LPENS
LABORATOIRE DE PHYSIQUE
DE L'ÉCOLE NORMALE SUPÉRIEURE

Stochastic inflation and applications to primordial black holes

Chiara Animali

Théorie, Univers et Gravitation

LPENS, Paris *12 October 2023*

ENS PSL UNIVERSITÉ PARIS cnrs SORBONNE UNIVERSITÉ Université Paris Cité

Primordial Black Holes

Primordial Black Holes

- Black holes which could have formed in the early Universe through a non-stellar way

  Hawking [1971] : *Gravitationally collapsed objects of very low mass*

  Carr & Hawking [1974]: *Black holes in the early Universe*

Primordial Black Holes

- Black holes which could have formed in the early Universe through a non-stellar way

  Hawking [1971] : *Gravitationally collapsed objects of very low mass*

  Carr & Hawking [1974]: *Black holes in the early Universe*

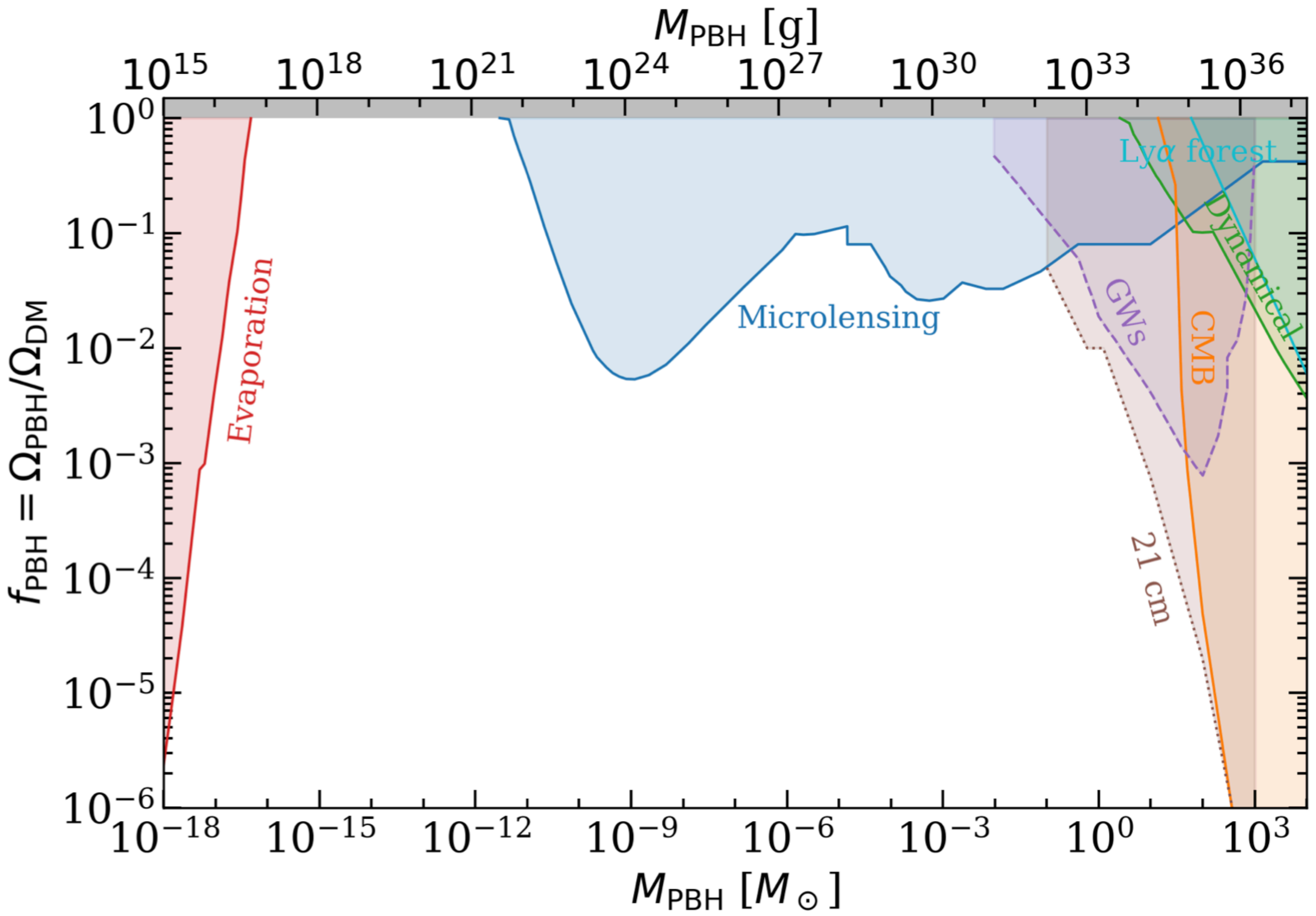
- They could solve several conundrums in astrophysics and cosmology

  - They could be the totality, or a fraction, of the Dark Matter

  - They may explain the existence of progenitors for the merging events observed by LIGO/VIRGO

  - They could be the seeds of supermassive black holes in galactic nuclei

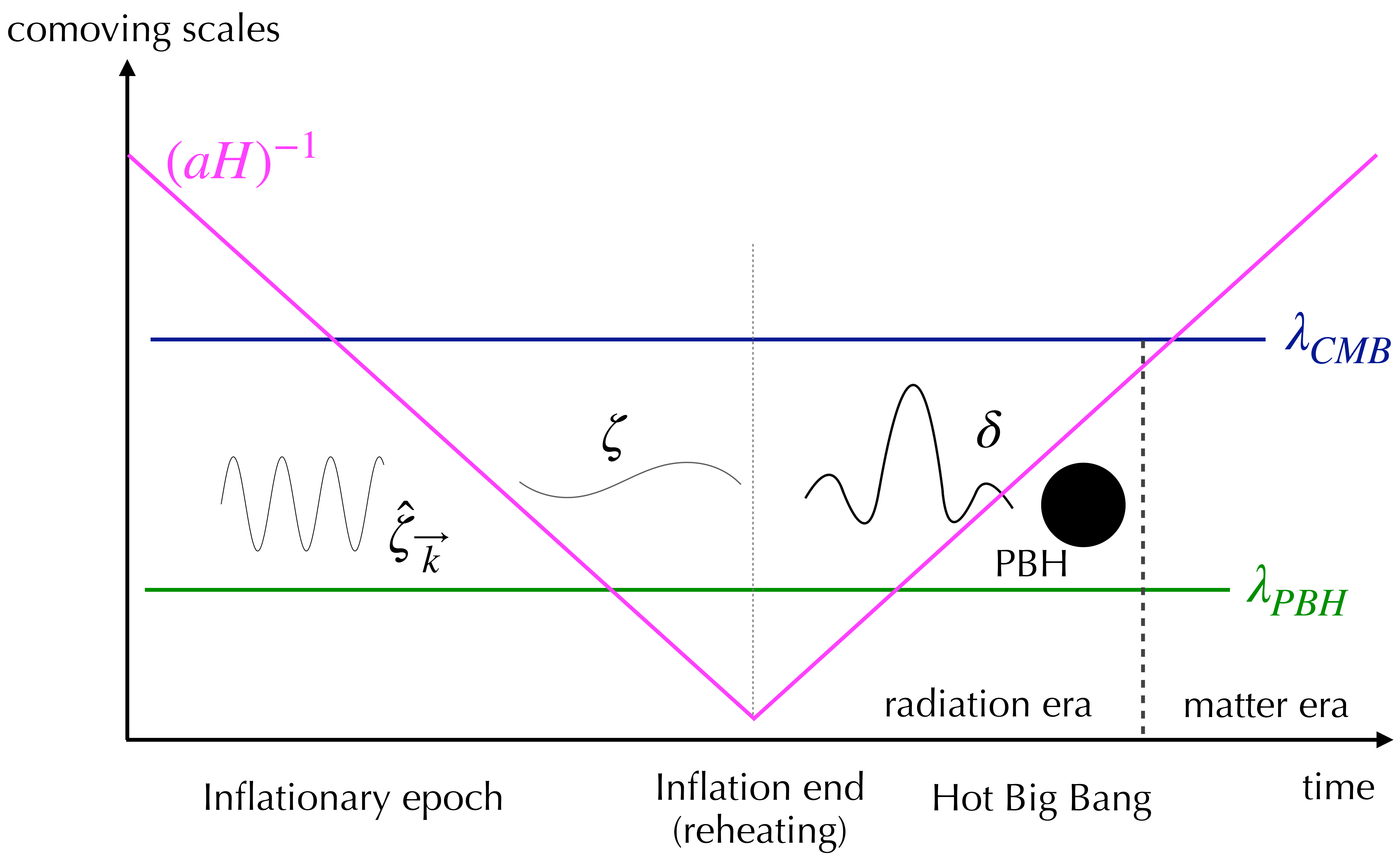
  - They could generate cosmological structures



P. Villanueva-Domingo, O. Mena, S. Palomares-Ruiz [2021]
A brief review on primordial black holes as dark matter

Primordial Black Holes : How?

- PBHs may be originated from peaks of the density perturbations generated in the early universe



$$\delta \sim \left. \frac{\delta\rho}{\rho} \right|_{k=aH} \sim \zeta > \zeta_c \sim 1$$

Primordial Black Holes and Quantum Diffusion

Primordial Black Holes and Quantum Diffusion

- Large fluctuations are needed to form PBHs

  They could backreact on the expansion dynamics

  Backreaction can be incorporated in an effective (stochastic) theory

Primordial Black Holes and Quantum Diffusion

- Large fluctuations are needed to form PBHs

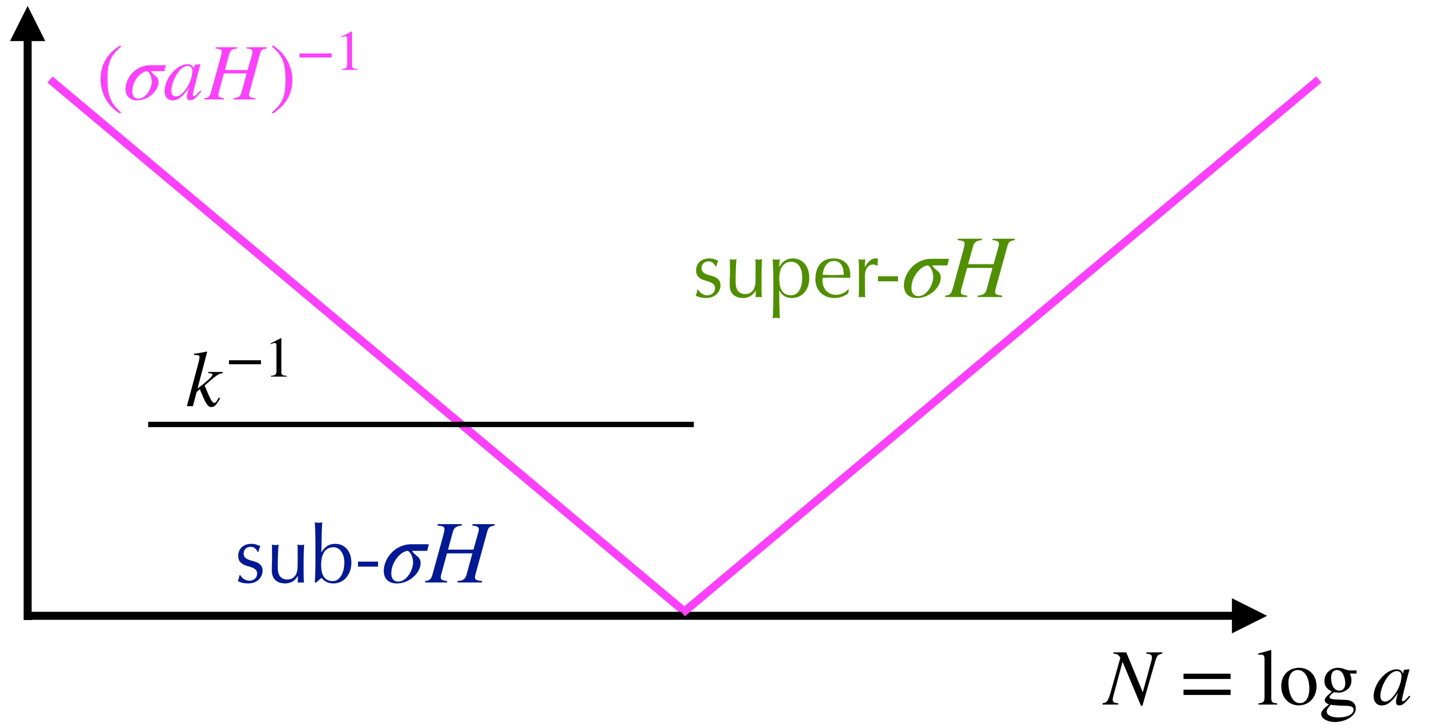
  They could backreact on the expansion dynamics

  Backreaction can be incorporated in an effective (stochastic) theory

- Stochastic inflation   A. Starobinsky [1986] *Stochastic de Sitter (inflationary) stage in the early universe*

  Splitting fields into UV and IR part: coarse-graining scale $k_{cg} = \sigma\, a\, H$

$$\phi(x) = \phi_{cg} + \int \frac{dk}{(2\pi)^{3/2}} W\left(\frac{k}{\sigma a H}\right) \left[\phi_k(N)\, e^{-i\vec{k}\cdot\vec{x}}\, \hat{a}_{\vec{k}} + h.c.\right]$$



Quantum subhorizon fluctuations source the background

Primordial Black Holes and Quantum Diffusion

- Large fluctuations are needed to form PBHs

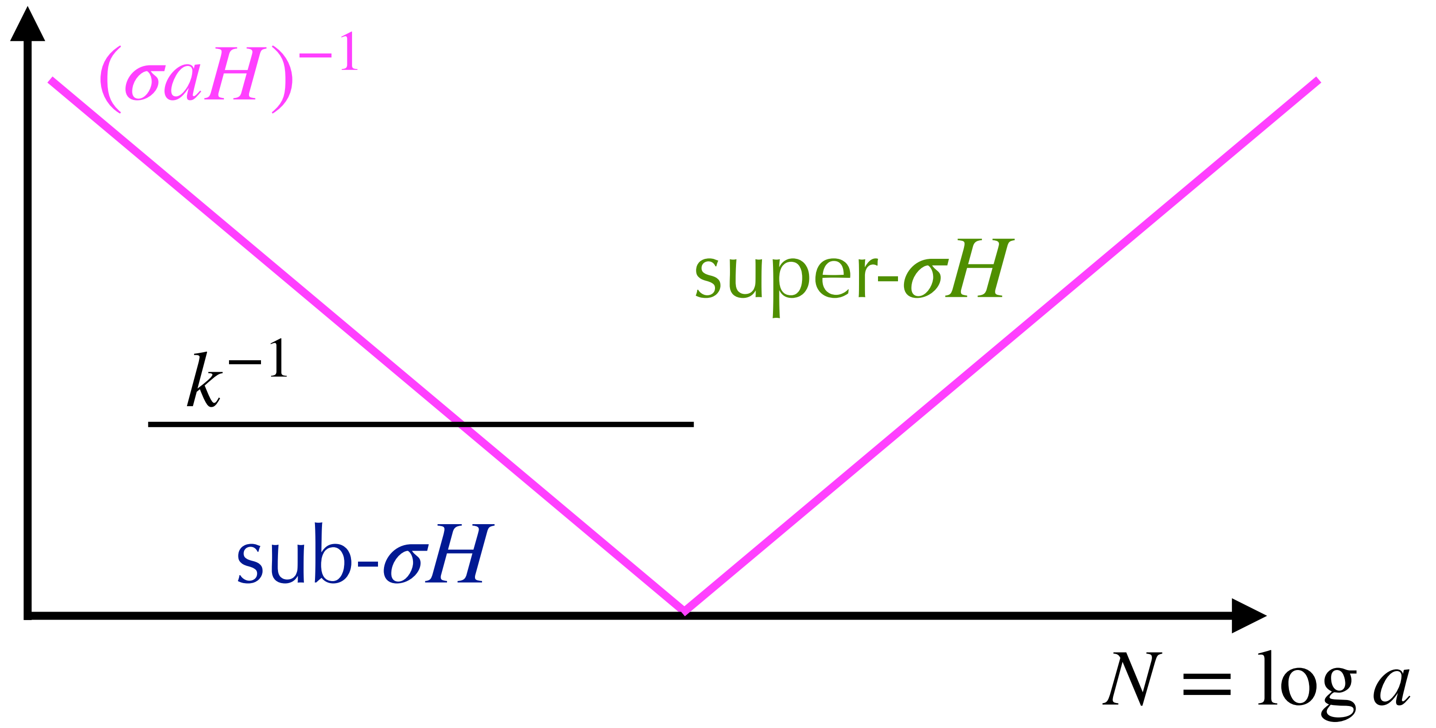
  They could backreact on the expansion dynamics

  Backreaction can be incorporated in an effective (stochastic) theory

- Stochastic inflation   A. Starobinsky [1986] *Stochastic de Sitter (inflationary) stage in the early universe*

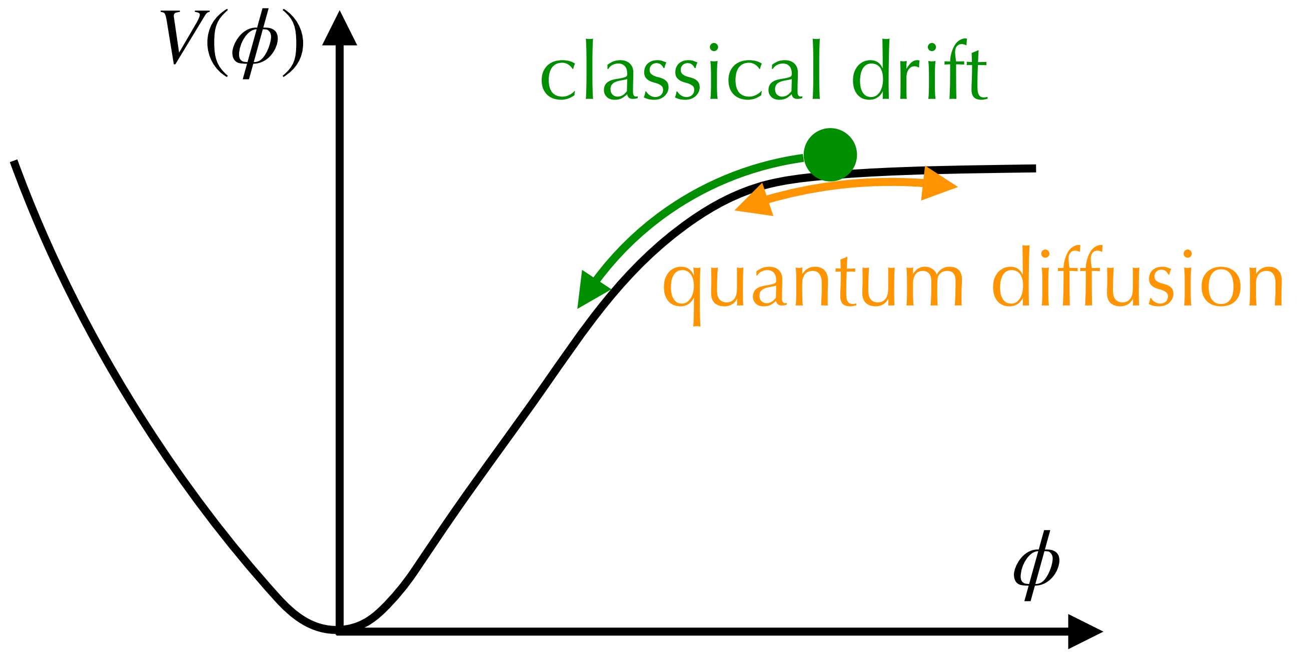
Splitting fields into UV and IR part: coarse-graining scale $k_{cg} = \sigma\, a\, H$

$$\phi(x) = \phi_{cg} + \int \frac{dk}{(2\pi)^{3/2}} W\left(\frac{k}{\sigma a H}\right) \left[\phi_k(N)\, e^{-i\vec{k}\cdot\vec{x}}\, \hat{a}_{\vec{k}} + h.c.\right]$$



Quantum subhorizon fluctuations source the background

Dynamics at leading order in slow roll: $$\frac{d}{dN}\phi_{cg} = -\frac{V_{,\phi}(\phi_{cg})}{3H^2(\phi_{cg})} + \frac{H(\phi_{cg})}{2\pi}\xi(N)$$



Primordial Black Holes and Quantum Diffusion

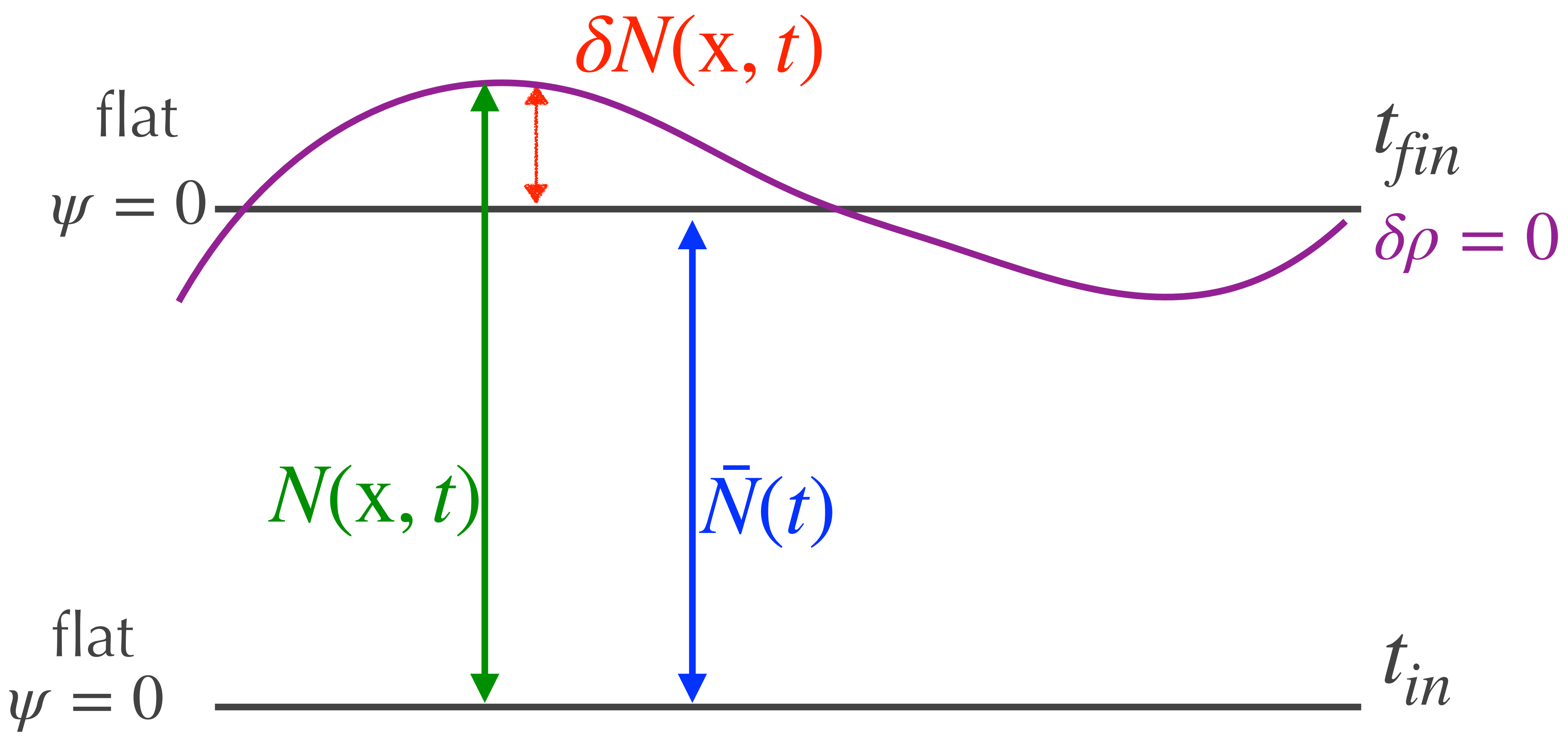
Primordial Black Holes and Quantum Diffusion

- How to reconstruct the statistics of ζ in presence of quantum diffusion?

Primordial Black Holes and Quantum Diffusion

- How to reconstruct the statistics of ζ in presence of quantum diffusion?

- δN formalism



$$\zeta(t, \mathrm{x}) = N(t, \mathrm{x}) - \bar{N}(t) \equiv \delta N$$

Lifshitz, Khalatnikov [1960]

Starobinsky [1983]

Wands, Malik, Lyth, Liddle [2000]

Quantum diffusion during inflation: properties of perturbations

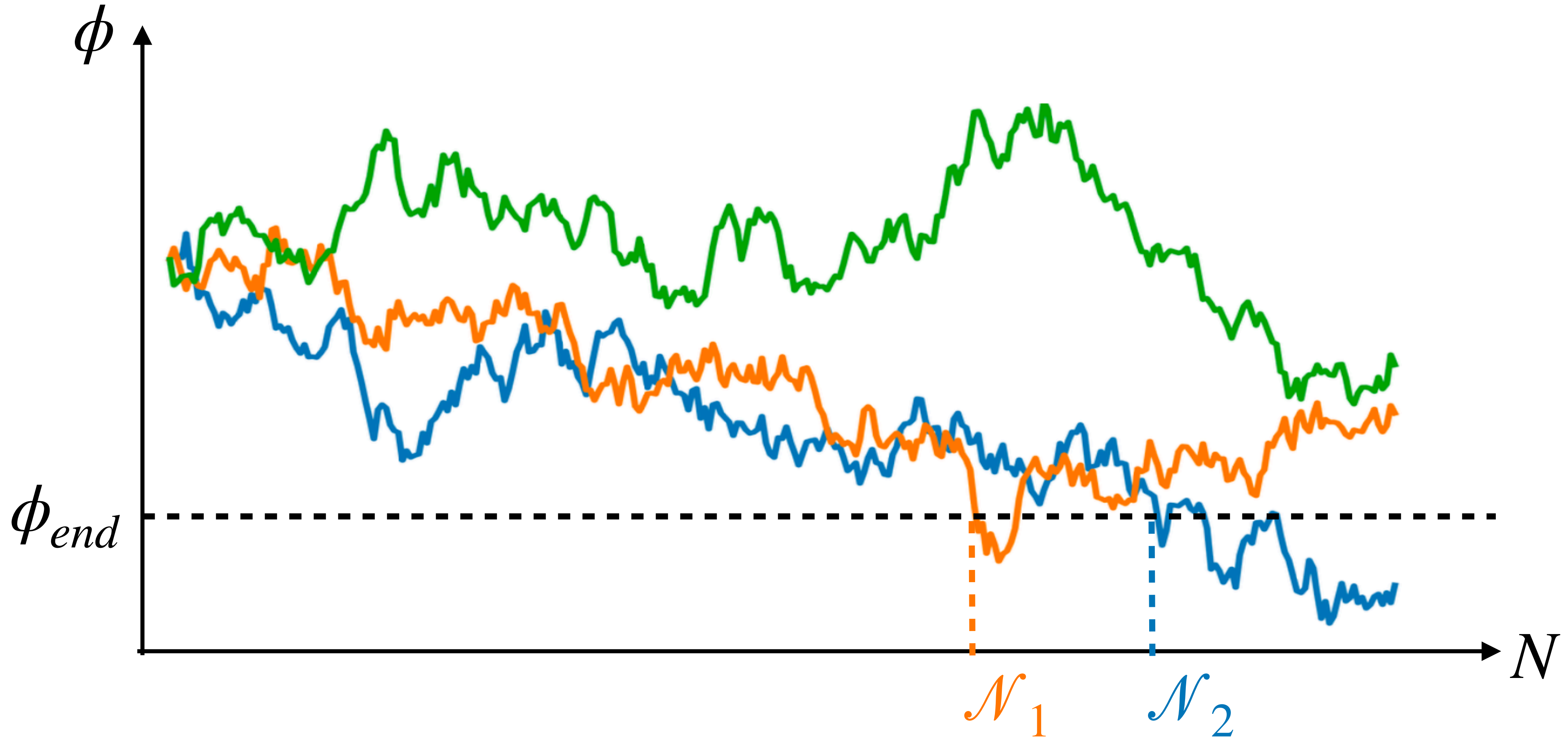
Quantum diffusion during inflation: properties of perturbations

- Stochastic-δN formalism

[Enqvist, Nurmi, Podolsky, Rigopoulos [2008]]

Vennin, Starobinsky [2015]

Number of e-folds is a stochastic variable $\mathcal{N}$



Statistics of ζ from the statistics of $\mathcal{N}$

$$\zeta_{cg}(\mathrm{x}) = \mathcal{N}(\mathrm{x}) - \langle \mathcal{N} \rangle$$

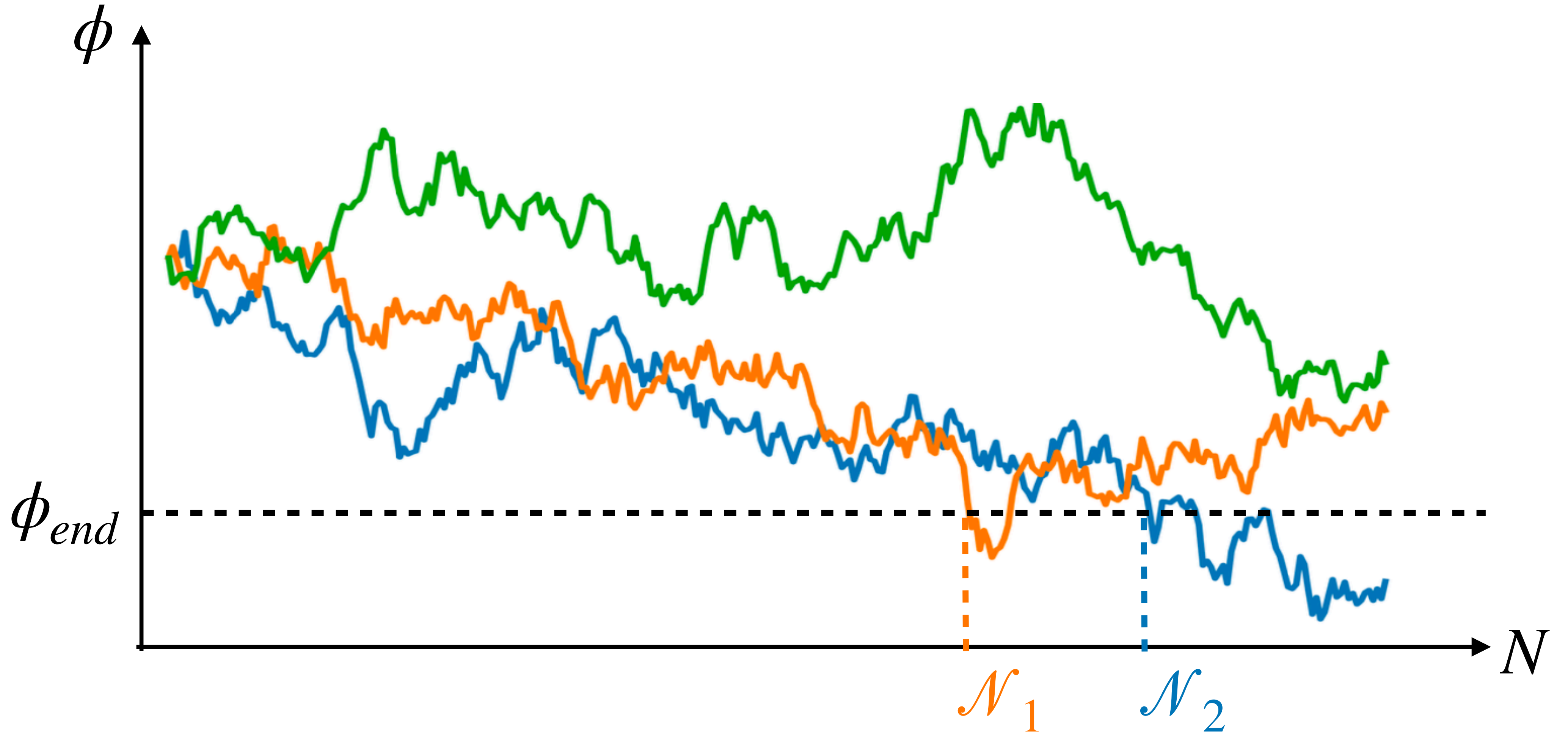
Quantum diffusion during inflation: properties of perturbations

- Stochastic-δN formalism

[Enqvist, Nurmi, Podolsky, Rigopoulos [2008]

Vennin, Starobinsky [2015]

Number of e-folds is a stochastic variable $\mathcal{N}$



Statistics of ζ from the statistics of $\mathcal{N}$

$$\zeta_{cg}(\mathrm{x}) = \mathcal{N}(\mathrm{x}) - \langle \mathcal{N} \rangle$$

Distribution function for the duration of inflation (first passage time)

$$\frac{\partial}{\partial \mathcal{N}} P(\mathcal{N}, \phi) = \mathcal{L}^{\dagger}_{FP}(\phi) \cdot P(\mathcal{N}, \phi) \qquad \frac{1}{M_{Pl}^2} \mathcal{L}^{\dagger}_{FP}(\phi) = -\frac{v'(\phi)}{v(\phi)} \frac{\partial}{\partial \phi} + v(\phi) \frac{\partial^2}{\partial \phi^2} \qquad v = \frac{V}{24\pi^2 M_{Pl}^4}$$

Stochastic-δN formalism: exponential tails

Stochastic-δN formalism: exponential tails

- Tail of the PDF for ζ has an exponential fall-off behaviour: $P(\mathcal{N}) \propto e^{-\Lambda_0 \mathcal{N}}$

Stochastic-δN formalism: exponential tails

- Tail of the PDF for ζ has an exponential fall-off behaviour: $P(\mathcal{N}) \propto e^{-\Lambda_0 \mathcal{N}}$

- Characteristic function (includes all moments)

$$\chi(t,\phi) \equiv \langle e^{it\mathcal{N}} \rangle = \int_{-\infty}^{\infty} e^{it\mathcal{N}} P(\mathcal{N},\phi)\, d\mathcal{N} \longrightarrow P(\mathcal{N},\phi) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-it\mathcal{N}} \chi(t,\phi)\, dt$$

Stochastic-δN formalism: exponential tails

- Tail of the PDF for ζ has an exponential fall-off behaviour: $P(\mathcal{N}) \propto e^{-\Lambda_0 \mathcal{N}}$

- Characteristic function (includes all moments)

$$\chi(t,\phi) \equiv \langle e^{it\mathcal{N}} \rangle = \int_{-\infty}^{\infty} e^{it\mathcal{N}} P(\mathcal{N},\phi)\, d\mathcal{N} \longrightarrow P(\mathcal{N},\phi) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{-it\mathcal{N}} \chi(t,\phi)\, dt$$

- Useful trick: pole expansion

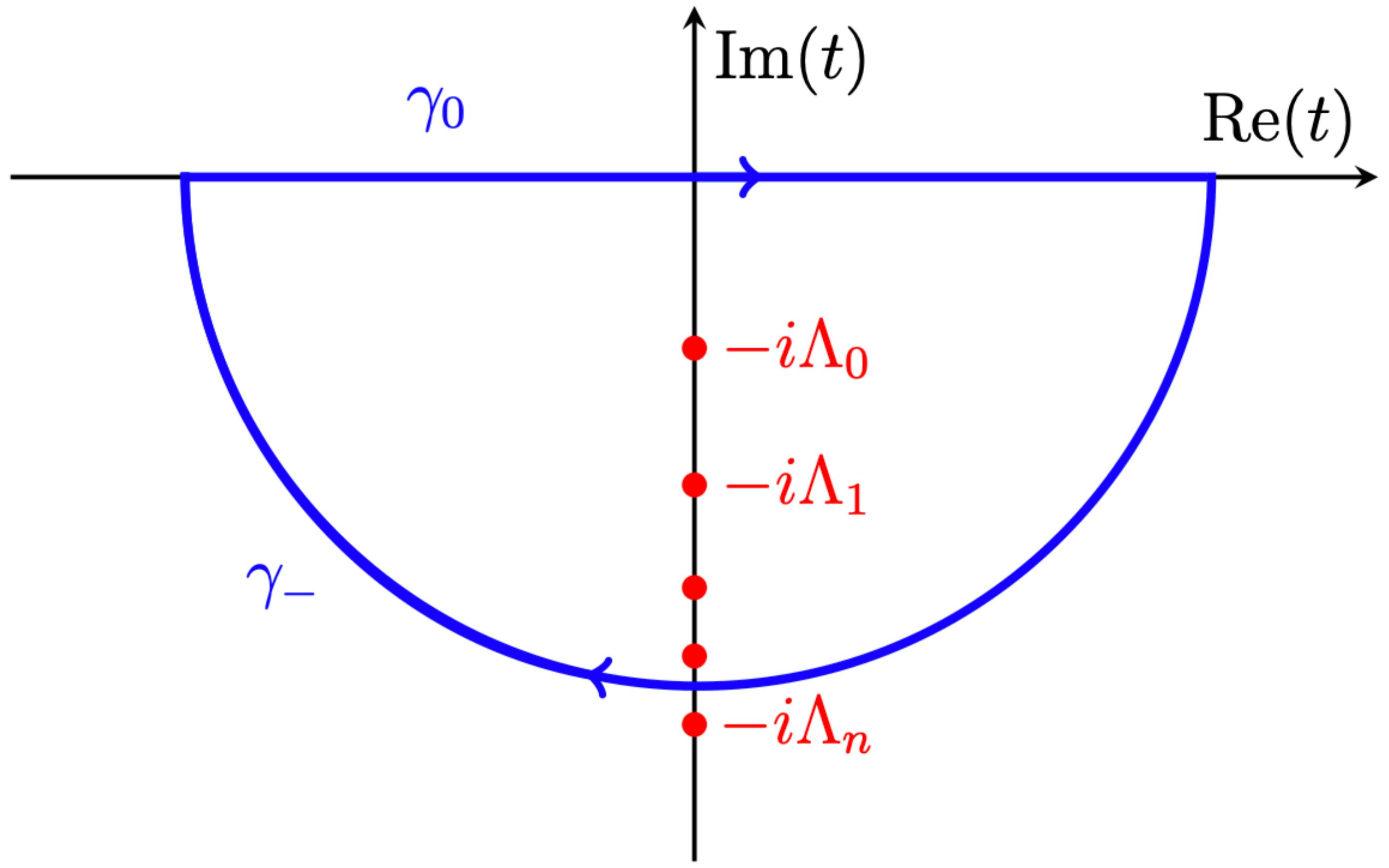
[Ezquiaga, Garcia-Bellido, Vennin (2020)]

