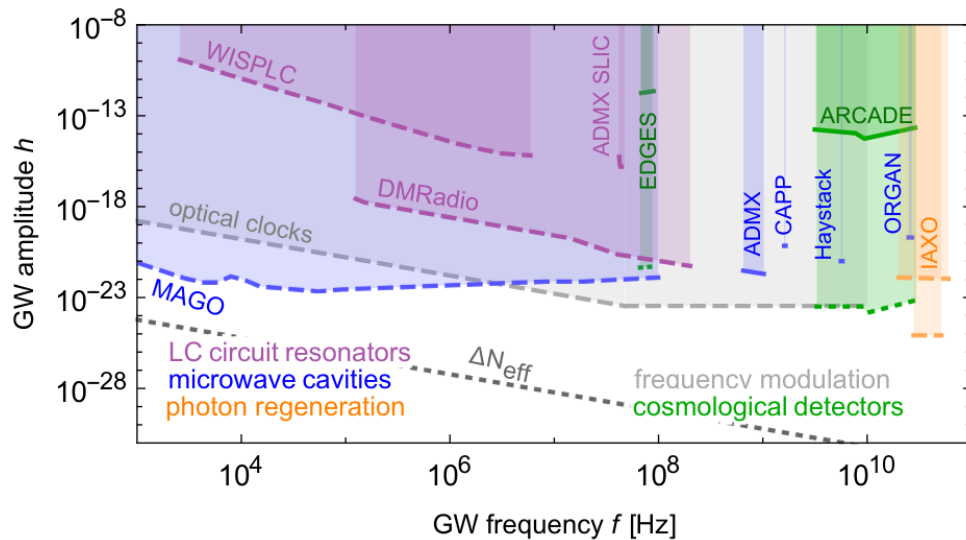


# Axions and gravitational waves



Valerie Domcke  
CERN

*Axions++*  
September 25 - 28, 2023

based in part on

[2011.12414](#)

Living Review on UHF GW searches,

[2202.00695](#), [2306.03125](#), [2306.04496](#)

w. Camilo Garcia-Cely, Sung Mook Lee  
and Nick Rodd

# Axions and GWs

# Axions and GWs

## Axions as a source of GWs

$\phi F_{\mu\nu} \tilde{F}^{\mu\nu}$  with  $\dot{\phi} \neq 0$  → tachyonic gauge field instability → GW source  
(eg axion inflation)  
Turner, Widrow '88; Garretson, Field, Carroll '92

axion superradiance around BHs → GW source  
review: Brito, Cardoso, Pani '15  
see talks by Francesca Day, Yifan Chen

axion strings, domain walls → GW source  
see talks by Marco Gorghetto, Yann Goutnoire

# Axions and GWs

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## Synergies between axion and GW searches ← this talk

- GW electrodynamics vs axion electrodynamics
- Searching for high-frequency GWs with axion haloscopes
- [Possible high-frequency GW sources]
- Photon regeneration experiments and cosmological detectors

# GW electrodynamics

Classical electrodynamics + linearized GR,  $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$  :

$$\partial_\nu F^{\mu\nu} = j_{\text{eff}}^\mu = (-\nabla \cdot \mathbf{P}, \nabla \times \mathbf{M} + \partial_t \mathbf{P})$$

$$\partial_\nu \tilde{F}^{\mu\nu} = 0$$

effective current  
effective polarization vector  
effective magnetization vector

with

$$P_i = -h_{ij}E_j + \frac{1}{2}hE_i + h_{00}E_i - \epsilon_{ijk}h_{0j}B_k,$$

$$M_i = -h_{ij}B_j - \frac{1}{2}hB_i + h_{jj}B_i + \epsilon_{ijk}h_{0j}E_k,$$

induced at linear order in h  
in presence of external E,B field  
VD, Garcia-Cely, Rodd `22

Direct analogy with axion electrodynamics

$$\mathcal{L} \supset g_{a\gamma\gamma} a \mathbf{E} \cdot \mathbf{B} \quad \rightarrow \quad \mathbf{P} = g_{a\gamma\gamma} a \mathbf{B}, \quad \mathbf{M} = g_{a\gamma\gamma} a \mathbf{E}$$

McAllister et al `18  
Tobar, McAllister, Goryachev `19  
Ouellet, Bogorad `19

effective source terms in Maxwell's equation due to GW

# [ a note on frames ]

GR is invariant under coordinate transformations, but linearized GR is not

## Transverse traceless (TT) gauge

- coordinates fixed by freely falling test masses
- GW takes very simple form  $h_{0\mu} = 0, h_i^i = 0, \partial_j h^{ij} = 0$
- rigid body seems to 'oscillate' in presence of GW

$$h_{ij}^{TT} = (h^+ e_{ij}^+(\phi_h, \theta_h) + h^\times e_{ij}^\times(\phi_h, \theta_h)) e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)}$$

## Proper detector frame

- coordinates fixed by laboratory frame
- GW takes a more involved form
- description of experimental setup and observables is straightforward

$$\begin{aligned} h_{00} &= \omega^2 F(\mathbf{k} \cdot \mathbf{r}) \mathbf{b} \cdot \mathbf{r}, & b_j &\equiv r_i h_{ij}^{TT} \Big|_{\mathbf{r}=0}, \\ h_{0i} &= \frac{1}{2} \omega^2 [F(\mathbf{k} \cdot \mathbf{r}) - iF'(\mathbf{k} \cdot \mathbf{r})] (\hat{\mathbf{k}} \cdot \mathbf{r} b_i - \mathbf{b} \cdot \mathbf{r} \hat{k}_i), \\ h_{ij} &= -i\omega^2 F'(\mathbf{k} \cdot \mathbf{r}) (|\mathbf{r}|^2 h_{ij}^{TT} \Big|_{\mathbf{r}=0} + \mathbf{b} \cdot \mathbf{r} \delta_{ij} - b_i r_j - b_j r_i), \end{aligned}$$

VD, Garcia-Cely, Rodd `22  
s.a. Berlin et al `21

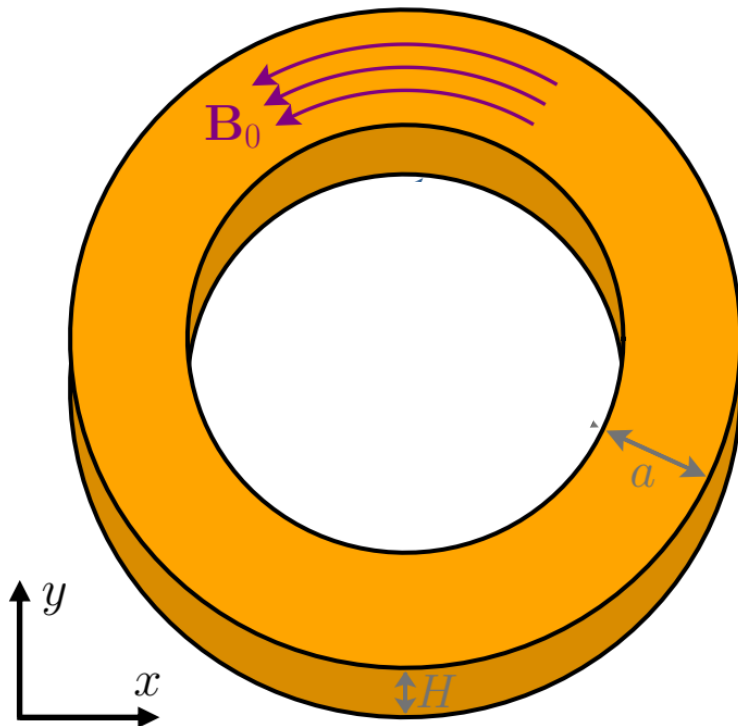
we will consider a plane wave and rigid detector in the proper detector frame

# GW signal in axion haloscopes

eg ABRACADABRA, SHAFT, DM Radio:

VD, Garcia-Cely, Rodd '22

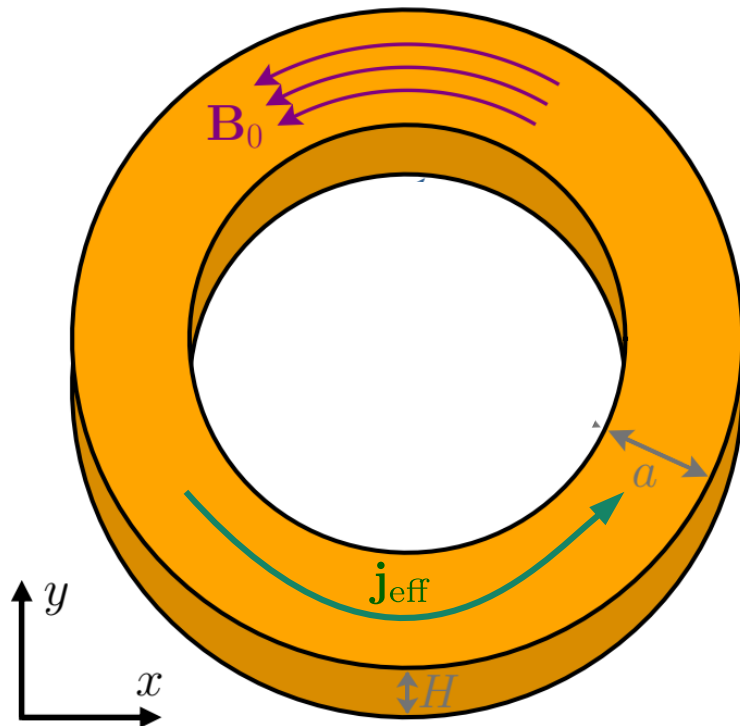
static magnetic field



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eg ABRACADABRA, SHAFT, DM Radio:

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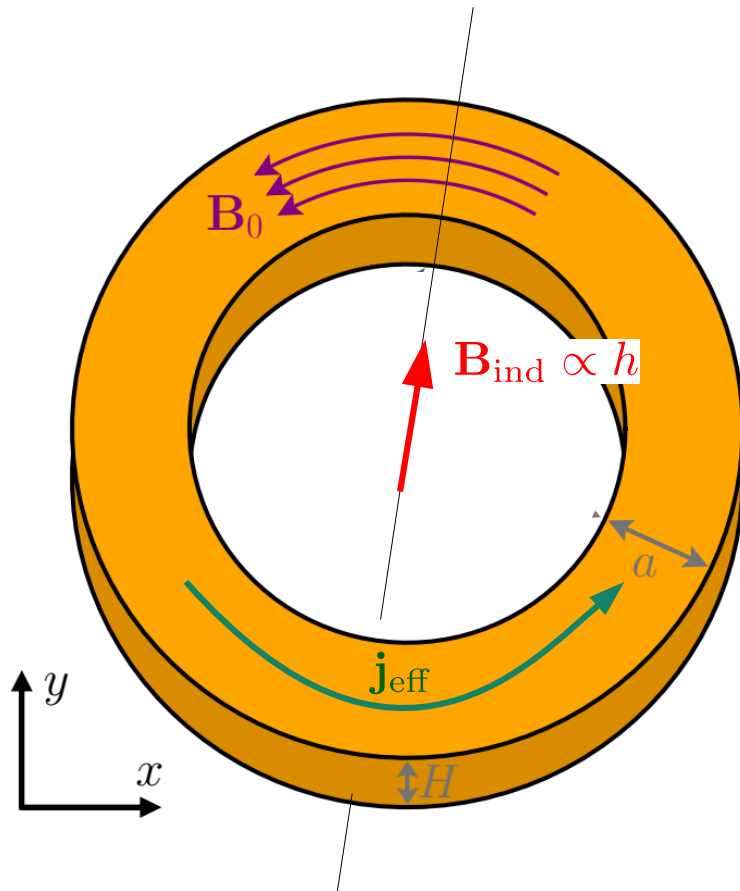
effective current



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static magnetic field

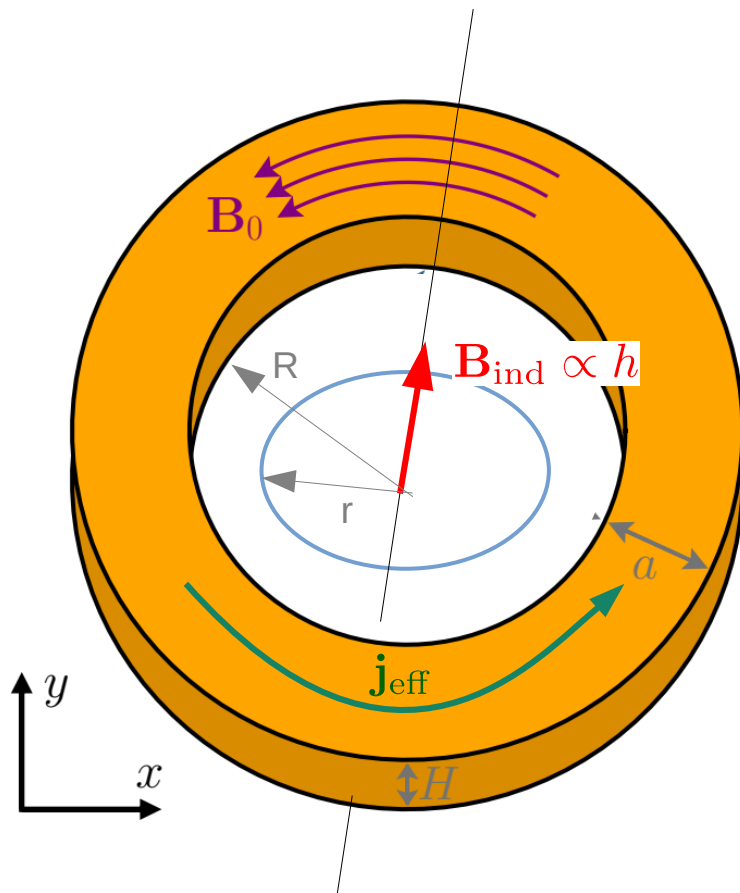
effective current

induced oscillating magnetic field

# GW signal in axion haloscopes

eg ABRACADABRA, SHAFT, DM Radio:

VD, Garcia-Cely, Rodd '22



static magnetic field

effective current

induced oscillating magnetic field

measure magnetic flux ( $\sim h$ )  
through pickup loop

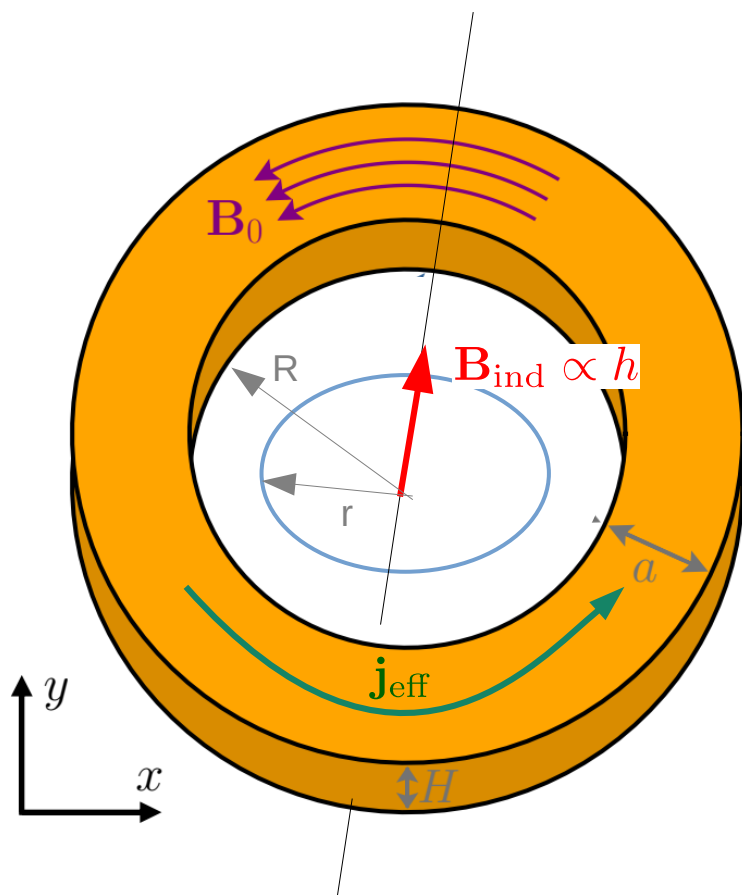
at leading order in  $(\omega R)$  :

$$\Phi_{\text{gw}} = \frac{i e^{-i\omega t}}{16\sqrt{2}} h \times \omega^3 B_0 \pi r^2 R a (a + 2R) s_{\theta_h}^2$$

# GW signal in axion haloscopes

eg ABRACADABRA, SHAFT, DM Radio:

VD, Garcia-Cely, Rodd '22



static magnetic field

effective current

induced oscillating magnetic field

measure magnetic flux ( $\sim h$ )  
through pickup loop

at leading order in  $(\omega R)$ :  $\sim (\omega L)^3 h B_0 L^2$

$$\Phi_{\text{gw}} = \frac{i e^{-i\omega t}}{16\sqrt{2}} h \times \omega^3 B_0 \pi r^2 R a (a + 2R) s_{\theta_h}^2$$

match to axion induced flux to recast  
axion-photon coupling bounds as GW bounds

$$\Phi_a = e^{-i\omega t} g_{a\gamma\gamma} \sqrt{2\rho_{\text{DM}}} B_0 \pi r^2 R \ln(1 + a/R)$$

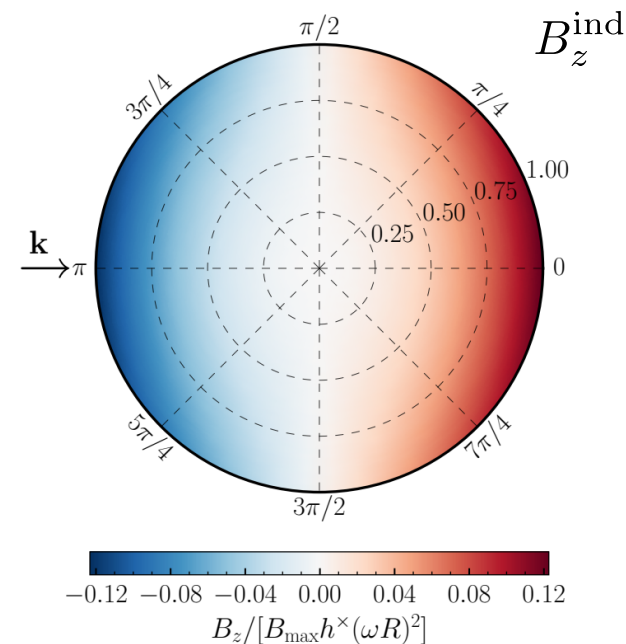
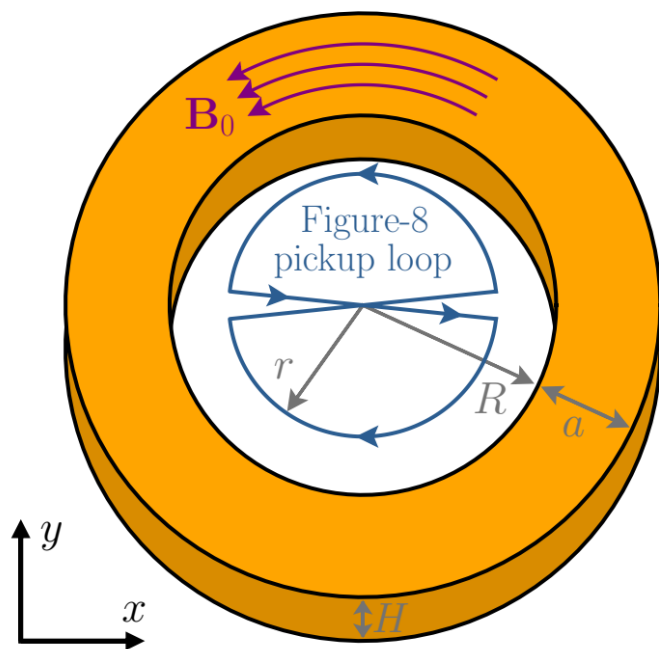
$$\sim (\omega L) g_{a\gamma\gamma} B_0 L^2$$

suppression at low frequencies as  $(\omega L)^3$   
implies very good volume scaling

