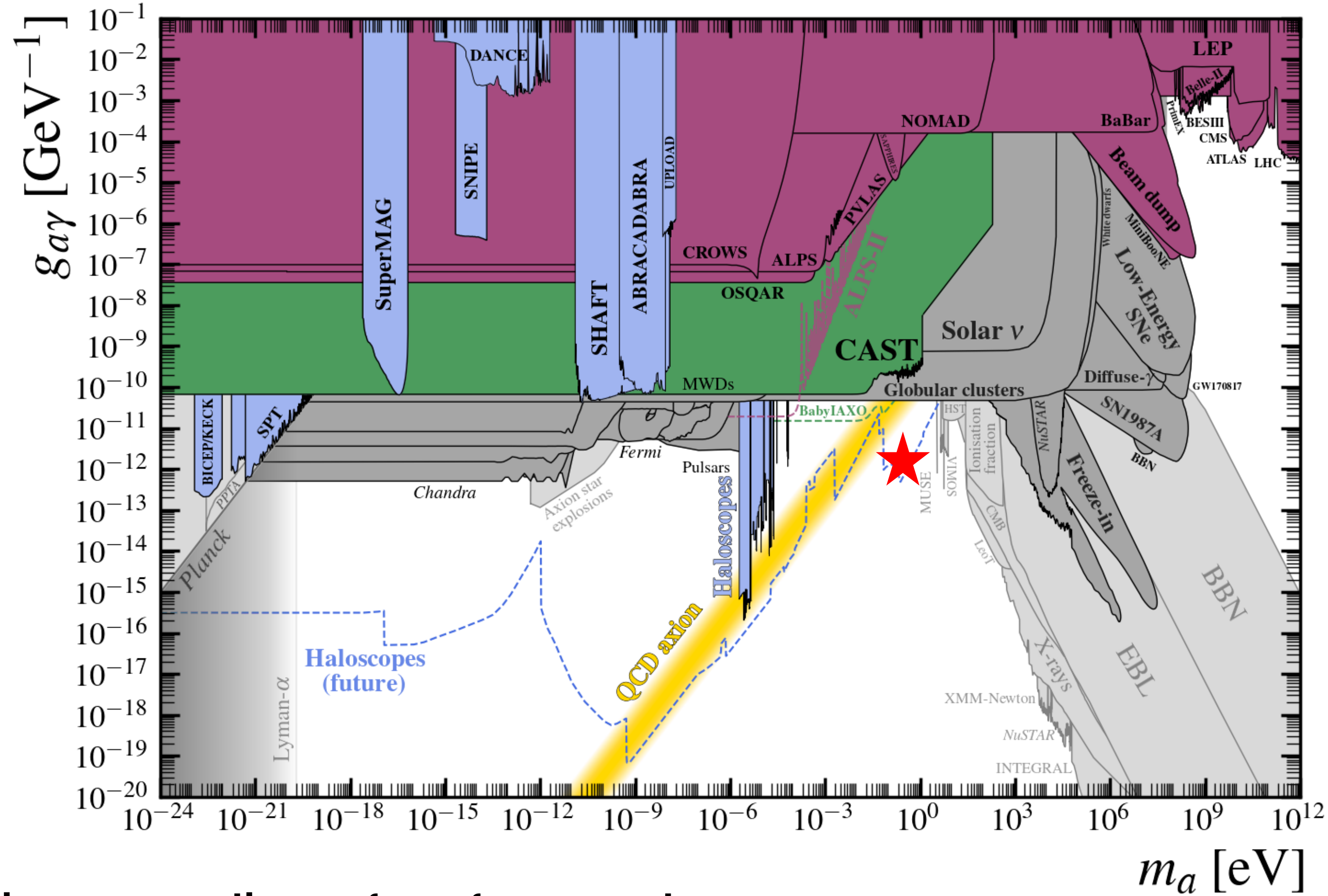
Motivation



Strong CP problem? Dark matter? Baryogenesis?

GUT? Extra dimensions? Composite theory?

Motivation



Revise assumptions of our frameworks.

Relate IR with UV physics.

Gavela, Quílez, MR, 2305.15465

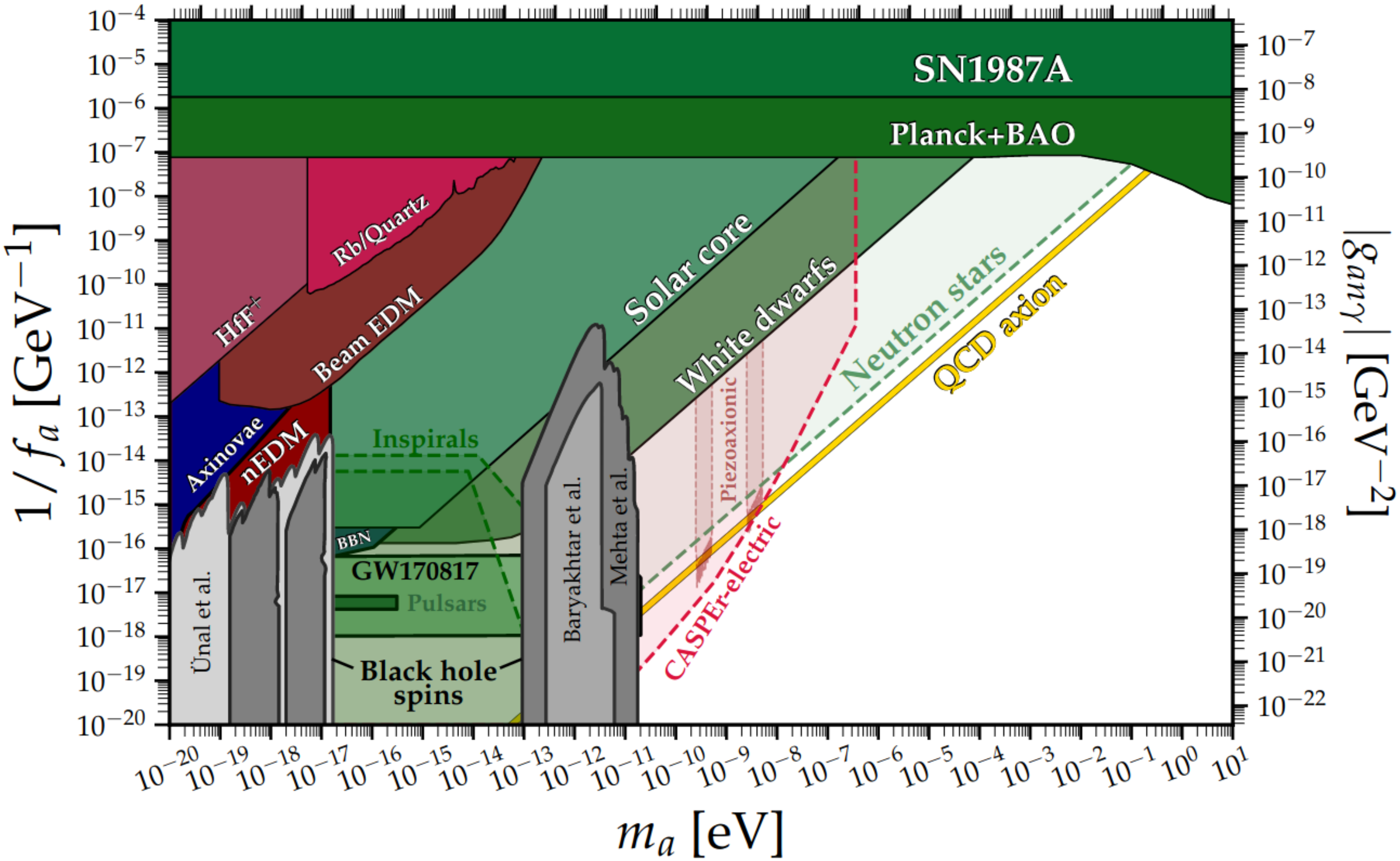
Revisiting the axion solution to the strong CP problem:

I. Can scalar mixing impact our predictions?

<u>Assuming the SM gauge group setting</u>

The QCD axion: minimal way

$$\mathcal{L} = \frac{\alpha_s}{8\pi}\left(\frac{\hat{a}_{G\widetilde{G}}}{f_a} - \bar{\theta}\right) G\widetilde{G} \rightarrow \quad m_a^2\, f_a^2 = \chi_{\rm QCD} \simeq m_\pi^2\, f_\pi^2\, \frac{m_u\, m_d}{(m_u + m_d)^2}$$



The QCD axion: non-minimal way

$$m_a^2\, f_a^2 = \chi_{\rm QCD}$$

[interaction basis = mass basis]

up to mixing with QCD resonances

But the axion may not be the only singlet scalar in Nature.

- Motivation from fundamental setups: e.g. string axiverse, extra dimensions

  Dienes, Dudas, Gherghetta 99
  Arvanitakia, Dimopoulos, Dubovskyc, Kalopere, Russell 09

- Axion-ALP mixing opens new regions of parameter space for dark matter Cyncynates, Giurgica-Tiron, Simon, Thompson 21, ...

$$\mathcal{L} = \frac{\alpha_s}{8\pi}\left(\frac{\hat{a}_{G\widetilde{G}}}{F} - \bar{\theta}\right) G\widetilde{G} - V'(\hat{a}_{G\widetilde{G}}, \ldots, \hat{a}_N)$$

$$\Rightarrow m_i^2\, f_i^2 = g_i \chi_{\rm QCD}$$

PQ condition

$$\mathcal{L} = \frac{\alpha_s}{8\pi}\left(\sum_{k=1}^{N}\frac{\hat{a}_k}{\hat{f}_k} - \bar{\theta}\right) G\widetilde{G} - V_B(\hat{a}_1,\, \hat{a}_2, \ldots, \hat{a}_N) \to \frac{\alpha_s}{8\pi}\left(\frac{\hat{a}_{G\widetilde{G}}}{F} - \bar{\theta}\right) G\widetilde{G} - V_B^{\mathrm{R}}(\hat{a}_{G\widetilde{G}}, \ldots)$$

$$\frac{1}{F^2} = \sum_{k=1}^{N}\frac{1}{\hat{f}_k^2}$$

A preferred basis.

$$\boxed{\mathbf{M}^2 \equiv \mathbf{R}\,\hat{\mathbf{M}}^2\mathbf{R}^T}$$

$$\mathbf{M}^2 = \mathbf{M}_A^2 + \mathbf{M}_B^2 = \begin{pmatrix} b_{11} & \mathbf{X}^\dagger \\ \mathbf{X} & \mathbf{M}_1^2 \end{pmatrix} = \frac{\chi_{\mathrm{QCD}}}{F^2}\begin{pmatrix} 1 & 0 \\ 0 & \mathbf{0} \end{pmatrix} + \begin{pmatrix} b_{11} - \frac{\chi_{\mathrm{QCD}}}{F^2} & \mathbf{X}^\dagger \\ \mathbf{X} & \mathbf{M}_1^2 \end{pmatrix},$$

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$$\boxed{\exists\, U(1)_{PQ} \implies \lim_{\chi_{\mathrm{QCD}}\to 0} \det \mathbf{M}^2 = 0 \implies \det \mathbf{M}_B^2 = 0} \quad \langle \hat{a}_0 | a_{G\widetilde{G}} \rangle \neq 0$$

PQ condition

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Applying Schur's formula.

$$\det \mathbf{M}_1^2\left(b_{11} - \frac{\chi_{\mathrm{QCD}}}{F^2} - \mathbf{X}^\dagger\mathbf{M}_1^{-2}\mathbf{X}\right) = 0$$

$$\Rightarrow \boxed{\frac{\det \mathbf{M}^2}{\det \mathbf{M}_1^2} = \left(b_{11} - \mathbf{X}^\dagger\mathbf{M}_1^{-2}\mathbf{X}\right) = \frac{\chi_{\mathrm{QCD}}}{F^2}}$$

PQ condition

$$\Rightarrow \quad \frac{\det \mathbf{M}^2}{\det \mathbf{M}_1^2} = \left(b_{11} - \mathbf{X}^\dagger \mathbf{M}_1^{-2} \mathbf{X}\right) = \frac{\chi_{\rm QCD}}{F^2}$$

Moving to the physical basis.

$$\mathcal{L} \supset \frac{\alpha_s}{8\pi} \frac{a_i}{f_i} G\widetilde{G} \qquad \text{with} \qquad \frac{1}{f_i} = \frac{\langle \hat{a}_{G\widetilde{G}} | a_i \rangle}{F} \equiv \frac{v_{i1}}{F} \implies \sum_{i=1}^{N} \frac{1}{f_i^2} = \frac{1}{F^2}$$

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Eigenvector-eigenvalue Th.
(generic A matrix)

$$\frac{\det\left(\lambda \mathbb{I}_{N-1} - M_j\right)}{\det\left(\lambda \mathbb{I}_N - A\right)} = \sum_{i=1}^{N} \frac{|v_{ij}|^2}{\lambda(A) - \lambda_i(A)}$$

