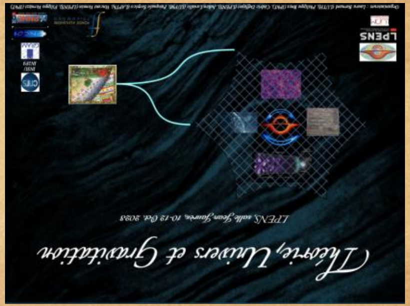

Neutrinos, Cosmological **Phase-Transition** and the **Matter-Antimatter** Asymmetry

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Rémi Faure, in collaboration with Stéphane Lavignac (IPhT, Saclay)

- Why is there no « anti »-version of us?
- Almost only particles are observed at all scales (no anti-particles)
- The CMB gives us the ratio baryons/photons:

$$\eta_B \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} \simeq \frac{n_B}{n_\gamma} = 6.10^{-10} \neq 0$$

→ Need for New Physics: Sterile Neutrinos

Outline

1. Phase-Transition
2. Propagation with time-dependent masses
3. The role of flavor
4. Lepton asymmetry

1. **Phase Transition**: going out of equilibrium

Sakharov conditions for generation of matter-antimatter asymmetry (1967):

- **Baryon/Lepton number violation**
- **C and CP violation**
- **Out-of-Equilibrium**

Andreï Sakharov

$$L = L_{SM} + i\bar{N}\gamma^{\mu}\partial_{\mu}N - M_N^I \bar{N}_I^c N_I + Y_{I\alpha} N_I \bar{l}_{\alpha}\tilde{\phi} + h.c.$$

Massive Neutrinos
Majorana mass M

→ Violates **Lepton number**

The Neutrinos interact with the Standard Model

→ Violates **CP symmetry**

$$L = L_{SM} + i\bar{N}\gamma^{\mu}\partial_{\mu}N - M_N^I \bar{N}_I^c N_I + Y_{I\alpha} N_I \bar{l}_\alpha \tilde{\phi} + h.c.$$

[Khoze, Ro, 2013]
[Fischer, Lindner, van der Woude, 2021]

[Rosauro-Alcaraz (2021)]
[Huang, Xie (2022)]

$$L = L_{SM} + L_S + i\bar{N}\gamma^{\mu}\partial_{\mu}N - \lambda_{NS}^I S \bar{N}_I^c N_I + Y_{I\alpha} N_I \bar{l}_\alpha \tilde{\phi} + h.c.$$

$$M_N^I = \lambda_{NS}^I S$$

$$L = L_{SM} + i\bar{N}\gamma^{\mu}\partial_{\mu}N - M_N^I \bar{N}_I^c N_I + Y_{I\alpha} N_I \bar{l}_{\alpha}\tilde{\phi} + h.c.$$

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$$L = L_{SM} + L_S + i\bar{N}\gamma^{\mu}\partial_{\mu}N - \lambda_{NS}^I S\bar{N}_I^c N_I + Y_{I\alpha} N_I \bar{l}_{\alpha}\tilde{\phi} + h.c.$$