Pattison, Vennin, Assadullahi, Wands [2017]

$$\chi(t,\phi) = \sum_n \frac{a_n(\phi)}{\Lambda_n - i\,t} + g(t,\phi)$$

$$P(\mathcal{N},\phi) = \sum_n a_n(\phi)\, e^{-\Lambda_n \mathcal{N}}$$

$$0 < \Lambda_0 < \Lambda_1 < \cdots \Lambda_n$$



Stochastic-δN formalism: exponential tails

- Tail of the PDF for ζ has an exponential fall-off behaviour: $P(\mathcal{N}) \propto e^{-\Lambda_0 \mathcal{N}}$

- Characteristic function (includes all moments)

$$\chi(t,\phi) \equiv \langle e^{it\mathcal{N}} \rangle = \int_{-\infty}^{\infty} e^{it\mathcal{N}} P(\mathcal{N},\phi)\, d\mathcal{N} \longrightarrow P(\mathcal{N},\phi) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{-it\mathcal{N}} \chi(t,\phi)\, dt$$

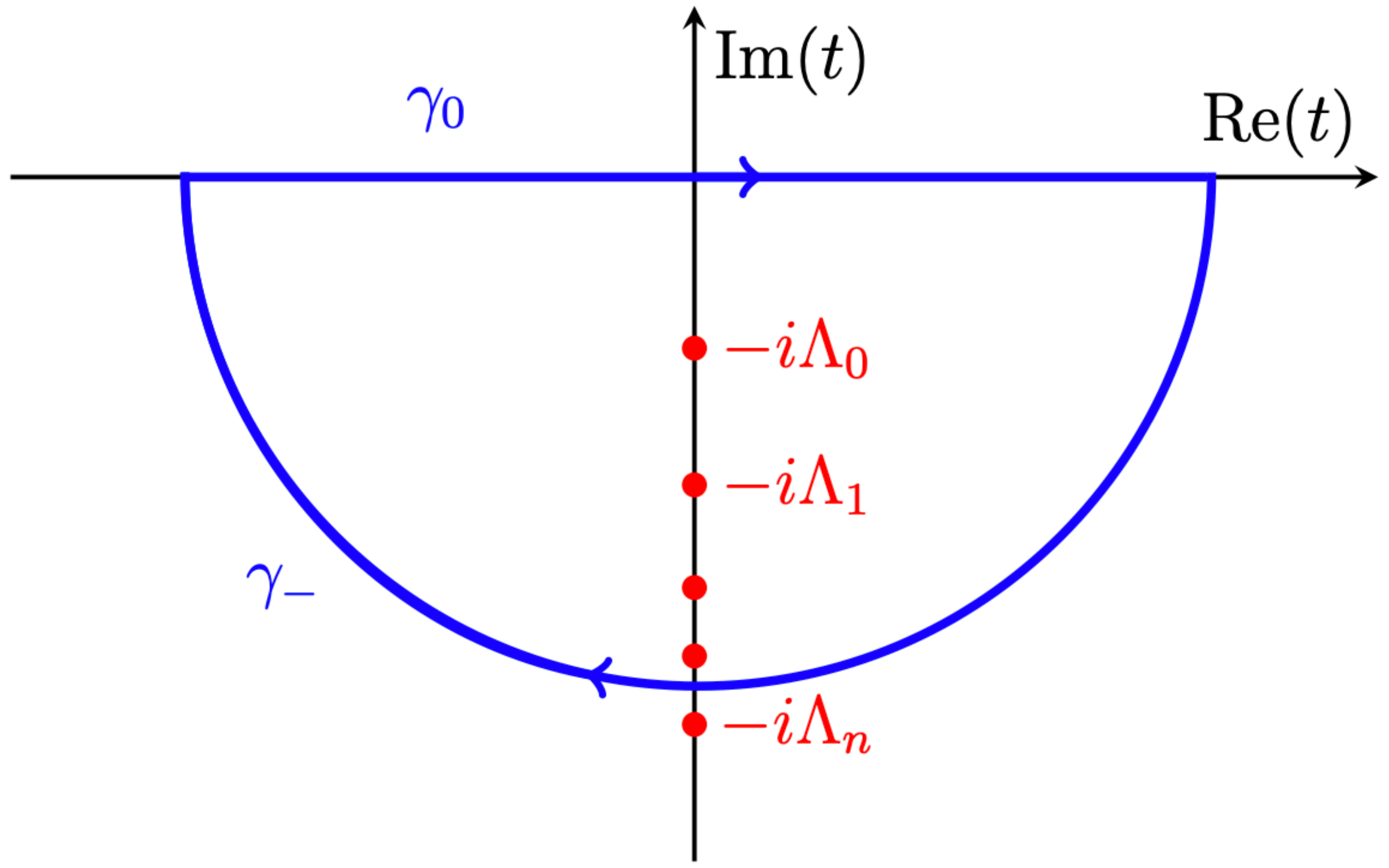
- Useful trick: pole expansion

[Ezquiaga, Garcia-Bellido, Vennin (2020)]

Pattison, Vennin, Assadullahi, Wands [2017]

$$\chi(t,\phi) = \sum_n \frac{a_n(\phi)}{\Lambda_n - i\,t} + g(t,\phi)$$

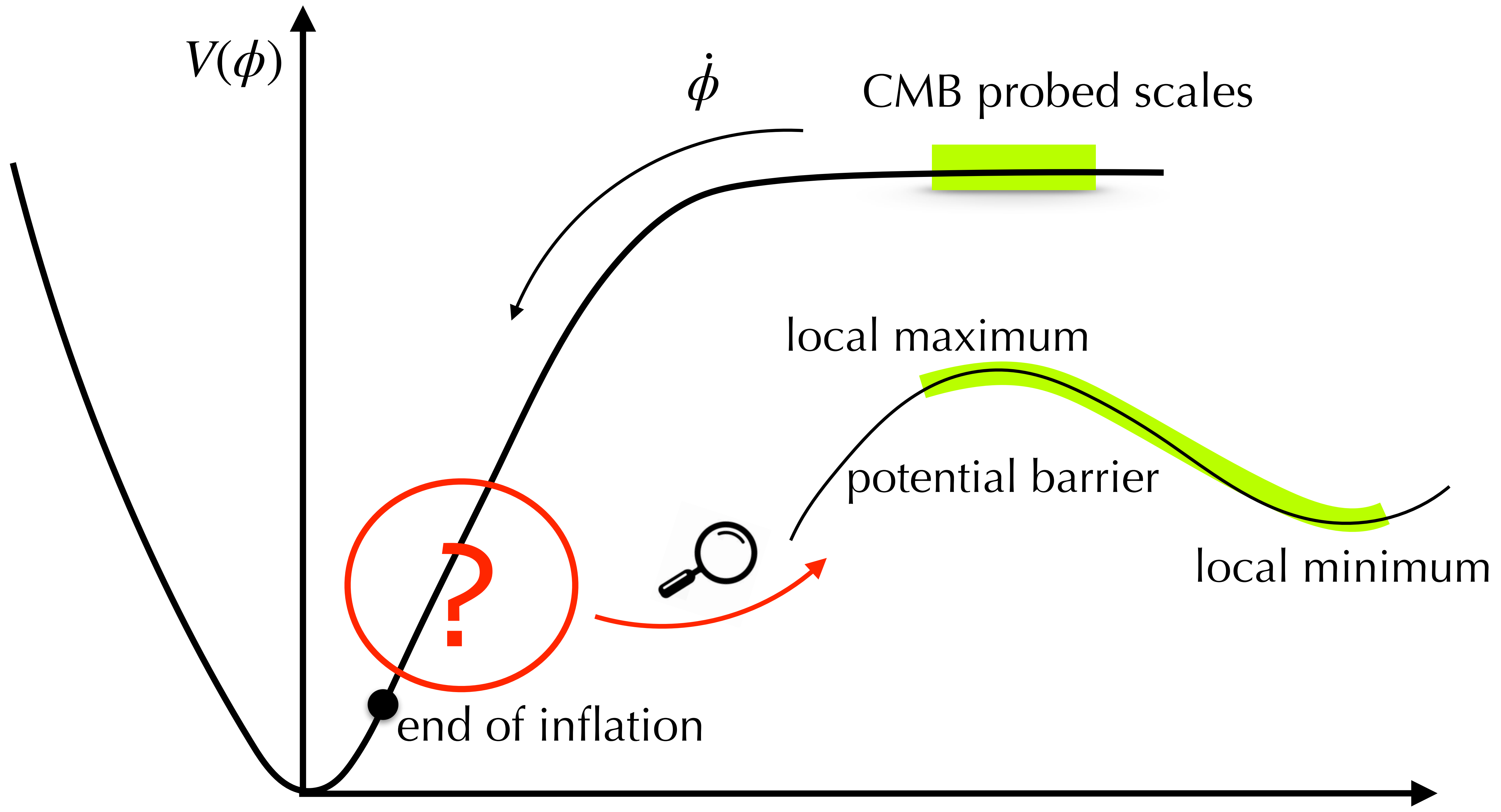
$$P(\mathcal{N},\phi) = \sum_n a_n(\phi)\, e^{-\Lambda_n \mathcal{N}} \qquad 0 < \Lambda_0 < \Lambda_1 < \cdots \Lambda_n$$



- This type of non-Gaussianities cannot be captured by perturbative parametrisations (such as the fNL expansion) !

Stochastic tunnelling

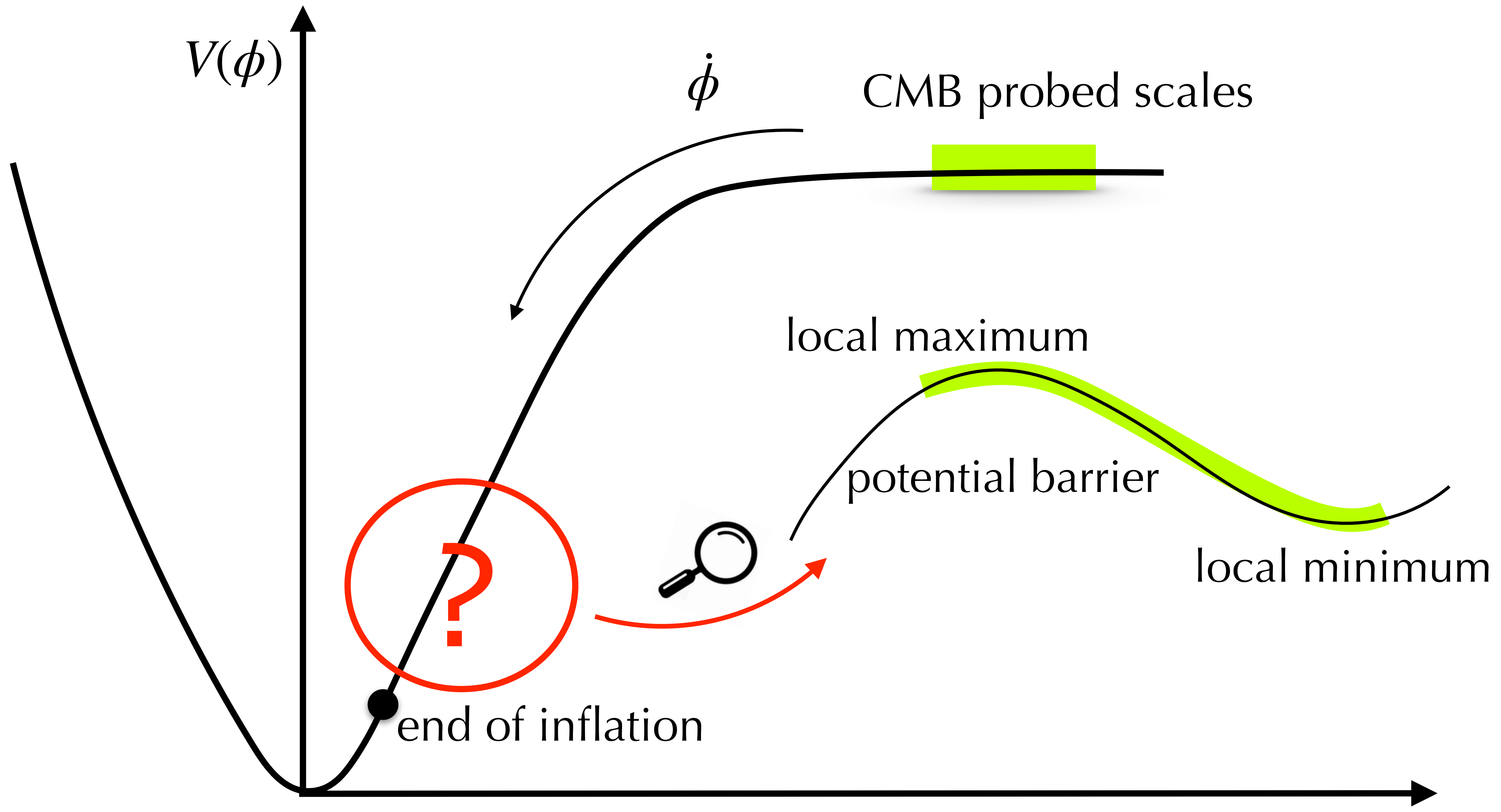
C.A., V. Vennin [2022]
"Primordial black holes from stochastic tunneling"
JCAP 02(2023) 043



Stochastic tunnelling

C.A., V. Vennin [2022]
"Primordial black holes from stochastic tunneling"
JCAP 02(2023) 043

- False vacuum state

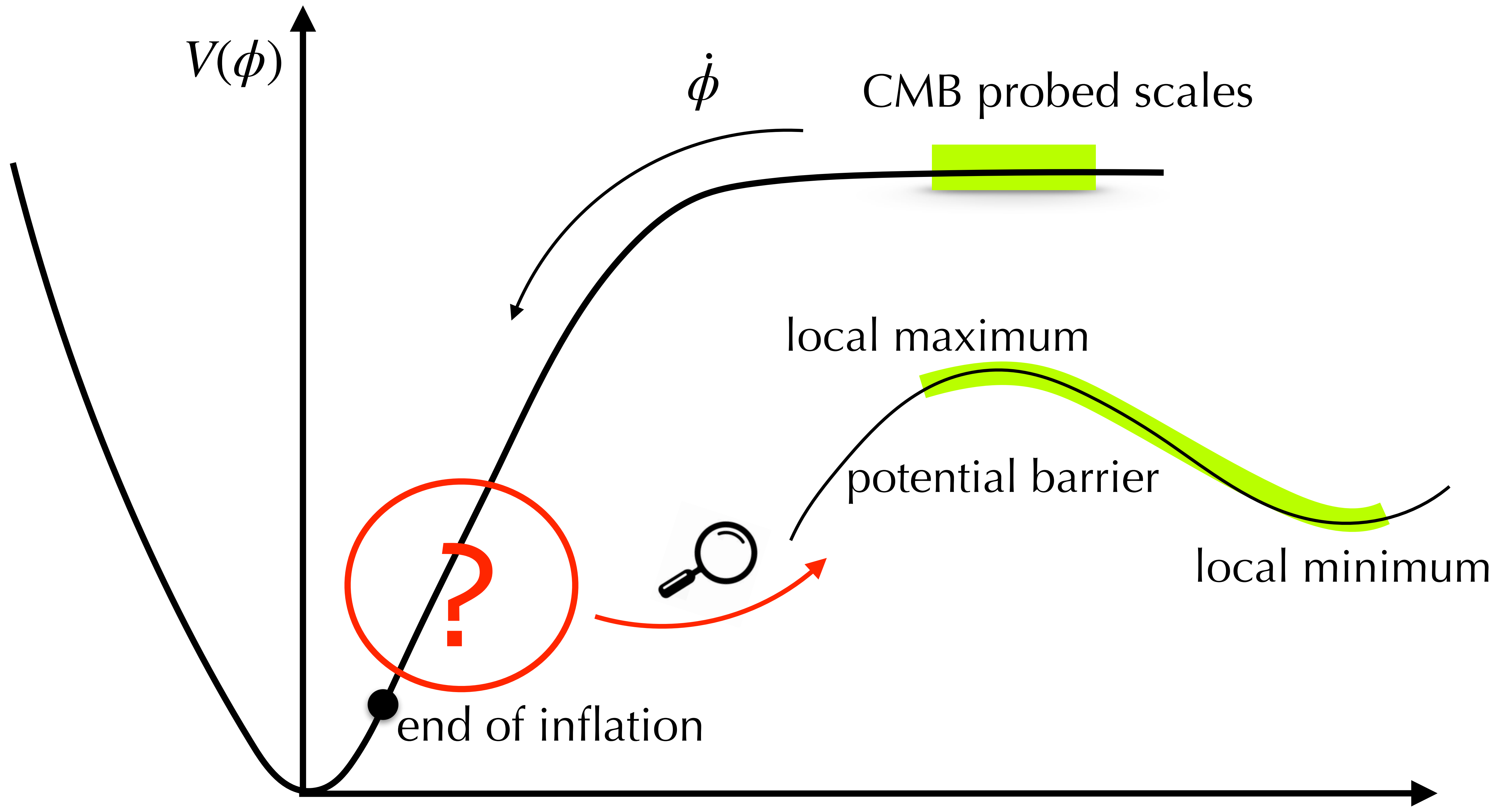


Stochastic tunnelling

C.A., V. Vennin [2022]
"Primordial black holes from stochastic tunneling"
JCAP 02(2023) 043

- False vacuum state

- Local minima naturally appear in various contexts: - high energy constructions (supersymmetry, supergravity)
  - breaking of flat-inflection point condition through radiative corrections
  - etc.



Stochastic tunnelling

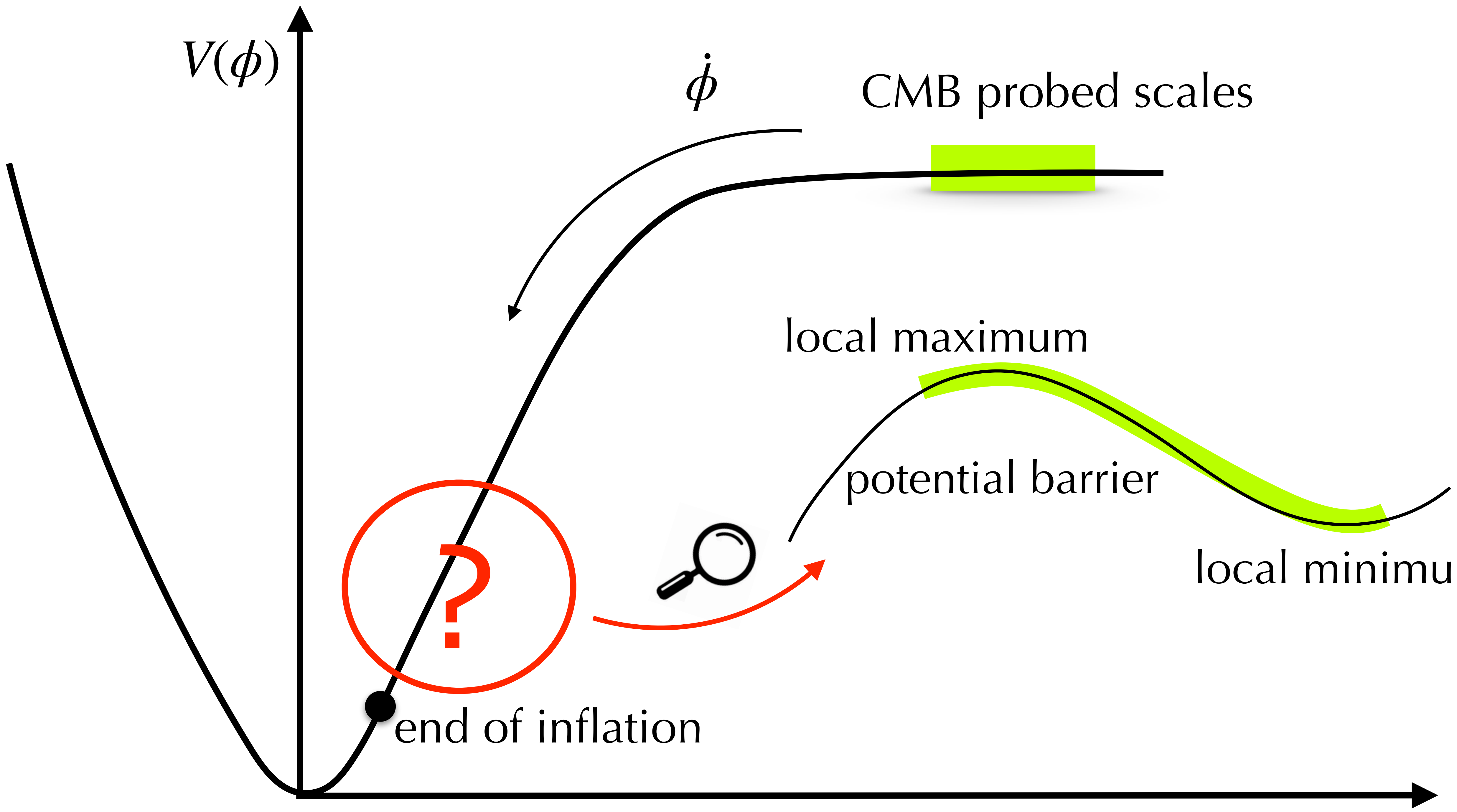
C.A., V. Vennin [2022]
"Primordial black holes from stochastic tunneling"
JCAP 02(2023) 043

- False vacuum state

- Local minima naturally appear in various contexts: - high energy constructions (supersymmetry, supergravity)
  - breaking of flat-inflection point condition through radiative corrections
  - etc.

- How to escape? 1) Large classical velocity

  2) "Stochastic tunnelling": quantum fluctuations jiggle the inflaton and push it outwards



Stochastic tunnelling

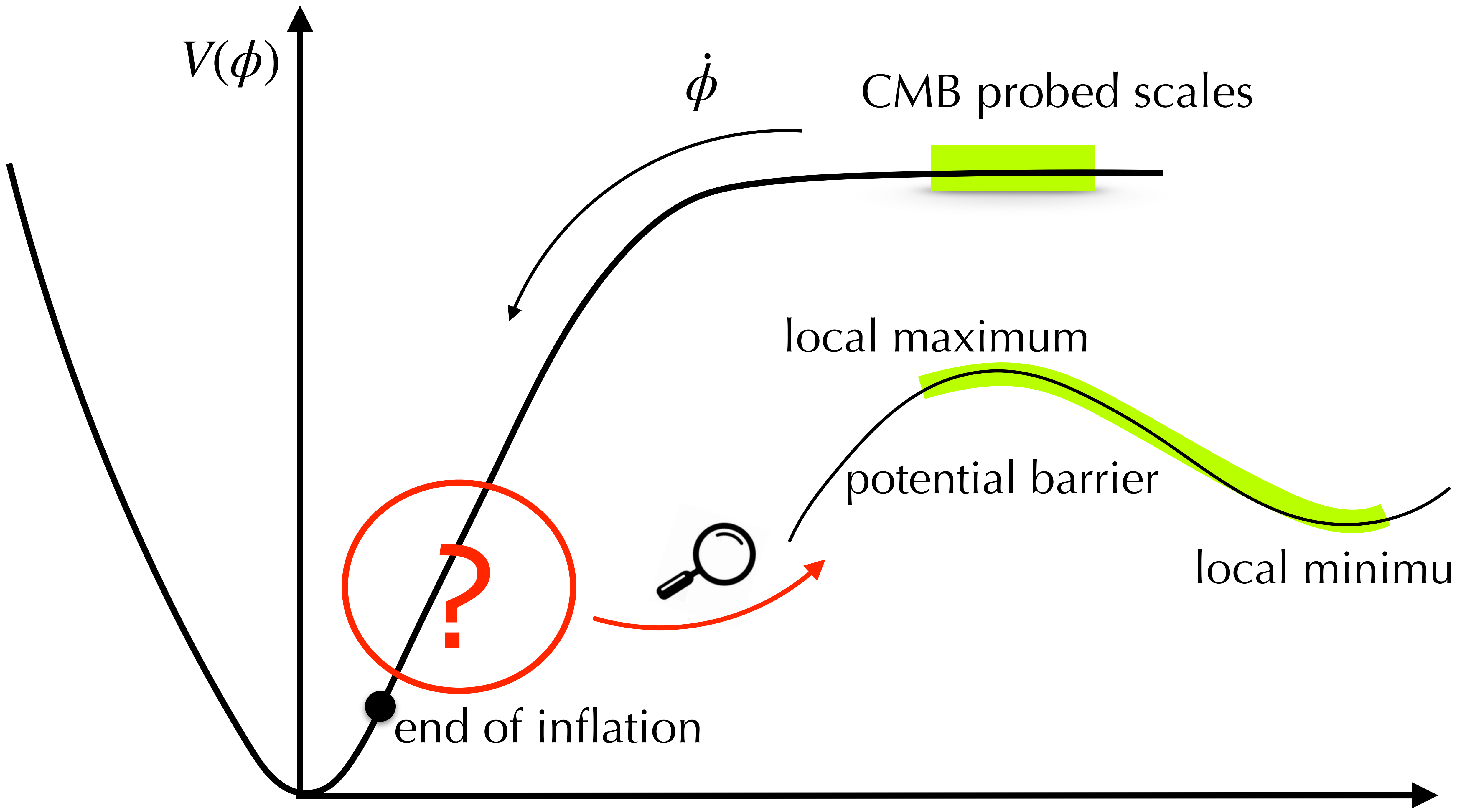
C.A., V. Vennin [2022]
"Primordial black holes from stochastic tunneling"
JCAP 02(2023) 043

- False vacuum state

- Local minima naturally appear in various contexts: - high energy constructions (supersymmetry, supergravity)
  - breaking of flat-inflection point condition through radiative corrections
  - etc.

- How to escape? 1) Large classical velocity

  2) "Stochastic tunnelling": quantum fluctuations jiggle the inflaton and push it outwards



Stochastic tunnelling

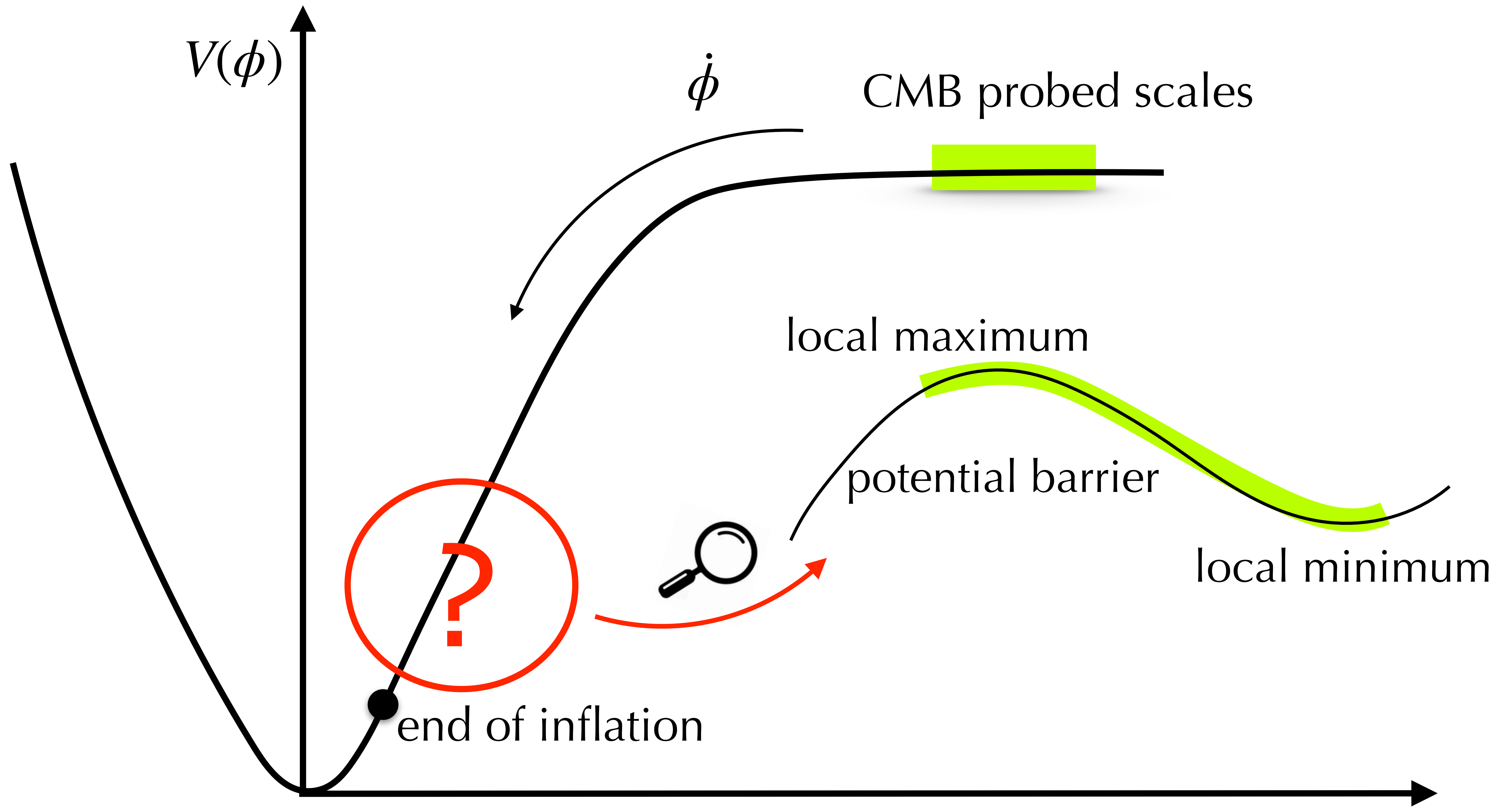
C.A., V. Vennin [2022]
"Primordial black holes from stochastic tunneling"
JCAP 02(2023) 043

- False vacuum state

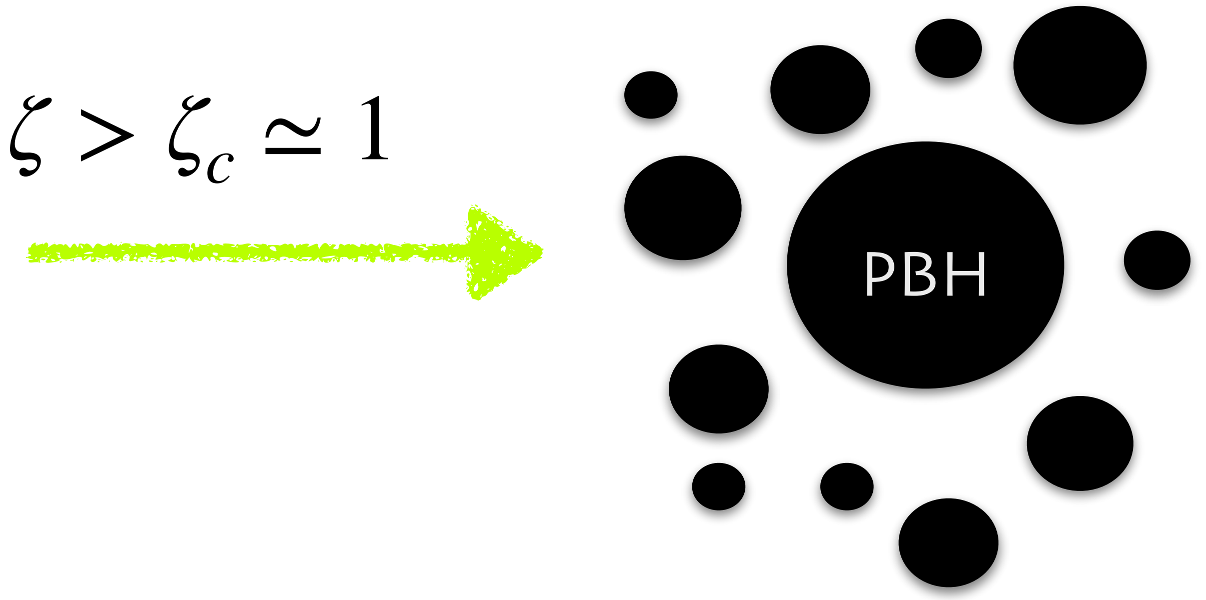
- Local minima naturally appear in various contexts:
  - high energy constructions (supersymmetry, supergravity)
  - breaking of flat-inflection point condition through radiative corrections
  - etc.

