



# Stellar Dynamics in Galactic Nuclei

Jean-Baptiste Fouvry

in collaboration with K. Tep, J. Giral Martinez, N. Magnan  
C. Pichon, P.-H. Chavanis, W. Dehnen, S. Tremaine, B. Bar-Or

TUG meeting, ENS Paris  
October 2023



# Stellar Dynamics in Galactic Nuclei

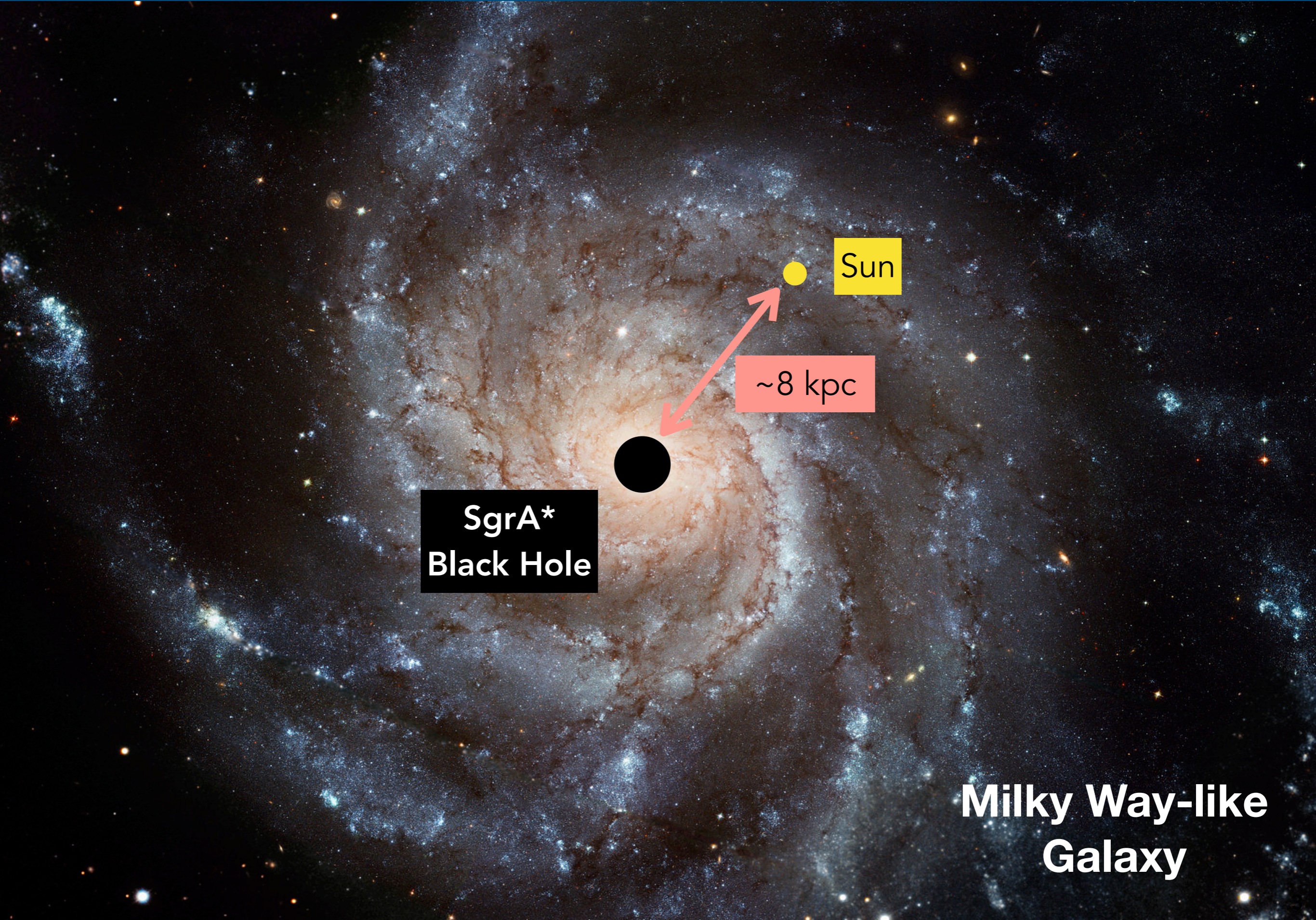
*and constraining IMBHs  
around SgrA\**

