# Non-relativistic corners in String Theory and AdS/CFT

ENS Summer Institute, Paris, July 3, 2023

Costas Kounnas/Eugene Cremmer memorial session

Niels Obers (Nordita & Niels Bohr Institute)

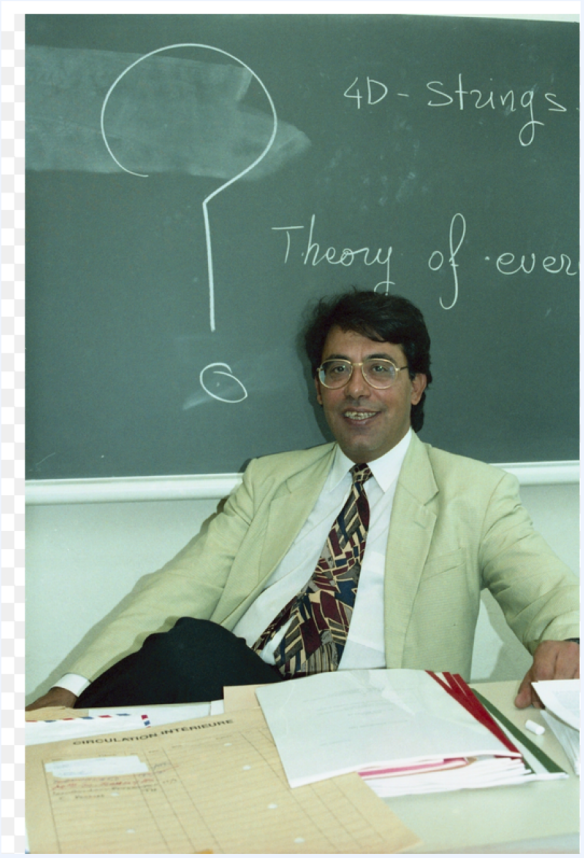



research part of the talk based on work:

to appear **230y.xxxxx** (Bidussi,Harmark,Hartong,NO,Oling)

**2107.006542** (JHEP) (Bidussi,Harmark,Hartong,NO,Oling)

and earlier papers 2011.02539 (JHEP), 1907.01663 (JHEP), 1810.05560 (JHEP), 1705.03535 (PRD)




1st encounter: Berkeley, 1988 (or 89?)
(while PhD student)
& later: Paris, CERN & numerous workshops in Europe
(from the Greek islands to Copenhagen)

1st encounter: early 1990's at ENS SI
& and many subsequent occasions, during which he always made me feel very welcome at ENS

## Costas as a close colleague/friend

- especially during CERN years I learned an amazing amount of string theory from him (and Elias) during weekly teaching sessions (for all interested Fellows) as well as private discussions

- generous with time/knowledge
- warm, honest and frank
- great at bringing people together/encouraging young researchers
- always full of passion/interest in both physics and beyond

for probably any past/current/future researcher
in string theory/gravity/field theory:

the work of both Costas and Eugene has made a huge impact and
has had a tremendous influence in our research field

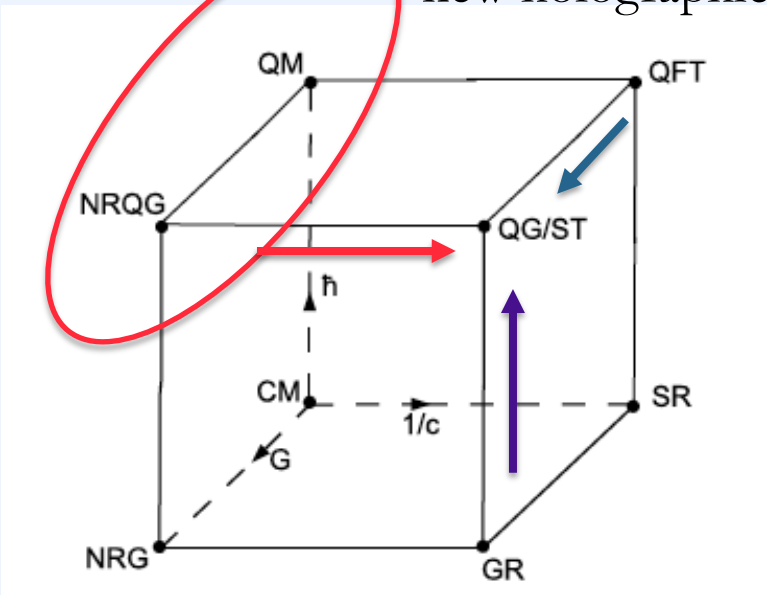
for me in particular, work on :
- (I)CFT
- threshold corrections in string theory (proud to have 1 paper with CK ! )
- string theory/supergravity dualities
- black (super)gravity solutions and AdS/CFT correspondence
- …
- current research

