Utrecht University

Theoretical Physics Department

# Quantum Foundations and the Constants in the Standard Model

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In memory of Claude Bouchiat,
ENS Paris

July 13, 2023

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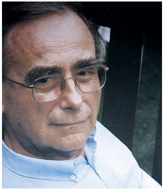


Orsay Aug 1972
,, Aug 1973
,, Aug 1974
Copenhagen Aug 1975

Crete Aug 1977
Paris Aug 1978
Paris Aug 1979
Copenhagen June 1980
Crete July 1980

"Triangle Meetings"

Paris Aug 1981
Utrecht April 1984
Paris Aug 1984
Copenhagen May 1985
Paris April 1986
Utrecht May 1987
Rome Aug 1990
Paris April 1991
Heraclion June 1992
Marseille July 1992
Utrecht April 1993

Milestones in physics:

Forces: Newton ($17^{\text{th}}$ century), and Maxwell ($19^{\text{th}}$ century),

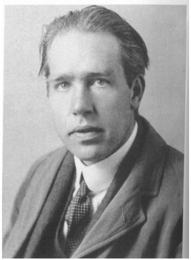
Atoms and molecules (end $19^{\text{th}}$ century)

Quantum mechanics and relativity (beginning $20^{\text{th}}$ century)

The Standard Model (second half $20^{\text{th}}$ century)

Should we now add ‘string theory’ and ‘CFT/AdS duality’ ?
Or are we again in a ‘crisis’ ?

Opinions are divided.
But we can do better. What are the weak points of our best theories today?



In the 1920s, a group of physicist, in their discussions at the Niels Bohr physics institute in Copenhagen, reached agreements as to what the theory of quantum mechanics says, and how to work with it. The most difficult issue:

> *What is really going on, in a quantum system as we describe it?*

Finally they did agree: It is amazing how well the theory predicts all probabilities *without* the need to answer this last question. Therefore:

Do not ask the question; there is no way to answer it by doing experiments.

*"Shut up and calculate!"*

This came to be known as the Copenhagen interpretation. It is entirely correct,

**But not all agree with the last dictum !**

*I do ask what it might be that is 'truly happening', hoping to learn more about our physical world.*

Take: the Standard Model of the Elementary Particles.

It has weak points. Usually scientists complain:

1. The theory does not fully include gravitation;
2. No explanation of the cosmological constant;
3. No clues for further unification of all forces.

We tried all possible alleys to address these problems head on. But these attempts were all made by using the same techniques over and over again. We should address the following weak points:

4. The field equations do not explain the values of *any* of the coupling constants.
5. The definitions of the theory are based on *divergent perturbation expansions!*

To find out how to address these issues, we have to go beyond Quantum Field Theory.

And that is possible !

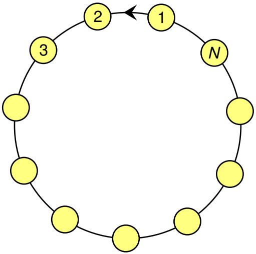
Let's assume:

God not only does not throw dice . . .
He makes his decisions with infinite accuracy!

Therefore, divergent perturbation expansions should not be at the basis of our theories.

Yet today, they are!
Why not try to be infinitely precise. Make theories that produce certain ("ontological') descriptions of what goes on. It is possible!

## Basic Models: combine *determinism* with *discreteness.*




1. The periodic chain. Ontological (= real) states:

$$|0\rangle,\ |1\rangle,\ \dots\ |N-1\rangle$$

Evolution law: $|k\rangle_{t+\delta t} = U(\delta t)\,|k\rangle_t$

$$U(\delta t)\,|k\rangle = |k+1 \mod N\rangle$$

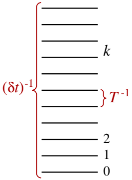
$$U(\delta t) = e^{-iH\,\delta t}\ , \qquad \frac{\mathrm{d}|\psi\rangle}{\mathrm{d}t} = -i\,H|\psi\rangle$$

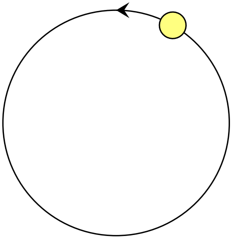
(Schrödinger Equation)

$$|n\rangle^E \stackrel{\text{def}}{=} \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} e^{2\pi ikn/N}|k\rangle^{\text{ont}}\ , \qquad k = 0,\cdots,\ N-1\ ; \quad n = 0,\cdots,\ N-1\ .$$

$$|k\rangle^{\text{ont}} = \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1} e^{-2\pi ikn/N}|n\rangle^E\ .$$

$$H = \frac{2\pi}{N\,\delta t}\,n = \omega n$$





## Step 1a. The continuum limit.

Ontological states: $|\phi\rangle$

Evolution law:

$\frac{\mathrm{d}}{\mathrm{d}t}|\phi\rangle_t = \omega$

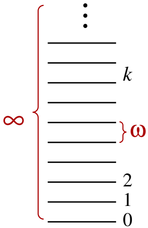
$$U(\delta t)|\phi\rangle = |\phi + \omega\delta t\rangle$$

$$U(\delta t) = e^{-iH\,\delta t}\ , \qquad \frac{\mathrm{d}|\psi\rangle}{\mathrm{d}t} = -i\,H|\psi\rangle$$

$$|n\rangle^E \ \stackrel{\mathrm{def}}{=}\ \frac{1}{\sqrt{2\pi}}\oint e^{i\phi n/N}|\phi\rangle^{\mathrm{ont}}\ , \qquad 0\le\phi<2\pi\,;\quad n=0,\cdots,\ \infty\ .$$

$$|\phi\rangle^{\mathrm{ont}} = \frac{1}{\sqrt{2\pi}}\sum_{n=0}^{\infty} e^{-i\phi n/N}|n\rangle^E\ .$$

*We generate exactly the spectrum of the harmonic oscillator*: $H = \omega\, n$



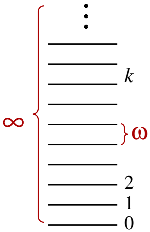
Important theorem: At integer time steps, this Schroedinger equation sends collapsed wave functions (delta peaks) into collapsed wave functions. It does not generate superpositions.

Important theorem: if system $A$ has the same spectrum of energy eigenvalues as system $B$ then a mapping $A \leftrightarrow B$ exists , so that the two systems are physically the same.

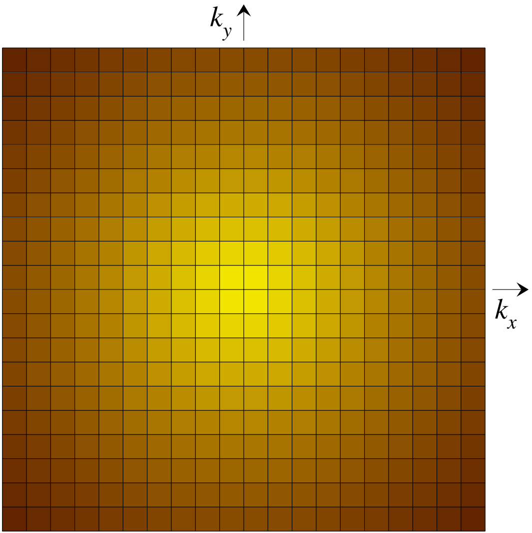
But to make contact with our experiences with today's physics, we may introduce perturbation expansions.
*They* do generate divergent perturbation expansions! And it is these, imperfect, equations that generate superpositions.

Thus, all systems that consist of *harmonic oscillators* can be formulated in a basis of Hilbert space that sends ontological states into ontological states.

With infinite precision.
Quantum field theories without interactions, consist of harmonic oscillators! It is here that we start with our improvements!




We first go to momentum space:




For a theory in a box, momentum space is discrete.

Free field theories:,

$$H = \frac{1}{2}(\vec{k}^{\,2} + M^2)\Phi^2 + \frac{1}{2}\Pi^2.$$

Single harmonic oscillator at every $\vec{k}$ value.

Apply what we did 3 slides ago, yo find the ontic variables $b(\vec{k})$. Fourier transform back: to get $b(\vec{x},\, t)$.

These $b(\vec{x},\, t)$ obey *classical equations !* Introduce interactions in terms of the ontic variables, and then return to the quantum variables.

This may well give the same quantum theory we started from,
But now it has an ontological interpretation.

Note that, as we start off with an unperturbed theory with short periodicities, we have in our perturbative formulation, intermediate states with high energies. Of these, perturbation expansions only use the lowest energy states. This may make perfect sense in the perturbative corrections, but it is not right if we try to "go beyond" perturbation theory.

The usual perturbation expansions do contain these emergent, divergent energy modes. This works to help us find good approximations, but they cannot be exact, because the exact solutions mostly have infinite periodicities (infinite recursion times).

About SM constants:

Earlier investigations suggested that in deterministic theories, interaction constants can only take rational values such as $1/N$. gravity theories suggest that entropies near black holes are bounded, which would suggest that $N$ values have to be small.

the end

**THANK YOU**