Non-perturbative Wavefunction of the Universe in Inflation with (Resonant) Features

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Work in progress

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Outline

I. Motivations

II. How to go beyond perturbation theory

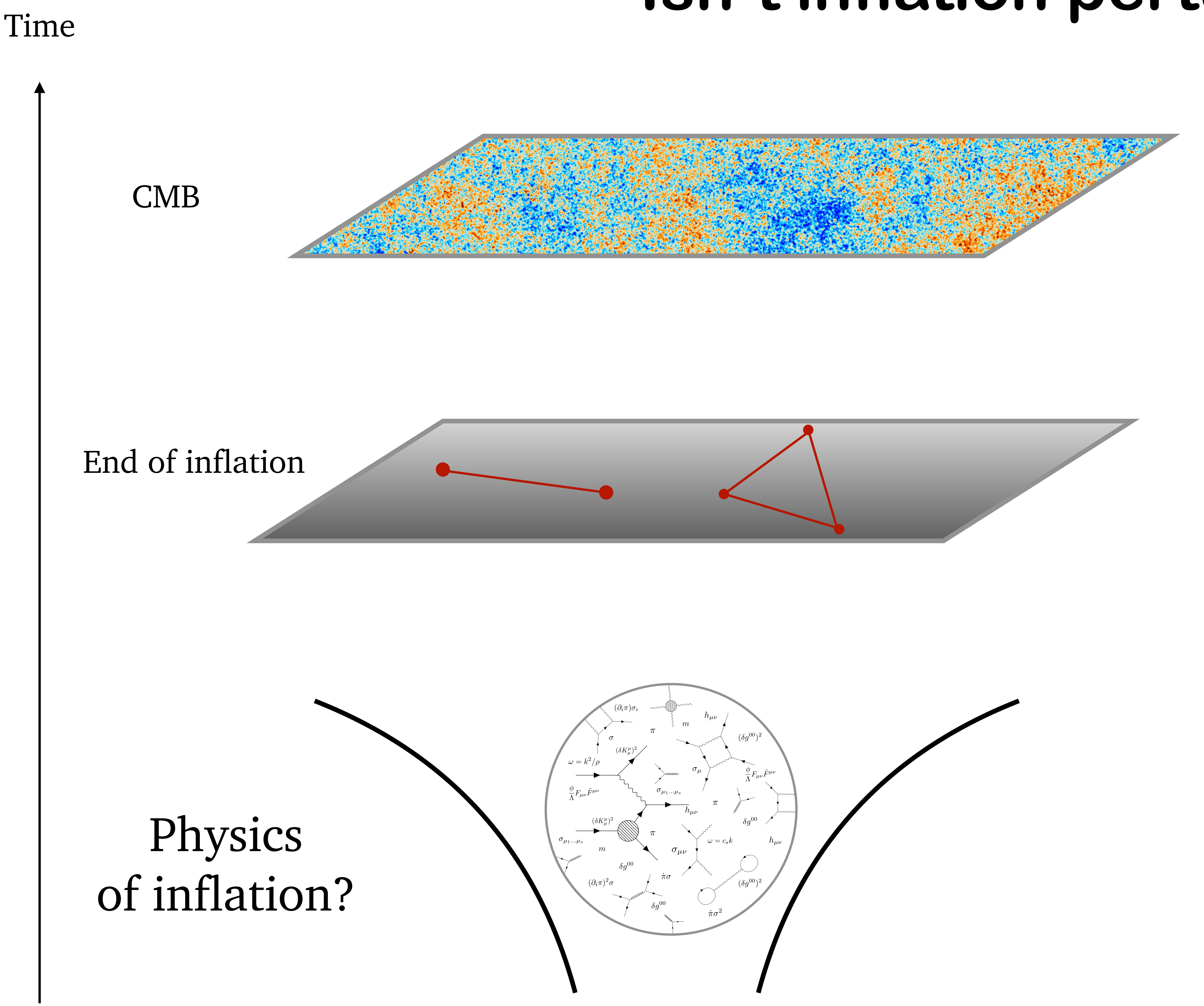
III. Wavefunction for resonant features

IV. Results and questions

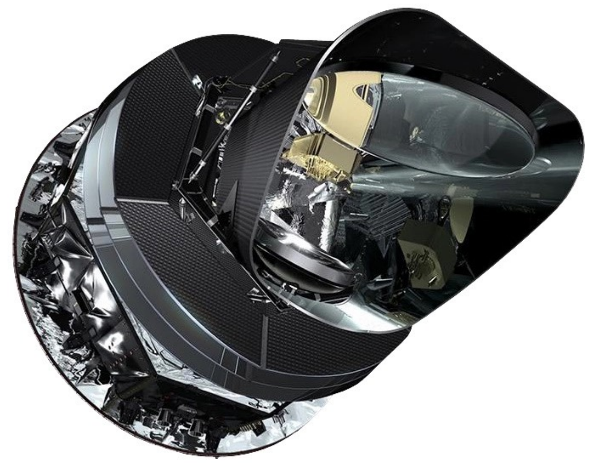
I. Motivations

- Phenomenological
- Theoretical

Isn’t inflation perturbative?!



Credit: Denis Werth



Planck satellite

$|f_{\mathrm{NL}}^{\mathrm{loc}}| < 5 \quad |f_{\mathrm{NL}}^{\mathrm{eq}}| < 40$

