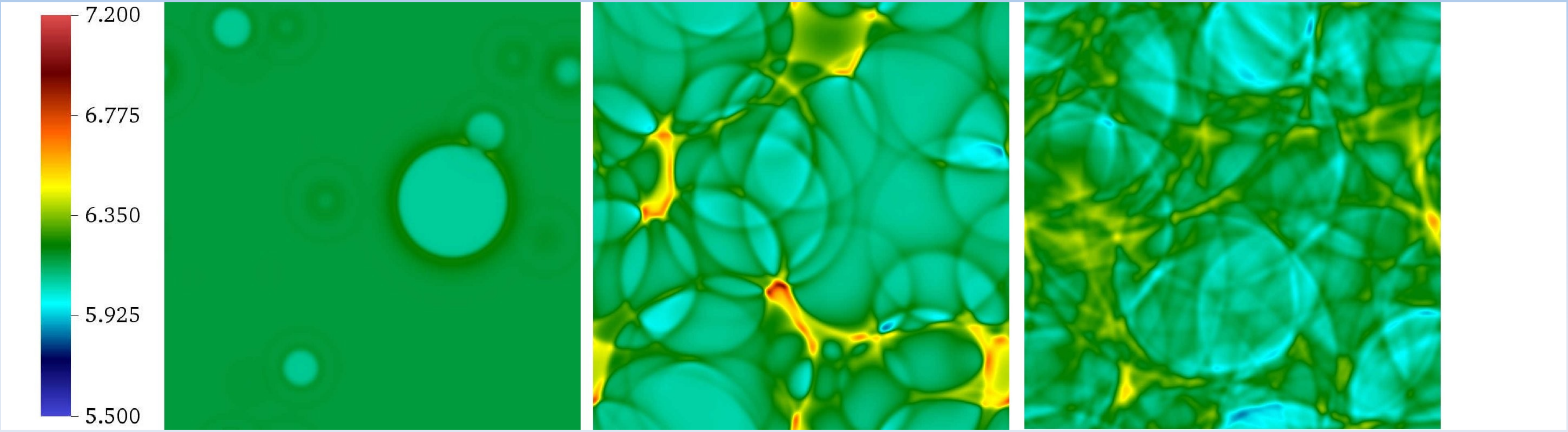
# Gravitational waves from cosmological phase transitions

Thomas Konstandin

ENS summer institute, July 10, 2023

# Gravitational waves from cosmological phase transitions

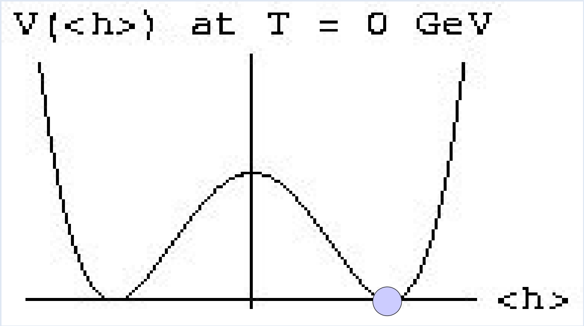



I. Introduction
II. Recent results
III. NanoGrav

# Electroweak symmetry breaking

The **Mexican hat** potential is designed to lead to a finite Higgs vacuum expectation value (VEV) and break the electroweak symmetry

$$V(h) = \frac{\lambda}{4}\left(h^2 - v^2\right)^2$$

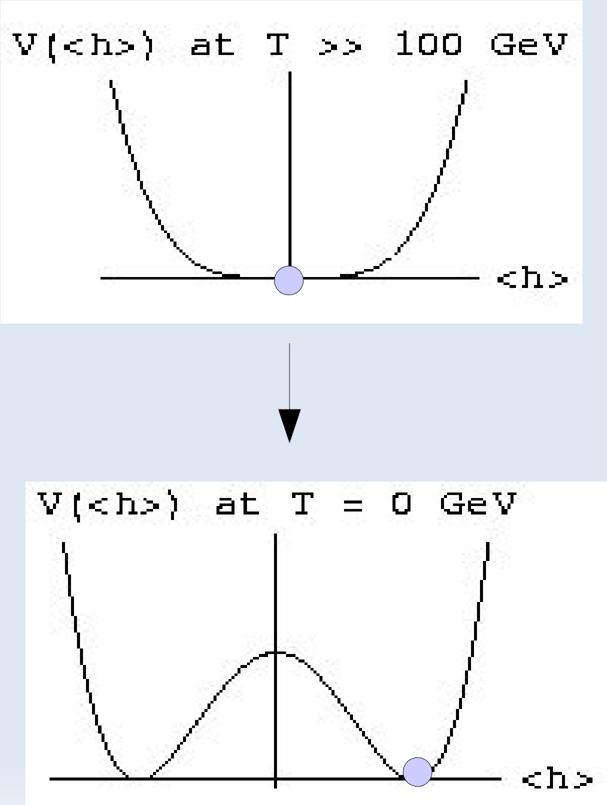


# Electroweak symmetry breaking

*[Weinberg '74]*

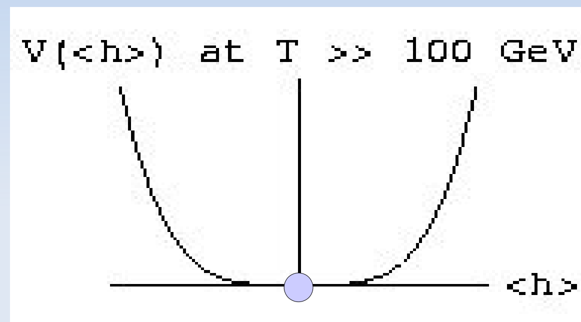
At large temperatures the symmetry is restored

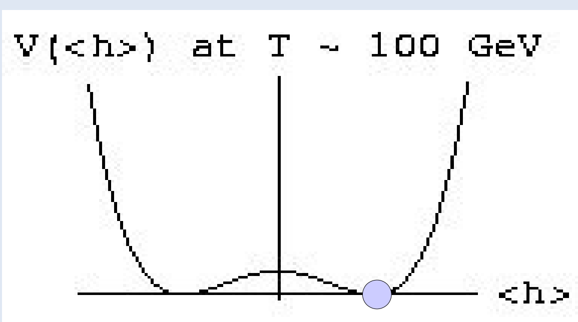
$$V(h,T) = \frac{\lambda}{4}\left(h^2 - v^2\right)^2 + \text{const} \times h^2 T^2 + \text{details}$$



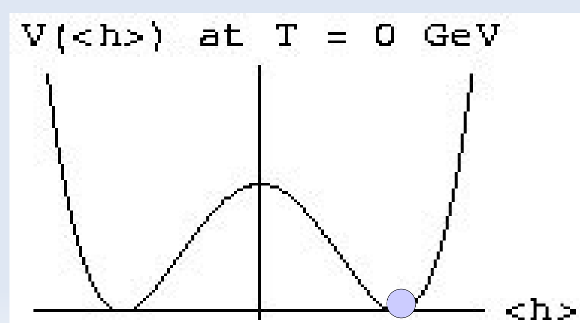
# Electroweak symmetry breaking

Depending on the details, the phase transition can be very weak or even a cross over



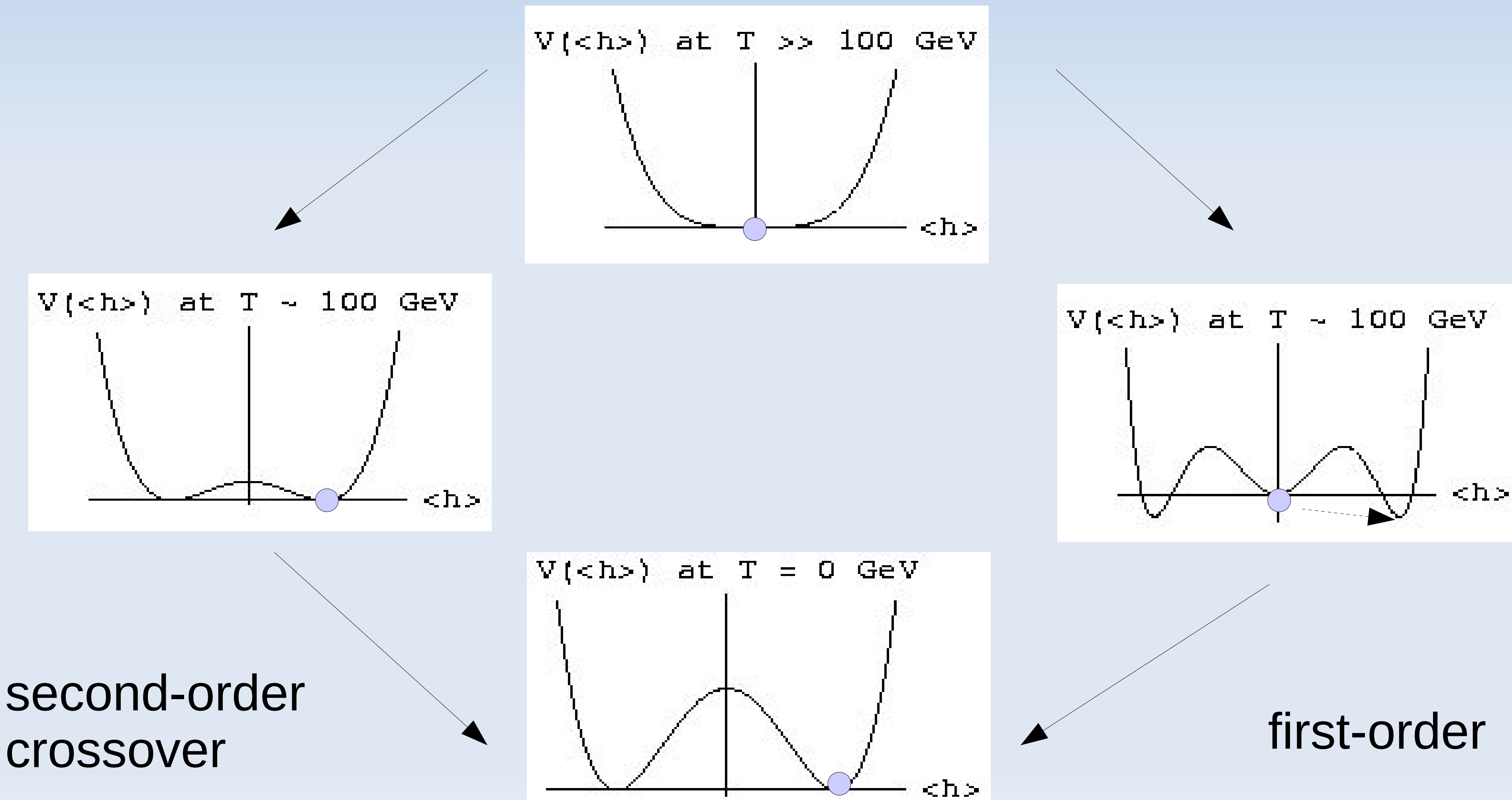




second-order
crossover



# Electroweak symmetry breaking

It can also be a strong phase transition if a **potential barrier** seperates the new phase from the old phase



