Axion-like Particles as Mediators for Dark Matter: Beyond Freeze-out

In collaboration with Felix Brümmer, Nishita Desai

Based on A. Bharucha, F. Brümmer, N. Desai and S. Mutzel, [arXiv:2209.03932 [hep-ph]], to be published in JHEP

Aoife Bharucha and Sophie Mutzel

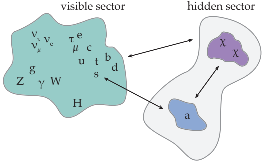
Centre de Physique Théorique Marseille

28th September 2023,
axions++

The Model

A new light pseudoscalar a might well be an axion-like particle (ALP), i.e. the pseudo-Goldstone boson of an approx. $\mathrm{U}(1)_{\mathrm{PQ}}$ global symmetry, spontaneously broken at a high scale f_a ($\Rightarrow$ light)

Axion-like particle (a) mediator between the SM fermions (f) and the DM (χ)a $\mathrm{U}(1)_{\mathrm{PQ}}$ charged Dirac fermion



$$\mathcal{L} \supset \frac{1}{2}\partial_\mu a \partial^\mu a + \bar{\chi}(i\partial\!\!\!/ - m_\chi)\chi - \frac{1}{2}m_a^2 a^2 + ia\sum_f \frac{m_f}{f_a} C_f \bar{f}\gamma_5 f + ia\frac{m_\chi}{f_a} C_\chi \bar{\chi}\gamma_5\chi + a\sum_f C_f \frac{y_f}{\sqrt{2}f_a} h\bar{f}i\gamma_5 f + \ldots$$

$g_{a\chi\chi} \equiv C_\chi/f_a$, $g_{aff} \equiv C_f/f_a$, no coupling to gauge bosons at tree-level but couple via loops

- a can emerge naturally from extended Higgs sector $\Rightarrow$ also expect dim-5 couplings
- If a was (DFSZ-like) QCD axion, multiple astrophysical and laboratory constraints (see [DiLuzio'20] for overview). (may be possible to circumvent these constraints by model-building)
- Therefore assume that the a mass is mainly due to some explicitly $\mathrm{U}(1)_{\mathrm{PQ}}$-breaking effect other than the anomaly (no m_a g_{aff} relation and evade constraints)

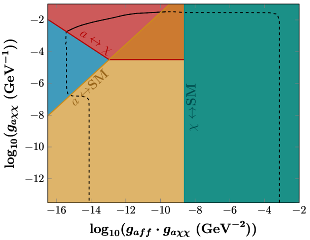
Dark matter generation beyond freeze-out

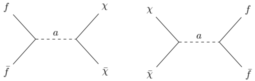
- Assuming initial abundance zero after reheating and DM and ALPs gradually produced by scattering processes in thermal plasma.
- We consider the following possibilities
  - Small $g_{a\chi\chi}$ and g_{aff}, DM never reaches thermal eq. with SM or ALPs $\Rightarrow$ freeze-in. Either directly via $f\bar{f} \to \chi\bar{\chi}$ or e.g. $f\bar{f} \to ag$ followed by $aa \to \chi\bar{\chi}$ (where the ALP may or may not be in equilibrium with the SM [Bélanger et al'20],) plus $2 \to 3$ scattering $f\bar{f} \to h\chi\bar{\chi}$
  - Intermediate g_{aff}, ALPs not in eq. with SM, but if $g_{a\chi\chi}$ sufficiently large, in eq. with DM $\Rightarrow$ freeze-out from a thermally decoupled dark sector, or to put it simply, decoupled freeze-out (DFO), see [Chu et al'12], and more recently [Hambye'19].
- Let's study how to solve the Boltmann equations in these different cases

Coupled Boltzmann equations

$$\langle \sigma_{aa\to\chi\bar\chi} v\rangle\, ({\color{red}T'})\, n_a^{\rm eq}({\color{red}T'}) \simeq H$$

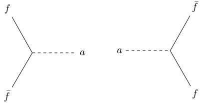
$$\langle \sigma_{ai\to jk} v\rangle\, n_i^{\rm eq} \simeq H$$

