Corner symmetry and quantum geometry
A paradigm for quantum gravity

TUG Paris
Oct 12 2023

Based on 2302.12799 with M. Geiller and W. Wieland
A chapter in ``Handbook of Quantum Gravity”

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M. Geiller, R.G. Leigh, F. Moosavian, R. Oliveri, D. Pranzetti,
A. Raclariu, S. Speziale, A. Speranza, N. Teh., W. Wieland.

QG Questions

- What are the fundamental QG degrees of freedom?
- What is the geometrical entropy counting?
- What are the fundamental observables?
- Can we provide a model of quantum gravity that respect the presence of a Planckian cutoff and the principle of general covariance
- Is there an optimal way to regulate covariant in way compatible with symmetry ?
- How does quantum geometry affect UV divergences ?

- Local Holography: A bottom-up perspective which comes from new questions:
- How do we decompose a gravitational sytems into subsystems?
- What is the nature of entanglement across subregions ?
- How do we understand the quantization of QFT and QG in finite regions?
- What are the symmetries of gravity ?
- What are the local quantum reference frames ?

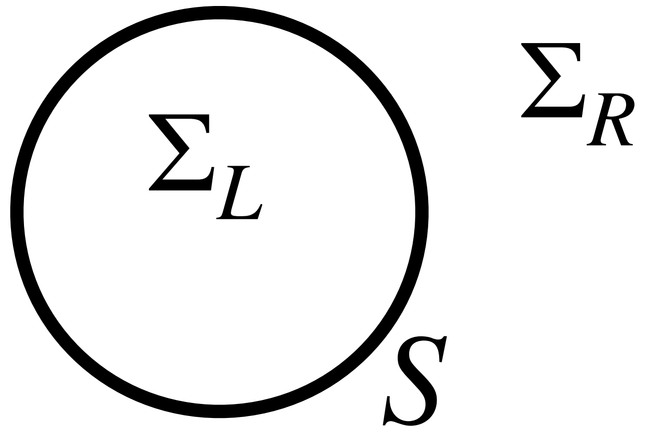
- Developing new tools: Covariant phase space, Coadjoint orbits
  Representation theory of sphere loop groups,
  Carrollian geometry, Hamiltonian Fluid dynamics

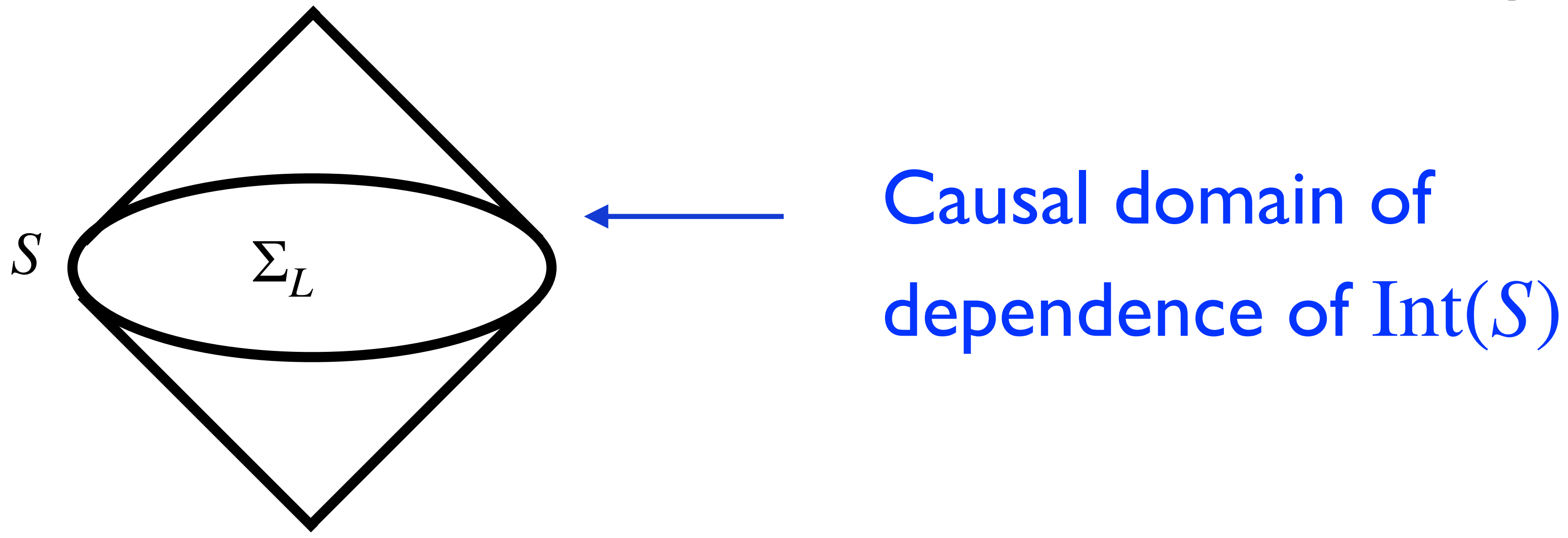
Local Holography

- In gravity the subregion entanglement is controlled by a symmetry group called the corner symmetry group, which follows from gauge invariance of the total space.
- This symmetry group is universal. It gives us semi-classical phase space tools to understand quantum geometry in the continuum
- At the quantum level finding the group representations amount to quantizing geometry. The symmetry generators and their Casimirs acts as geometrical operators
- Einstein's equations are charge conservation laws for the symmetry generators
- Recent developments are showing that the area operator action is connected to the modular group action
- Taking the limit of infinitely large regions naturally connects this approach with S-matrix quantization, asymptotic symmetries and soft theorems through celestial holography

Space entanglement

- Given Σ a Cauchy slice. We chose a 2d surface that divide the slice into 2 subregions $\Sigma = \Sigma_L \cup \Sigma_R$
- S is the entangling surface it defines the codimension 2 corner of the sustaining causal diamond





- We denote $\mathscr{A}_\Sigma$ the algebra of observable associated with the region Σ and $\mathscr{H}_\Sigma$ the corresponding Hilbert space obtained by acting with $\mathscr{A}_\Sigma$ on a vacuum state

- In Quantum mechanics we have double factorizability.
$\mathscr{A}_\Sigma = \mathscr{A}_{\Sigma_L} \vee \mathscr{A}_{\Sigma_R}$ and $\mathscr{H}_\Sigma = \mathscr{H}_{\Sigma_L} \otimes \mathscr{H}_{\Sigma_R}$

Modular Hamiltonian

- The modular group is the one dimensional group generated by $\dfrac{2\pi}{\hbar} K_\Psi = -\ln \rho_L$