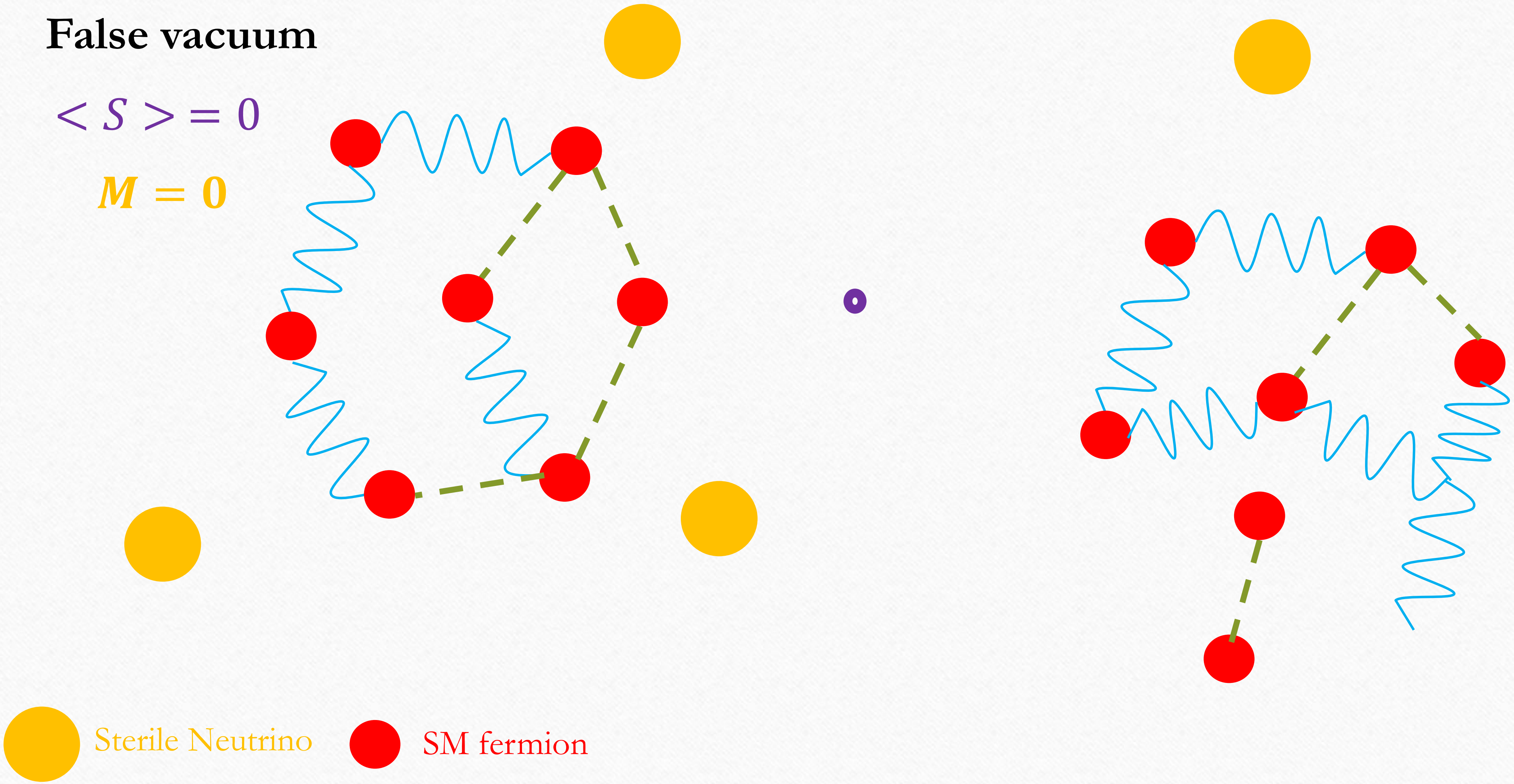
$$M_N^I = \lambda_{NS}^I S$$

New dynamics for the sterile sector: **phase-transition**

False vacuum

$< S > = 0$

$\boldsymbol{M = 0}$