# optimized pickup loop geometry

spin 2 structure of GW induces angular modulation of induced B field

leading order  $(\omega R)^2$  contribution can be captured if cylindrical symmetry is broken, here by using a figure-8 geometry for the pickup loop



$$\Phi_{\text{gw},8} = \frac{e^{-i\omega t}}{3\sqrt{2}} \omega^2 B_0 r^3 R \ln(1 + a/R) s_{\theta_h} \times (h^\times s_{\phi_h} - h^+ c_{\theta_h} c_{\phi_h}) \sim (\omega L)^2 h B_0 L^2$$

parametric improvement for modified pickup loop

# geometry and time scales

VD, Garcia-Cely, Lee, Rodd `23

## Symmetries and selection rules:

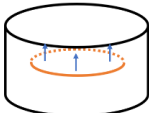
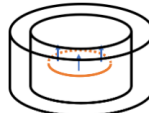
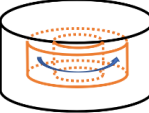
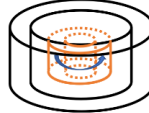
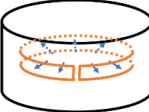
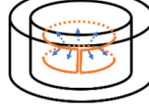
- For an instrument with azimuthal symmetry,  $\Phi_h \propto h^+$  at  $\mathcal{O}[(\omega L)^2]$
- For an instrument with azimuthal symmetry, the flux is proportional to either  $h^+$  or  $h^\times$
- For an instrument with full cylindrical symmetry,  $\phi_h$  contains only even or odd powers of  $\omega$

# geometry and time scales

VD, Garcia-Cely, Lee, Rodd `23

## Symmetries and selection rules:

- For an instrument with azimuthal symmetry,  $\Phi_h \propto h^+$  at  $\omega \rightarrow 0$
- For an instrument with azimuthal symmetry, the flux is proportional to  $\omega^2$  at  $\omega \rightarrow \infty$
- For an instrument with full cylindrical symmetry,  $\phi_h$  contains only  $h^+$  and  $h^\times$

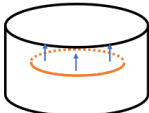
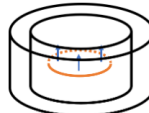
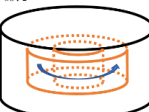
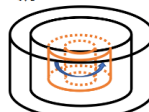
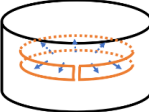
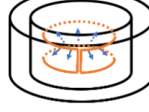
	Solenoid: $\mathbf{B}_0 \propto \hat{\mathbf{e}}_z$ ( $\eta_b = +1, \eta = -1$ )	Toroid: $\mathbf{B}_0 \propto \hat{\mathbf{e}}_\phi$ ( $\eta_b = -1, \eta = +1$ )
$\hat{\mathbf{n}}' \propto \hat{\mathbf{e}}_z$ ( $\kappa = +1, \kappa = -1$ )	$h^+, n \text{ even} \Rightarrow \mathcal{O}[(\omega L)^2]$ $\Phi_h = \frac{e^{-i\omega t}}{48\sqrt{2}} h^+ \omega^2 B_0 s_h^2 \pi r^2 (11r^2 + 14R^2 + 16R^2 \ln \frac{R}{r})$ 	$h^\times, n \text{ odd} \Rightarrow \mathcal{O}[(\omega L)^3]$ $\Phi_h = \frac{e^{-i\omega t}}{48\sqrt{2}} h^\times \omega^3 B_{\max} \pi r^2 a R (a + 2R) s_h^2$ 
$\hat{\mathbf{n}}' \propto \hat{\mathbf{e}}_\phi$ ( $\kappa = -1, \kappa = +1$ )	$h^\times, n \text{ odd} \Rightarrow \mathcal{O}[(\omega L)^3]$ $\Phi_h = \frac{e^{-i\omega t}}{96\sqrt{2}} h^\times \omega^3 B_0 \pi r^2 l (12R^2 - 5r^2) s_h^2$ 	$h^+, n \text{ even} \Rightarrow \mathcal{O}[(\omega L)^2]$ $\Phi_h = \frac{3e^{-i\omega t}}{4\sqrt{2}} h^+ \omega^2 B_{\max} \frac{\pi r^2 a R [(a+2R)l]}{R^2} s_h^2$ 
$\hat{\mathbf{n}}' \propto \hat{\mathbf{e}}_\rho$ ( $\kappa = +1, \kappa = +1$ )	$h^+, n \text{ odd} \Rightarrow \mathcal{O}[(\omega L)^3]$ $\Phi_h = \frac{e^{-i\omega t}}{96\sqrt{2}} h^+ B_0 \omega^3 c_{\eta_b} s_h^2 \times \pi r^2 l (3l^2 - 22(r^2 + 2R^2) - 36R^2 \ln \frac{R}{r})$ 	$h^\times, n \text{ even} \Rightarrow \mathcal{O}[(\omega L)^4]$ $\Phi_h = \frac{e^{-i\omega t}}{32\sqrt{2}} h^\times \omega^4 B_{\max} \pi r^2 a R l (a + 2R) c_{\eta_b} s_h^2$ 

# geometry and time scales

VD, Garcia-Cely, Lee, Rodd '23

## Symmetries and selection rules:

- For an instrument with azimuthal symmetry,  $\Phi_h \propto h^+$  at  $\kappa = +1$
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$\hat{\mathbf{n}}^l \propto \hat{\mathbf{e}}_\phi$ ( $\kappa_b = -1, \kappa = +1$ )	$h^\times, n \text{ odd} \Rightarrow \mathcal{O}[(\omega L)^3]$ $\Phi_h = \frac{\epsilon^{-1-\eta}}{36\sqrt{2}} h^\times \omega^3 B_0 \pi r^2 (12R^2 - 5r^2) s_h^2$ 	$h^+, n \text{ even} \Rightarrow \mathcal{O}[(\omega L)^2]$ $\Phi_h = \frac{3\epsilon^{-1-\eta}}{32\sqrt{2}} h^+ \omega^2 B_{\text{max}} \frac{\pi r^2 a R [(a+2R)]}{4R} s_h^2$ 
$\hat{\mathbf{n}}^l \propto \hat{\mathbf{e}}_\phi$ ( $\kappa_b = +1, \kappa = +1$ )	$h^+, n \text{ odd} \Rightarrow \mathcal{O}[(\omega L)^3]$ $\Phi_h = \frac{\epsilon^{-1-\eta}}{36\sqrt{2}} h^+ B_0 \omega^3 c_{\eta_b} s_h^2 \times \pi r^2 l (3l^2 - 22(r^2 + 2R^2) - 36R^2 \ln \frac{R}{r})$ 	$h^\times, n \text{ even} \Rightarrow \mathcal{O}[(\omega L)^4]$ $\Phi_h = \frac{\epsilon^{-1-\eta}}{32\sqrt{2}} h^\times \omega^4 B_{\text{max}} \pi r^2 a R l (a + 2R) c_{\eta_b} s_h^2$ 

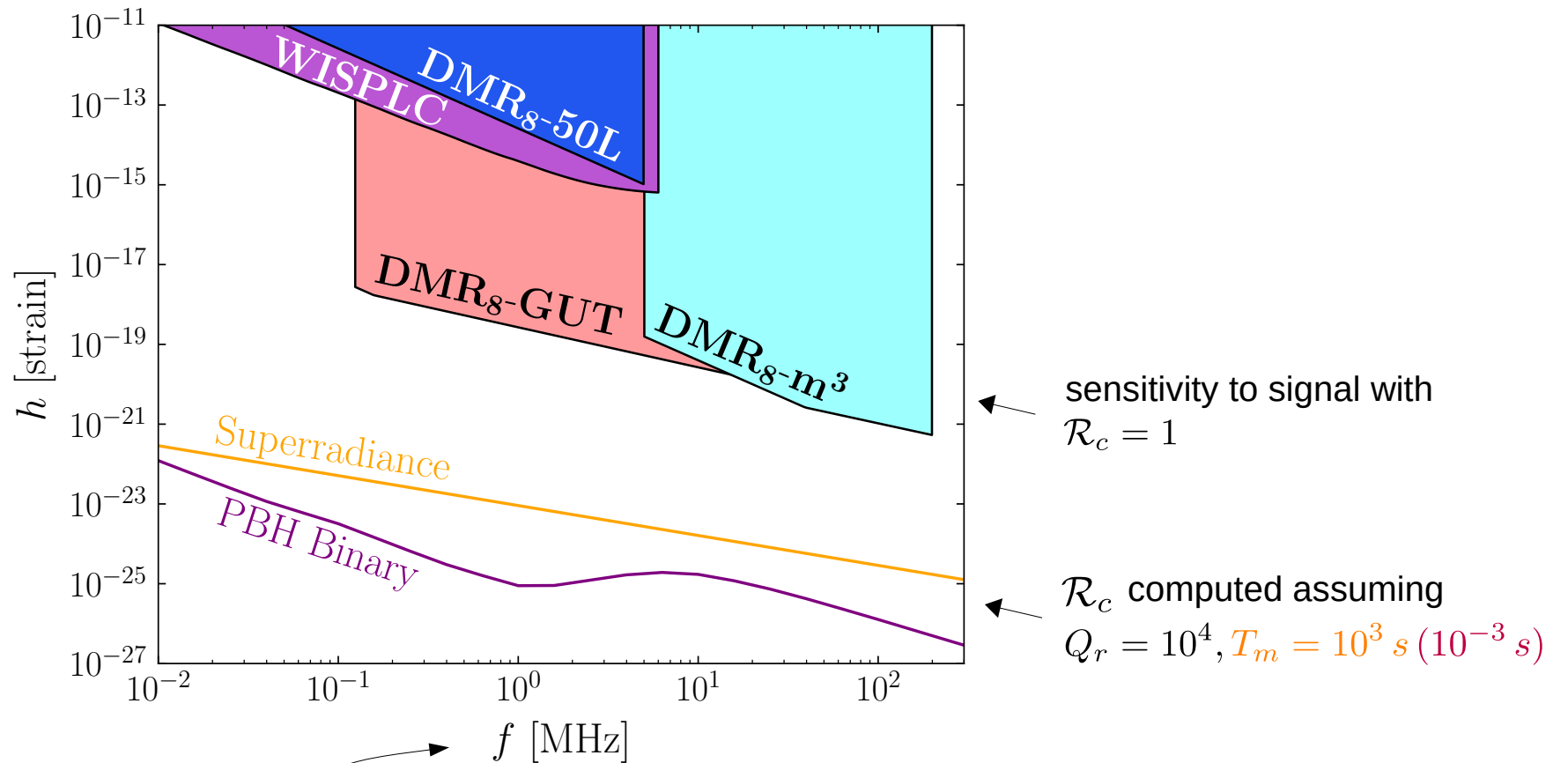
## Time scales:

$$\Phi_h(h^+, h^\times; \phi_h, \theta_h) = \mathcal{R}_c \Phi_a(g_{a\gamma\gamma}), \quad \mathcal{R}_c = \left(\frac{T_m}{\tau_h}\right)^{1/4} \left(\frac{Q_a}{Q_h}\right)^{1/4} \begin{cases} 1 & Q_r < Q_a, Q_h, \\ (Q_a/Q_r)^{1/4} & Q_a < Q_r < Q_h, \\ Q_r/Q_h & Q_h < Q_r < Q_a, \\ (Q_a/Q_r)^{1/4} Q_r/Q_h & \text{otherwise.} \end{cases}$$

signal duration, coherence time < ring up time, axion coherence time, measurement time

➔ will reduce detectability

# bounds and prospects



axion haloscopes as **high frequency** GW detectors

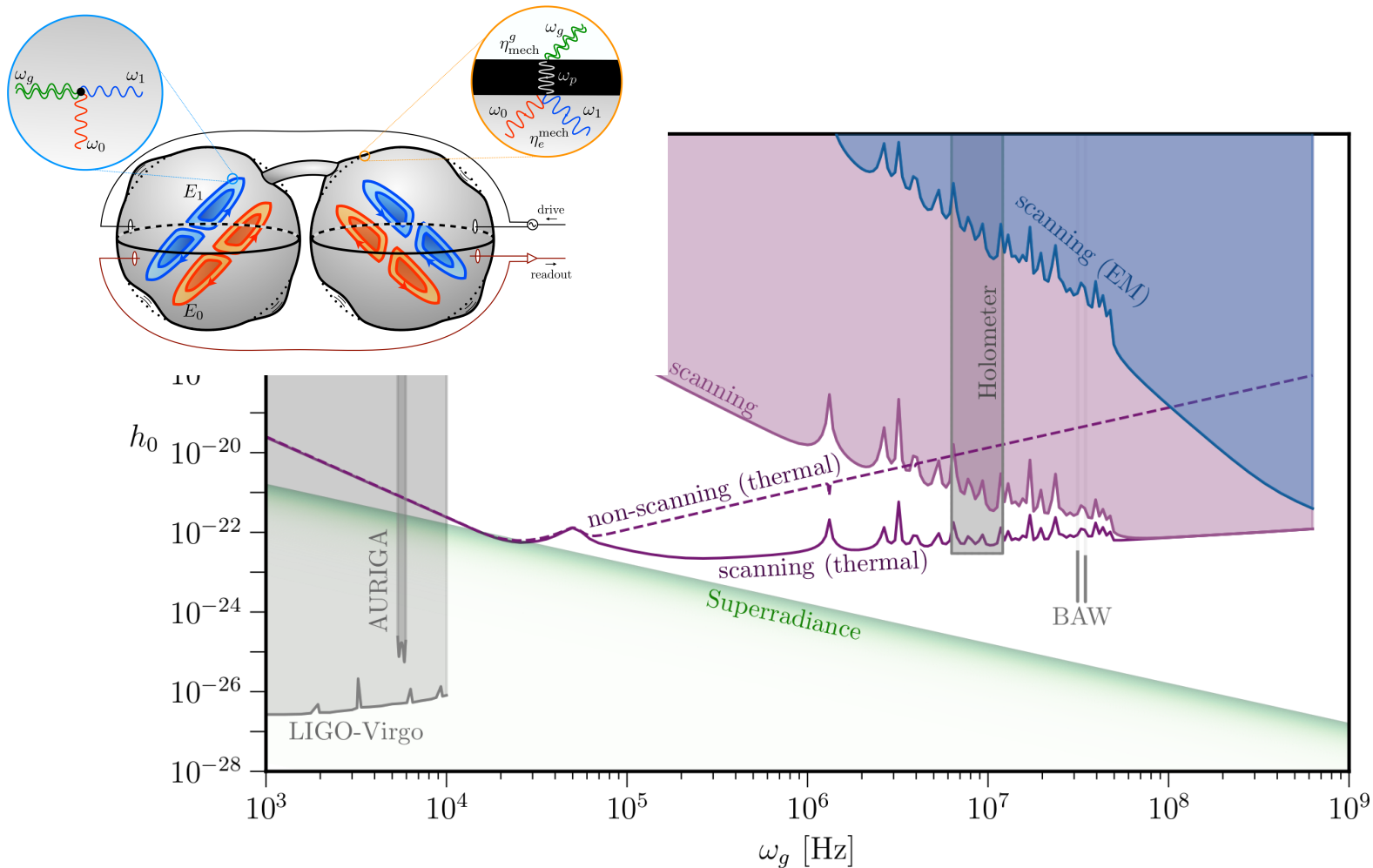
still far away from BBN bound, but clear synergies of UHF GW and axion searches



# microwave cavities

effective current can also induce power in microwave cavities, in addition consider mechanical deformation of cavity walls:

Berlin, Blas, D'Agnolo et al '23



# Axions and GWs

- GW electrodynamics vs axion electrodynamics
- Searching for high-frequency GWs with axion haloscopes
- [Possible high-frequency GW sources]
- Photon regeneration experiments and cosmological detectors

# high frequency ( $> \text{kHz}$ ) GW sources

## Cosmological

- sourced by violent cosmological event in the early Universe
- stochastic GW background (SGWB): stationary, isotropic, broad spectrum
- GW frequency determined by Hubble horizon at sourcing time  
→ high frequency = early Universe
- observationally bounded by BBN and CMB (extra radiation)
- vanilla cosmology: SGWB from cosmic inflation & CGWB very small. But in many BSM models, saturating BBN bound is easy

## Astrophysical

# high frequency ( $> \text{kHz}$ ) GW sources

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## Astrophysical

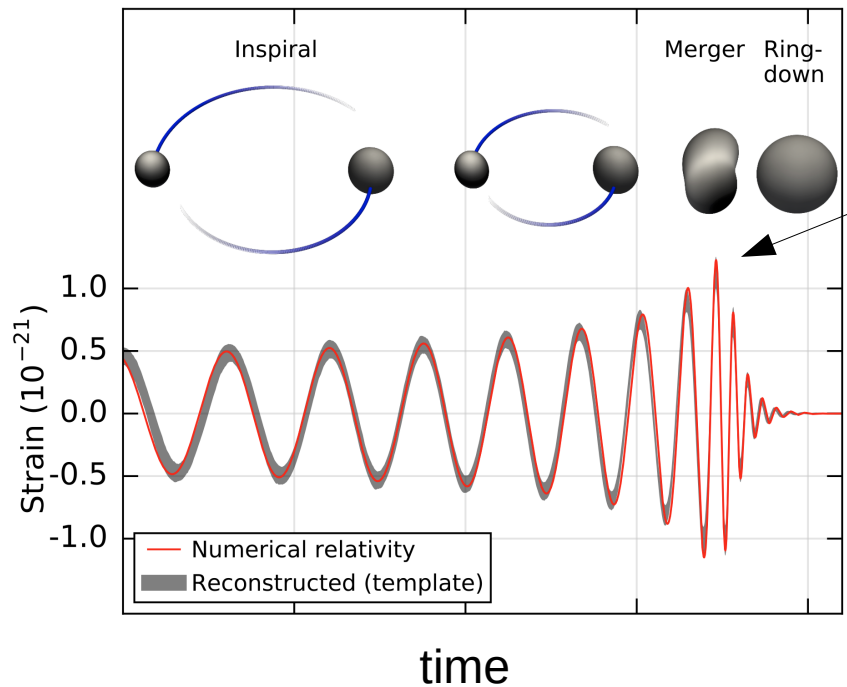
- localized GW sources, both coherent and incoherent signals possible
- no known astrophysical objects emit (significantly) in UHF band
- eg mergers of light primordial black holes or exotic compact objects, superradiance
- large signals require near-by events  
→ rare events with GW strain far above BBN bound are possible
- SGWB from unresolved sources, typically harder to detect

UHF GW searches are always a search for New Physics

# astrophysical sources

Example:  
mergers of light primordial black holes

$$h_{+, \times}^{\text{PBH}} \simeq 10^{-23} \left( \frac{10 \text{ kpc}}{D} \right) \left( \frac{m_{\text{PBH}}}{10^{-5} M_{\odot}} \right)^{5/3} \left( \frac{f}{100 \text{ MHz}} \right)^{2/3}$$



$$f_{\text{ISCO}} = 220 \text{ MHz} \left( \frac{10^{-5} M_{\odot}}{m_{\text{PBH}}} \right)$$

# astrophysical sources

Example:  
mergers of light primordial black holes

$$h_{+, \times}^{\text{PBH}} \simeq 10^{-23} \left( \frac{10 \text{ kpc}}{D} \right) \left( \frac{m_{\text{PBH}}}{10^{-5} M_{\odot}} \right)^{5/3} \left( \frac{f}{100 \text{ MHz}} \right)^{2/3}$$