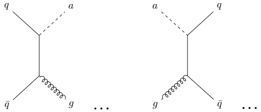




$$\frac{\mathrm{d}n_\chi}{\mathrm{d}t} + 3Hn_\chi = \sum_f \left\langle \sigma_{\chi\bar\chi\to f\bar f} v\right\rangle \left(\left(n_\chi^{\rm eq}(T)\right)^2 - n_\chi^2 \right) + \langle \sigma_{aa\to\chi\bar\chi} v\rangle\, (T^{(\prime)}) n_a^2 - \langle \sigma_{\chi\bar\chi\to aa} v\rangle\, (T^{(\prime)}) n_\chi^2$$

$$\frac{\mathrm{d}n_a}{\mathrm{d}t} + 3Hn_a = -\langle \sigma_{aa\to\chi\bar\chi} v\rangle\, (T^{(\prime)}) n_a^2 + \langle \sigma_{\chi\bar\chi\to aa} v\rangle\, (T^{(\prime)}) n_\chi^2 + \langle \Gamma_a\rangle \left(n_a^{\rm eq}(T) - n_a \right) + \sum_{i,j,k} \left\langle \sigma_{ai\to jk} v\right\rangle \left(n_a^{\rm eq}(T) n_i^{\rm eq}(T) - n_a n_i^{\rm eq}(T) \right)$$

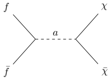




Coupled Boltzmann equations

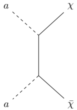
$$\langle \sigma_{aa\to\chi\bar\chi} v\rangle\,(T')\; n_a^{\mathrm{eq}}(T') \simeq H$$

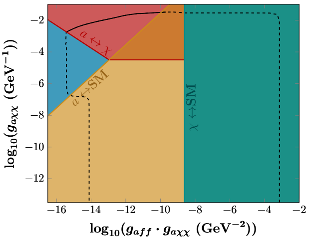
$$\langle \sigma_{ai\to jk} v\rangle\; n_i^{\mathrm{eq}} \simeq H$$



$$\frac{\mathrm{d}n_\chi}{\mathrm{d}t} + 3Hn_\chi = \sum_f \left\langle \sigma_{\chi\bar\chi\to f\bar f} v\right\rangle \overbrace{\left(\left(n_\chi^{\mathrm{eq}}(T)\right)^2\right.}$$

$$+ \underbrace{\langle \sigma_{aa\to\chi\bar\chi} v\rangle\,(T) n_a^2}$$

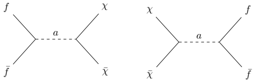




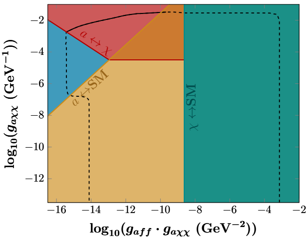
Coupled Boltzmann equations

$$\langle\sigma_{aa\to\chi\bar\chi}v\rangle\,(T')\,n_a^{\mathrm{eq}}(T')\simeq H$$

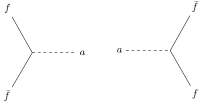
$$\langle\sigma_{ai\to jk}v\rangle\,n_i^{\mathrm{eq}}\simeq H$$

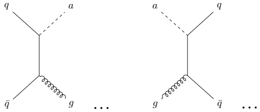


$$\frac{\mathrm{d}n_\chi}{\mathrm{d}t}+3Hn_\chi=\sum_f\left\langle\sigma_{\chi\bar\chi\to f\bar f}v\right\rangle\left(\left(n_\chi^{\mathrm{eq}}(T)\right)^2-n_\chi^2\right)+\langle\sigma_{aa\to\chi\bar\chi}v\rangle\,(T')n_a^2-\langle\sigma_{\chi\bar\chi\to aa}v\rangle\,(T')n_\chi^2$$



$$\frac{\mathrm{d}n_a}{\mathrm{d}t}+3Hn_a=-\langle\sigma_{aa\to\chi\bar\chi}v\rangle\,(T')n_a^2+\langle\sigma_{\chi\bar\chi\to aa}v\rangle\,(T')n_\chi^2+\langle\Gamma_a\rangle\left(n_a^{\mathrm{eq}}(T)-n_a\right)+\sum_{i,j,k}\langle\sigma_{ai\to jk}v\rangle\left(n_a^{\mathrm{eq}}(T)n_i^{\mathrm{eq}}(T)-n_an_i^{\mathrm{eq}}(T)\right)$$





Determining the temperature of the hidden sector-I

Solving the Energy transfer Boltzmann equation we can obtain the HS energy density ρ'. The temperature T' of HS calculated from ρ' via the equation of state:

$$\frac{\partial \rho'}{\partial t} + 3H\,(\rho' + P') = \int \frac{d^3p}{(2\pi)^3} C[f(p,t)] \qquad \text{using} \qquad P' = \tfrac{1}{3}\langle p \tfrac{\partial E}{\partial p}\rangle.$$

where $\rho' + P' = \rho_a + \rho_\chi + P_a + P_\chi$

In our model, complication as not $f\bar{f} \to aa$ (see [Chu et al'12]) but $f\bar{f} \to ag$, $gf \to af$ $+ f\bar{f} \to a$.

