INFORMATION RECOVERY FROM BLACK HOLES

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ENS Summer Institute (Paris).

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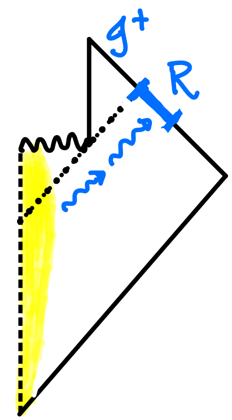
OUTLINE

- Page curve : review
- Island mechanism + BCFTs .
- Throwing in + recovering diaries / shockwaves .
- Summary .

BLACK HOLES $\leftrightarrow$ INFORMATION PROCESSORS.



Collapsing matter $|\Psi_0\rangle$

* Hawking rad. thermal : entropy grows monotonically

(Hawking '76)

* von Neumann entropy : $S_{vN}(R) = \min\left(S_{rad}(t), S_{BH}(t)\right)$ (Page '93, 2003)

$S_{rad}(t) \to \ln d_R$, $S_{BH}(t) \to \ln d_B$.

* **Page curve**:

Radial wave eq. + Regge-Wheeler potential barrier
(ℓ ≠ 0 harmonics transmitted at high freq.)

$\Downarrow$

- Focus on s-wave of rad.

- Slow evaporation

$\Downarrow$

Effective 1+1-d (Eg. JT gravity)

Quasi-equilibrium.

- $$S_{rad} = \underset{\text{\# d.o.f.}}{c}\, \frac{\pi}{6}\, \alpha \int_0^t T(t)\, dt$$

- $$\frac{dM_{BH}}{dt} = -c \int_0^\infty \frac{d\omega}{2\pi} \cdot \frac{\omega\, \Gamma(\omega)}{(e^{\beta\omega} - 1)} = -\frac{\pi c}{12} T^2(t) \cdot \eta$$

Transmitted flux

- $$\dot{M}_{BH} = T\, \dot{S}_{BH} \implies \dot{S}_{rad} = -\xi\, \dot{S}_{BH}$$

- $$\frac{dS_{total}}{dt} = -(\xi - 1)\, |\dot{S}_{BH}|$$

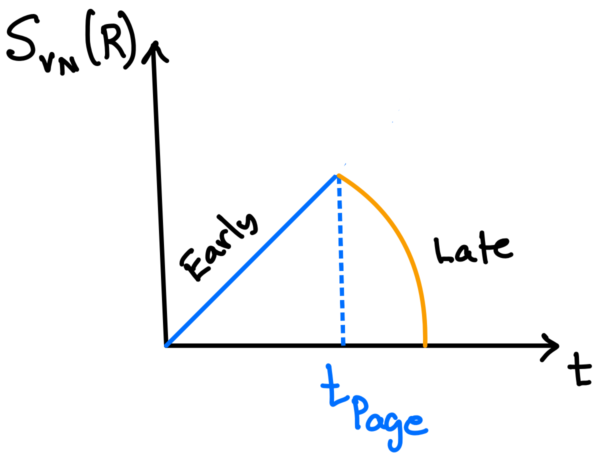
$1 \leq \xi \leq 2$

$\xi = 2$ perfect transmission

$\xi = 1$ Reversible.

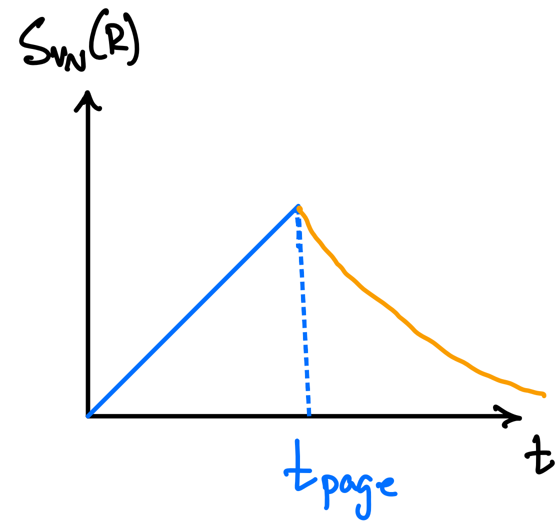
Page curves:

Schwarzschild $\left(M_{BH} = \frac{1}{8\pi T}\right)$



Near extremal RN

$M_{BH} = M_* + \# T^2$



$$t_{Page}: \quad S_{BH}(t_{page}) = \frac{\xi}{\xi+1} S_{BH}(0)$$

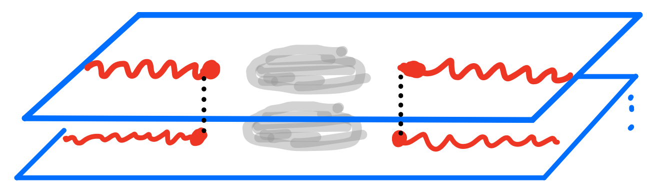
- Late and early modes are correlated.