but of course their work touches upon a much wider universe !

# Non-relativistic physics and cube of physical theories

$(\hbar, G_N, 1/c)$

new holographic dualities ?




a third route towards
(relativistic) quantum gravity

how does this fit with
string theory/holography ?

already (classical) non-relativistic gravity (NRG)
is more than just Newtonian gravity

# Non-Lorentzian geometries

recent progress in understanding non-relativistic corners of:
gravity, quantum field theory and string theory:

→ builds on improved understanding of non-Lorentzian geometries
= spacetimes with local symmetries other than Lorentz

NL geometries appear in:

- bdry geometries in non-AdS holography (e.g. Lifshitz  flat space)
- covariant formulations of PN approximation in GR
- covariant formulations of non-Lorentzian fluids and CMT systems (FQHE, fractons, .. )
- Horava–Lifshitz gravity,  non-relativistic versions of CS, JT
- cosmology, black hole horizons, ultra-local GR (Carroll)
- double field theory
- **non-relativistic corners of String Theory**
- **near-BPS limits of string theory on AdS5 × S5**

# Why non-relativistic (NR) string theory ?

- can we learn more about ordinary string theories from NR techniques

- what is landscape of UV complete non-Lorentzian theories (certain NR strings contained in double field theory)

- D-branes/open strings in non-Lorentzian string theory?

- can we build explicit examples of holographic dualities ?

- is NRQG a well-defined corner and does it have a ST description ?

# Main messages

NR string theory is self-consistent corner of ST
- from near-critical B-field (Gomis,Ooguri/Danielsson et al, 2000)

→ curved spacetime generalization (still 2D CFTs): (since 2017)

strings with Lorentzian worldsheet/probing non-Lorentzian target space time

Further world-sheet limit gives:
Galilean string (with non-relativistic worldsheet)
→ novel class of sigma models that includes NR AdS/CFT corners

(Note: dual limit gives Carrollian string - relevant for AdS/CFT ? )

- Galilean strings naturally appear in the AdS/CFT correspondence
- NR strings from near-BPS limits in N=4 SYM

# NR strings

NR strings on flat spacetime = Gomis-Ooguri string

Gomis,Ooguri(2000); Danielsson et al.(2000);

→ Non-Lorentzian (stringy Newton-Cartan) geometries
when spacetime is curved

Andringa et al (2012), Harmark,Hartong,NO(2017); Bergshoeff,Gomis,Yan(2018);
Harmark,Hartong,Menculini,NO,Yan(2018); Gomis,Oh,Yan(2019);
Gallegos,Gursoy,Zinnato(2019), Harmark,Hartong,Menculini,NO,Oling(2019);
Bergshoeff,Gomis,Rosseel,Simsek,Yan(2019); Kluson (2018/19), …….
Yan (2021), Bergshoeff,,Lahnsteiner,,Romano,Rosseel,Simsek(2021);
Bidussi, Harmark,Hartong,NO,Oling(2021); …. (& many more refs since then)

also:

- tensionless strings — e.g.Lindstrom,Sundborg,Theodoridis(1991) Bagchi,Gopakumar(2009) Bagchi,Banerjee,Parekh(2019)
- Galilean strings — Battle,Gomis,Not(2016))
- relation to double field theory — Ko,Melby-Thompson,Meyer,Park(2015) Morand,Park(2017);Berman,Blair,Otsuki(2019);Blair(2019)

# NR strings (on flat spacetime)

Gomis,Ooguri(2000); Danielsson et al.(2000)

zero Regge slope limit of relativistic string theory in near-critical B-field

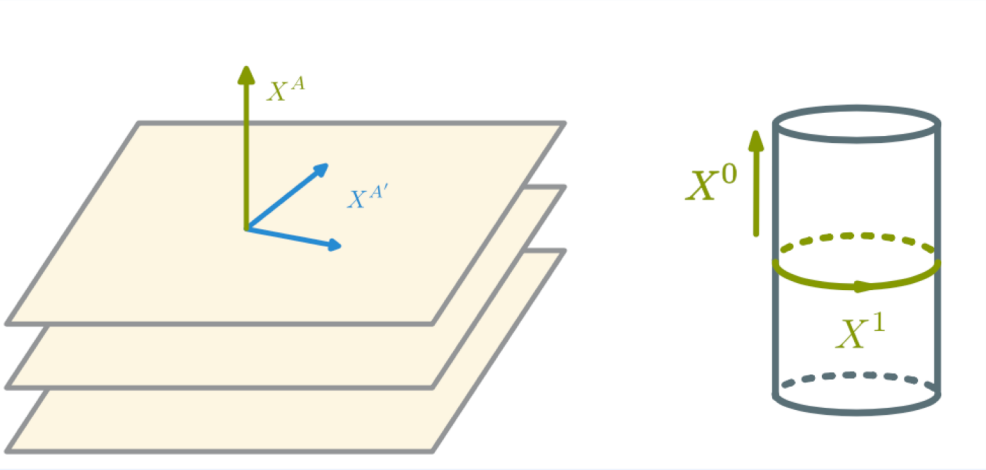
$$S = \frac{1}{4\pi\alpha'} \int_\Sigma d^2\sigma \sqrt{h} \left( h^{\alpha\beta}\, \partial_\alpha X^{A'}\, \partial_\beta X_{A'} + \lambda\, \bar{\mathcal{D}} X + \bar{\lambda}\, \mathcal{D} \overline{X} \right),$$

in conformal gauge:

$$S = \frac{1}{4\pi\alpha'} \int_\Sigma d^2\sigma \left( \partial_\alpha X^{A'}\, \partial^\alpha X_{A'} + \lambda\, \bar{\partial} X + \bar{\lambda}\, \partial \overline{X} \right),$$

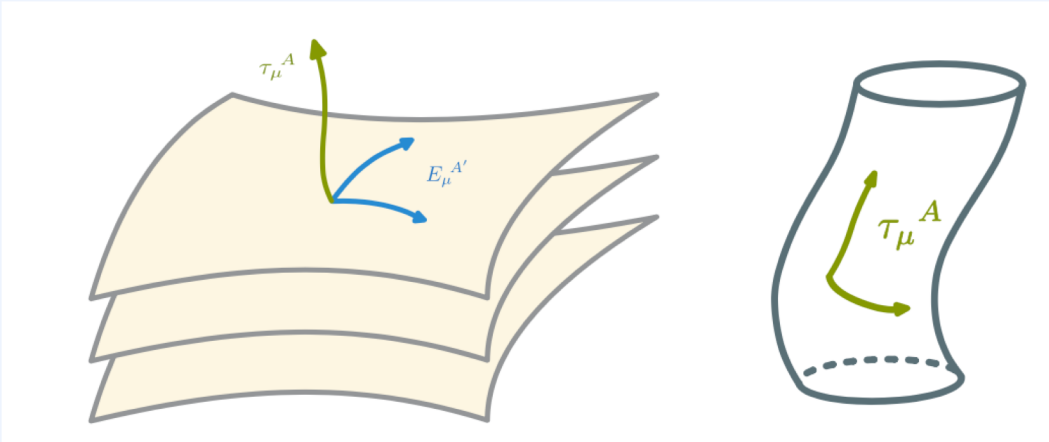
- Galilean invariant dispersion relation
- no massless physical states
- low-energy effective theory described by Newton-like gravity
- all asymptotic states carry non-zero winding along (compact) X1
- space-time S-matrix with NR symmetry