$\rho_L = \mathrm{Tr}_{H_L} |\Psi\rangle\langle\Psi|$

- Entropy $S = \langle \Psi | K_\Psi | \Psi \rangle$

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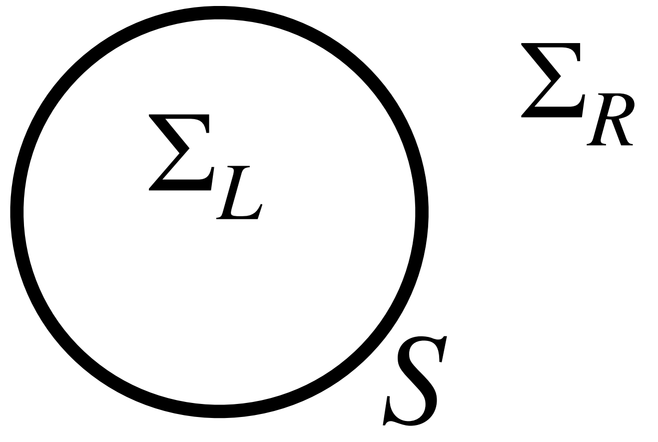

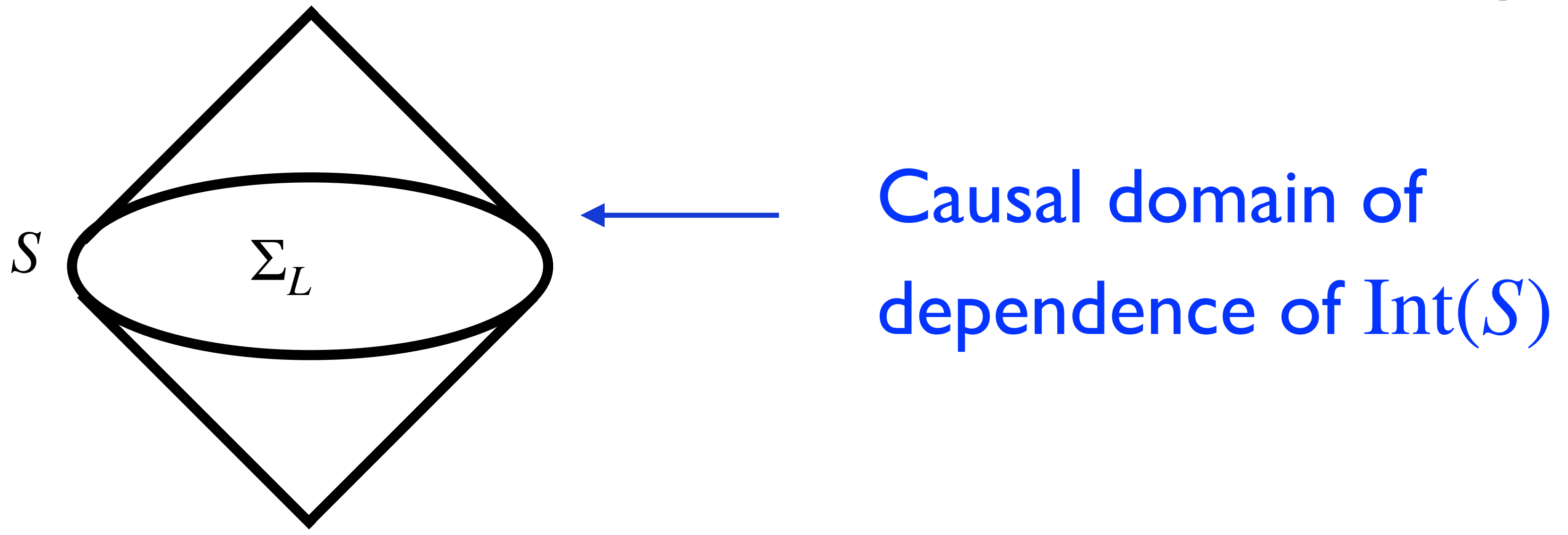

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- In Relativistic QFT we loose factorizability of the Hilbert space
$\mathscr{A}_\Sigma = \mathscr{A}_{\Sigma_L} \vee \mathscr{A}_{\Sigma_R}$ and $\mathscr{H}_\Sigma \neq \mathscr{H}_{\Sigma_L} \otimes \mathscr{H}_{\Sigma_R}$ since $G(\mathscr{H}_{\Sigma_L}, \mathscr{H}_{\Sigma_R}) \neq 0$

Infinite vaccuum entanglement

Reeh-Schlieder theorem

- Bisognano-Wichman theorem $\longrightarrow K_{|0\rangle}$ is equal to the boost generator for S a symmetry axis
- UV divergences implies that $\Delta K_\Psi = \infty$

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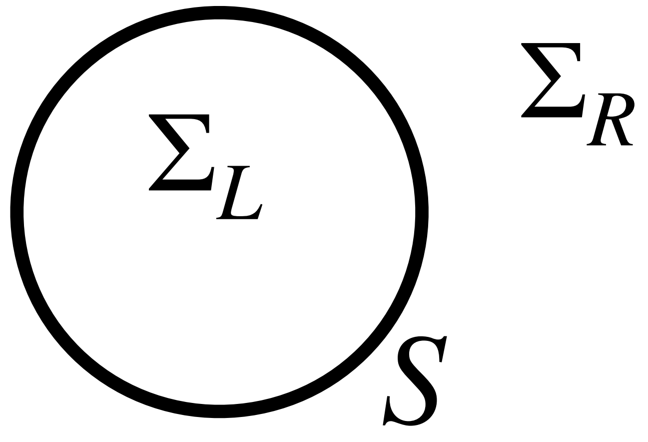

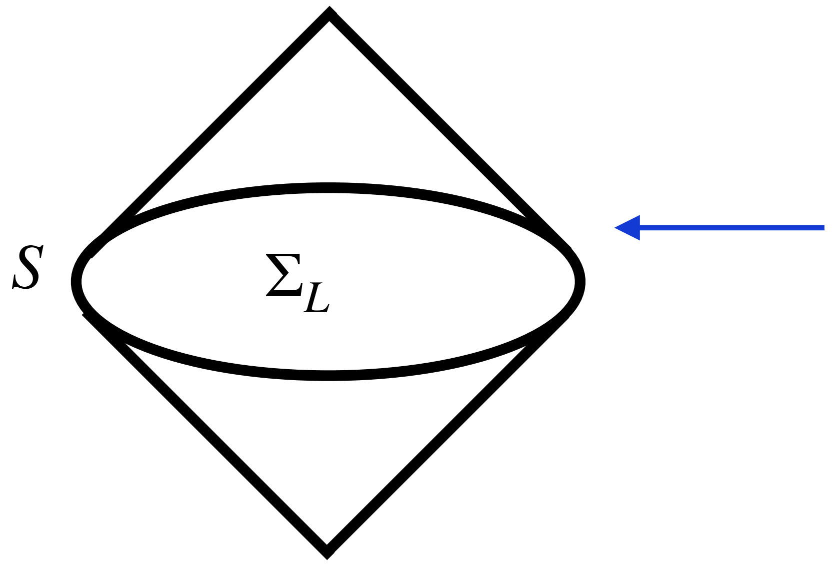

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- In Gravity and Gauge theory we also loose factorizability of observable algebra $\mathscr{A}_\Sigma \supsetneq \mathscr{A}_{\Sigma_L} \vee \mathscr{A}_{\Sigma_R}$ and $\mathscr{H}_\Sigma \neq \mathscr{H}_{\Sigma_L} \otimes \mathscr{H}_{\Sigma_R}$