Page curve from first principles.

- Replica trick : $S_{EE}(R) = \lim_{n \to 1} \frac{\ln \mathrm{Tr}\, \rho_R^n}{1-n}$ Reduced density matrix

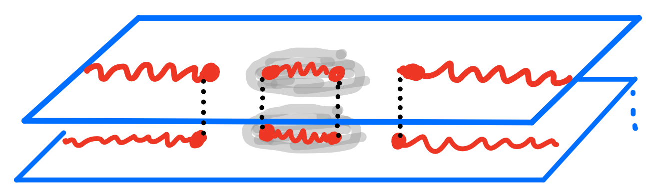
- $\mathrm{Tr}\, \rho_R^n$: Semiclassical gravity saddle points AHMST PSSY (2019)

1. "Hawking" or "no-island" saddle.



(Euclidean picture).

2. Replica wormhole or "island" saddle.



(Lorentzian: Marolf-Maxfield)

* Generalized entropy / Island formula.

[Engelhardt-Wall (2014) ...
AHMST (2019) ; PSSY (2019)

$$S_{gen}(R) = \underset{\substack{I \\ \{\partial I\}}}{\text{Min ext}} \left[\sum_{\partial I} \frac{\text{Area}(\partial I)}{4G_N} + S_{rad/QFT}(R \cup I) \right]$$

Islands

Quantum extremal surface (QES)

Island $\subset$ Hilbert space of R

* Generalized Renyi entropies ? $(n \neq 1)$

↓

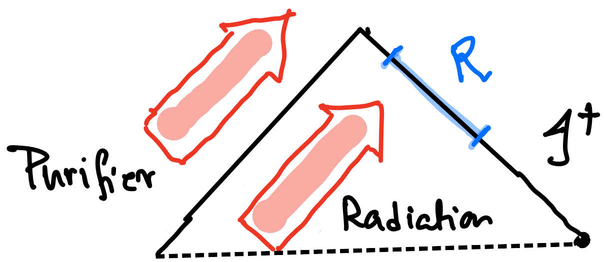
- Replica wormhole problem in AdS_2 + bath (JT)

- Uniformisation problem + Liouville conformal blocks.

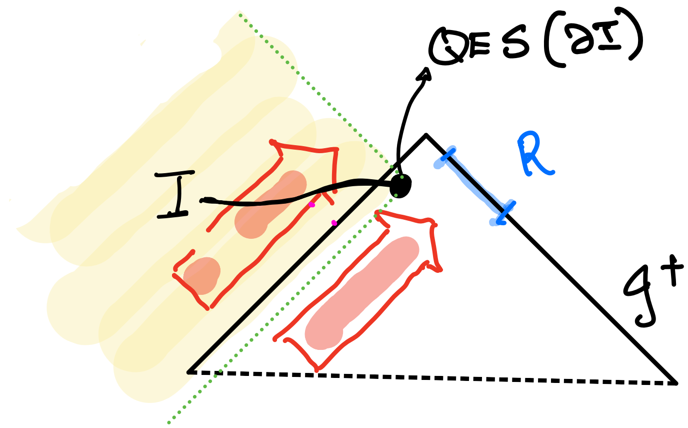
- Generalised Renyi entropies in certain (factorisation) limits.

(Hollowood - SPK - Piper in preparation).

• Post t_{Page} : Island $\mathcal{I}$ gathers purifiers of R.



Early time



Late / old BH

"NO-ISLAND" SADDLE

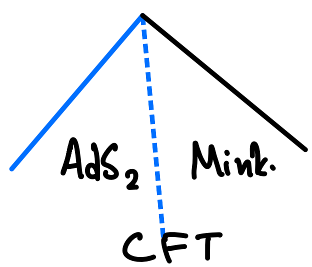
"ISLAND" SADDLE

Purifier of R gathered in $\mathcal{I}$

Islands + BCFT

Semiclassical picture

(JT + bath).

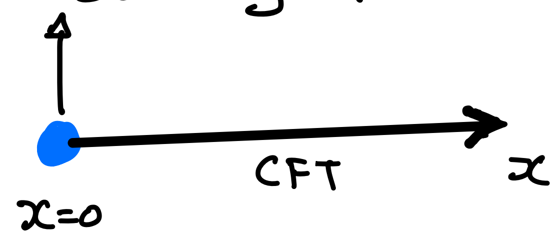


AdS_2 BH + CFT bath.