# Gomis/Ooguri NR string lives in flat space




figures from
review on NRST
Gerben Oling & Ziqi Yan
(2202.12698)

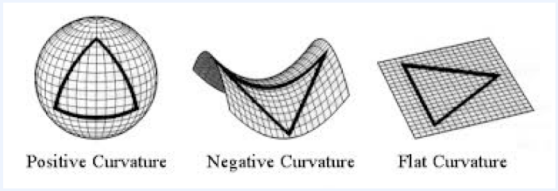
Q: what is the general target space probed by NR strings ?

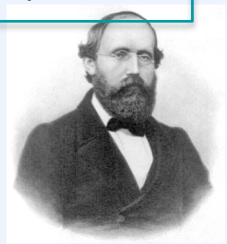


# Space-Time symmetries and Geometry

local symmetries of space and time ←→ geometry of space and time

Einstein: Poincare ←→ (pseudo-)Riemannian geometry



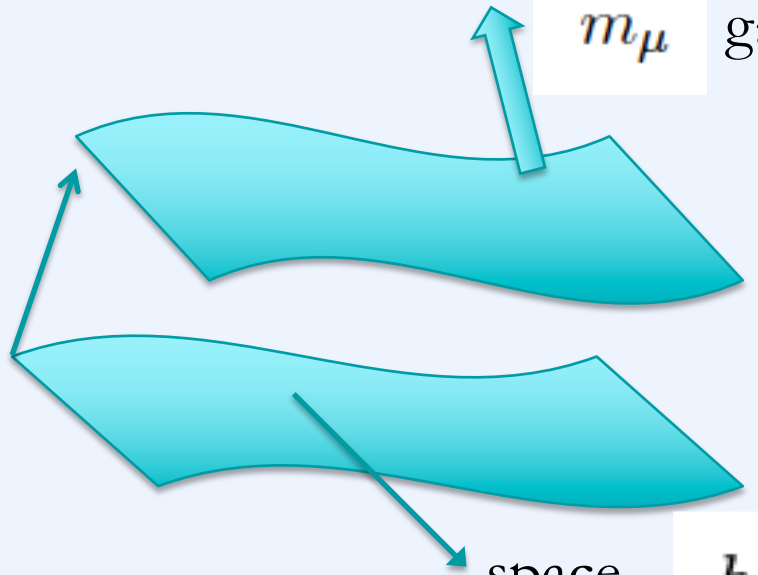



Cartan: Galilean/Bargmann ←→ Newton-Cartan geometry

[Eisenhart,Trautman,Dautcourt,Kuenzle,Duval,Burdet,Perrin,Gibbons,Horvathy,Julia,Nicolai,…] ..

- geometrize Poisson equation of Newtonian gravity
  falling observers see Galilean laws of physics

# torsional Newton-Cartan geometry (NR particles)




gauge field (of mass generator)

$$\nabla_\mu \tau_\nu = 0$$

gives connection with intrinsic torsion $\sim d\tau$

NC = no torsion → $d\tau = 0$ absolute time

TTNC = twistless torsion → $\tau \wedge d\tau = 0$ preferred foliation equal time slices

TNC no condition on $\tau_\mu$

Christensen,Hartong,NO,Rollier(2013)

NC in FQHE: Son(2013);Geracie,Son,Wu,Wu (2014)

# Coupling of non-relativistic particle to TNC geometry

two routes:

- null-reduction of relativistic particle
- c → infinity of extremal charged particle

$$q = mc^2.$$

$$S = \frac{m}{2}\int \frac{h_{\mu\nu}\dot{X}^\mu \dot{X}^\nu}{\tau_\rho \dot{X}^\rho} d\lambda - m\int m_\mu \dot{X}^\mu d\lambda$$

[Kuchar],
[Bergshoeff et al]

kinetic term

potential term:

coupling to $m_\mu$

$m_0$ ~ Newtonian potential

$$T^\mu = m\int d\tau\, \partial_\tau X^\mu \delta(x - X(\tau))$$

mass current

- action has TNC local target space symmetries

# Non-relativistic string on curved spacetime

two routes:

- null reduction of relativistic string action
  (`light-like T-duality' along null isometry)
- c → infinity limit in near-critical B-field of relativistic string action
  (F-strings extremally charged under B-field: tension = charge)

→ NR strings move in torsional string Newton-Cartan geometry:

- Riemannian geometry (transverse) fibered over 2-dimensional Lorentzian base (longitudinal)

- pullback of longitudinal base on the worldvolume
  = Lorentzian metric on worldsheet

# NRST action on TSNC target space

- Nambu-Goto form:

kinetic potential

$$S_{\rm NR} = -\frac{T}{2}\int d^2\sigma \left[\sqrt{-\tau}\,\eta^{AB}\tau^{\alpha}_{A}\tau^{\beta}_{B}h_{\alpha\beta} + \epsilon^{\alpha\beta}m_{\alpha\beta}\right],$$

A = 0,1 : longitudinal directions
a= 2,…D-1: transverse directions
alpha = sigma,tau: world-sheet

$$h_{\mu\nu} = e^{a}_{\alpha}e^{b}_{\beta}\delta_{ab}$$

torsional string Newton–Cartan geometry : $\tau^{A}_{\mu}, \quad h_{\mu\nu}, \quad m_{\mu\nu}.$

$m_{\mu\nu}.$ couples to worldsheet tension current

$$J^{\mu\nu}_{\rm T} = T\int d^2\sigma\,\epsilon^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu \delta(x - X(\sigma^\alpha)),$$

TSNC Polyakov action:

$$S = -\frac{T}{2}\int d^2\sigma\Big[\sqrt{-\gamma}\,\gamma^{\alpha\beta}\partial_\alpha X^M \partial_\beta X^N h_{MN} + \epsilon^{\alpha\beta}\partial_\alpha X^M \partial_\beta X^N m_{MN} + \lambda\epsilon^{\alpha\beta}e^{+}{}_{\alpha}\tau^{+}{}_{M}\partial_\beta X^M + \bar{\lambda}\epsilon^{\alpha\beta}e^{-}{}_{\alpha}\tau^{-}{}_{M}\partial_\beta X^M\Big],$$

# Symmetries of the TSNC action

- 2-form gauge syms: $\bar{\delta} m_{\mu\nu} = 2\partial_{[\mu}\lambda_{\nu]}.$

- transverse string Galilean boosts:

$$\bar{\delta} h_{\mu\nu} = -\lambda_{Ab}\left(\tau^{A}_{\mu} e^{b}_{\nu} + \tau^{A}_{\nu} e^{b}_{\mu}\right) \quad , \quad \bar{\delta} m_{\mu\nu} = -2\epsilon_{AB}\lambda^{B}{}_{c}\tau^{A}_{[\mu} e^{c}_{\nu]}.$$

A = 0,1 : longitudinal directions
a= 2,…D-1: transverse directions

- longitudinal Lorentz boosts

→ string analogue of the symmetries of NR particle coupling to Newton-Cartan

# Remarks

- beta functions/effective spacetime actions for the NR string obtained in various different formulations/using different methods

Gomis,Oh,Yan(2019); Bergshoeff,Gomis,Rosseel,Simsek,Yan(2019); Yan,Yu(2019)
Bergshoeff et al (2021); Yan (2021); Gallegos,Gursoy,Zinnato(2019);
Gallegos,Gursoy,Verma,Zinnato(2020);….