False vacuum

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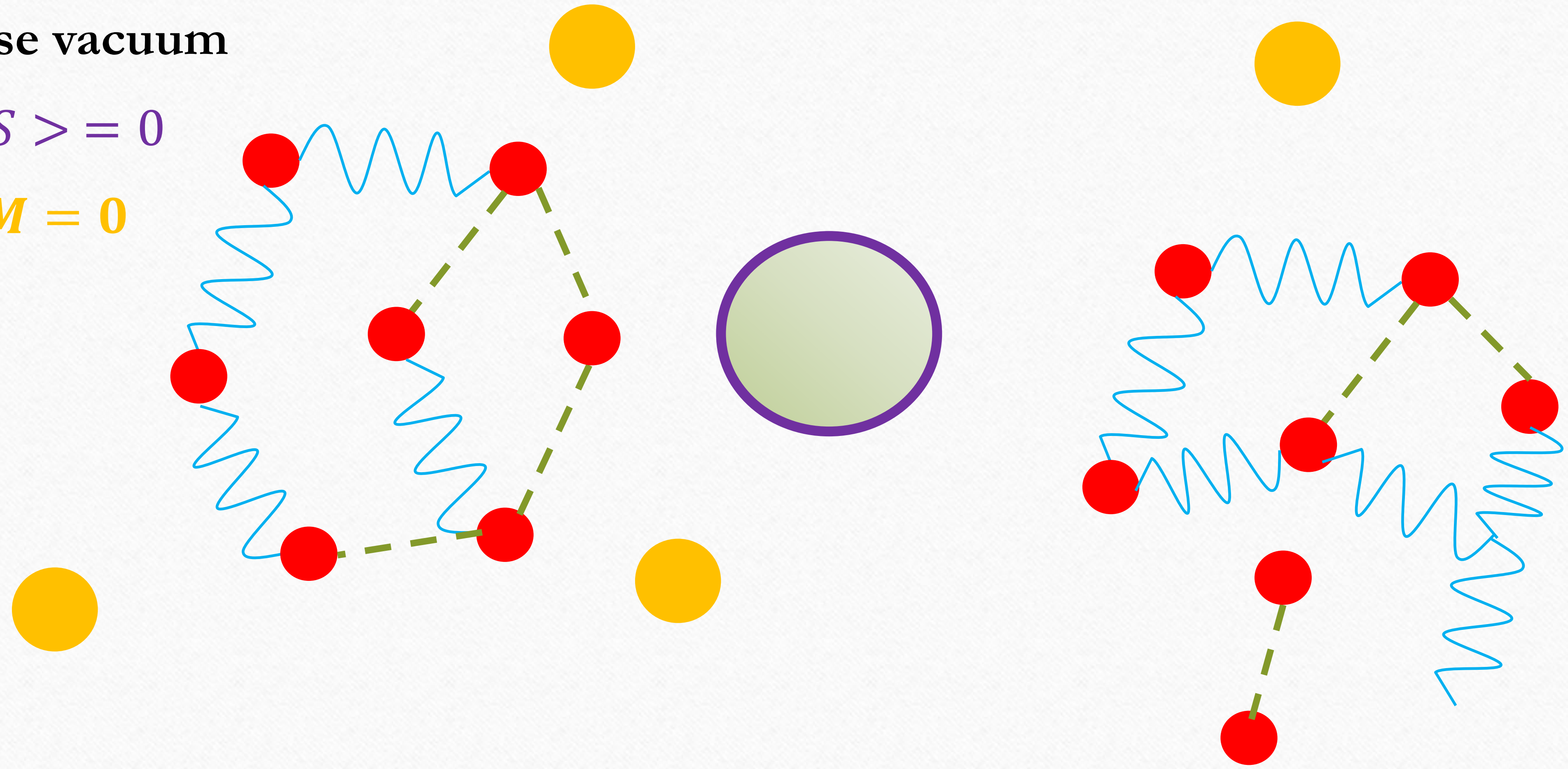
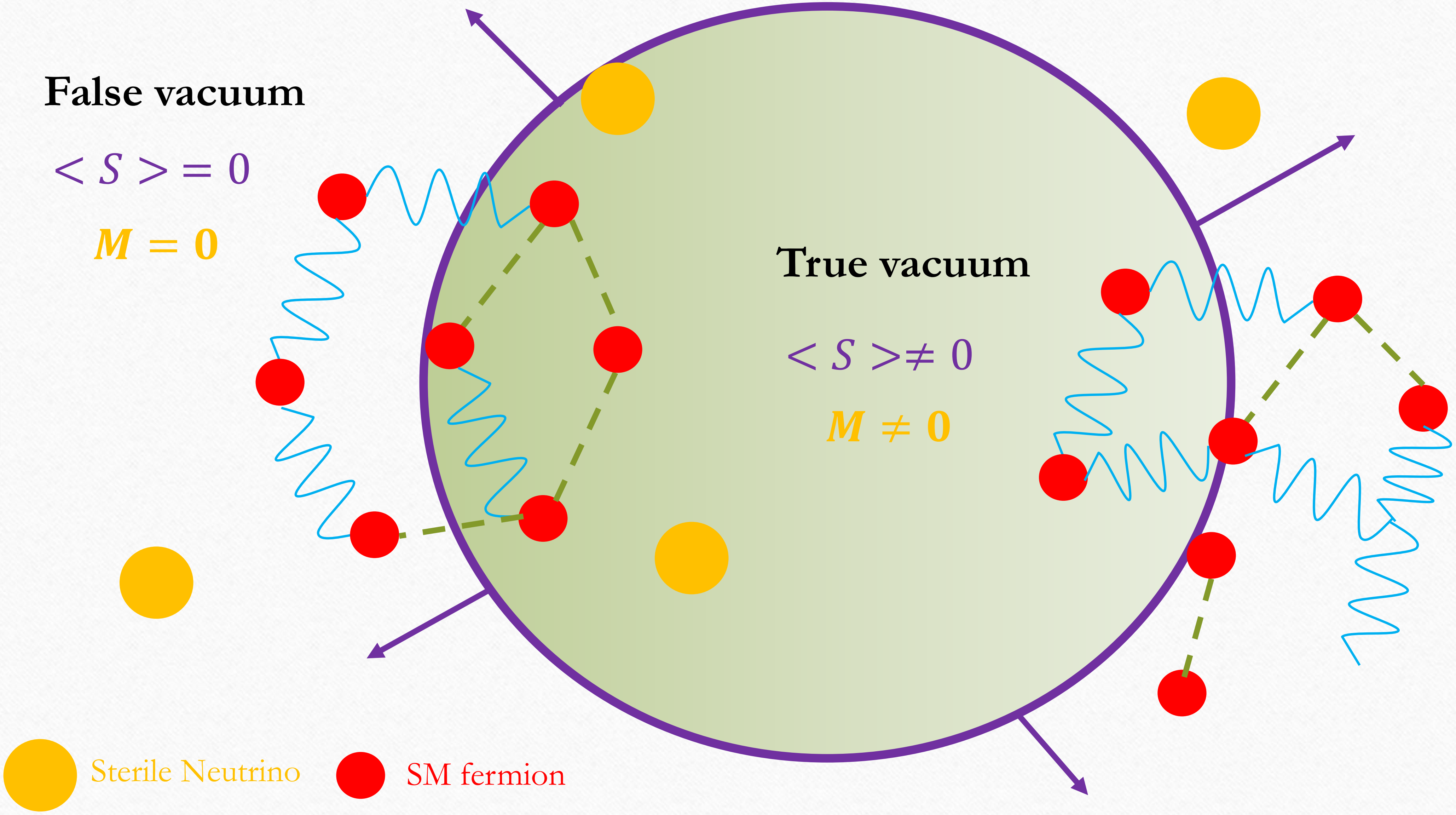
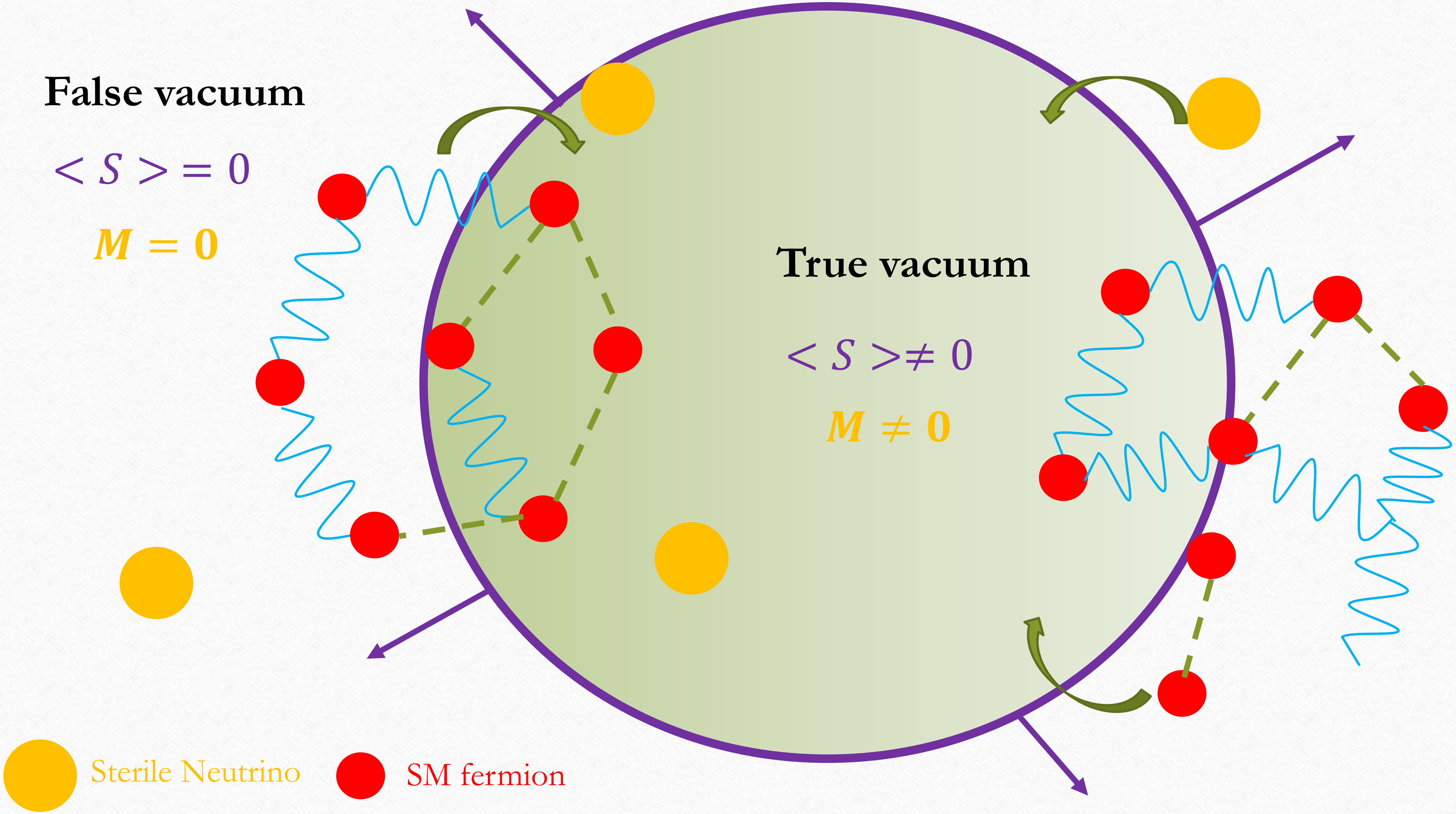