- How to escape? 1) Large classical velocity

  2) "Stochastic tunnelling": quantum fluctuations jiggle the inflaton and push it outwards



$\zeta > \zeta_c \simeq 1$

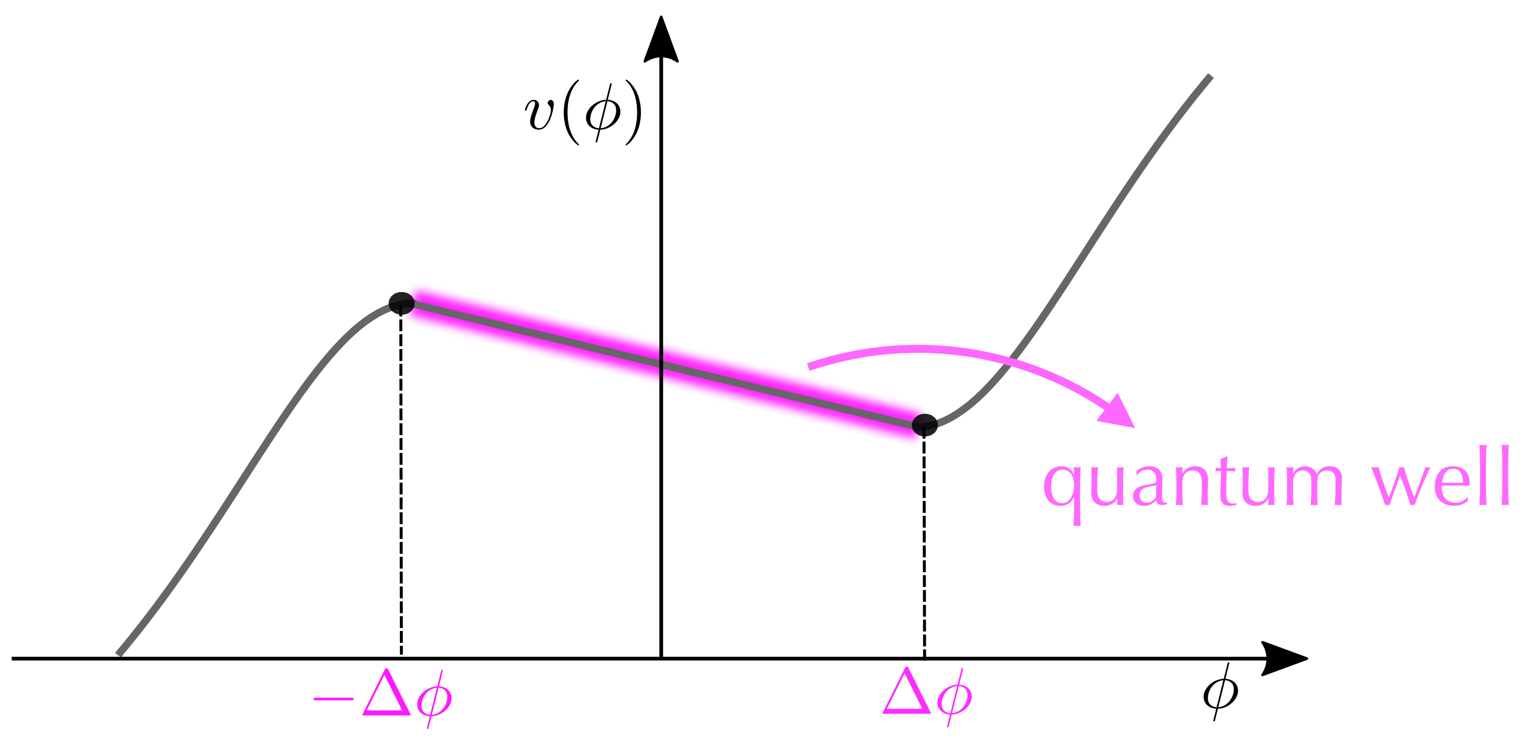


False vacuum: simple toy models

False vacuum: simple toy models

- Linear model

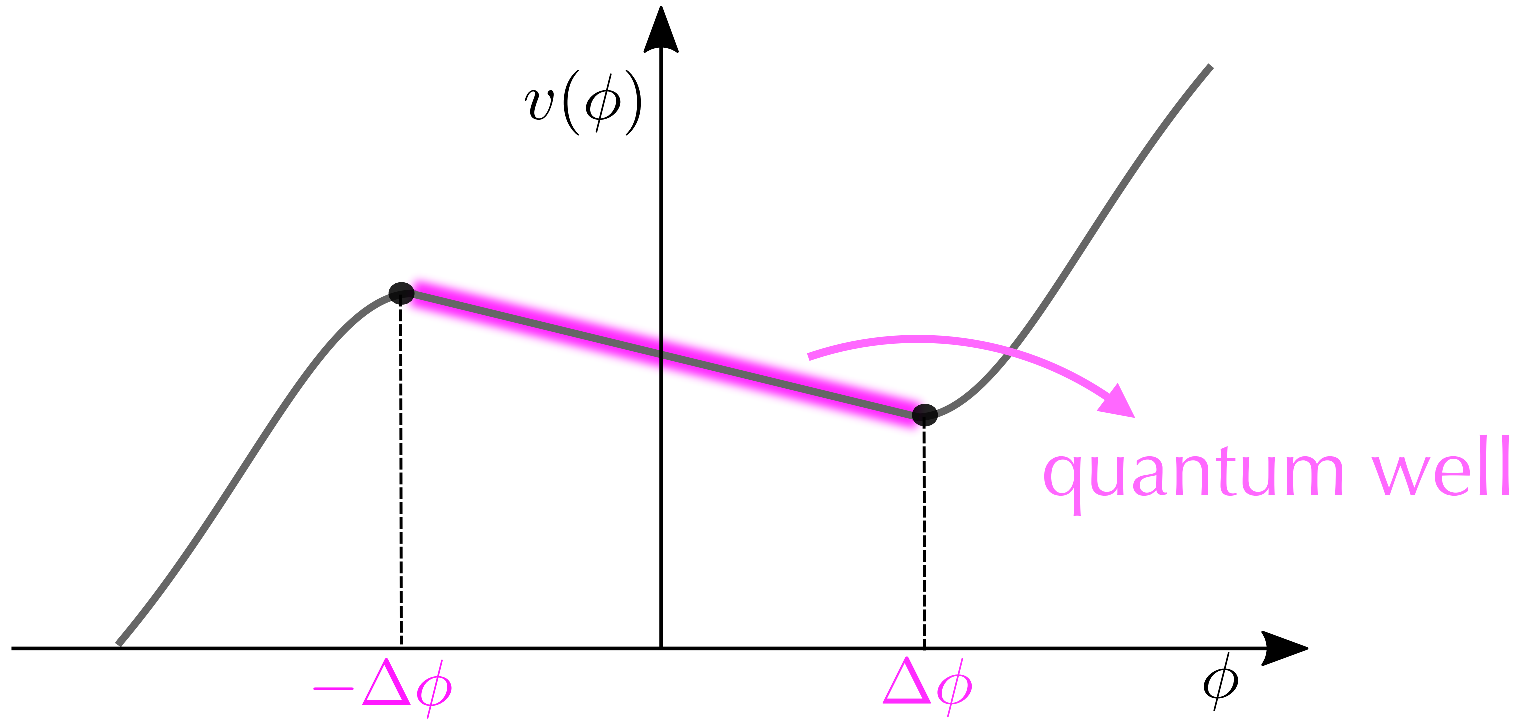
$$v(\phi) = v_0 \left(1 - \alpha \frac{\phi}{\Delta\phi} \right)$$



False vacuum: simple toy models

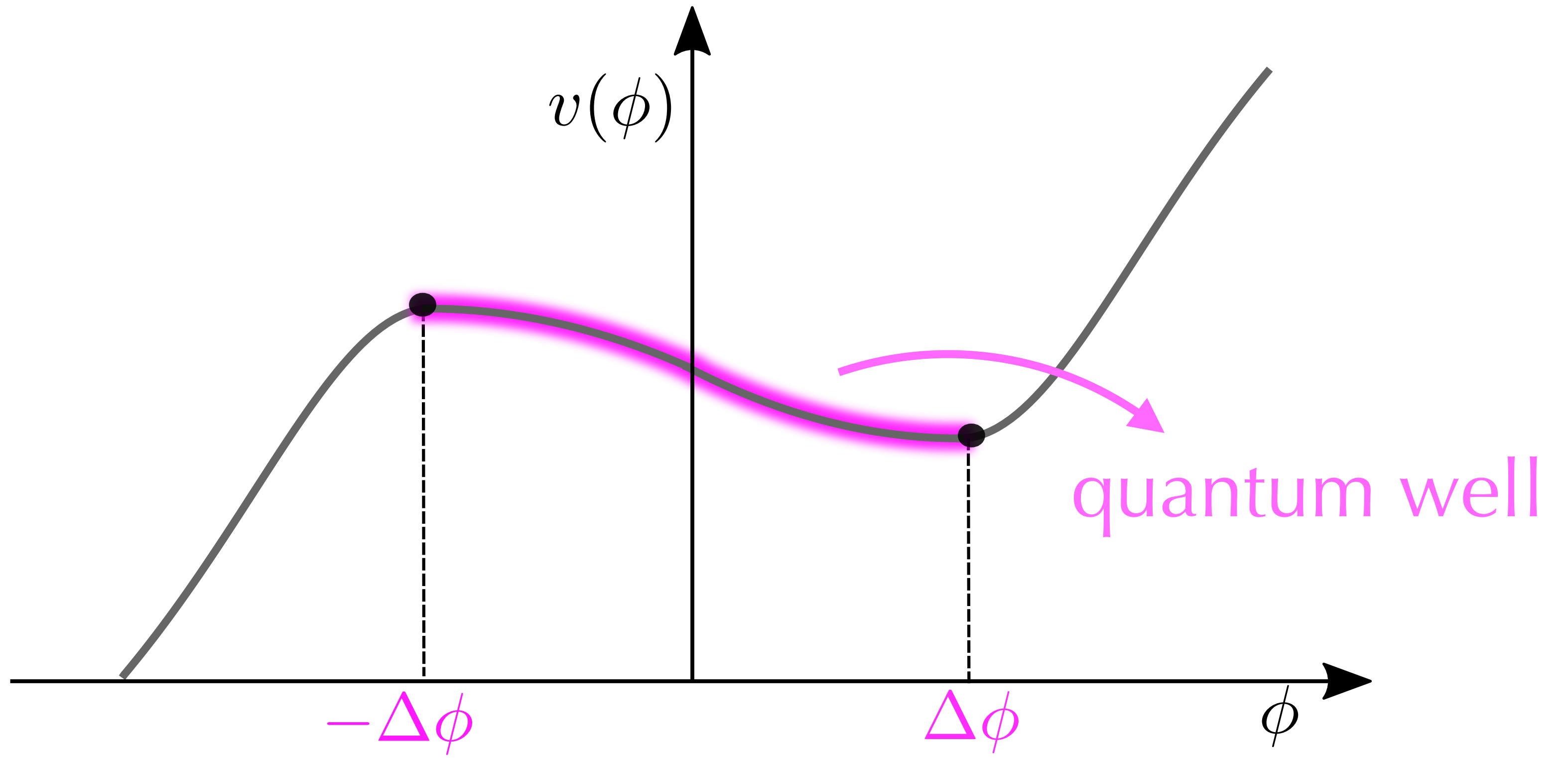
- Linear model

$$v(\phi) = v_0 \left(1 - \alpha \frac{\phi}{\Delta\phi}\right)$$



- Quadratic model ("two-parabola approximation")

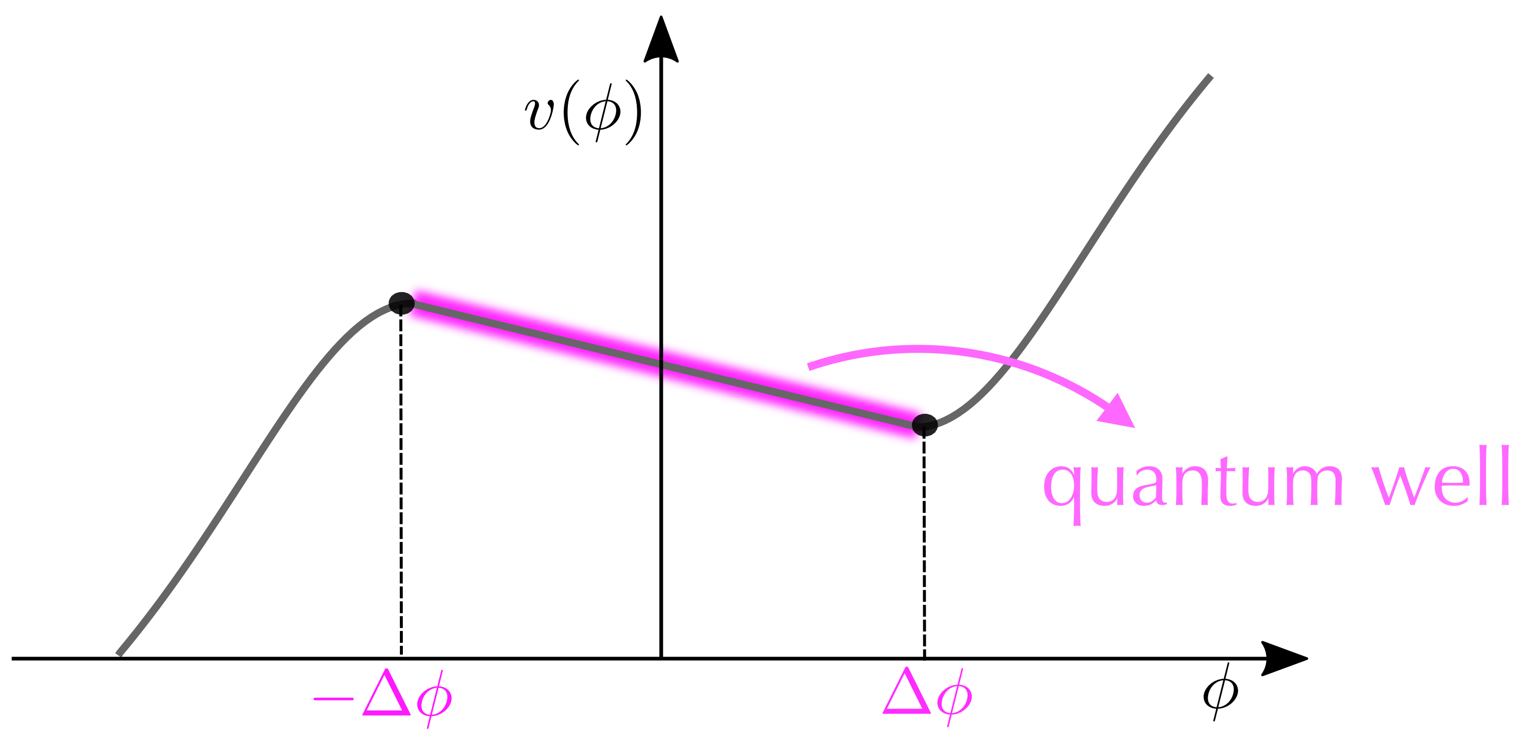
$$v(\phi) = v_0 \begin{cases} 1 + \alpha \left[\left(\frac{\phi}{\Delta\phi} - 1\right)^2 - 1\right] & \text{if} \quad 0 \leq \phi \leq \Delta\phi \\ 1 - \alpha \left[\left(\frac{\phi}{\Delta\phi} + 1\right)^2 - 1\right] & \text{if} \quad -\Delta\phi \leq \phi \leq 0 \end{cases}$$



False vacuum: simple toy models

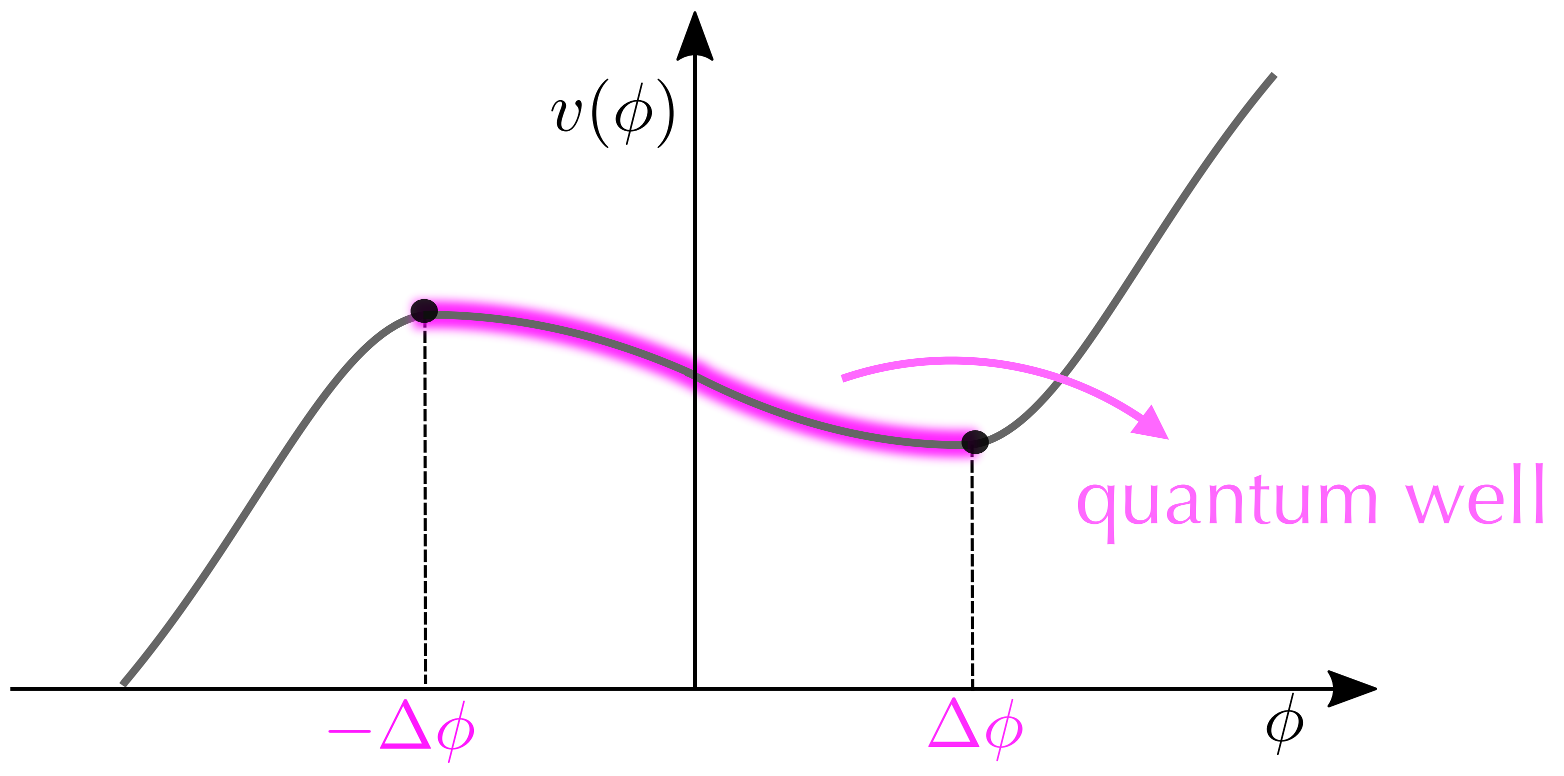
- Linear model

$$v(\phi) = v_0 \left(1 - \alpha \frac{\phi}{\Delta\phi}\right)$$



- Quadratic model (“two-parabola approximation”)

$$v(\phi) = v_0 \begin{cases} 1 + \alpha\left[\left(\frac{\phi}{\Delta\phi} - 1\right)^2 - 1\right] & \text{if} \quad 0 \le \phi \le \Delta\phi \\ 1 - \alpha\left[\left(\frac{\phi}{\Delta\phi} + 1\right)^2 - 1\right] & \text{if} \quad -\Delta\phi \le \phi \le 0 \end{cases}$$



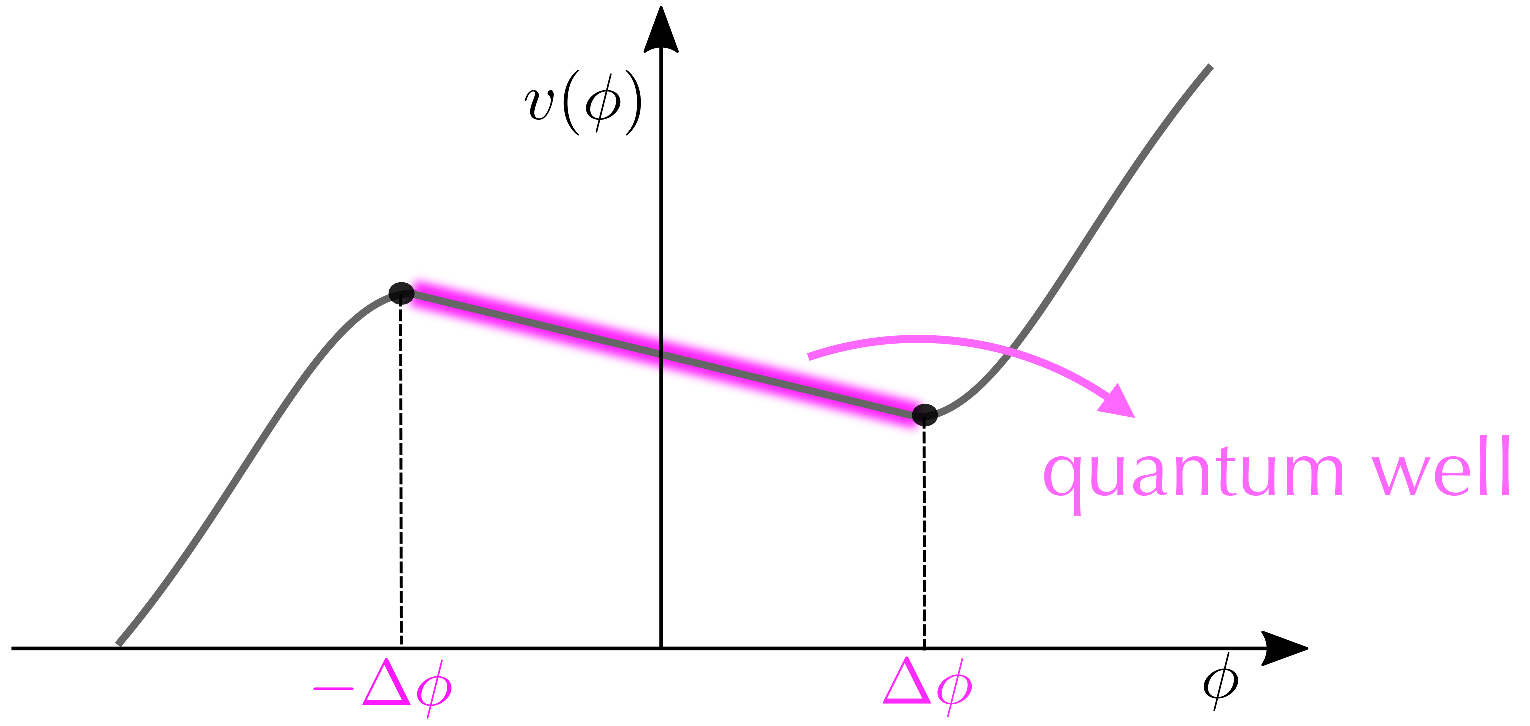
$$\mu^2 = \frac{(2\,\Delta\phi)^2}{v_0 M_{Pl}^2} \propto \frac{M_{Pl}^2\,\Delta\phi^2}{V}$$

$$a = \frac{\alpha}{v_0} \propto \frac{M_{Pl}^4\,\Delta V}{V^2}$$

False vacuum: simple toy models

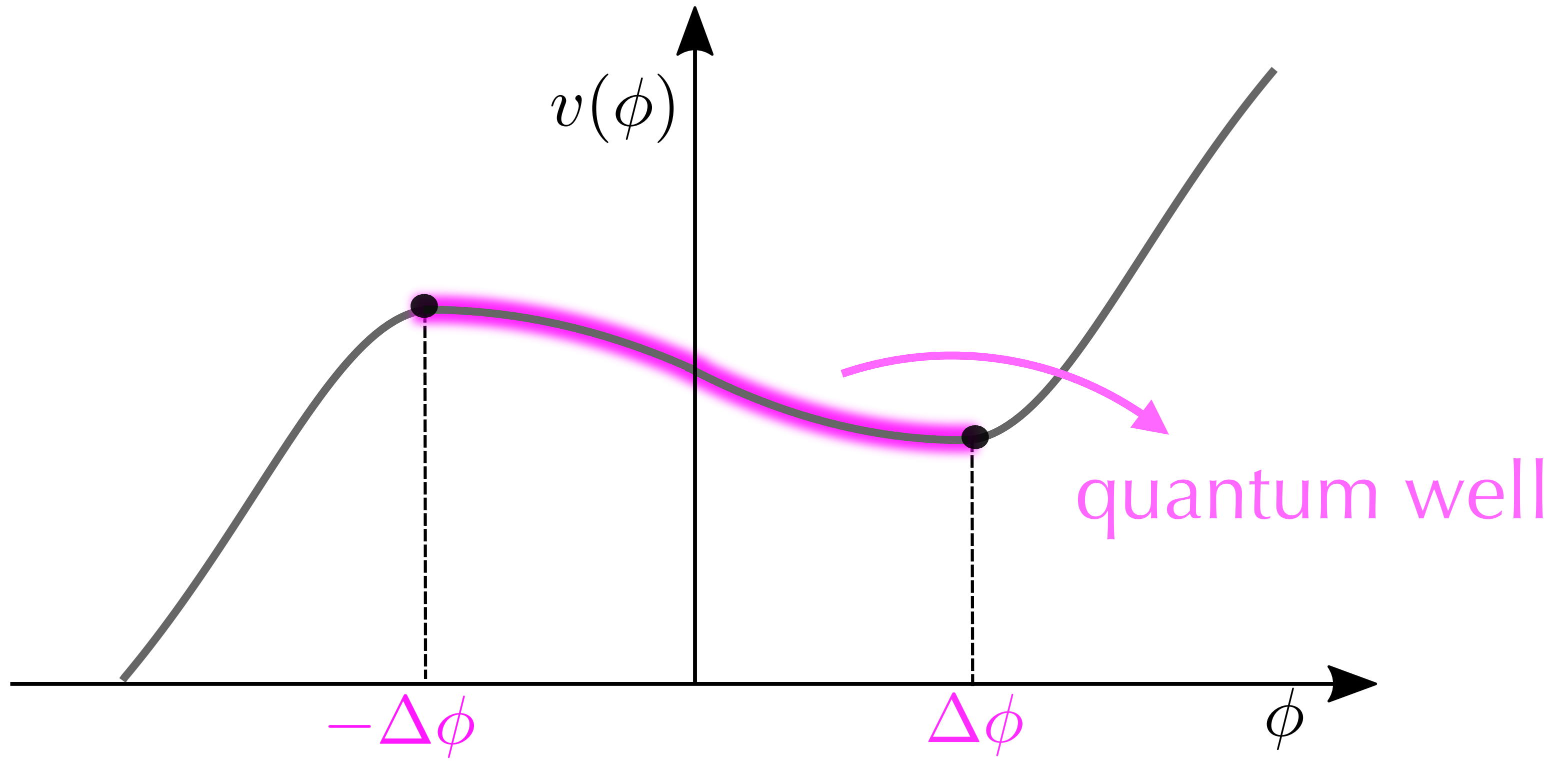
- Linear model

$$v(\phi) = v_0 \left(1 - \alpha \frac{\phi}{\Delta\phi}\right)$$



- Quadratic model (“two-parabola approximation”)

$$v(\phi) = v_0 \begin{cases} 1 + \alpha\left[\left(\frac{\phi}{\Delta\phi} - 1\right)^2 - 1\right] & \text{if} \quad 0 \le \phi \le \Delta\phi \\ 1 - \alpha\left[\left(\frac{\phi}{\Delta\phi} + 1\right)^2 - 1\right] & \text{if} \quad -\Delta\phi \le \phi \le 0 \end{cases}$$



$$\mu^2 = \frac{(2\,\Delta\phi)^2}{v_0 M_{Pl}^2} \propto \frac{M_{Pl}^2 \Delta\phi^2}{V}$$

$$a = \frac{\alpha}{v_0} \propto \frac{M_{Pl}^4 \Delta V}{V^2}$$

Quantum diffusion in highlighted regions, potential gradient elsewhere

Slow roll preserved: $\epsilon = \frac{M_{Pl}^2}{2}\left(\frac{v'}{v}\right)^2 \ll 1,\ |\eta| = \left| M_{Pl}^2 \frac{v''}{v} \right| \ll 1$

$\langle \mathcal{N} \rangle$ smaller than ~ 50: $\Delta v = v(-\Delta\phi) - v(\Delta\phi) \ll v_0$

False vacuum: linear model

False vacuum: linear model

- shallow-well limit

$$\Lambda_n^{shallow} = \frac{1}{\mu^2}\left[\pi^2\left(n+\frac{1}{2}\right)^2 - 2a + \mathcal{O}(a^2)\right]$$

$$P^{shallow}(\mathcal{N}, \phi = \Delta\phi) \simeq \frac{\pi}{\mu^2}(1-a)\, e^{-\left(\frac{\pi^2}{4} - 2a\right)\frac{\mathcal{N}}{\mu^2}}$$

$\sim e^{2a\mathcal{N}/\mu^2} \simeq e^{a\mathcal{N}/\langle\mathcal{N}\rangle}$ enhancement on the tail:

large for $\mathcal{N} \sim 1/a$ σ-away from the mean

False vacuum: linear model

- shallow-well limit

$$\Lambda_n^{shallow} = \frac{1}{\mu^2}\left[\pi^2\left(n+\frac{1}{2}\right)^2 - 2a + \mathcal{O}(a^2)\right]$$

$$P^{shallow}(\mathcal{N}, \phi = \Delta\phi) \simeq \frac{\pi}{\mu^2}(1-a)\, e^{-\left(\frac{\pi^2}{4} - 2a\right)\frac{\mathcal{N}}{\mu^2}}$$

$\sim e^{2a\mathcal{N}/\mu^2} \simeq e^{a\mathcal{N}/\langle\mathcal{N}\rangle}$ enhancement on the tail:
large for $\mathcal{N} \sim 1/a$ σ-away from the mean

- deep-well limit

$$\Lambda_0^{deep} = \frac{4\,a^2 e^{-2a}}{\mu^2}\left[1 + 2\,(2a-1)e^{-2a} + \mathcal{O}(e^{-4a})\right]$$

$$\Lambda_{n+1}^{deep} = \frac{a^2}{\mu^2} + \frac{\pi^2}{\mu^2}(n+1)^2\left[1 + \frac{2}{a} + \mathcal{O}\left(\frac{1}{a^2}\right)\right]$$

$$P^{deep}(\mathcal{N}, \phi = \Delta\phi) \simeq 4\frac{a^2}{\mu^2}e^{-2a}e^{-\frac{4a^2}{\mu^2}e^{-2a}\mathcal{N}}$$

"super-exponential" dependence on a

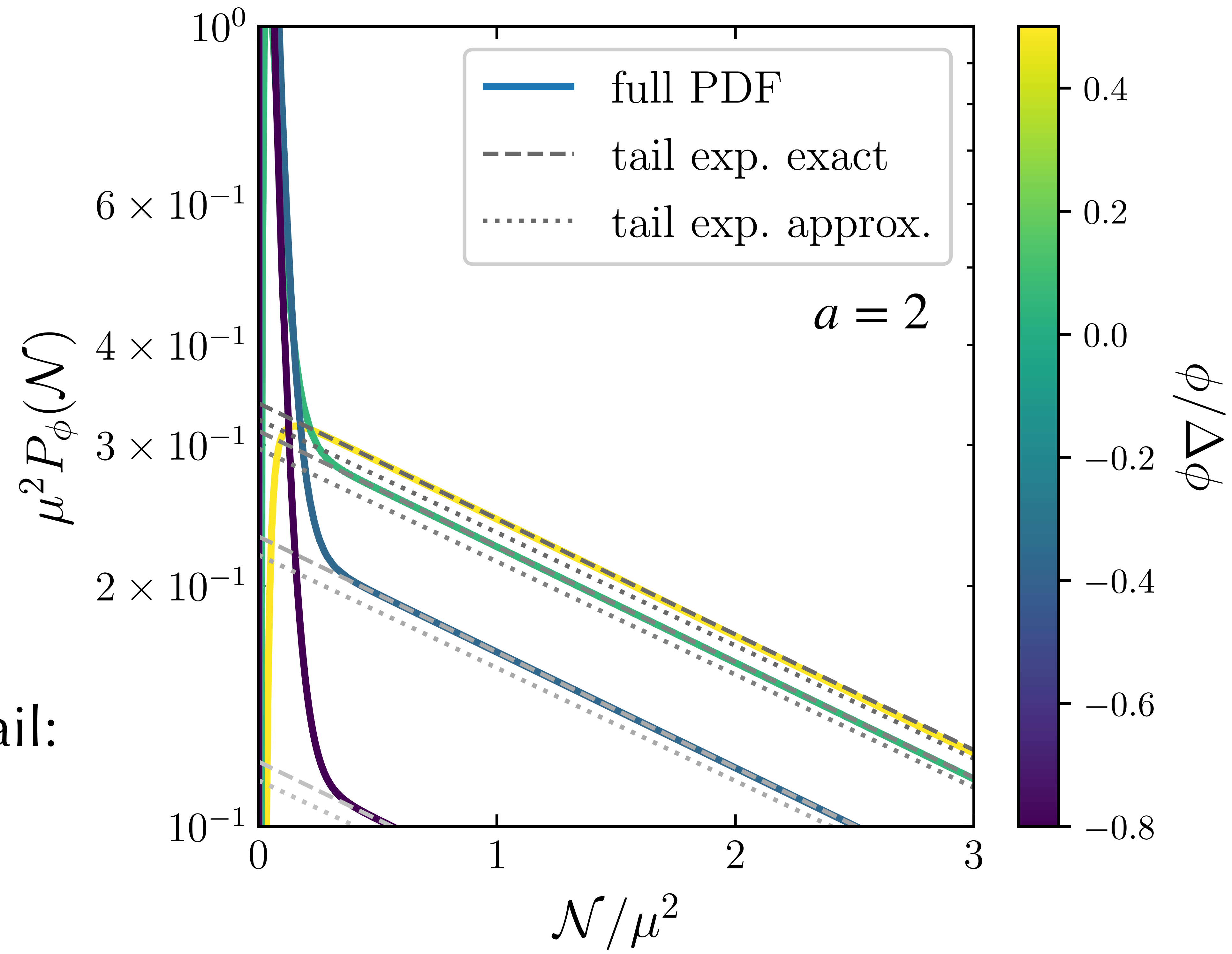
False vacuum: linear model

- shallow-well limit

$$\Lambda_n^{shallow} = \frac{1}{\mu^2}\left[\pi^2\left(n+\frac{1}{2}\right)^2 - 2a + \mathcal{O}(a^2)\right]$$

$$P^{shallow}(\mathcal{N}, \phi = \Delta\phi) \simeq \frac{\pi}{\mu^2}(1-a)\, e^{-\left(\frac{\pi^2}{4}-2a\right)\frac{\mathcal{N}}{\mu^2}}$$

$\sim e^{2a\mathcal{N}/\mu^2} \simeq e^{a\mathcal{N}/\langle\mathcal{N}\rangle}$ enhancement on the tail: large for $\mathcal{N} \sim 1/a$ σ-away from the mean



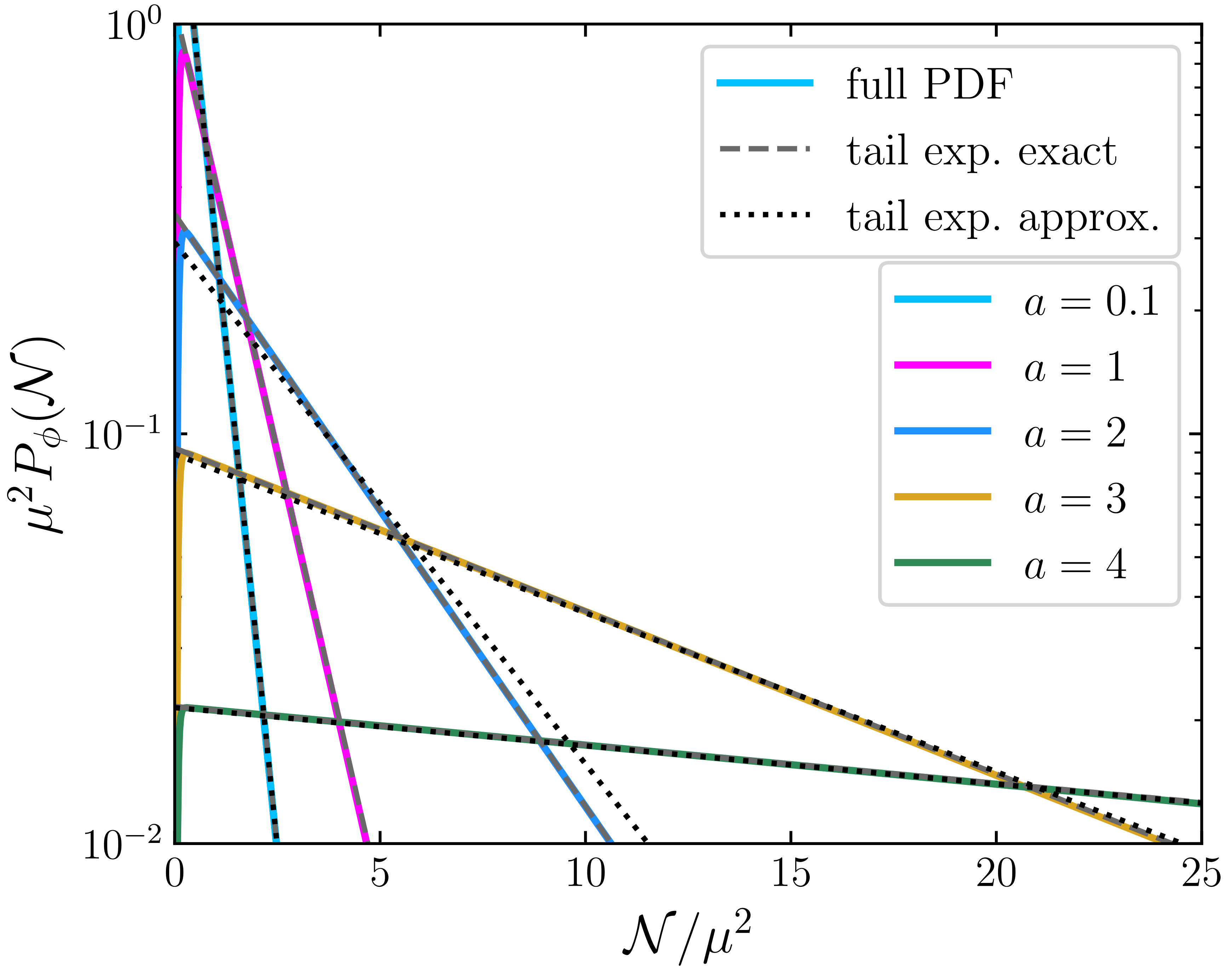
- deep-well limit

$$\Lambda_0^{deep} = \frac{4\,a^2 e^{-2a}}{\mu^2}\left[1 + 2\,(2a-1)e^{-2a} + \mathcal{O}(e^{-4a})\right]$$

$$\Lambda_{n+1}^{deep} = \frac{a^2}{\mu^2} + \frac{\pi^2}{\mu^2}(n+1)^2\left[1 + \frac{2}{a} + \mathcal{O}\left(\frac{1}{a^2}\right)\right]$$

$$P^{deep}(\mathcal{N}, \phi = \Delta\phi) \simeq 4\frac{a^2}{\mu^2}e^{-2a}e^{-\frac{4a^2}{\mu^2}e^{-2a}\mathcal{N}}$$