Gauge invariant
Observables are non local

- One expect UV divergences resolution from Quantum gravity $\Delta K_\Psi = \dfrac{\langle A_S \rangle}{4G}$

K.Zurek, E.Verlinde 19

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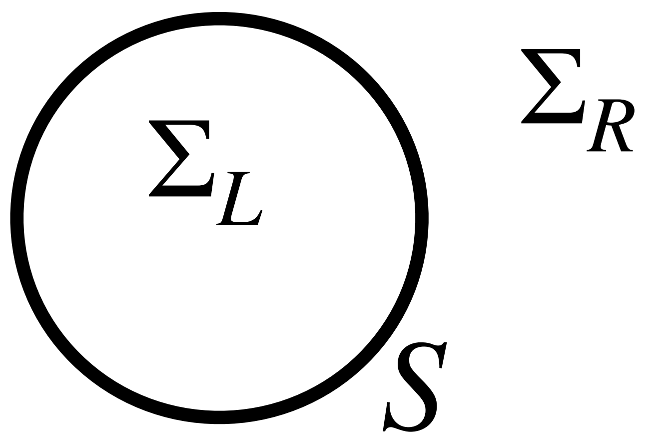

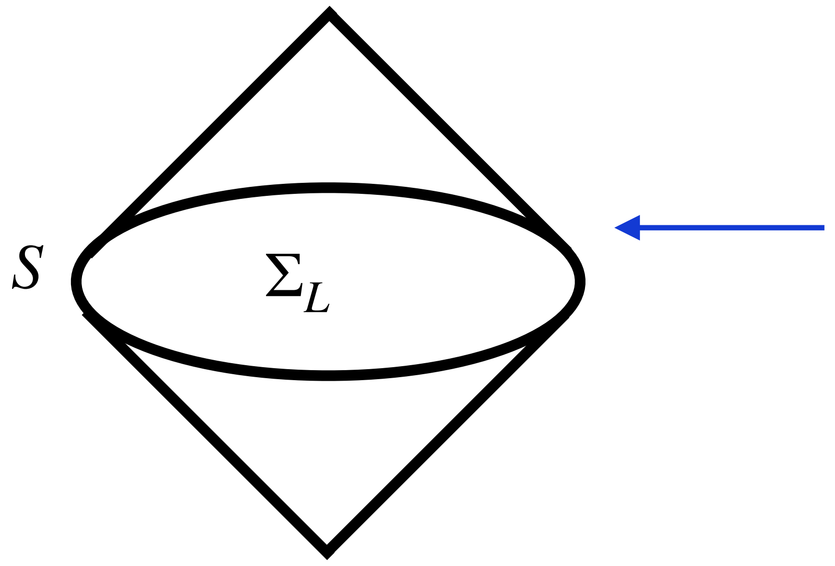

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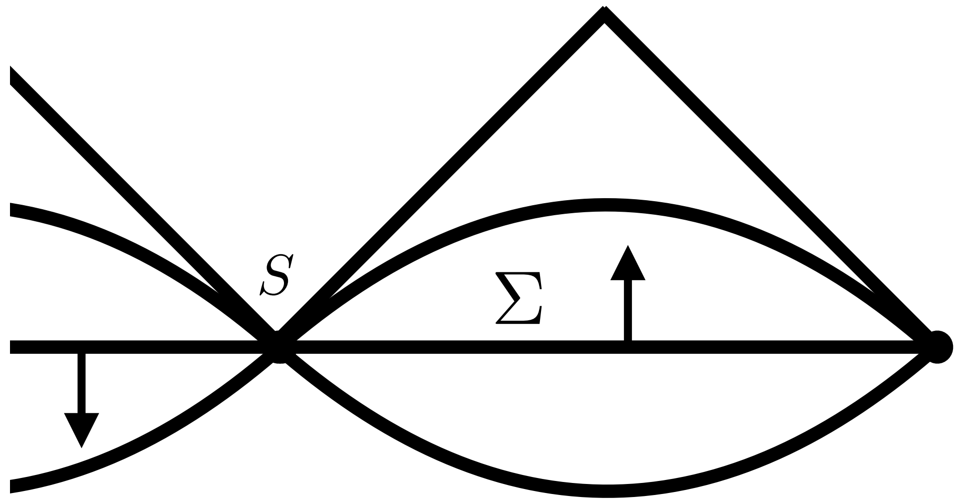
The factorability property of the algebra of observable in QG is governed by a symmetry group.

Gauge symmetry resolves entanglement



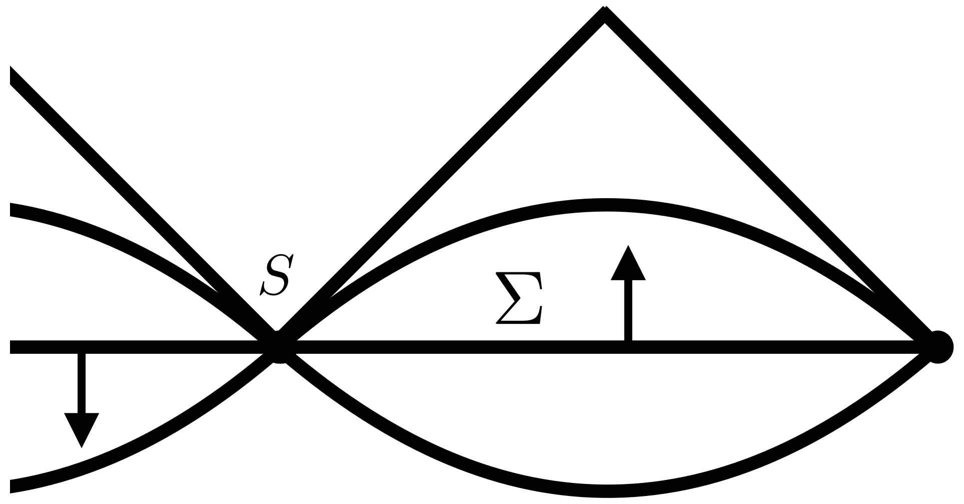
- In gauge theory and gravity, when no boundary exists the time evolution generator is a constraints $C_\xi \hat{=} 0$.
- The situation changes in the presence of a spacetime boundary or a spacetime corner
- In the presence of a spacetime boundary or a spacetime corner the time evolution operator is entirely supported on codimension 2 corners.