Initially for $T' > m_\chi, m_a$ ALPs and DM will be ultra-relativistic, $P' = \rho'/3$, and universe radiation-dominated $\rho \propto T^4$:

$$\frac{\partial \rho'}{\partial t} + 4H\,\rho' = -H\left(T\frac{\partial \rho'}{\partial T} - 4\rho'\right) = -HT\rho\frac{\partial}{\partial T}\left(\frac{\rho'}{\rho}\right) = \int \frac{d^3p}{(2\pi)^3} C[f(p,t)]\,.$$

where $\rho' = \rho_a^{\rm eq}(T') + \rho_{\rm DM}^{\rm eq}(T')$ and $P' = P_a^{\rm eq}(T') + P_{\rm DM}^{\rm eq}(T')$

$\Rightarrow$ Here the equation for T' can be solved independently

Determining the temperature of the hidden sector-II

For $T' \lesssim m_a, m_\chi$, HS particles become non-relativistic, and interactions will freeze out $\Rightarrow$ T' determined together with n_χ and n_a. [1]:

$$\rho_\chi = \frac{\rho_\chi^{\rm eq}(T')}{n_\chi^{\rm eq}(T')}\, n_\chi\,, \qquad P_\chi = \frac{P_\chi^{\rm eq}(T')}{n_\chi^{\rm eq}(T')}\, n_\chi = T' n_\chi\,,$$

and similarly for the ALP

The hidden sector equation of state is then given by

$$\rho' + P' = \frac{\rho_\chi^{\rm eq}(T')}{n_\chi^{\rm eq}(T')}\, n_\chi + \frac{\rho_a^{\rm eq}(T')}{n_a^{\rm eq}(T')}\, n_a + T'\left(n_\chi + n_a\right)\,.$$

$\Rightarrow$ solve three coupled differential equations:

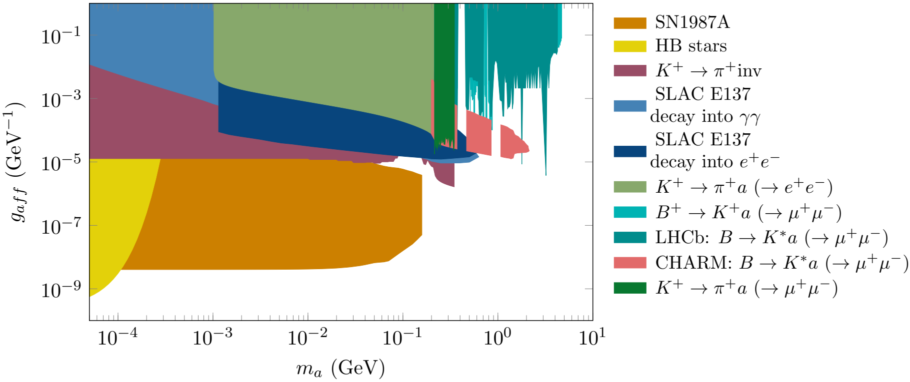
$$z\frac{{\rm d}\rho'}{{\rm d}T'}\frac{{\rm d}T'}{{\rm d}z} = -3(\rho' + P') + \frac{1}{H}\int \frac{d^3p}{(2\pi)^3} C[f(p,t)]$$

$$Hz\frac{{\rm d}n_\chi}{{\rm d}z} + 3Hn_\chi = \sum_f \left\langle \sigma_{\chi\bar\chi\to f\bar f} v\right\rangle(T) n_\chi^{\rm eq}(T)^2 + \sum_{i,j,k}\langle \sigma_{\chi\bar\chi i\to jk} v^2\rangle\, n_i^{\rm eq}\left(n_\chi^{\rm eq}\right)^2$$
$$+ \langle \sigma_{aa\to\chi\bar\chi} v\rangle(T') n_a^2 - \langle \sigma_{\chi\bar\chi\to aa} v\rangle(T') n_\chi^2$$

$$Hz\frac{{\rm d}n_a}{{\rm d}z} + 3Hn_a = \langle \Gamma_a\rangle\, n_a^{\rm eq}(T) + \sum_{i,j,k}\left\langle \sigma_{ia\to jk} v\right\rangle(T) n_a^{\rm eq}(T) n_i^{\rm eq}(T)$$
$$- \langle \sigma_{aa\to\chi\bar\chi} v\rangle(T') n_a^2 + \langle \sigma_{\chi\bar\chi\to aa} v\rangle(T') n_\chi^2$$