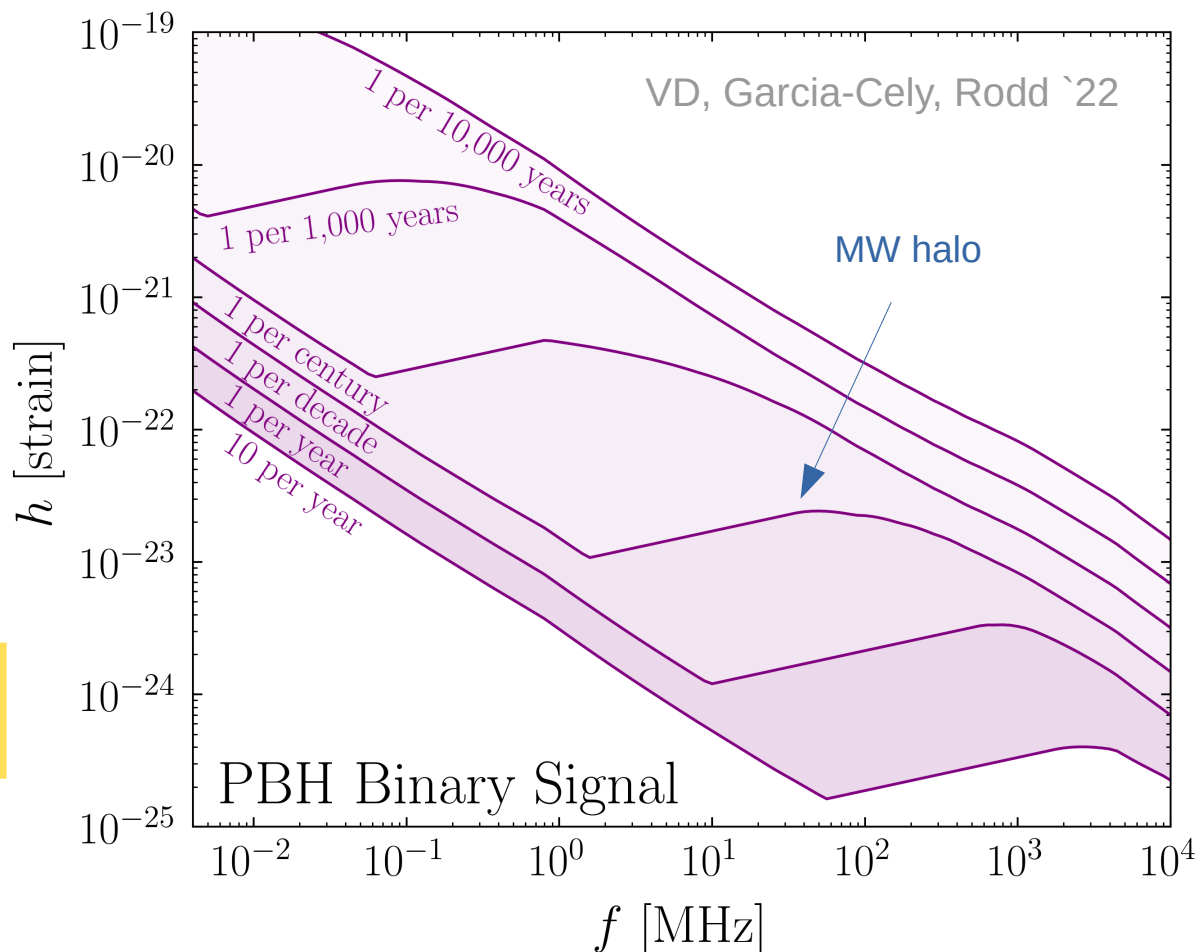
event rate:

$$\langle \Gamma \rangle = \int_0^{\infty} dr 4\pi r^2 \delta(r) R_0(m_{\text{PBH}}, f_{\text{PBH}})$$

MW halo      merger rate

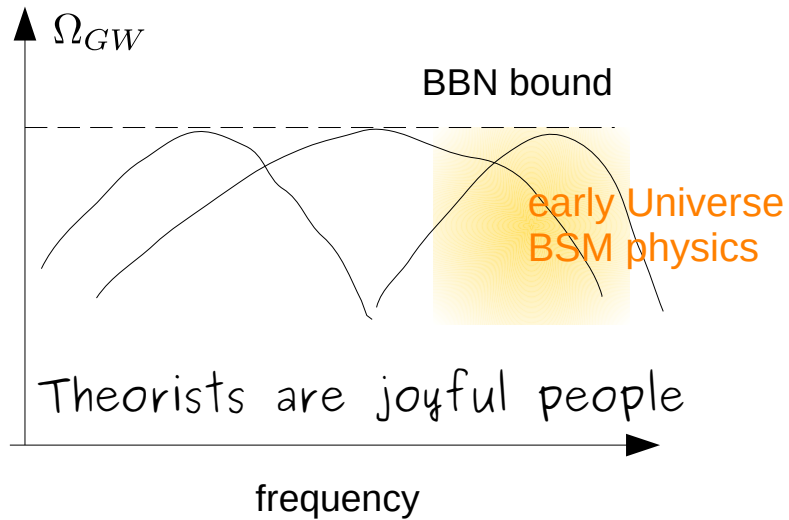
$$\times \Theta \left[ Q^{1/4} h_{+, \times}^{\text{PBH}}(f, m_{\text{PBH}}, r) - h_{\text{th}} \right]$$

large GW amplitudes possible  
for rare events



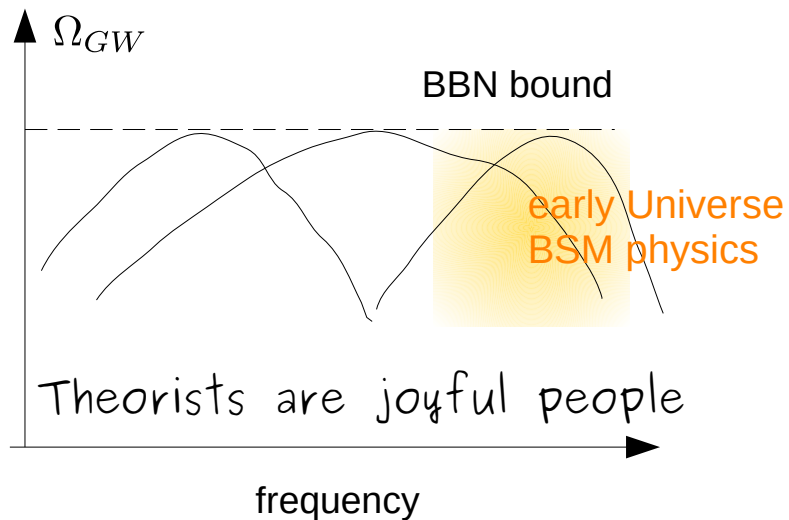
see also Franciolini, Maharana, Muia `22