"super-exponential" dependence on a

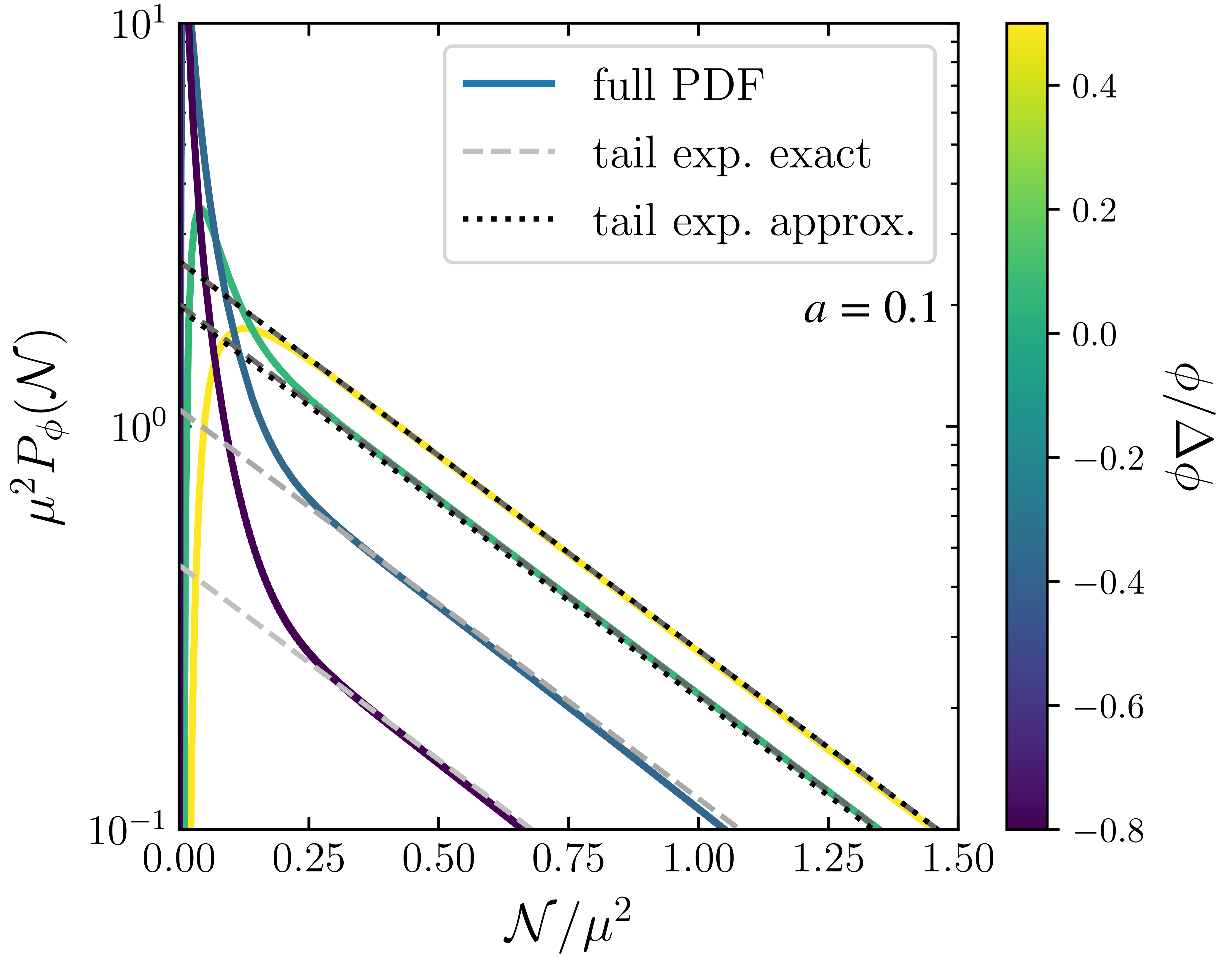


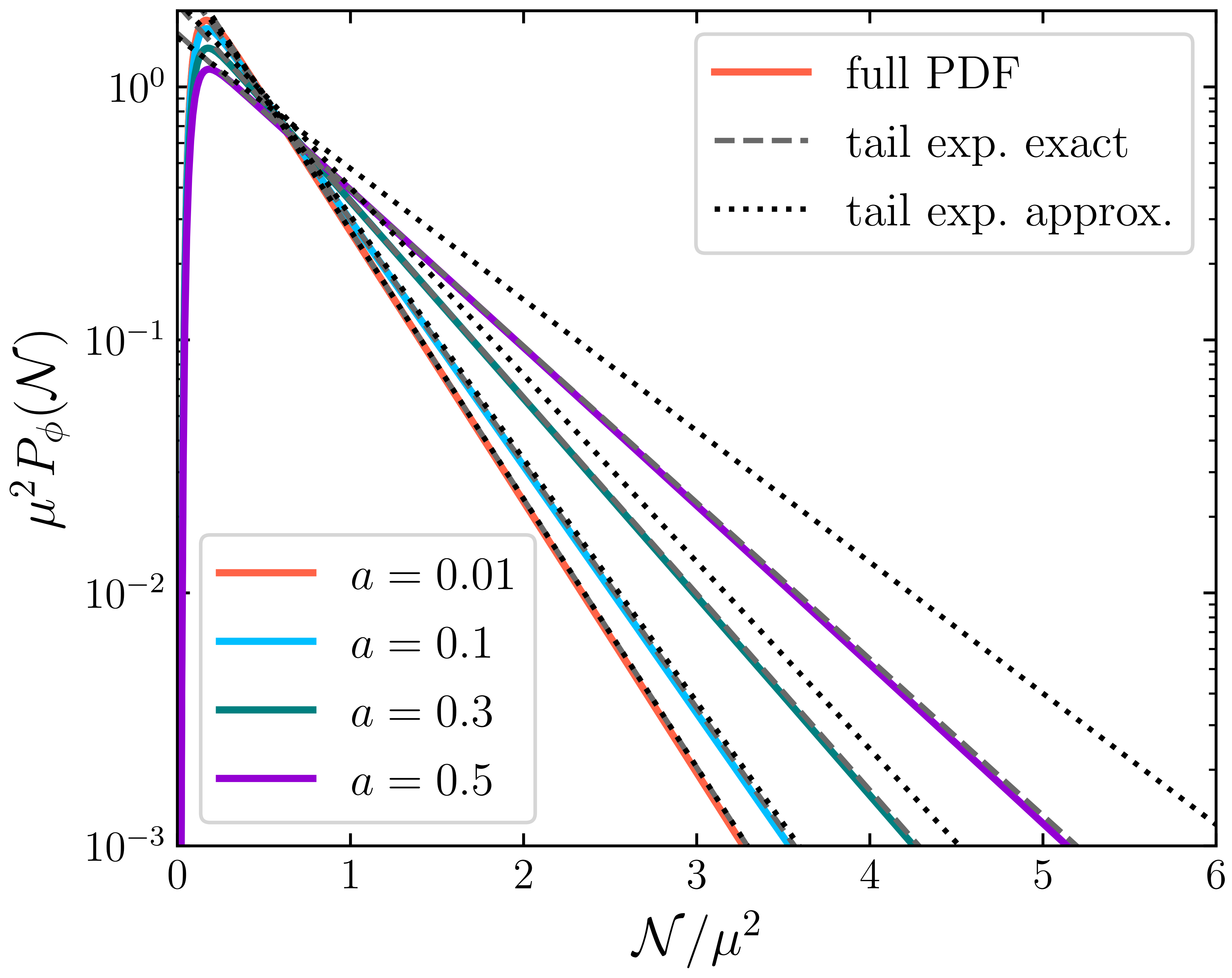
False vacuum: quadratic model

- shallow-well limit

$$\Lambda_n^{shallow} = \frac{\pi^2}{\mu^2}\left[\left(n+\frac{1}{2}\right)^2 + \frac{4\,a^2}{3\pi^2} - (-1)^n \frac{8a}{\pi^3\,(2\,n+1)}\right] + \mathcal{O}\left[(n+1/2)^{-2}\right]$$

$$P^{shallow}(\mathcal{N}, \phi = \Delta\phi) \simeq \frac{\pi}{\mu^2}(1-a)\,e^{-\left(\frac{\pi^2}{4}-\frac{8}{\pi}a\right)\frac{\mathcal{N}}{\mu^2}}$$





False vacuum: implications for Primordial Black Holes

- Typical abundance: Press-Schechter estimate

$$\beta \sim \int_{\zeta_c}^{\infty} P(\zeta)\, d\zeta \longrightarrow \beta \sim \int_{\langle \mathcal{N} \rangle + \zeta_c}^{\infty} P(\mathcal{N}, \phi = \Delta\phi)\, d\mathcal{N} \qquad \zeta_c \sim 1$$

False vacuum: implications for Primordial Black Holes

- Typical abundance: Press-Schechter estimate

$$\beta \sim \int_{\zeta_c}^{\infty} P(\zeta)\, d\zeta \longrightarrow \beta \sim \int_{\langle \mathcal{N} \rangle + \zeta_c}^{\infty} P(\mathcal{N}, \phi = \Delta\phi)\, d\mathcal{N} \qquad \zeta_c \sim 1 \longrightarrow$$

$$\beta = \sum_n \frac{a_n(\Delta\phi)}{\Lambda_n}\, e^{-\Lambda_n \left[\zeta_c + \langle \mathcal{N} \rangle(\Delta\phi) \right]}$$

$$\langle \mathcal{N} \rangle(\phi) = \sum_n \frac{a_n(\phi)}{\Lambda_n}$$

False vacuum: implications for Primordial Black Holes

- Typical abundance: Press-Schechter estimate

$$\beta \sim \int_{\zeta_c}^{\infty} P(\zeta)\, d\zeta \longrightarrow \beta \sim \int_{\langle \mathcal{N} \rangle + \zeta_c}^{\infty} P(\mathcal{N}, \phi = \Delta\phi)\, d\mathcal{N} \qquad \zeta_c \sim 1 \longrightarrow$$

$$\beta = \sum_n \frac{a_n(\Delta\phi)}{\Lambda_n} e^{-\Lambda_n [\zeta_c + \langle \mathcal{N} \rangle(\Delta\phi)]}$$

$$\langle \mathcal{N} \rangle(\phi) = \sum_n \frac{a_n(\phi)}{\Lambda_n}$$

$$\beta^{lin,shallow} \simeq \frac{4}{\pi}\left[1 + \left(\frac{8}{\pi^2} - \frac{\pi^2}{12}\right) a\right] e^{-\frac{\pi^2}{8} - \left(\frac{\pi^4}{4} - 2a\right)\frac{\zeta_c}{\mu^2}}$$

$$\beta^{quad,shallow} \simeq \frac{4}{\pi}\left[1 + \left(\frac{32}{\pi^3} + \frac{4}{\pi} - \frac{5\,\pi^2}{48} - 1\right) a\right] e^{-\frac{\pi^2}{8} - \left(\frac{\pi^4}{4} - \frac{8}{\pi}a\right)\frac{\zeta_c}{\mu^2}}$$

Similar abundances :

$$\beta \sim \beta(a=0)\, e^{A\, a \frac{\zeta_c}{\mu^2}} \qquad A^{lin} = 2 \qquad A^{quad} = \frac{8}{\pi}$$

exponential enhancement

False vacuum: implications for Primordial Black Holes

- Typical abundance: Press-Schechter estimate

$$\beta \sim \int_{\zeta_c}^{\infty} P(\zeta)\, d\zeta \longrightarrow \beta \sim \int_{\langle \mathcal{N} \rangle + \zeta_c}^{\infty} P(\mathcal{N}, \phi = \Delta\phi)\, d\mathcal{N} \qquad \zeta_c \sim 1 \longrightarrow$$

$$\beta = \sum_n \frac{a_n(\Delta\phi)}{\Lambda_n} e^{-\Lambda_n [\zeta_c + \langle \mathcal{N} \rangle(\Delta\phi)]}$$

$$\langle \mathcal{N} \rangle(\phi) = \sum_n \frac{a_n(\phi)}{\Lambda_n}$$

$$\beta^{lin,shallow} \simeq \frac{4}{\pi}\left[1 + \left(\frac{8}{\pi^2} - \frac{\pi^2}{12}\right) a\right] e^{-\frac{\pi^2}{8} - \left(\frac{\pi^4}{4} - 2a\right)\frac{\zeta_c}{\mu^2}}$$

$$\beta^{quad,shallow} \simeq \frac{4}{\pi}\left[1 + \left(\frac{32}{\pi^3} + \frac{4}{\pi} - \frac{5\,\pi^2}{48} - 1\right) a\right] e^{-\frac{\pi^2}{8} - \left(\frac{\pi^4}{4} - \frac{8}{\pi}a\right)\frac{\zeta_c}{\mu^2}}$$

Similar abundances :

$$\beta \sim \beta(a = 0)\, e^{A\, a \frac{\zeta_c}{\mu^2}} \qquad A^{lin} = 2 \qquad A^{quad} = \frac{8}{\pi}$$

exponential enhancement

What the slow-roll assumption implies?

quadratic model: $\mu \gg \sqrt{a}$ ⟶ exponential factor negligible ⟶ flat-well limit applies where slow roll satisfied

linear model: $\mu \gg a\sqrt{v_0}$ ⟶ exponential factor large even at small a values

False vacuum: implications for Primordial Black Holes

- Typical abundance: Press-Schechter estimate

$$\beta \sim \int_{\zeta_c}^{\infty} P(\zeta)\, d\zeta \longrightarrow \beta \sim \int_{\langle \mathcal{N} \rangle + \zeta_c}^{\infty} P(\mathcal{N}, \phi = \Delta\phi)\, d\mathcal{N} \qquad \zeta_c \sim 1 \longrightarrow$$

$$\beta = \sum_n \frac{a_n(\Delta\phi)}{\Lambda_n} e^{-\Lambda_n [\zeta_c + \langle \mathcal{N} \rangle(\Delta\phi)]}$$

$$\langle \mathcal{N} \rangle(\phi) = \sum_n \frac{a_n(\phi)}{\Lambda_n}$$

$$\beta^{lin,shallow} \simeq \frac{4}{\pi}\left[1 + \left(\frac{8}{\pi^2} - \frac{\pi^2}{12}\right) a\right] e^{-\frac{\pi^2}{8} - \left(\frac{\pi^4}{4} - 2a\right)\frac{\zeta_c}{\mu^2}}$$

$$\beta^{quad,shallow} \simeq \frac{4}{\pi}\left[1 + \left(\frac{32}{\pi^3} + \frac{4}{\pi} - \frac{5\,\pi^2}{48} - 1\right) a\right] e^{-\frac{\pi^2}{8} - \left(\frac{\pi^4}{4} - \frac{8}{\pi}a\right)\frac{\zeta_c}{\mu^2}}$$

Similar abundances :

$$\beta \sim \beta(a=0)\, e^{A\, a \frac{\zeta_c}{\mu^2}} \qquad A^{lin} = 2 \qquad A^{quad} = \frac{8}{\pi}$$

exponential enhancement

What the slow-roll assumption implies?

quadratic model: $\mu \gg \sqrt{a}$ ⟶ exponential factor negligible ⟶ flat-well limit applies where slow roll satisfied

linear model: $\mu \gg a\sqrt{v_0}$ ⟶ exponential factor large even at small a values

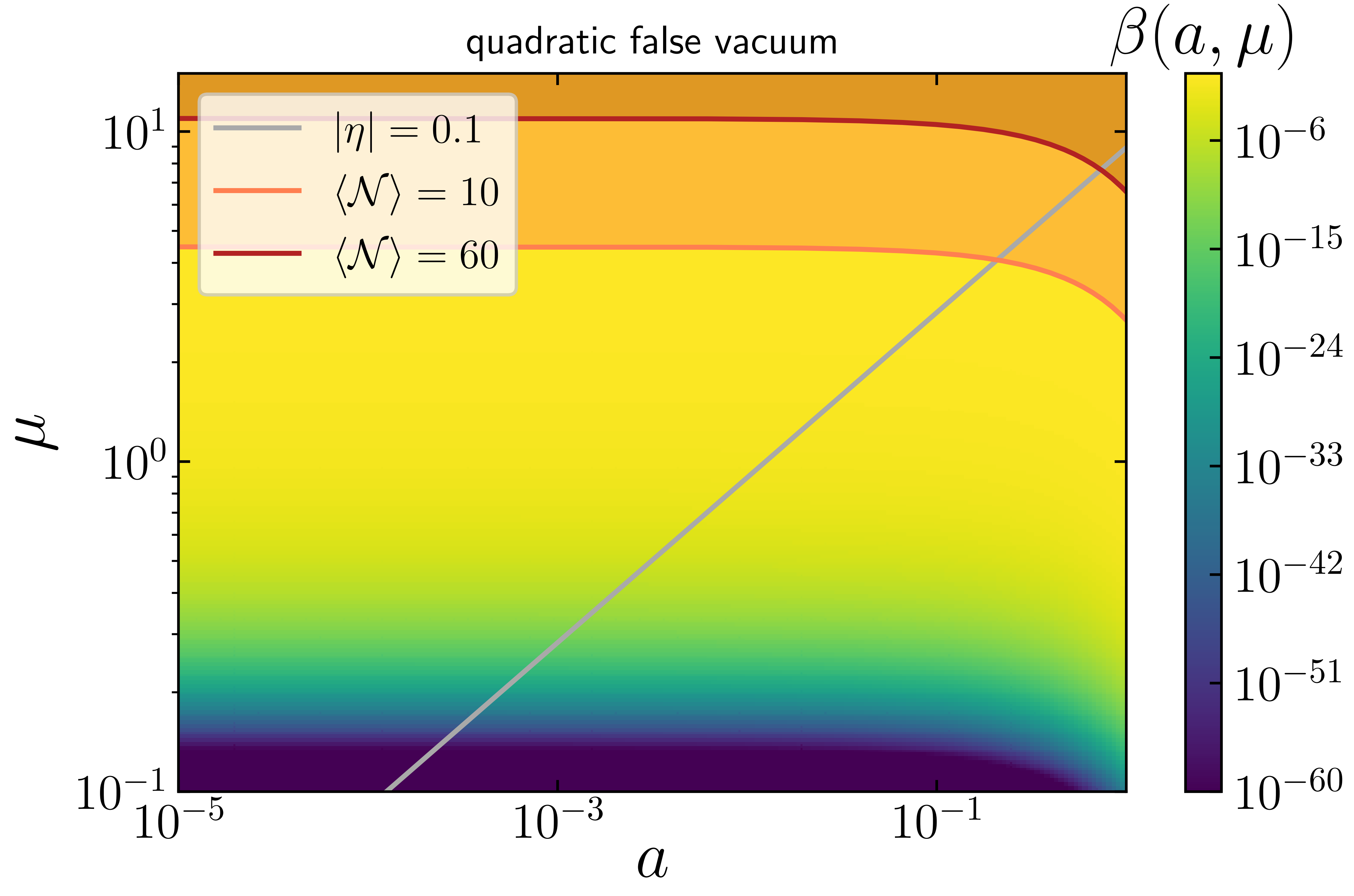
$$\beta^{lin,deep} \simeq e^{-1} e^{-(2\,a\,e^{-a})^2 \frac{\zeta_c}{\mu^2}}$$

⟶ super-exponential dependence on a

PBHs are overproduced when $a \gtrsim 8$

False vacuum: implications for Primordial Black Holes

Quadratic model

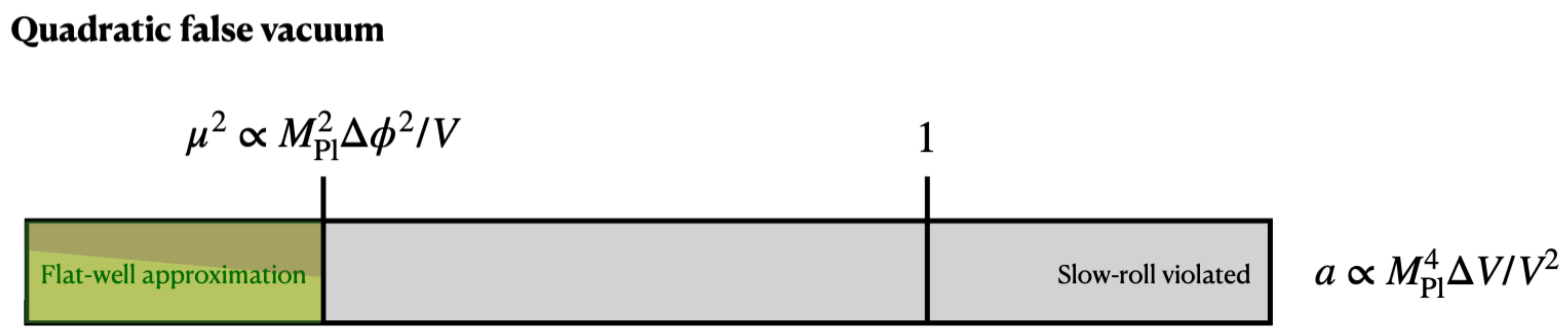


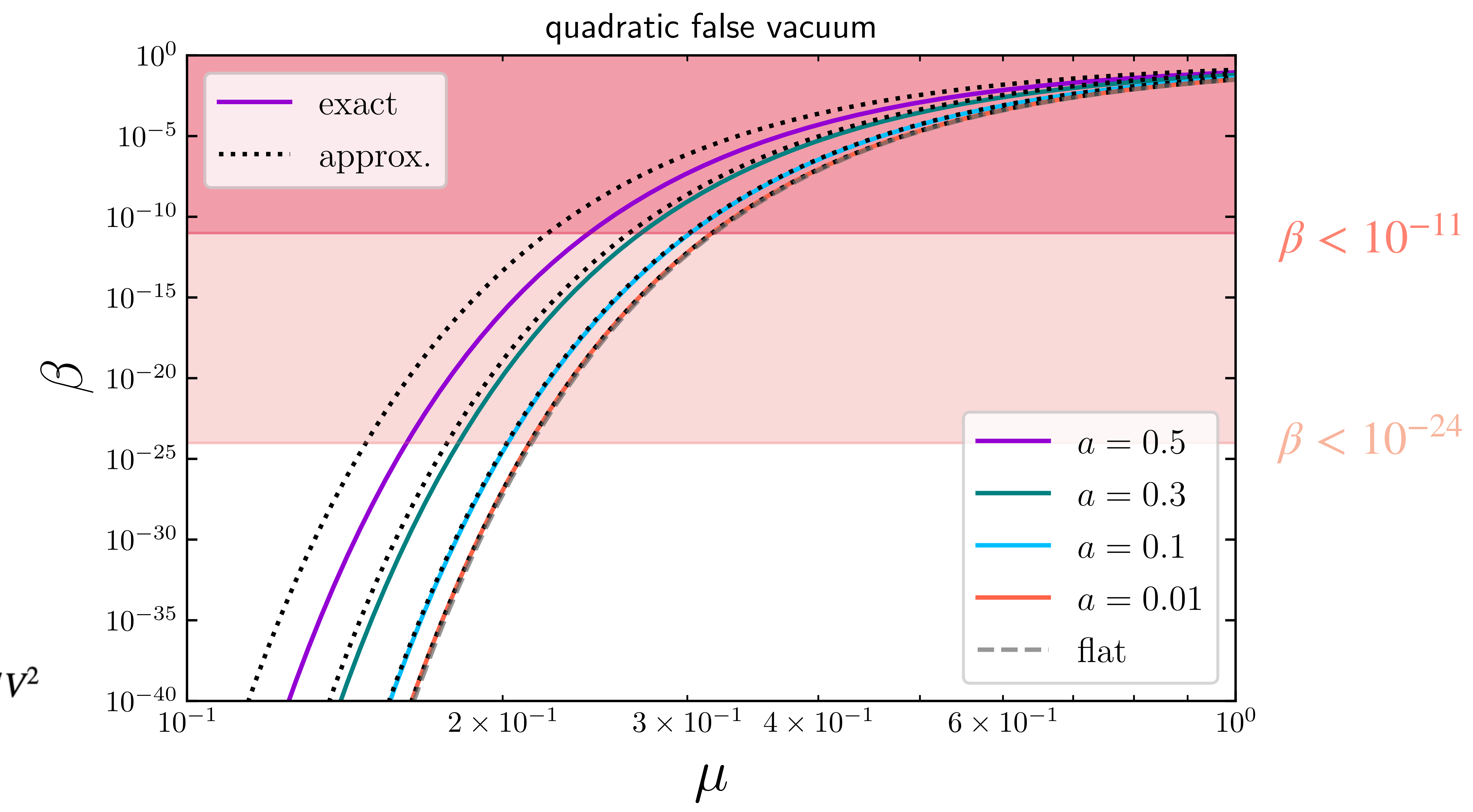
PBH abundance well captured by a flat-well limit ($a = 0$)

If $\mu \ll 1$, tiny amount of PBHs produced

If $\mu \lesssim 1$, PBHs produced with sizable abundance

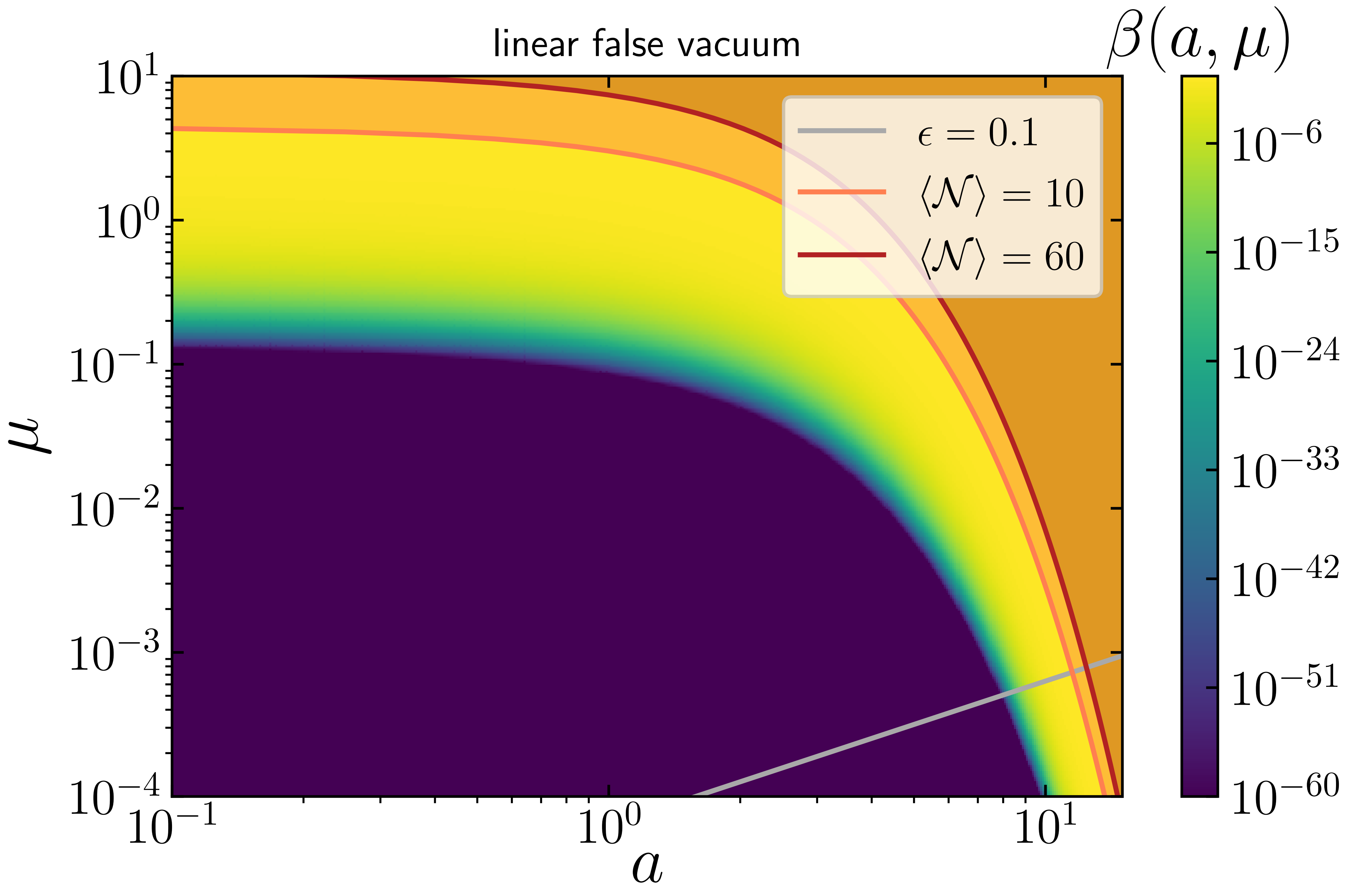
If $\mu \gtrsim 1$, PBHs overproduced

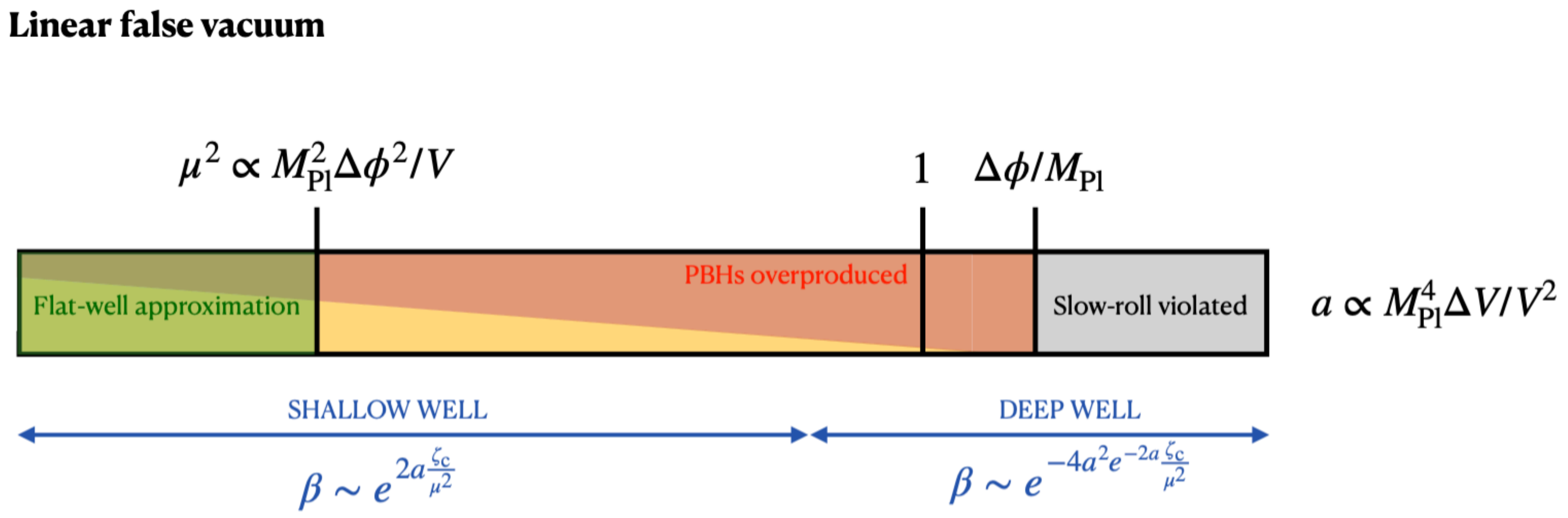




False vacuum: implications for Primordial Black Holes

Linear model

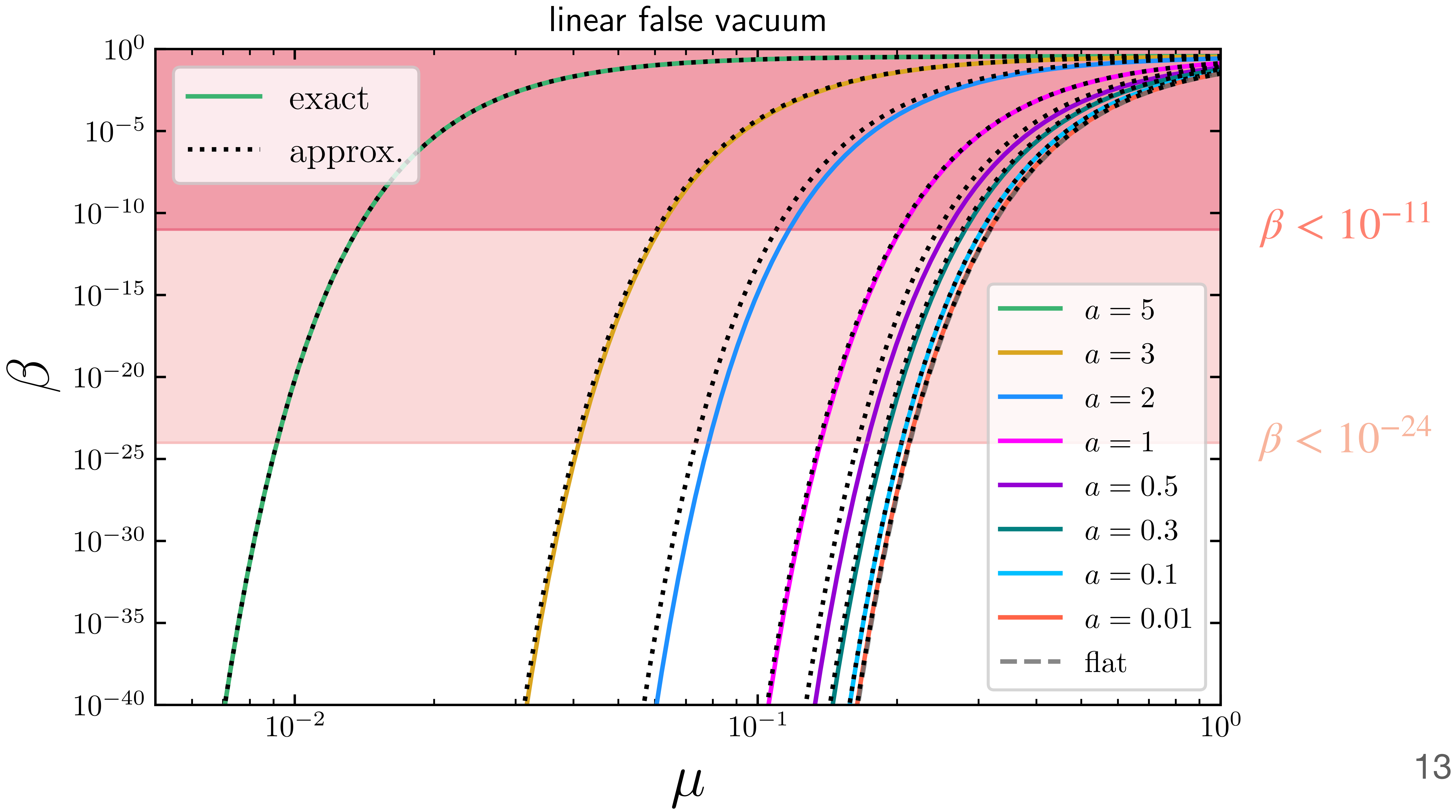




Additional regimes:

If $\mu^2 \ll a \ll 1$ (μ small):
large deviations from flat-well, still shallow-well domain;
non-trivial imprint of the false-vacuum profile

If $a \sim \mathcal{O}(1)$: large PBH production



Coarse graining in stochastic inflation

C.A., V. Vennin
In preparation

Coarse graining in stochastic inflation

C.A., V. Vennin
In preparation

- As it stands, the stochastic formalism delivers the one-point statistics of the curvature perturbation when coarse-grained at the Hubble scale at the end of inflation $R = (\sigma H_{\rm end})^{-1}$

Coarse graining in stochastic inflation

C.A., V. Vennin
In preparation

- As it stands, the stochastic formalism delivers the one-point statistics of the curvature perturbation when coarse-grained at the Hubble scale at the end of inflation $R = (\sigma H_{\mathrm{end}})^{-1}$

- Studying the formation of a given structure of mass M usually requires to coarse grain the perturbation field over a scale determined by M (which roughly corresponds to the Hubble scale at the time when the Hubble mass equals M).

Y.Tada, V. Vennin
JCAP02(2022)021

Coarse graining in stochastic inflation

C.A., V. Vennin
In preparation

- As it stands, the stochastic formalism delivers the one-point statistics of the curvature perturbation when coarse-grained at the Hubble scale at the end of inflation $R = (\sigma H_{\mathrm{end}})^{-1}$

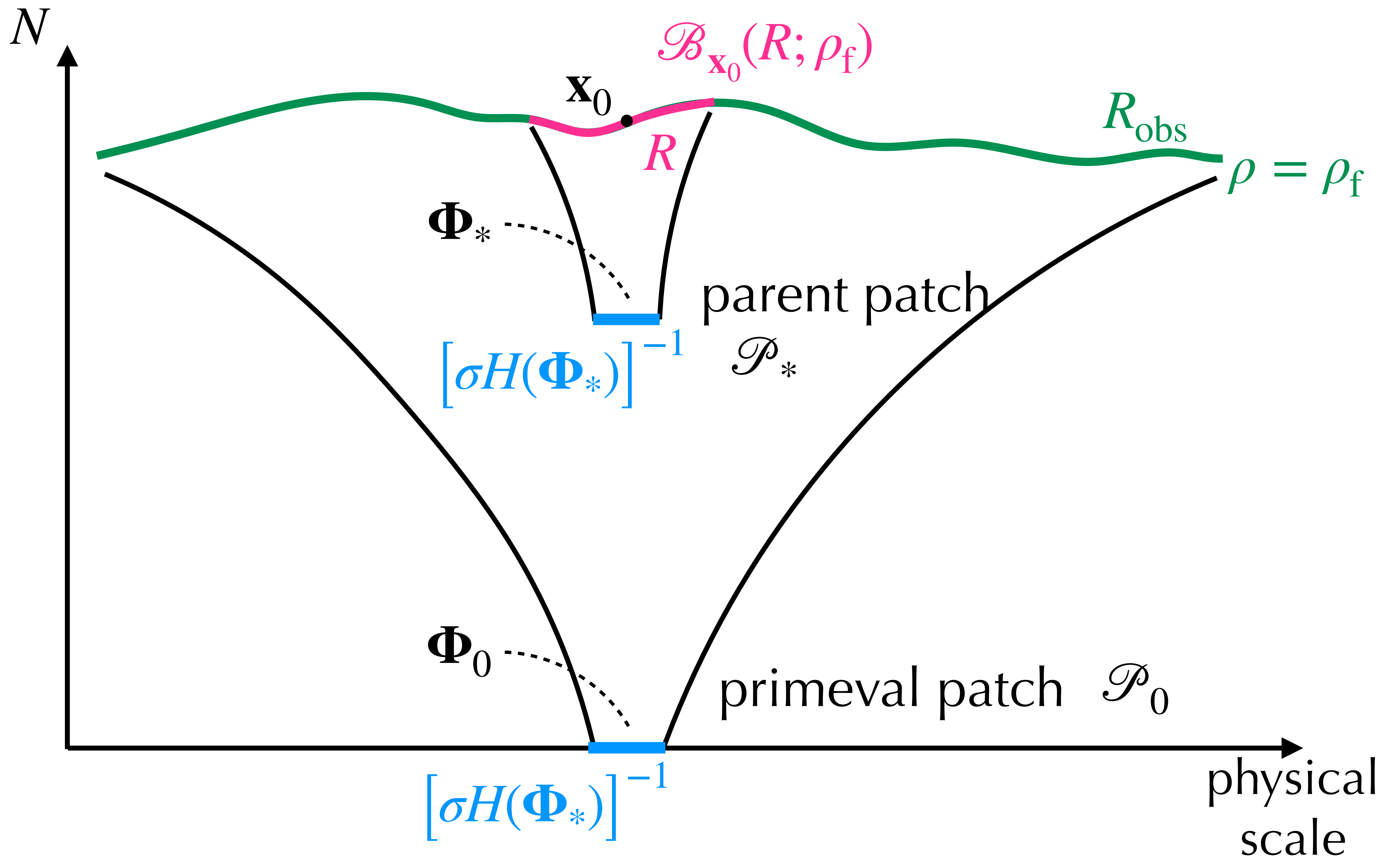
- Studying the formation of a given structure of mass M usually requires to coarse grain the perturbation field over a scale determined by M (which roughly corresponds to the Hubble scale at the time when the Hubble mass equals M).