PQ condition

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$$\frac{\det \mathbf{M}_1^2}{\det \mathbf{M}^2} = \sum_{i=1}^{N} \frac{|v_{1i}|^2}{m_i^2} = \frac{F^2}{\chi_{\rm QCD}} \sum_{i=1}^{N} \frac{1}{g_i}$$

$$\boxed{g_i = \frac{m_i^2 f_i^2}{\chi_{\rm QCD}}}$$

The QCD axion sum rule

$$\Rightarrow \frac{\det \mathbf{M}^2}{\det \mathbf{M}_1^2} = \left(b_{11} - \mathbf{X}^\dagger \mathbf{M}_1^{-2} \mathbf{X}\right) = \frac{\chi_{\rm QCD}}{F^2}$$

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$$\frac{\det \mathbf{M}_1^2}{\det \mathbf{M}^2} = \sum_{i=1}^{N} \frac{|v_{1i}|^2}{m_i^2} = \frac{F^2}{\chi_{\rm QCD}} \sum_{i=1}^{N} \frac{1}{g_i} \xRightarrow{\exists\, U(1)_{\rm PQ}} \boxed{\sum_{i=1}^{N} \beta_i = 1,\ \ \beta_i \equiv \frac{1}{g_i}}$$

$$\boxed{g_i = \frac{m_i^2 f_i^2}{\chi_{\rm QCD}}}$$

***axionness* is shared!**

The QCD axion sum rule

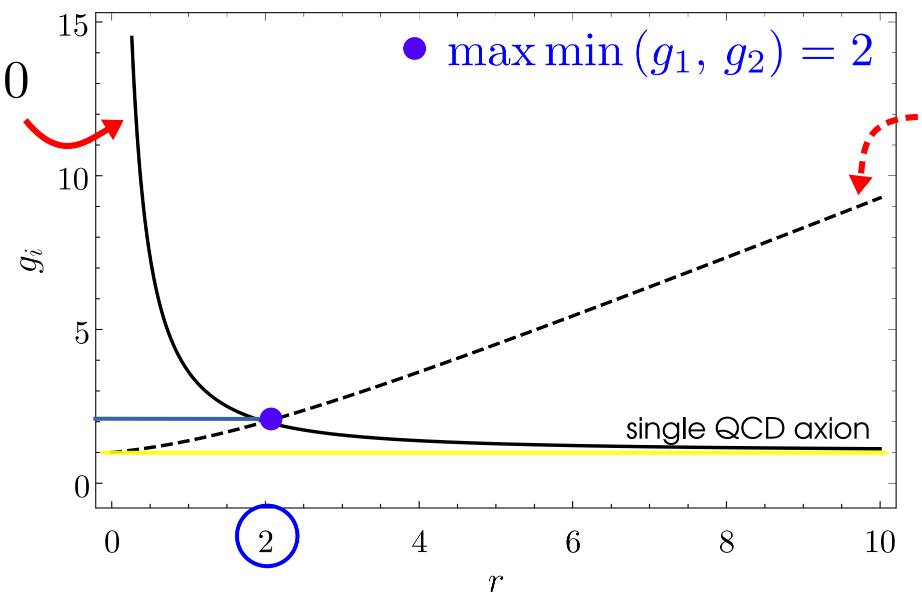
$$\beta_i = \frac{\langle \hat{a}_{PQ} \mid a_i \rangle \langle a_i \mid \hat{a}_{G\widetilde{G}} \rangle}{\langle \hat{a}_{PQ} \mid \hat{a}_{G\widetilde{G}} \rangle}$$

Toy example:

$$\mathcal{L}_{N=2} = \frac{\alpha_s}{8\pi}\left(\frac{\hat{a}_1}{\hat{f}_1} + \frac{\hat{a}_2}{\hat{f}_2} + \bar{\theta}\right) G\widetilde{G} - \mu^2 \hat{a}_2^2 \implies \quad \hat{a}_{GG} = \frac{1}{2}(\hat{a}_1 + \hat{a}_2) \quad \text{and} \quad \hat{a}_{PQ} = \hat{a}_1$$

$$\beta_1 = 1 \times 0$$

$$r = \mu^2 \frac{\hat{f}^2}{\chi_{QCD}}$$

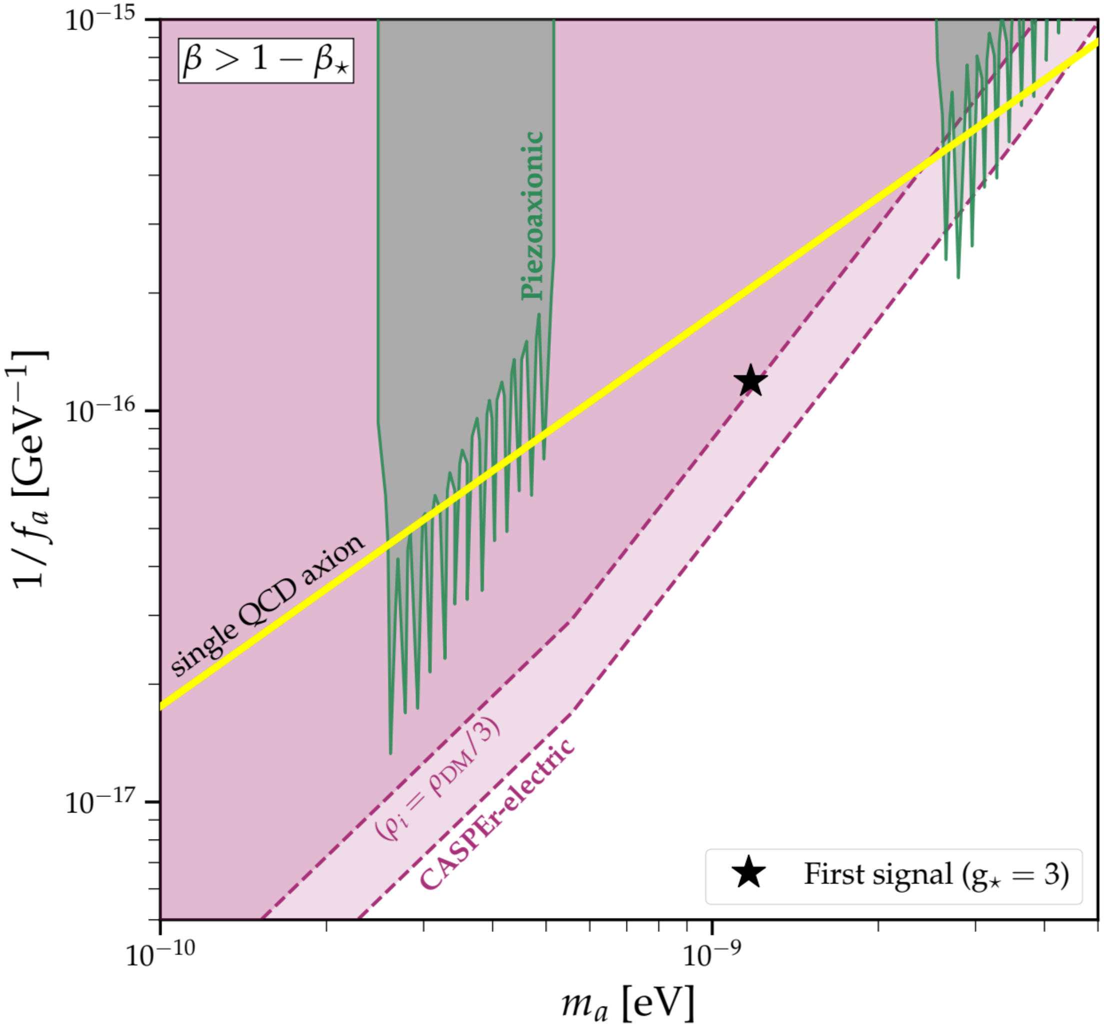


$$\beta_2 = 0 \times 1$$

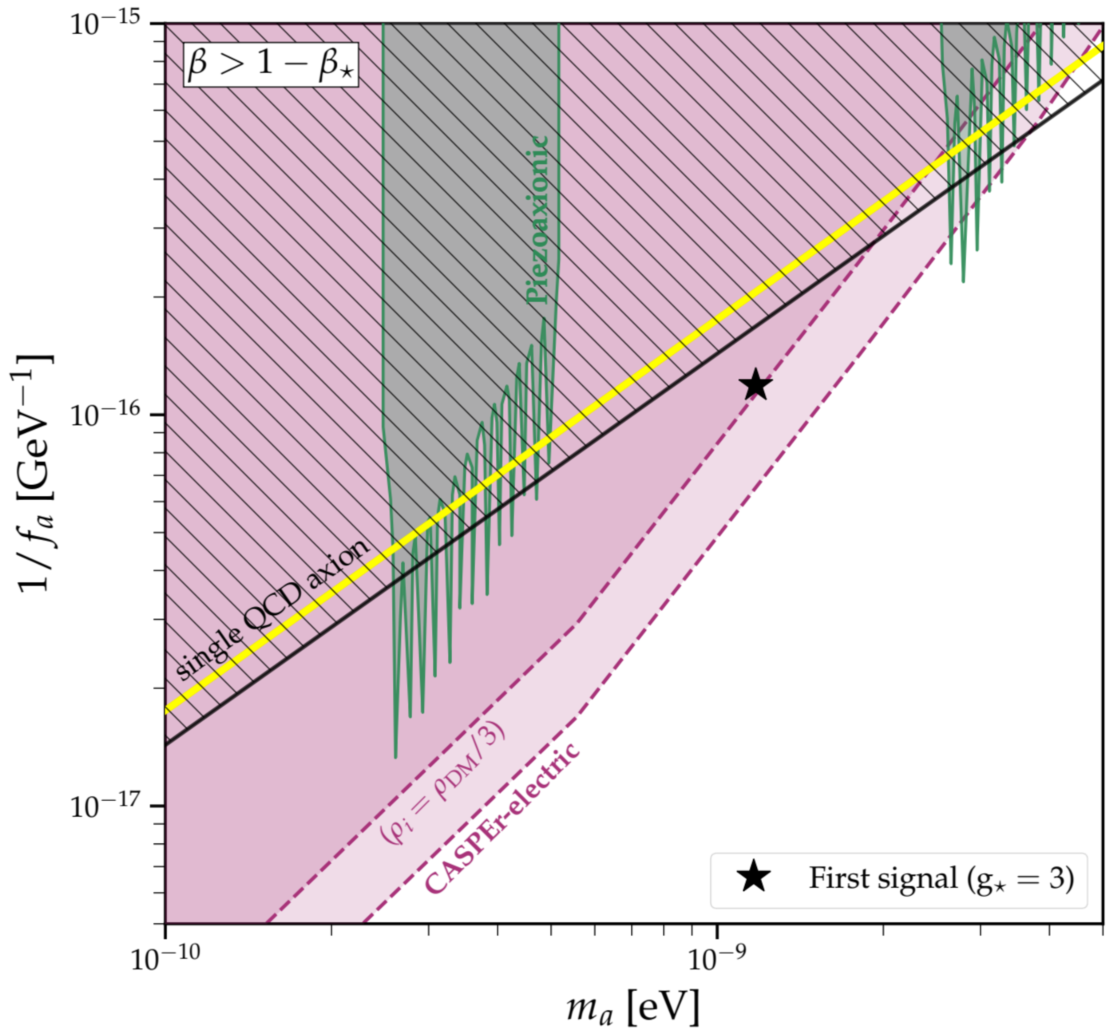
Kim et al, 04
Agrawal et al, 17 and 22
Farina et al, 17
Coy, Frigerio, 17
Choe et al, 14 and 16
Kaplan et al, 16
Giudice et al, 16