# challenges in UHF GW detection

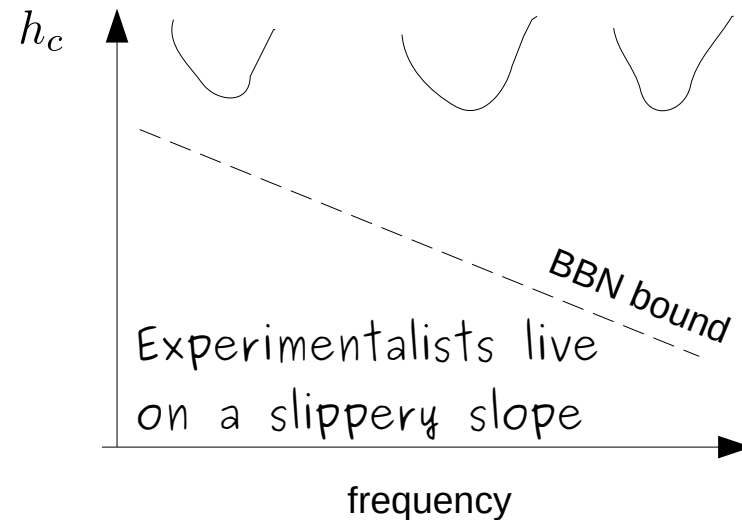


CMB/BBN bound constrains energy

# challenges in UHF GW detection



CMB/BBN bound constrains energy

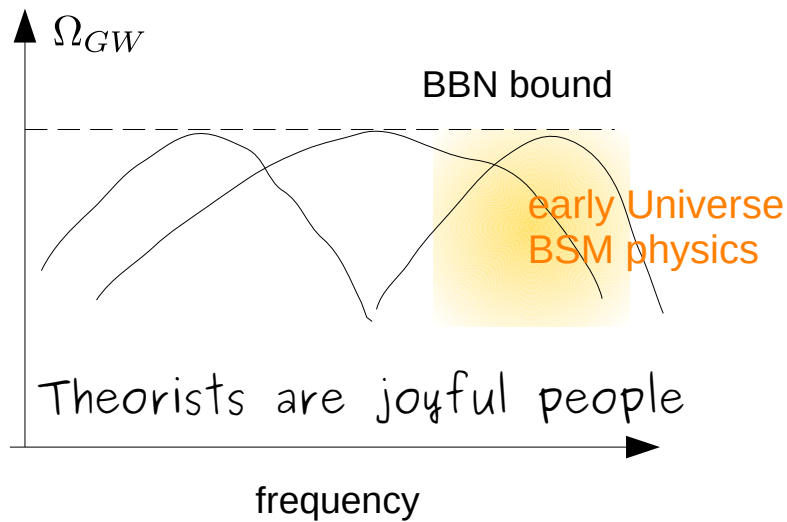


experiments measure displacement

$$\Omega_{GW} \propto f^2 h_c^2$$

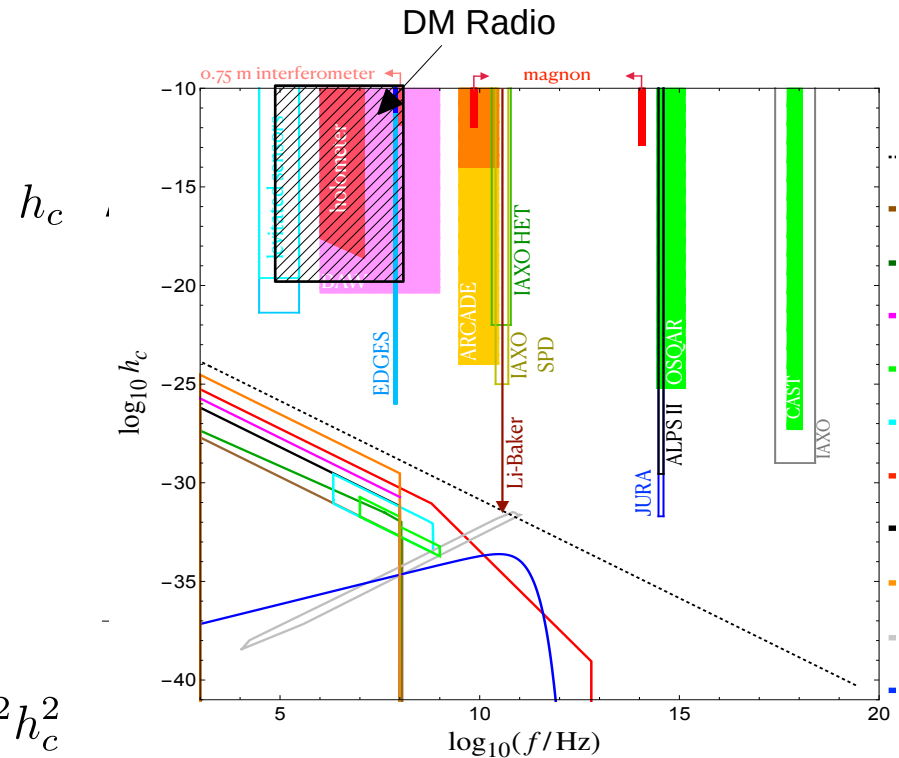


# challenges in UHF GW detection



$$\Omega_{GW} \propto f^2 h_c^2$$

CMB/BBN bound constrains energy



experiments measure displacement

Living Review on sources & detectors: <https://arxiv.org/abs/2011.12414>

# Axions and GWs

- GW electrodynamics vs axion electrodynamics
- Searching for high-frequency GWs with axion haloscopes
- [Possible high-frequency GW sources]
- Photon regeneration experiments and cosmological detectors

# GW to photon conversion

(inverse) Gertsenshtein effect:

[Gertsenshtein '62, Boccaletti et al '70, Raffelt, Stodolsky '88]

$$(\square + \omega_{\text{pl}}^2/c^2) A_\lambda = -B \partial_z h_\lambda, \quad \square h_\lambda = \kappa^2 B \partial_z A_\lambda$$

$A_\lambda =$  photon

$h_\lambda =$  GW

$B =$  ext. transv. B - field

$\omega_{\text{pl}} =$  plasma frequency

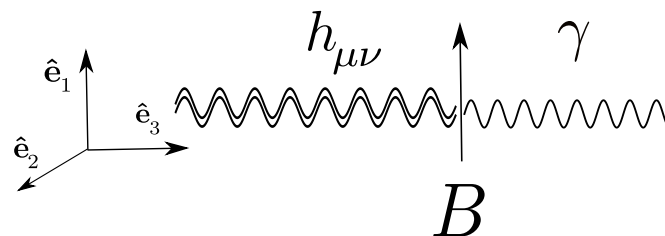
$$\mu^2 = 1 - \omega_{\text{pl}}^2/\omega^2$$

plane waves:

$$\rightarrow \psi(t, z) \equiv \begin{pmatrix} \sqrt{\mu} A_\lambda \\ \frac{1}{\kappa} h_\lambda \end{pmatrix} = e^{-i\omega t} e^{iKz} \psi(0, 0),$$

$$K = \begin{pmatrix} \frac{\mu}{c} \sqrt{\omega^2 + \left(\frac{\kappa B}{1+\mu}\right)^2} & -i \frac{\sqrt{\mu} \kappa B}{1+\mu} \\ i \frac{\sqrt{\mu} \kappa B}{1+\mu} & \frac{1}{c} \sqrt{\omega^2 + \left(\frac{\kappa B}{1+\mu}\right)^2} \end{pmatrix}$$

EM wave in curved space time  
(i.e. classical linearized general  
relativity)  $\rightarrow$  purely SM process

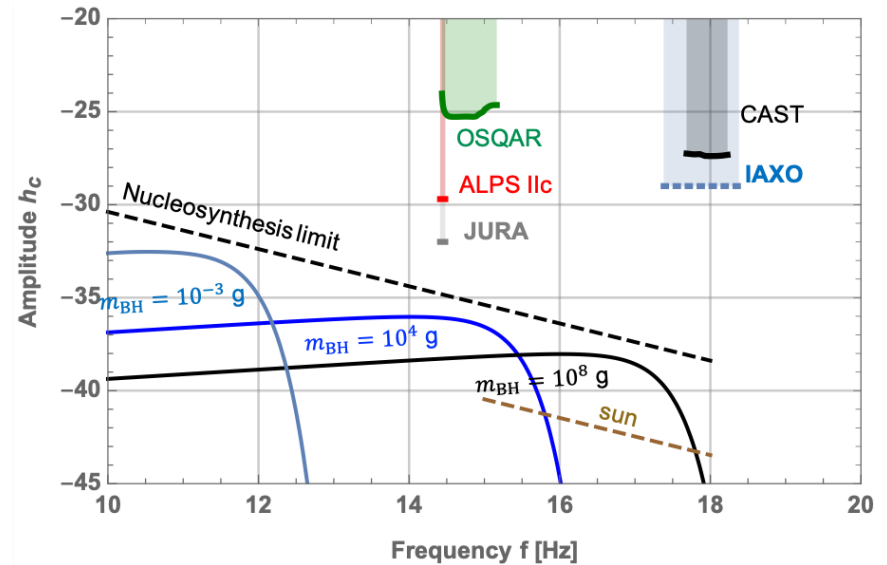
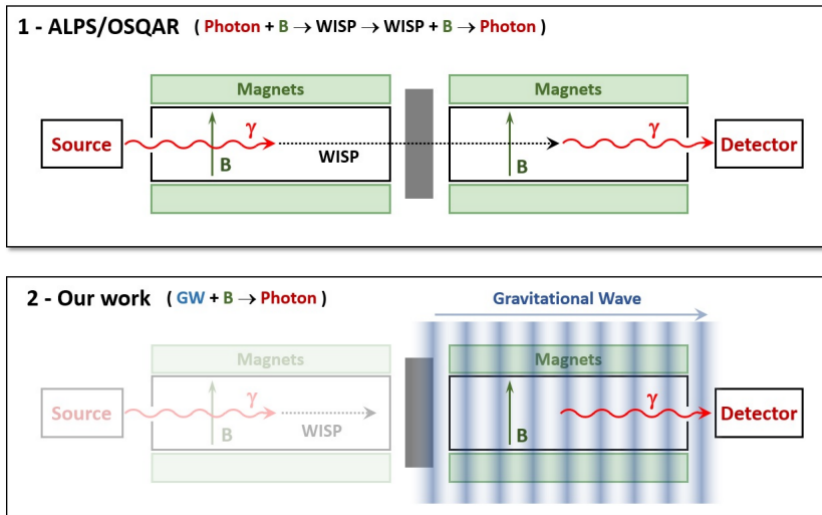


analogous to axion to photon conversion

# LSW experiments

Light-shining-through-the-wall (LSW) experiments:

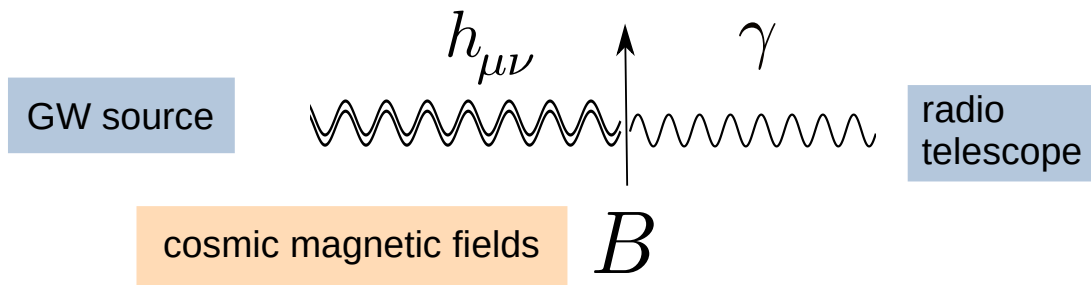
[Ejilli et al `19]