# Electroweak symmetry breaking

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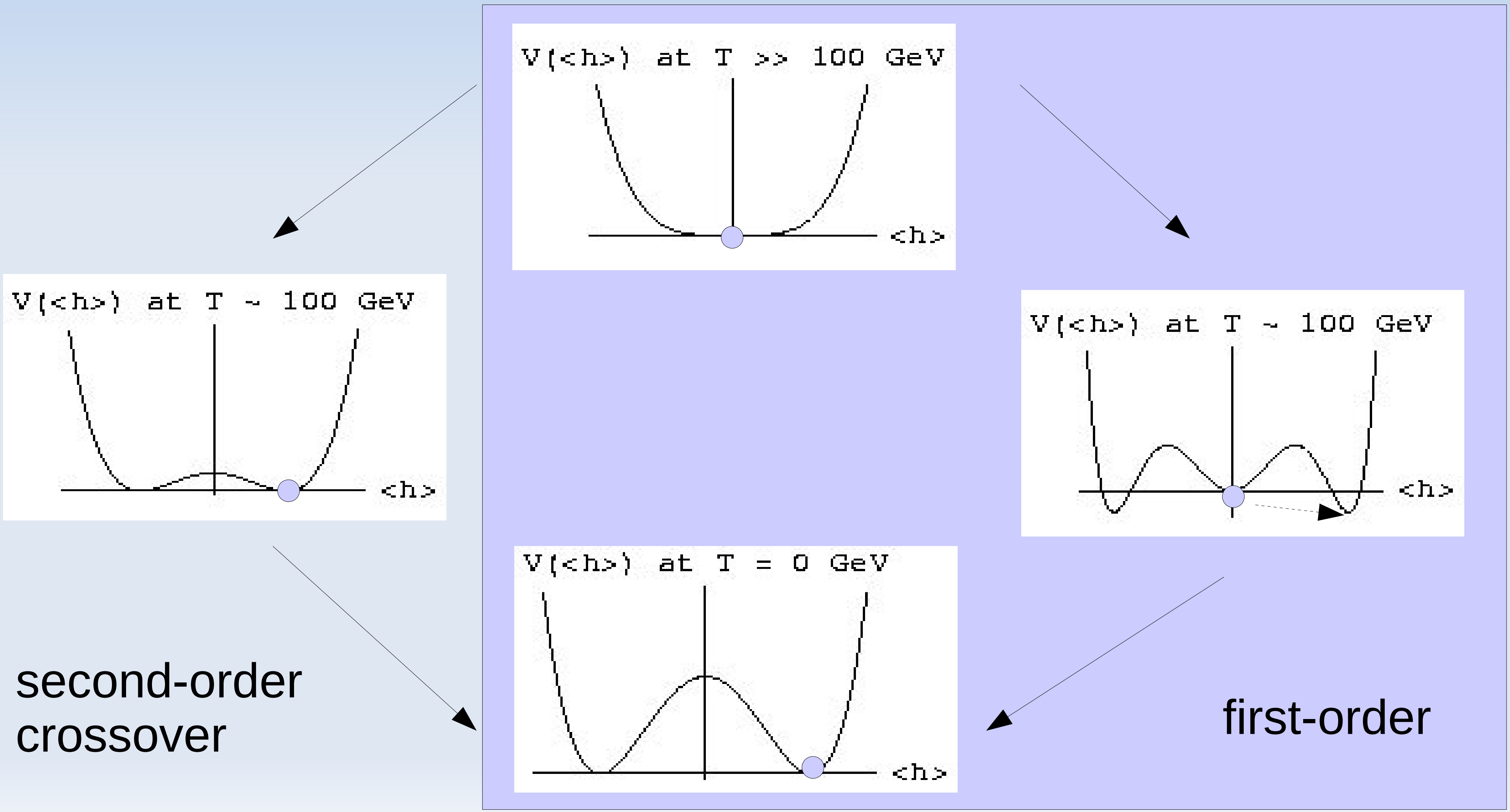


# Electroweak phase transition

gravitational
waves

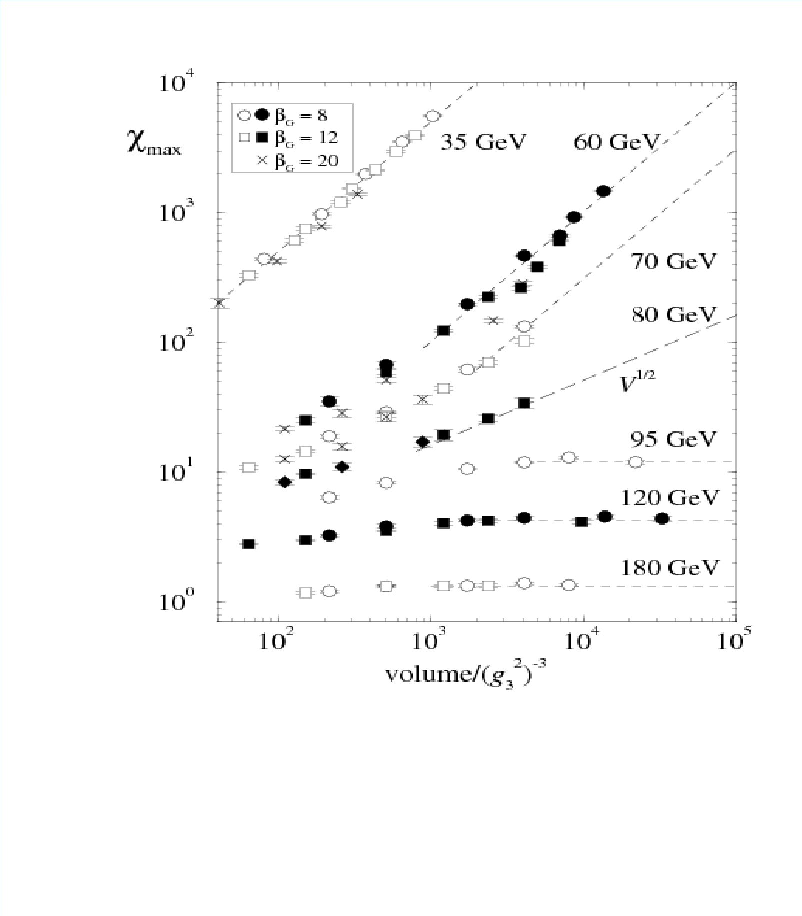
baryogenesis

# Electroweak phase transition in the SM

The effective potential is the standard tool to study phase transition at finite temperature.

Lattice studies show that there is a crossover in the SM.

A light Higgs would lead to a 1st-order PT.



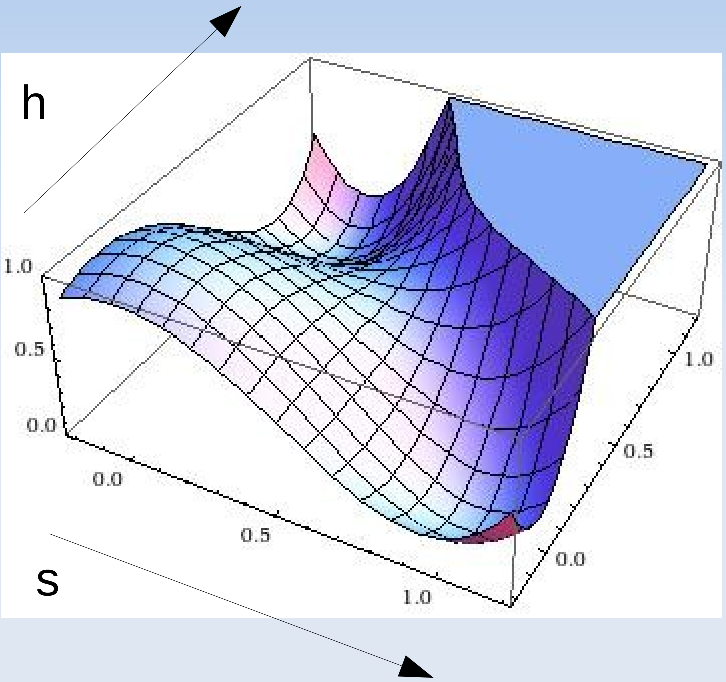

[Kajantie, Laine, Rummukainen, Shaposhnikov '96]

# Singlet extension

The Standard Model only features a electroweak crossover.

A potential barrier and hence first-order phase transitions are quite common in extended scalar sectors:

$$V(h,s) = \frac{\lambda}{4}\left(h^2 - v^2\right)^2 + m_s^2 s^2 + \lambda_s s^4 + \lambda_m s^2 h^2$$




The singlet field has an additional $\mathbb{Z}_2$ symmetry and is a viable DM candidate.

The phase transition proceeds via

$$(h,s) = (0,w) \rightarrow (h,s) = (v,0)$$

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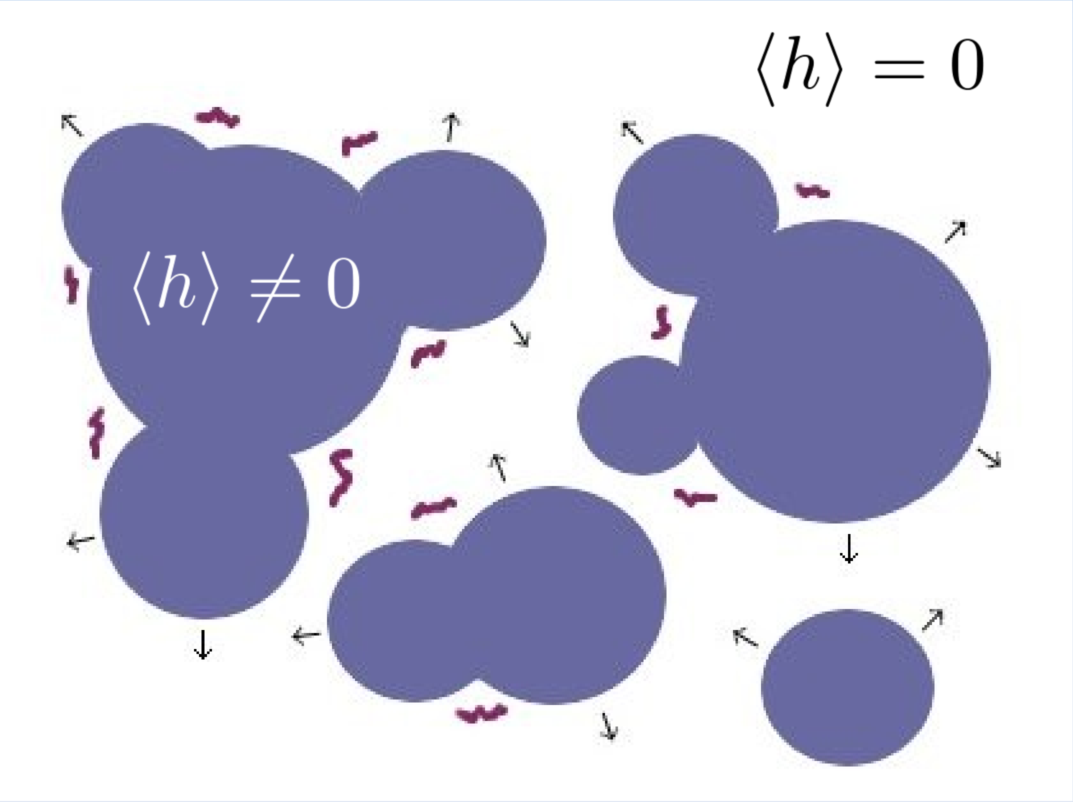
# First-order phase transitions

- first-order phase transitions proceed by bubble nucleations

- in case of the electroweak phase transition, the ”Higgs bubble wall” separates the symmetric from the broken phase

- this is a violent process $(v_{wall} \simeq O(c))$ that drives the plasma out-of-equilibrium and sets the fluid into motion

# Gravitational waves

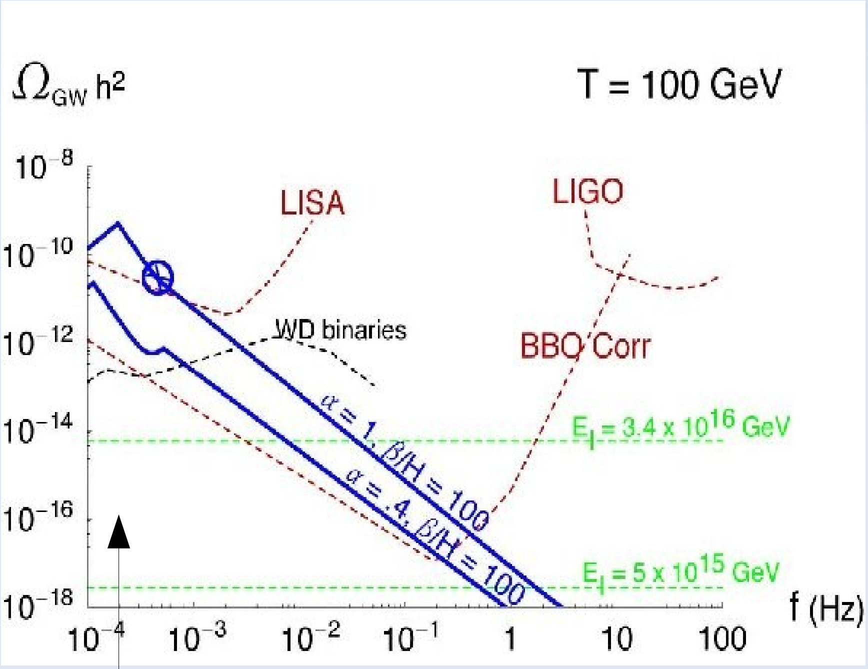



During the first-order phase transitions, the nucleated bubbles expand. Finally, the colliding bubbles break spherical symmetry and generate stochastic gravitational waves.