Large deviations require new scales close to the QCD generated mass

Experimental consequences

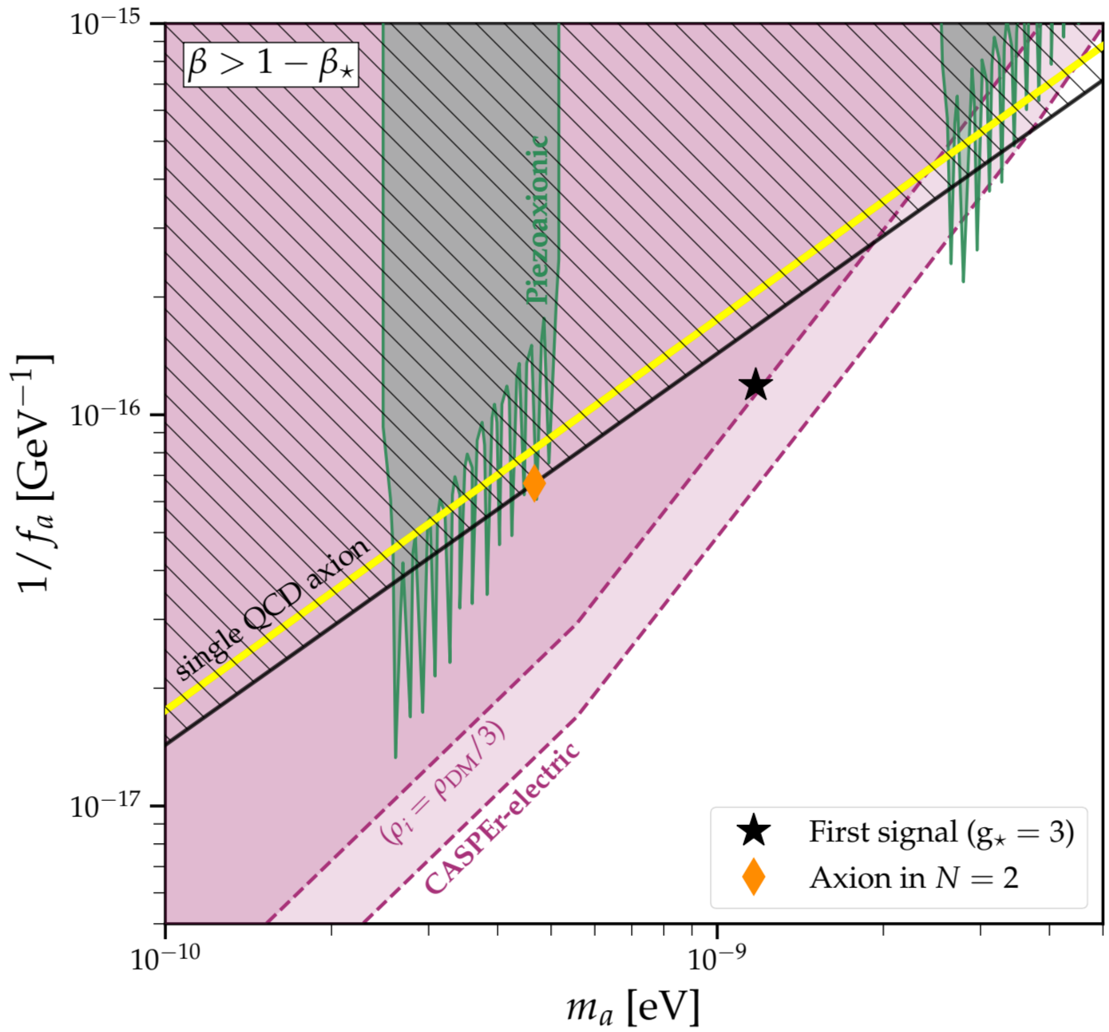


Experimental consequences



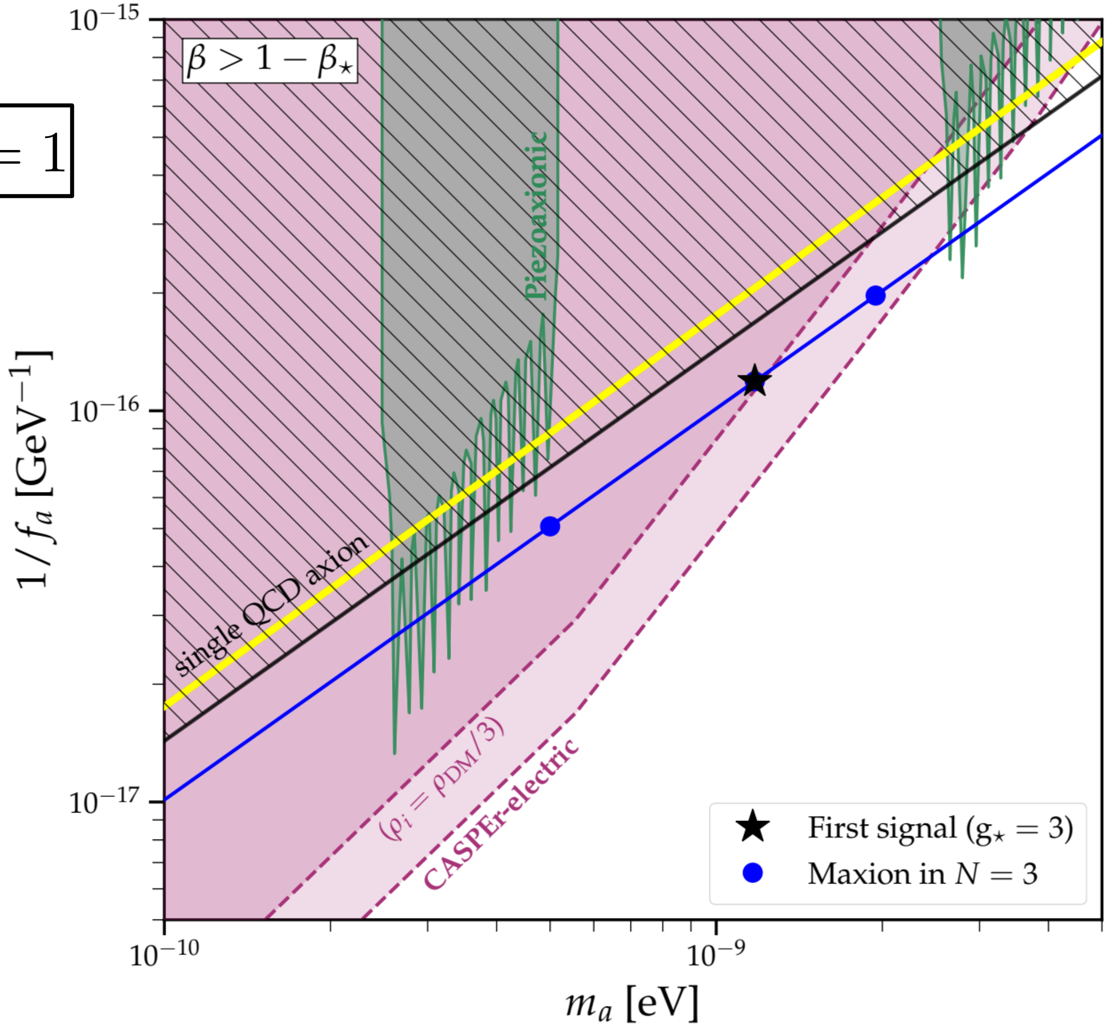
Experimental consequences

$$\beta_\star + \beta_2 = 1$$



Experimental consequences

$$\beta_\star + \beta_2 + \beta_3 = 1$$



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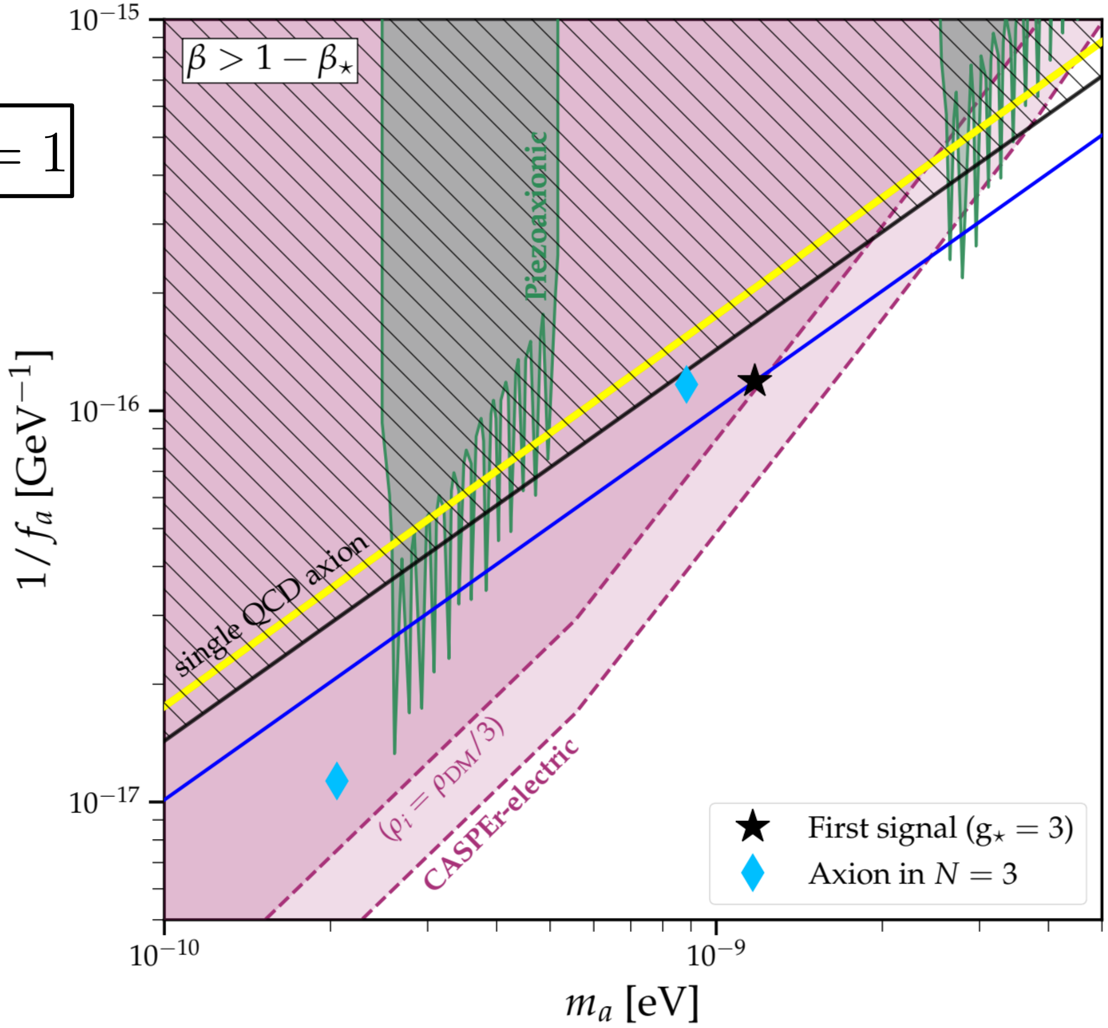
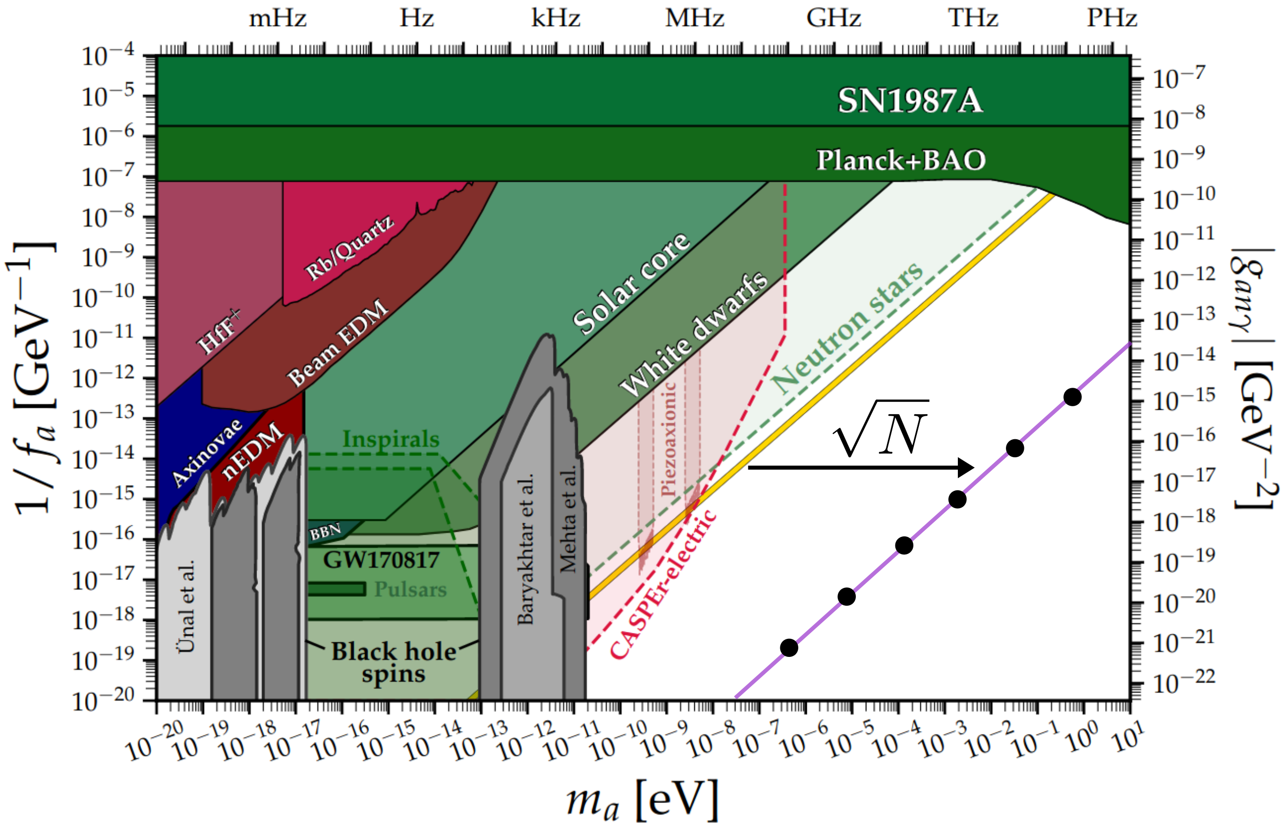