Sterile Neutrino

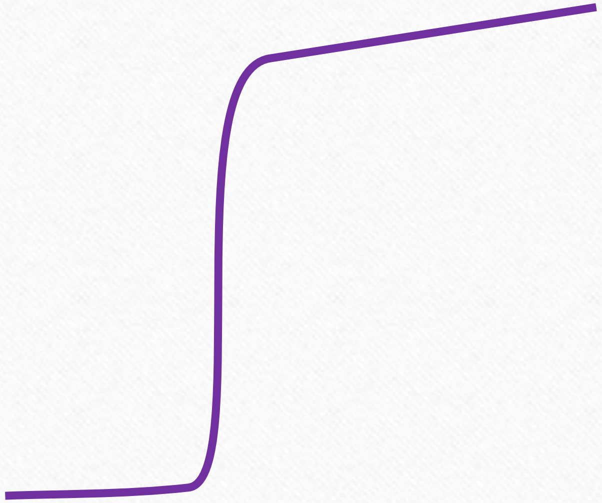
SM fermion


False vacuum
$< S > = 0$
$M = 0$
True vacuum
$< S > \neq 0$
$M \neq 0$
Sterile Neutrino
SM fermion


False vacuum
$< S > = 0$
$\boldsymbol{M = 0}$
True vacuum
$< S > \neq 0$
$\boldsymbol{M \neq 0}$
Sterile Neutrino
SM fermion

During the **phase-transition**, the masses of the Neutrinos are **time-dependent**. All quantities will have an explicit time-dependence along the **wall**.

$< S > (t)$



$$M_N^I = \lambda_{NS}^I < S > (t) = M_N^I(t)$$

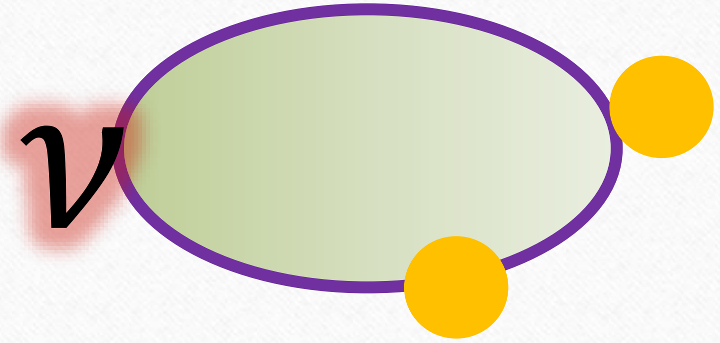
A direct brut-force QFT calculation turns out to be complicated. Our strategy will rather be to consider the **Dirac equation** and solve it directly.

$$N(x_1) \longrightarrow S_N(x_1, x_2) = -i\langle \bar{N}(x_1) N(x_2) \rangle$$

$$\downarrow$$

Lepton asymmetry ⟵ Correction to the self-energy of the leptons

2. Propagation with **time-dependent masses**



Consider the left- and right-handed parts of the Majorana Neutrino (one flavor for simplicity):