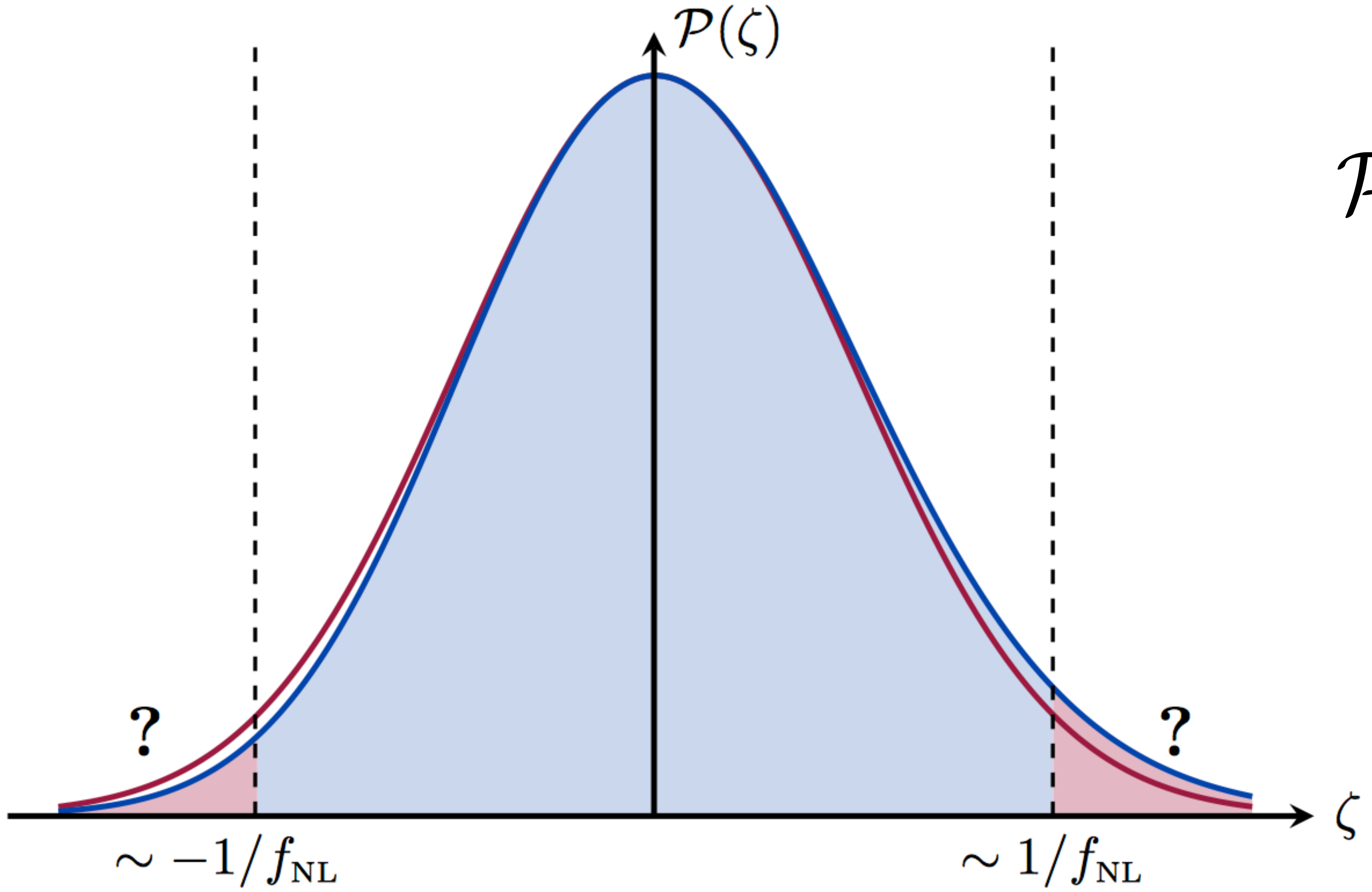
↓

Primordial fluctuations:
Gaussian + small corrections
(at most 0.1%)

↓

Simplest explanation: (very) weakly coupled theory

Tail of distribution



$$\mathcal{P}(\zeta) \sim \exp\left[-\frac{\zeta^2}{2P_\zeta}\left(1+\frac{\langle\zeta\zeta\zeta\rangle}{P_\zeta^2}\zeta+\frac{\langle\zeta\zeta\zeta\zeta\rangle}{P_\zeta^3}\zeta^2+\dots\right)\right]$$

$$\frac{\langle\zeta\zeta\zeta\rangle}{P_\zeta^2}\zeta \sim f_{\rm NL}\zeta \qquad \frac{\langle\zeta\zeta\zeta\zeta\rangle}{P_\zeta^3}\zeta^2 \sim g_{\rm NL}\zeta^2$$

Expansion parameter depends on size of ζ

$\zeta \sim \frac{\delta\rho}{\rho}$ primordial density fluctuations

Perturbation theory OK for correlation functions but Non-perturbative method needed for the tail

Motivations

Black hole formation
sensitive to $\zeta \sim 1$

Unexpected regimes?
(later in the talk)

Surprise in the data
on the tails?

One can compute the
wavefunction of the universe!

Eternal inflation:
can the tails be relevant?

II. How to go beyond perturbation theory

- Wavefunction approach
- Semi-classical method

Main idea

Since fluctuations are proportional to $\hbar^{1/2}$

Looking at unlikely events corresponds to the semi-classical limit $\hbar \to 0$

The tail of the distribution is amenable to a semiclassical calculation

The wavefunction

$$\Psi[\bar{\zeta}(\boldsymbol{x})] = \int_{\mathrm{BD}}^{\bar{\zeta}(\boldsymbol{x})} \mathcal{D}\zeta e^{iS[\zeta]/\hbar}$$

Transition amplitude, from Bunch-Davies vacuum to given configuration

$$\langle\zeta(\boldsymbol{x}_1)\ldots\zeta(\boldsymbol{x}_n)\rangle = \int \mathcal{D}\zeta\,\zeta(\boldsymbol{x}_1)\ldots\zeta(\boldsymbol{x}_n)|\Psi(\zeta)|^2$$

