TRIeste: Topological Recursion and Integrability

lundi 11 septembre 2023 - samedi 16 septembre 2023

Trieste (IT)

Programme Scientifique

Mini-courses

Introduction to integrable hierarchies: Lax representation and bi-Hamiltonian systems

Guido Carlet, U Bourgogne

After reviewing the classical setting for finite dimensional Hamiltonian integrable systems, we will introduce some examples of integrable hierarchies of PDEs, focusing on their Lax representation. We will then discuss the bi-Hamiltonian formulation of integrable hierarchies and the main results on the classification of Hamiltonian and bi-Hamiltonian structures.

Fock space formalism and applications

Séverin Charbonnier

I will start the mini-course by introducing two Fock spaces: on the one hand, the fermionic Fock space with the semi-infinite wedge formalism (also called the free fermions formalism); on the other hand, the bosonic Fock space with the algebra of symmetric functions. Both formalisms are connected by the Boson-Fermion correspondence. On the fermionic side, I will describe the creation and annihilation operators, the gl(∞) algebra and the Plücker embedding of the Sato Grassmannian.

I will then focus on two important applications of the Fock space formalism. First, it is a convenient tool to express solutions (encoded in tau-functions) to integrable hierarchies such as the Kadomtsev–Petviashvili hierarchy. We will concentrate on polynomial and hypergeometric tau-functions of the KP hierarchy, and show how the Plücker relations defining the Sato Grassmannian are equivalent to the Hirota bilinear equation.

Second, Fock spaces are often used in Hurwitz theory: using Frobenius's formula, one can express the partition function of branched coverings of the sphere in terms of Schur polynomials. Hence, we will describe various types of Hurwitz numbers in the bosonic Fock space and the properties satisfied by their generating functions. Doing so, we will have a glimpse towards other areas, such as intersection theory on the moduli space of Riemann surfaces, topological recursion, and free probability.

Riemann-Hilbert problems: an introduction, and applications

Giordano Cotti, U Lisboa

Riemann–Hilbert problems concern the surjectivity of the monodromy map: does there exist a linear ordinary differential equation with a prescribed monodromy phenomenon?

In these lectures, after briefly recalling basic notions in the theory of linear ODES, the speaker will provide a gentle introduction to the analytic theory of RH problems.

The final part of the mini-course will be devoted to applications of RH problems to enumerative geometry, namely Gromov–Witten theory (closed and open), and even more general F-cohomological field theories.

Introduction to topological recursion

Bertrand Eynard, IPhT

Research talks

Isomonodromic deformations on a torus, Fredholm determinants, and their modular properties

Pavlo Gavrylenko, SISSA

Isomonodromic systems are more complicated non-autonomous versions of some integrable systems. Their solutions can be written in terms of the Fourier series' of Nekrasov partition functions, or equivalently in terms of conformal blocks in 2d CFT, whereas solutions of autonomous systems are expressed in terms of the theta functions.

We will consider the simplest isomonodromic system on a torus, which is the deautonomization of 2-particle elliptic Calogero–Moser system. To study its solution we will express it as a Fredholm determinant given in terms of solution of some auxiliary 3-point problem on a sphere. Expressions in terms of conformal blocks/Nekrasov functions follow from the minor expansion of this determinant, and moreover, it can also be used to find the relation between the tau function of □ and –1/□.

Volumes of moduli spaces of super hyperbolic surfaces

Paul Norbury, U Melbourne

Mirzakhani produced recursion relations between polynomials that give Weil–Petersson volumes of moduli spaces of hyperbolic surfaces. Stanford and Witten described an analogous construction for moduli spaces of super hyperbolic surfaces producing Mirzakhani-like recursion relations between polynomials that give super volumes, achieved in the so-called Neveu–Schwarz case. Both of these situations can be described via corresponding spectral curves where the volumes give the topological recursion correlators. In this lecture I will describe what occurs in the Ramond case of the super construction. It produces deformations of the Neveu–Schwarz volume polynomials again satisfying Mirzakhani-like recursion relations, and corresponding deformations of the topological recursion correlators.

Recursion relations for gauge theories with defects

Alessandro Tanzini, SISSA

We show that the partition function of four dimensional supersymmetric gauge theories in presence of surface defects satisfy non-autonmous Toda equations and show how to use them to get recurrence relations which fully determine the non-perturbative contributions. We present Fredholm determinant solutions for an (infinite)-class of the above equations. If time permits we will also discuss five-dimensional gauge theories, which satisfy q-difference equations.

Integrable features of sine and Airy kernels at finite temperature

Sofia Tarricone, IPhT

The notion of integrable kernels, in which the sine and Airy kernel fit, was introduced in the pioneering work by Its–Izergin–Korepin–Slavnov where Riemann–Hilbert problems were used to study the Fredholm determinants associated to such integrable kernels. They are now considered as universal objects, appearing in limiting behavior of different classical models in integrable probability as random matrix theory, statistical mechanics models, random partitions models and so on. Recently, "finite temperature" versions of these kernels and associated Fredholm determinants appeared in new models. In this talk we explain how classical results known for the sine and Airy kernel extend to their finite temperature analogue, using the Riemann-Hilbert techniques. In particular we provide generalizations of the classical Tracy–Widom formulae in terms of integro-differential Painlevé type equations also connected to integrable PDEs. To conclude, we explain how the Riemann–Hilbert technique is suited also to study other quantities than the Fredholm determinants, known as Jànossy densities. The talk is based on the work ([math-ph] 2303.09848) with T. Claeys, G. Glesner, G. Ruzza and some ongoing work with T. Claeys.