axion bounds recast as HFGW bounds

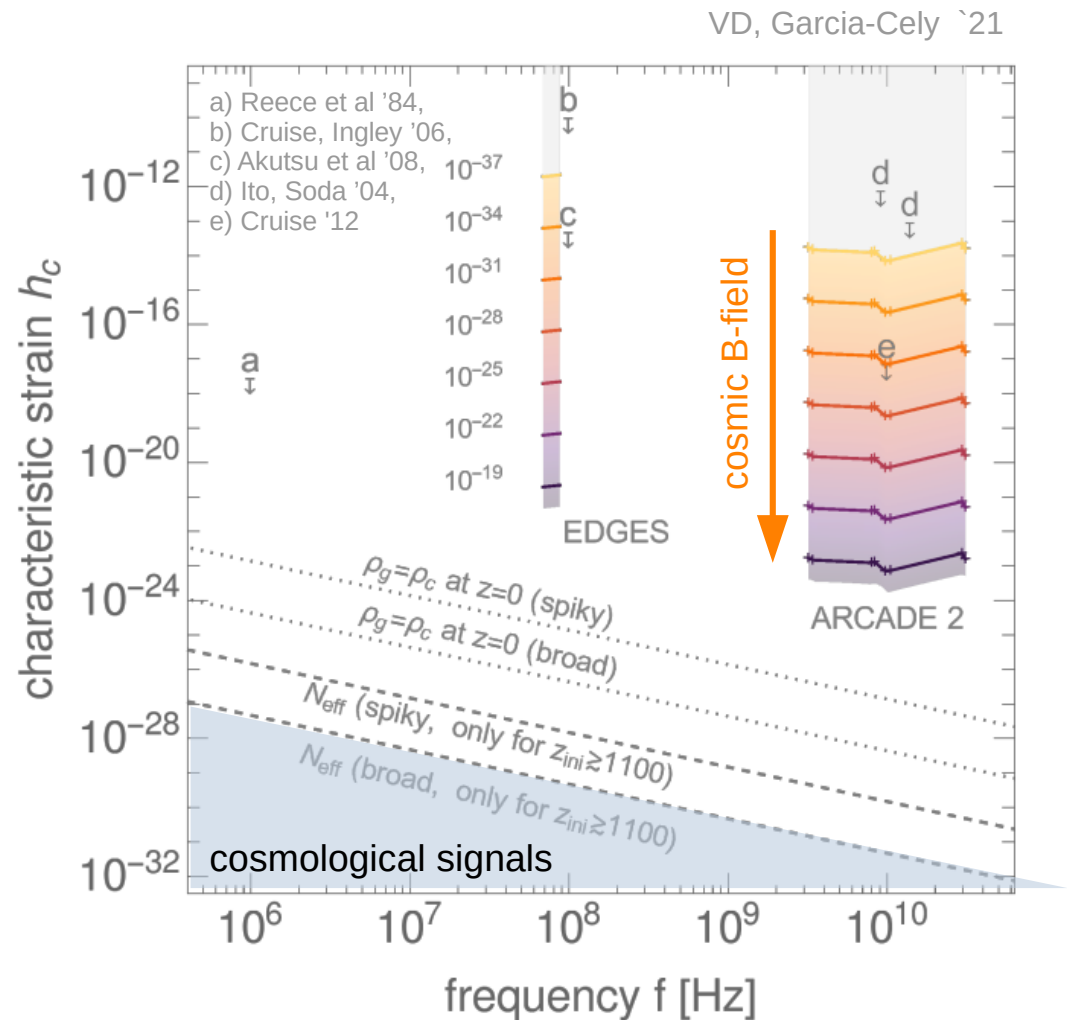
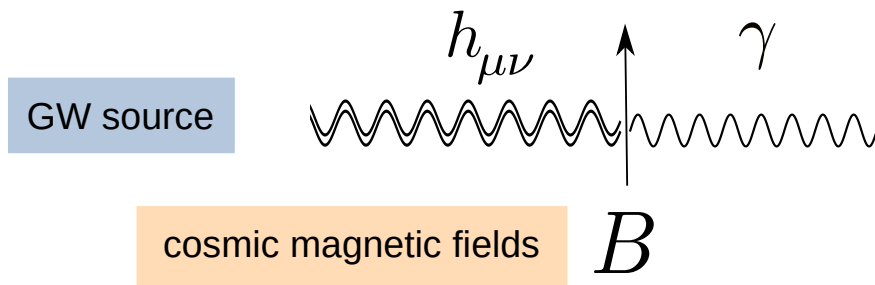
# a cosmic GW detector

idea: compensate small GW to EM coupling with cosmologically big detector:



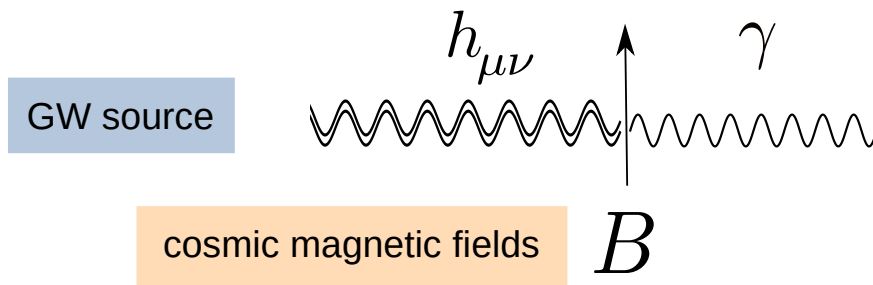
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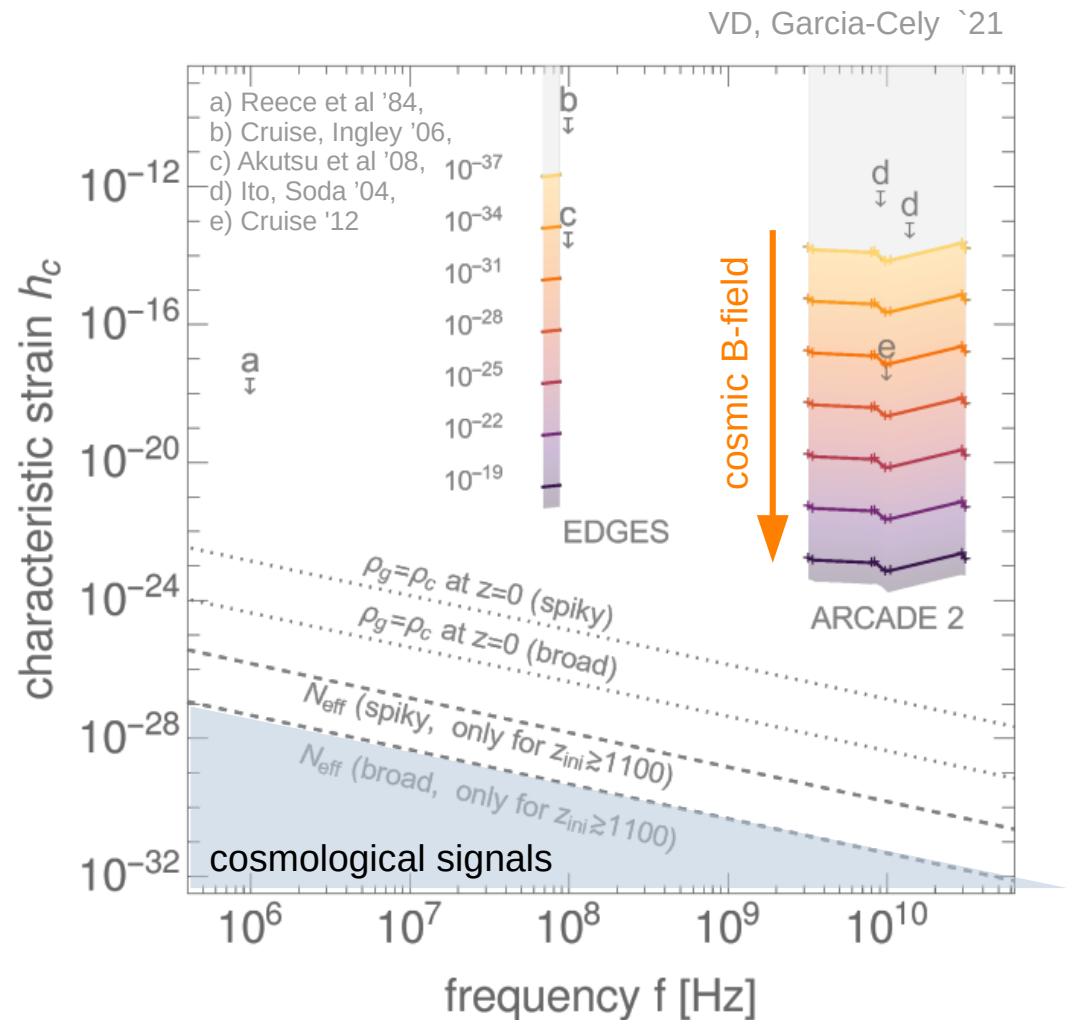
- promising, but significant improvements needed
- a lot of room for new ideas (laboratory & cosmo)

See also :

neutron stars: Feng et al `22, Liu et al `23, Ito et al `23

Milky Way: Ramazanov et al `23

CMB depletion: Fujita et al `20



# Conclusions and Outlook

## **Synergies between GW and axion searches**

- GW electrodynamics has clear similarities with axion electrodynamics:  
Important synergies between axion searches and UHF GW searches
- New bounds and prospects for low-mass axion haloscopes as GW detectors
- Also SRF cavities, LSW experiments, cosmological detectors,....

## **GW sources at high frequencies**

- GW signals  $\gg$  kHz would be a smoking gun of BSM physics
- Cosmological signals well motivated, but amplitude constrained by BBN and CMB
- Larger astrophysical signals from rare exotic events possible, e.g. light PBHs



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**Thank you!**

backup slides

# cosmological sources

## Amplitude: BBN / CMB bound

$$\frac{\rho_{GW}^0}{\rho_c^0} = \Omega_\gamma^0 \left( \frac{g_s^0}{g_s(T)} \right)^{4/3} \underbrace{\frac{\rho_{GW}(T)}{\rho_\gamma(T)}}_{\lesssim 10\%} \Big|_{T_{\text{CMB, BBN}}} \leq 10^{-5} \Delta N_{eff} \simeq 10^{-6}$$

for a broadband SGWB:  $\rightarrow h_{c,sto} \lesssim 10^{-29} (100 \text{ MHz}/f) \Delta N_{\text{eff}}^{1/2}$

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during radiation era:  $f \sim 100 \text{ MHz}/\epsilon_* (T_*/10^{15} \text{ GeV}), \quad \epsilon_* \lesssim 1$

during inflation:  $f \sim 10^{-18} \text{ Hz } e^{N_{\text{CMB}} - N} \lesssim 10^8 \text{ Hz } e^{-N}, \quad N_{\text{CMB}} \lesssim 60$

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**Examples:** (Axion) inflation, (p)reheating, relic cosmic GW background, phase transitions (first order PT and/or topological defects from PTs) ,...