Y.Tada, V. Vennin
JCAP02(2022)021

$$\mathscr{B}_{\mathbf{x}_0}(R;\rho_{\mathrm{f}}) = \{\mathbf{x}\,|\,r_{\mathrm{ph}}(\mathbf{x},\mathbf{x}_0;\rho_{\mathrm{f}}) \le R\}$$

$$\zeta_R(\mathbf{x}_0) = \frac{1}{V[\mathscr{B}_{\mathbf{x}_0}(R;\rho_{\mathrm{f}})]}\int_{\rho=\rho_{\mathrm{f}}} \mathrm{d}\mathbf{x}\,\zeta(\mathbf{x})\,W\left[\frac{r_{\mathrm{ph}}(\mathbf{x},\mathbf{x}_0\,;\rho_{\mathrm{f}})}{R}\right]$$

$$\frac{4}{3}\pi R^3 = \frac{1}{\sigma^3 H^3(\mathbf{\Phi}_*)}\int_{\mathscr{B}} e^{3\mathcal{N}(\mathbf{x})}\,\mathrm{d}\mathbf{x}$$



Coarse graining in stochastic inflation

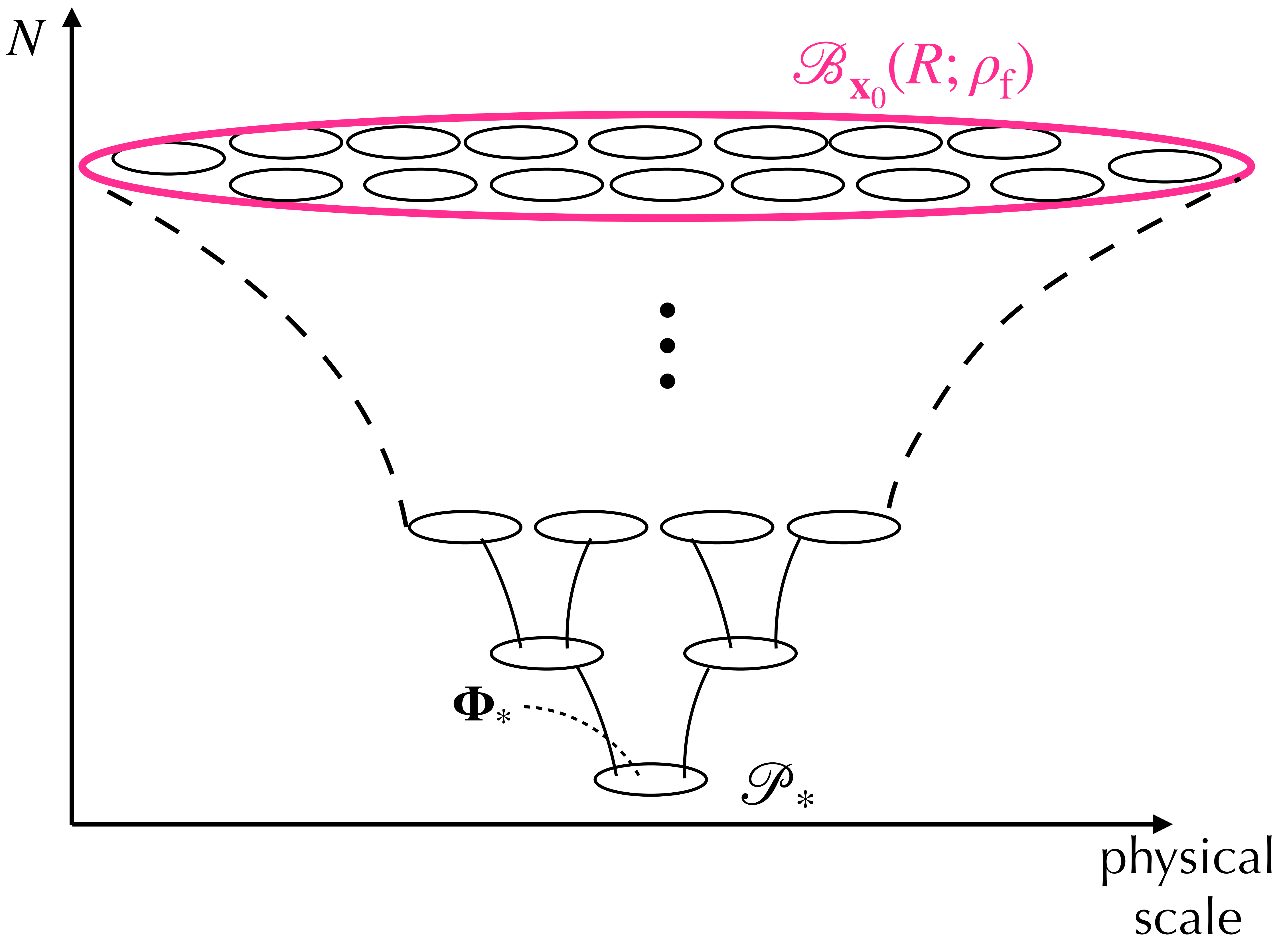
Coarse graining in stochastic inflation

$$\frac{4}{3}\pi R^3 = \frac{1}{\sigma^3 H^3(\mathbf{\Phi}_*)}\int_{\mathscr{B}} e^{3\mathcal{N}(\mathbf{x})}\mathrm{d}\mathbf{x}$$

- Approximating the emerging volume: $R \gg (\sigma H_{\mathrm{end}})^{-1}$

$$e^{3\mathcal{N}(\mathbf{x})} \to \langle e^{3\mathcal{N}}\rangle(\mathbf{\Phi}_*) = \int P_{\mathrm{FPT},\mathbf{\Phi}_*}(\mathcal{N})e^{3\mathcal{N}}\mathrm{d}\mathcal{N}$$

$$R^3 = \frac{1}{\sigma^3 H^3(\mathbf{\Phi}_*)}\langle e^{3\mathcal{N}}\rangle(\mathbf{\Phi}_*)$$



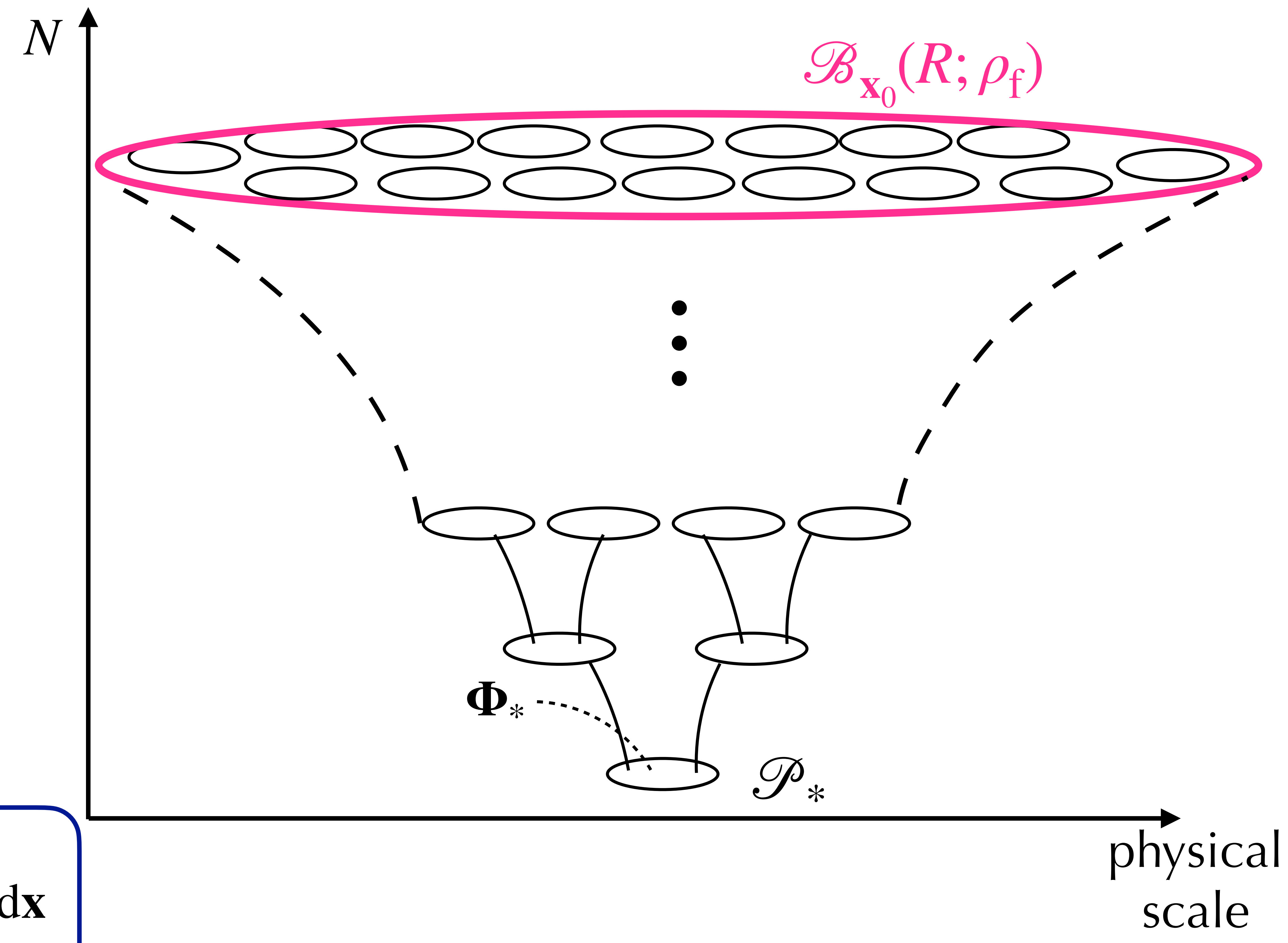
Coarse graining in stochastic inflation

$$\frac{4}{3}\pi R^3 = \frac{1}{\sigma^3 H^3(\mathbf{\Phi}_*)}\int_{\mathscr{B}} e^{3\mathcal{N}(\mathbf{x})}\mathrm{d}\mathbf{x}$$

- Approximating the emerging volume: $R \gg (\sigma H_{\mathrm{end}})^{-1}$

$$e^{3\mathcal{N}(\mathbf{x})} \to \langle e^{3\mathcal{N}}\rangle(\mathbf{\Phi}_*) = \int P_{\mathrm{FPT},\mathbf{\Phi}_*}(\mathcal{N})e^{3\mathcal{N}}\mathrm{d}\mathcal{N}$$

$$R^3 = \frac{1}{\sigma^3 H^3(\mathbf{\Phi}_*)}\langle e^{3\mathcal{N}}\rangle(\mathbf{\Phi}_*)$$



$$\zeta_R = \int_{\mathscr{B}}\left[\mathcal{N}_{\mathbf{\Phi}_0} - \langle\mathcal{N}\rangle(\mathbf{\Phi}_0)\right]\mathrm{d}\mathbf{x} = \int_{\mathscr{B}}\left[\mathcal{N}_{\mathbf{\Phi}_0\to\mathbf{\Phi}_*}(\mathbf{x}_0) + \mathcal{N}_{\mathbf{\Phi}_*}(\mathbf{x}) - \langle\mathcal{N}\rangle(\mathbf{\Phi}_0)\right]\mathrm{d}\mathbf{x}$$

$$= \mathcal{N}_{\mathbf{\Phi}_0\to\mathbf{\Phi}_*} - \langle\mathcal{N}\rangle(\mathbf{\Phi}_0) + \int_{\mathscr{B}}\mathcal{N}_{\mathbf{\Phi}_*}(\mathbf{x})\,\mathrm{d}\mathbf{x} = \mathcal{N}_{\mathbf{\Phi}_0\to\mathbf{\Phi}_*} - \langle\mathcal{N}\rangle(\mathbf{\Phi}_0) + \langle\mathcal{N}\rangle(\mathbf{\Phi}_*)$$

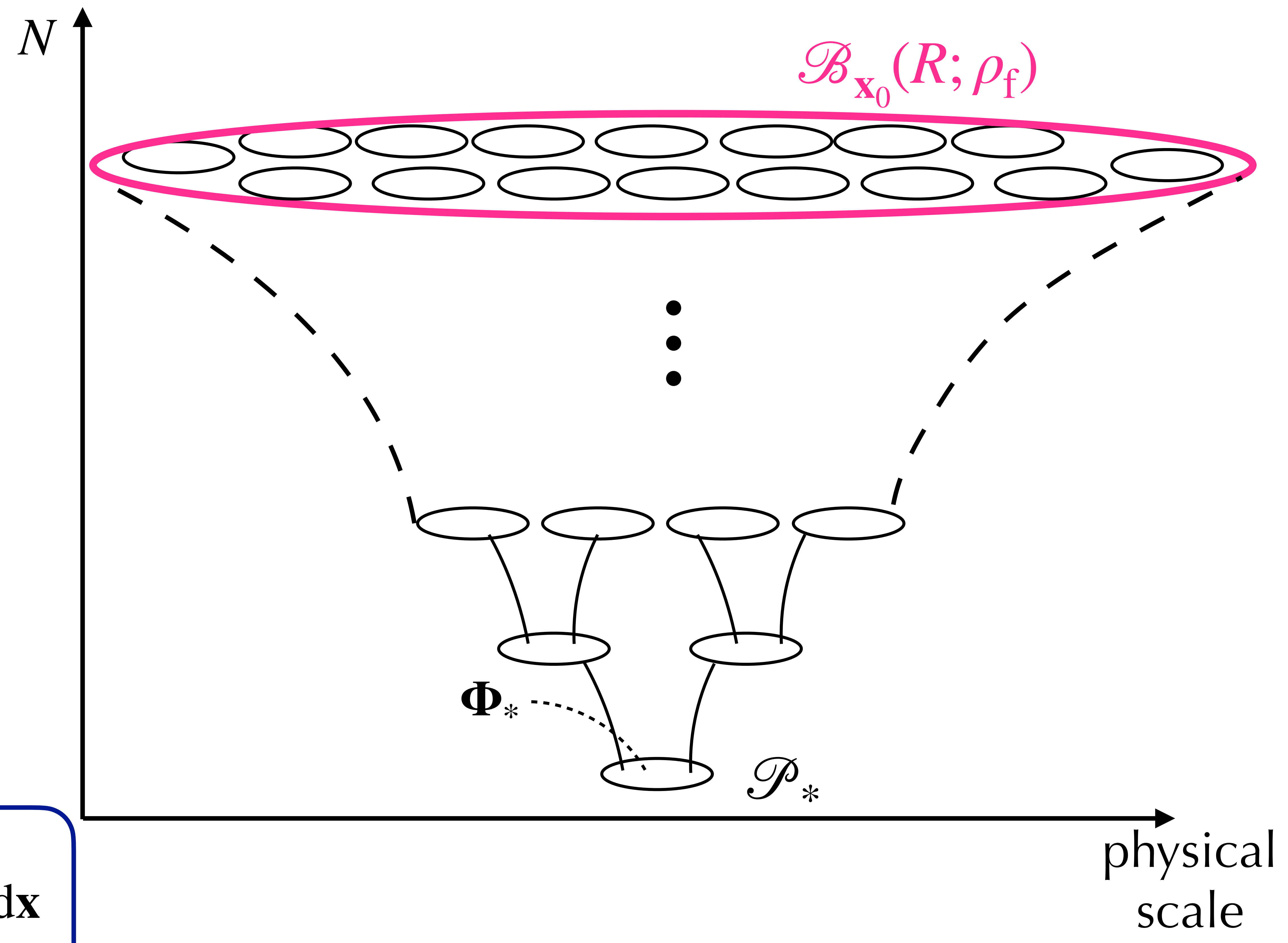
Coarse graining in stochastic inflation

$$\frac{4}{3}\pi R^3 = \frac{1}{\sigma^3 H^3(\mathbf{\Phi}_*)}\int_{\mathscr{B}} e^{3\mathcal{N}(\mathbf{x})}\mathrm{d}\mathbf{x}$$

- Approximating the emerging volume: $R \gg (\sigma H_{\mathrm{end}})^{-1}$

$$e^{3\mathcal{N}(\mathbf{x})} \to \langle e^{3\mathcal{N}}\rangle(\mathbf{\Phi}_*) = \int P_{\mathrm{FPT},\mathbf{\Phi}_*}(\mathcal{N})e^{3\mathcal{N}}\mathrm{d}\mathcal{N}$$

$$R^3 = \frac{1}{\sigma^3 H^3(\mathbf{\Phi}_*)}\langle e^{3\mathcal{N}}\rangle(\mathbf{\Phi}_*)$$



$$\zeta_R = \int_{\mathscr{B}}\left[\mathcal{N}_{\mathbf{\Phi}_0} - \langle\mathcal{N}\rangle(\mathbf{\Phi}_0)\right]\mathrm{d}\mathbf{x} = \int_{\mathscr{B}}\left[\mathcal{N}_{\mathbf{\Phi}_0\to\mathbf{\Phi}_*}(\mathbf{x}_0) + \mathcal{N}_{\mathbf{\Phi}_*}(\mathbf{x}) - \langle\mathcal{N}\rangle(\mathbf{\Phi}_0)\right]\mathrm{d}\mathbf{x}$$

$$= \mathcal{N}_{\mathbf{\Phi}_0\to\mathbf{\Phi}_*} - \langle\mathcal{N}\rangle(\mathbf{\Phi}_0) + \int_{\mathscr{B}}\mathcal{N}_{\mathbf{\Phi}_*}(\mathbf{x})\,\mathrm{d}\mathbf{x} = \mathcal{N}_{\mathbf{\Phi}_0\to\mathbf{\Phi}_*} - \langle\mathcal{N}\rangle(\mathbf{\Phi}_0) + \langle\mathcal{N}\rangle(\mathbf{\Phi}_*)$$

$$P(\zeta_R) = \int \mathrm{d}\mathbf{\Phi}_* P(\mathbf{\Phi}_*)P_{\mathrm{FPT},\mathbf{\Phi}_0\to\mathbf{\Phi}_*}\left[\zeta_R + \langle\mathcal{N}\rangle(\mathbf{\Phi}_0) - \langle\mathcal{N}\rangle(\mathbf{\Phi}_*)\right]$$

Spatial correlation in stochastic inflation

C.A., V. Vennin
In preparation

Spatial correlation in stochastic inflation

C.A., V. Vennin
In preparation

- Information on the relative distance of patches encoded in the time they become statistically independent

Spatial correlation in stochastic inflation

C.A., V. Vennin
In preparation

- Information on the relative distance of patches encoded in the time they become statistically independent

- Stochastic inflation formalism also allows to describe the spatial correlation between durations of inflation at different points

K.Ando, V. Vennin
JCAP 04 (2021) 057

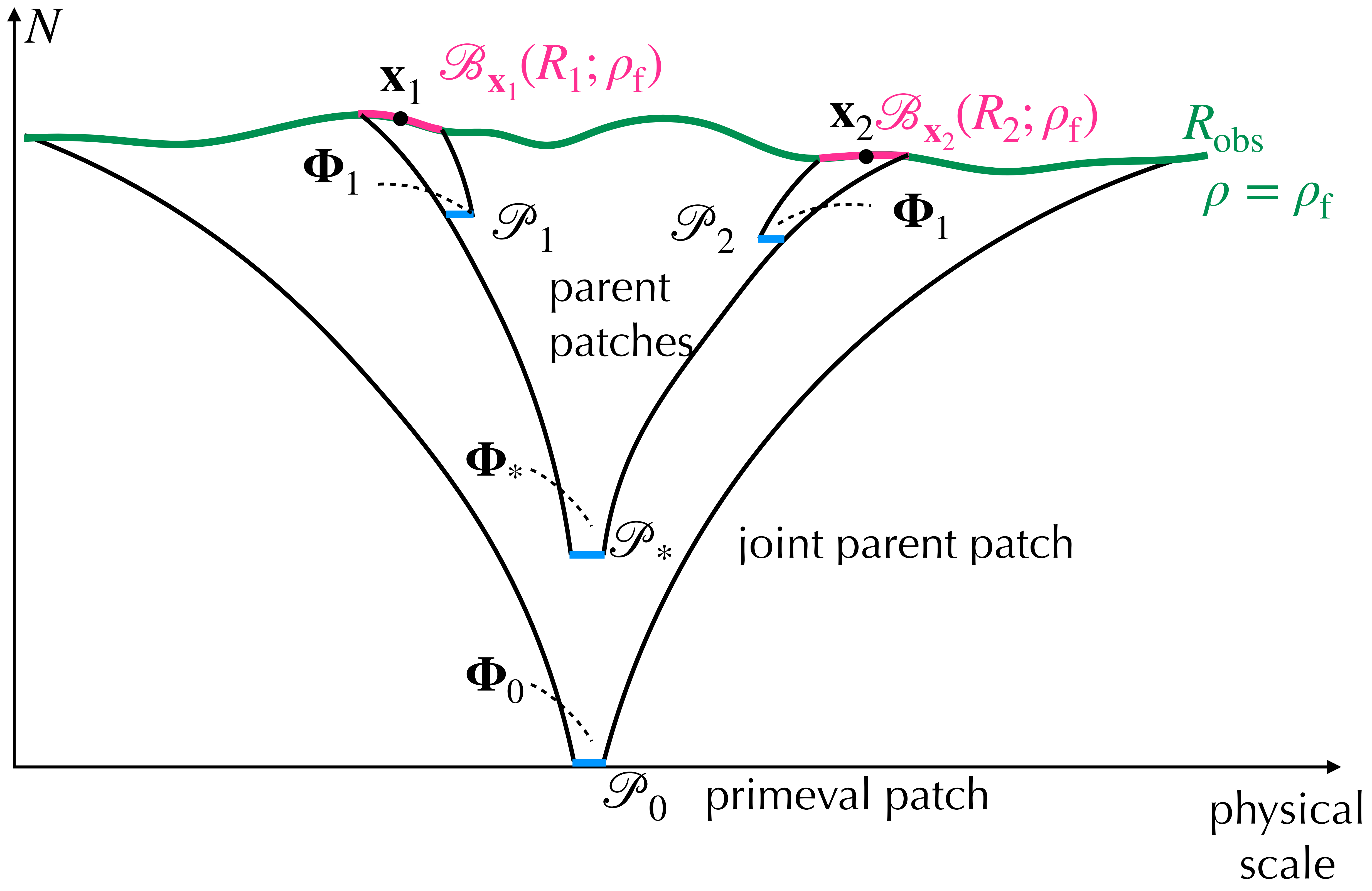
Spatial correlation in stochastic inflation

C.A., V. Vennin
In preparation

- Information on the relative distance of patches encoded in the time they become statistically independent

- Stochastic inflation formalism also allows to describe the spatial correlation between durations of inflation at different points

K.Ando, V. Vennin
JCAP 04 (2021) 057



Spatial correlation in stochastic inflation

C.A., V. Vennin
In preparation

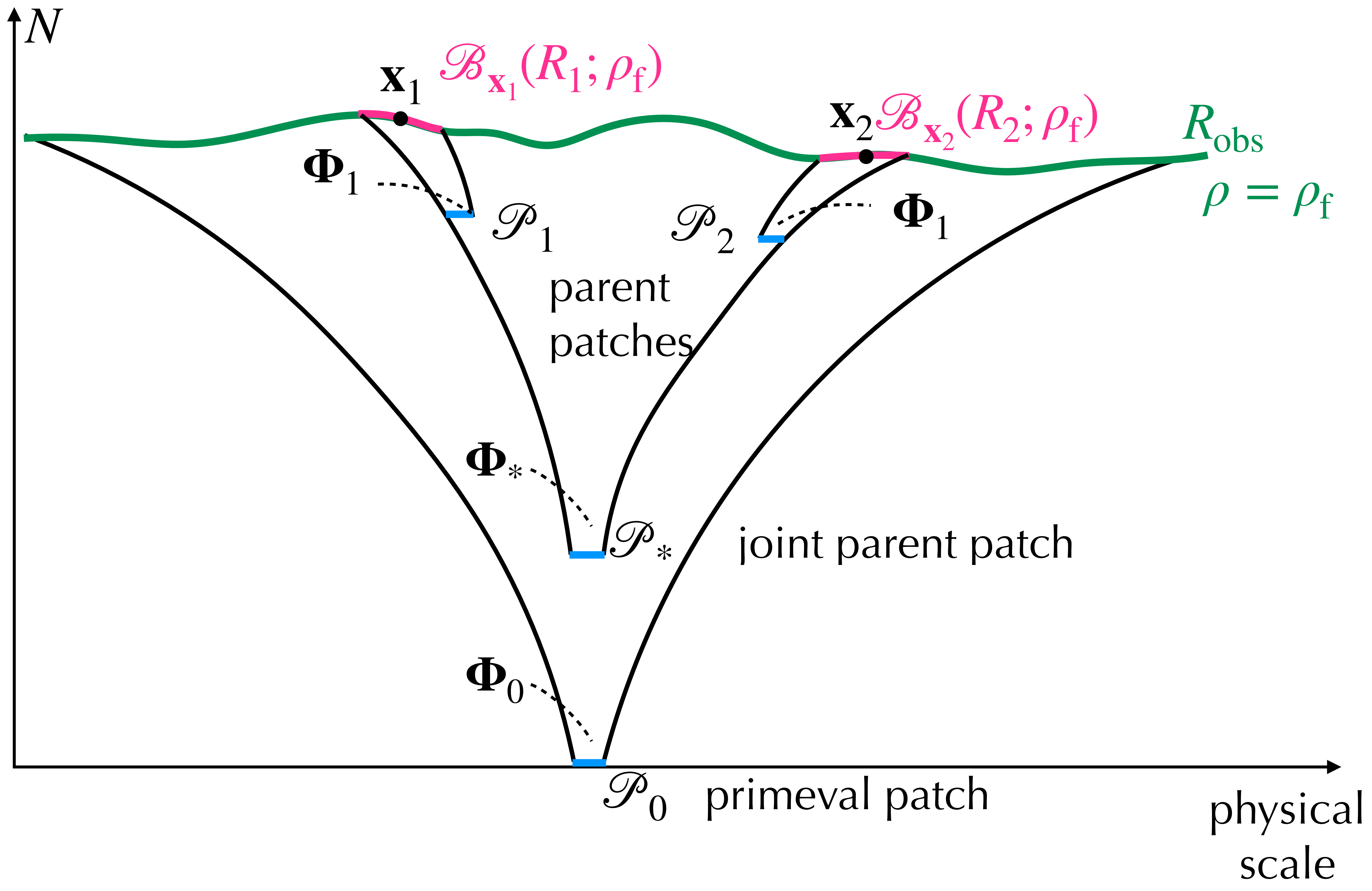
- Information on the relative distance of patches encoded in the time they become statistically independent

- Stochastic inflation formalism also allows to describe the spatial correlation between durations of inflation at different points

K.Ando, V. Vennin
JCAP 04 (2021) 057

$$\zeta_{R_1} = \mathcal{N}_{\mathbf{\Phi}_0\to\mathbf{\Phi}_*} + \mathcal{N}_{\mathbf{\Phi}_*\to\mathbf{\Phi}_1} - \langle\mathcal{N}\rangle(\mathbf{\Phi}_0) + \langle\mathcal{N}\rangle(\mathbf{\Phi}_1)$$

$$\zeta_{R_2} = \mathcal{N}_{\mathbf{\Phi}_0\to\mathbf{\Phi}_*} + \mathcal{N}_{\mathbf{\Phi}_*\to\mathbf{\Phi}_2} - \langle\mathcal{N}\rangle(\mathbf{\Phi}_0) + \langle\mathcal{N}\rangle(\mathbf{\Phi}_2)$$



Spatial correlation in stochastic inflation

C.A., V. Vennin
In preparation

- Information on the relative distance of patches encoded in the time they become statistically independent

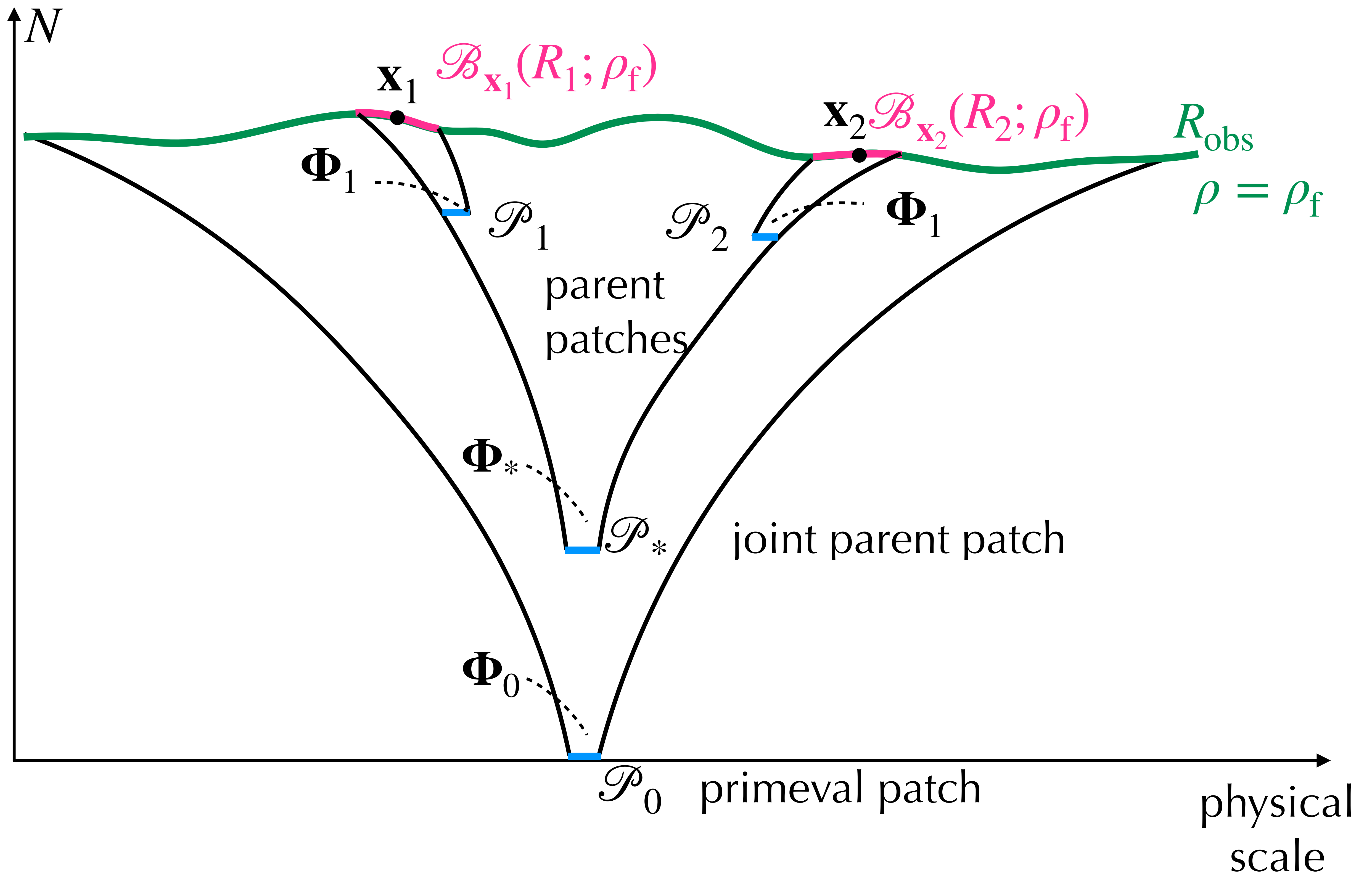
- Stochastic inflation formalism also allows to describe the spatial correlation between durations of inflation at different points

K.Ando, V. Vennin
JCAP 04 (2021) 057

$$\zeta_{R_1} = \mathcal{N}_{\mathbf{\Phi}_0 \to \mathbf{\Phi}_*} + \mathcal{N}_{\mathbf{\Phi}_* \to \mathbf{\Phi}_1} - \langle \mathcal{N} \rangle(\mathbf{\Phi}_0) + \langle \mathcal{N} \rangle(\mathbf{\Phi}_1)$$

$$\zeta_{R_2} = \mathcal{N}_{\mathbf{\Phi}_0 \to \mathbf{\Phi}_*} + \mathcal{N}_{\mathbf{\Phi}_* \to \mathbf{\Phi}_2} - \langle \mathcal{N} \rangle(\mathbf{\Phi}_0) + \langle \mathcal{N} \rangle(\mathbf{\Phi}_2)$$

$$\begin{aligned} P(\zeta_{R_1}, \zeta_{R_2}) = \int & \mathrm{d}\mathbf{\Phi}_* \, \mathrm{d}\mathbf{\Phi}_1 \, \mathrm{d}\mathbf{\Phi}_2 \, \mathrm{d}\mathcal{N}_{\mathbf{\Phi}_0 \to \mathbf{\Phi}_*} P(\mathbf{\Phi}_*) P(\mathbf{\Phi}_1 \mid \mathbf{\Phi}_*) P(\mathbf{\Phi}_2 \mid \mathbf{\Phi}_*) \\ & \times P_{\mathrm{FPT}, \mathbf{\Phi}_0 \to \mathbf{\Phi}_*}(\mathcal{N}_{\mathbf{\Phi}_0 \to \mathbf{\Phi}_*}) \\ & \times P_{\mathrm{FPT}, \mathbf{\Phi}_* \to \mathbf{\Phi}_1} \left[\zeta_{R_1} - \mathcal{N}_{\mathbf{\Phi}_0 \to \mathbf{\Phi}_*} + \langle \mathcal{N} \rangle(\mathbf{\Phi}_0) - \langle \mathcal{N} \rangle(\mathbf{\Phi}_1) \right] \\ & \times P_{\mathrm{FPT}, \mathbf{\Phi}_* \to \mathbf{\Phi}_2} \left[\zeta_{R_2} - \mathcal{N}_{\mathbf{\Phi}_0 \to \mathbf{\Phi}_*} + \langle \mathcal{N} \rangle(\mathbf{\Phi}_0) - \langle \mathcal{N} \rangle(\mathbf{\Phi}_2) \right] \end{aligned}$$



Single clock models

Single clock models

- Values of fields at the parent patches are deterministic quantities: $P(\mathbf{\Phi}_*), P(\mathbf{\Phi}_1 | \mathbf{\Phi}_*), P(\mathbf{\Phi}_2 | \mathbf{\Phi}_*)$ are Dirac distributions

Single clock models

- Values of fields at the parent patches are deterministic quantities: $P(\boldsymbol{\Phi}_*)$, $P(\boldsymbol{\Phi}_1 | \boldsymbol{\Phi}_*)$, $P(\boldsymbol{\Phi}_2 | \boldsymbol{\Phi}_*)$ are Dirac distributions

- Scale-field values relation: $R^3 = \frac{1}{\sigma^3 H^3(\boldsymbol{\Phi}_*)} \langle e^{3\mathcal{N}} \rangle (\boldsymbol{\Phi}_*)$

Single clock models

- Values of fields at the parent patches are deterministic quantities: $P(\boldsymbol{\Phi}_*), P(\boldsymbol{\Phi}_1 \,|\, \boldsymbol{\Phi}_*), P(\boldsymbol{\Phi}_2 \,|\, \boldsymbol{\Phi}_*)$ are Dirac distributions