Maximally deviated QCD axions=**Maxions**

$$\max\left\{\min_i\{g_i\}\right\} = N \qquad \Longrightarrow \quad g_i = N,\ \forall\, i$$



Maximally deviated QCD axions=**Maxions**

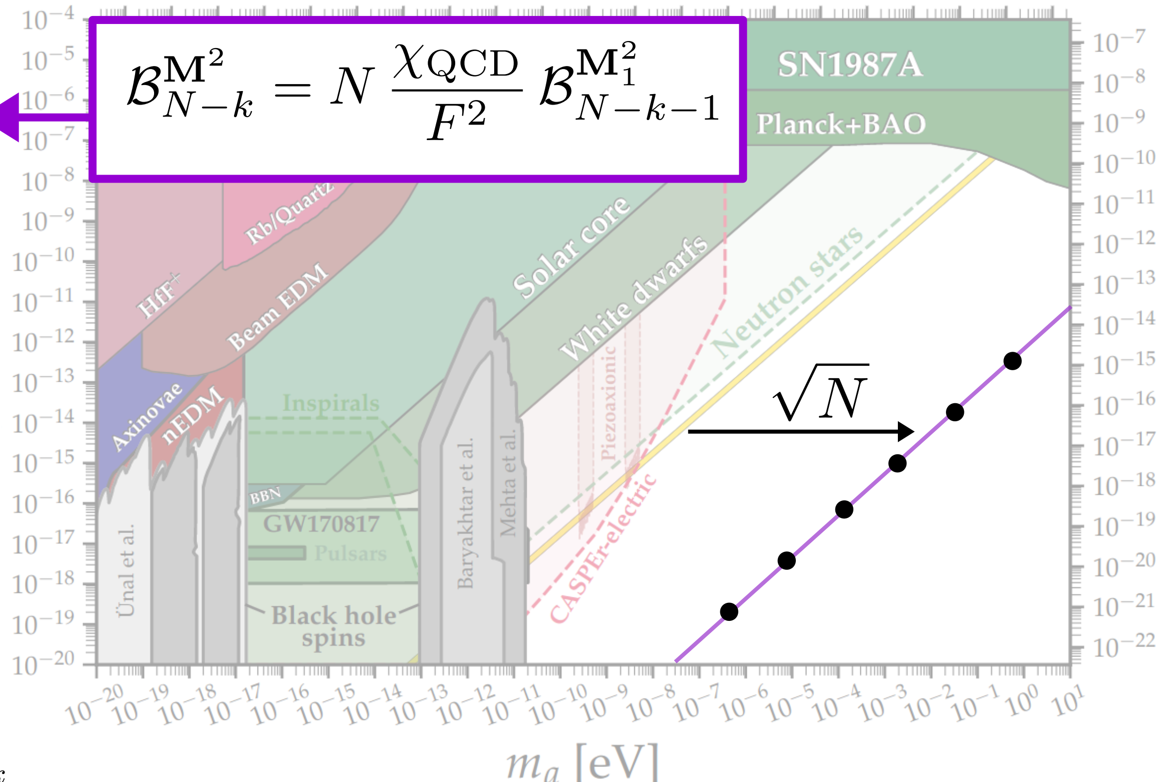
$$\max_i \left\{ \min_i \{g_i\} \right\} = N \quad \Longrightarrow \quad g_i = N,\ \forall\, i$$

m-parameter family of maxions: $m = N(N+1)/2$

$$\mathrm{tr}\,\mathbf{M}^2 = N \frac{\chi_{\rm QCD}}{F^2}$$

$$\frac{\det \mathbf{M}^2}{\det \mathbf{M}_1^2} = \frac{\chi_{\rm QCD}}{F^2}$$

$\ldots$



$$p_{\mathbf{M}^2}(\lambda) = \sum_{k=0}^{N} \frac{(-1)^{N-k}}{(N-k)!} \mathcal{B}^{\mathbf{M}^2}_{N-k} \lambda^k$$

Coupling to photons

Assuming **universal** anomaly factors,

$$\mathcal{L} \supset \frac{\alpha_{em}}{8\pi} \sum_{k=1}^{N} \frac{E_k}{\mathcal{N}_k} \frac{\hat{a}_k}{\hat{f}_k} F\widetilde{F} \implies \frac{\alpha_{em}}{8\pi} \frac{E}{\mathcal{N}} \frac{a_{G\tilde{G}}}{F} F\widetilde{F}$$

Making an axion-dependent rotation, $q = \begin{pmatrix} u \\ d \end{pmatrix} \to e^{i\gamma_5 a_{G\tilde{G}}/(2F) Q_a} \begin{pmatrix} u \\ d \end{pmatrix}$:

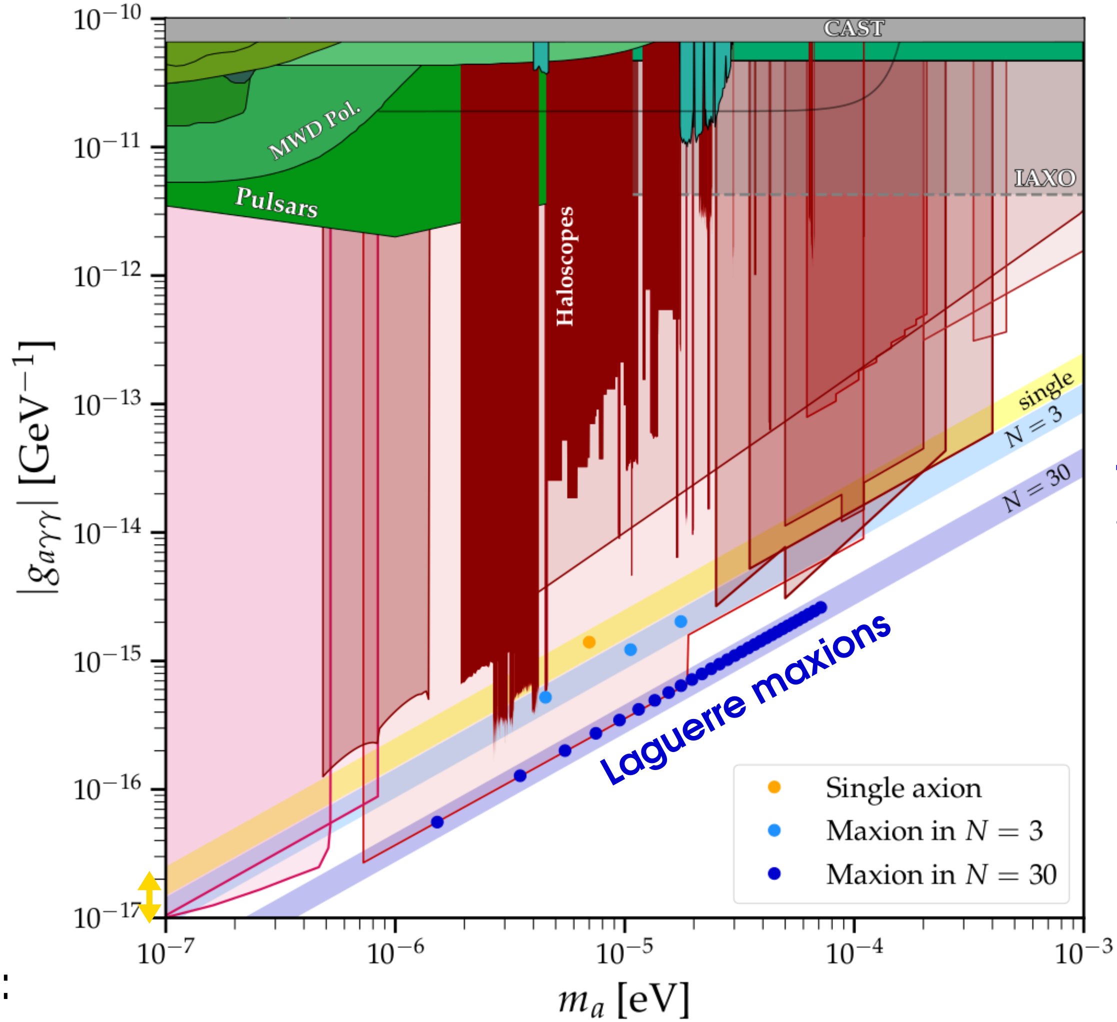
Di Cortona, Hardy, Vega, Villadoro 15

$$\mathcal{L} \supset \frac{\alpha_{em}}{2\pi} \left[\frac{E}{\mathcal{N}} - 1.92\right] \sum_i \frac{a_i}{f_i} F\widetilde{F}$$

$$\boxed{\frac{m_i^2}{g_{a_i\gamma\gamma}^2} = \frac{m_a^2}{g_{a\gamma\gamma}^2}\Bigg|_{\text{single QCD axion}} \times g_i}$$

$$\frac{(2\pi)^2}{\alpha_{em}^2} \left[\frac{E}{\mathcal{N}} - 1.92\right]^{-2} \sum_{i=1}^{N} \frac{g_{a_i\gamma\gamma}^2}{m_i^2} = 1$$

Coupling to photons



Mass splitting is not fixed!

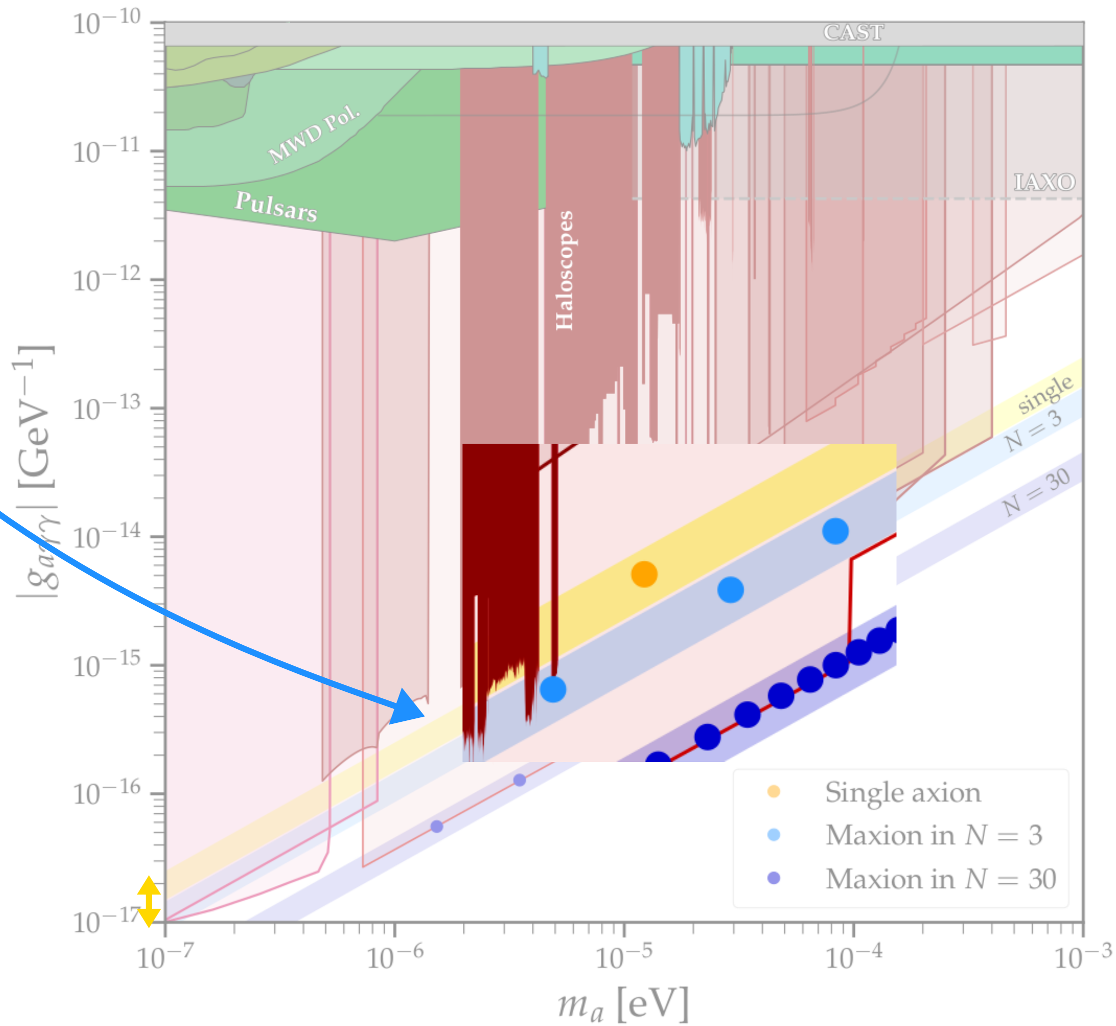
(Laguerre are **one type** of maxion solutions)