# Observation

*[Grojean&Servant '06]*

The produced gravitational waves can be observed with laser interferometers in space

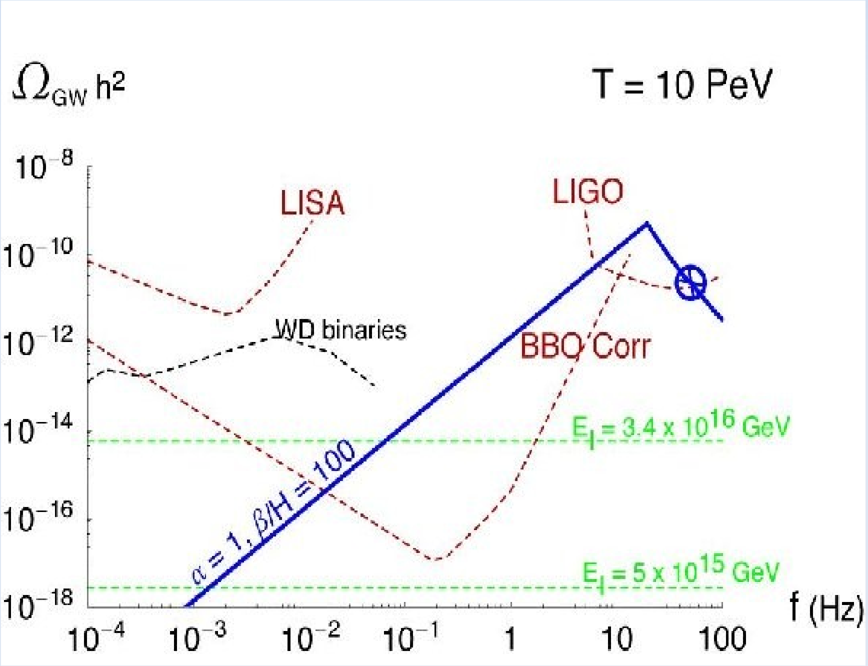



redshifted **Hubble horizon** during a phase transition at T ~ 100 GeV

# Observation

*[Grojean&Servant '06]*

… or on the ground




Strong phase transition at larger temperatures produce the same energy fraction of gravitational waves but at higher frequencies.

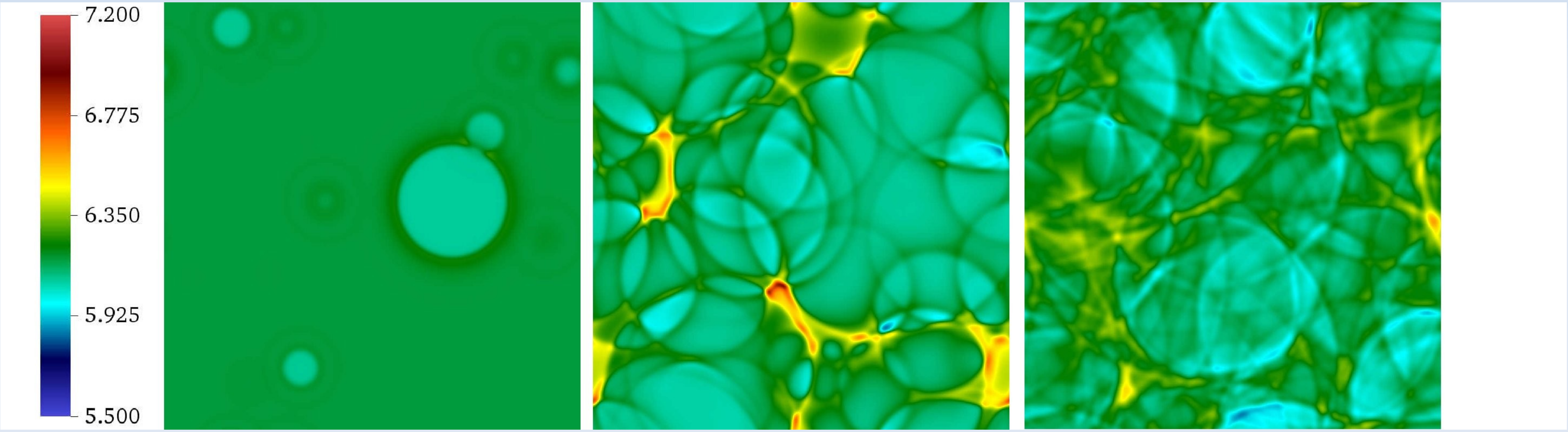
# Sources of GWs from PTs

During and after the phase transition, several sources of GWs are active

- Collisions of the scalar field configurations / initial fluid shells
- Sound waves after the phase transition (long-lasting → dominant source)
- Turbulence
- Magnetic fields

In the last 10 years, simulations became the main tool to incorporate all these effects.

# GWs from cosmological phase transitions




*[Hindmarsh, Huber, Rummukainen, Weir '15]*

# Back of the envelope

There are several quantities that can enter in the determination of the GW spectrum:

The temperature of the phase transition T.

The (inverse) duration of the phase transition

$P \propto \exp(\beta t)$ and typically $\beta / H \sim O(100)$

The bubble size is $R_* \sim 3/\beta$ and the wall velocity $v_{w}$.

The amount of latent heat $\Lambda$ that is transformed into kinetic energy *K* in the plasma:

$$\Lambda \to K \,, \quad \alpha = \frac{K}{\rho_{\rm tot}}$$

# Back of the envelope

The peak frequency at production is linked to the bubble size or the duration of the phase transition

$$\omega^*_{\rm peak} \simeq \beta \simeq O(100)\, H$$

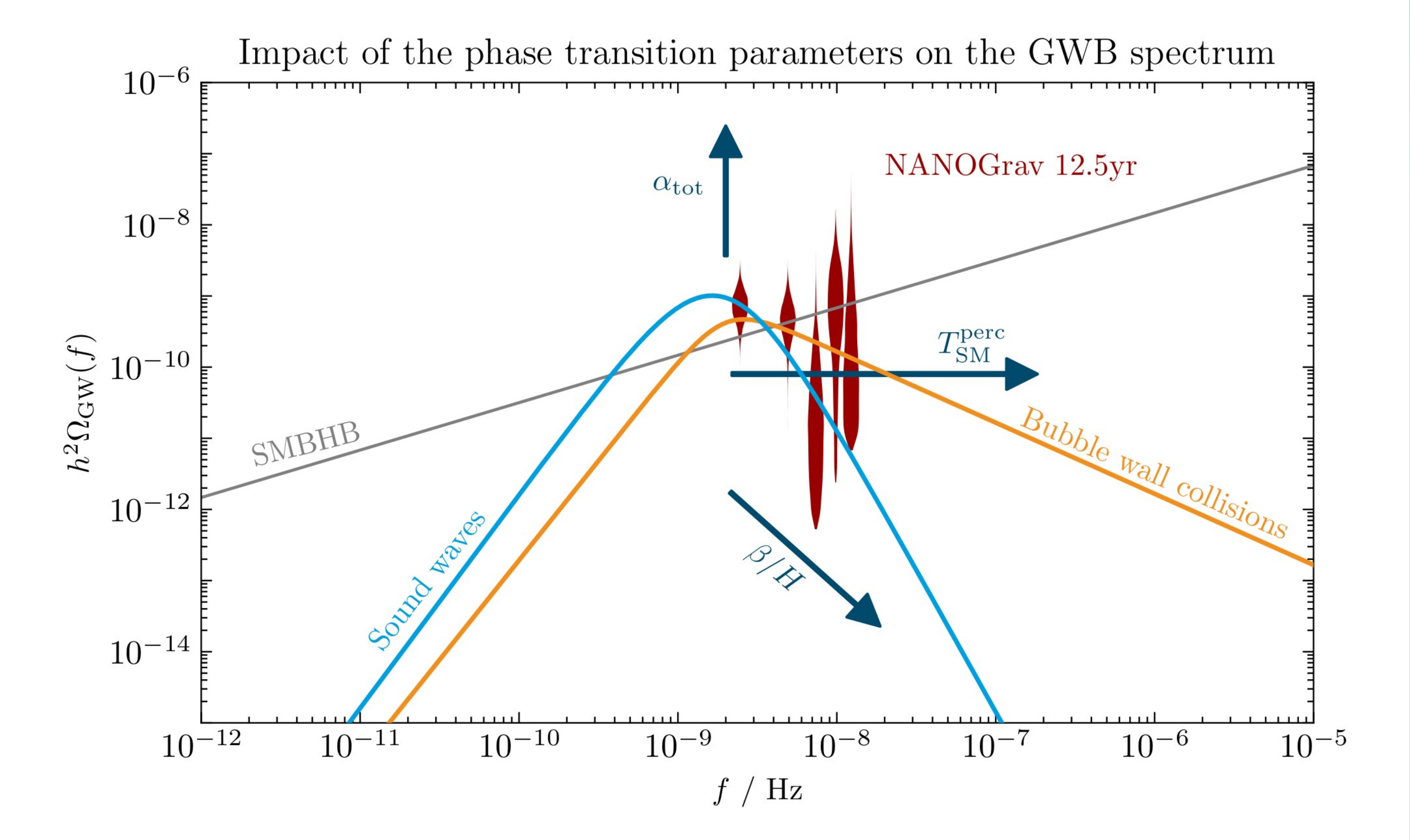
After the redshift, this amounts to

$$\omega_{\rm peak} \simeq \frac{\beta}{100\, H} \frac{T}{100 \rm GeV} \ \rm mHz$$

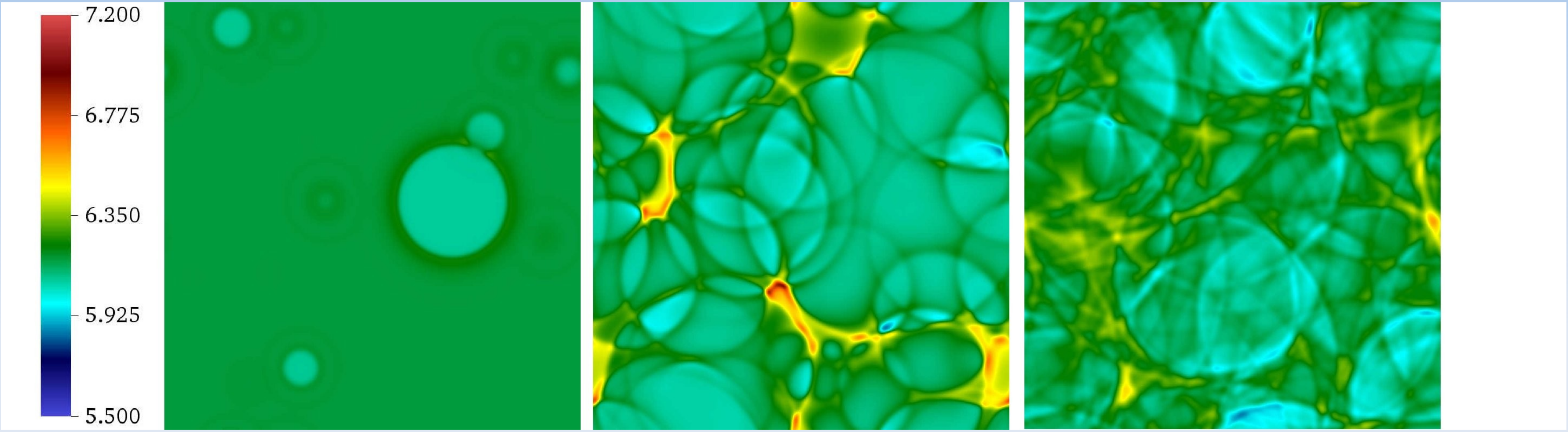
Since GWs behave as radiation, $\Omega_{GW}$ is only redshifted after the transition to matter domination.