[1] Kinetic equilibrium is maintained (due to efficient $\chi a \to \chi a$ scattering)

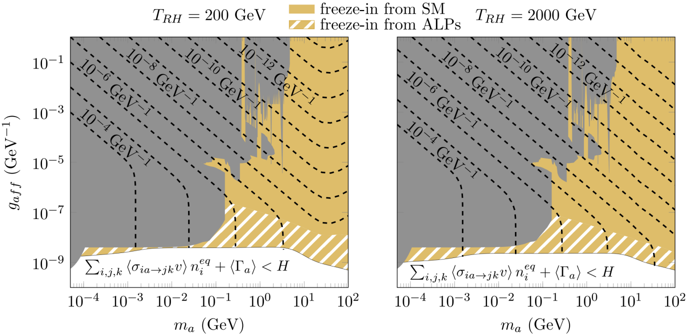
Constraints on our ALP

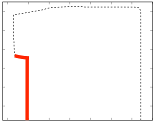


- Revisit constraints from electron beam dumps, rare *B* and *K* decays, astrophysics, dark matter searches and cosmology.
- In particular, for our specific ALP scenario we (re)calculate and improve beam dump, flavour and supernova constraints.

Freeze-in vs. constraints on our ALP

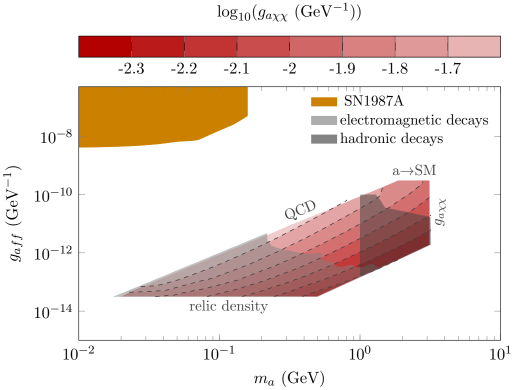
$m_\chi / m_a = 10$

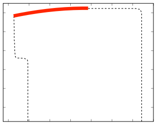




DFO vs. constraints on our ALP

$m_\chi / m_a = 10$

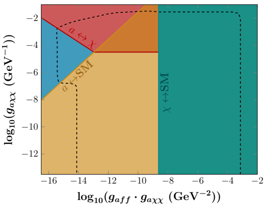




- Tiny g_{aff} ⇒ ALP relatively long lived ⇒ Consequences for BBN
- For $m_a \lesssim 2m_\mu$ constraints are very similar, see [Kawasaki et al.'20] for very long-lived ALPs with sub-GeV m_a excluding $\tau_a \sim 10^3 - 10^5$ s
- For $2m_\mu \lesssim m_a \lesssim 1$ GeV, EM bounds probably apply too, llifetimes not excluded.
- For hadronic decays $\tau \sim 0.1$ s can be excluded, see [Kawasaki et al'17]
- ALPs decaying into photons can re-equilibrate with the SM, see [Millea et al'15], excluding much shorter lifetimes, but these should be alleviated as our dark sector is at T', to be studied in more detail

Conclusion

What we have done



- Our simple framework of an axion-like particle mediating DM leads to various alternative DM genesis scenarios
- Performed a detailed numerical calculation of full region of parameter space giving the correct relic density in various regimes, presented DFO and Freeze-in regions today
- Re-analysed relevant constraints (normally constraints for ALPs for photon coupling) to verify if these regions of parameter space are allowed

Future work

$$E\left(\partial_t - Hp\partial_p\right) f = C\left[f\right]$$

- Want to assess the basic assumption of a negligible relic density after reheating
- Improve accuracy, in particular in sequential freeze-in region but also DFO, by solving unintegrated Boltzmann equation
- Assess the potential sensitivity of future experiments to the region of interest, already inspired by the workshop and interested hearing more ideas!