Correlators

Wavefunction

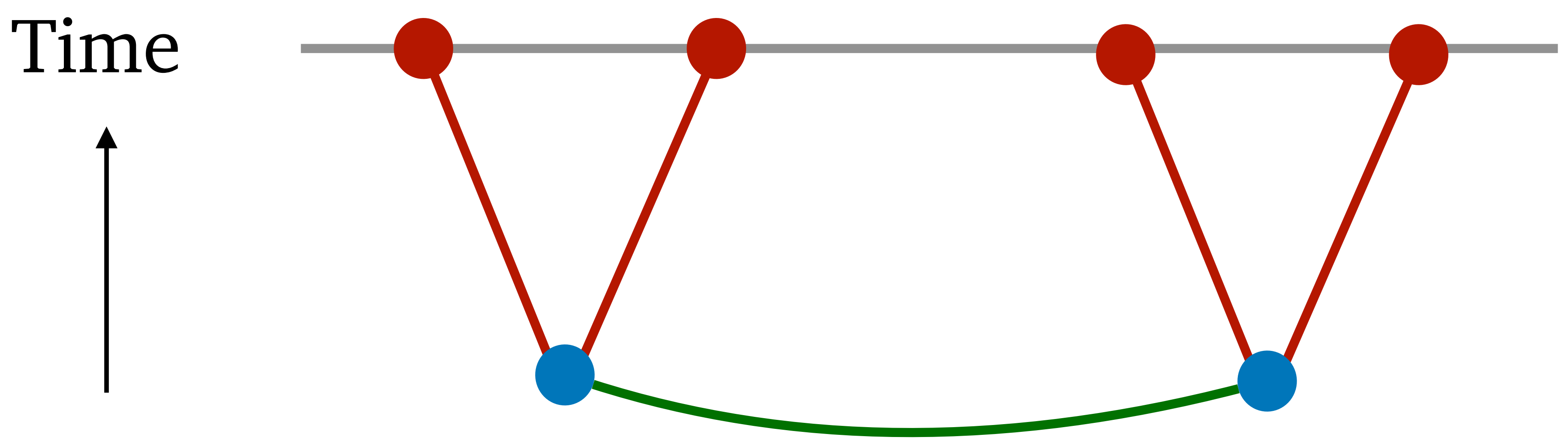
The perturbative wavefunction

$$\Psi[\bar{\zeta}(\boldsymbol{x})] = \int_{\mathrm{BD}}^{\bar{\zeta}(\boldsymbol{x})} \mathcal{D}\zeta e^{iS[\zeta]/\hbar}$$

$$\Psi(\zeta) = \exp\Big[\sum_{n\geq 2} \frac{1}{n!} \int \prod \mathrm{d}\boldsymbol{k}_i \delta\big(\sum \boldsymbol{k}_i\big)\, \zeta_{\boldsymbol{k}_1} \ldots \zeta_{\boldsymbol{k}_n} \psi_n(\boldsymbol{k}_i)\Big]$$

Wavefunction coefficients

Feynmann-Witten diagrams
with diagrammatic rules



The non-perturbative wavefunction

$$\Psi[\bar{\zeta}(\boldsymbol{x})] = \int_{\mathrm{BD}}^{\bar{\zeta}(\boldsymbol{x})} \mathcal{D}\zeta\, e^{iS[\zeta]/\hbar} \simeq e^{iS[\zeta_{\mathrm{cl}}]/\hbar}$$

ζ_{cl} : solution to the classical (non-linear) equation of motion with prescribed boundary conditions (like instantons)

Resummation of all tree-level non-linearities
with negligible loop effects

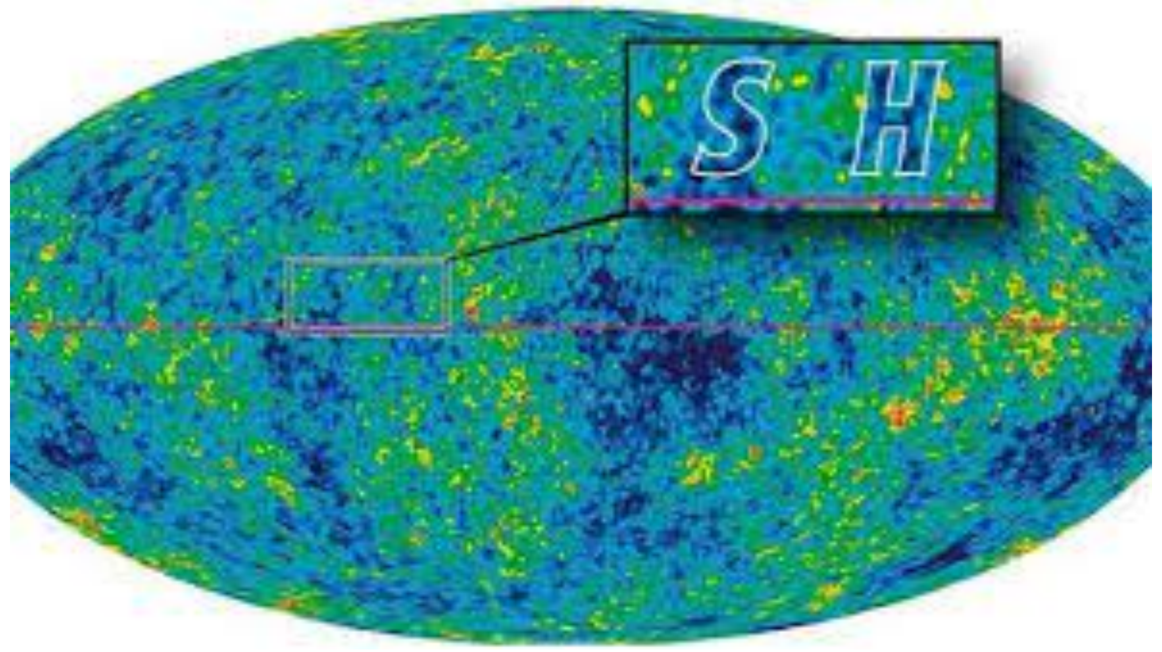
Celoria, Creminelli, Tambalo, Yingcharoenrat 2021

Some remarks

- Different from other non-perturbative approaches like stochastic inflation:

takes into account non-trivial quantum physics inside the Hubble radius

- Not one point pdf, full morphological information of density profile

e.g. $\mathbb{P}\Bigg[$ 

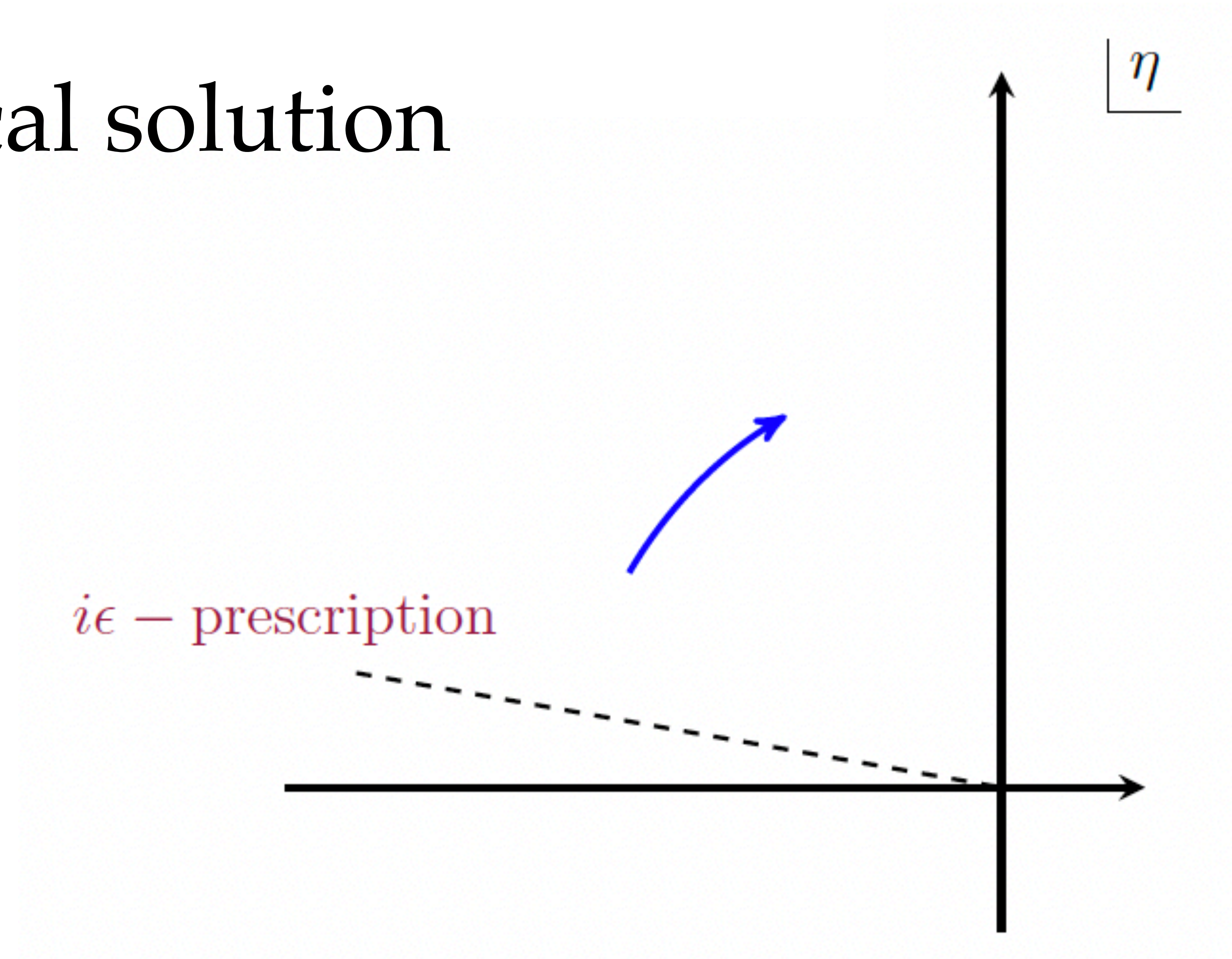
$\Bigg]$

Gaussian wavefunction

- For free theory, semi-classical result is exact, with classical solution

$$\zeta_{\rm cl}(\eta, \boldsymbol{x}) = \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3} \bar{\zeta}(\boldsymbol{k}) e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \frac{(1 - ik\eta)e^{ik\eta}}{(1 - ik\eta_{\rm f})e^{ik\eta_{\rm f}}}$$

decays exponentially after $i\epsilon$ rotation



- Result: $iS = \dfrac{i}{2P_\zeta} \displaystyle\int \mathrm{d}^3\boldsymbol{x} \frac{1}{\eta_{\rm f}^2} \zeta_{\rm cl}(\eta_{\rm f}, \boldsymbol{x}) \zeta'_{\rm cl}(\eta_{\rm f}, \boldsymbol{x}) \simeq \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3} \left(i\frac{k^2}{\eta_{\rm f}} - k^3 \right) \frac{\bar{\zeta}(\boldsymbol{k})\bar{\zeta}(-\boldsymbol{k})}{2P_\zeta}$

$$P_\zeta \equiv \frac{H_\star^2}{2\epsilon_\star M_{\rm Pl}^2}$$

Pure phase: does not affect probability

Scale-invariant power spectrum

III. Wavefunction for resonant features

- Full Nonlinear Action
- Resonant features
- Expression

Fully nonlinear action for fluctuations?

New non-perturbative expression

$$S = \int \mathrm{d}t\, \mathrm{d}^3x\, a^3 M_{\mathrm{Pl}}^2 \dot{H}(t+\pi)(\partial_\mu \pi)^2$$

Full nonlinear action valid in all models of canonical single-field inflation

Reformulation of EFT of inflation, valid in the decoupling limit $\epsilon \ll 1$