E. Noether 1918

$$Q_\xi(\Sigma_L) \hat{=} \int_{\partial\Sigma_L} q_\xi$$

- Corners unlike boundaries do not need the specification of boundary conditions

Gauge symmetry resolves entanglement



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- This property is, when we have no out-going radiation at infinity, the fundamental expression of gravitational holography D. Marolf 2008

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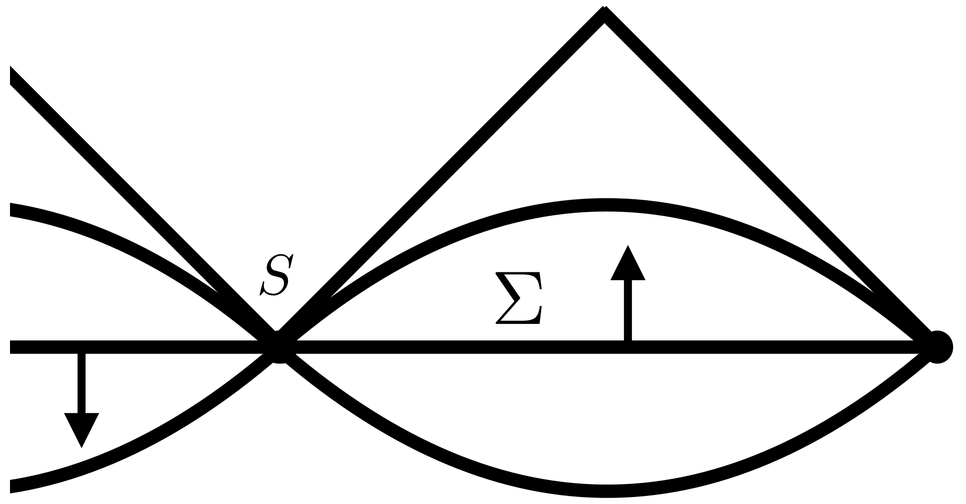

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A. Strominger 2014

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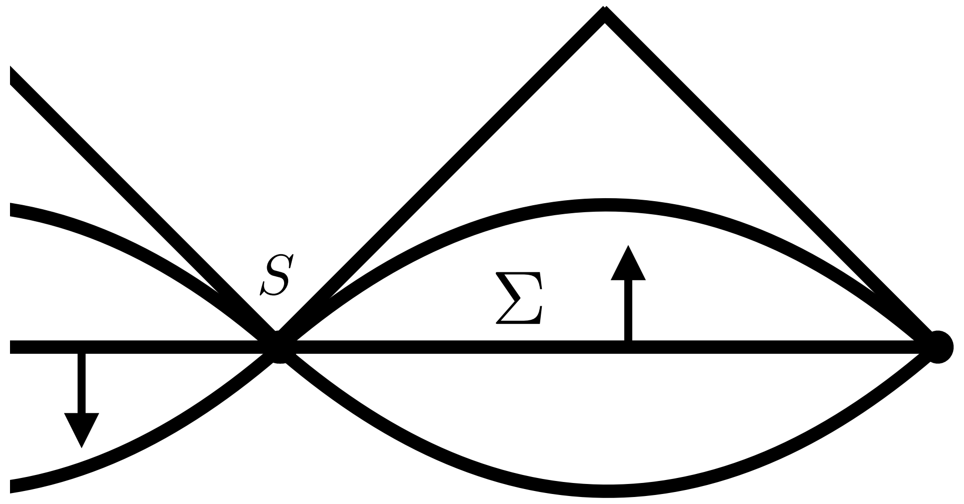

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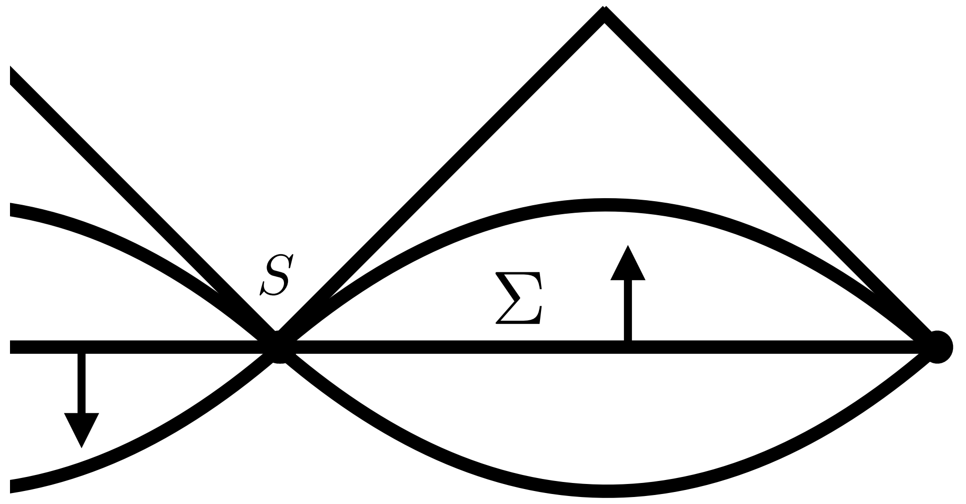

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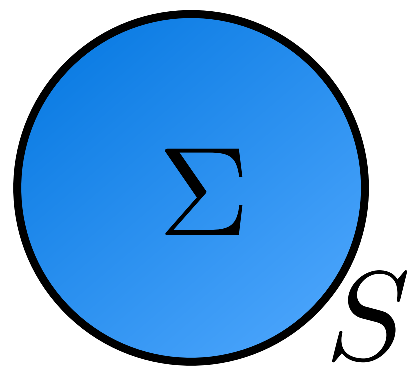
  Entangling corners carries the representation of a fundamental group of symmetry the corner symmetry group G_S. The modular group which contains boost hinging along S is a distinguished subgroup of G_S
- Noether theorem tells us that the charges represents elements of the spacetime geometry ⟶ Non-commutativity of the corner metric components

$$[Q_\xi, Q_\chi] = iQ_{[\xi,\chi]}$$

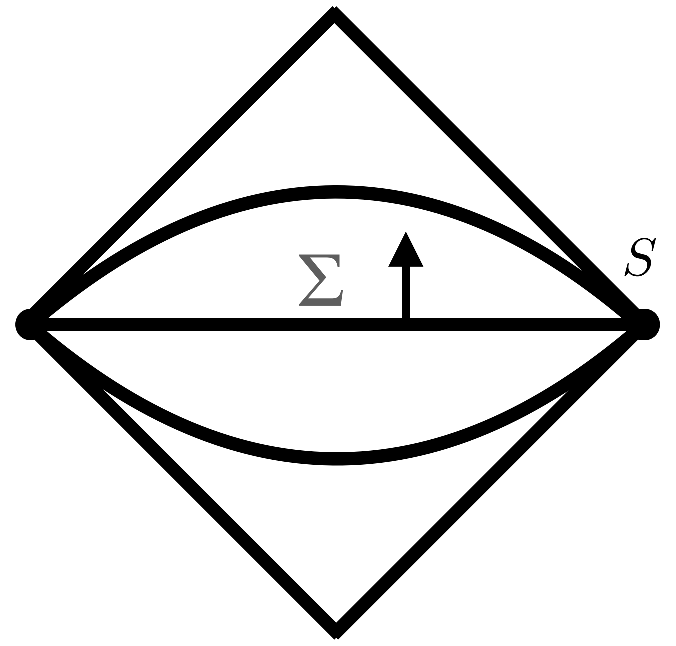
- Finding the quantum representation of G_S is equivalent to quantizing geometry

⟶ Understanding the quantum causal diamond

Symmetries and Gravity



- Given a region R with slice Σ the symmetry charges are supported on codimension 2 corners S= entangling sphere



- The extended corner symmetry group G_S is the subgroup of Diff(M) which and possesses non zero Noether charges in the presence of S, its with kinematical subgroup $G_S \subset \hat{G}_S$ preserves the region R.