$\longleftrightarrow$ Holography

Microscopic description

Boundary QM



$\big\downarrow$ IR

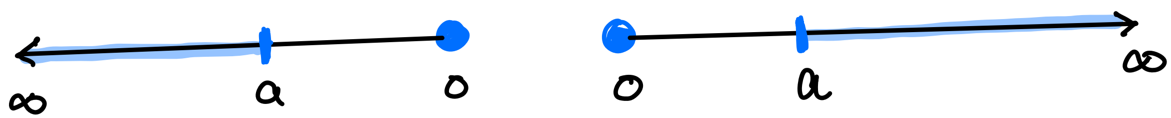
CFT with boundary

(BCFT).

BCFT in Thermofield double state

- $$|\Psi_{TFD}(t)\rangle = \sum_n \underbrace{e^{-2iE_n t}}_{e^{-i(H_L+H_R)t}} \, e^{-\beta E_n/2} \, |E_n\rangle_L \otimes |E_n\rangle_R$$

- ### Two semi-infinite intervals



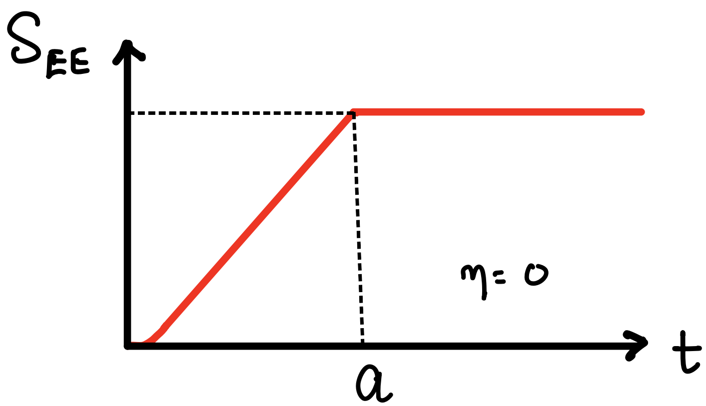
$$A = A_L \cup A_R .$$

- $$S_{EE}(A) = \lim_{n\to 1} \frac{1}{(n-1)} \ln \langle \bar{T}_n(x_L) \; T_n(x_R) \rangle$$

Twist - antitwist

BCFT Doubling trick.

- $\langle \bar{\tau}^L \tau^R \rangle_{BCFT} \longrightarrow \langle \bar{\tau}^L \tau^R \bar{\tau}^R \tau^L \rangle_{Plane}$. Chiral correlator on plane.

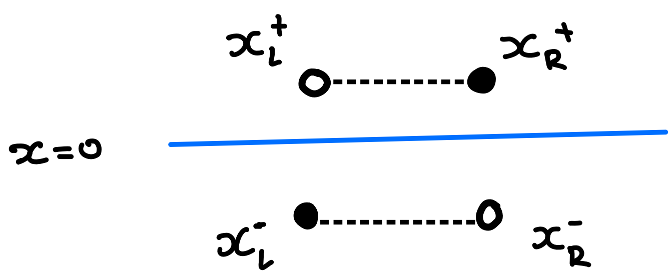
- 

  Exact CFT result smoothly interpolates between 2 channels.

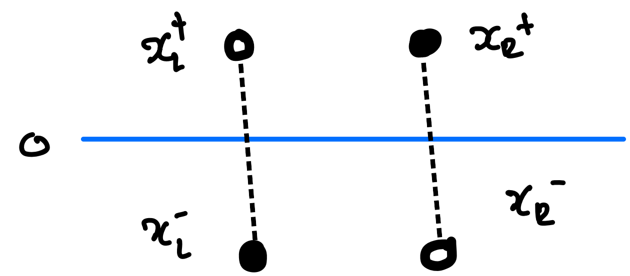
- High T limit: $S_{EE}(A) = \min\left(\frac{2\pi c t}{3\beta} , \underbrace{2 \ln g_b}_{\text{boundary entropy.}} + \frac{2\pi c}{3\beta} a \right)$

Page time $\longleftrightarrow$ Exchange of OPE channels.