# BBN bound

radiation energy after electron decoupling:

$$\rho_{rad} = \frac{\pi^2}{30} \left( 2 + \frac{7}{4} \left( \frac{4}{11} \right)^{4/3} (3.046 + \Delta N_{eff}) \right) T^4$$

photons
neutrinos
BSM

at BBN or CMB decoupling:

$$\rho_{GW}(T) < \Delta \rho_{rad}(T) \quad \Rightarrow \quad \left( \frac{\rho_{GW}}{\rho_\gamma} \right)_{T_{BBN,CMB}} \leq \frac{7}{8} \left( \frac{4}{11} \right)^{4/3} \Delta N_{eff} \simeq 0.05$$

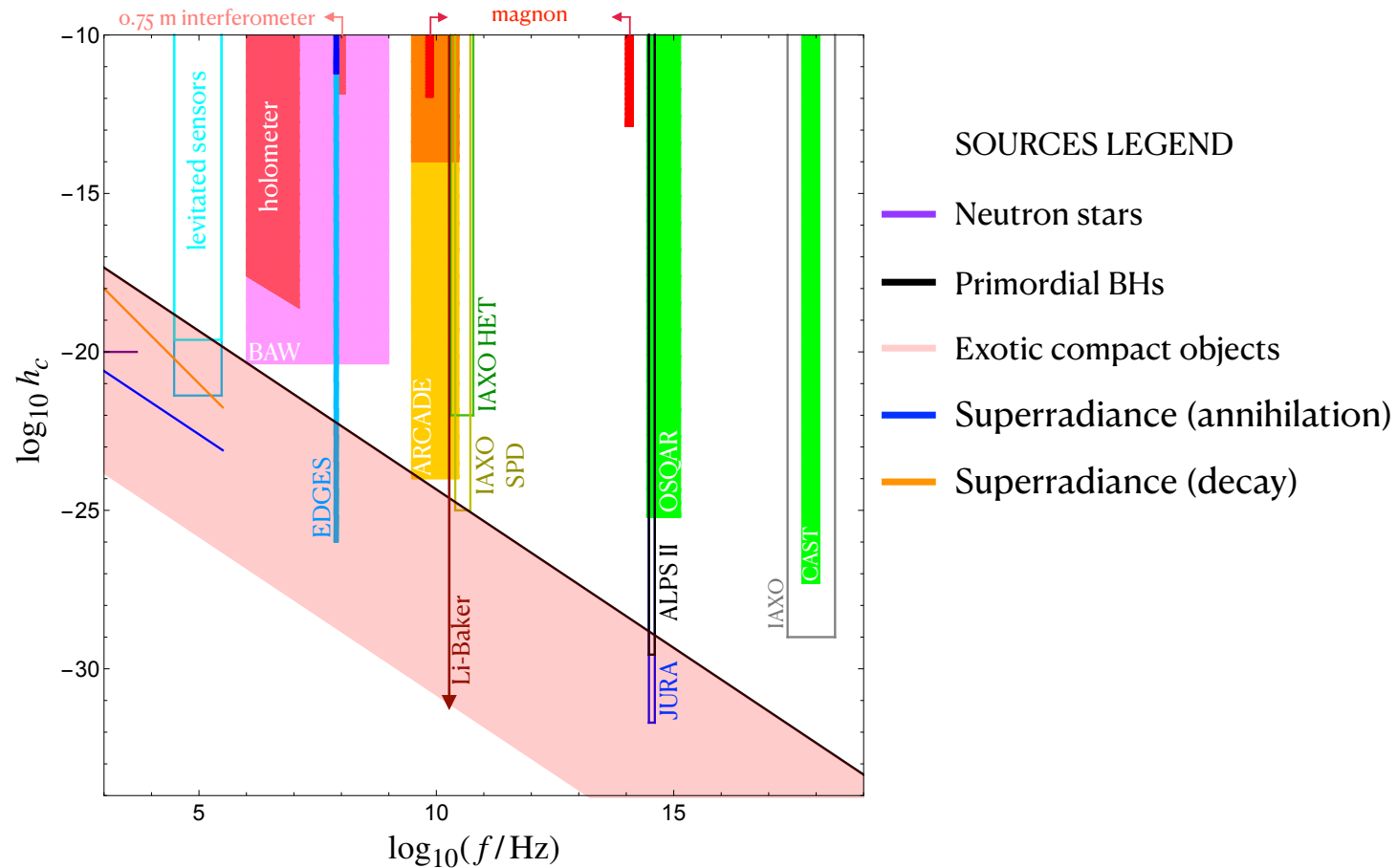
→ at BBN, CMB decoupling ~ 5 % GW energy density allowed

today:  $\frac{\rho_{GW}^0}{\rho_c^0} = \Omega_\gamma^0 \left( \frac{g_s^0}{g_s(T)} \right)^{4/3} \frac{\rho_{GW}(T)}{\rho_\gamma(T)} \leq 10^{-5} \Delta N_{eff} \simeq 10^{-6}$

note: constraint on *total* GW energy

→ today, energy fraction < 10<sup>-6</sup> (for GWs present at BBN / CMB decoupling)

# astrophysical sources



# GW electrodynamics

homogeneous Maxwell equation

$$0 = \nabla_{\mu} F_{\nu\rho} + \nabla_{\nu} F_{\rho\mu} + \nabla_{\rho} F_{\mu\nu} = \partial_{\mu} F_{\nu\rho} + \partial_{\nu} F_{\rho\mu} + \partial_{\rho} F_{\mu\nu}$$

$$\rightarrow F_{\alpha\beta} = \partial_{\alpha} A_{\beta} - \partial_{\beta} A_{\alpha} \quad \text{independent of background metric}$$

inhomogeneous Maxwell equation

$$\nabla_{\nu} (g^{\alpha\mu} F_{\alpha\beta} g^{\beta\nu}) = j^{\mu} \quad \rightarrow \partial_{\nu} (\sqrt{-g} g^{\alpha\mu} F_{\alpha\beta} g^{\beta\nu}) = \sqrt{-g} j^{\mu}$$

$$\text{expand in } h: \quad g^{\alpha\mu} F_{\alpha\beta} g^{\beta\nu} \simeq F^{\mu\nu} - F_{\alpha}^{\nu} h^{\alpha\mu} - F^{\mu}_{\beta} h^{\beta\nu}, \quad \sqrt{-g} \simeq 1 + h/2$$

$$\partial_{\nu} \left( \left( 1 + \frac{h}{2} \right) F^{\mu\nu} - F_{\alpha}^{\nu} h^{\alpha\mu} - F^{\mu}_{\beta} h^{\beta\nu} \right) = \left( 1 + \frac{h}{2} \right) j^{\mu} + \mathcal{O}(h^2),$$

$$\partial_{\nu} F^{\mu\nu} = \left( 1 + \frac{1}{2} h \right) j^{\mu} + \partial_{\nu} \left( -\frac{1}{2} h F^{\mu\nu} + F_{\alpha}^{\nu} h^{\alpha\mu} + F^{\mu}_{\beta} h^{\beta\nu} \right) + \mathcal{O}(h^2)$$

---

$j_{\text{eff}}^{\mu}$



# GW to photon conversion

(inverse) Gertsenshtein effect:

[Gertsenshtein '62, Boccaletti et al '70, Raffelt, Stodolsky '88]

$$(\square + \omega_{\text{pl}}^2/c^2) A_\lambda = -B \partial_z h_\lambda, \quad \square h_\lambda = \kappa^2 B \partial_z A_\lambda$$

$A_\lambda =$  photon

$h_\lambda =$  GW

$B =$  ext. transv. B - field

$\omega_{\text{pl}} =$  plasma frequency

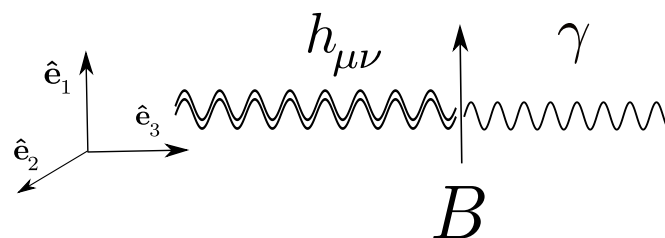
$$\mu^2 = 1 - \omega_{\text{pl}}^2/\omega^2$$

plane waves:

$$\rightarrow \psi(t, z) \equiv \begin{pmatrix} \sqrt{\mu} A_\lambda \\ \frac{1}{\kappa} h_\lambda \end{pmatrix} = e^{-i\omega t} e^{iKz} \psi(0, 0),$$

$$K = \begin{pmatrix} \frac{\mu}{c} \sqrt{\omega^2 + \left(\frac{\kappa B}{1+\mu}\right)^2} & -i \frac{\sqrt{\mu} \kappa B}{1+\mu} \\ i \frac{\sqrt{\mu} \kappa B}{1+\mu} & \frac{1}{c} \sqrt{\omega^2 + \left(\frac{\kappa B}{1+\mu}\right)^2} \end{pmatrix}$$

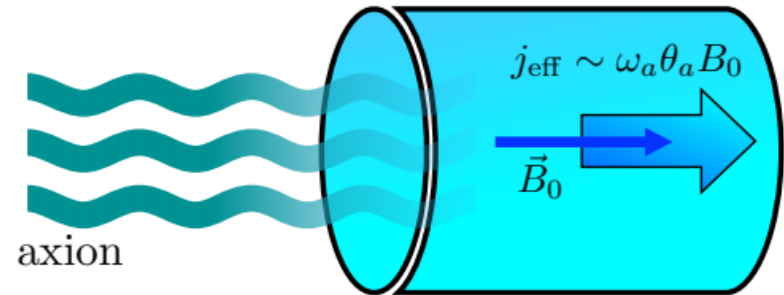
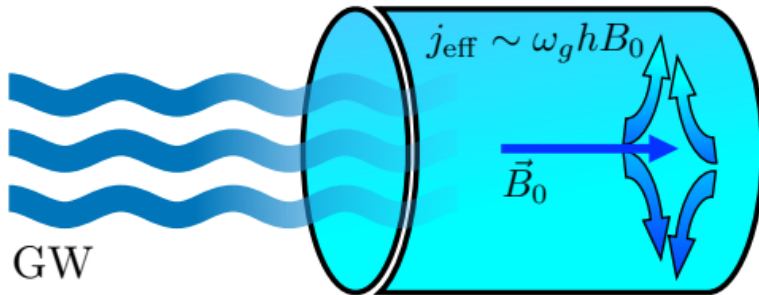
EM wave in curved space time  
(i.e. classical linearized general  
relativity)  $\rightarrow$  purely SM process



analogous to axion to photon conversion

# microwave cavities

[Berlin et al `21]



Projected Sensitivities of Axion Experiments