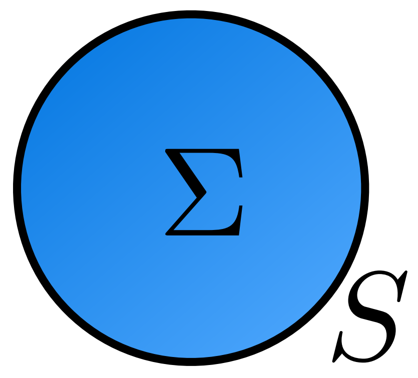
- In metric gravity

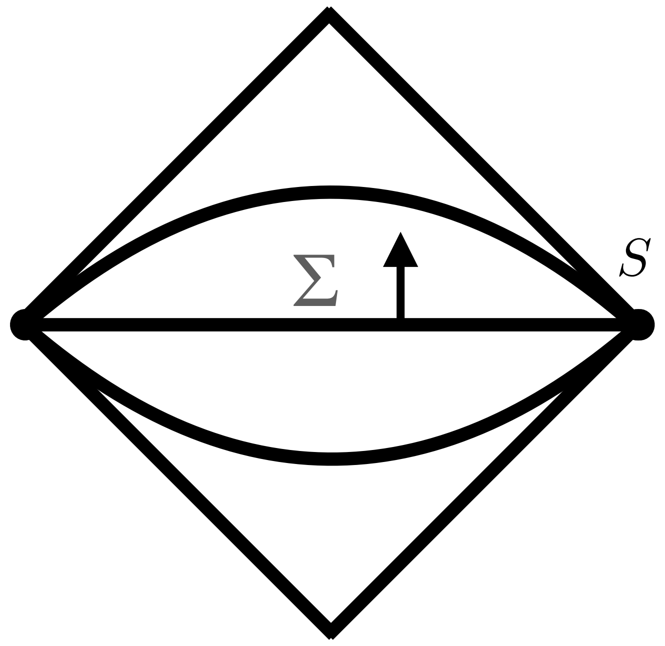
$$\hat{G}_S = (\mathrm{Diff}(S) \ltimes \mathrm{SL}(2,\mathbb{R})^S) \ltimes \mathbb{R}^{2\bar{S}}$$

Group = Kinematical + dynamical

W. Donnelly, L.F 2016
L.F, Leigh, Ciambelli' 21

Symmetries and Gravity





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W. Donnelly, L.F 2016
L.F, Leigh, Ciambelli' 21

- Double Universality of $\hat{G}_S$ for metric gravity!
  - -Same group for infinitesimal diamond or very large ones
  - -Same group for Einstein gravity or any other higher derivative formulation of gravity no matter how many extra derivative

Wald, Speranza'17

- What changes is either the choice of representation or the canonical representation of the symmetry generators: By Noether theorem symmetry generator are expressed in terms of metric components pulled back to S

Symmetry on null surfaces

- Local gravitational symmetries are attached to codimension 2 corner: In metric gravity this group is the extended corner symmetry group (Universal)

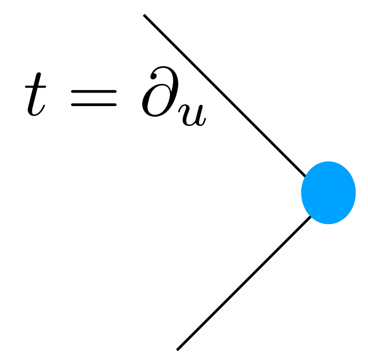
$$\hat{G}_S = (\mathrm{Diff}(S) \ltimes \mathrm{SL}(2,\mathbb{R})^S) \ltimes \mathbb{R}^{2S}$$

- When we study Horizon, asymptotic infinity or the nature of quantum radiation one focuses our attention onto a specific null surface. In that case the subgroup preserving the preserving the null structure (Thermal Carrollian structure) is

$$\mathrm{BMSW} = (\mathrm{Diff}(S) \ltimes \mathrm{Weyl}) \ltimes \mathbb{R}^S$$

Barnich-Trossaert'10,
Chandrasekar, Flanagan, Prabhu'18
LF, Oliveri, Pranzetti Speziale '21

$$\xi = T\partial_u + Y^A\partial_A + W(u\partial_u - r\partial_r)$$



- At infinity, same group, conservation law are associated with GBMS

$$W = \frac{1}{2} D_A Y^A$$

Barnich Troesseart '11 Campiglia, Ladha '16
Compere, Fiorucci, Ruzziconi'18

Quantum Corner symmetry

$$G_S = \mathrm{Diff}(S) \ltimes \mathrm{SL}(2,\mathbb{R})^S$$

Donnelly, Moosavian, Speranza, LF'

- What are the reps? what are the Casimirs?
- The little group is the group that preserves $C_{\mathrm{SL}(2,\mathbb{R})_\perp} = \det(q) > 0$
- The subgroup generated by $\sqrt{q}$ is the local modular boost group
- In gravity we have that for states representing minimal surfaces $K = \frac{A}{4G}$
- Representations are classified by representations of the area preserving Diffeomorphism subgroup: Coadjoint orbits
- The **outer curvature** generator Ω form a representation of the area preserving diffeomorphisms algebra

- The Casimirs are then given by

$$C_n = \int_S \sqrt{q}\,\Omega^n$$

$$C_0 = \mathrm{Area}$$
$$C_1 = \mathrm{NUT\ charge}$$
$$C_2 = \mathrm{Fluid\ enstrophy}$$

Quantum fluid

G_S is isomorphic to the symmetry groups of 2d hydrodynamics

W. Donnelly, A. Speranza, F.M Moosavian, L.F 2020

Classification of quantum representation of corner group isomorphic to the representation theory of

- Analogy: the area density $\sqrt{q}$ plays the role of the fluid density ρ
  The outer curvature Ω plays the role of the fluid vorticity w