Determining the HS equation of state

$$3(\rho' + P') = 3(\rho_a + \rho_\chi + P_a + P_\chi)\ .$$

Radiation dominated $3(\rho' + P') = 4\rho'$, we can change variables using $\frac{\partial}{\partial t} \approx -HT\frac{\partial}{\partial T}$

$$\frac{\partial \rho'}{\partial t} + 4H\rho' = -H\left(T\frac{\partial \rho'}{\partial T} - 4\rho'\right) = -HT\rho\frac{\partial}{\partial T}\left(\frac{\rho'}{\rho}\right) = \int \frac{d^3p}{(2\pi)^3} C[f(p,t)]\,.$$

- $T' > m_\chi, m_a$

$$\rho' = \rho_a^{\rm eq}(T') + \rho_{\rm DM}^{\rm eq}(T') \qquad P' = P_a^{\rm eq}(T') + P_{\rm DM}^{\rm eq}(T')$$

- $T' \lesssim m_\chi$

$$\rho_\chi = \left(\frac{\rho}{n}\right)_\chi^{\rm eq}(T')\ n_\chi \qquad P_\chi = \left(\frac{P}{n}\right)_\chi^{\rm eq}(T')\ n_\chi = T' n_\chi\ ,$$

- $T' \lesssim m_a$

$$3(\rho' + P') = 3\left[\left(\frac{\rho}{n}\right)_\chi^{\rm eq}(T')\ n_\chi + \left(\frac{\rho}{n}\right)_a^{\rm eq}(T')\ n_a + T'(n_\chi + n_a)\right]\ .$$

Boundaries of DFO region

- **Relic density** For connector couplings, g_{aff}, which are too small, the hidden sector does not become sufficiently populated. Although $g_{a\chi\chi}$ might be large enough to establish equilibrium between the hidden sector particles, $n_\chi^{\rm eq}(T')$ can never reach the amount of DM density observed today. This is indicated by the lower boundary.
- **$a \leftrightarrow$ SM** On the other hand, if the connector coupling g_{aff} is too large, the interactions between the hidden sector and the SM become strong enough to establish thermal equilibrium. Depending on the hidden sector coupling, the DM is then either produced by freeze-in (see section **??**) or thermal freeze-out. We remark that the numerical solution close to the transition between the freeze-in and the DFO regime is challenging and we chose this upper boundary conservatively.
- **$g_{a\chi\chi}$** The DM-mediator interaction is (cf. eq. (**??**))

$$C_\chi \frac{m_\chi}{f_a} a\bar{\chi}\gamma_5\chi \equiv g_{a\chi\chi} m_\chi a\bar{\chi}\gamma_5\chi \,. \tag{1}$$

  Our effective theory is valid only below the scale f_a. Thus, the reheating temperature T_{RH} should be below this scale to safely ignore UV contributions. On the other hand, T_{RH} has to be higher than m_χ. We consequently need a hierarchy $f_a \gtrsim T_{RH} \gtrsim m_\chi$, i.e. small $g_{a\chi\chi} = C_\chi/f_a$, and this is the reason why the DM (the ALP for a fixed mass ratio) should not be too heavy.
- **QCD** Finally, we employ a perturbative description of the strong interactions, only convergent at high enough energies. The DM abundance should therefore be set by interactions happening at temperatures before the QCD phase transition. In practice, we set the upper boundary labelled "QCD" by requiring that the χ-particles freeze out at temperatures above the threshold $T_{\rm pert} = 600$ MeV.

Flavour Constraints

For $2m_e \lesssim m_a \lesssim 5$ GeV, best chance of detecting ALP could be via heavy mesons decays

Consider constraints from B and K FCNC decays ($b \to sa$ or $s \to da$): $B \to K^{(*)}\ell^+\ell^-$ and $K \to \pi\ell^+\ell^-$ or $B \to K^{(*)}\nu\bar{\nu}$ and $K \to \pi\nu\bar{\nu}$. For $m_a < 2m_e$, $a \to \gamma\gamma$ and a long lived $\Rightarrow$ invisible
The main experimental constraints that we consider in this work are therefore:

- $B^+ \to K^+X(\to \mu^+\mu^-)$ for long-lived scalar X (LHCb [LHCb'16]), the 95% C.L. upper limits on the branching ratio are given as a function of the lifetime of X in the range 0.1 to 1000 ps.
- $B^0 \to K^{*0}X(\to \mu^+\mu^-)$ where X is a scalar particle with mass in the range 214 to 4350 MeV (LHCb [LHCb'15]), the 95% C.L. upper limits on the branching ratio are given as a function of the mass and lifetime of X. The limit is of the order 10^{-9} over the majority of this range.
- $B^0 \to K^{(*)}X(\to \mu^+\mu^-)$ at fixed target experiments, limits can be extracted from CHARM results as described in ref. [Dobrich'18].
- $K^+ \to \pi^+\nu\bar{\nu}$ from NA62 [NA62'21], where 90% C.L. upper limits are given for the $K^+ \to \pi^+X$ branching ratio, where X is a long-lived scalar or pseudoscalar particle decaying outside the detector, for lifetimes longer than 100 ps.
- $K^\pm \to \pi^\pm e^+e^-$, where NA48/2 [NA482'09] provides a 90% C.L. upper limit (here we assume the lifetime should be less than 10 ns).
- $K^\pm \to \pi^\pm X(\to \mu^+\mu^-)$ for long-lived X (NA48/2 [NA482'16]) , the 90% C.L. upper limits on the branching ratio are given as a function of the lifetime of X in the range 100 ps to 100 ns.
- CHARM constraints [CHARM:1985anb,CHARM:1983ayi] from [Dobrich'18jyi][2]

Calculate $B \to K^{(*)}a$ and $K \to \pi a$ BR (see [Gavela'19] and [MartinCamalich'20]):

- FFs for $B \to K$ from [Bharucha'10im] and for $B \to K^*$ from [Bharucha'15].
- For $K \to \pi a$, follow [Alves'17], accounting for octet enhancement in non-leptonic K decays.

[2] We are very grateful to the authors of ref. [Dobrich'18jyi] for providing us with the bounds they obtained in the $m_a - g_{aff}$ plane via private communication.

BBN constraints

If a substantial number of ALPs are produced in the very early universe, they can affect the successful predictions for big bang nucleosynthesis

For heavier ALPs, electromagnetic showers produced in ALP decays during or after BBN can destroy the newly created nuclei and thus directly alter the light element abundances, see [Kawasaki'20,Depta'20] for recent studies. For ALP masses above the GeV scale, hadronic showers give rise to additional constraints, excluding abundant hadronically decaying particles with lifetimes down to about $\mathcal{O}(0.1\ \mathrm{s})$. This is because cascade hadrons can scatter off background protons, which once again increases the neutron-to-proton ratio [Reno'87]; see [Kawasaki'17] for a recent numerical analysis.[3]

- For $m_a \lesssim 2m_\mu$, i.e. dominantly decaying into $\gamma\gamma$ and e^+e^-, constraints are very similar. Use [Kawasaki'20qxm] on very long-lived ALPs in sub-GeV mass range. Generically speaking, they exclude sufficiently abundant particles decaying electromagnetically with lifetimes $\tau_a \sim 10^{3} - 10^{5}$ s. The bound labelled "electromagnetic decays" in figure ?? was obtained by applying the bounds on the ALP's lifetime from figures 4 and 5 in ref. [Kawasaki'20], interpolating between the mass of the decaying particle and the dominant decay channel of the ALP.
- For $2m_\mu \lesssim m_a \lesssim 1$ GeV, $a \to \mu^+\mu^-$ dominates. In principle, the applicable constraints are the electromagnetic ones here – see also the discussion in ref. [cosmobounds] – and the lifetime is short enough for them not to matter.
- For hadronic decays bounds more severe, and lifetimes above $\tau \sim 0.1$ s can be excluded. In ref. [Kawasaki'17] bounds are provided on hadronically decaying particles with masses in the GeV-TeV range. The smallest mass for which results are available is 30 GeV. We apply the corresponding bound to our model, extrapolating from the given shapes that the bounds will remain approximately constant for lower masses.

---

[3] Taking into account the branching ratios of the various decay channels, the energy injection from ALPs is sufficient in the DFO region for these constraints to apply. However, photo-dissociation is clearly not the only possible scenario. For instance, ALPs decaying into photons can re-equilibrate with the SM, a scenario which was studied in depth in ref. [Millea'15], excluding much shorter lifetimes. However, since the temperature of our hidden sector T' is in general well below T, we expect these bounds to be alleviated in our case [Depta'20].