and with manifest super-Hubble conservation of π

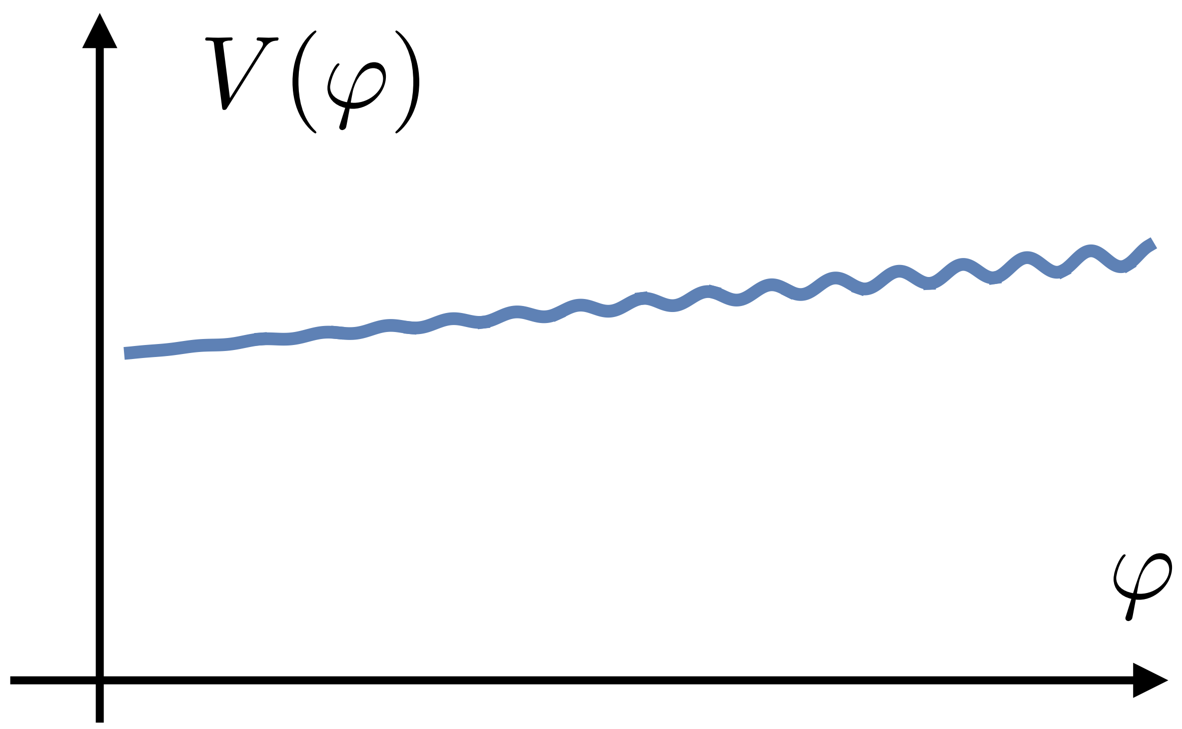
Boundary value $\bar{\zeta} = -H_\star \bar{\pi}$

Notation $\zeta = -H_\star \pi$ in the following

Resonant features

Chen, Easther, Lim 08, Flauger, Pajer 2010, Leblond, Pajer 11, Behbahani, Dymarsky, Mirbabayi, Senatore 11…

Small but fast oscillations (here in $\dot{H}(t)$)



$$V(\varphi) = V_{\rm sr}(\varphi) + \Lambda^4 \cos(\varphi/f)$$

$$\dot{H}(t) = \dot{H}_\star \left(1 - \tilde{b}\cos(\omega t + \delta)\right)$$

- Small amplitude $\tilde{b} \ll 1$
- Large frequency $\alpha = \dfrac{\omega}{H_\star} \gtrsim 1$

Resonance between:

background oscillations $e^{i\omega t}$
and quantum modes oscillations $e^{ik\eta}$

→

non-Gaussianities are enhanced with a peculiar shape

$$\langle \zeta_{\boldsymbol{k}_1}\zeta_{\boldsymbol{k}_2}\zeta_{\boldsymbol{k}_3}\rangle \propto \tilde{b}\alpha^{5/2}\sin\left(\alpha \log((k_1+k_2+k_3)/k_\star)\right)$$

Wavefunction for resonant features

- We work at first order in amplitude of the feature → we can evaluate the action on the solution with no feature

$$S[\zeta = \zeta_0 + \tilde{b}\zeta_1] = S[\zeta_0] + 0 + \mathcal{O}(\tilde{b}^2)$$

- Subtraction of divergent unobservable part (cf free theory)

- Rotation to Euclidean time (everything analytic): $\Psi[\bar{\zeta}] = e^{-S_g} e^{-\tilde{b}\Delta S_{E,1}}$

$$\Delta S_{\mathrm{E},1}[\bar{\zeta}] = \int_{-\infty}^{0} \mathrm{d}\tau \int \mathrm{d}^3\boldsymbol{x}\, \frac{1}{2\tau^2 P_\zeta} \Big\{ [\zeta'^2 + (\partial_i\zeta)^2] \cos\Big(\alpha\left(\log(\tau/\eta_\star) + \zeta\right) - \tilde{\delta} - i\alpha\pi/2\Big) - (\partial_i\bar{\zeta})^2 \cos\Big(\alpha\left(\log(\tau/\eta_\star) + \bar{\zeta}\right) - \tilde{\delta} - i\alpha\pi/2\Big) \Big\}$$

Explicit result.

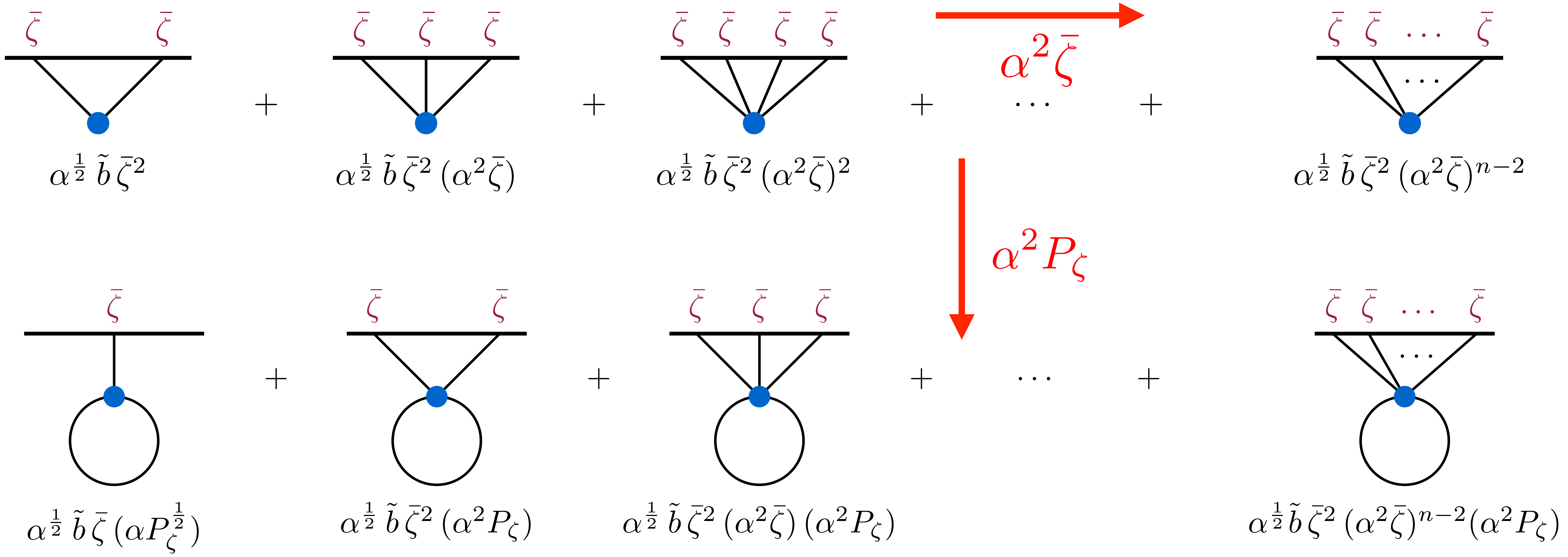
Numerical integration or analytical results for $\alpha \gg 1$

with $\zeta(\tau, \boldsymbol{x}) = \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3} \bar{\zeta}(\boldsymbol{k}) e^{i\boldsymbol{k}\cdot\boldsymbol{x}} (1 - k\tau) e^{k\tau}$

Actually, full wavefunction!