→ describe the dynamics of (versions of) non-relativistic (super-) gravity

- limits vs. expansions

- limit geometry: type I (cancellation of divergent term)
- geometry from expansion: type II (each term in the action generates more gauge fields)

van den Bleeken (2018), Hansen,Hartong,NO (2019, 2020)

Hartong,Have (2021,2022)

# subsequent worldsheet scaling limit: two new classes of sigma models from NRST

non-relativistic strings on curved spactime:
torsional string Newton-Cartan geometry

$\tilde{c} \to \infty$

'worldsheet speed of light' limit

$\tilde{c} \to 0$

Galilean string

Carrrollian string

Galilean worldsheet structure

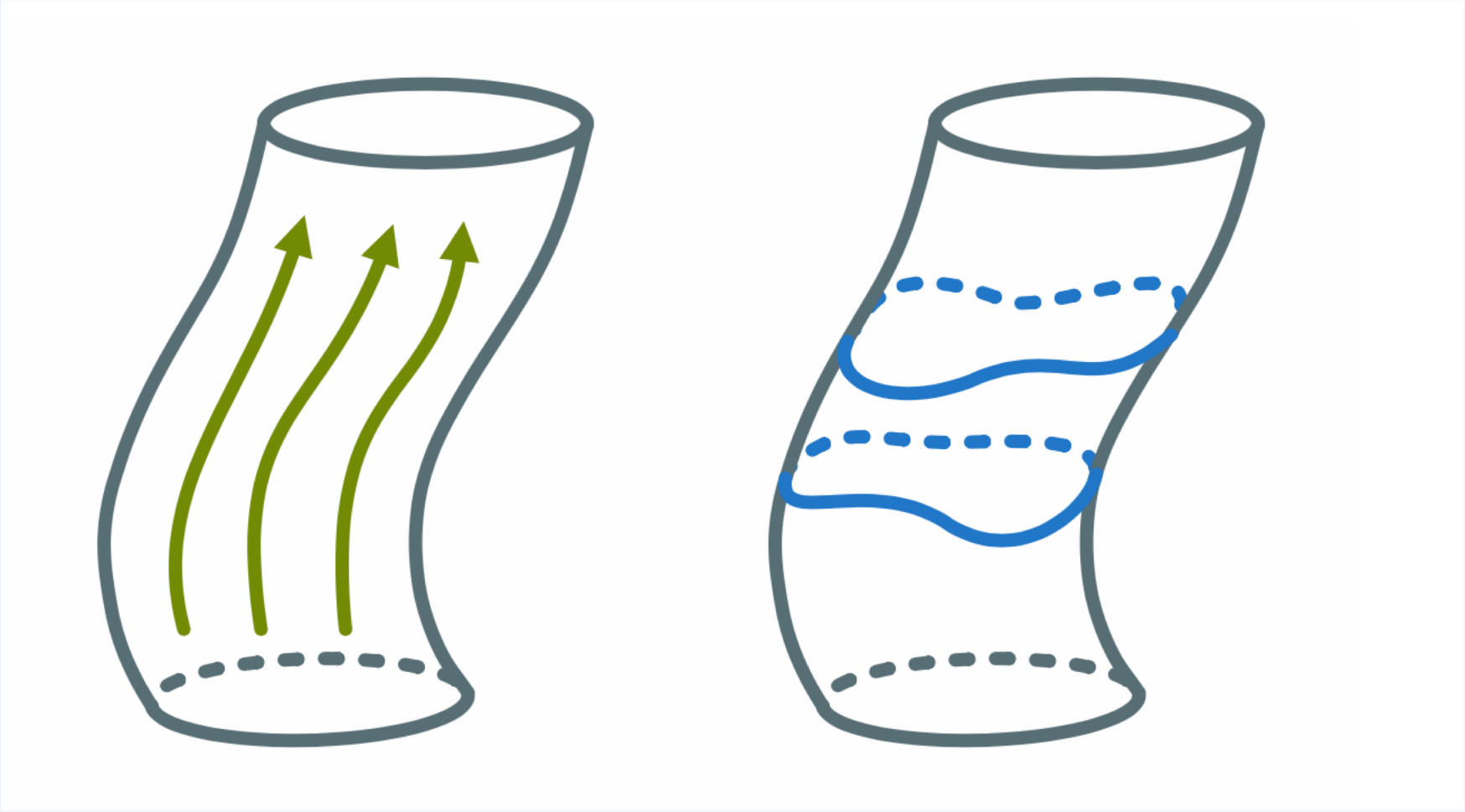
Virasoro x R (subalgebra of GCA)