# Observation

dark sector phase transition, T ~ MeV



# Gravitational waves from cosmological phase transitions

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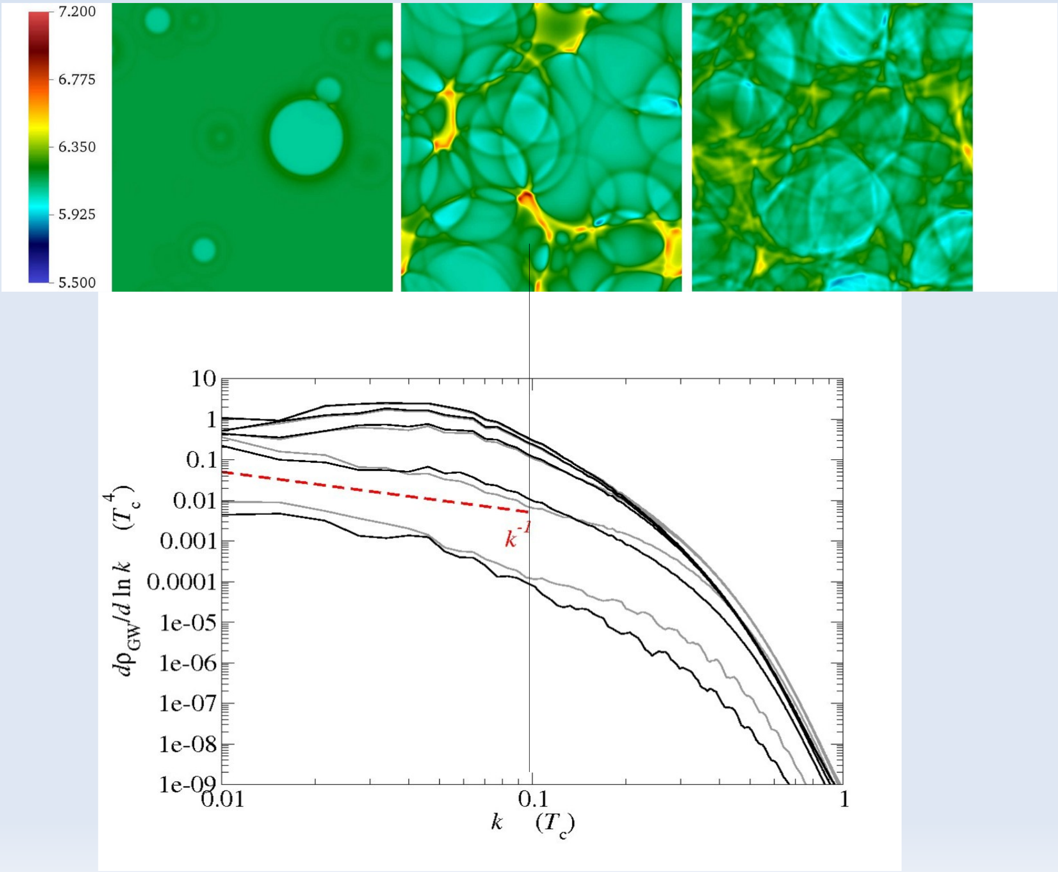
# State-of-the-art: simulations

*[Hindmarsh, Huber , Rummukainen, Weir '13, ’15, ’17]*
*[Weir ’16] [Gould, Sukuvaara,Weir ’21] [Cutting, Hindmarsh, Weir ’18&’19]*
*[Cutting, Escartin, Hindmarsh, Weir ’20]*

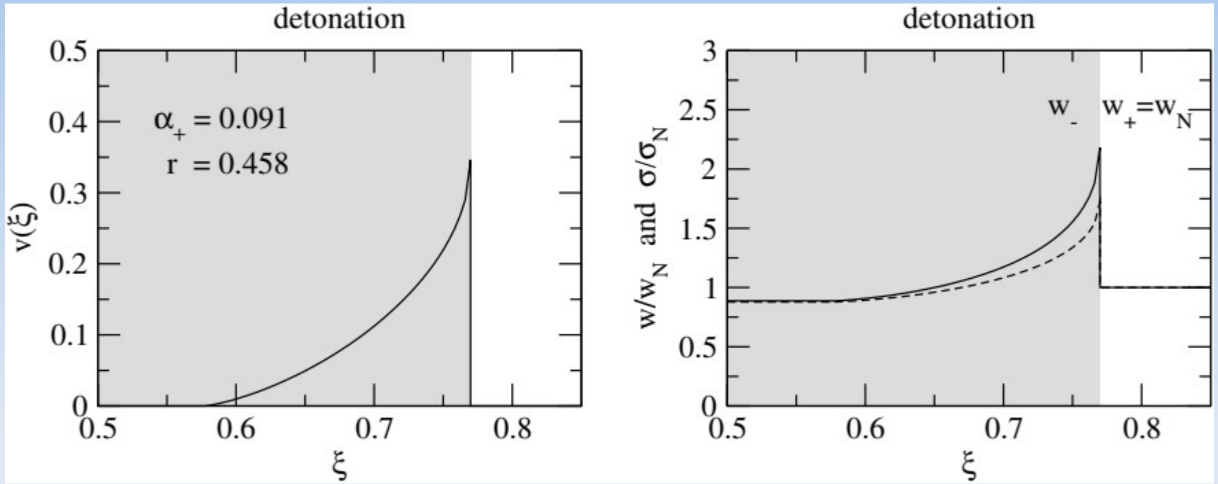
Depending on the context, the system can be descibed using hydrodynamics (fluid + Higgs) or just a scalar field




The produced GW spectrum can be read off from the simulation.

Really robust results, not many a priori assumptions.
But very costly.
How to extrapolate to other models and parameters?

# Single bubble: spherical solutions




Many insights can be gained by studying the expansion of a single bubble.

Hydrodynamics dictates how the latent heat sets the plasma into motion and how much energy is transformed into bulk motion vs heating the plasma.

# Bubble wall thickness

The main challenge in the hydrodynamic simulation is to cover very different length scales.

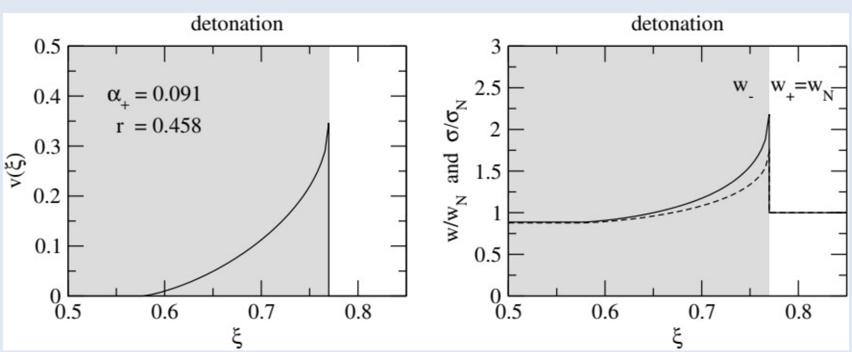
In the physical phase transition

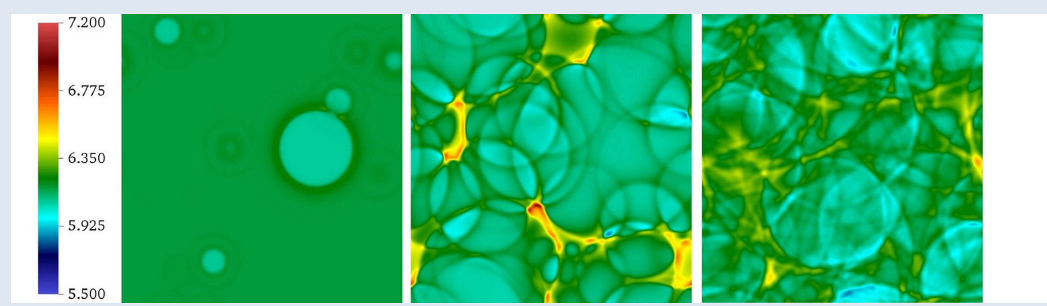
wall thickness <<<<<<< fluid shell thickness < bubble size

1/100GeV % of Hubble radius

In simulations:

grid spacing < (wall thickness < fluid shell thickness < bubble size) < box size

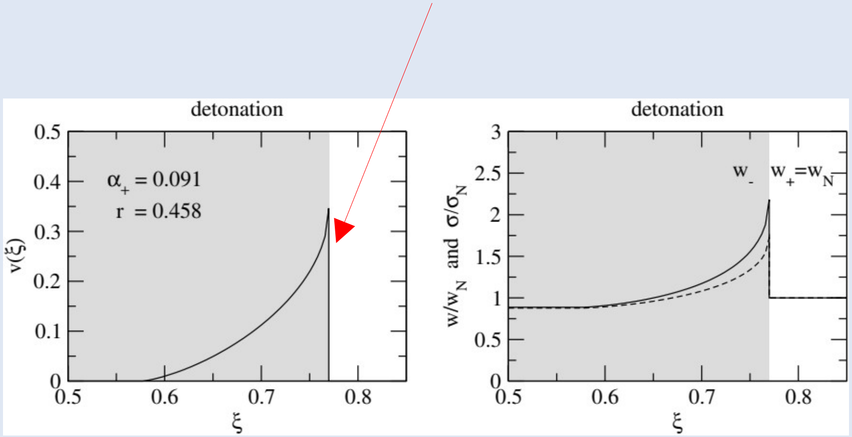


# Higgsless simulations

In order to avoid this issue, we want to perform simulations that are agnostic about the wall thickness. This would resemble an *EFT* where the Higgs field was integrated out.

However, this requires a hydrodynamic numerical framework that can deal with *shocks* and other discontinuities:



# Higgsless simulations

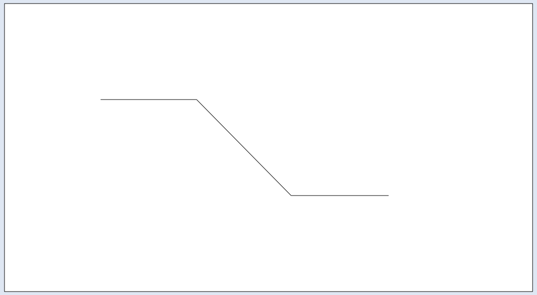
Consider the differential equation of a right-mover

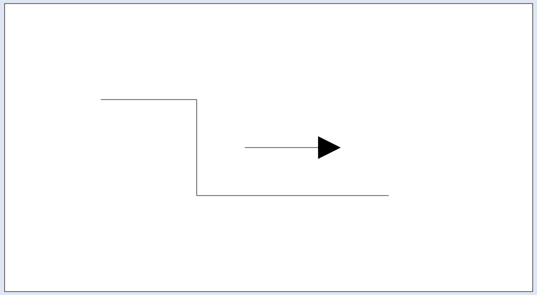
$$(\partial_t + \partial_x)\, f(t,x) = 0$$

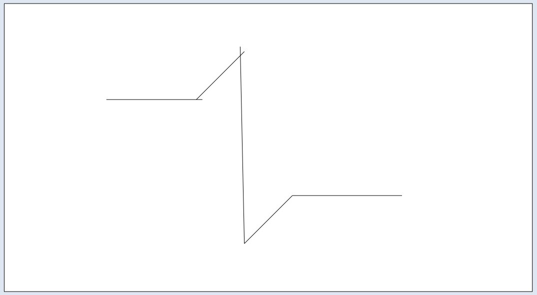
With the solution

$$f(t,x) = g(x-t)$$

When this equation is numerically solved, typically one of two issues occurs







too much viscosity
damping

not enough viscosity
Gibbs oscillations

# Higgsless simulations

Ideally one wants to have a scheme that abides to total variation diminishing to avoid oscillations.