CONNECTED / BULK CHANNEL

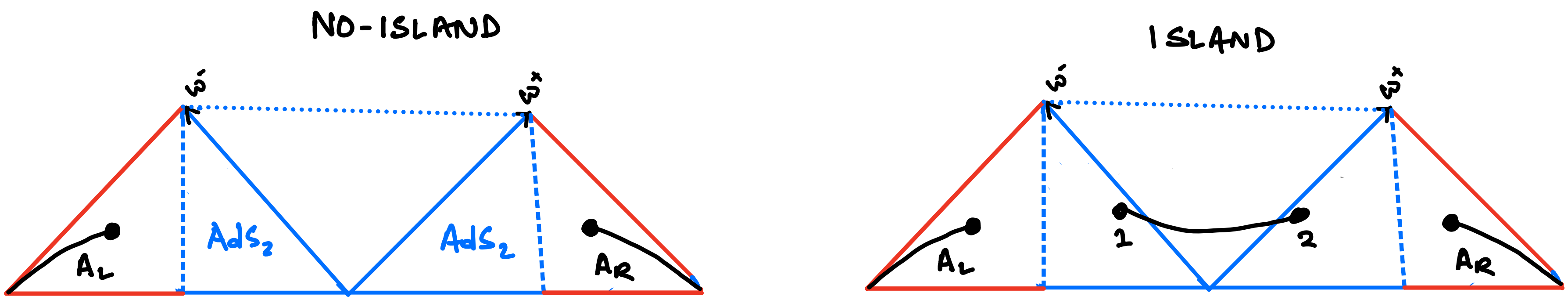


BOUNDARY CHANNEL



- BCFT images $\longleftrightarrow$ Quantum extremal surfaces
- Boundary entropy $\ln g_b$ $\longleftrightarrow$ S_{BH}

GRAVITY PICTURE (JT + bath)

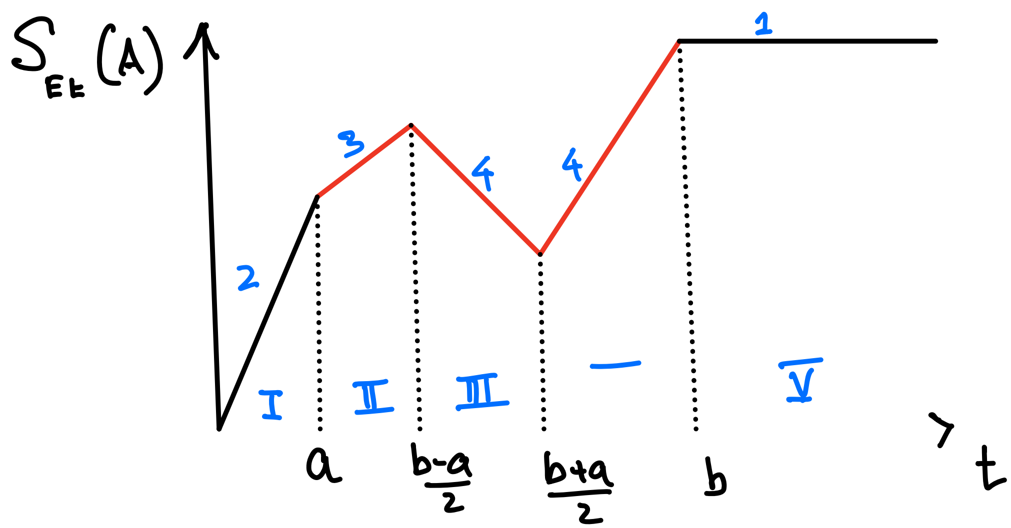


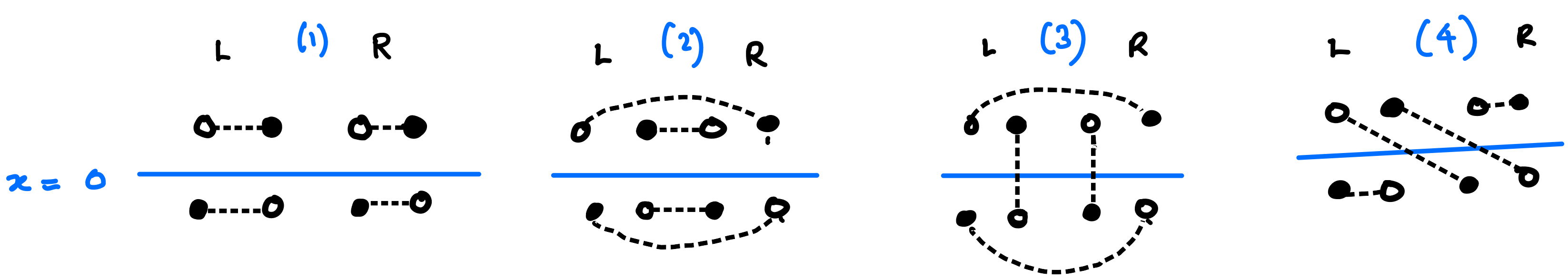
$$S_{gen}(A_L \cup A_R) = \underset{1,2}{\text{ext}} \left[\sum_{1,2} \frac{\phi(w^\pm)}{4G_N} + S_{CFT}(A_L \cup A_R \cup I) \right]$$

$$S_{EE}(A) = \min \left(\underbrace{\frac{2\pi c}{3\beta} t}_{\text{no-island}} , \; \underbrace{2 S_{BH} + \frac{2\pi c}{3\beta}}_{\text{island}} \cdot a \right)$$

- ISLANDS → More intricate features of info. recovery

- E.g. 2 finite intervals in BCFT: $(a,b)_L \cup (a,b)_R$





(Hollowood - SPK - Legramandi - Talwar arXiv: 2109.01895)