$\sigma^0 = Id, \sigma^i =$ Pauli matrices

$$N = \begin{pmatrix} N_L^c = i\,\sigma^2 N_R \\ N_R \end{pmatrix}$$

$$i\,\sigma^\mu \partial_\mu N_R - M_N(t) N_L^c = 0$$

$$i\,\bar{\sigma}^\mu \partial_\mu N_L^c - M_N(t) N_R = 0$$

We decompose the Majorana field in (spatial) momentum modes:

$$N(\boldsymbol{x},t) = \sum_{h=\pm} \int d^3\boldsymbol{k}\, e^{i\,\boldsymbol{k}\cdot\boldsymbol{x}} \left(\begin{pmatrix} \boldsymbol{L_h} \\ \boldsymbol{R_h} \end{pmatrix} \otimes \xi_{\boldsymbol{k},h} a_{\boldsymbol{k},h} + \begin{pmatrix} \boldsymbol{h\,R_h^*} \\ \boldsymbol{-hL_h^*} \end{pmatrix} \otimes \xi_{-\boldsymbol{k},-h} a^{\dagger}_{-\boldsymbol{k},h} \right)$$

$\xi_{\boldsymbol{k},h}$ = helicity eigenvectors

$a_{\boldsymbol{k},h}, a^{\dagger}_{\boldsymbol{k},h}$ = annihilation and creation operators **defined at** $\boldsymbol{t = -\infty}$

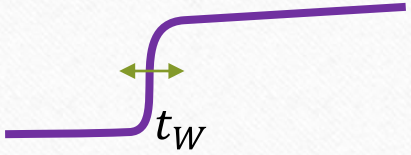
Free case: $L_h, R_h \propto e^{-\,i\omega t}$, $L_h^*, R_h^* \propto e^{+\,i\omega t}$
(valid at $\boldsymbol{t} = -\infty$)

$$i\,\partial_t L_h + h\,k\,L_h - M_N(t) R_h = 0$$

$$i\,\partial_t R_h - h\,k\,R_h - M_N(t) L_h = 0$$

For a time-dependence of the mass $M_N(t) = M_N^0(1 + \tanh(t/t_W))/2$, one can get analytical results.

[Prokopec, Schmidt, Weenink, 2013]



$$Z \equiv \frac{1 + \tanh(t/t_W)}{2}$$

$$\gamma \equiv 1/t_W$$

$$\frac{d}{dt} = \frac{dZ}{dt}\frac{d}{dZ} = \frac{1}{2\,t_W}(1 - \tanh(t/t_W)^2)\frac{d}{dZ} = 2\,\gamma Z(1 - Z)\frac{d}{dZ}$$

$$i\,\partial_t L_h + h\,k\,L_h - M_N(t) R_h = 0$$

$$i\,\partial_t R_h - h\,k\,R_h - M_N(t) L_h = 0$$

$$\Rightarrow \left[\partial_t^2 + 2ihk\,\frac{\partial_t M_N}{M_N}\,\partial_t + \left(k^2 + M_N(t)^2\right)\right] L_h = 0$$

$$L_h \equiv Z^{\alpha}(1-Z)^{\beta}\chi_h(Z)$$

$$\Downarrow$$

$$Z(1-Z)\chi_h'' + \left(c_h - (a_h + b_h + 1)Z\right)\chi_h' - a_h b_h\,\chi_h = 0$$

$$i\,\partial_t L_h + h\,k\,L_h - M_N(t) R_h = 0$$
$$i\,\partial_t R_h - h\,k\,R_h - M_N(t) L_h = 0$$

$$\left[\partial_t^2 + 2ihk\,\frac{\partial_t M_N}{M_N}\,\partial_t + \left(k^2 + M_N(t)^2\right)\right] L_h = 0$$

$$L_h \equiv Z^{\alpha}(1-Z)^{\beta}\chi_h(Z)$$

$$Z(1-Z)\chi_h'' + \left(c_h - (a_h + b_h + 1)Z\right)\chi_h' - a_h b_h\,\chi_h = 0$$

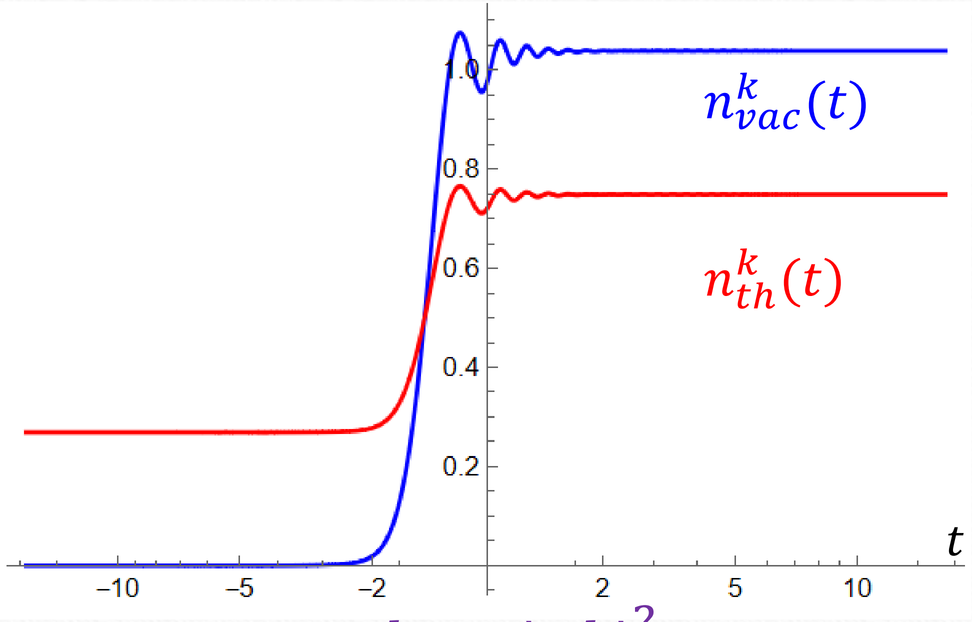
$$\ldots \qquad L_h \sim \lambda_h\ {}_2F_1(a_h, b_h, c_h, Z) + \mu_h\ {}_2F_1(a_h', b_h', c_h', Z)$$