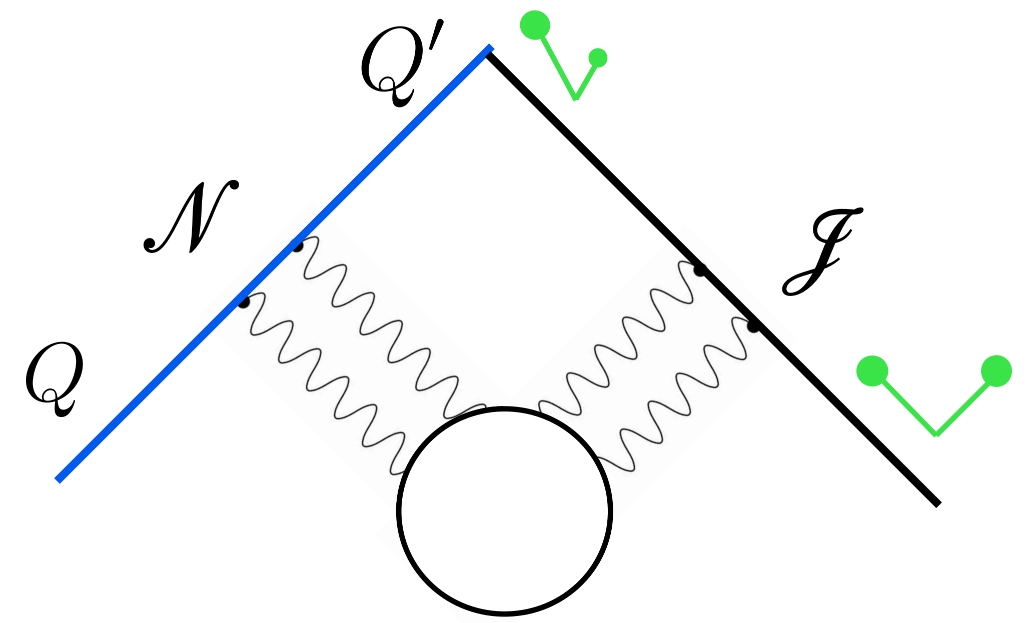
Arnold'66; Marsden, Ratiu'95 Khesin'17

- The quantum representations are classified by a choice of area and vorticity densities (ρ, w) on S. isomorphic
- (ρ, w) can be related to labels of the coadjoint orbits (hence representation) of the `fluid group' G_S

- Classical fluid corresponds to a choice of density density measure $\rho > 0$ which is absolutely continuous with respect to the Lebesgue measure
- Quantum fluid corresponds to a choice where both ρ and w are counting measures.
  This gives a constituent picture to the fluid
- Area constituent in the continuum from quantization!

LF, Geiller, Wieland '22

Dynamics along null surfaces



- The gravitational evolution along null surfaces can be entirely formulated as conservation laws for the corner charges ⟶ Quantization of the Einstein equation

Three main results for dynamics along causal Horizons:
Carollian structure (ℓ^a, q_{ab}) such that $\ell^a q_{ab} = 0$

- The Gravitational dynamics projected on $\mathcal{N}$ can be recast as a set of Null conservation Laws $D_b T_a{}^b = 0$ $\quad T_a{}^b = \tau_a \ell^b + \tau_a{}^b$

Carollian connection

Carollian energy-momentum tensor

Donnay, Marteau '19
LF, Hopfmueller,'19; Sheikh-Jabbari'20
Speranza, Flanagan, Chandrasekaran 21
Ashtekar, Khera, Kolanowski, Lewandowski 22
LF, P Jai-Akson 22

- This dynamics can be understood as the conservation of charges for a universal null surface symmetry group BMSW
- The dynamics can be understood in terms of a canonical structure associated with

$$\Theta^{\text{can}} = \frac{1}{8\pi G} \int_{\mathcal{N}} \left(\tfrac{1}{2} \tau^{ab} \delta q_{ab} - \tau_a \delta \ell^a \right) \epsilon_{\mathcal{N}}$$

- Conservation laws are the expression of diffeomorphism invariance

⟶ Quantum Raychauduri $\qquad$ Ciambelli, Leigh, F 23

Quantum Raychauduri

Ciambelli, Leigh, F 23

- The symplectic structure on null surfaces contains 3 geometric elements:

Spin 0 pair: (Ω, μ) where $\Omega = \sqrt{q}$ is the area form, and $\mu = \kappa + \frac{\theta}{2}$ (inaffinity +expansion)
Spin 1 pair: (π_a, ℓ^a) where $\pi_a = k_b D_a \ell^b$ is the boost connection
Spin 2 pair: $(\bar{q}_{ab}, \sigma^{ab})$ where $\bar{q}_{ab} = q_{ab}/\Omega$ is the conformal metric and $\sigma_{ab} = \frac{1}{2}\partial_v \bar{q}_{ab}$ is the shear

$$\Theta^{\text{can}} = \frac{1}{8\pi G}\int_{\mathcal{N}} \left(\tfrac{1}{2}\sigma^{ab}\delta\bar{q}_{ab} - \mu\delta\Omega\right)\epsilon_{\mathcal{N}}$$

- Spin 0 and spin 1 represents the quantum geometry degrees of freedom (geometric backreaction), while spin2 and matter represents the hard (EFT) degrees of freedom

- Raychaudhuri equation $$\partial_v^2\Omega = \mu\Omega - \Omega\left(\frac{1}{2}\sigma^2 + 8\pi G T_{vv}^{\text{matter}}\right)$$

can be understood as a balance equation between a sum of stress tensors

$$T^{(0)} + T^{(2)} + T^{\text{matter}} = 0$$

Balance equation

- Raychaudhuri equation can be understood as a balance equation between a sum of stress tensors

$$C := T^{(0)} + T^{(2)} + T^{\rm matter} = 0$$

$$T^{(0)} = \frac{1}{8\pi G}\left(\partial_v^2\Omega - \mu\partial_v\Omega\right) \qquad T^{(2)} = \frac{1}{16\pi G}\sigma^{ab}\sigma_{ab} \qquad T^{\rm matt} = (\partial_v\phi)^2$$

beta-gamma twisted CFT — Spin2 CFT — Matter CFT

- Given $T^{(i)}[f] = \int_{\mathcal{N}} f(v,\sigma)T^{(i)}(v,\sigma)$ they satisfy a Virasoro algebra above each point on the sphere

$$\{T^{(i)}[f], T^{(j)}[g]\} = \delta^{ij}T[f\dot{g} - g\dot{f}] + O(G)$$

- Gravity manifest itself in the presence of a non-zero stress tensor for geometry itself (spin 0). $T^{(0)} \neq 0$ is necessary to sustain the balance equation as $T^{(2)}, T^{\rm matt} \geq 0$
- At the quantum level the UV divergences shows itself as the fact that the total central charge is ∞
- Regulating UV divergences means allowing $c_{\rm total}$ to be finite

Quantum Time

- The spin 0 momenta represent a choice of dynamical time variable: $(v, \sigma^A) \to (V(v,\sigma), \sigma^A), \partial_v V \neq 0$

$$\mu = \frac{\partial_v^2 V}{\partial_v V}$$

- Raychaudhuri can be solved non-perturbatively away from caustic $\partial_v \Omega \neq 0$ as $V = \bar{V}(\Omega, \sigma)$

$$\partial_\Omega \bar{V} = \exp \int^\Omega d\Omega' \Omega' \left(\sigma^2 + 8\pi G T^{\text{matt}}_{\Omega\Omega} \right)$$