Our result:
resummation of
tree-level diagrams
at order $\tilde{b}$

Loop diagrams
at order $\tilde{b}$



$\bar{\zeta} \gtrsim \dfrac{1}{\alpha^2}$ perturbation theory breaks down

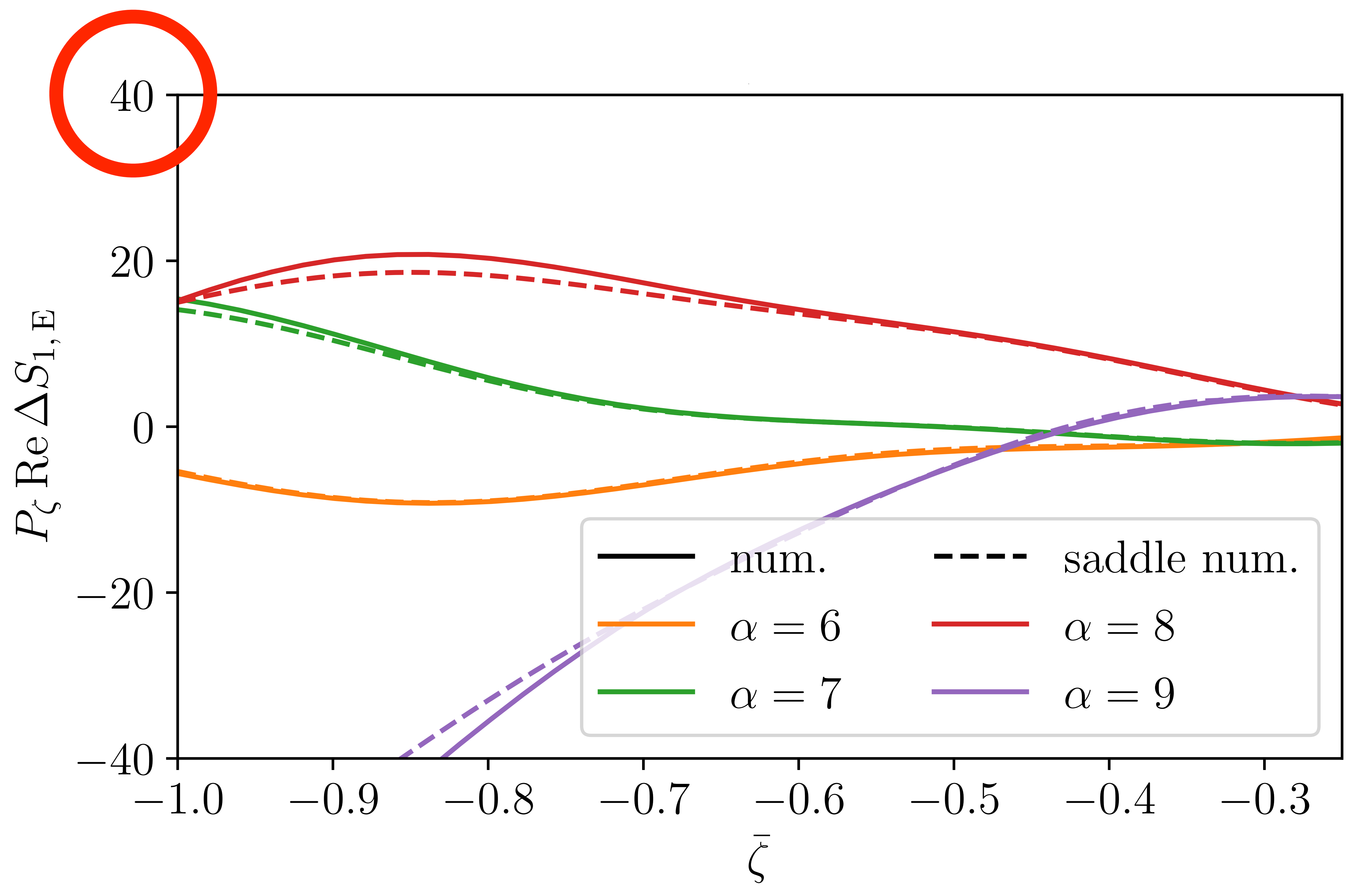
$\alpha^2 P_\zeta \ll 1$ Computation of WFU also valid for typical fluctuations