↕ Band width spans:

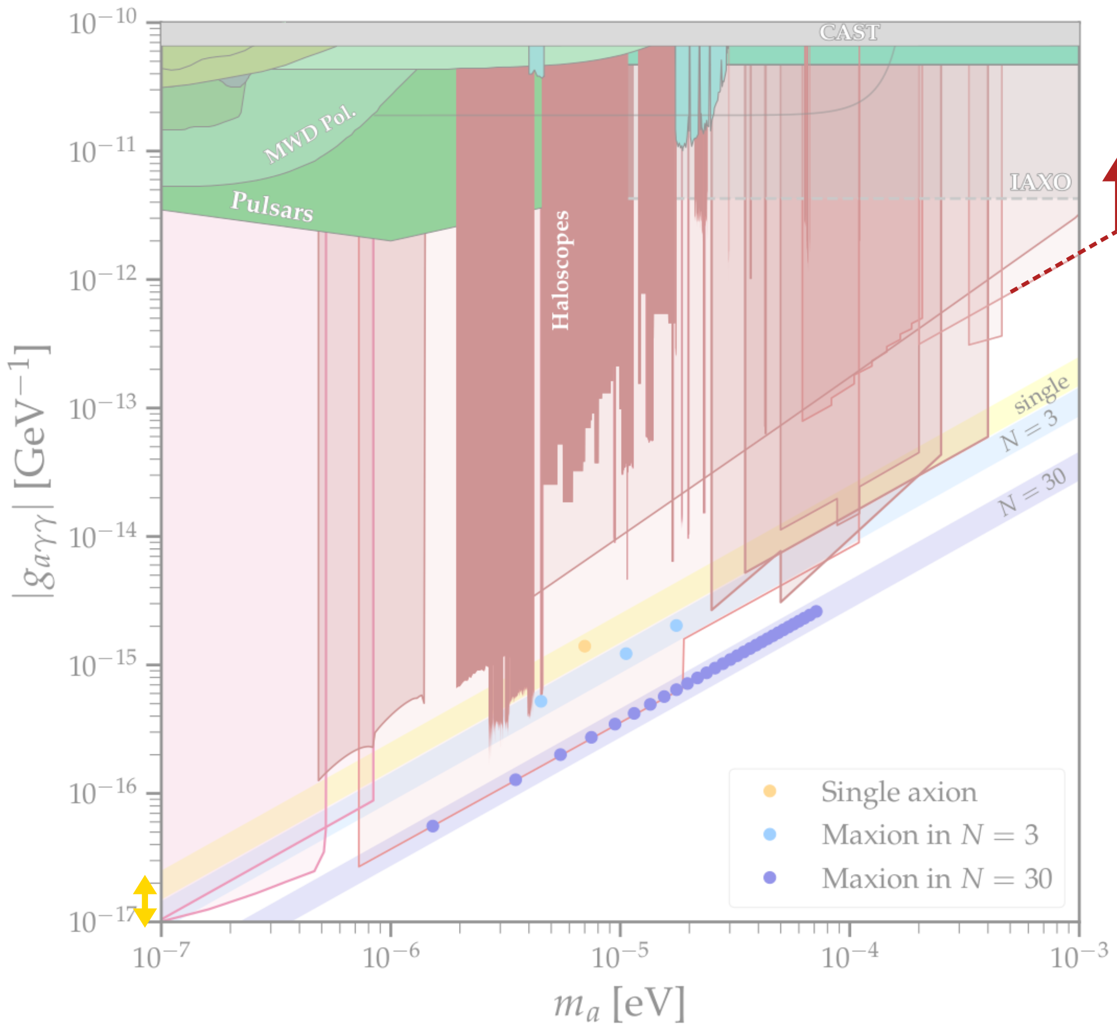
$$E/\mathcal{N} \in [2/3, 8/3]$$

Coupling to photons

Multiplicity of signals might be the smoking gun



Coupling to photons



$\sqrt{N}$ *if* $\rho_i = \rho_{\rm DM}/N$

How does each scalar contribute to the **dark matter** abundance?

Chala, Guedes, Santiago, MR, 2012.09017
Machado, Das Bakshi, MR, 2306.08036

Revisiting the axion/ALP EFT:

II. Can operator mixing impact our predictions?

The minimal basis

$$L_4 = L_{\rm SM} + \frac{1}{2}(\partial_\mu s)(\partial^\mu s) - \frac{1}{2}m_s^2 s^2 - \frac{\kappa_s}{3!}s^3 - \frac{\lambda_s}{4!}s^4 - \kappa_{s\phi}s\phi^\dagger\phi - \frac{\lambda_{s\phi}}{2}s^2\phi^\dagger\phi$$

$$L_5 = \mathcal{L}_{\rm SM+s} + s\left[i\,\overline{q_L}a_{su\phi}\widetilde{\phi}u_R + i\,\overline{q_L}a_{sd\phi}\phi d_R + i\,\overline{l_L}a_{se\phi}\phi e_R + \text{h.c.}\right] + a_{s^5}s^5$$
$$+ a_{s^3}s^3(\phi^\dagger\phi) + a_s s(\phi^\dagger\phi)^2 + a_{s\widetilde{G}}sG^A_{\mu\nu}\widetilde{G}^{A\mu\nu} + a_{s\widetilde{W}}sW^a_{\mu\nu}\widetilde{W}^{a\mu\nu}$$
$$+ a_{s\widetilde{B}}sB_{\mu\nu}\widetilde{B}^{\mu\nu} + a_{sG}sG^A_{\mu\nu}G^{A\mu\nu} + a_{sW}sW^a_{\mu\nu}W^{a\mu\nu} + a_{sB}sB_{\mu\nu}B^{\mu\nu}$$

CP-odd

To quantify the stability of UV scenarios:

$$16\,\pi^2\mu\frac{\mathrm{d}a_i}{\mathrm{d}\mu} = \gamma^{(1)}_{ij}a_j\,, \qquad 16\pi^2\mu\frac{\mathrm{d}\kappa_i}{\mathrm{d}\mu} = \gamma^{(2)}_{ij}\kappa_j + \gamma^{(3)}_{ij}m^2a_j$$

Comparison with axion basis

$$L_{\rm ALP} = \frac{1}{2}(\partial_\mu s)^2 + \sum_\psi \frac{\partial_\mu s}{f_s}\overline{\psi} c_\psi \gamma^\mu \psi + \sum_X c_X \frac{g_X^2}{16\pi^2}\frac{s}{f_s} F_{\mu\nu}\widetilde{F}^{\mu\nu}$$

periodicity: $c_X \in \mathbb{Z}$ Fraser, Reece 22

$$L_5 \supset s\left[i\,\overline{q_L} a_{su\phi}\widetilde{\phi} u_R + i\,\overline{q_L} a_{sd\phi}\phi d_R + i\,\overline{l_L} a_{se\phi}\phi e_R + \text{h.c.}\right]$$

Shift symmetry (perturbative level)

$$a_{su\phi} = \frac{\mathrm{i}}{f_s}\left(y^u c_u - c_q y^u\right)\,, \quad a_{sd\phi} = \frac{\mathrm{i}}{f_s}\left(y^d c_d - c_q y^d\right)\,, \quad a_{se\phi} = \frac{\mathrm{i}}{f_s}\left(y^e c_e - c_l y^e\right)$$

See also Bonilla, Brivio, Gavela, Sanz 21 and Bauer, Neubert, Renner, Schnubel, Thamm 21

Shift-symmetry invariants → Jonathan's talk

$$I_e^{(1)} = \mathrm{Re}\,\mathrm{Tr}\left(a_{se\phi} y_e^\dagger\right)\,, \quad I_e^{(2)} = \mathrm{Re}\,\mathrm{Tr}\left(x_e a_{se\phi} y_e^\dagger\right)\,, \quad I_e^{(3)} = \mathrm{Re}\,\mathrm{Tr}\left(x_e^2 a_{se\phi} y_e^\dagger\right)$$

Bonnefoy, Grojean, Kley 22

Renormalization results

	s^5	$s^3\phi^\dagger\phi$	$s(\phi^\dagger\phi)^2$	$s\overline{\Psi_L}\phi\psi_R$	sXX	$sX\widetilde{X}$
s^5	λ_s	$\lambda_{s\phi}$	0	0	0	0
$s^3\phi^\dagger\phi$	$\lambda_{s\phi}$	$\lambda_s + \lambda_{s\phi} + \lambda + y_t^2$	$\lambda_{s\phi}$	$\lambda_{s\phi}y_t$	$\lambda_{s\phi}g_2^2$	0
$s(\phi^\dagger\phi)^2$	0	$\lambda_{s\phi}$	$\lambda_{s\phi} + \lambda + y_t^2$	$y_t^3 + \lambda y_t$	λg_2^2	0
$s\overline{\Psi_L}\phi\psi_R$	0	0	0	$\lambda_{s\phi} + y_t^2$	$g_3^2 y_t$	$g_3^2 y_t$
sXX	0	0	0	0	g_3^2	0
$sX\widetilde{X}$	0	0	0	0	0	g_3^2

trivial zeros

Renormalization results

	s^5	$s^3\phi^\dagger\phi$	$s(\phi^\dagger\phi)^2$	$s\overline{\Psi_L}\phi\psi_R$	sXX	$sX\widetilde{X}$
s^5	λ_s	$\lambda_{s\phi}$	0	0	0	0
$s^3\phi^\dagger\phi$	$\lambda_{s\phi}$	$\lambda_s + \lambda_{s\phi} + \lambda + y_t^2$	$\lambda_{s\phi}$	$\lambda_{s\phi}y_t$	$\lambda_{s\phi}g_2^2$	0
$s(\phi^\dagger\phi)^2$	0	$\lambda_{s\phi}$	$\lambda_{s\phi} + \lambda + y_t^2$	$y_t^3 + \lambda y_t$	λg_2^2	0
$s\overline{\Psi_L}\phi\psi_R$	0	0	0	$\lambda_{s\phi} + y_t^2$	$g_3^2 y_t$	$g_3^2 y_t$
sXX	0	0	0	0	g_3^2	0
$sX\widetilde{X}$	0	0	0	0	0	g_3^2

Only due to WFR
(not required by quantization argument)

Renormalization results

	s^5	$s^3\phi^\dagger\phi$	$s(\phi^\dagger\phi)^2$	$s\overline{\Psi_L}\phi\psi_R$	sXX	$sX\widetilde{X}$
s^5	λ_s	$\lambda_{s\phi}$	0	0	0	0
$s^3\phi^\dagger\phi$	$\lambda_{s\phi}$	$\lambda_s + \lambda_{s\phi} + \lambda + y_t^2$	$\lambda_{s\phi}$	$\lambda_{s\phi}y_t$	$\lambda_{s\phi}g_2^2$	0
$s(\phi^\dagger\phi)^2$	0	$\lambda_{s\phi}$	$\lambda_{s\phi} + \lambda + y_t^2$	$y_t^3 + \lambda y_t$	λg_2^2	0
$s\overline{\Psi_L}\phi\psi_R$	0	0	0	$\lambda_{s\phi} + y_t^2$	$g_3^2 y_t$	$g_3^2 y_t$
sXX	0	0	0	0	g_3^2	0
$sX\widetilde{X}$	0	0	0	0	0	g_3^2

Large deviations from NDA

Renormalization results

	s^5	$s^3(\phi^\dagger\phi)$	$s(\phi^\dagger\phi)^2$	$s\overline{\Psi_L}\phi\psi_R$	sXX	$sX\widetilde{X}$
s^3	m_s^2	μ^2	0	0	0	0
$s(\phi^\dagger\phi)$	0	m_s^2	μ^2	$y_t\mu^2$	$g_2^2\mu^2$	0
s^4	κ_s	$\kappa_{s\phi}$	0	0	0	0
$s^2(\phi^\dagger\phi)$	0	$\kappa_s + \kappa_{s\phi}$	$\kappa_{s\phi}$	$y_t\kappa_{s\phi}$	$g_2^2\kappa_{s\phi}$	0
$(\phi^\dagger\phi)^2$	0	0	$\kappa_{s\phi}$	0	0	0

CP-odd effect: mixing between different mass dimensions

Matching at the EW scale

$$\begin{pmatrix} s \\ h \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \hat{s} \\ \hat{h} \end{pmatrix}$$

$$\tan 2\theta = \frac{-2a_s v^3 + 40 a_{s^5} v_s^4/v - \frac{4}{3} v_s/v \left(6m_s^2 + 3\kappa_s v_s + \lambda_s v_s^2\right)}{\left(4a_s - 6a_{s^3}\right) v^2 v_s - 40 a_{s^5} v_s^3 + 2m_s^2 + \left(-4\lambda + \lambda_{s\phi}\right) v^2 + v_s \left(2\kappa_s + \lambda_s v_s\right)}$$