← Boost operator in areal time

- Using the dressing time one can construct gauge invariant observables $\tilde{q}_{ab}(v, \sigma^A) = q_{ab}(V, \sigma^A)$
- After resolution of the constraint the dressing time is a corner observable
- At the quantum level quantum fluctuations of the time variables help resolve some of the UV singularity: Edge mode = cross product that reduces type III to type II

$$\Delta K \Delta T \geq \hbar \text{ and } \Delta T > 0 \Rightarrow \Delta K < \infty$$

A. Connes 1968-1970
Witten, Chandrasekaran, Longo 23
Speranza, Sorce, Jensen 23
E.Gesteau, L.F 23-24

Entropy

- We can now compute the generator of the local boost generator hinging at a cut $V = 0$
- This canonical generator is $K(\sigma) := \frac{1}{4G}(\Omega - V\partial_V \Omega)$

$$\partial_v K = V \left(\frac{\sigma^2}{4G} + T^{\text{matt}}_{VV} \right) \epsilon_{\mathcal{N}} \geq 0$$

- This suggests that K is the proper modular hamiltonian

Quantum Radiation

- Quantum Radiation can be described as elementary transitions between representation of G_S

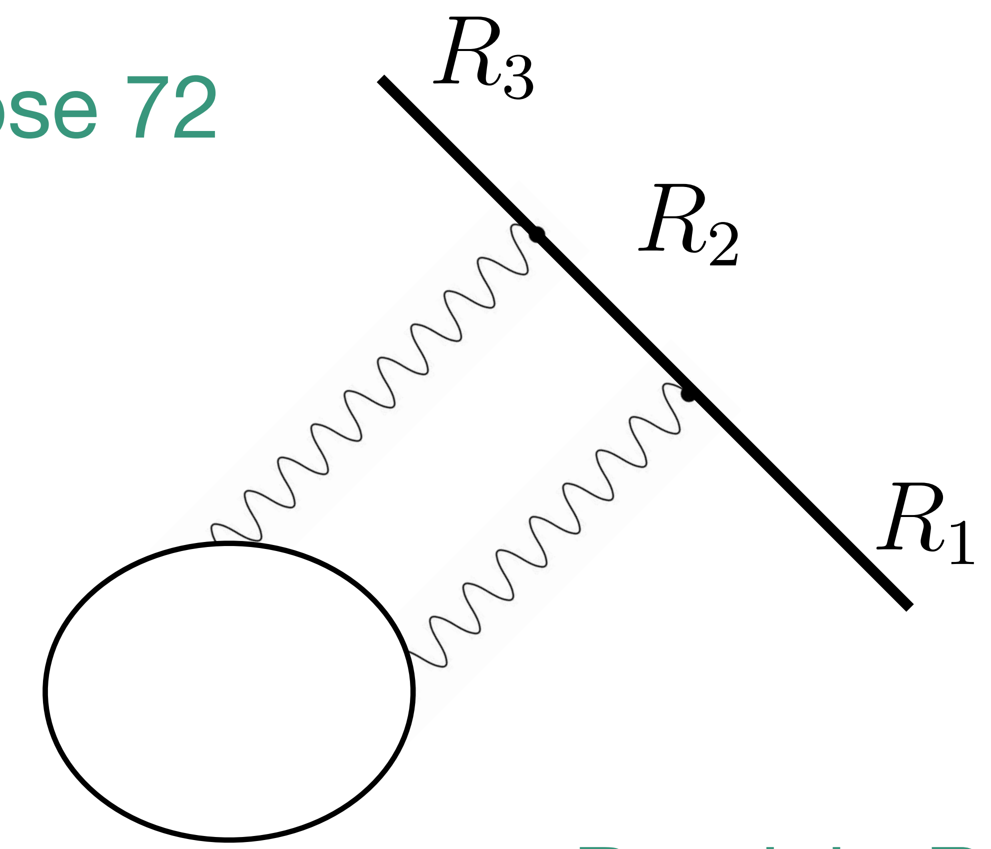
Wieland
Barrett

- These transitions are controlled by the extended symmetry group $\hat{G}_S$
- They can be represented classically in terms of spacetimes carrying Impulsive waves

LF Pranzetti 21

- Impulsive waves are solution of EE which carries non trivial radiations but no energy-momentum tensor along null sheets.
- Obtained through simple cut and paste techniques Penrose 72

$$\mathcal{N}^{AB} = \Delta n^{AB} \delta(u - u_0)$$



- Representations R corresponds to coadjoint orbits of GBMS

Barnich, Ruzziconi' 21
LF, Moosavian, Pranzetti '23

- At the quantum level we expect these transitions are intertwinners of the extended group G_s: A new picture of quantum dynamics.
- Asymptotic states form representation of G_S

Summary:

- The profound consequences of Noether theorem for gravitational theories leads to a new picture of quantum geometry as a state of representation of the corner symmetry group which capture the essence of subregions entanglement.

- It encodes the non-commutativity of geometrical observables associated with subregions representing the quantization of geometry.
- It leads discretization of space from the representation of continuous non-commutative infinite dimensional algebras represented as quantum fluid at the corner.

- Dynamics along null surfaces is encoded into Carrolian conservation laws for the symmetry charges and activated at the quantum level by the representation of the dynamical charges and transitions between corners

- These concepts can be extended to asymptotic Dynamics which connects to S-matrix calculations and reveals a new tower of higher spin symmetry responsible for all known soft theorems

Thank You !

Diffferent Corner symmetry

- For a different formulation of gravity we have $L_F = L_{EH} + d\ell_{F/EH}$ LF, Geiller, Pranzetti '20

$$\Omega_F = \Omega_{EH} + \mathrm{d}\Omega_{F/EH}$$

- Different formulation have different symmetry groups ⟶ **Inequivalent quantization**

$$\hat{G}_S = (\mathrm{Diff}(S) \ltimes K_S) \ltimes \mathbb{R}^{2\bar{S}}$$

Perez, Engle, Noui '10
Bodendorfer '13
Perez, LF '15

$$\{q_{AB}(\sigma), q_{CD}(\sigma')\} = \gamma(\epsilon_{AC}q_{BD} + \cdots)\delta^{(2)}(\sigma, \sigma')$$