(Gaussian hypergeometric function, with a_h, b_h, c_h functions of k and M_N)

$$N_R(\boldsymbol{x},t) = \sum_{h=\pm} \int d^3\boldsymbol{k}\, e^{i\,\boldsymbol{k}\cdot\boldsymbol{x}} \left(\boldsymbol{L_h} \otimes \xi_{\boldsymbol{k},h} a_{\boldsymbol{k},h} + \boldsymbol{h}\, \boldsymbol{R_h^*} \otimes \xi_{-\boldsymbol{k},-h} a^\dagger_{-\boldsymbol{k},h} \right)$$

$$L_h \sim {}_2F_1(\ldots) \sim_{t\to+\infty} \alpha_h^{\boldsymbol{k}} e^{-i\omega_+ t} + \beta_h^{\boldsymbol{k}} e^{+i\omega_+ t} \neq \text{Free case}$$

The modes that correspond to positive frequencies at initial times ($-\infty$) end up being a **linear combination** of positive and negative frequencies in the far future ($+\infty$). This corresponds to making a **Bogolyubov transformation** of the annihilation and creation operators.



$$n_{th}^{\boldsymbol{k}} = \left|\beta_h^{\boldsymbol{k}}\right|^2$$

3. The role of flavor

If we add interactions to the story,

$$i\,\sigma^{\mu}\partial_{\mu}N_{RI} - M_N^I(t)N_{LI}^c = Y_{I\alpha}\bar{l}_{\alpha}\tilde{\phi}$$

$$i\,\bar{\sigma}^{\mu}\partial_{\mu}N_{LI}^c - M_N^I(t)N_{RI} = Y_{I\alpha}^{*}\bar{l}_{\alpha}^c\phi$$

…(thermal average)

$$\left[\partial_t^2 + 2ihk\,\frac{\partial_t M_N^I}{M_N^I}\partial_t + (k^2 + M_N^I(t)^2)\right]L_h^{IJ} + M_{th,IK}^2 L_h^{KJ} = 0$$

$$M_{th,IK}^2 = (Y^{*}Y^{T})_{IK}T^2/12$$

Order-by-order: $M^2_{th,IK}$ is small $\qquad L_h^{IJ} = L_h^{IJ\,(0)} + L_h^{IJ\,(1)} + \cdots$

$$\left[\partial_t^2 + 2ihk\,\frac{\partial_t M_N^I}{M_N^I}\partial_t + \left(k^2 + M_N^I(t)^2\right)\right] L_h^{IJ\,(0)} = 0$$

Diagonal in flavor

$$\left[\partial_t^2 + 2ihk\,\frac{\partial_t M_N^I}{M_N^I}\partial_t + \left(k^2 + M_N^I(t)^2\right)\right] L_h^{IJ(1)} = -M^2_{th,IK} L_h^{KJ(0)}$$

Same homogeneous part ⇒ hypergeometric functions

4. **Lepton asymmetry**

ν $\qquad$ L

L_h, R_h

$N(x_1)$ → $S_N(x_1, x_2) = -i\langle \bar{N}(x_1) N(x_2) \rangle$

✓

↓

Lepton asymmetry ← Correction to the self-energy of the leptons

The propagator can equivalently be written with the modes or with **phase-space distributions**:

$$S_N = \mathbb{1} - \sum_h \begin{pmatrix} L_h L_h^\dagger & L_h R_h^\dagger \\ R_h L_h^\dagger & R_h R_h^\dagger \end{pmatrix} \otimes P_{\boldsymbol{k},h}$$

$$= \sum_{h,s} \underbrace{P_{\boldsymbol{k},h} P_I^s \gamma^0 P_J^s}\, f_{IJ}^{m,s} + \underbrace{P_{\boldsymbol{k},h} P_I^s \gamma^0 P_J^{-s}}\, f_{IJ}^{c,s}$$

$$= \left[\sum_{h,s} \mathcal{P}_{\boldsymbol{k},h}^{m,s}\, f_h^{m,s} + \mathcal{P}_{\boldsymbol{k},h}^{c,s}\, f_h^{c,s} \right]_{IJ}$$

By projecting on the correct subspaces, we have linear relations between the modes L_h/R_h and the distribution functions f^c/f^m,

$$f^{m,s}_{\boldsymbol{k},h} = \mathrm{Tr}\left(\mathcal{P}^{m,s}_{\boldsymbol{k},h} S_N\right) \qquad f^{c,s}_{\boldsymbol{k},h} = \mathrm{Tr}\left(\mathcal{P}^{c,s}_{\boldsymbol{k},h} S_N\right)$$