Viscosity should be minimal to reduce damping.

This can be achieved via hybridization (adding non-linear terms) in a semi-discrete scheme.

For conservation laws, this is for example possible via the Kurganov-Tadmor method.

## New High-Resolution Central Schemes for Nonlinear Conservation Laws and Convection–Diffusion Equations

Alexander Kurganov* and Eitan Tadmor†

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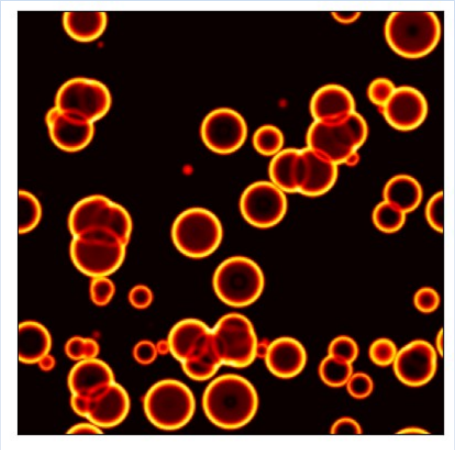
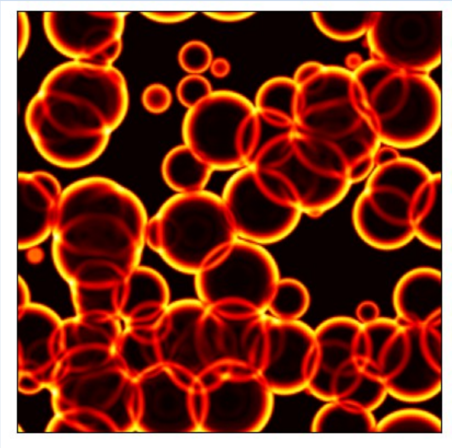
Central schemes may serve as universal finite-difference methods for solving nonlinear convection–diffusion equations in the sense that they are not tied to the specific eigenstructure of the problem, and hence can be implemented in a straightforward manner as black-box solvers for general conservation laws and related equations governing the spontaneous evolution of large gradient phenomena. The first-order Lax–Friedrichs scheme (P. D. Lax, 1954) is the forerunner for such central schemes. The central Nessyahu–Tadmor (NT) scheme (H. Nessyahu and E. Tadmor, 1990) offers higher resolution while retaining the simplicity of the Riemann-solver-free approach. The numerical viscosity present in these central schemes is of order $\mathcal{O}((\Delta x)^{2r}/\Delta t)$. In the convective regime where $\Delta t \sim \Delta x$, the improved resolution of the NT scheme and its generalizations is achieved by lowering the amount of numerical viscosity with increasing $r$. At the same time, this family of central schemes suffers from excessive numerical viscosity when a sufficiently small time step is enforced, e.g., due to the presence of degenerate diffusion terms.

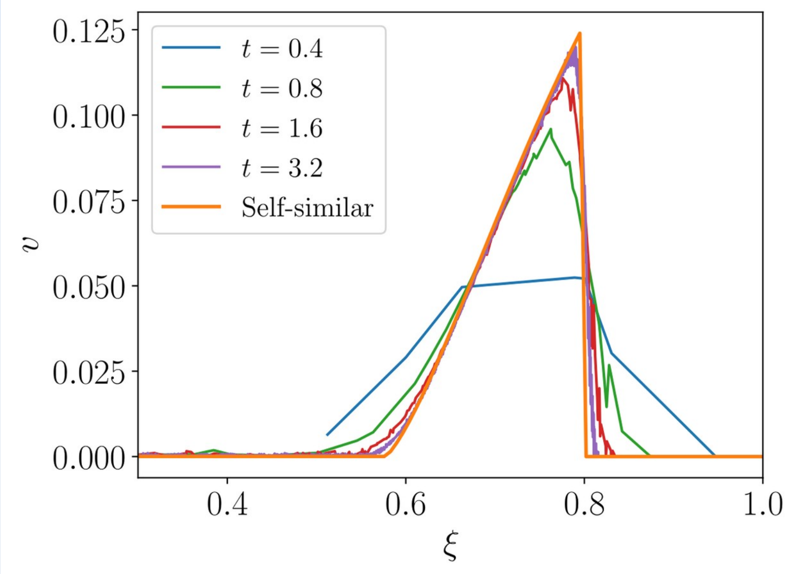
In this paper we introduce a new family of central schemes which retain the sim-

# Simulation of cosmological phase transitions

We recently developed a highly efficient scheme to simulate relativistic hydrodynamics during cosmological first-order phase transitions.



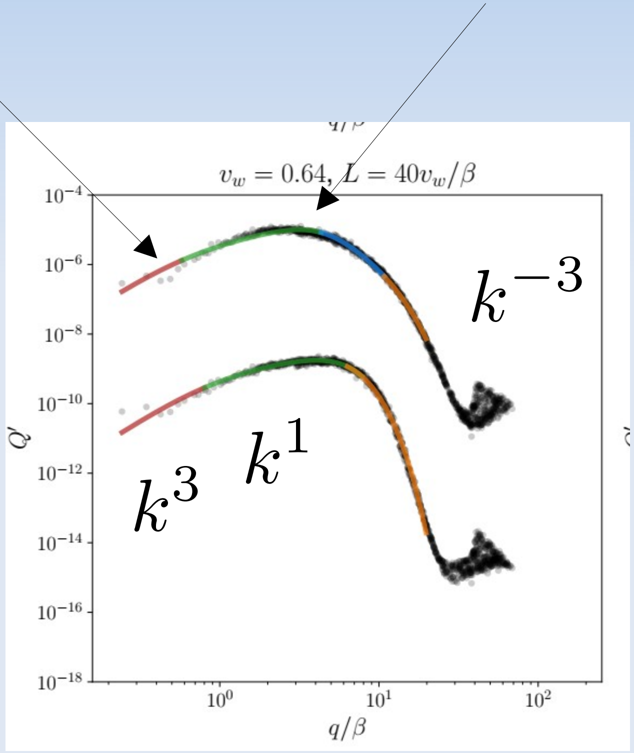





These simulations allow to extract GW spectra from the phase transition in a few hours instead of weeks (factor **2000 speed improvement** compared to former approaches)

# Simulation of cosmological phase transitions

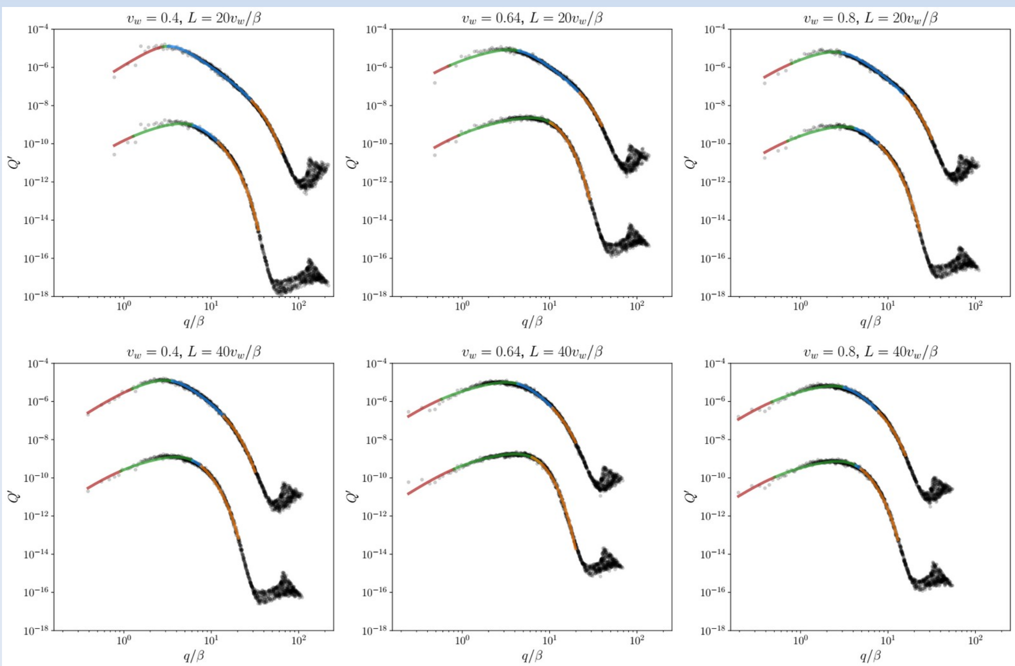
The spectra have two features due to the bubble size and the shell thickness.

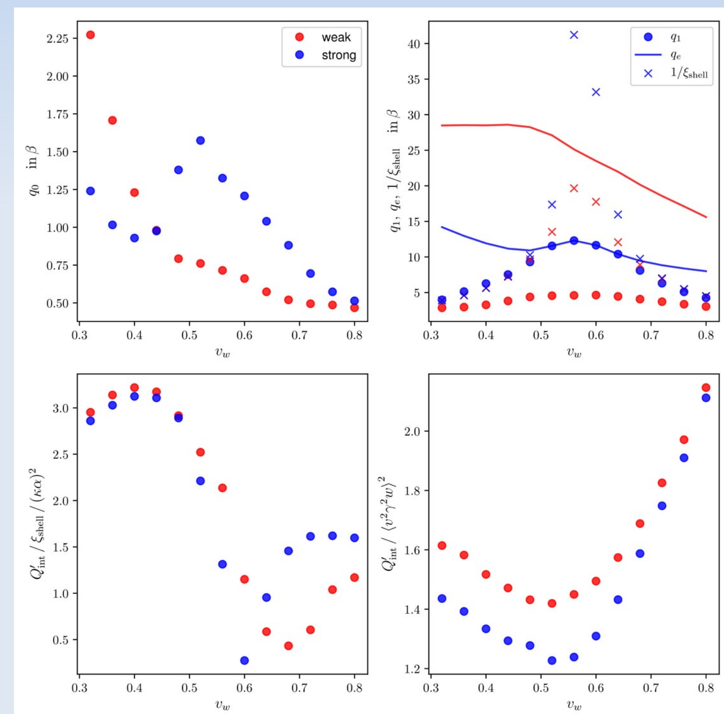



[Jinno, TK, Rubira, Stomberg 2022]
[Hindmarsh 2016]

# Simulation of cosmological phase transitions

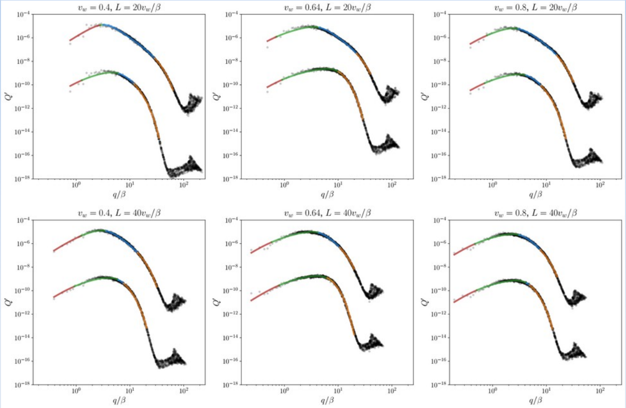
The setup allows to run many simulations a day and to extract the GW spectra as functions of the PT properties: wall velocity $v_w$, PT strength α