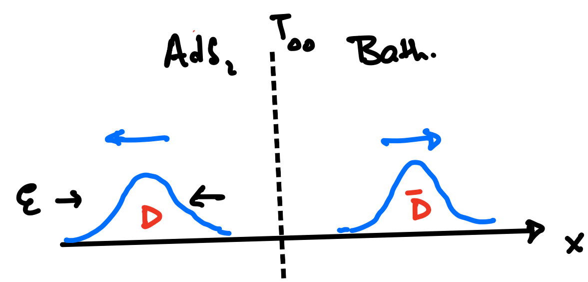
RECOVERING A "DIARY"

(Gyongyosi-Hollowood-SPK-Legramandi-Talwar; 2209....)

- "DIARY" = Shockwave carrying energy + entropy.

- CFT Local Quench:

$$\rho_{\theta} = \mathcal{O}(t_0 \pm i\varepsilon)\ \rho_0\ \mathcal{O}^{+}(t_0 \mp i\varepsilon)$$

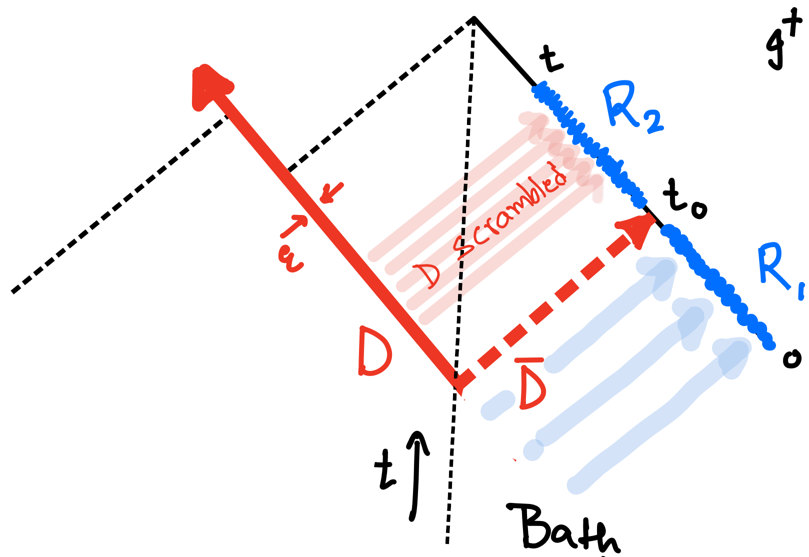


$\bar{D}$ purifier of D

$S_D = S_{\bar{D}}$

- $$T^{\text{shock}}_{\substack{++\\--}} \approx \frac{\Delta_\theta}{\varepsilon}\, \delta(x^{\pm} \mp t_0)$$

- $E_D = \dfrac{\Delta_\theta}{\varepsilon}$
- $S_D << S_{BH}(t_0)$.



- How is D returned in $R = R_1 \cup R_2$?

- MUTUAL INFORMATION:

$$I(R, \bar{D}) = S(R) + S(D) - S(R \cup \bar{D})$$

- Given: $S(R \cup BH \cup \bar{D}) = 0$

SOLVING BACKREACTED JT background

- Energy balance at boundary:

$$\frac{dE}{dt} = T_{--}^{\text{shock}} + \frac{c}{24\pi}\{f(t), t\}$$

AdS$_2$ boundary "particle"

$$E = -\frac{c}{24\pi k}\{f(t), t\}$$

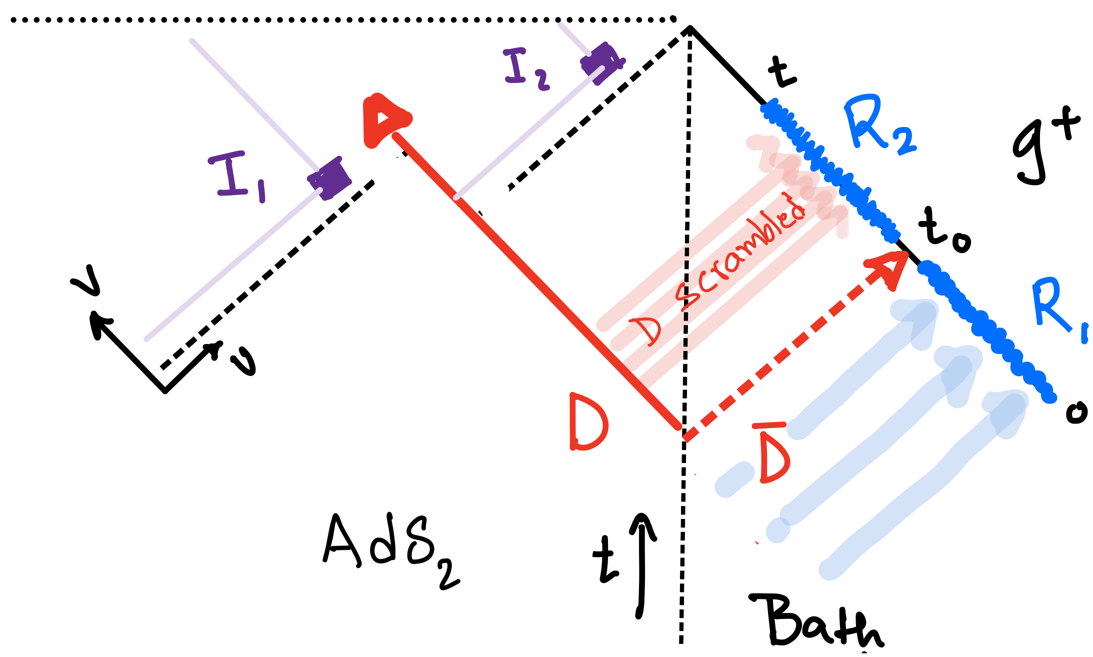
- Solve for $f(t)$, then obtain dilaton Φ (area term)