$f^{m,s}_{\boldsymbol{k},h}$ is called the **mass-shell** distribution function ~ particle-particle transitions ~ $e^{i(\omega_I - \omega_J)t}$ **Slow mode**

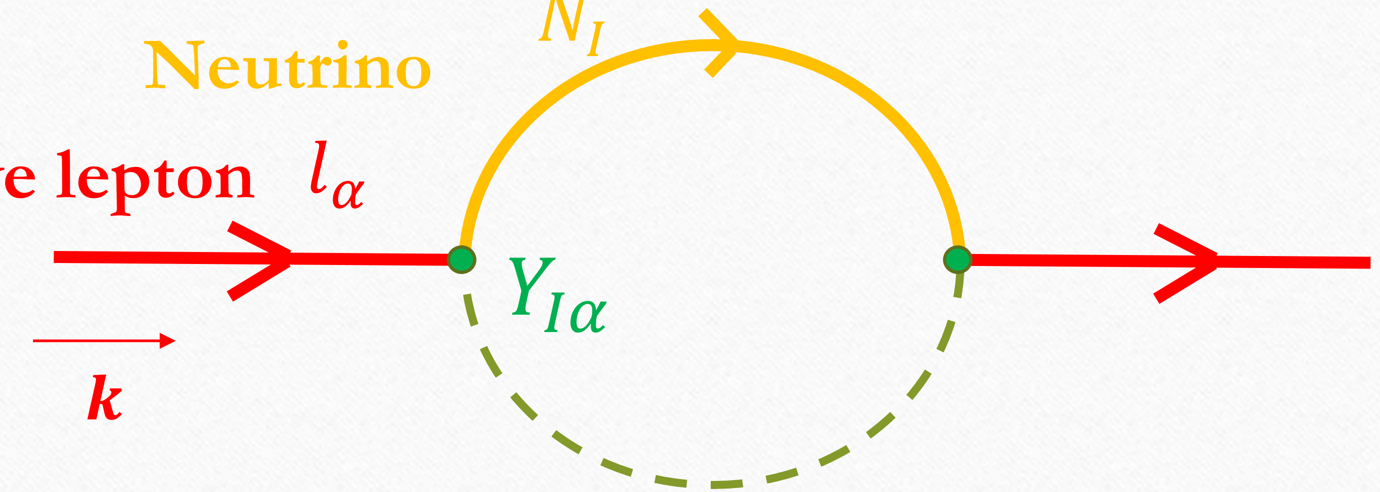
$f^{c,s}_{\boldsymbol{k},h}$ is called the **coherence-shell** distribution function ~ particle-antiparticle transitions ~ $e^{i(\omega_I + \omega_J)t}$ **Fast mode**

$N(x_1)$ $\xrightarrow{L_h, R_h}$ $S_N(x_1, x_2) = -i\langle \bar{N}(x_1) N(x_2) \rangle$

✓ ✓

$\downarrow$ f^c, f^m

Lepton asymmetry $\leftarrow$ Correction to the self-energy of the leptons



$\Delta_L^{\boldsymbol{k}}$= asymmetry in leptons

$$\partial_t \Delta_L^{\boldsymbol{k}} \approx \left[\mathrm{Im}\left(YY^\dagger\right)_{IJ} \mathrm{Im}\left(f_{\boldsymbol{k},+}^m + f_{\boldsymbol{k},-}^m\right)_{JI} + \mathrm{Re}\left(YY^\dagger\right)_{IJ} \mathrm{Re}\left(f_{\boldsymbol{k},+}^m - f_{\boldsymbol{k},-}^m\right)_{JI}\right] \times \hat{\Sigma}_{\mathcal{A}}(\boldsymbol{k})$$

Phase-space distribution of Neutrinos

[Jukkala, Kainulainen, Rahkila, 2021]

[Drewes, Garbrecht, 2012]

$\hat{\Sigma}_{\mathcal{A}}(\boldsymbol{k})$ = self-energy of the Neutrino

$$\partial_t \Delta_L^{\boldsymbol{k}} \approx \left[\mathrm{Im}\left(YY^\dagger\right)_{IJ} \mathrm{Im}\left(f_{\boldsymbol{k},+}^{\boldsymbol{m}} + f_{\boldsymbol{k},-}^{\boldsymbol{m}}\right)_{JI} + \mathrm{Re}\left(YY^\dagger\right)_{IJ} \mathrm{Re}\left(f_{\boldsymbol{k},+}^{\boldsymbol{m}} - f_{\boldsymbol{k},-}^{\boldsymbol{m}}\right)_{JI}\right] \times \hat{\Sigma}_{\mathcal{A}}(\boldsymbol{k})$$

The phase-space distributions were solved to first-order in $YY^\dagger$ and lead to many terms. As an example,

$$\left[\mathrm{Im}\left(YY^\dagger\right)_{IJ} \mathrm{Im}\left(f_{\boldsymbol{k},+}^{\boldsymbol{m}} + f_{\boldsymbol{k},-}^{\boldsymbol{m}}\right)_{JI} + \mathrm{Re}\left(YY^\dagger\right)_{IJ} \mathrm{Re}\left(f_{\boldsymbol{k},+}^{\boldsymbol{m}} - f_{\boldsymbol{k},-}^{\boldsymbol{m}}\right)_{JI}\right]$$

$$\ni \frac{M_I^2 - M_J^2}{8\,k\,\langle M \rangle_{IJ}} \mathrm{Im}\left[\left(YY^\dagger\right)_{IJ}^2\right]\left[\mathrm{Im}\left(L_+^{(0)}\chi_-^* - L_-^{(0)}\chi_+^*\right)_{JI} - \mathrm{Re}\left(L_+^{(0)}\chi_-^* + L_-^{(0)}\chi_+^*\right)_{JI}\right]$$