Carrollian world-sheet structure

BMS3 symmetry

includes for subclass of spacetimes:
ST description of near-BPS SMT decoupling limits of AdS/CFT

# Visualisation of world-sheet



(drawing: courtesy of Gerben Oling)

# NR strings from N=4 SYM

- Spin Matrix theory (SMT) limits of AdS/CFT obtained by zooming in on unitarity bounds of N=4 SYM on RxS3: Harmark/Orselli (2014)

$$\lambda \to 0 \quad , \quad \frac{E-Q}{\lambda} = \text{fixed}$$

Q = linear sum of Cartan charges of PSU(2,2|4)

→ N=4 SYM simplifies and becomes QM theory

- reduces to nearest-neighbor spin chains in planar N limit

- low energy excitations of spin chains = magnons

$$E - Q = \sqrt{1 + \frac{\lambda}{\pi^2}\sin^2\frac{p}{2}} - 1$$

[Beisert(0511)]

becomes in SMT limt:

$$H - Q = \frac{g}{2\pi^2}\sin^2\frac{p}{2}$$

non-relativistic

- semi-classical limits of spin chains become sigma models: Kruczenski (0311)

e.g. Landau-Lifshitz model for SU(2) sector

$$\mathcal{L}_{\text{LL}} = \frac{J}{4\pi}\left[\sin\theta\dot{\phi} - \frac{1}{4}\left((\theta')^2 + \cos^2\theta(\phi')^2\right)\right]$$

# Stringy side of SMT gives NR sigma models

- using AdS/CFT dictionary: SMT (near-BPS) limit can be formulated as limit of type IIB string theory on AdS5xS5

  - correspond to non-relativistic world-sheet strings !

→ LL model (and generalizations for other near-BPS sectors) is example of a novel class of non-relativistic worldsheet strings with a non-Lorentzian target spacetime

- one of target space dimensions = position along the spin chain (zero momentum because of cyclicity of trace)

- strongly suggests: bulk description of SMT is a type of NR gravity

  - new class of flat-fluxed backgrounds obtained recently: analogue of flat Minkowski space using Penrose type limits
  → natural starting point to quantize the theory

# Further developments (NRST)

- open strings and branes:
  - non-relativistic open string sector and DBI actions
    Gomis,Yan,Yu (2020)
  - connection to NR D/M-branes
    Kluson/Blair,Gallegos,Zinnato (2021)/Ebert,Sun,Yan(2021)
  - strings/branes as background solutions
    Bergshoeff,Lahnsteiner,Romano,Rosseel(2022)
  - generalize procedure to non-relativistic limit of extremal p-branes
    TSNC analogue for p-branes (incl. D/M)
    Bidussi,Harmark,Hartong,NO,Oling (in progress)

- SUSY generalization of (include RR fields)
  NR limit & relations to DFT/exceptional FT
  - non-perturbative dualities in NR string theory

- connection to integrable models
  Gomis,Gomis,Kamimura(2005)/Roychowdhury(2019/
  Fontanella,NietoGarcia,Torielli(2021),Fontanello,van Tongeren(2022)

# Further developments (NR worldsheets)

- Hamiltonian analysis

  Kluson (2021), Bidussi,Harmark,Hartong,NO,Oling (to appear)