(Hollowood - SPK 2004.14944)

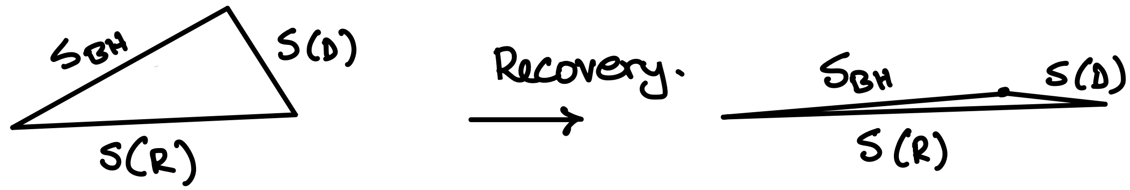
- Find QES for $S(R)$ & $S(R \cup \bar{D})$ after extremizing S_{gen}

Two relevant saddles

- I_1 : QES in front of shockwave
  I_2 : QES behind shockwave.



- Recovery when $I(R, \bar{D}) = 2\,S(D)$

- 

3 saddles

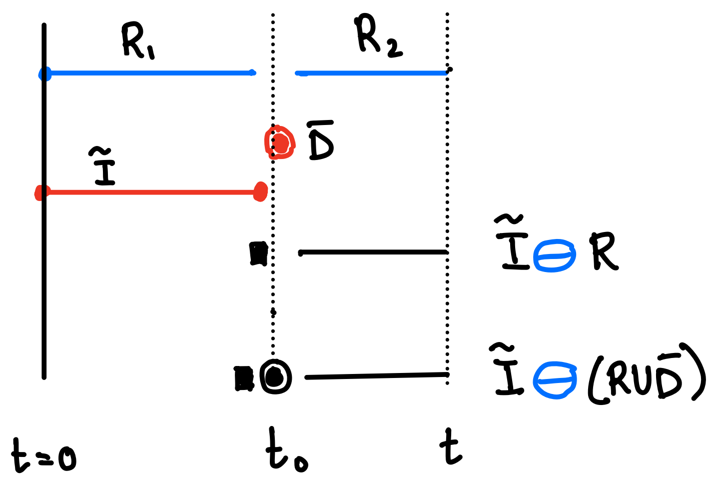
- No island: Φ

$$S(R) = S_{rad}(R_1) + S_{rad}(R_2) + S(D)$$
$$S(R \cup \bar{D}) = S_{rad}(R_1) + S_{rad}(R_2).$$

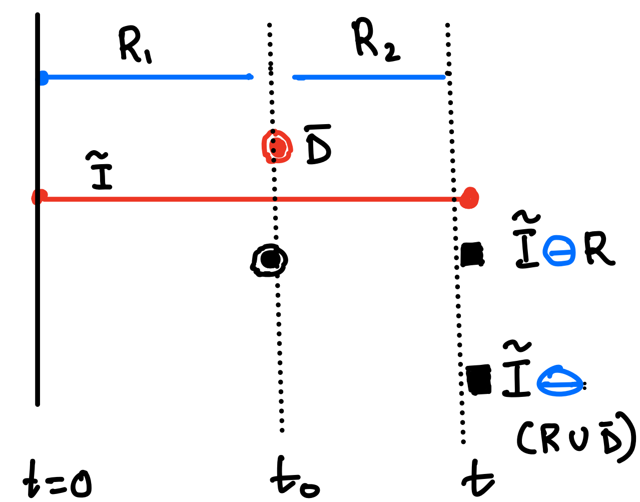
- Island 1: I_1

- Island 2: I_2

I_1



I_2



$$S_{I_1}(R) = S_{rad}(R_2) + S_{BH}(t_0)$$

$$S_{I_2}(R) = S_{BH}(t) + S(D)$$

$$S_{I_1}(R \cup \bar{D}) = \partial \tilde{I} + S_{rad}(R \cup \bar{D} \ominus \tilde{I})$$

$$= S_{rad}(R_2) + S_{BH}(t_0) + S(D)$$

$$S_{I_2}(R \cup \bar{D}) = S_{BH}(t)$$

DIARY RECOVERY SEQUENCE

Case I: $t_0 < t_{Page}$.