Einstein-Hilbert $K_S = \mathrm{SL}(2,\mathbb{R})^S_\perp$

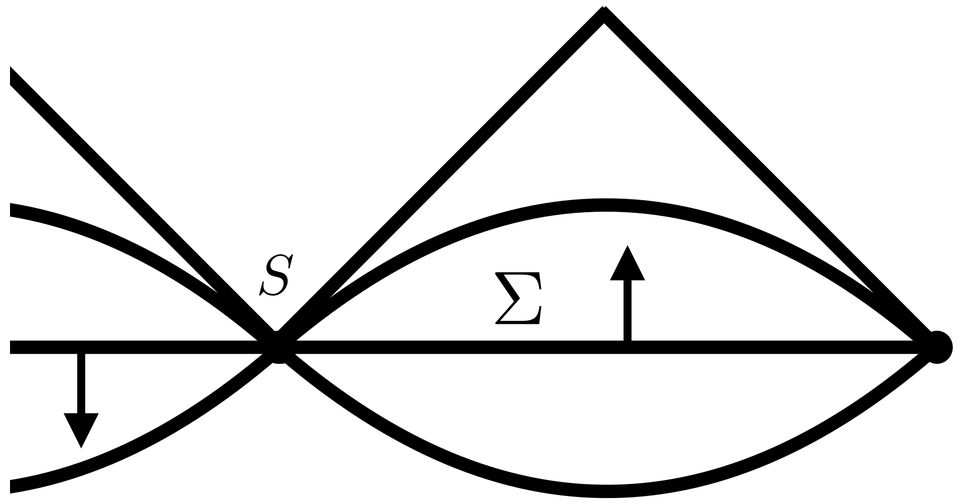
Einstein-Cartan-Holst $K_S = \mathrm{SL}(2,\mathbb{C})^S_\parallel \times \mathrm{SL}(2,\mathbb{R})^S_\parallel$ ⟵ Tangential metric q_{AB}

Electric Flux ↑

- In all cases we have that $\sqrt{\mathrm{Casimir}_2(K_S)} \propto \sqrt{q}$ Area form! Loop gravity input

- $\mathrm{SU}(2)^S$ is a subgroup of Gs

Edge modes and symmetry



W. Donnelly, LF 2016
Speranza 2017
Leigh, Ciambelli 2021
LF 2021

- Entangling corners carries the representation of a fundamental group of symmetry the corner symmetry group G_S. The Tomita-Takesaki modular group which represents boost hinging along S is a distinguished subgroup of G_S
- In order to define Gravity in finite region we need a field that tells us where the corner is situated $X : B_3 \to \Sigma$
- This field, called the embedding field or edge mode field is part of the gravitational phase space
- It can be used to dress physical operators and allows us to define an extended algebra of observables $\mathscr{A}^{\text{ext}}_{\Sigma_L} = \mathscr{A}^{\text{ext}}_{\Sigma_L} \vee \mathscr{A}_S$
- $\mathscr{A}_S = B(L^2(G_S))$ is the algebra of operators acting on functions on the corner symmetry group
- The extended algebra of observable is the cross product algebra
- It carries a representation of the corner symmetry group
- The full algebra can then be obtained by fusion of extended algebra $\mathscr{A}_\Sigma = \mathscr{A}_{\Sigma_L} \boxtimes_{G_S} \mathscr{A}_{\Sigma_R}$
- This mechanism can be extended to more than one subregion $\Sigma = \cup_i \Sigma_i$ .The observable algebra is then obtained from the gluing of many extended subalgebras

Connes 1973
Venkatesa, Witten et al 2023
Speranza et al. 2023
LF, E Gesteau 2022

Charge and Flux

- Dynamical symmetries carry **Flux**

$$I_\xi \Omega = \delta Q_\xi + \mathscr{F}_\xi$$

Canonical variation = Noether + Flux

- Local conservation Law
  = Equation of motion
  $\ell = \partial_t$

$$\dot{Q}_\xi = Q_{[\xi,\ell]} + I_\xi \mathscr{F}_\ell$$

Evolution = Rotation + dissipation

Ashtekar Streubel '81
Wald, Zoupas' 00
Barnich Troessaert '10
Pasterski, Strominger, Zhib '18
Ladha, Campiglia '18
Pranzetti, Oliveri, Speziale,LF '21
Ciambelli, Leigh '21
Wieland'22

- The charges splits into kinematical charges $\mathscr{F}_\xi = 0$ and dynamical charges.
- The kinematical charges are canonical bracket that form a quantizable algebra:
  Corner symmetry group G_S

$$[Q_\xi, Q_\chi] = iQ_{[\xi,\chi]}$$

- The Dynamical charges form an extended corner symmetry group $\hat{G}_S$ also quantizable in a **extended phase space**

Ciambelli, Leigh '21
LF'21

- Einstein's equation are recasted as quasi-local conservation laws on causal diamond around the entangling corner

Charge and Flux

- Dynamical symmetries carry **Flux**

$$I_\xi \Omega = \delta Q_\xi + \mathscr{F}_\xi$$

Canonical variation = Noether + Flux

- Local conservation Law: Flux balance

$$\delta_\ell Q_\xi = Q_{[\xi,\ell]} + I_\xi \mathscr{F}_\ell$$

Evolution = Rotation + dissipation

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- The Dynamical charges form an extended corner symmetry group $\hat{G}_S$ also quantizable in a **extended phase space**
- The Charges represents the non-commutative geometry

⟶ Non-commutativity of the corner metric components

- At the quantum level physical observables form representations of $G_S \subset \hat{G}_S$

⟶ Quantising $G_S, \hat{G}_S$ = Quantizing geometry.

Quantum fluid

W. Donnelly, A. Speranza, F.M Moosavian, L.F 2020

G_S is isomorphic to the symmetry groups of 2d hydrodynamics

Landau; Arnold; Marsden, Ratiu,

- Analogy: the area density $\sqrt{q}$ plays the role of the fluid density ρ

  The outer curvature plays the role of the fluid vorticity w

- This provides a **constituent picture** where

M. Geiller, D. Pranzetti, L.F 2021

Fluid molecularization = Area constituent
Vortex quantization = momenta quantization

$$\rho = \sum_i \rho_i \delta^{(2)}(\sigma, \sigma_i)$$

- Each constituent carries a density, weight and spin (ρ_i, Δ_i, s_i)

$$P_A = \sum_i \delta^{(2)}(\sigma, \sigma_i) D_A + (\Delta_i \delta_A^B + s_i \epsilon_A{}^B) \partial_B \delta^{(2)}(\sigma, \sigma_i)$$

LF, Geiller, Wieland '22
L.F '23

- Area constituent in the continuum from quantization!

- Einstein Cartan gravity with an Immirzi parameter implies that $\rho_i = \gamma\sqrt{j_i(j_i+1)}$.
  Area gap in the continuum!

Wieland '19

Quantum fluid

W. Donnelly, A. Speranza, F.M Moosavian, L.F 2020

H_S is isomorphic to the symmetry groups of 2d hydrodynamics

Landau; Arnold; Marsden, Ratiu,

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Geiller, Wieland, L.F 2022

- Area constituent in the continuum from quantization!

- The area preserving diffeomorphisms arises as the large N limit of SU(N)

  → Matrix model deformation of Gravity and its symmetry.

W. Donnelly, A. Speranza, F.M Moosavian, L.F 2022