- obtain beta functions for Galilean string

- connection with explicit construction of SMT using classical reduction of N=4 SYM & suitable quantization method

  Harmark,Wintergerst (2019),Baiguera,Harmark,Wintergerst(2020)
  Baiguera,Harmark,Lei,Wintergerst (2020)

- connections to Carrollian (small speed of light) gravity

Henneaux (1979), Bergshoeff,Gomis,Rollier,ter Veldhuis(2017),Hartong(2015)
Henneaux,Salgado-Rebolledo(2021),
de Boer,Hartong,NO,Sybesma,Vandoren(2021),Perez(2021),Hansen,NO,Oling,Soegaard(2021)

# Outlook

- NR (a la GO) describes closed subsector of relativistic ST
  has covariant string Newton-Cartan formulation (still Lor. CFT2 on ws)

what can we add to 90s ST?
- covariant formulation of DLCQ of strings
- further connection with Matrix string theory/M-theory
- other expansions ?

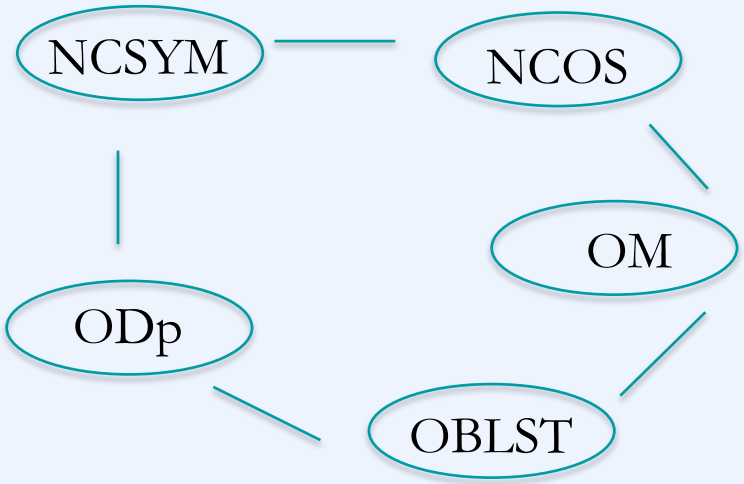
- Spin Matrix limits give
  - tractable subsector of N=4 SYM
  - strings with Galilean structure on worldsheet
  → quantize

- similar Carrollian worldsheet models exist
  - from usual tensionless limit
  - BMS3 residual symmetries
  → role in AdS/CFT ?

# Duality web of `non-Lorentzian' string theories?

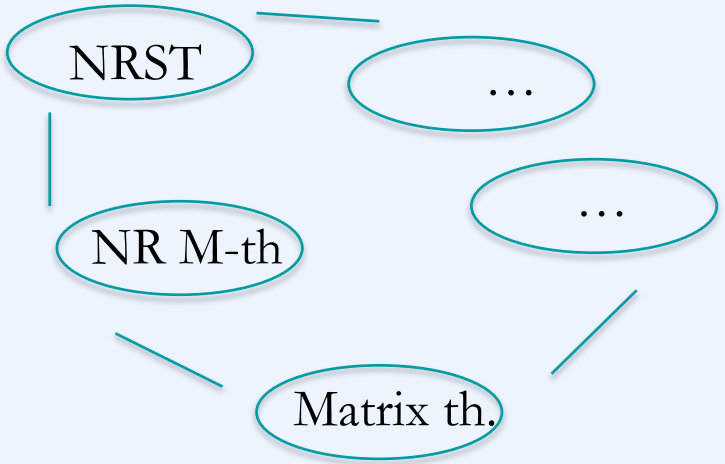
web of decoupled non-gravitational theories

(`open string sector')




web of non-Lorentzian gravitational string theories

(`closed string sector'_)




back to 2000s…

- self-contained corners of ST w. own geometry
- new window on non-perturbative effects ?

Thanks to Costas and Eugene for all they meant for our field and everything around it !

They are dearly missed

the end