- Early times: $t_0 < t < t_{Page}$. No-island Φ

No correlations

$$S(R) = S_{rad}(R_1) + S_{rad}(R_2)$$

$$S(R \cup \bar{D}) = S_{rad}(R_1) + S_{rad}(R_2) + S(D)$$

$$I(R, \bar{D}) = 0 .$$

- Little before t_{Page}:

$S(R \cup \bar{D})$ transits to I_2 : $S_{I_2}(R \cup \bar{D}) = S_{BH}(t)$.

but still $S(R) = S_{\Phi}(R)$

When: $S_{rad}(R) = S_{BH}(t) - S(D)$.

In this phase $S_{I_2}(R \cup \bar{D}) \leq S_{\phi}(R \cup \bar{D})$

$$\therefore \quad I(R, \bar{D}) = S_{rad}(R) + S(D) - S_{I_2}(R \cup \bar{D})$$

$$= S_{rad}(R) + S(D) - S_{BH}(t) \geq 0$$

* Mutual info rises continuously

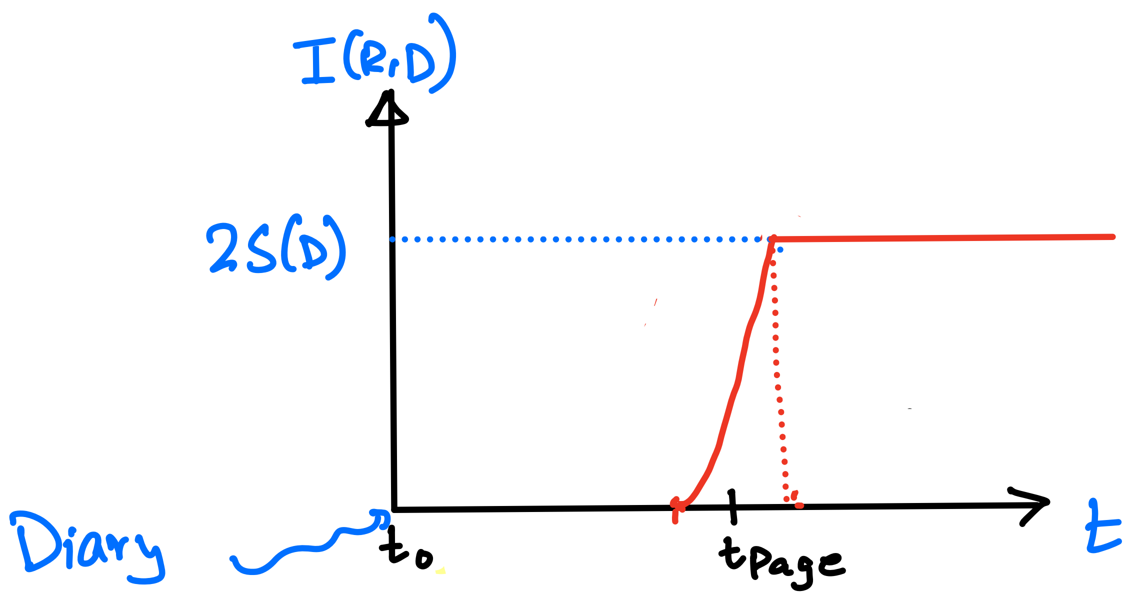
Info recovery begun!

- Final step: little after t_{page}.

$S(R)$ also transits to I_2

$$S_{I_2}(R) = S_{BH}(t) + S(D) \rightarrow I(R, \bar{D}) = 2S(D)$$

(Δ-inequality saturated).



Case II : $t_0 > t_{Page}$

- Now $S(R)$ is already at island I_1

  and so is $S(R \cup \bar{D})$

$$I(R, \bar{D}) = S_{I_1}(R) + S(D) - S_{I_1}(R \cup \bar{D}) = 0$$

No immediate correlations.