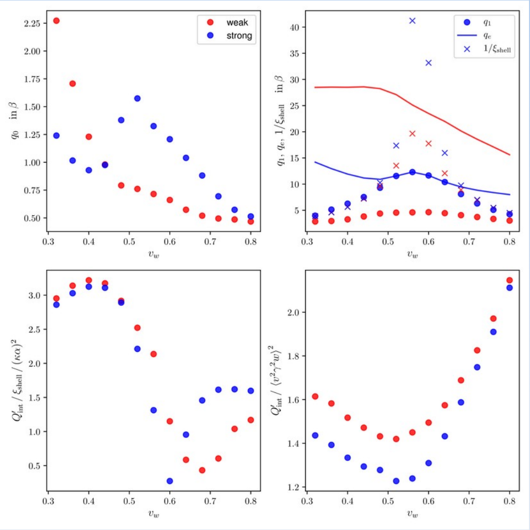



*[Jinno, TK, Rubira, Stomberg 2022]*

# Some conclusions

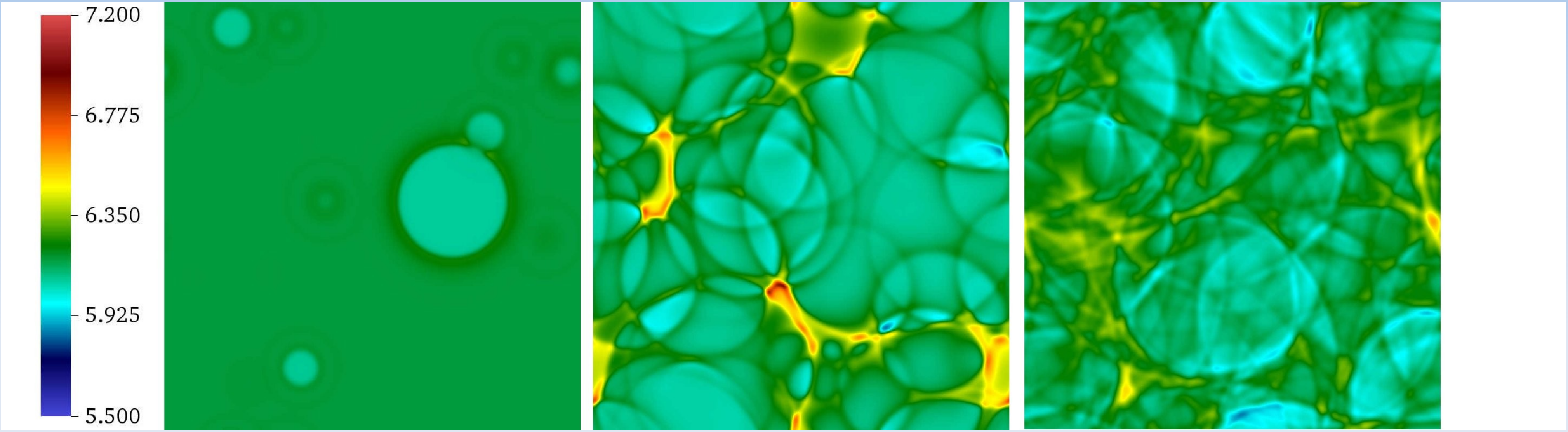





Using these simultions, by now, we have very accurate and reliable predictions of GW spectra from cosmological phase transitions.

There are still some loose end though: deep IR, role of turbulence

# Gravitational waves from cosmological phase transitions

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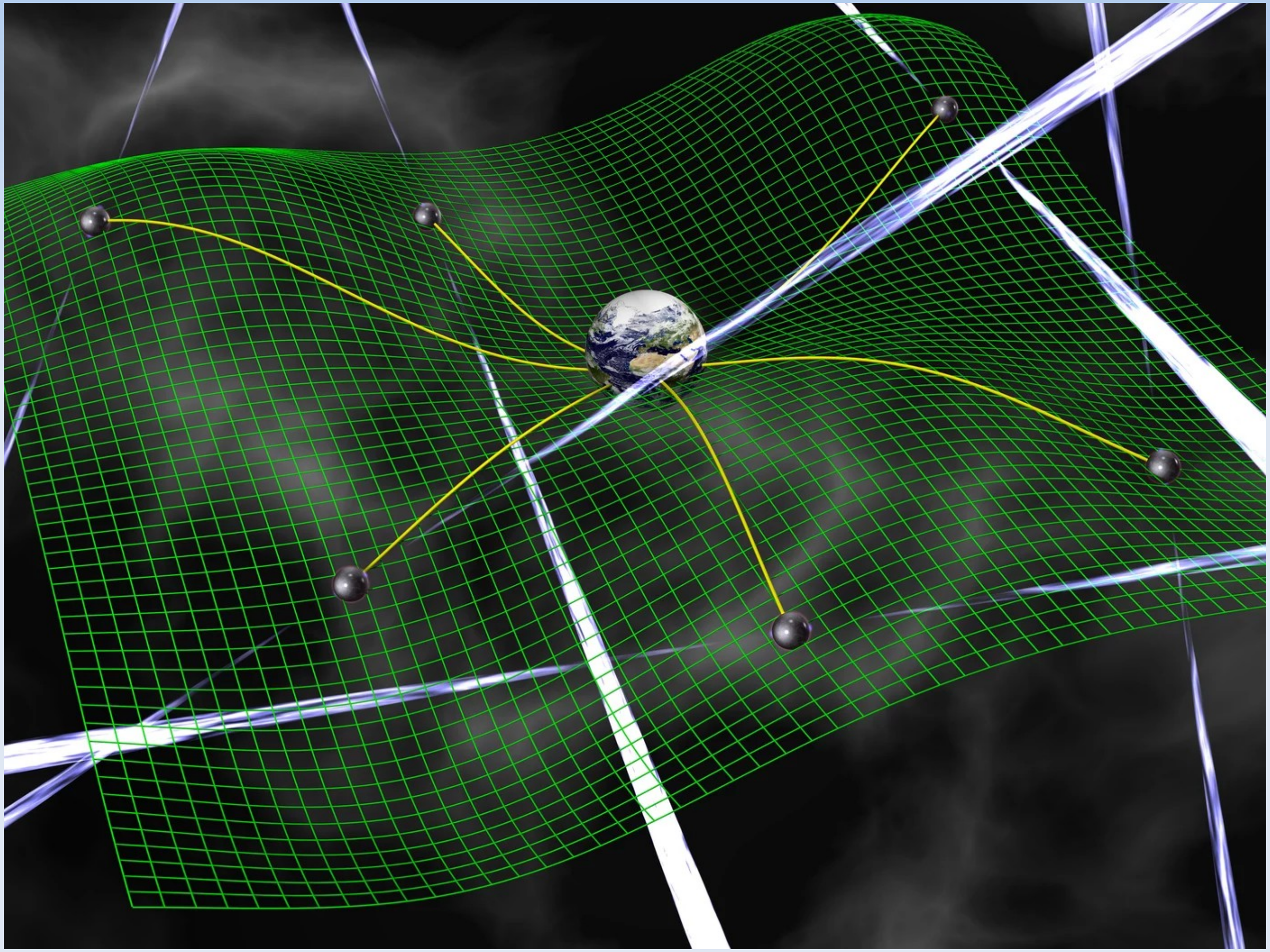
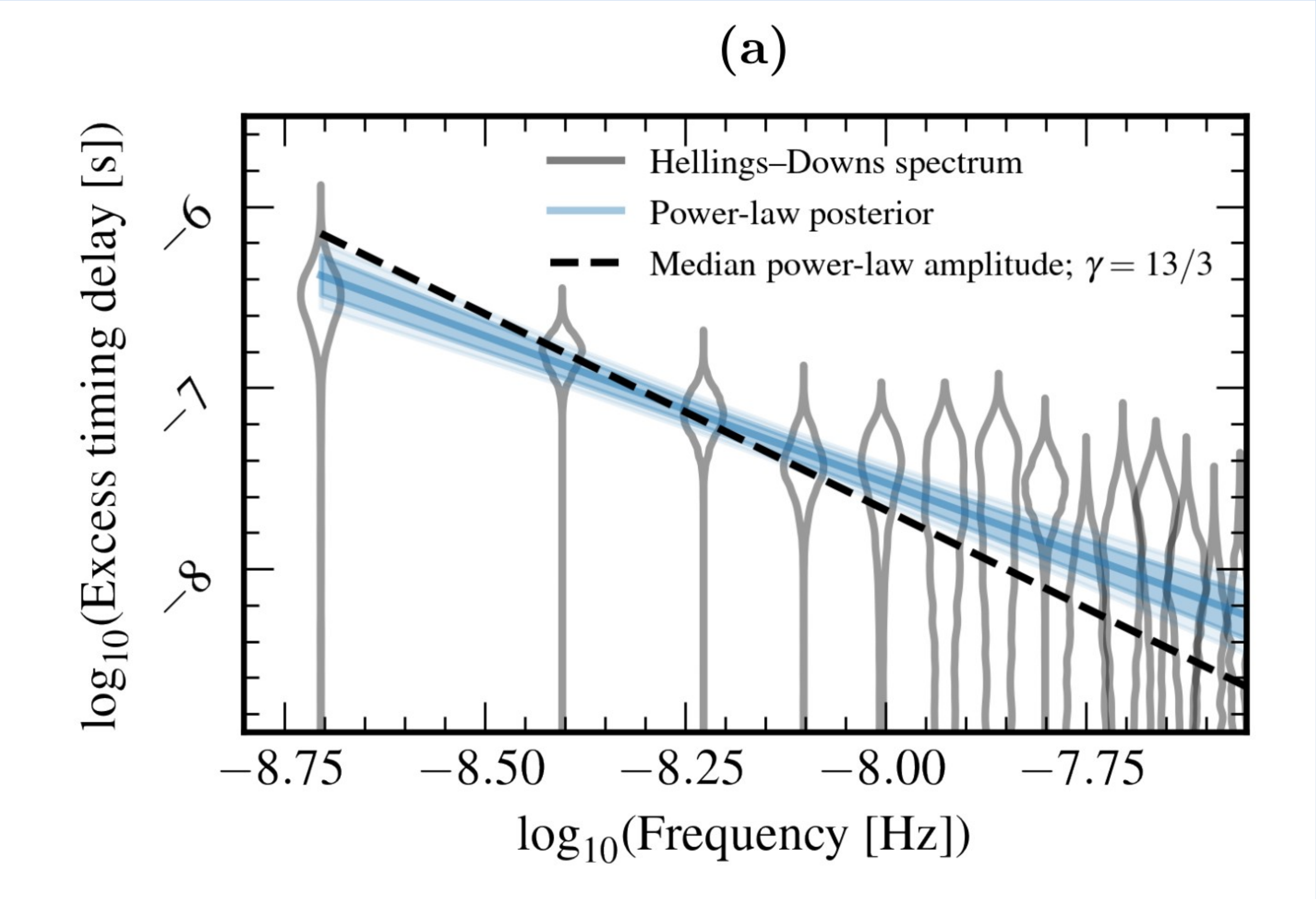
# Pulsar timing array



Credit: David Champion/Max Planck Institute for Radio Astronomy

# New data release

NanoGrav measured a common red noise spectrum in the nHz regime (1/10 year)

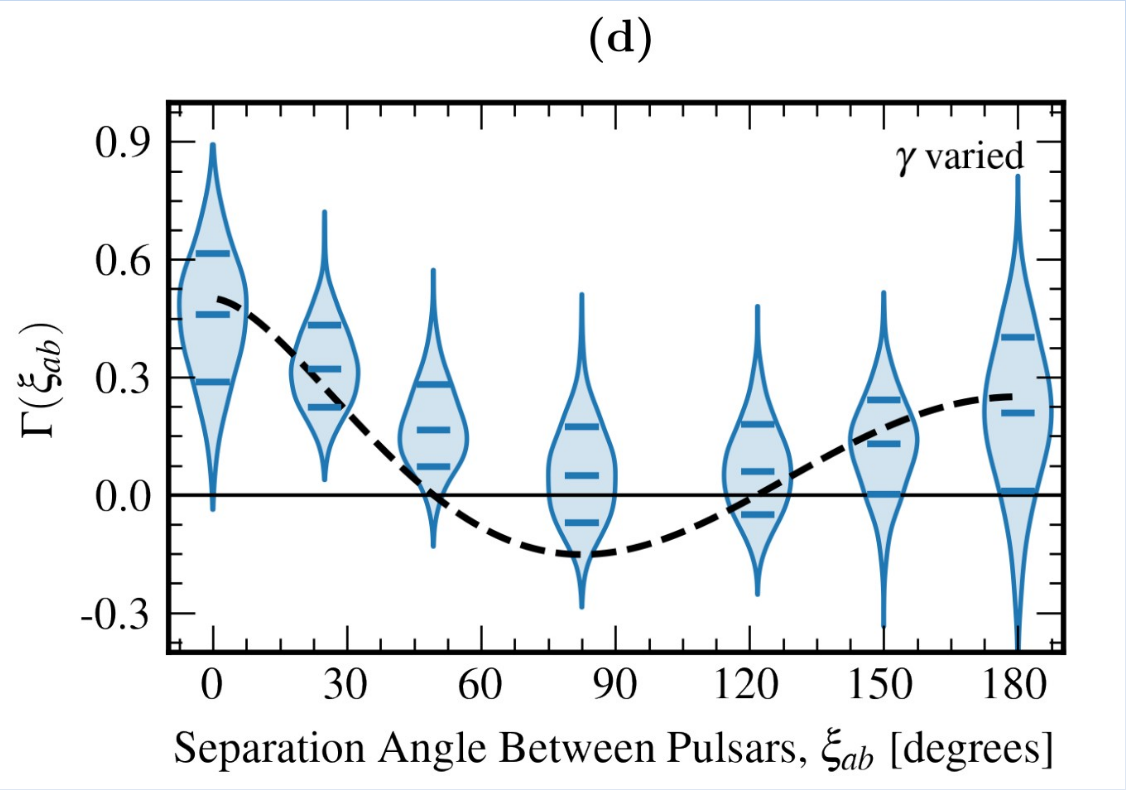



Other PTA experiements had similar results with somewhat less statistics (EPTA, PPTA, CPTA)