$v_s = 0 \implies \mu^2 = \lambda v^2 \not\Longrightarrow \theta = 0$ **@ non-renormalizable level**

Other effective effects:

$$\hat{\lambda}_{s^2h^2} v - \hat{\kappa}_{s^2h} = \theta \left[\hat{\kappa}_s + 4v^2 \left(6\hat{a}_{s^3h^2} - 7\hat{a}_{sh^4}\right) \right]$$

$$\frac{1}{2}\hat{\lambda}_h v - \hat{\kappa}_h = -\frac{3}{2}\frac{\hat{m}_h^2}{v} + 24\theta \hat{a}_{sh^4} v^2$$

The low-energy EFT

$$L_{\rm LEFT} = \frac{1}{2}(\partial_\mu s)(\partial^\mu s) - \frac{1}{2}\tilde{m}_s^2 s^2 - \frac{\tilde{\kappa}_s}{3!}s^3 - \frac{\tilde{\lambda}_s}{4!}s^4 - \frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} - \frac{1}{4}A_{\mu\nu}A^{\mu\nu} + \tilde{\theta}_{\rm QCD}G^A_{\mu\nu}\widetilde{G}^{A\mu\nu}$$
$$+ \sum_{\psi=u,d,e}\left[\overline{\psi} i \not{D}\psi - \overline{\psi_L}\tilde{m}_\psi \psi_R + i\, s\overline{\psi_L}\tilde{c}_\psi\psi_R + s^2\overline{\psi_L}\tilde{a}_\psi\psi_R + \text{h.c.}\right] + \tilde{a}_{s^5}s^5$$
$$+ \tilde{a}_{sA}sA_{\mu\nu}A^{\mu\nu} + \tilde{a}_{sG}sG^A_{\mu\nu}G^{A\mu\nu} + \tilde{a}_{s\tilde{A}}sA_{\mu\nu}\widetilde{A}^{\mu\nu} + \tilde{a}_{s\tilde{G}}sG^A_{\mu\nu}\widetilde{G}^{A\mu\nu}$$
$$+ \sum_{\psi=u,d,e}\left[\overline{\psi_L}\tilde{a}_{\psi A}\sigma^{\mu\nu}\psi_R A_{\mu\nu} + \overline{\psi_L}\tilde{a}_{\psi G}\sigma^{\mu\nu}T_A\psi_R G^A_{\mu\nu} + \text{h.c.}\right]$$

$$\tilde{a}_\psi \sim \lambda_{s\phi}\frac{y^\psi}{v} \sim \lambda_{s\phi}\frac{m_\psi}{v^2}$$

e.g. $\tilde{c}_\psi = \frac{1}{\sqrt{2}}V^\dagger_{\psi_L}\left[i\,\theta y_\psi + a_{s\psi\phi}\,(v+\theta v_s)\right]V_{\psi_R}$

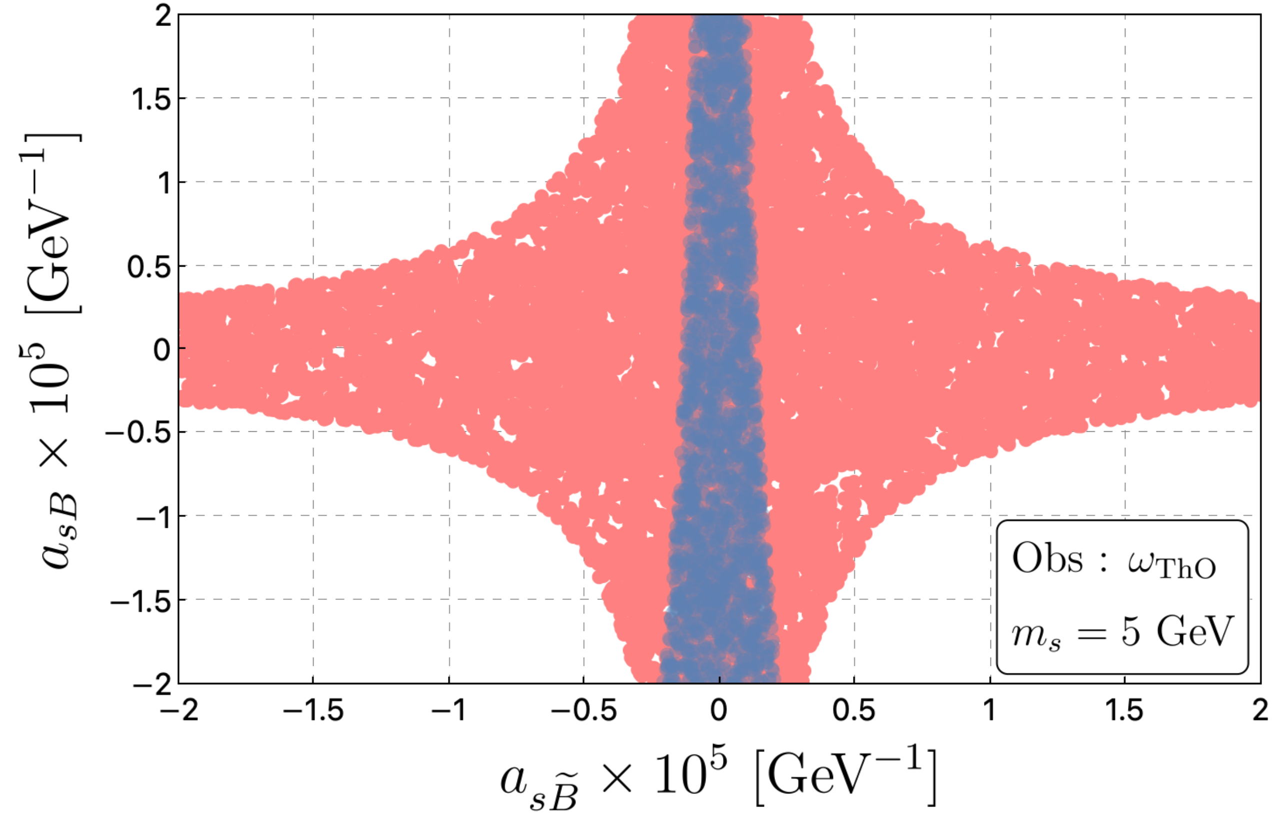
Then we run.

$$\beta_{\tilde{a}_{s\tilde{G}}} = \left(-\frac{46}{3}\tilde{g}_3^2 + 2\text{Tr}\left[\tilde{c}_e\tilde{c}_e{}^\dagger + 3\tilde{c}_d\tilde{c}_d{}^\dagger + 3\tilde{c}_u\tilde{c}_u{}^\dagger\right]\right)\tilde{a}_{s\tilde{G}}$$
$$- 2g_3\text{Tr}\left[\tilde{a}_{dG}\tilde{c}_d{}^\dagger + \tilde{a}_{uG}\tilde{c}_u{}^\dagger + \text{h.c.}\right]$$

$$\beta_{\tilde{c}_e} = -6\tilde{e}^2\tilde{c}_e - 24\tilde{e}^2\tilde{m}_e\tilde{a}_{s\tilde{A}} + 24i\tilde{e}^2\tilde{m}_e\tilde{a}_{sA} + 2\text{Tr}\left[\tilde{c}_e\tilde{c}_e^\dagger + 3\left(\tilde{c}_u\tilde{c}_u^\dagger + \tilde{c}_d\tilde{c}_d^\dagger\right)\right]\tilde{c}_e$$
$$+ 3\tilde{c}_e\tilde{c}_e^\dagger\tilde{c}_e + 2\text{Tr}\left[\tilde{m}_e\tilde{c}_e^\dagger\tilde{a}_e - 2\tilde{a}_e\tilde{m}_e^\dagger\tilde{c}_e + \tilde{a}_e\tilde{c}_e^\dagger\tilde{m}_e - 2\tilde{c}_e\tilde{m}_e^\dagger\tilde{a}_e\right]$$
$$- 2i\tilde{\kappa}_s\tilde{a}_e - 12\tilde{e}^2\left(\tilde{m}_e\tilde{c}_e^\dagger\tilde{a}_{eA} - \tilde{a}_{eA}\tilde{m}_e^\dagger\tilde{c}_e + \tilde{a}_{eA}\tilde{c}_e^\dagger\tilde{m}_e - \tilde{c}_e\tilde{m}_e^\dagger\tilde{a}_{eA}\right)$$

Phenomenological implications

Impact of scalar mixing: $\kappa_{s\phi} \sim 0.1 \to \theta \sim \mathcal{O}(10^{-4})$

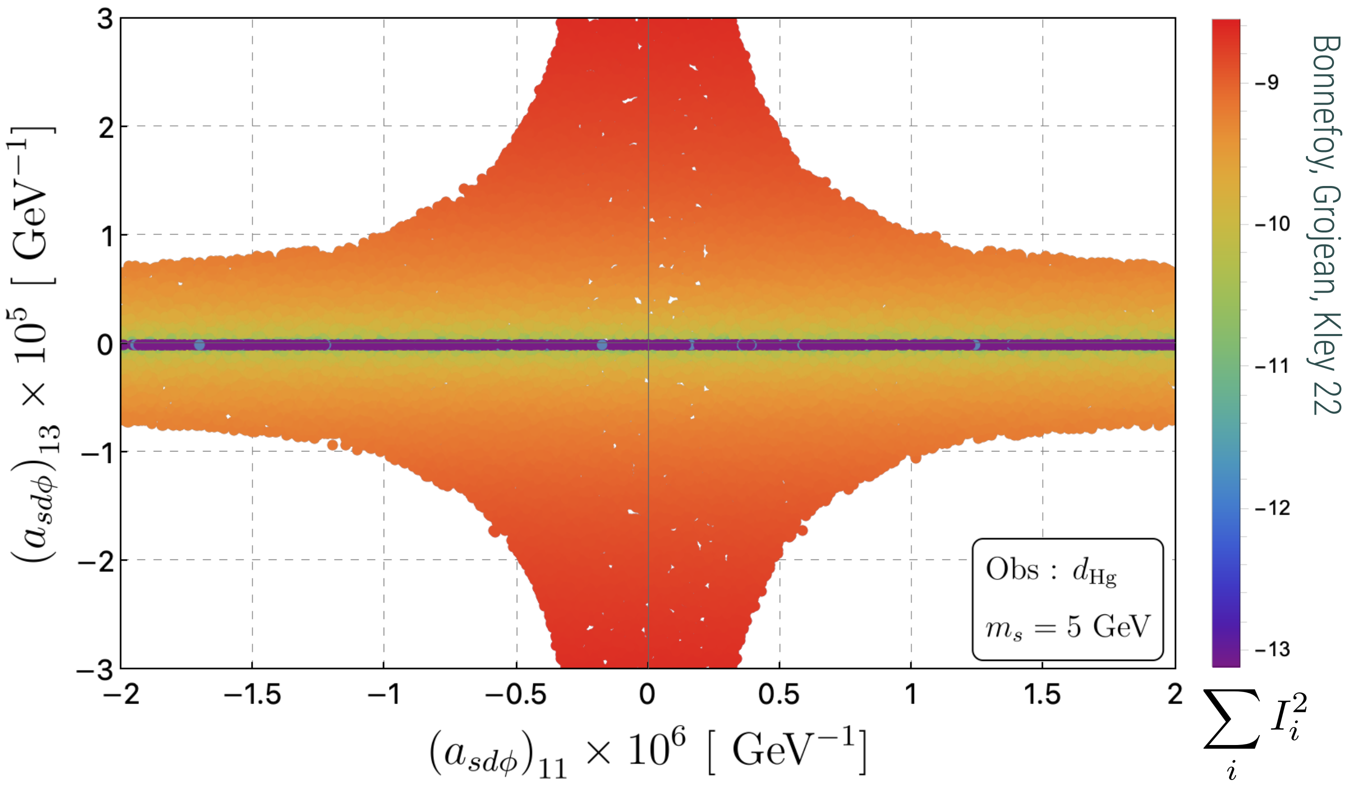


From △ diagram with: $c_{se} \propto i\theta y_e + a_{se\phi}(v + \theta v_s)$

EDM expressions in Di Luzio, Grober, Paradisi 10

Phenomenological implications

Assuming only Re couplings in the UV



Phenomenological implications

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{2}\partial_\mu s \partial^\mu s + \frac{1}{2}\tilde{m}^2 s^2 + \frac{a_{s\widetilde{Z}}}{c_\omega^2 - s_\omega^2} s \left(c_\omega^2 W_{\mu\nu}\widetilde{W}^{\mu\nu} - s_\omega^2 B_{\mu\nu}\widetilde{B}^{\mu\nu} \right)$$

Direct constraints from mono-*Z*: $a_{s\tilde{Z}} < 0.2\ (0.04)\ \mathrm{TeV}^{-1}$