- D-recovery begins when:

$$S_{rad}(R_2) \geq \Delta S_{BH}(t) - S(D)$$

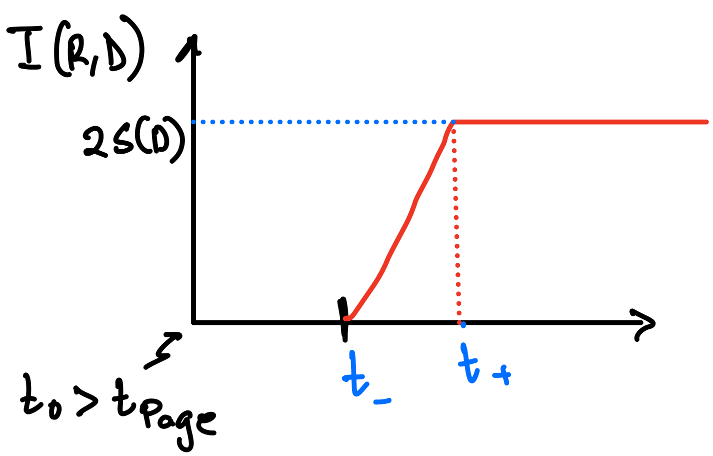
RHS > 0 requires. Gen. Second Law.

$$S(R \cup \bar{D}) = S_{I_2}(R \cup \bar{D})$$

- $I(R,\bar{D}) \geqslant 0$ & Recovery begins after non-zero delay

- $t_{delay} \sim \dfrac{\Delta S_{BH} - S(D)}{T}$ = "processing time" on top of scrambling time.

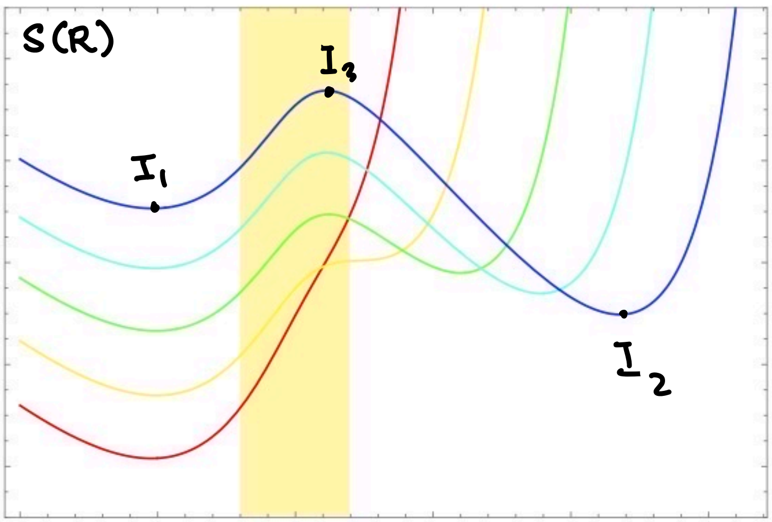
- D-recovery complete when $S(R) = S_{I_2}(R)$

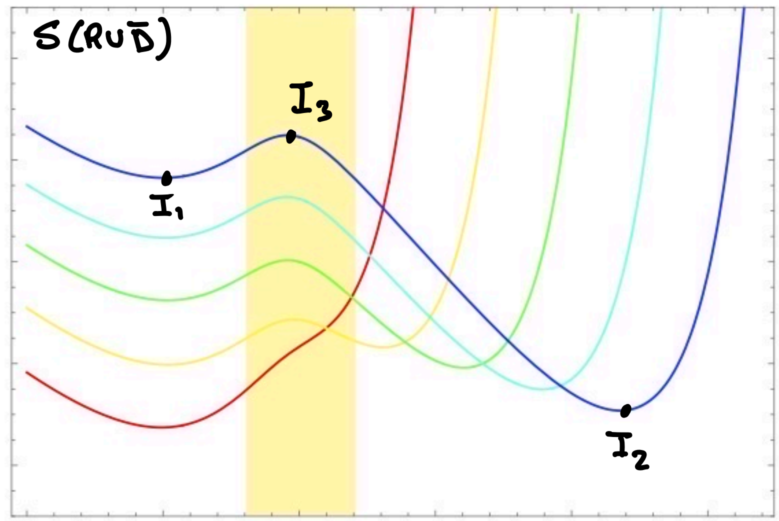


$$t_{\pm} = \frac{\Delta S_{BH} \pm S(D)}{T}$$

S_{gen} for finite pulse (numerical)

"OFF-shell" sweep





- I_3 (local maximum) : Python's snack

Complexity of decoding D. : $C(U_{dec.}) \sim e^{\frac{(S_{I_3} - S_{I_1})}{2}} = e^{\frac{\Delta S_{BH} + S(D)}{2}}$

Summary / Conclusions

- Island mechanism & BCFT channels/factorisation.
  See more generally, e.g. SYK + CFT (Y. Chen-Qi-Zhang)

- Islands capture intricate features of info. recovery. Microscopic picture?

- Can we identify corresponding mechanism in a microscopic framework e.g. fuzzballs?

- What do we expect physics behind horizon is for post Page time BHs?

THANK YOU!