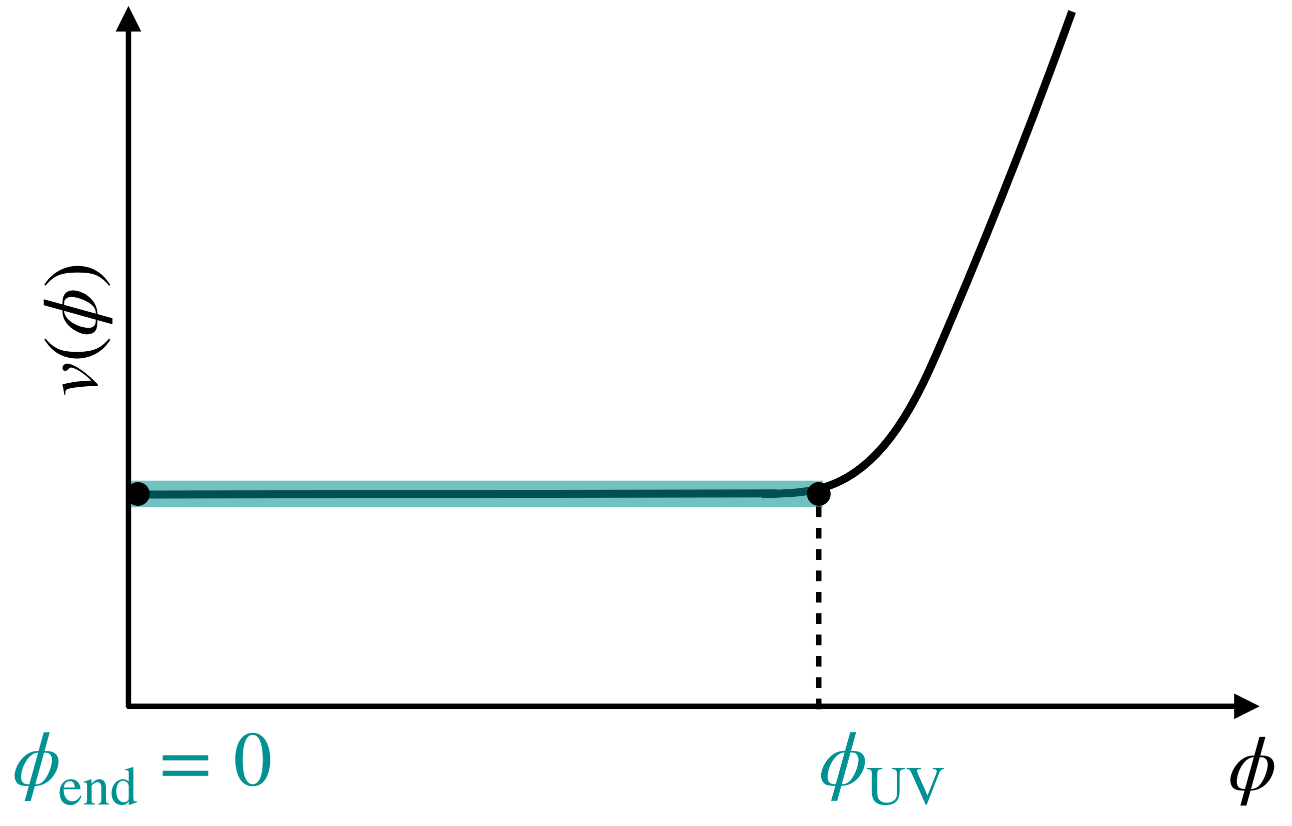
- Scale-field values relation: $R^3 = \frac{1}{\sigma^3 H^3(\boldsymbol{\Phi}_*)} \langle e^{3\mathcal{N}} \rangle (\boldsymbol{\Phi}_*)$

$$P(\zeta_R) = P_{\mathrm{FPT},\boldsymbol{\Phi}_0 \to \boldsymbol{\Phi}_*} \left[\zeta_R + \langle \mathcal{N} \rangle (\boldsymbol{\Phi}_0) - \langle \mathcal{N} \rangle (\boldsymbol{\Phi}_*) \right]$$

$$P(\zeta_{R_1}, \zeta_{R_2}) = \int \mathrm{d}\mathcal{N}_{\boldsymbol{\Phi}_0 \to \boldsymbol{\Phi}_*} P_{\mathrm{FPT},\boldsymbol{\Phi}_0 \to \boldsymbol{\Phi}_*}(\mathcal{N}_{\boldsymbol{\Phi}_0 \to \boldsymbol{\Phi}_*}) P_{\mathrm{FPT},\boldsymbol{\Phi}_* \to \boldsymbol{\Phi}_1} \left[\zeta_{R_1} - \mathcal{N}_{\boldsymbol{\Phi}_0 \to \boldsymbol{\Phi}_*} + \langle \mathcal{N} \rangle (\boldsymbol{\Phi}_0) - \langle \mathcal{N} \rangle (\boldsymbol{\Phi}_1) \right]$$
$$\times P_{\mathrm{FPT},\boldsymbol{\Phi}_* \to \boldsymbol{\Phi}_2} \left[\zeta_{R_2} - \mathcal{N}_{\boldsymbol{\Phi}_0 \to \boldsymbol{\Phi}_*} + \langle \mathcal{N} \rangle (\boldsymbol{\Phi}_0) - \langle \mathcal{N} \rangle (\boldsymbol{\Phi}_2) \right]$$

Single clock models

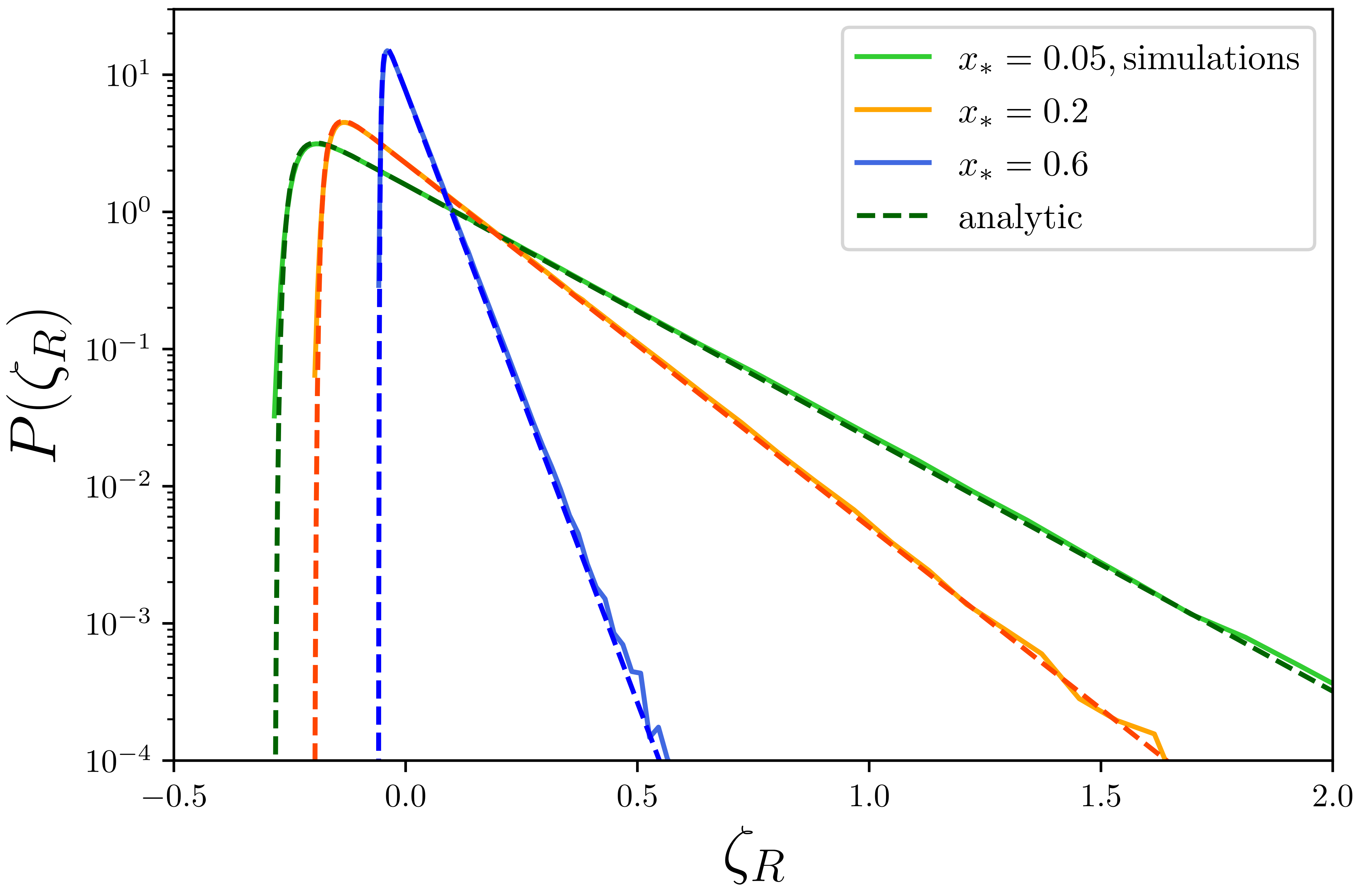
- Constant potential

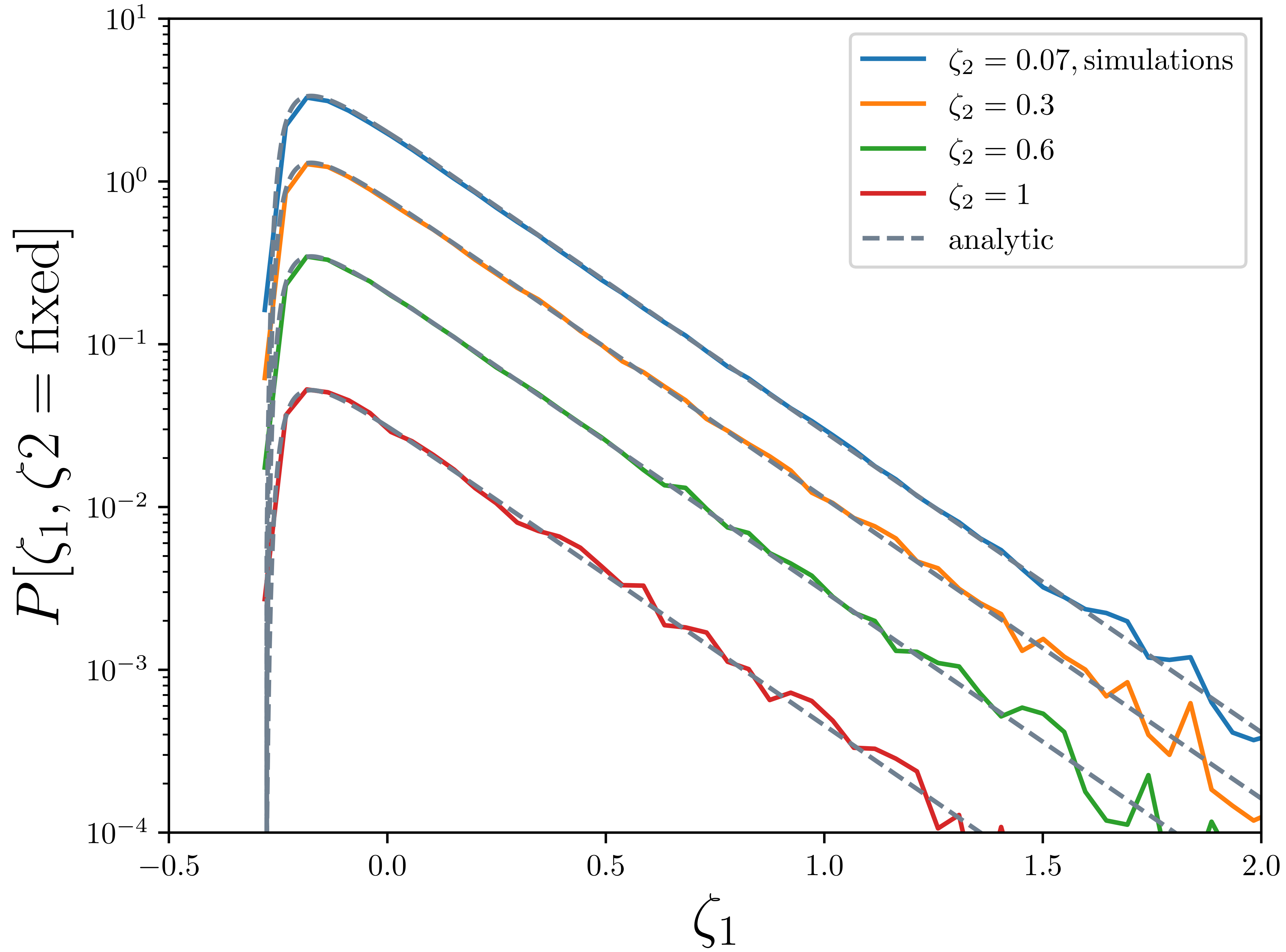


$$P(\zeta_R) = -\frac{\pi}{2\mu^2(1-x_*)^2}\vartheta_2'\left[\frac{\pi}{2}, e^{-\frac{\pi^2}{\mu^2(1-x_*)^2}\left[\zeta_R + \frac{\mu^2}{2}(1-x_*)^2\right]}\right]$$

$$x_* = \phi_*/\Delta\phi = \phi_*/\phi_{\rm UV}$$

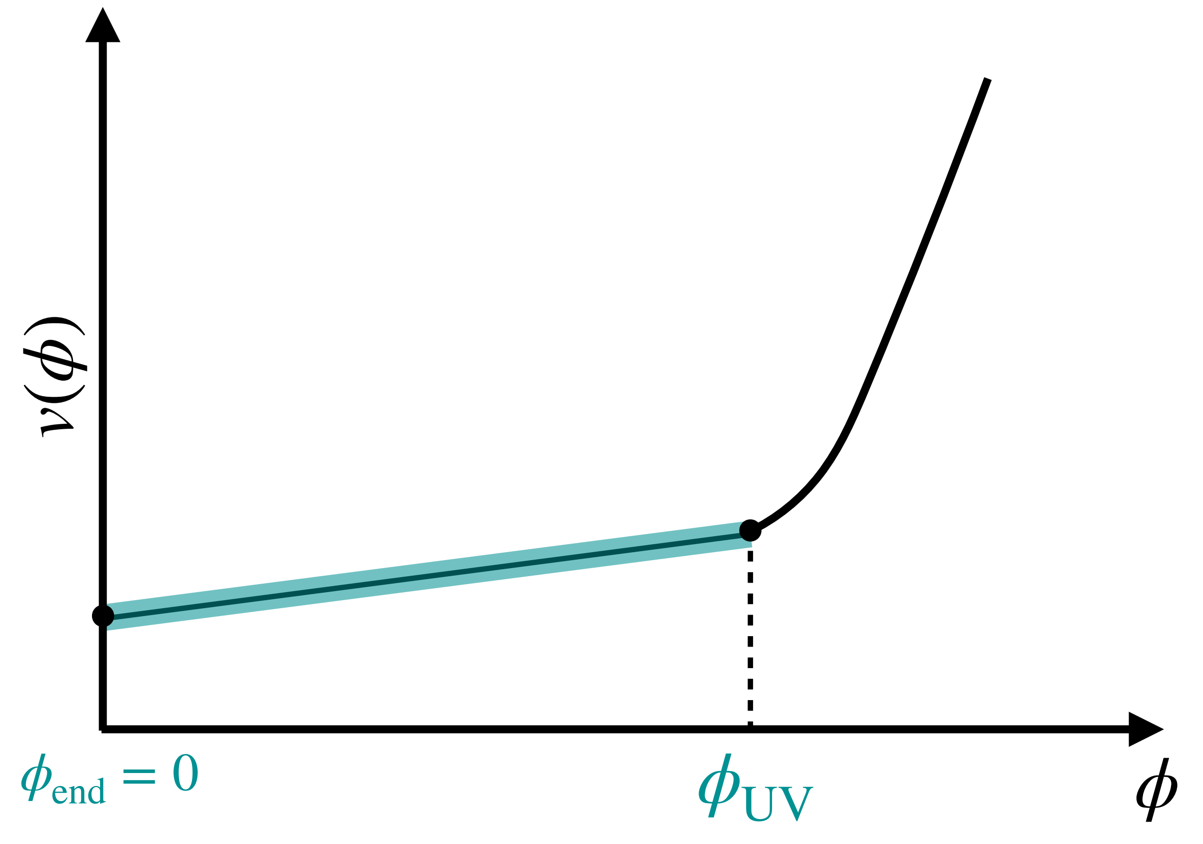
$$P(\zeta_{R_1},\zeta_{R_2}) = -\frac{\pi^3}{8\mu^6(1-x_*)^2(1-x_1)^2(1-x_2)^2}\int \mathrm{d}\mathcal{N}_{x_0\to x_*}\vartheta_2'\left[\frac{\pi}{2}, e^{-\frac{\pi^2}{\mu^2(1-x_*)^2}\mathcal{N}_{x_0\to x_*}}\right]$$
$$\times\,\vartheta_2'\left[\frac{\pi}{2}x_*, e^{-\frac{\pi^2}{\mu^2(1-x_1)^2}\left[\zeta_{R_1} - \mathcal{N}_{x_0\to x_*} + \frac{\mu^2}{2}(1-x_1)^2\right]}\right]\vartheta_2'\left[\frac{\pi}{2}x_*, e^{-\frac{\pi^2}{\mu^2(1-x_2)^2}\left[\zeta_{R_2} - \mathcal{N}_{x_0\to x_*} + \frac{\mu^2}{2}(1-x_2)^2\right]}\right]$$

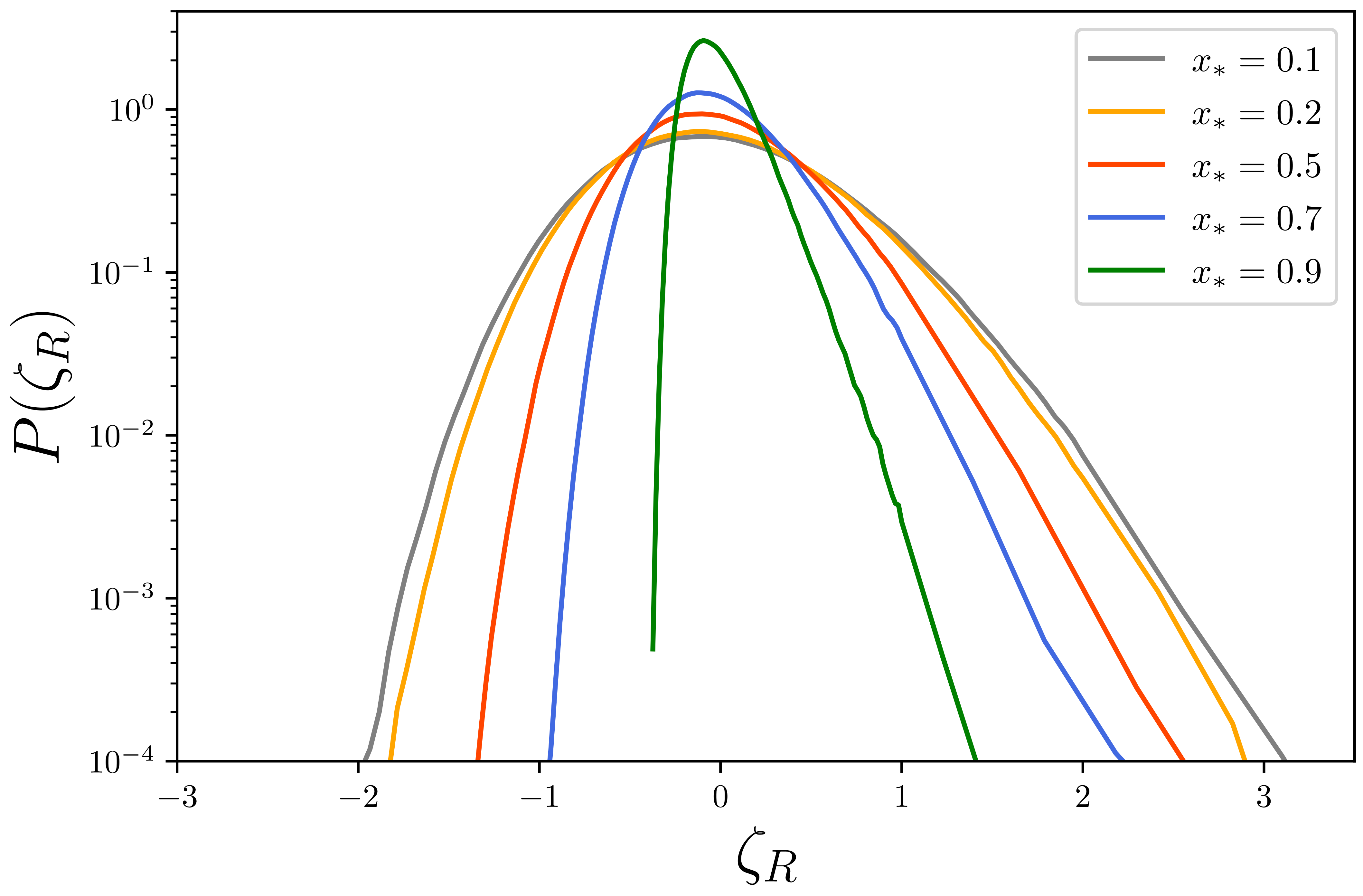


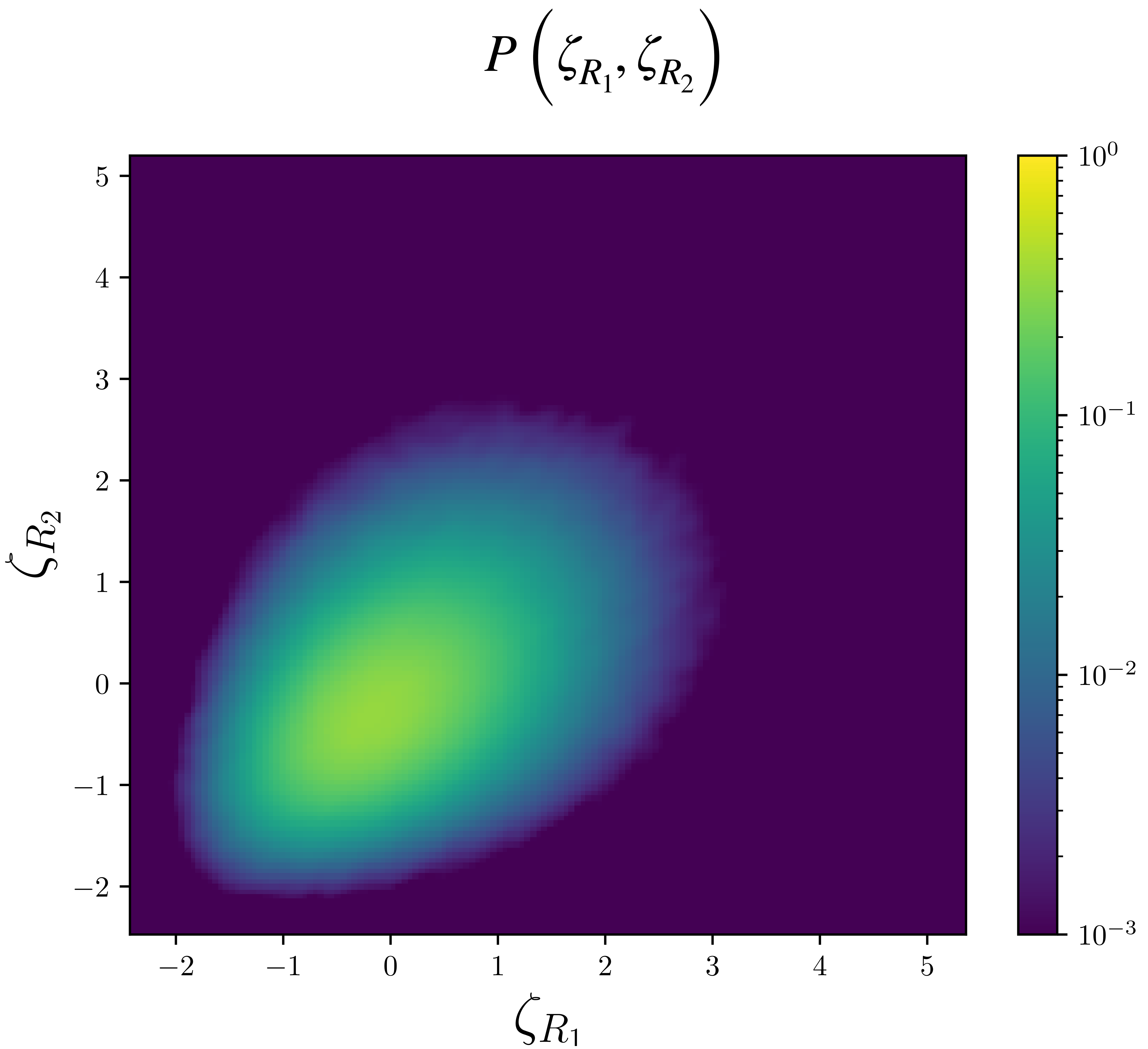


Single clock models

- Linear potential positive slope







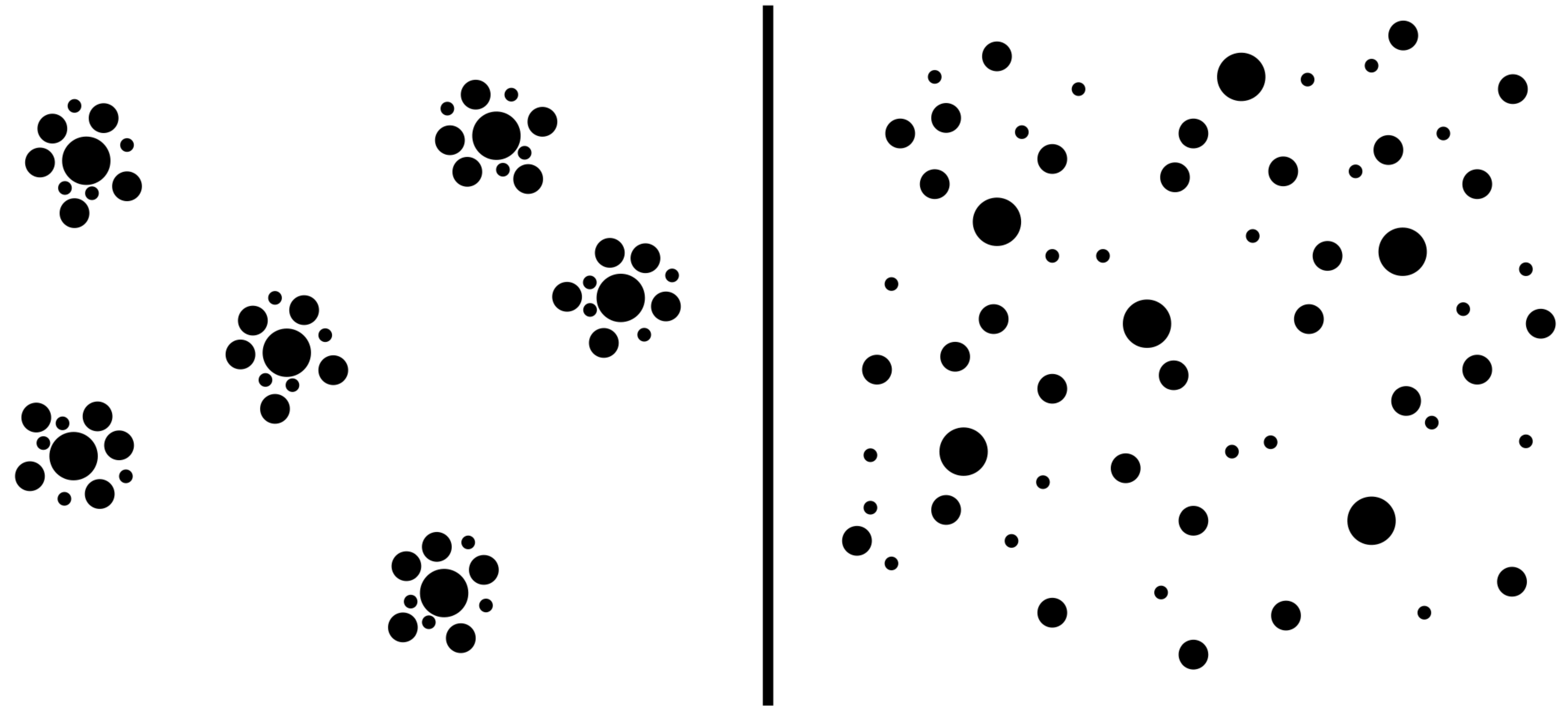
Are primordial black holes clustered at formation?

Are primordial black holes clustered at formation?

- Two-point correlation function of overdensities

$$1 + \xi(r) = \frac{p\left(\zeta(0) > \zeta_c, \zeta(r) > \zeta_c\right)}{p^2\left(\zeta(0) > \zeta_c\right)} \equiv \frac{P_2}{P_1^2}$$

$$\xi(r) \equiv \xi_{\rm PBH}(r) \equiv \xi_{\rm red}(r) \qquad \text{for} \qquad r \gtrsim C R$$



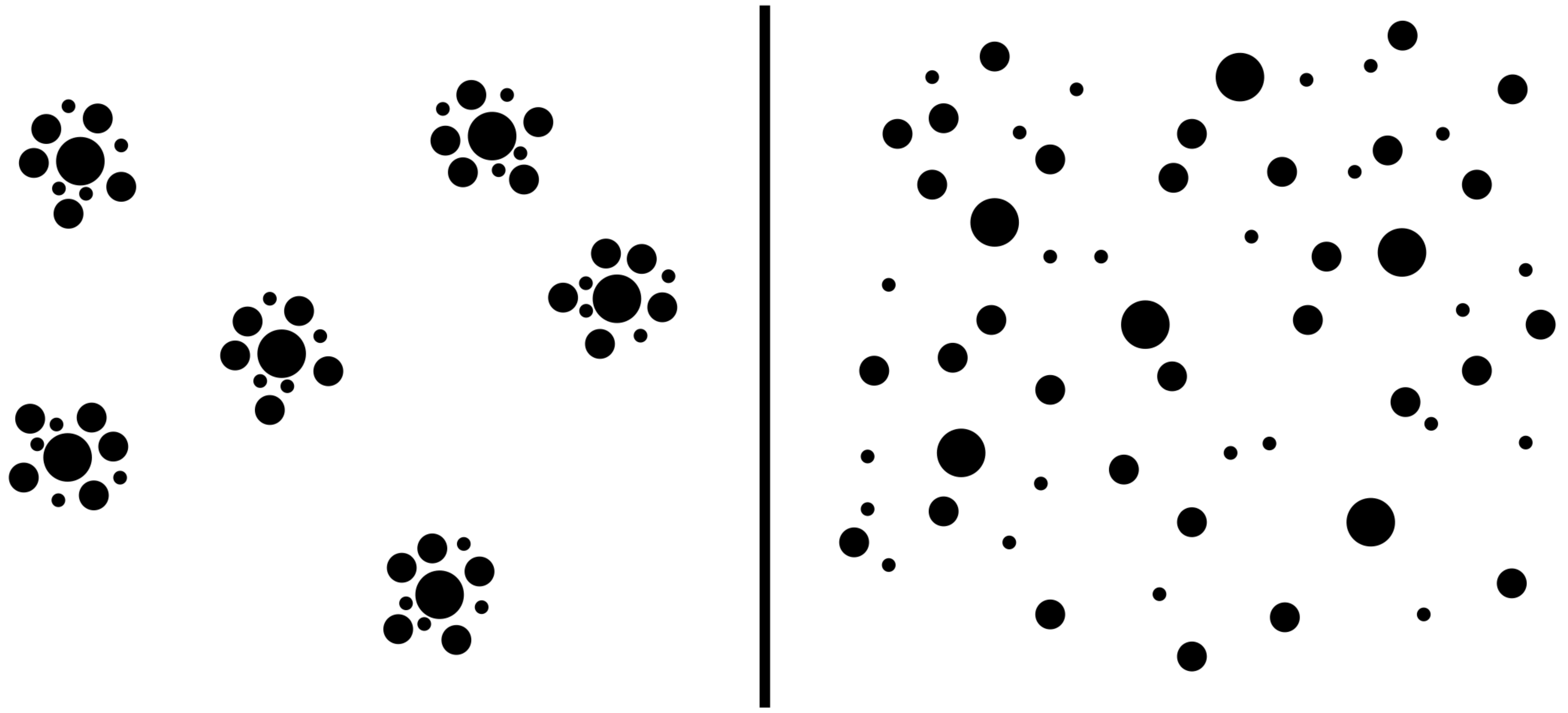
clustered vs non-clustered spatial distribution

Are primordial black holes clustered at formation?

- Two-point correlation function of overdensities

$$1 + \xi(r) = \frac{p\left(\zeta(0) > \zeta_c, \zeta(r) > \zeta_c\right)}{p^2\left(\zeta(0) > \zeta_c\right)} \equiv \frac{P_2}{P_1^2}$$

$$\xi(r) \equiv \xi_{\rm PBH}(r) \equiv \xi_{\rm red}(r) \qquad \text{for} \qquad r \gtrsim CR$$



clustered vs non-clustered spatial distribution

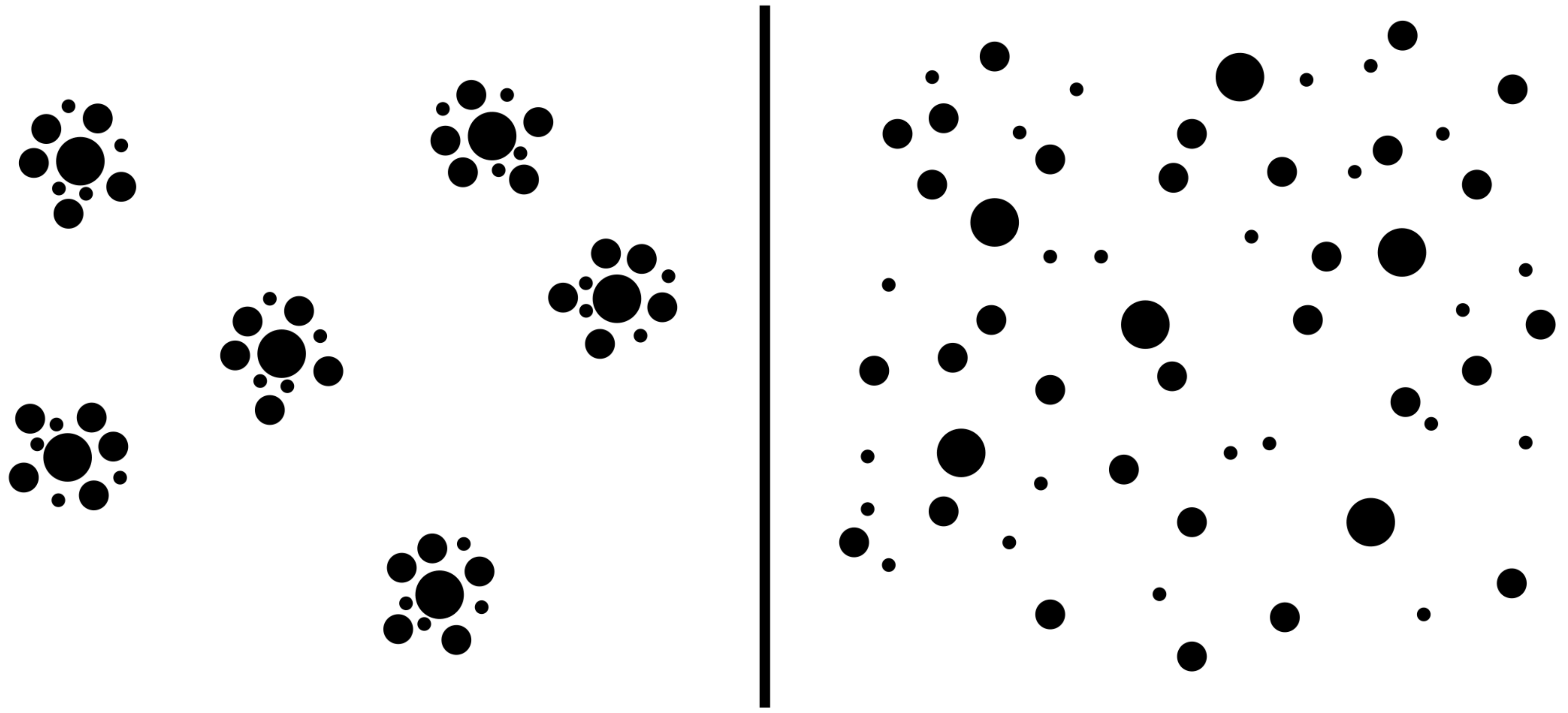
- If ζ is a gaussian field: Poissonian distribution Y. Ali-Hamoud [2018], PRL 121,081304

Are primordial black holes clustered at formation?