Brivio, Gavela, Merlo, Mimasu, No, Rey, Sanz 17

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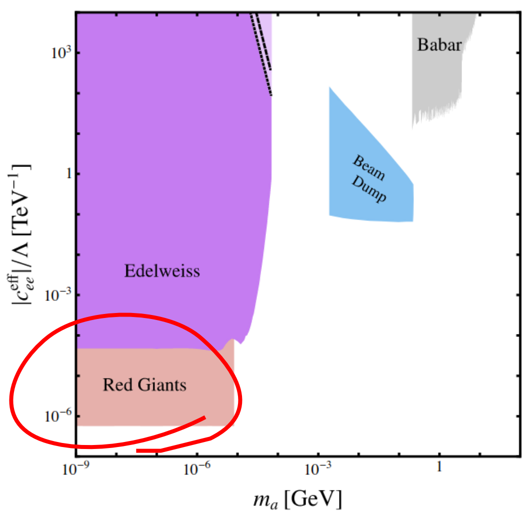
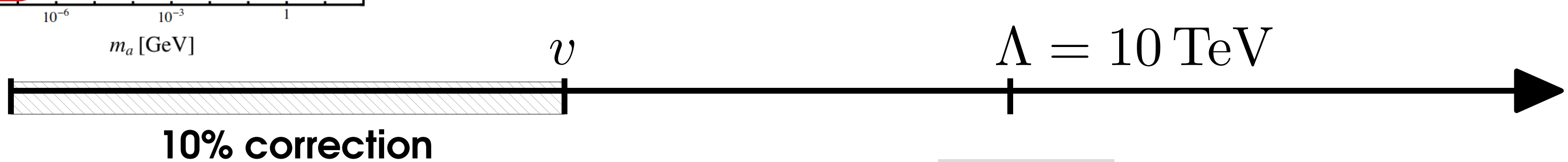
Brivio, Gavela, Merlo, Mimasu, No, Rey, Sanz 17

Bauer, Neubert, Thamm 17



$$\beta_{a_{se\phi}} = 2\Big[a_{se\phi}\Big(\lambda_{s\phi} - \frac{15g_1^2}{8} - \frac{9g_2^2}{8} + \frac{1}{2}\gamma_\phi^{(Y)}\Big) + \frac{5}{4}y^e y^{e\dagger} a_{se\phi} + a_{se\phi} y^{e\dagger} y^e - \Big(\frac{15g_1^2}{2} a_{s\widetilde{B}} + \frac{9g_2^2}{2} a_{s\widetilde{W}} - \mathrm{Tr}\left[y^e a_{se\phi}^{\mathsf{T}} + 3y^d a_{sd\phi}^{\mathsf{T}} - 3a_{su\phi}y^{u\dagger})\right]\Big) y^e\Big]$$



Other cases in Bonilla, Brivio, Gavela, Sanz 21

Phenomenological implications

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{2}\partial_\mu s \partial^\mu s + \frac{1}{2}\tilde{m}^2 s^2 + \frac{a_{s\tilde{Z}}}{c_\omega^2 - s_\omega^2} s \left(c_\omega^2 W_{\mu\nu}\widetilde{W}^{\mu\nu} - s_\omega^2 B_{\mu\nu}\widetilde{B}^{\mu\nu} \right)$$

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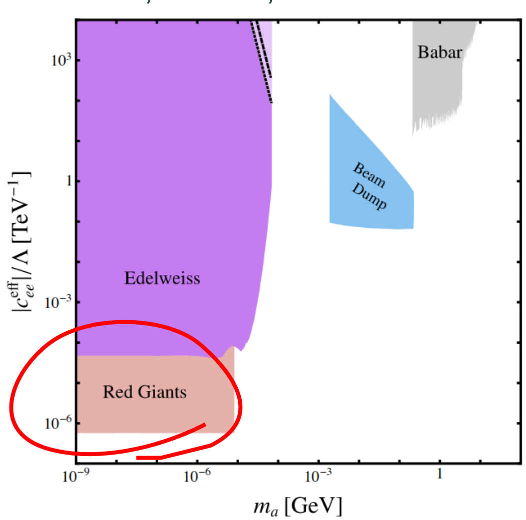
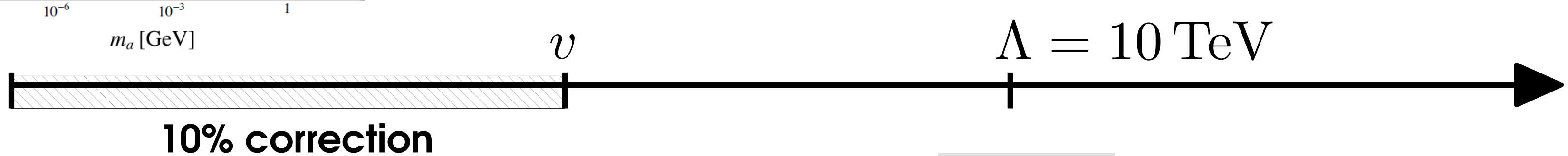


$$\beta_{a_{se\phi}} = 2\Bigg[a_{se\phi}\left(\lambda_{s\phi} - \frac{15 g_1^2}{8} - \frac{9 g_2^2}{8} + \frac{1}{2}\gamma_\phi^{(Y)}\right) + \frac{5}{4} y^e y^{e\dagger} a_{se\phi} + a_{se\phi} y^{e\dagger} y^e$$
$$- \left(\frac{15 g_1^2}{2} a_{s\widetilde{B}} + \frac{9 g_2^2}{2} a_{s\widetilde{W}} - \text{Tr}\left[y^e a_{se\phi}^{\mathsf{T}} + 3 y^d a_{sd\phi}^{\mathsf{T}} - 3 a_{su\phi} y^{u\dagger}) \right] \right) y^e \Bigg]$$

$$a_{s\tilde{Z}} < 4.8 \times 10^{-6}\ \text{TeV}^{-1}$$



Other cases in Bonilla, Brivio, Gavela, Sanz 21

Conclusions

I. Scalar mixing effects: The PQ mechanism leads in all generality to multiple axion signals, which are linked by an exact sum rule. The maximum deviation of *N* axions is $\sqrt{N}$. The main experimental impact is from scales not far from the QCD contribution.

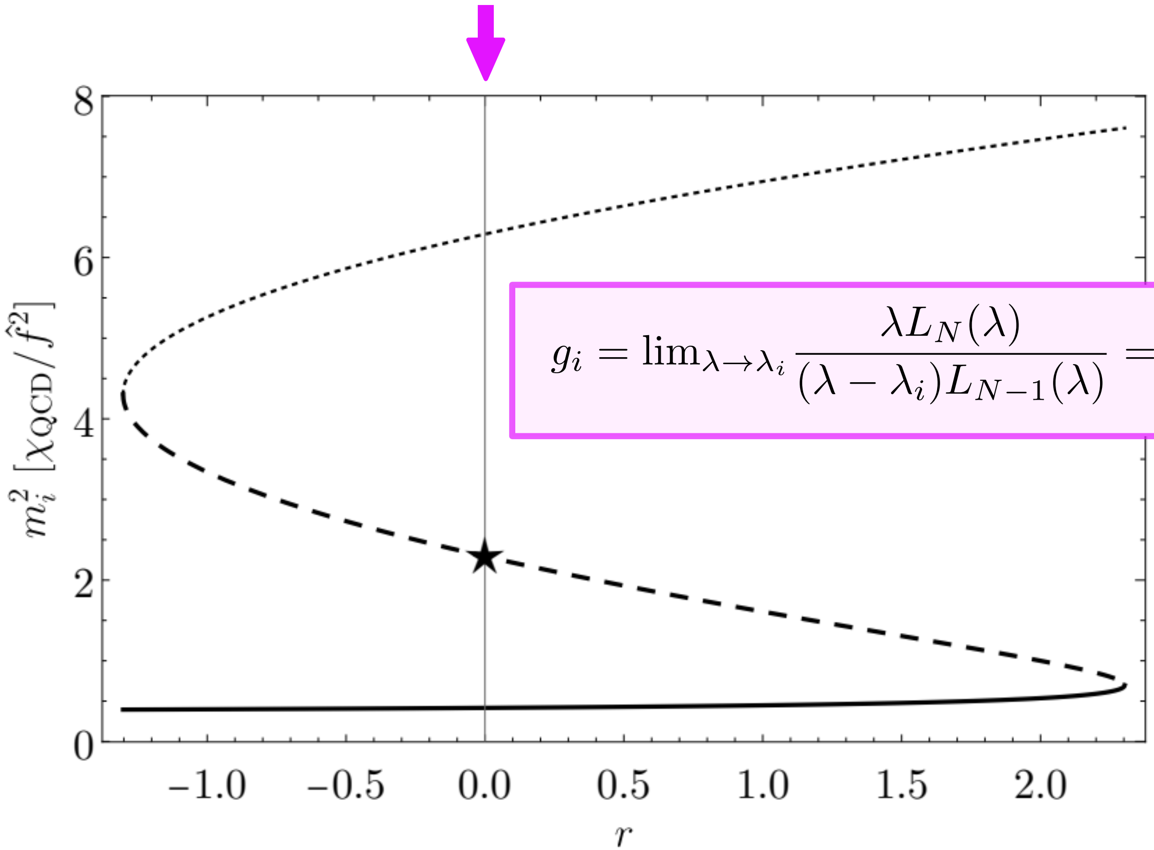
II. Operator mixing effects: The full RGE effects should be taken into account to correctly interpret low-energy bounds in terms of Wilson coefficients generated in the UV. Shift-breaking interactions typically source sizable mixings and open novel signatures for ALPs. For example, new CP-odd phases can be produced, which are absent in more shift-symmetric scenarios.

Thanks!

backup

Laguerre maxions

$$\hat{\mathbf{M}}^2 = \frac{\chi_{\rm QCD}}{\hat{f}^2}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 4-\sqrt{3+r-r^2} & 1+r \\ 1 & 1+r & 4+\sqrt{3+r-r^2} \end{pmatrix}$$



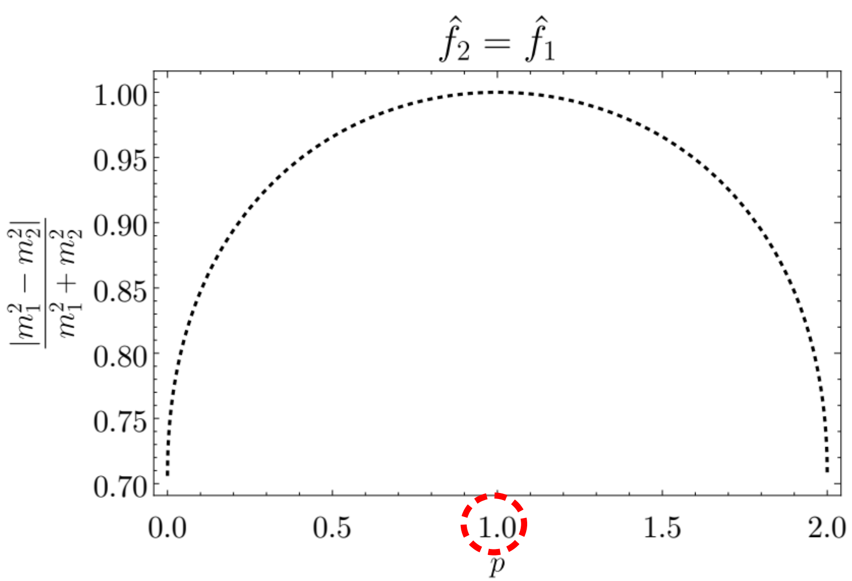
$$g_i = \lim_{\lambda\to\lambda_i} \frac{\lambda L_N(\lambda)}{(\lambda-\lambda_i)L_{N-1}(\lambda)} = N$$

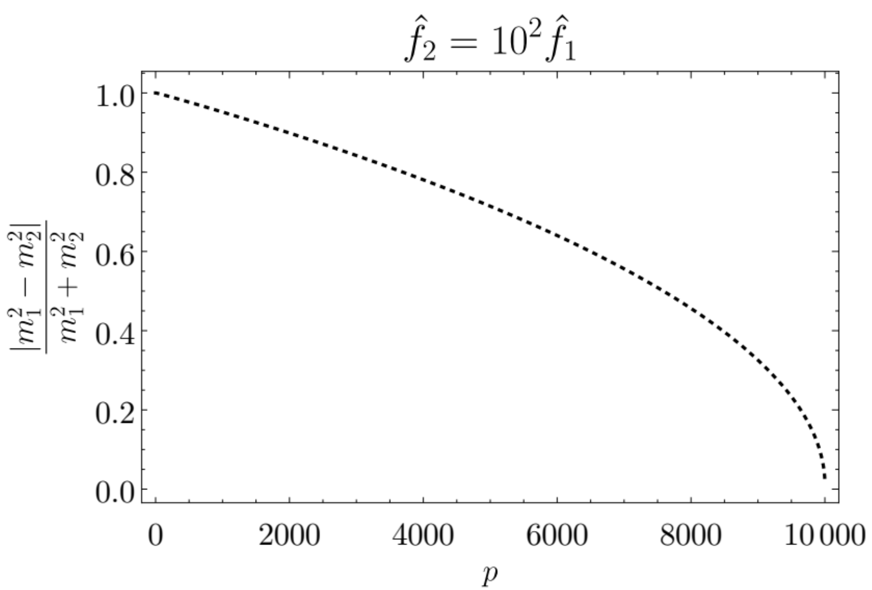
backup

Eigenvalues dispersion

All families of maxions (with same scale) for N=2:

$$\mathbf{M}^2_{N=2} = \frac{\chi_{\rm QCD}}{\hat{f}^2}\begin{pmatrix} 2-p & 1+\sqrt{p(2-p)} \\ 1+\sqrt{p(2-p)} & 1+p \end{pmatrix}$$