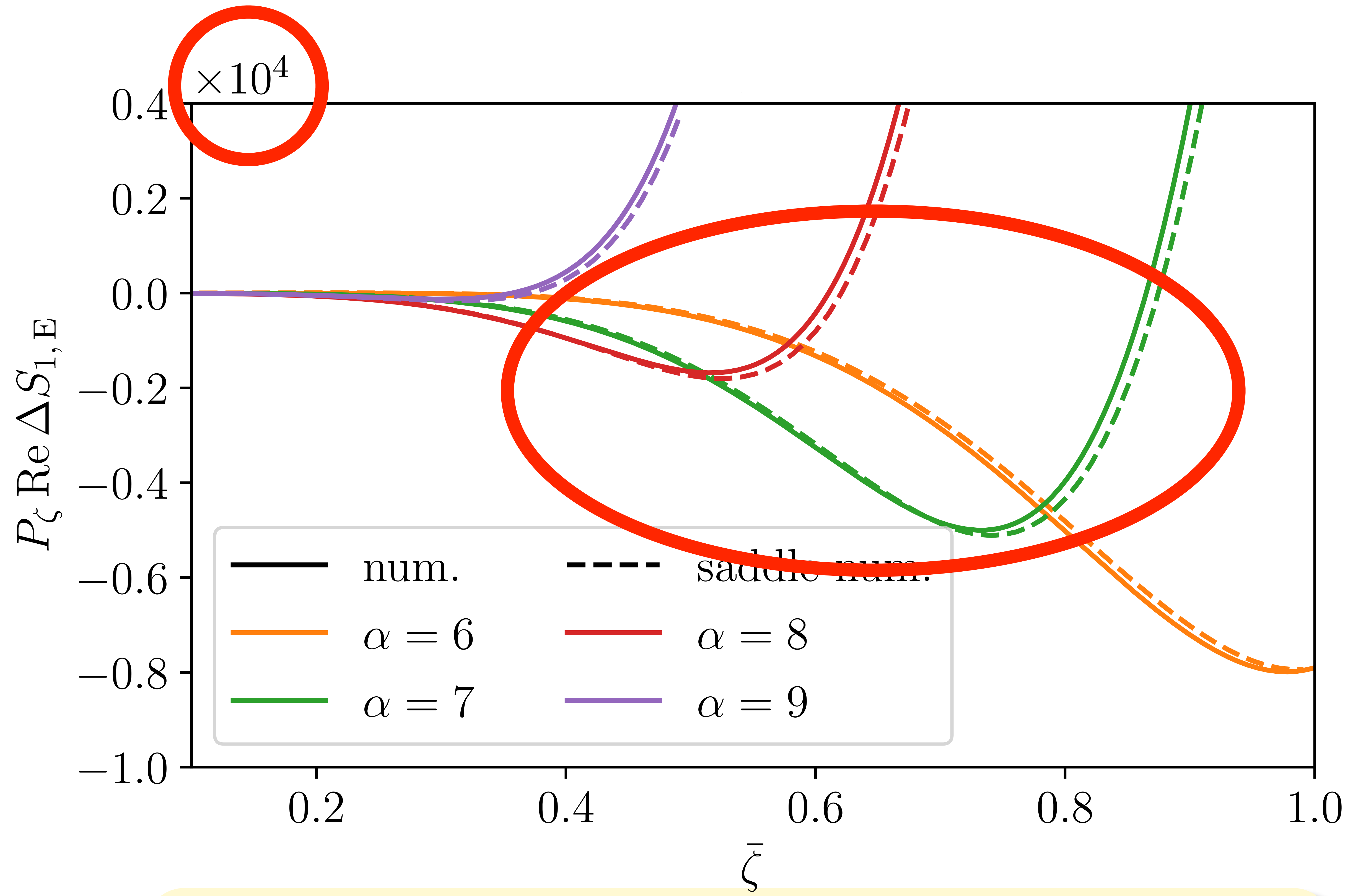
IV. Results and questions

Some results

We choose a given spherically symmetric profile, and vary its overall amplitude

e.g. $\bar{\zeta}(r) = \bar{\zeta} e^{-(r/r_0)^2}$

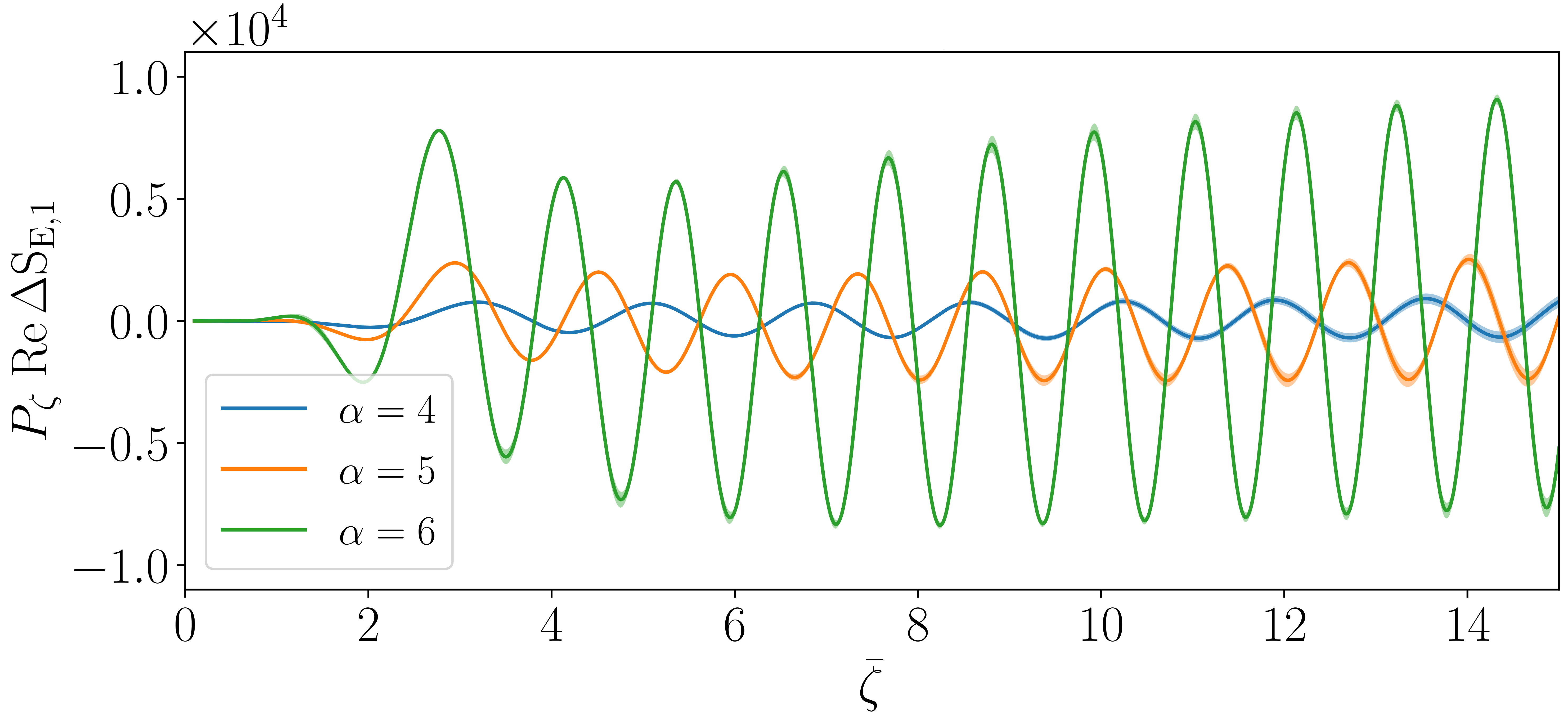




Asymmetry between minima and maxima, unexpected from perturbation theory

Exponential growth for $\bar{\zeta} \gtrsim 1/\alpha^2$

Some results



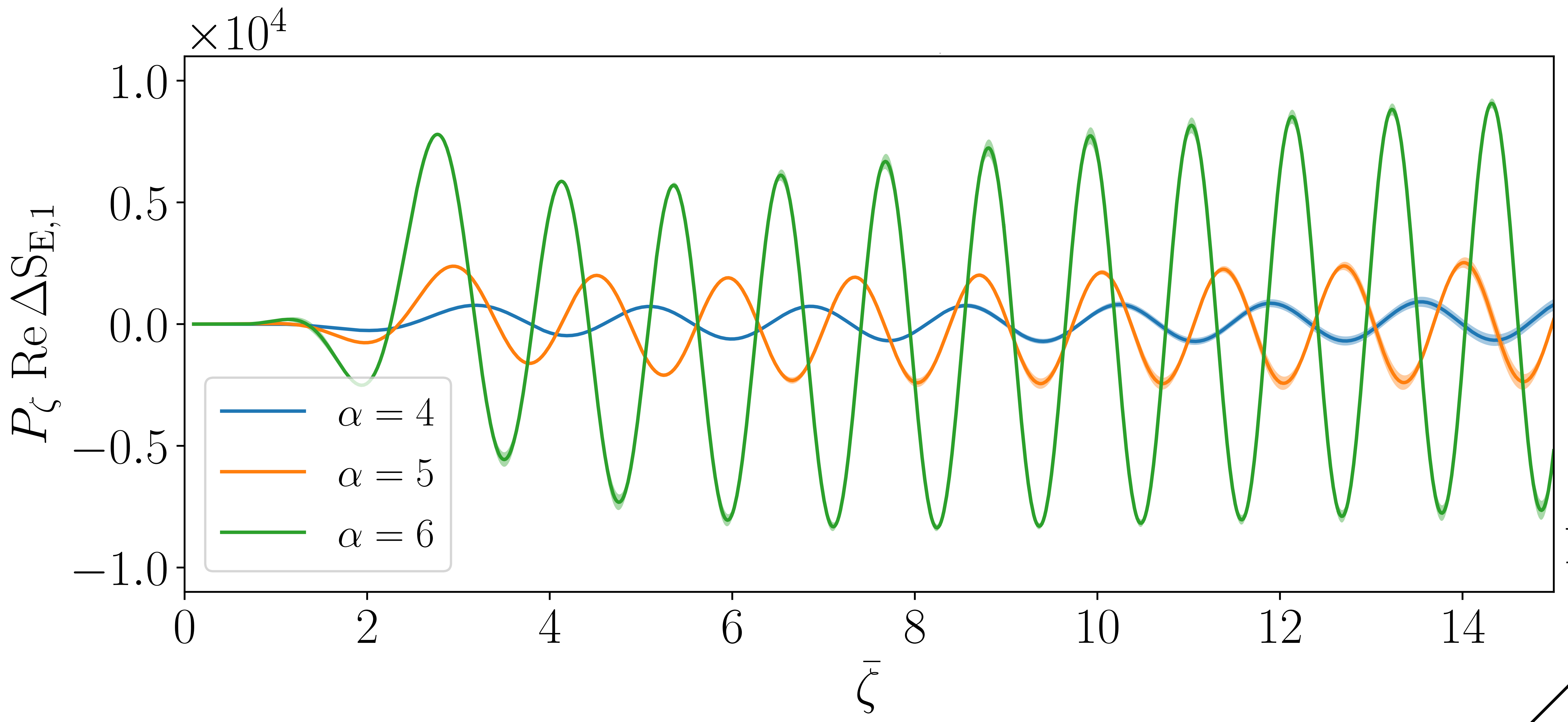
For large $\bar{\zeta}$, the growth saturates and one gets oscillations on the tail

Analytical understanding

for $\alpha \gg 1$ (saddle-point)

$$P_\zeta \Delta S_{E,1} \propto \frac{e^{\frac{\pi\alpha}{2}}}{\alpha^2}\, e^{i\sigma\alpha(\bar{\zeta} - \log(\sqrt{|\nabla^2 \bar{\zeta}|}))}$$

Some results



Exponential enhancement absent for local minimum

For large $\bar{\zeta}$, the growth saturates and one gets oscillations on the tail

Analytical understanding

for $\alpha \gg 1$ (saddle-point)

$$P_\zeta \Delta S_{\mathrm{E},1} \propto \frac{e^{\frac{\pi\alpha}{2}}}{\alpha^2} e^{i\sigma\alpha(\bar{\zeta}-\log(\sqrt{|\nabla^2\bar{\zeta}|}))}$$

Questions

- Is all this real or an artefact of working at $O(\tilde{b})$?

  We think it is real, full numerics can tell: $e^{iS[\zeta_{\rm cl},b]}$

- Physical understanding of asymmetry and super-resonance $\sim e^{\alpha}$

  Analogy with simple quantum mechanical problem? in progress

Intriguing new regime

$$\alpha^2 P_\zeta^{1/2} \gtrsim 1 \ ?$$ Resummation is needed even for typical fluctuations

$\longleftrightarrow$

$$\omega > 4\pi f$$ Beyond regime of validity of EFT for scalar field?, with $\cos(\varphi/f)$

Fine actually: $\Lambda_{\rm cutoff} \sim 4\pi f \log^{1/2}(1/\tilde{b})$ Hook, Rattazzi 2023

Negligible 3-, 4- pt function… but a lot of large n-point functions To study more!

Conclusion

- Generic method to compute the non-perturbative tail of WFU (sometimes full WFU)
- First analytical results of non-perturbative phenomena from inside-the-horizon interactions
- Simple expression for any small feature
- Typical fluctuations in non-perturbative regime?!
- Generalizations: DBI, eternal inflation, tensor modes
- Developments to relate to observations