- Two-point correlation function of overdensities

$$1 + \xi(r) = \frac{p\left(\zeta(0) > \zeta_c, \zeta(r) > \zeta_c\right)}{p^2\left(\zeta(0) > \zeta_c\right)} \equiv \frac{P_2}{P_1^2}$$

$$\xi(r) \equiv \xi_{\rm PBH}(r) \equiv \xi_{\rm red}(r) \qquad \text{for} \qquad r \gtrsim CR$$



clustered vs non-clustered spatial distribution

- If ζ is a gaussian field: Poissonian distribution Y. Ali-Hamoud [2018], PRL 121,081304

- Small perturbative non gaussianities: generation of little amount of clustering at formation

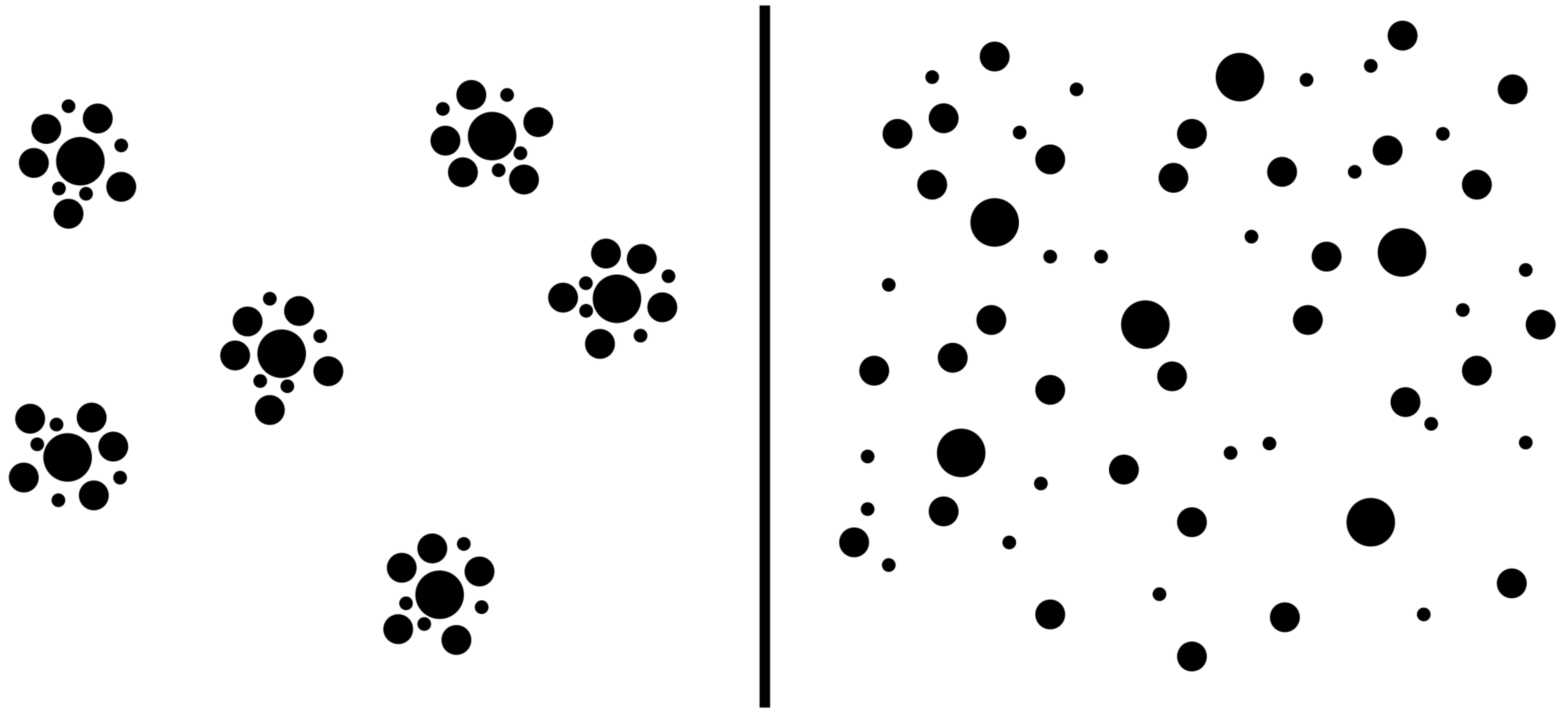
  Franciolini, Kehagias, Matarrese, Riotto [2018],JCAP 03016 T. Suyama, S. Yokoyama [2019],PTEP, 103E02

Are primordial black holes clustered at formation?

- Two-point correlation function of overdensities

$$1+\xi(r)=\frac{p\left(\zeta(0)>\zeta_c,\zeta(r)>\zeta_c\right)}{p^2\left(\zeta(0)>\zeta_c\right)}\equiv\frac{P_2}{P_1^2}$$

$$\xi(r)\equiv\xi_{\rm PBH}(r)\equiv\xi_{\rm red}(r)\qquad\text{for}\qquad r\gtrsim CR$$



clustered vs non-clustered spatial distribution

- If ζ is a gaussian field: Poissonian distribution   Y. Ali-Hamoud [2018], PRL 121,081304

- Small perturbative non gaussianities: generation of little amount of clustering at formation

  Franciolini, Kehagias, Matarrese, Riotto [2018],JCAP 03016   T. Suyama, S. Yokoyama [2019],PTEP, 103E02

- Non perturbative non gaussianities?

$$P_1=\int_{\zeta_c}\mathrm{d}\zeta\,P(\zeta)\qquad P_2=\int_{\zeta_c}\mathrm{d}\zeta_1\mathrm{d}\zeta_2\,P(\zeta_1,\zeta_2)$$

C.A., V. Vennin
In preparation

In summary

- PBHs are a useful probe of inflation beyond tested regimes

- PBHs may be produced by large fluctuations during inflation: quantum diffusion cannot be neglected;
  it can be incorporated by the stochastic δN formalism: non gaussian tails

- We can extend the stochastic delta N formalism to arbitrary coarse graining scales, and to multiple point statistics

- Do non-perturbative non gaussianities also affect the spatial distribution of PBHs?

LPENS

LABORATOIRE DE PHYSIQUE
DE L'ÉCOLE NORMALE SUPÉRIEURE

Many thanks for the attention!

chiara.animali@phys.ens.fr

ENS | PSL UNIVERSITÉ PARIS

cnrs

SORBONNE UNIVERSITÉ

Université Paris Cité

Inflation

Inflation

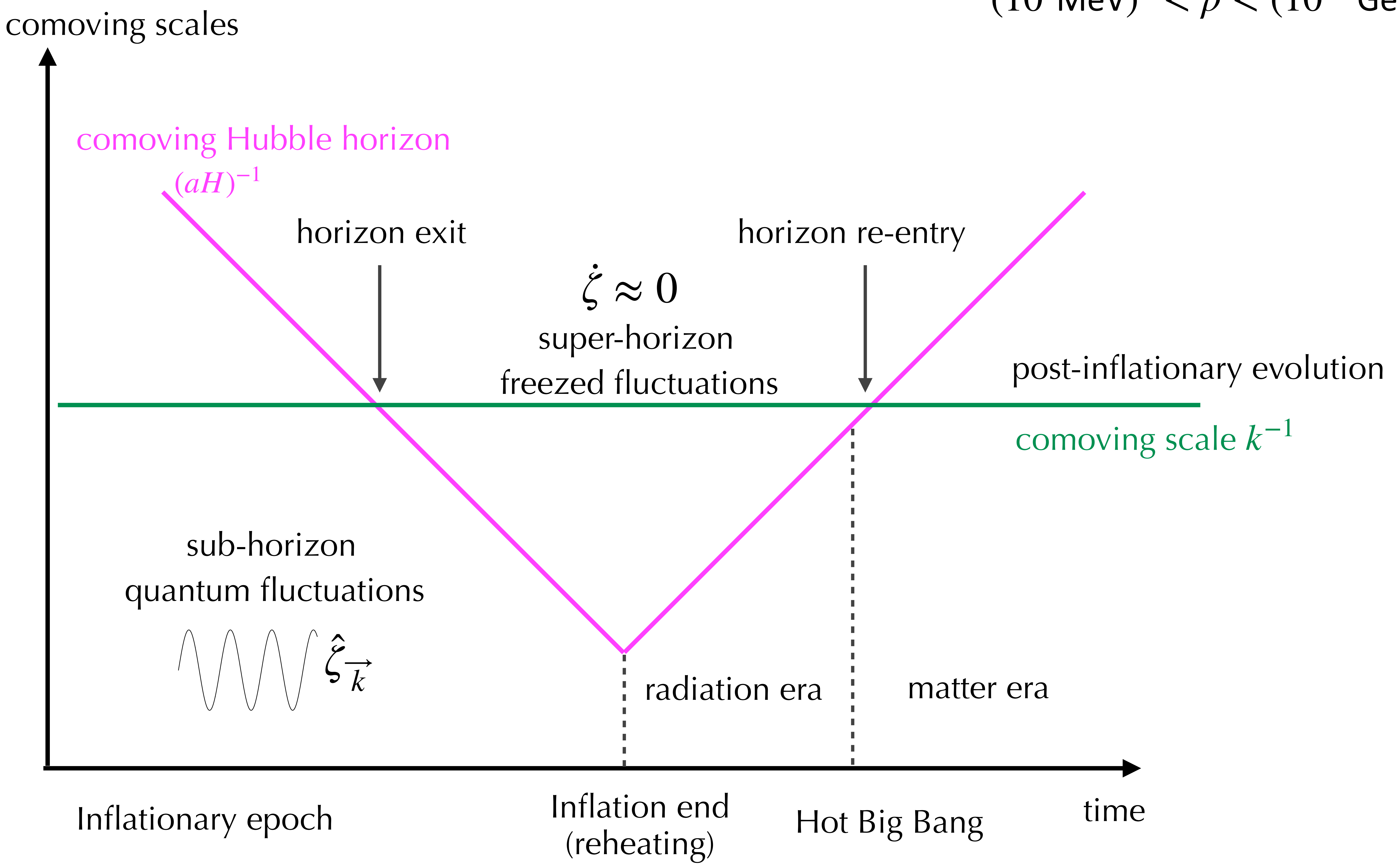
- High energy phase of accelerated expansion of spacetime $ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)d\vec{x}^2$ $\dot{a}, \ddot{a} > 0$

$(10\ \text{MeV})^4 < \rho < (10^{16}\ \text{GeV})^4$

Inflation

- High energy phase of accelerated expansion of spacetime $ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)d\vec{x}^2$ $\dot{a}, \ddot{a} > 0$

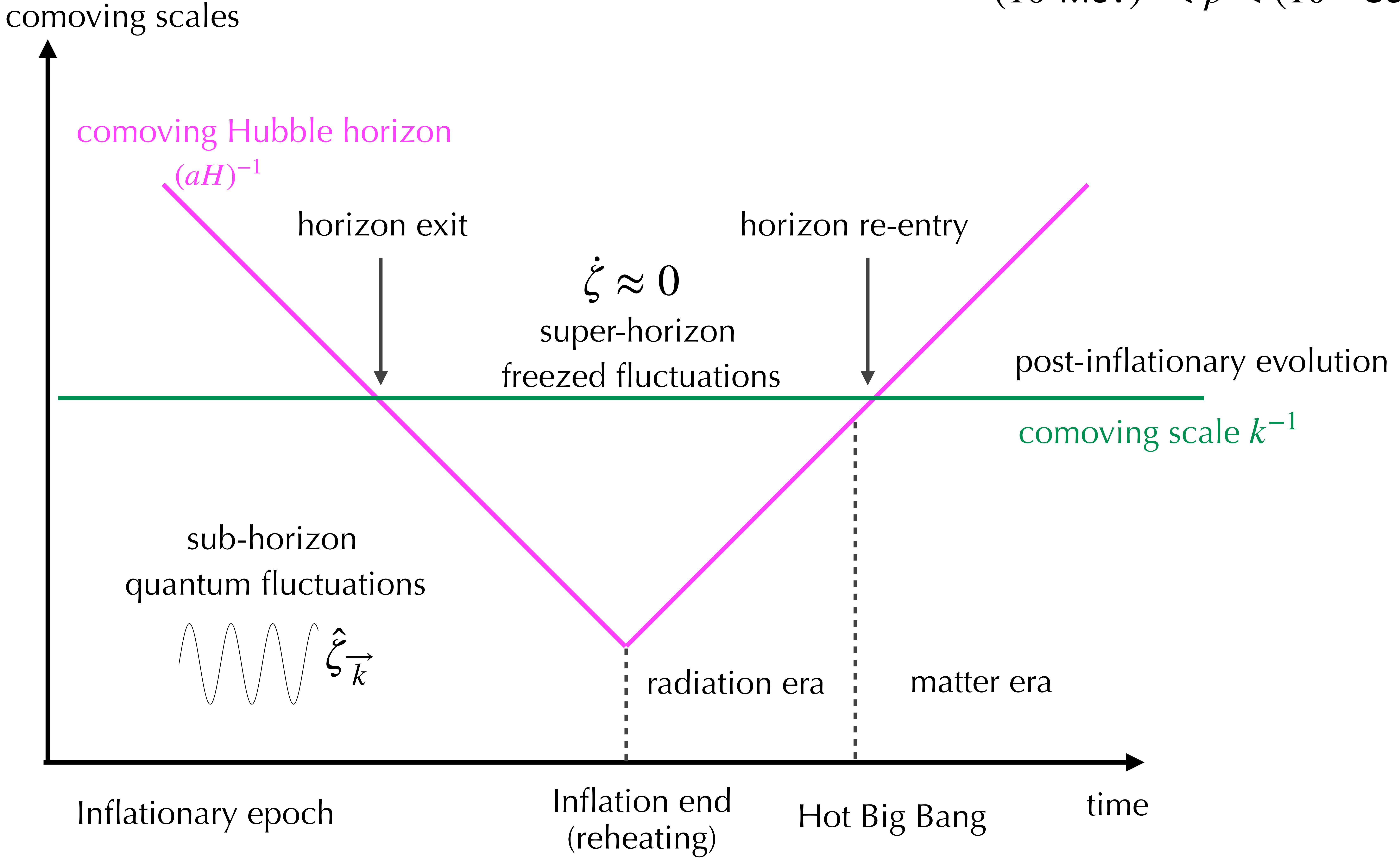
$(10\ \text{MeV})^4 < \rho < (10^{16}\ \text{GeV})^4$

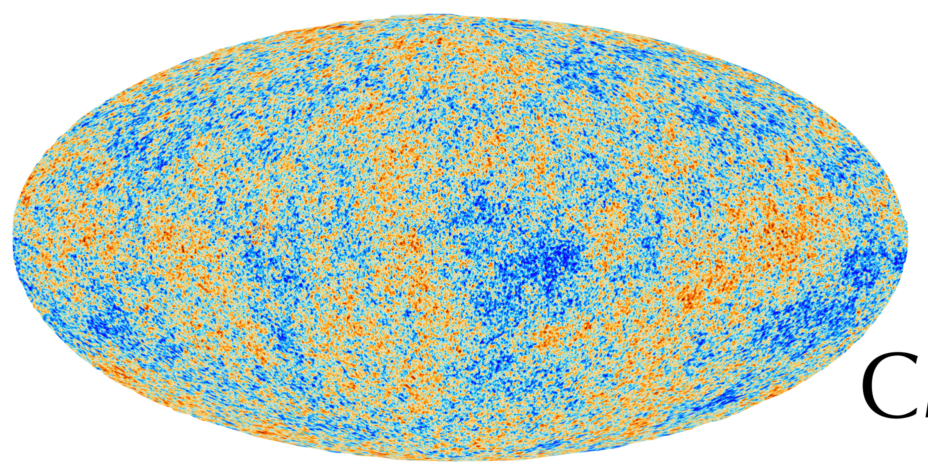


Inflation

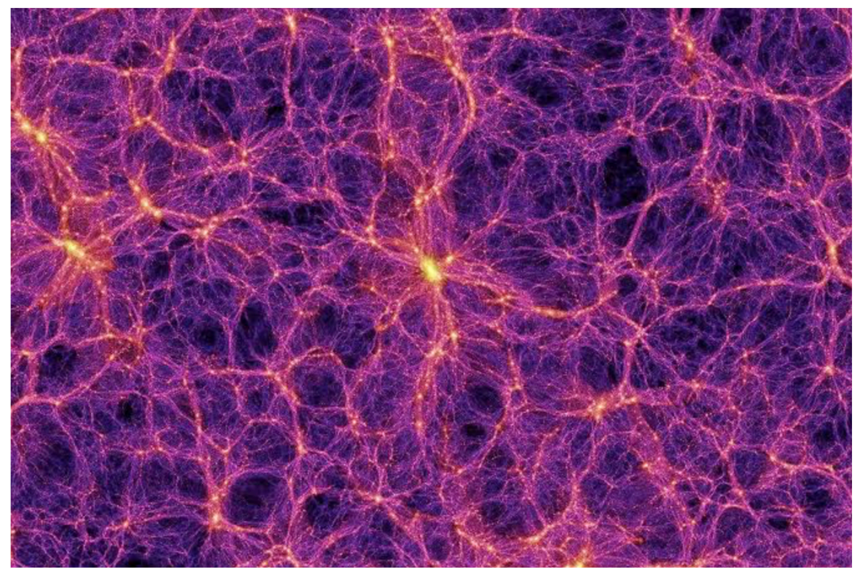
- High energy phase of accelerated expansion of spacetime $ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)d\vec{x}^2$ $\quad \dot{a}, \ddot{a} > 0$

$$(10\ \text{MeV})^4 < \rho < (10^{16}\ \text{GeV})^4$$





CMB



LSS

Inflation

Inflation

- Simplest realisation: slow-roll inflation

scalar field ϕ (inflaton) slowly rolling towards the minimum of its potential

$$S_\phi = \int d^4x \sqrt{-g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right)$$

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{1}{16\pi G} \left(\frac{V_{,\phi}}{V} \right)^2$$

$$\eta = \frac{\dot{\epsilon}}{H\epsilon} = \frac{1}{8\pi G} \left(\frac{V_{\phi\phi}}{V} \right)$$

$$\{\epsilon, |\eta|\} \ll 1$$

Inflation

- Simplest realisation: slow-roll inflation

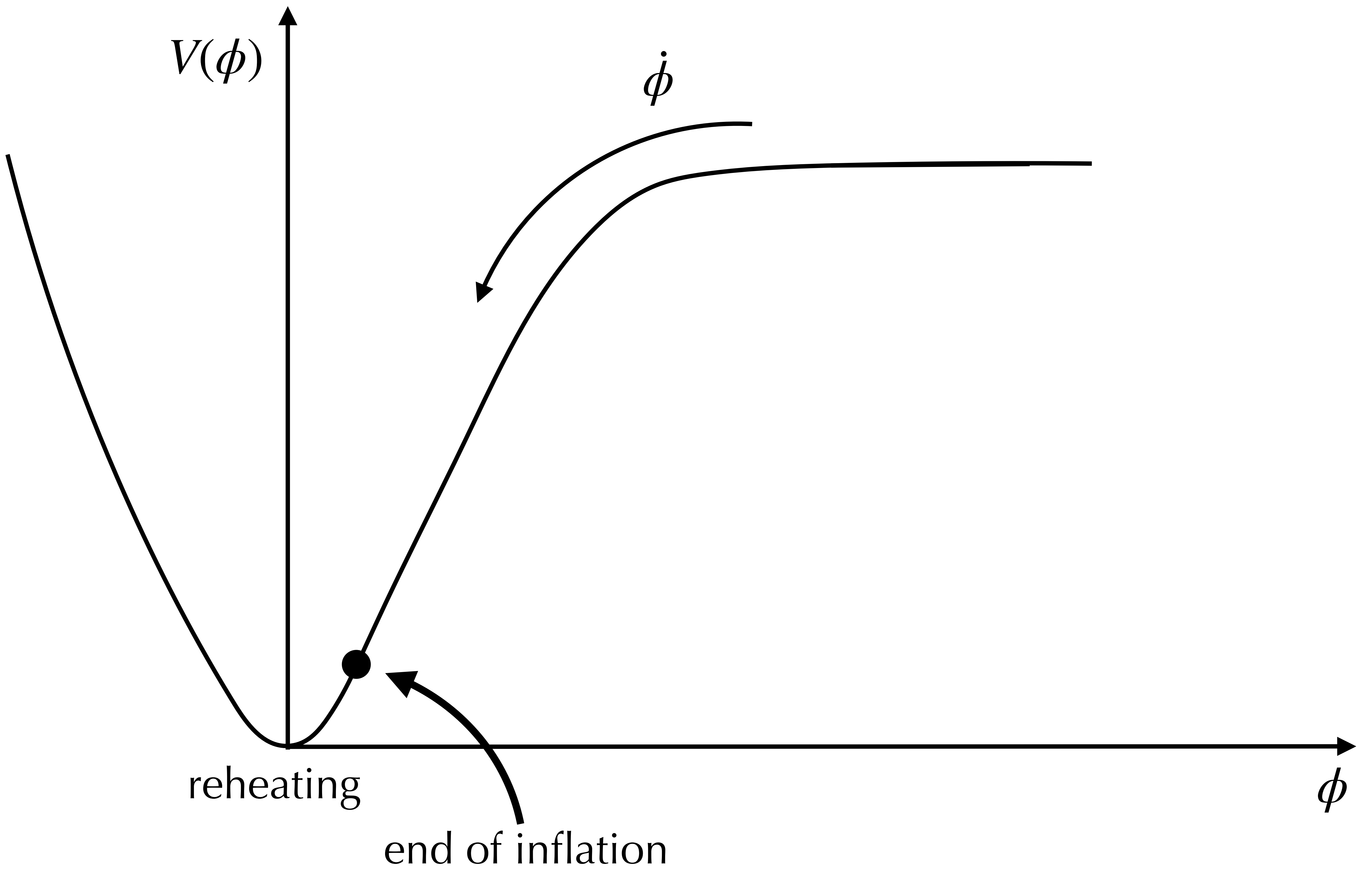
scalar field ϕ (inflaton) slowly rolling towards the minimum of its potential

$$S_\phi = \int d^4x \sqrt{-g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right)$$

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{1}{16\pi G} \left(\frac{V_{,\phi}}{V} \right)^2$$

$$\eta = \frac{\dot{\epsilon}}{H\epsilon} = \frac{1}{8\pi G} \left(\frac{V_{\phi\phi}}{V} \right)$$

$$\{\epsilon, |\eta|\} \ll 1$$



Inflation

- Simplest realisation: slow-roll inflation

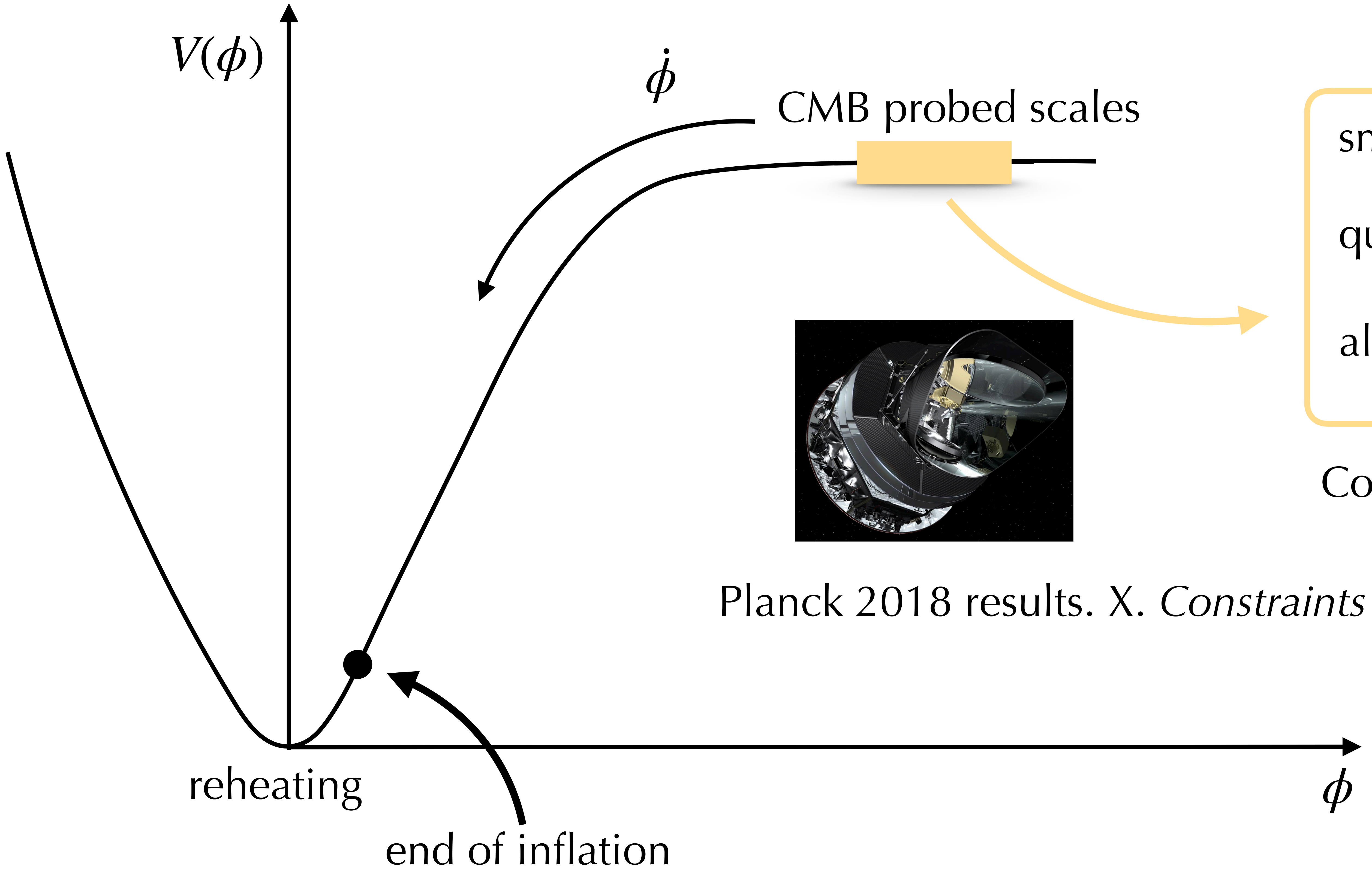
scalar field ϕ (inflaton) slowly rolling towards the minimum of its potential

$$S_\phi = \int d^4x \sqrt{-g}\, \left(\frac{1}{2} g^{\mu\nu}\, \partial_\mu \phi \partial_\nu \phi - V(\phi) \right)$$

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{1}{16\,\pi\, G} \left(\frac{V_{,\phi}}{V} \right)^2$$

$$\eta = \frac{\dot{\epsilon}}{H\epsilon} = \frac{1}{8\,\pi\, G} \left(\frac{V_{\phi\phi}}{V} \right)$$

$$\{\epsilon, |\eta|\} \ll 1$$



small perturbations $\zeta \simeq 10^{-5}$

quasi-Gaussian

almost scale invariant

Constrained window ~ 7 *e*-folds

Planck 2018 results. X. *Constraints on inflation*

Inflation

- Simplest realisation: slow-roll inflation

scalar field ϕ (inflaton) slowly rolling towards the minimum of its potential

$$S_\phi = \int d^4x \sqrt{-g}\,\left(\frac{1}{2} g^{\mu\nu}\,\partial_\mu\phi\partial_\nu\phi - V(\phi)\right)$$

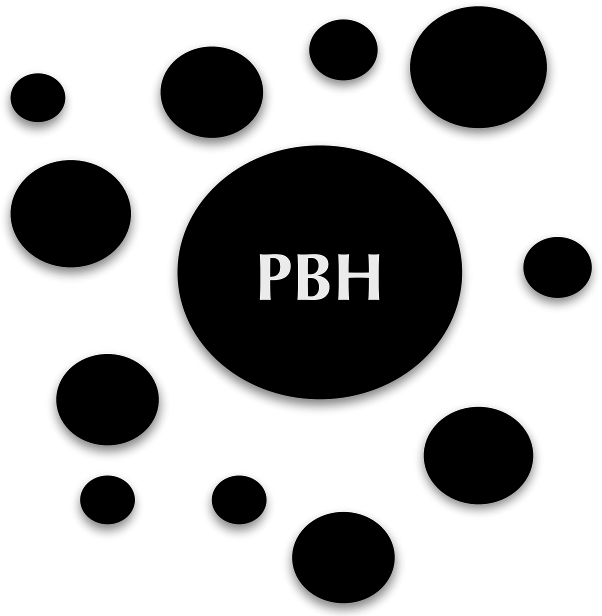
$$\epsilon = -\frac{\dot H}{H^2} = \frac{1}{16\,\pi\,G}\left(\frac{V_{,\phi}}{V}\right)^2$$

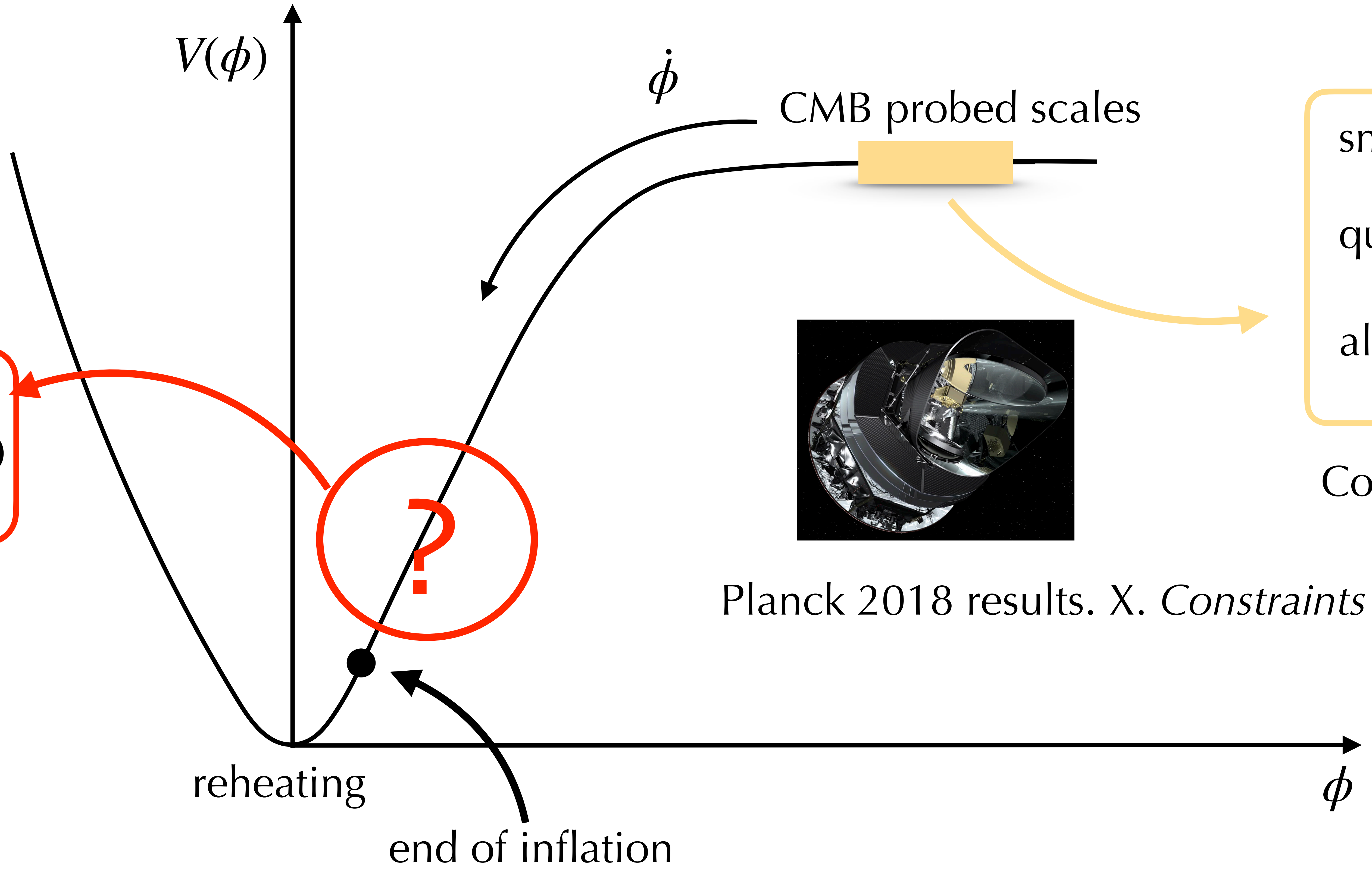
$$\eta = \frac{\dot\epsilon}{H\epsilon} = \frac{1}{8\,\pi\,G}\left(\frac{V_{\phi\phi}}{V}\right)$$

$$\{\epsilon, |\eta|\} \ll 1$$

small scales unconstrained

possible enhancement $\zeta \sim \mathcal{O}(1)$





small perturbations $\zeta \simeq 10^{-5}$

quasi-Gaussian

almost scale invariant

Constrained window ~ 7 e-folds

Planck 2018 results. X. *Constraints on inflation*

Primordial Black Holes

Primordial Black Holes

- Black holes which could have formed in the early Universe through a non-stellar way

Hawking [1971] : *Gravitationally collapsed objects of very low mass*

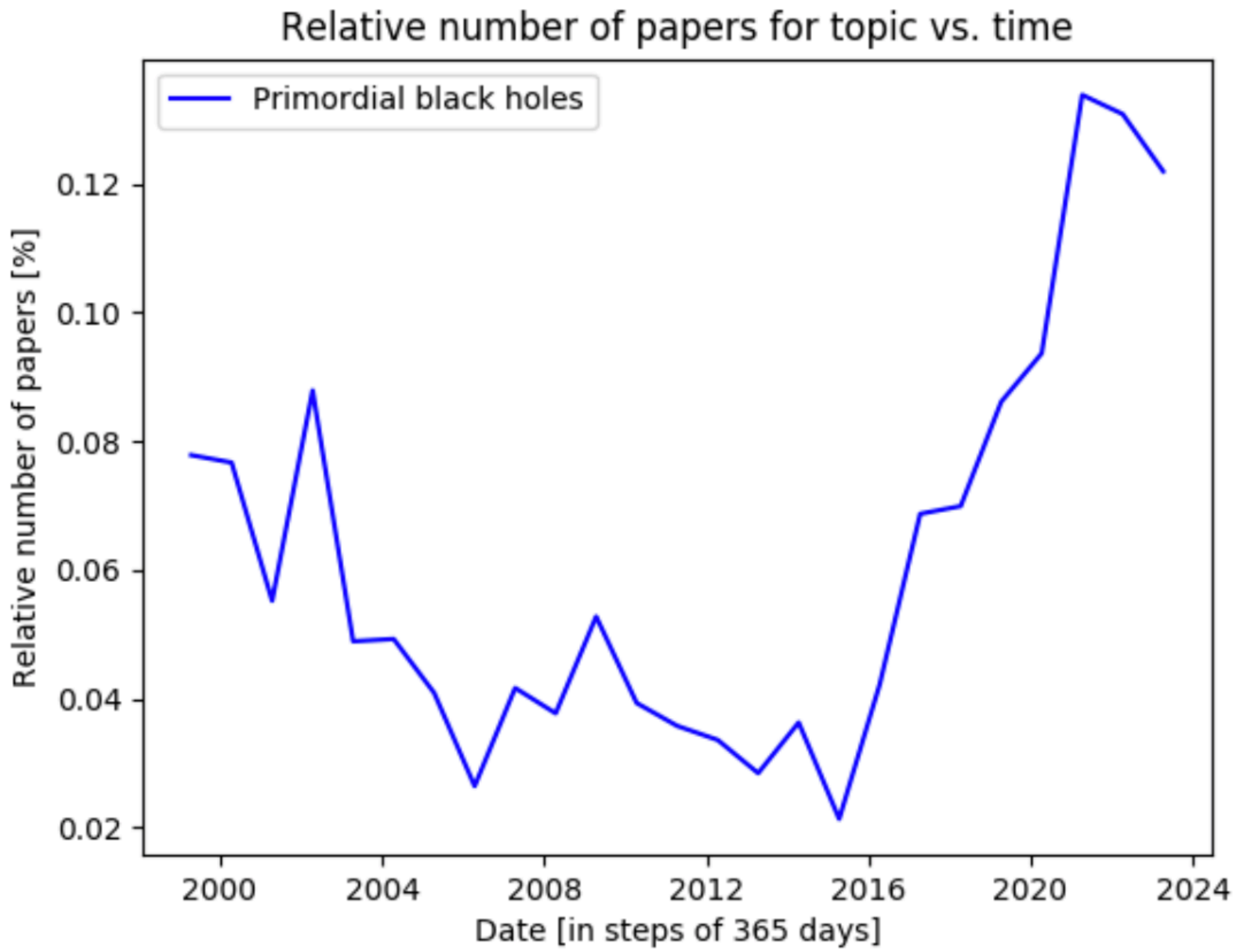
Carr & Hawking [1974]: *Black holes in the early Universe*

Primordial Black Holes

- Black holes which could have formed in the early Universe through a non-stellar way

Hawking [1971] : *Gravitationally collapsed objects of very low mass*

Carr & Hawking [1974]: *Black holes in the early Universe*

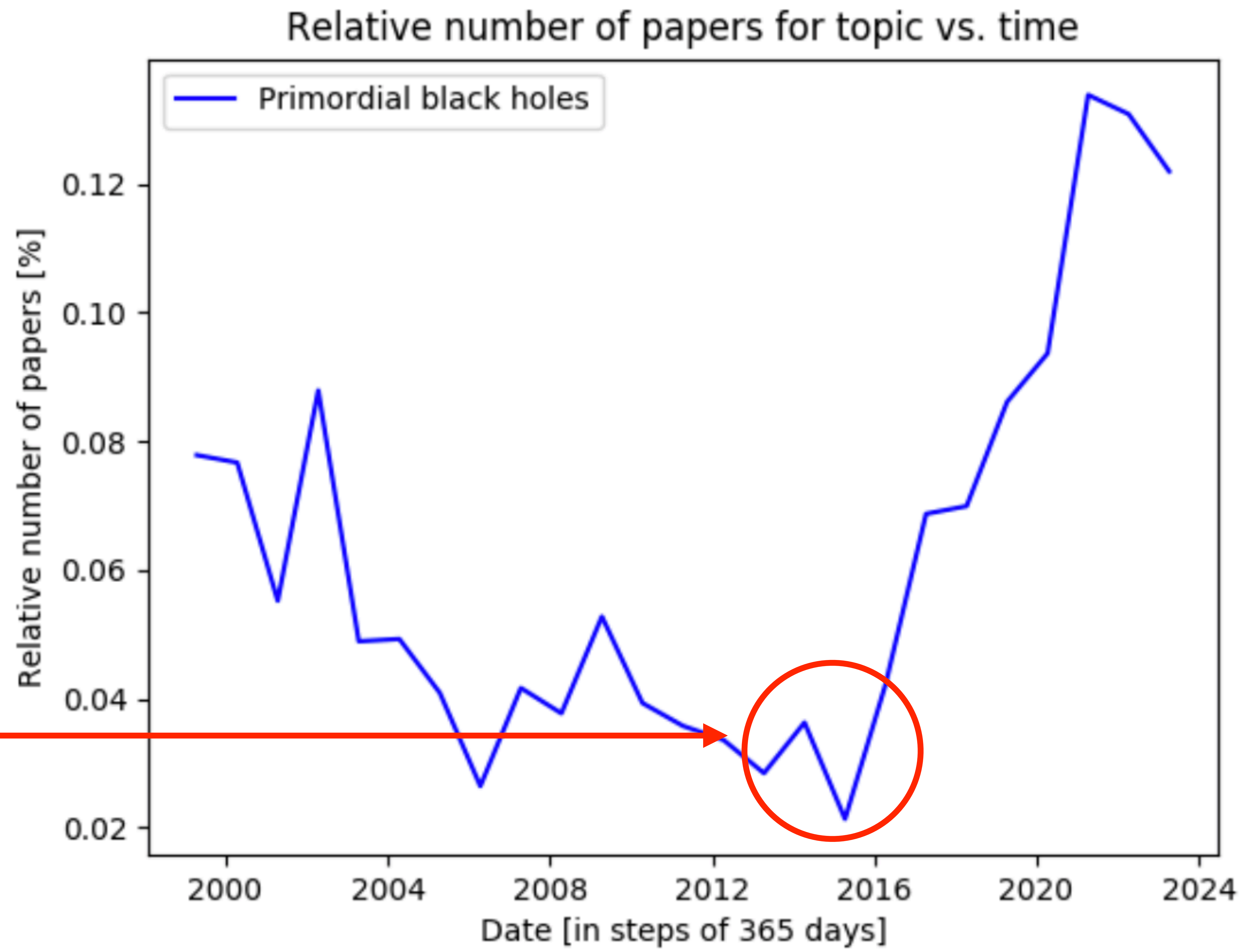


Primordial Black Holes

- Black holes which could have formed in the early Universe through a non-stellar way

Hawking [1971] : *Gravitationally collapsed objects of very low mass*

Carr & Hawking [1974]: *Black holes in the early Universe*



LIGO SCIENTIFIC, VIRGO collaboration [2016]:
Observation of gravitational waves from a binary black hole merger

S. Bird, I. Cholis, J.B. Muñoz, Y. Ali-Haïmoud,
M. Kamionkowski, E. D. Kovetz, A. Raccanelli, A. G. Riess [2016]:
Did LIGO detect dark matter?