Limiting case: Massless state has no mixing with gluons, the heavy one with mass $\sim 4\frac{\chi_{\rm QCD}}{\hat{f}^2}$

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One UV completion

$$\mathcal{L}_{\rm UV} = |\partial_\mu S_1|^2 + |\partial_\mu S_2|^2 + \overline{\Psi}_1 {\rm i} D\Psi_1 + \overline{\Psi}_2 {\rm i} D\Psi_2 - \left[y_1 \overline{\Psi}_1 \Psi_1 S_1 + y_2 \overline{\Psi}_2 \Psi_2 S_2 + {\rm h.c.}\right] - V(S_{1,2})$$

$$U(1)_{\rm PQ}: \quad \Psi_{j,L} \to e^{{\rm i}\alpha_j/2}\Psi_{j,L}\,,\ \Psi_{j,R} \to e^{-{\rm i}\alpha_j/2}\Psi_{j,R}\,,\ S_j \to e^{{\rm i}\alpha_j} S$$

After SSB, $$S_{1,2} = \frac{1}{\sqrt{2}}\left(\hat{f}_{1,2} + \rho_{1,2}\right) e^{{\rm i}\hat{a}_{1,2}/\hat{f}_{1,2}}\,.$$

$V(S_{1,2}) = \lambda S_1^3 S_2 + {\rm h.c.}$ reduces the symmetry to just one PQ

After QCD confinement,

$$V_{\rm eff} = \frac{1}{2}\chi_{\rm QCD}\left(\frac{\hat{a}_1 + \hat{a}_2}{\hat{f}} - \bar{\theta}\right)^2 + \frac{\lambda}{4}\hat{f}^4\left(\frac{3\hat{a}_1 + \hat{a}_2}{\hat{f}}\right)^2$$

$r = \lambda\frac{\hat{f}^4}{2\chi_{\rm QCD}}$

$$\mathbf{M}^2 = \frac{\chi_{\rm QCD}}{\hat{f}^2}\begin{pmatrix} 2+8r & -4r \\ -4r & 2r \end{pmatrix}, \quad \text{with } 1/F^2 = 2/\hat{f}^2$$

which contains maxion solutions ($r = 1/5$).

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Clockwork scenario

Farina, Pappadopulo, Rompineve, Tesi 17

$$\hat{\mathbf{M}}^2 = \frac{\chi_{\rm QCD}}{\hat{f}^2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + r\frac{\chi_{\rm QCD}}{\hat{f}^2}\begin{pmatrix} 1 & -q & 0 \\ -q & 1+q^2 & -q \\ 0 & -q & q^2 \end{pmatrix}$$

Correspondingly, $v_{j0} \propto \frac{1}{q^j}$ *leads* to decay constant exponentially enhanced

PQ: $\frac{\det \mathbf{M}^2}{\det \mathbf{M}_1^2} = \frac{\chi_{\rm QCD}}{F^2}$ ✓

Maxions:
$$\begin{cases} \mathrm{tr}\,\mathbf{M}^2 = N\frac{\chi_{\rm QCD}}{F^2} \Leftrightarrow r = \frac{1}{10} \\ \mathrm{tr}^2\,\mathbf{M}^2 - \mathrm{tr}\,\mathbf{M}^2\cdot\mathbf{M}^2 = N\frac{\chi_{\rm QCD}}{F^2}\mathrm{tr}\,\mathbf{M}_1^2 \Leftrightarrow r = 0 \vee r = \frac{11}{182} \end{cases}$$
✗

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Potential scales

In the basis where the extra potential is diagonal, $\mathbf{M}_B^2 = \mathrm{diag}(\tilde{\lambda}_1, \ldots, \tilde{\lambda}_N)$

$$g_i = \frac{m_i^2\, F^2}{\left|\langle a_{G\tilde{G}}|a_i\rangle\right|^2 \chi_{\rm QCD}} = \frac{m_i^2}{\left|\langle a_{\rm PQ}|a_i\rangle/f_{\rm PQ} + \sum_j^{N-1}\langle \tilde{a}_j|a_i\rangle/\tilde{f}_j\right|^2 \chi_{\rm QCD}}$$

For $\tilde{\lambda}_j \gg \chi_{\rm QCD}/F^2$:

$$\frac{1}{g_j} \sim \frac{\left|\langle a_{G\tilde{G}}|\tilde{a}_j\rangle\right|^2 \chi_{\rm QCD}}{\tilde{\lambda}_j\, F^2} = \frac{(F/\tilde{f}_j)^2\, \chi_{\rm QCD}}{\tilde{\lambda}_j\, F^2} \leq \frac{\chi_{\rm QCD}}{\tilde{\lambda}_j\, F^2} \longrightarrow 0$$

For $\tilde{\lambda}_j \ll \chi_{\rm QCD}/F^2$:

$$a_\varepsilon = \frac{a_{\rm PQ}}{f_{\rm PQ}} - \frac{\tilde{a}_j}{\tilde{f}_j} + \mathcal{O}(\varepsilon), \quad m_\epsilon^2 \sim \tilde{\lambda}_j = \varepsilon\, \chi_{\rm QCD}/F^2$$

$$\frac{1}{g_j} \sim \frac{\left|\langle a_{G\tilde{G}}|\tilde{a}_\varepsilon\rangle\right|^2 \chi_{\rm QCD}}{\tilde{\lambda}_j\, F^2} \sim \frac{\varepsilon^2}{\varepsilon} \longrightarrow 0$$

Whenever one scale is very different from the QCD induced mass, one state decouples.

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Mixing effects in DM abundance

$$\mathcal{L} \supset \frac{1}{2}(\partial\phi_a)^2 + \frac{1}{2}(\partial\phi_S)^2 - m_a^2(T) f_a^2 \left[1 - \cos\left(\frac{\phi_a}{f_a} + \frac{\phi_S}{f_S}\right)\right] - m_S^2 f_S^2 \left[1 - \cos\left(\frac{\phi_S}{f_S}\right)\right]$$

$$m_a^2(T) = m_{a,0}^2 \max\left\{1, \left(\frac{T}{T_{\rm QCD}}\right)^{-n}\right\}$$

$$V \approx \begin{pmatrix} \phi_a & \phi_S \end{pmatrix} \begin{pmatrix} m_a^2 & \frac{f_a}{f_S} m_a^2 \\ \frac{f_a}{f_S} m_a^2 & m_S^2 + \frac{f_a^2}{f_S^2} m_a^2 \end{pmatrix} \begin{pmatrix} \phi_a \\ \phi_S \end{pmatrix}$$

Would typically dominate the late-time energy density

Assume $f_s \gg f_a$:

At early times,

$$m_a(T) \ll m_S \to \phi_H \sim \phi_S \,, \;\; \phi_L \sim \phi_a$$

At late times,

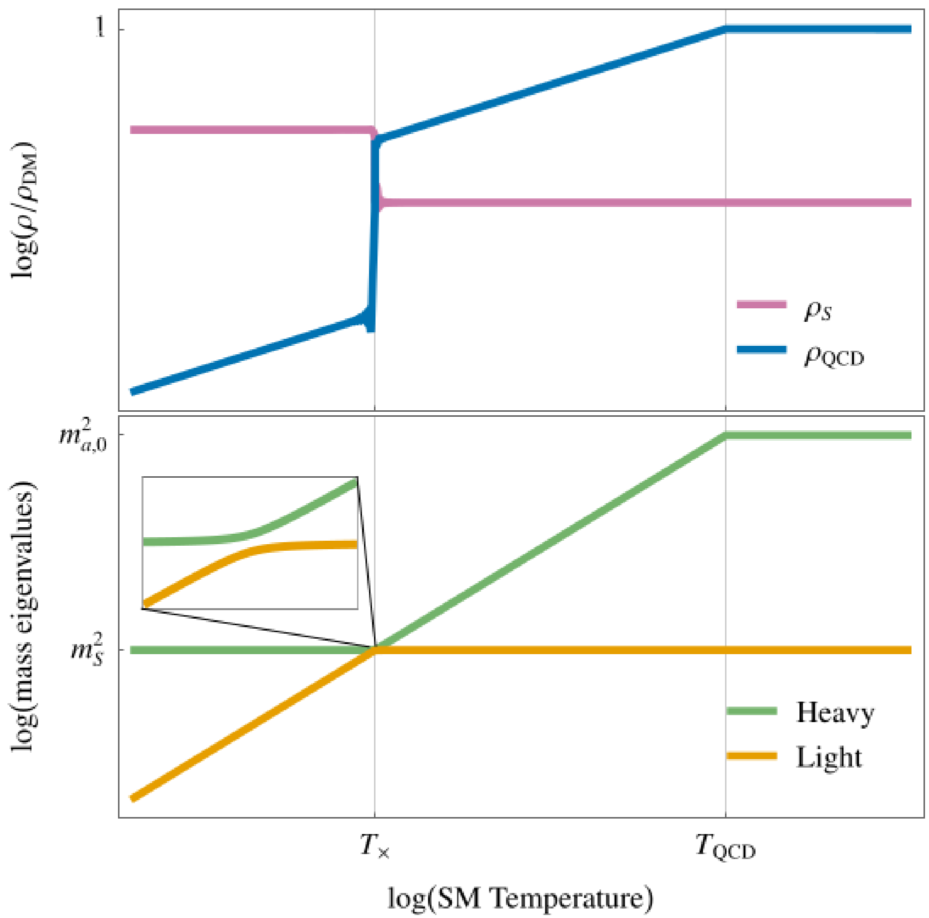
$$m_a(T) \gg m_S \to \phi_H \sim \phi_a \,, \;\; \phi_L \sim \phi_S$$

so energy is transferred into the QCD axion...

Cyncynates, Thompson 23
[other examples from Takahashi et al]

backup

Mixing effects in DM abundance



Would typically dominate the late-time energy density

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At early times,

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Cyncynates, Thompson 23
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Off-shell renormalization

- Compute **one-loop divergent amplitudes** generated by 1P1 diagrams

  `Feynrules + FeynArts + FormCalc + Matchmakereft`

- Match to the **Green's basis**
- Project onto the minimal basis, using field redefinitions

Scalar	Yukawa	Gauge	Derivative
$\mathcal{O}_{s^5} = s^5$	$\mathcal{O}_{su\phi} = is\overline{q_L}\tilde{\phi}u_R$	$\mathcal{O}_{s\widetilde{B}} = sB_{\mu\nu}\widetilde{B}^{\mu\nu}$	$\mathcal{R}_{s\phi\Box} = is\phi^\dagger D^2\phi$
$\mathcal{O}_{s^3} = s^3(\phi^\dagger\phi)$	$\mathcal{O}_{sd\phi} = is\overline{q_L}\phi d_R$	$\mathcal{O}_{s\widetilde{W}} = sW_{\mu\nu}\widetilde{W}^{\mu\nu}$	$\mathcal{R}_{s\Box} = s^2\partial^2 s$
$\mathcal{O}_{s} = s(\phi^\dagger\phi)^2$	$\mathcal{O}_{se\phi} = is\overline{l_L}\phi e_R$	$\mathcal{O}_{s\widetilde{G}} = sG_{\mu\nu}\widetilde{G}^{\mu\nu}$	$\mathcal{R}_{\phi s\Box} = \phi^\dagger\phi\partial^2 s$
		$\mathcal{O}_{sB} = sB_{\mu\nu}B^{\mu\nu}$	$\mathcal{R}_{sq} = is\overline{q_L}\not{D}q_L$
		$\mathcal{O}_{sW} = sW_{\mu\nu}W^{\mu\nu}$	$\mathcal{R}_{sl} = is\overline{l_L}\not{D}l_L$
		$\mathcal{O}_{sG} = sG_{\mu\nu}G^{\mu\nu}$	$\mathcal{R}_{su} = is\overline{u_R}\not{D}u_R$
			$\mathcal{R}_{sd} = is\overline{d_R}\not{D}d_R$
			$\mathcal{R}_{se} = is\overline{e_R}\not{D}e_R$