*[NanoGrav 2023]*

# Are these really GWs?

The smoking gun for a stochastic GW source is a correlation that follows the Hellings-Downs curve

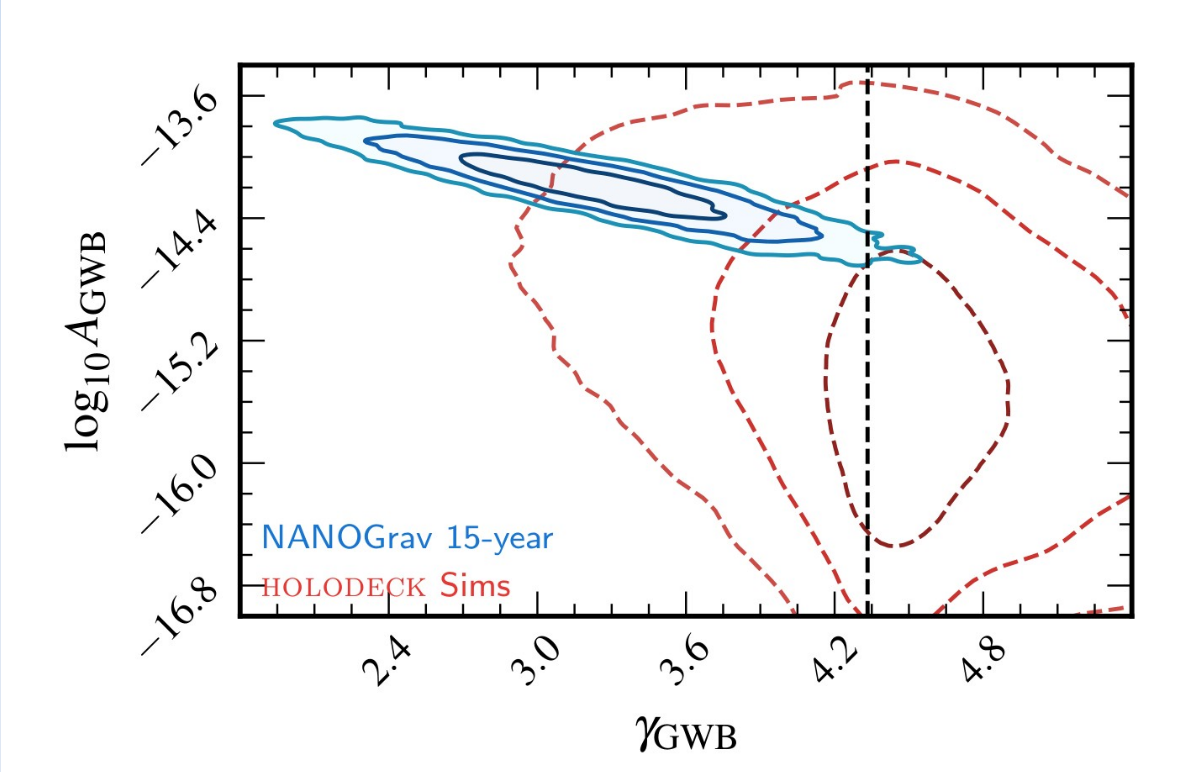



The data seems to support a Hellings-Downs curve, even though there is also a quite large monopole.

*[NanoGrav 2023]*

# Where do they come from?

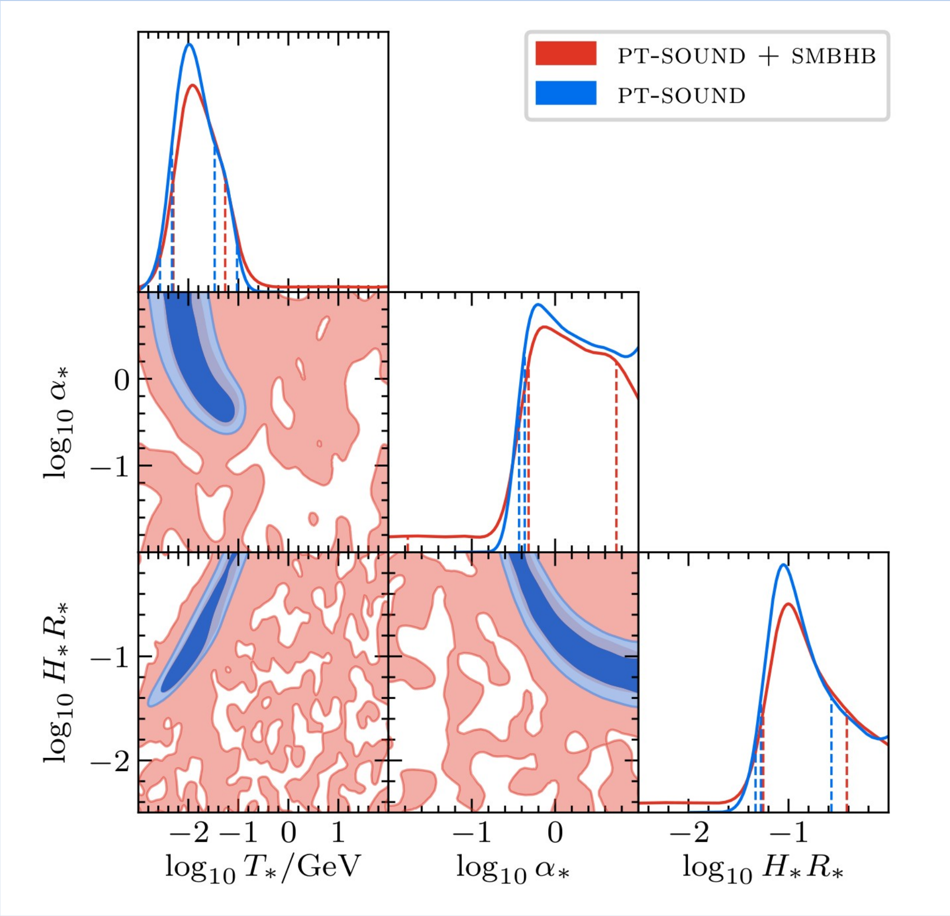
The currently favored interpretation is in terms of
a population of supermassive black hole mergers.
Still, the amplitude is on the low side and the spectrum seems a bit steep.




*[NanoGrav 2023]*

# Can GWs from phase transitions fit the data?

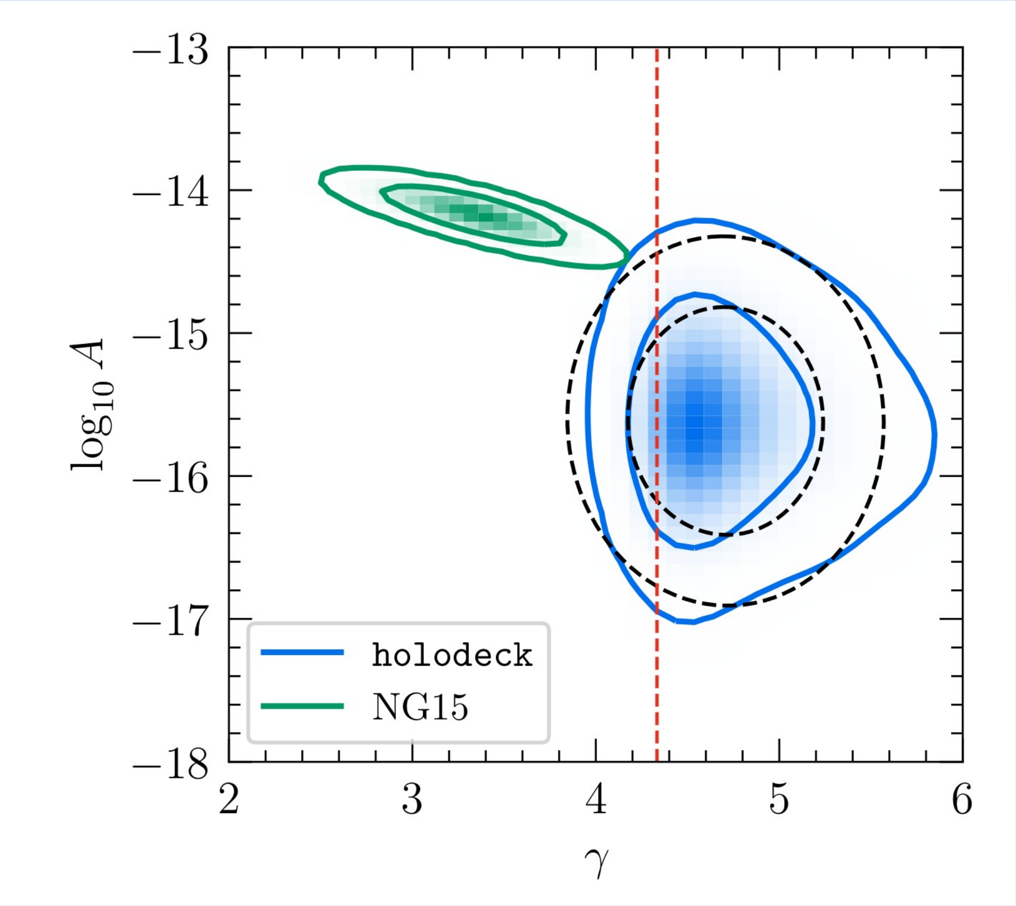
Yes!




But so can cosmic strings, domain walls, PBHs from inflation ...