Primordial black holes: observational constraints

Depends on the mass at which PBHs form

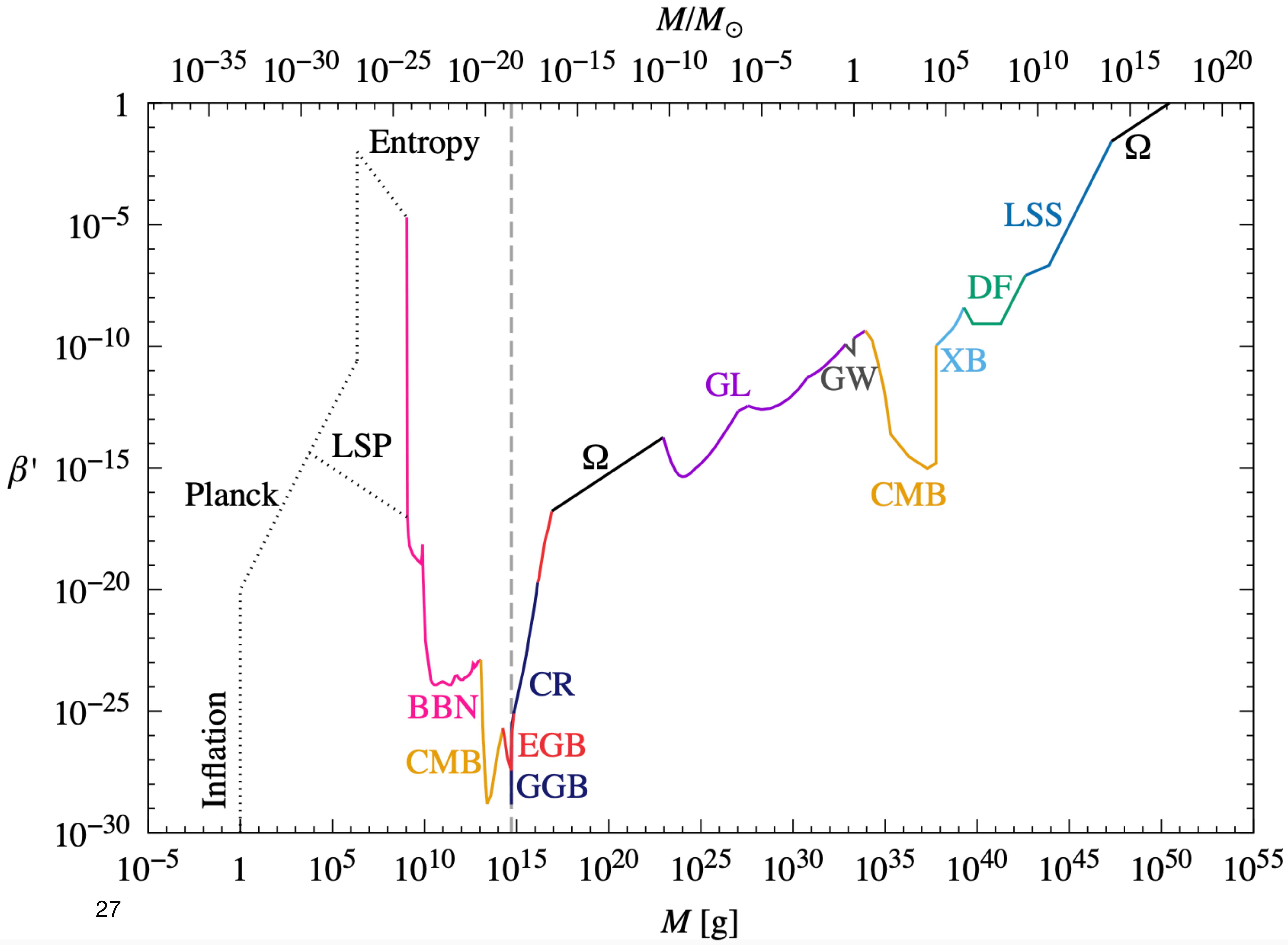
$10^9 g < M_{PBH} < 10^{16} g \longrightarrow$ from $\beta < 10^{-24}$ to $\beta < 10^{-17}$

PBH Hawking evaporation on Big Bang Nucleosynthesis and on the extragalactic photon background

$10^{16} g < M_{PBH} < 10^{50} g \longrightarrow$ from $\beta < 10^{-11}$ to $\beta < 10^{-5}$

Gravitational and astrophysical effects

$M_{PBH} < 10^9 g$ Evaporate before BBN: no direct imprint
no constraints

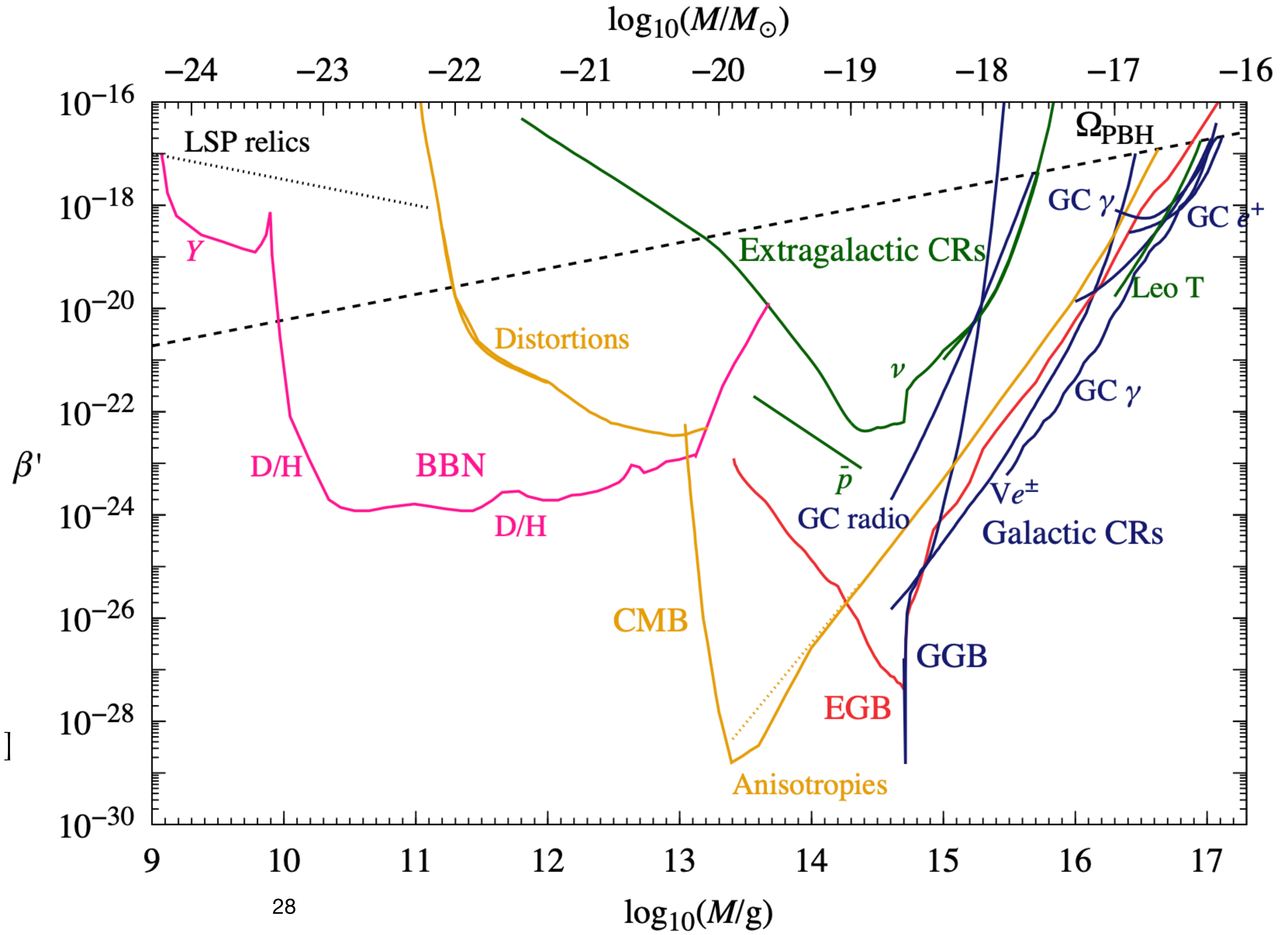


B. Carr, K. Kohri, Y. Sendouda, J. Yokoyama [2021]
Constraints on Primordial Black Holes

Primordial black holes: observational constraints

$10^9 g < M_{PBH} < 10^{16} g$ ⟶ from $\beta < 10^{-24}$ to $\beta < 10^{-17}$

PBH Hawking evaporation on Big Bang Nucleosynthesis and on the extragalactic photon background



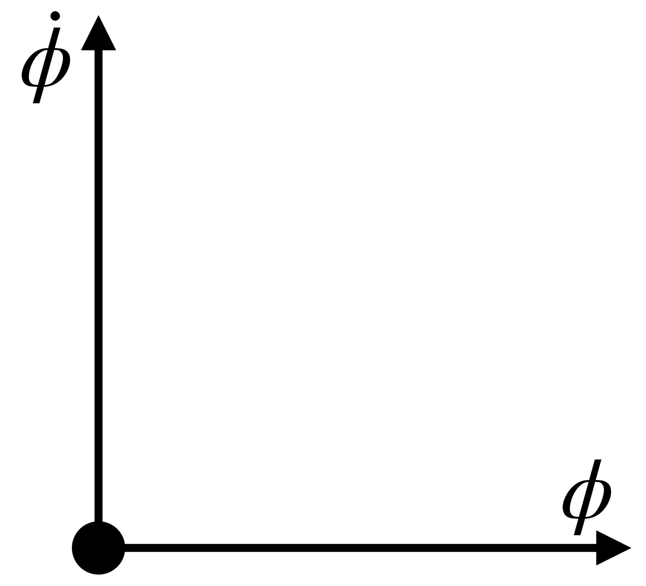
B. Carr, K. Kohri, Y. Sendouda, J. Yokoyama [2021]
Constraints on Primordial Black Holes

False vacuum: preserving slow roll

Slow roll requires: $|\ddot{\phi}| \ll 3H|\dot{\phi}|, |V_{,\phi}|$

What happens if $|V_{,\phi}| = 0$?

$$\ddot{\phi} + 3\,H(\phi,\dot{\phi})\,\dot{\phi} + V_{,\phi} = 0 \qquad H^2(\phi,\dot{\phi}) = \frac{1}{3M_{Pl}^2}\left(V(\phi) + \frac{\dot{\phi}^2}{2}\right)$$



minimum $(\phi = 0, \dot{\phi} = 0)$

$V(\phi) \simeq V_0 + m^2\phi^2/2$

Linearised Klein-Gordon equation

$$\ddot{\phi} + 3H_0\,\dot{\phi} + m^2\phi = 0 \qquad H_0^2 = \frac{V_0}{3\,M_{Pl}^2}$$

$$\phi = A\exp\left[-\frac{3}{2}\left(1 + \sqrt{1 - \frac{4\,m^2}{9\,H_0^2}}\right)H_0\,t\right] + B\exp\left[-\frac{3}{2}\left(-1 - \sqrt{1 - \frac{4\,m^2}{9\,H_0^2}}\right)H_0\,t\right]$$

$m \gg 3H_0/2$: damped oscillations, friction term $3H\dot{\phi}$ subdominant: far from slow-roll regime

$$m \ll 3H_0/2 \qquad \phi \simeq A\exp\left(-3H_0t\right) + B\exp\left(-\frac{1}{3}\frac{m^2}{H_0^2}H_0t\right) \simeq B\exp\left(-\frac{m^2t}{3H_0}\right)$$

$$3H\dot{\phi} \simeq -m^2\phi = -V_{,\phi}(\phi) \qquad \ddot{\phi} \simeq \frac{m^4}{9\,H_0^2}\phi = \frac{m^2}{9\,H_0^2}V_{,\phi} \ll V_{,\phi}(\phi)$$

slow-roll regime: acceleration term subdominant

(m^2/H_0^2 - suppressed)

False vacuum: parameter space

False vacuum: parameter space

- $\langle \mathcal{N} \rangle$ features quadratic dependence on μ and exponential dependence on a
- μ constrained from below by slow-roll conditions

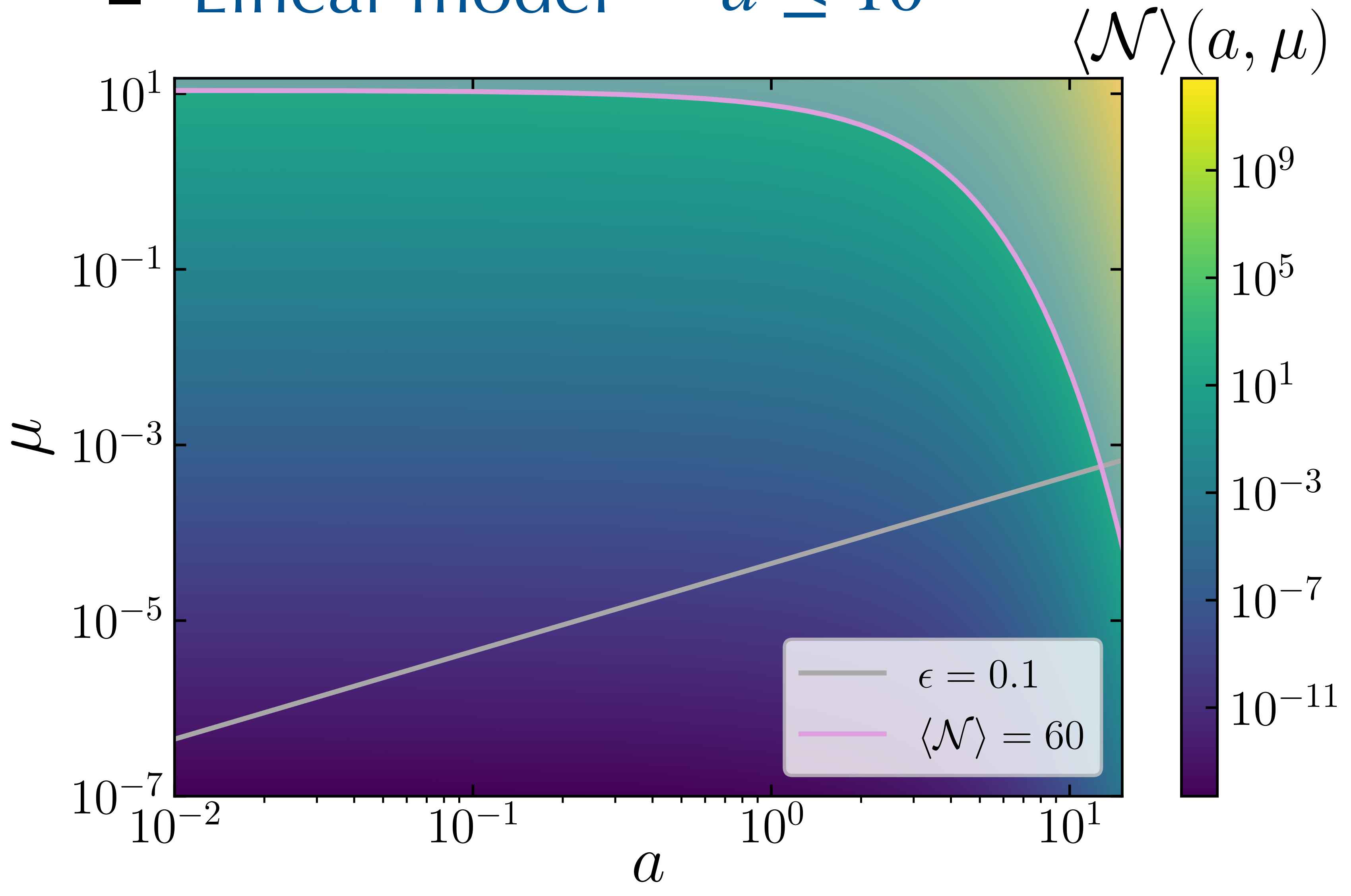
a not much larger than 1

False vacuum: parameter space

- $\langle \mathcal{N} \rangle$ features quadratic dependence on μ and exponential dependence on a
- μ constrained from below by slow-roll conditions

a not much larger than 1

- Linear model $a \leq 10$



- $\epsilon \ll 1 \quad \Rightarrow \quad \alpha \ll \dfrac{\Delta\phi}{M_{Pl}}$
- Two regimes: "shallow well" ($a \lesssim 1$)

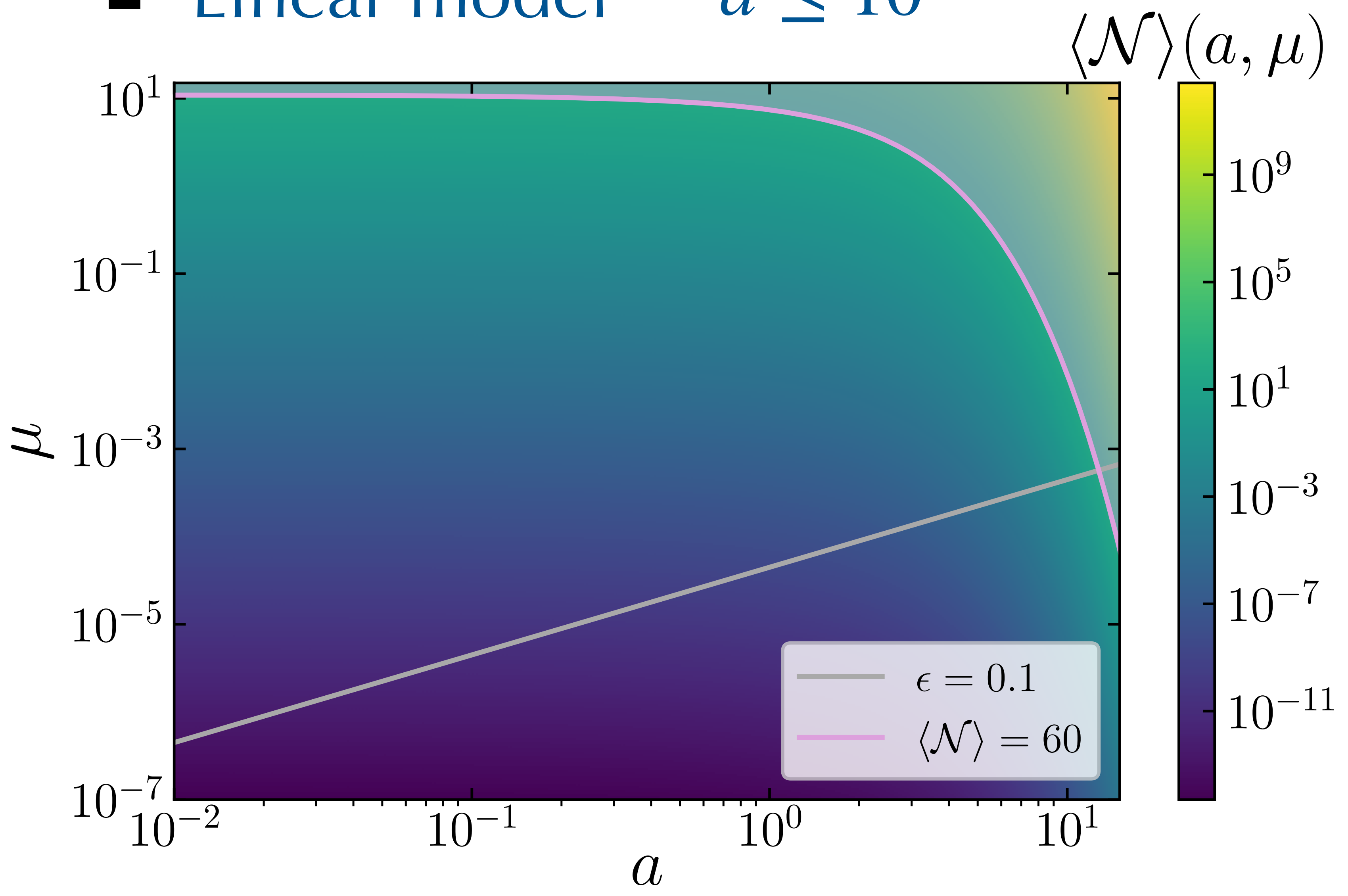
  "deep well" ($a \gtrsim 1$)

False vacuum: parameter space

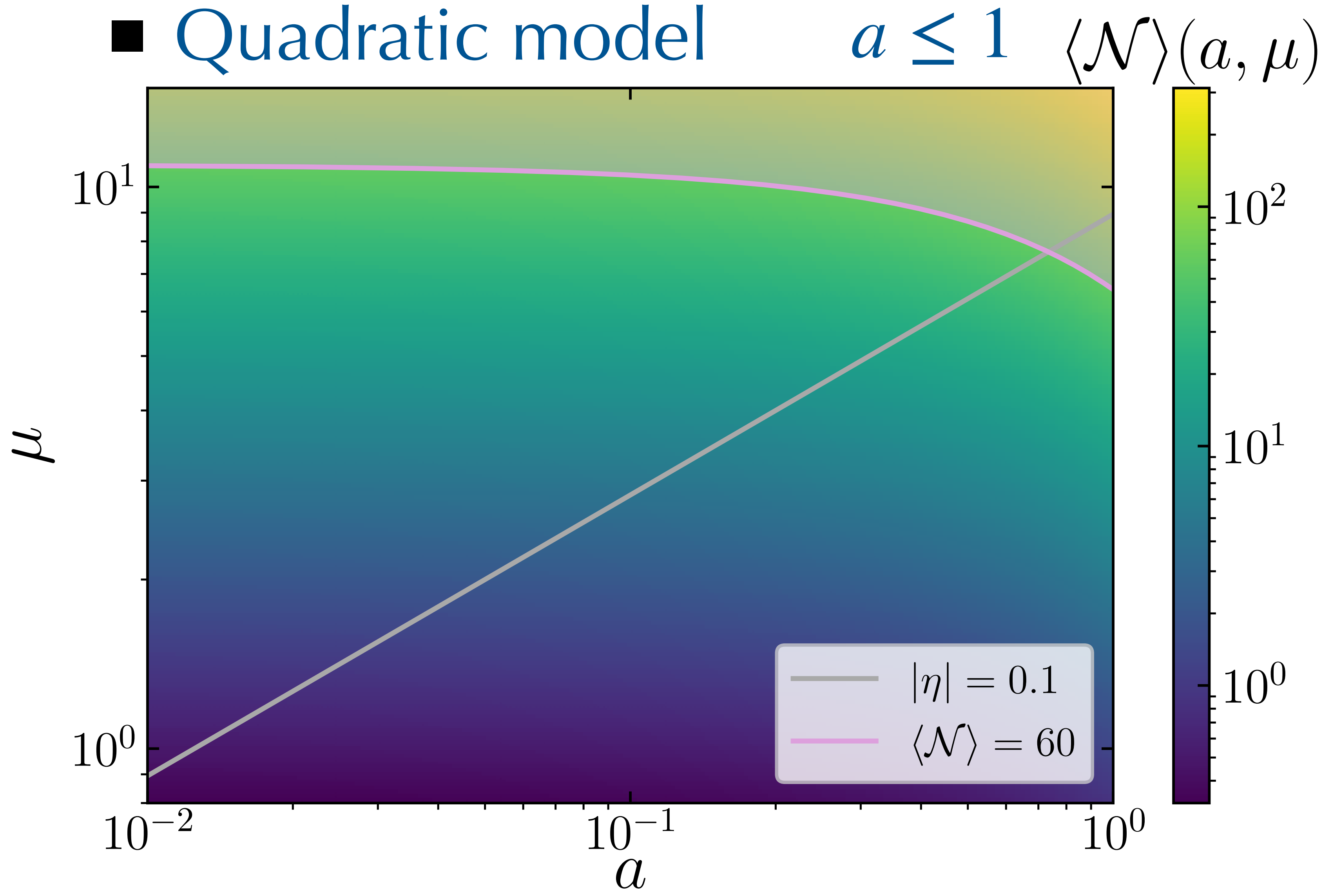
- $\langle \mathcal{N} \rangle$ features quadratic dependence on μ and exponential dependence on a
- μ constrained from below by slow-roll conditions

a not much larger than 1

- Linear model $a \leq 10$



- Quadratic model $a \leq 1$



- $\epsilon \ll 1 \;\Rightarrow\; \alpha \ll \dfrac{\Delta\phi}{M_{Pl}}$
- Two regimes: "shallow well" ($a \lesssim 1$)

  "deep well" ($a \gtrsim 1$)

- $\epsilon \ll 1 \;\Rightarrow \alpha \;\ll \dfrac{\Delta\phi}{M_{Pl}}$, $|\eta| \ll 1 \;\Rightarrow\; \alpha \ll \left(\dfrac{\Delta\phi}{M_{Pl}}\right)^2$

  $\Delta\phi \ll M_{Pl} \;\Rightarrow\; |\eta| \gg \epsilon$

- Only a "shallow-well" regime

δN formalism

FLRW metric: $ds^2 = -\,dt^2 + a^2(t)\,\delta_{ij}\,dx^i\,dx^j$

deviations from homogeneity and isotropy: $ds^2 = -\,dt^2 + a^2(t)\,e^{2\zeta(t,\vec{x})}\,\gamma_{ij}$

$t-$slices of uniform energy density

$x-$worldlines comoving

local scale factor: $\tilde{a}(t,\vec{x}) = a(t)\,e^{\zeta(t,\vec{x})}$

expansion from flat slice at time t_{in} to a slice of uniform energy density:

$$N(t,\vec{x}) = \log\left[\frac{\tilde{a}(t,\vec{x})}{a(t_{in})}\right] \qquad \zeta(t,\vec{x}) = N(t,\vec{x}) - N_0(t) \equiv \delta N \qquad N_0(t) = \log\left[\frac{a(t)}{a(t_{in})}\right]$$

Stochastic-δN formalism

Phase space field vector: $\Phi = (\phi_1, \pi_1, \cdots \phi_n, \pi_n)$

$$\Phi_{cg} = \frac{1}{(2\pi)^{3/2}} \int_{k<k_\sigma} d^3k \Phi_k e^{-ik\vec{x}}$$

$$\frac{d\Phi_{cg}}{dN} = F(\Phi_{cg}) + G(\Phi_{cg}) \cdot \xi$$

$$\langle \xi_i(\vec{x}_i, N_i)\, \xi_i(\vec{x}_j, N_j) \rangle = \delta_{ij}\, \delta(N_i - N_j)$$

$$(G^2)_{ij} = \frac{d \log k_\sigma}{dN} \mathscr{P}_{\Phi_i, \Phi_j} \left[k_\sigma(N), N \right]$$

$$\delta N_{cg}(\vec{x}) = \mathcal{N}(\vec{x}) - \langle \mathcal{N} \rangle = \zeta_{cg}(\vec{x}) = \frac{1}{(2\pi)^{3/2}} \int_{k_{in}}^{k_{end}} d\vec{k}\, \zeta_{\vec{k}}\, e^{i\vec{k}\cdot\vec{x}}$$

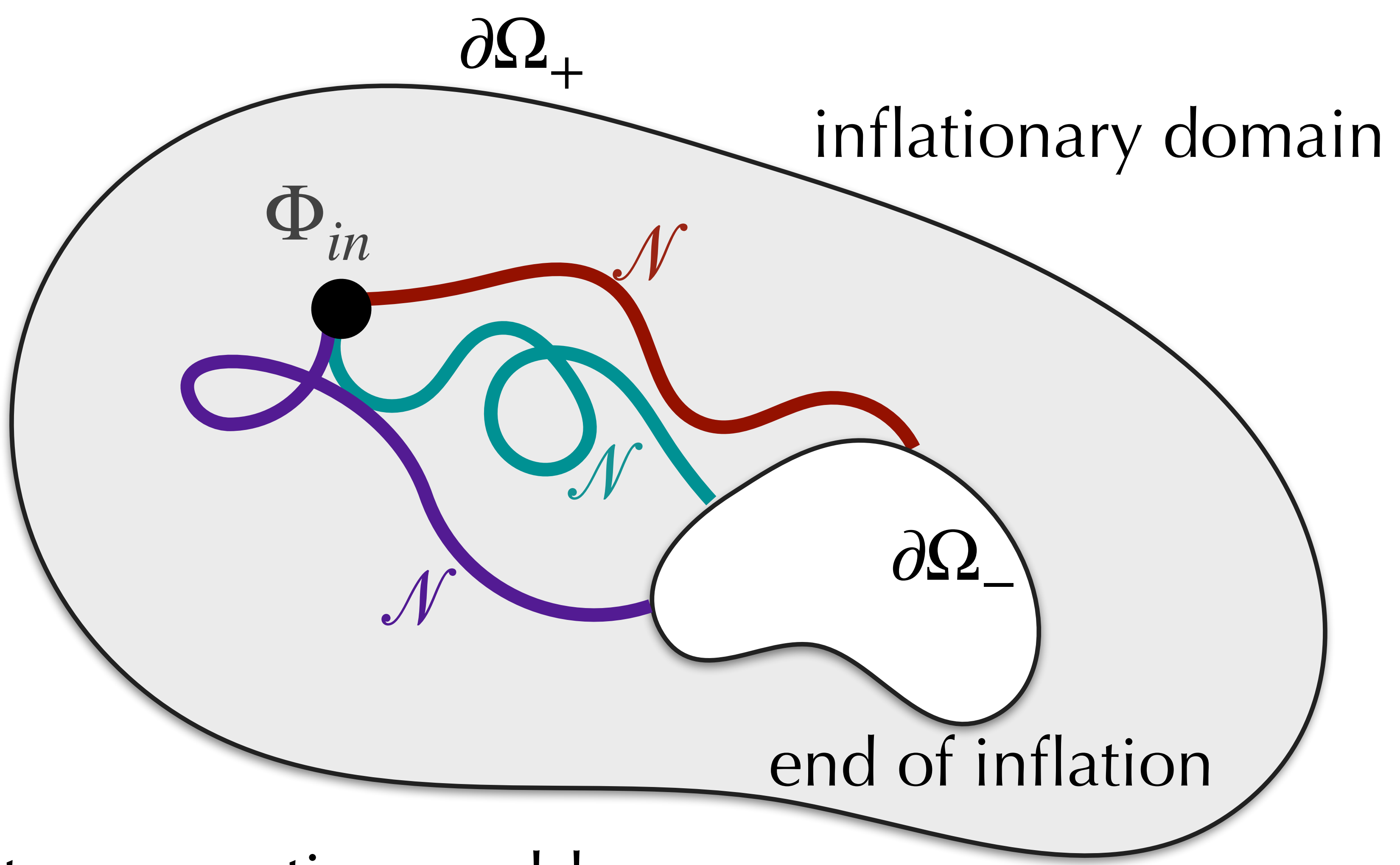
Curvature perturbation coarse grained between:
the scale that crosses the Hubble radius at initial time (k_{in}) and the scale that crosses the Hubble radius at final time k_{end}

First passage time distribution

$$\mathcal{L}^{\dagger}_{FP}(\Phi) = F_i(\Phi)\frac{\partial}{\partial \Phi_i} + \alpha G_{il}(\Phi)\frac{\partial G_{lj}(\Phi)}{\partial \Phi_l}\frac{\partial}{\partial \Phi_i} + \frac{1}{2}G_{il}(\Phi)G_{jl}(\Phi)\frac{\partial^2}{\partial \Phi_i \Phi_j}$$

$$\frac{\partial}{\partial \mathcal{N}}P(\mathcal{N},\Phi) = \mathcal{L}^{\dagger}_{FP}(\Phi)\cdot P(\mathcal{N},\Phi)$$

$$\int d\Phi f_1(\Phi)\left[\mathcal{L}_{FP}(\Phi)\cdot f_2(\Phi)\right] = \int d\Phi\left[\mathcal{L}^{\dagger}_{FP}(\Phi)\cdot f_1(\Phi)\right]f_2(\Phi)$$



first passage time problem

Boundary conditions $\partial\Omega = \partial\Omega_- \cup \partial\Omega_+$

$\partial\Omega_-$: all moments of the FPT vanish on $\partial\Omega_-$ (absorbing boundary)

Sometimes additional conditions required on $\partial\Omega_+$: absorbing or reflective boundary (gradients of moments projected onto the orthogonal direction to the tangent surface of $\partial\Omega_+$ vanish)

hierarchy of coupled differential equations: $\mathcal{L}^{\dagger}_{FP}(\Phi^{in})\cdot\langle\mathcal{N}^n\rangle(\Phi^{in}) = -n\langle\mathcal{N}^{n-1}\rangle(\Phi^{in})$

Fokker-Planck equation

Evolution given by the Langevin equation: $$\Phi(N+\delta N) = \Phi(N) + F(\Phi)\delta N + G(\Phi)\cdot\int_N^{N+\delta N} d\tilde{N}\,\xi(\tilde{N})$$

Where to evaluate F and G? At $\Phi(N)$ or at $\Phi(N+\delta N)$?

$$\Phi_\alpha(N) = (1-\alpha)\Phi(N) + \alpha\Phi(N+\delta N) \qquad 0 \leq \alpha \leq 1$$

Itô prescription: $\alpha = 0$

Stratonovitch prescription: $\alpha = \dfrac{1}{2}$

$$\Phi(N+\delta N) = \Phi(N) + F[\Phi_\alpha(N)]\delta N + G[\Phi_\alpha(N)]\cdot\int_N^{N+\delta N} d\tilde{N}\,\xi(\tilde{N})$$

Fokker-Planck equation: $$\frac{\partial}{\partial N}P(\Phi, N\,|\,\Phi^{in}, N_{in}) = \mathscr{L}_{FP}(\Phi)P(\Phi, N\,|\,\Phi^{in}, N_{in})$$

$$\mathscr{L}_{FP}(\Phi) = -\frac{\partial}{\partial\Phi_i}\left[F_i(\Phi) + \alpha G_{lj}(\Phi)\frac{\partial G_{ij}(\Phi)}{\partial\Phi_l}\right] + \frac{1}{2}\frac{\partial^2}{\partial\Phi_i\Phi_j}G_{il}(\Phi)G_{jl}(\Phi)$$