Lepton number violation + CP-violation + Out-of-equilibrium modes = Lepton asymmetry source

Summary and conclusion

$N(x_1)$ ✓

L_h, R_h → **Hypergeometric equation**

$$S_N(x_1, x_2) = -i\langle \bar{N}(x_1) N(x_2) \rangle$$ ✓

↓ f^c, f^m **Spinorial projectors**

Correction to the self-energy of the leptons

…

← $\Delta_L^{\boldsymbol{k}}$ **Momentum integrals**

Lepton asymmetry

…

Further prospects

- Numerical implementation
- Dependence on the parameters of the Phase Transition (thickness of the wall)
- Washout of the asymmetries before electroweak PT

True vacuum

ν

Thank you for your attention!

Homogeneous and isotropic Universe

$$ds^2 = dt^2 - a(t)^2 d\vec{x}^2$$

Scale factor = « Size » of the Universe

Its evolution is driven by the **matter content**

$$f_h(\vec{p}, \vec{x}, t) = f_h(\vec{p}, t)$$

$$n_h = \int d^3\vec{p}\, f_h(\vec{p}, t) = n_h(t) = n^{eq} + \delta n_h$$

The constants λ_h and μ_h are determined from initial conditions + normalization:

$$N(\boldsymbol{x},t) = \sum_{h=\pm} \int d^3\boldsymbol{k}\, e^{i\,\boldsymbol{k}\cdot\boldsymbol{x}} \left(\begin{pmatrix} \boldsymbol{L_h} \\ \boldsymbol{R_h} \end{pmatrix} \otimes \xi_{\boldsymbol{k},h} a_{\boldsymbol{k},h} + \begin{pmatrix} \boldsymbol{h\,R_h^*} \\ \boldsymbol{-hL_h^*} \end{pmatrix} \otimes \xi_{\boldsymbol{k},-h} a^\dagger_{-\boldsymbol{k},h} \right)$$

$\xi_{\boldsymbol{k},h}$ = helicity eigenvectors

$a_{\boldsymbol{k},h}, a^\dagger_{\boldsymbol{k},h}$ = annihilation and creation operators **defined at $\boldsymbol{t} = -\infty$**

<u>$\boldsymbol{t = -\infty}$</u> $L_h, R_h \propto e^{-\,i\omega_- t}$, $L_h^*, R_h^* \propto e^{+\,i\omega_- t}$

<u>**Normalization:**</u> $|L_h|^2 + |R_h|^2 = 1$

Multiflavor field decomposition:

$$N_I(\boldsymbol{x},t) = \sum_{h=\pm} \int d^3\boldsymbol{k}\, e^{i\,\boldsymbol{k}\cdot\boldsymbol{x}} \left(\begin{pmatrix} \boldsymbol{L_h^{IJ}} \\ \boldsymbol{R_h^{IJ}} \end{pmatrix} \otimes \xi_{\boldsymbol{k},h} a_{\boldsymbol{k},hJ} + h.c. \right)$$

$\xi_{\boldsymbol{k},h}$ = helicity eigenvectors

$a_{\boldsymbol{k},h}, a^{\dagger}_{\boldsymbol{k},h}$ = annihilation and creation operators **defined at $\boldsymbol{t} = -\infty$**

<u>$\boldsymbol{t = -\infty}$</u>

$$L_h^{IJ}, R_h^{IJ} \propto e^{-i\omega_- t}$$

<u>Normalization:</u>

$$L_h L_h^{\dagger} + R_h R_h^{\dagger} = 1$$

$$L_h^{IJ(1)} \equiv -M_{th,IJ}^2 Z^{\alpha}(1-Z)^{\beta_I}\chi_h^{IJ}(Z) \qquad L_h^{JJ(0)} \equiv Z^{\alpha}(1-Z)^{\beta_J}\chi_h^{J(0)}(Z)$$

$$Z(1-Z)\chi_h^{IJ''} + (c_I - (a_I + b_I + 1)Z)\chi_h^{IJ'} - a_I b_I\, \chi_h^{IJ} = (1-Z)^{\beta_J-\beta_I}\chi_h^{J(0)}(Z)$$

The general solution is the sum of an **homogeneous** and a **particular** solutions. The particular solution can be found from the source using the **Wronskian.**

$$\chi_h^{IJ} = \chi_p^{IJ} + \chi_{hom}^{I}$$

[Akhmedov, Rubakov, Smirnov (1998)]

Leptogenesis via neutrino oscillations **(ARS)**

[See for example Drewes, Garbrecht, Gueter, Klaric (2010)]

$$T_{osc} \equiv \left(M_{Pl} \Delta M^2_{2,1}\right)^{1/3}$$

$M_{Pl} \simeq 7 \times 10^{17}\text{GeV}$ is the Planck Mass

$$T_{osc} = \left(7 \times 10^{17} \times (110^2 - 100^2)\right)^{1/3}\text{GeV} \simeq 10^7\text{GeV} > T_{PT}$$

$M_1 = 100 \text{ GeV}$ $M_2 = 110 \text{ GeV}$ $T_{PT} = 10^6\text{GeV}$