*[NanoGrav 2023]*

# Actually, very well ...

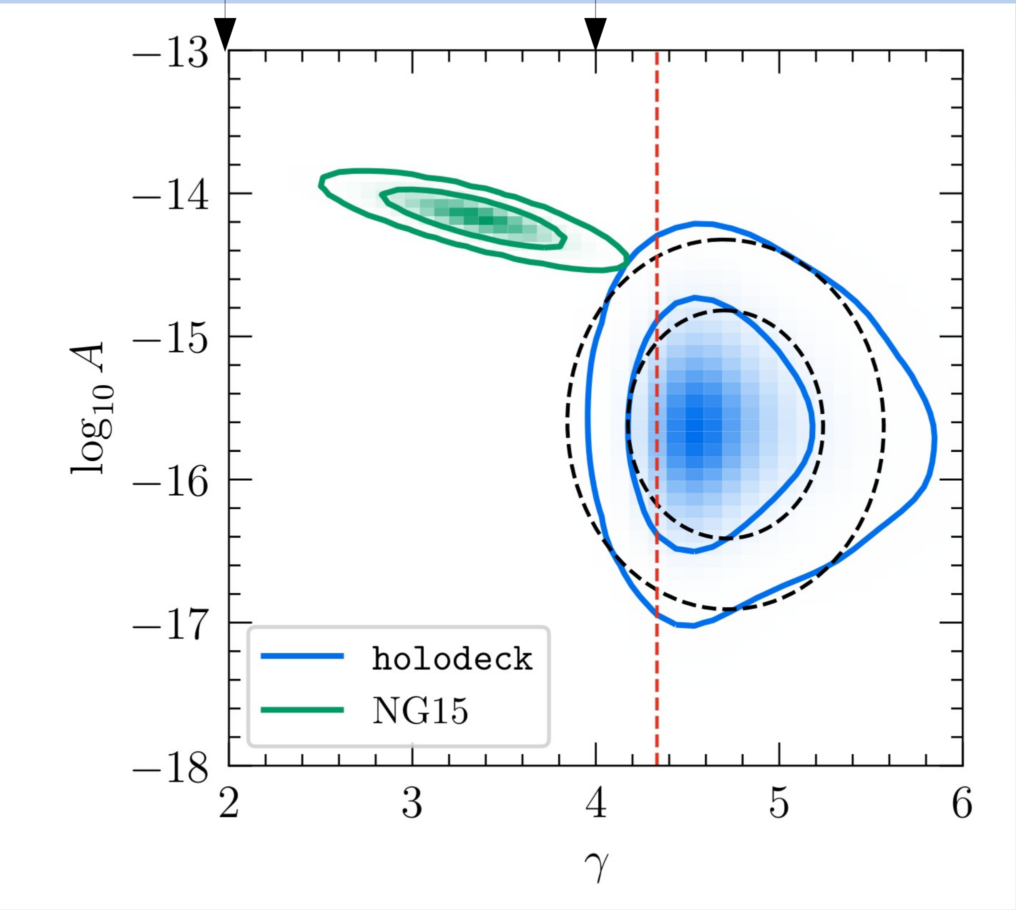



The power law fit is somewhere between the
IR tail and plateau. So the fit will probably further improve with the
new spectra.

*[NanoGrav 2023]*

# Actually, very well ...

IR tail $k^3$ plateau $k^1$

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*[NanoGrav 2023]*

# How to distiguish SMBHs from cosmological backgrounds?

There are in principle different ways to distinguish a background from supermassive black holes from a stochastic cosmological background

1) In principle the shape of the power spectrum can provide information.

2) For a SMBH background, isolated point sources should be at some point identifyable

3) more general, one would expect some anisotropies for SMBHs

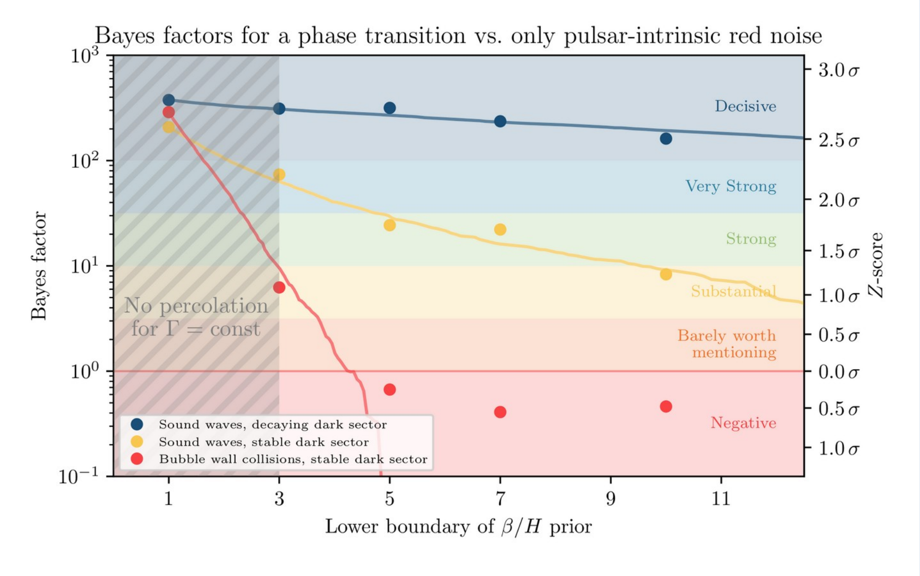
4) specific cosmological models might have additional signatures

5) Signal in LISA/LIGO

# Cosmological constraints on a MeV dark phase transition

Even if the phase transition happens in a dark sector, the GW signal is constrained by CMB and BBN observations.

If the dark sector is stable, a strong phase transition implies a large deviation for $N_{eff}$

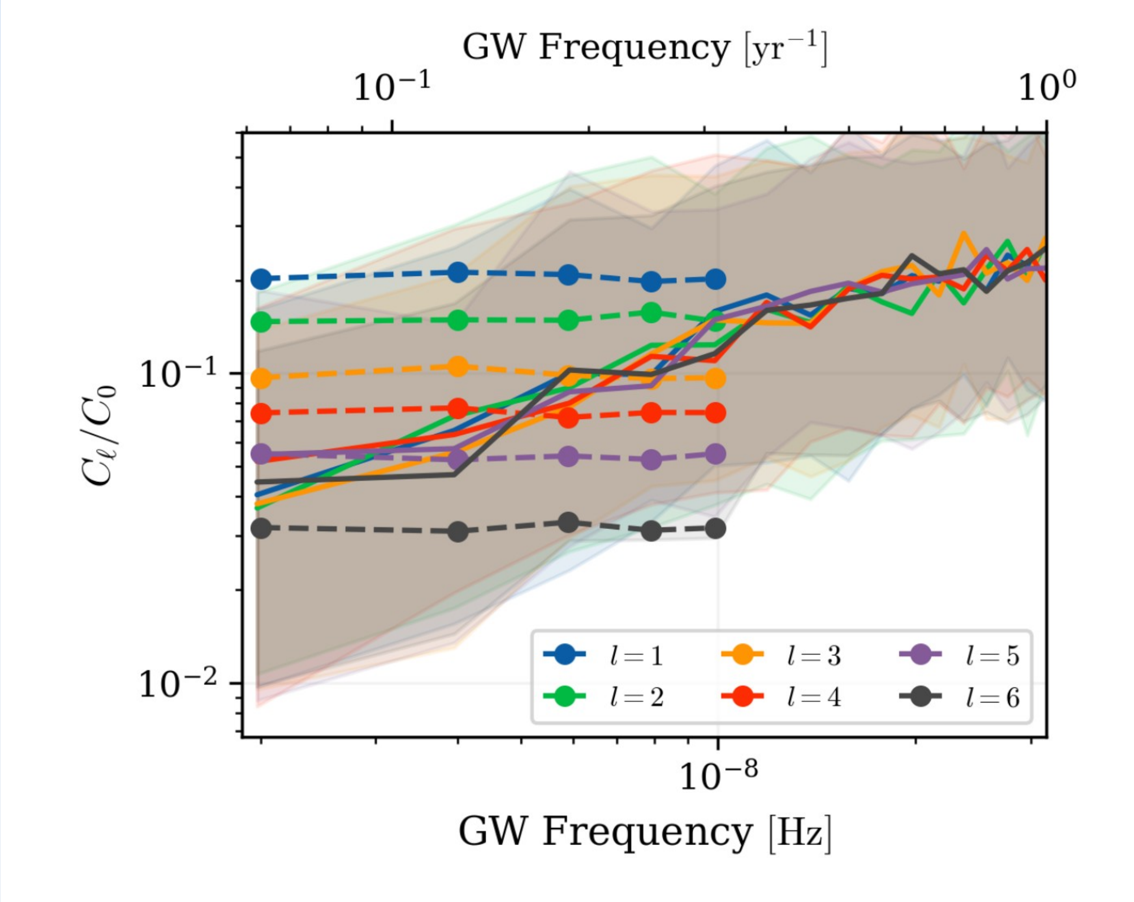



If the dark sector is unstable, the coupling to the SM can be potentially seen in beam dump experiments.

*[Bringmann, Depta, TK, Schmidt-Hoberg, Tasillo 2023]*

# Anisotropies

No anisotropies have been found so far.

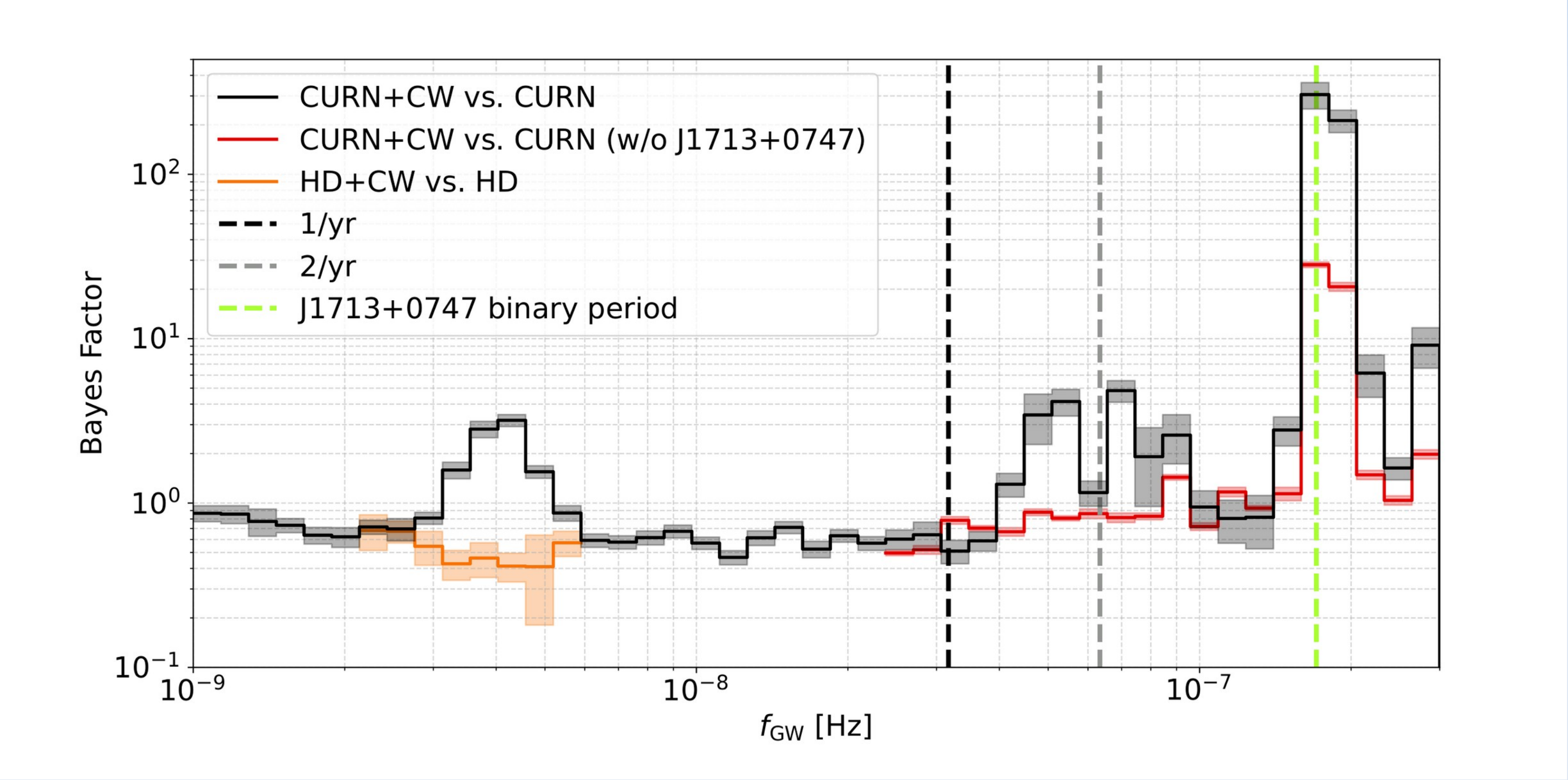



The bands denote expectations from SMBH.
The measurements are upper limits.

*[NanoGrav 2023]*

# Isolated sources

A fit to a GW background + isolated sources does not favor adding an isolated source




*[NanoGrav 2023]*

# Conclusions

Evidence for a stochastic GW background is building up in all PTA experiments. A combined result will further improve the statistics.

The mergers of supermassive black holes are a plausible interpretation of this signal.

But there are also some indications that the origin might be cosmological.

Dark phase transitions at the MeV are a possible candidate but require a prompt decay of the energy from the dark sector into the SM. This can lead to additional signatures.