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**SgrA\***  
**Black Hole**

**Sun**

**~8 kpc**

**Milky Way-like  
Galaxy**



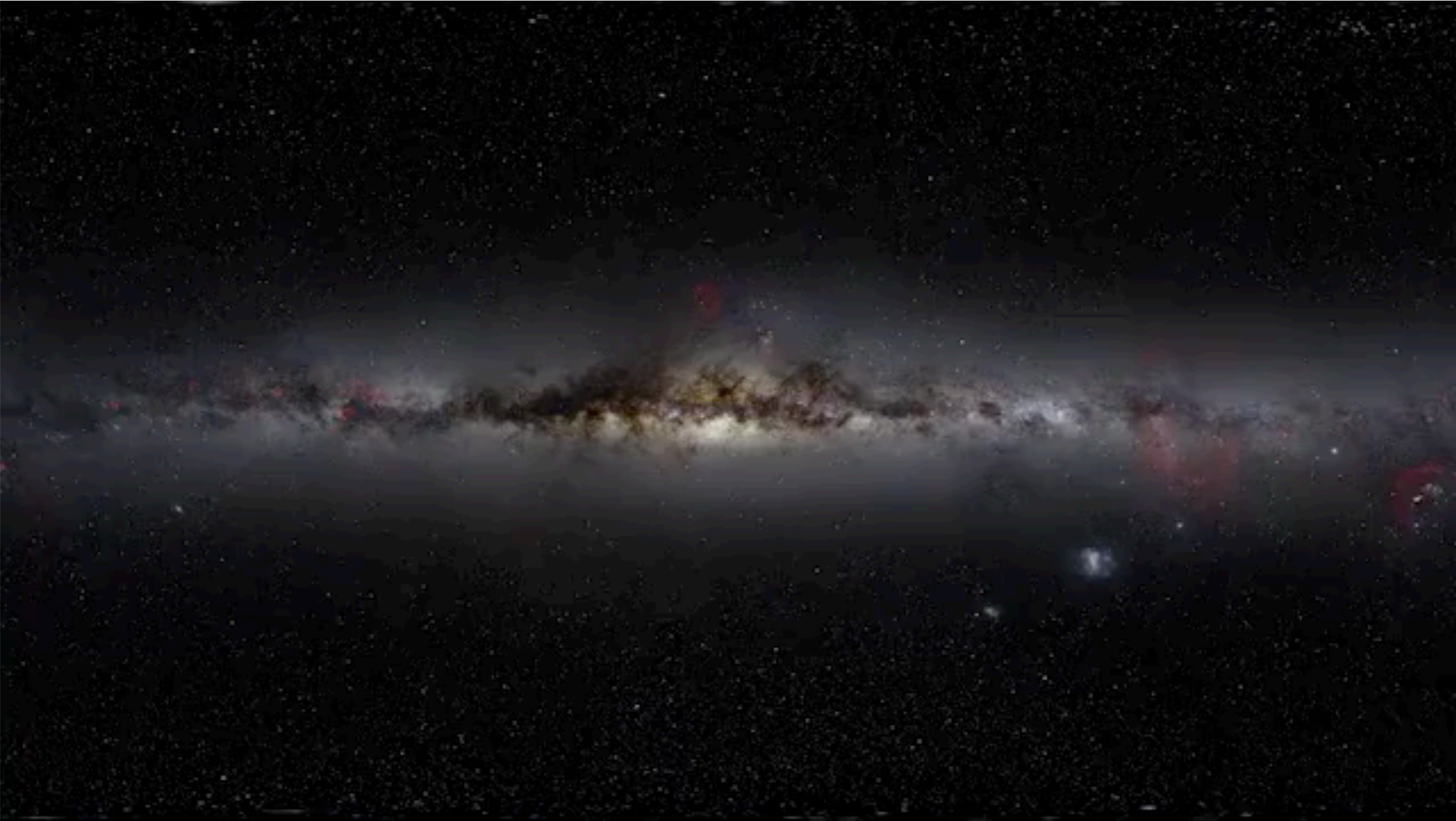
# SgrA\*, our Galactic Centre



← Milky Way (10<sup>17</sup> km) →



# SgrA\*, our Galactic Centre

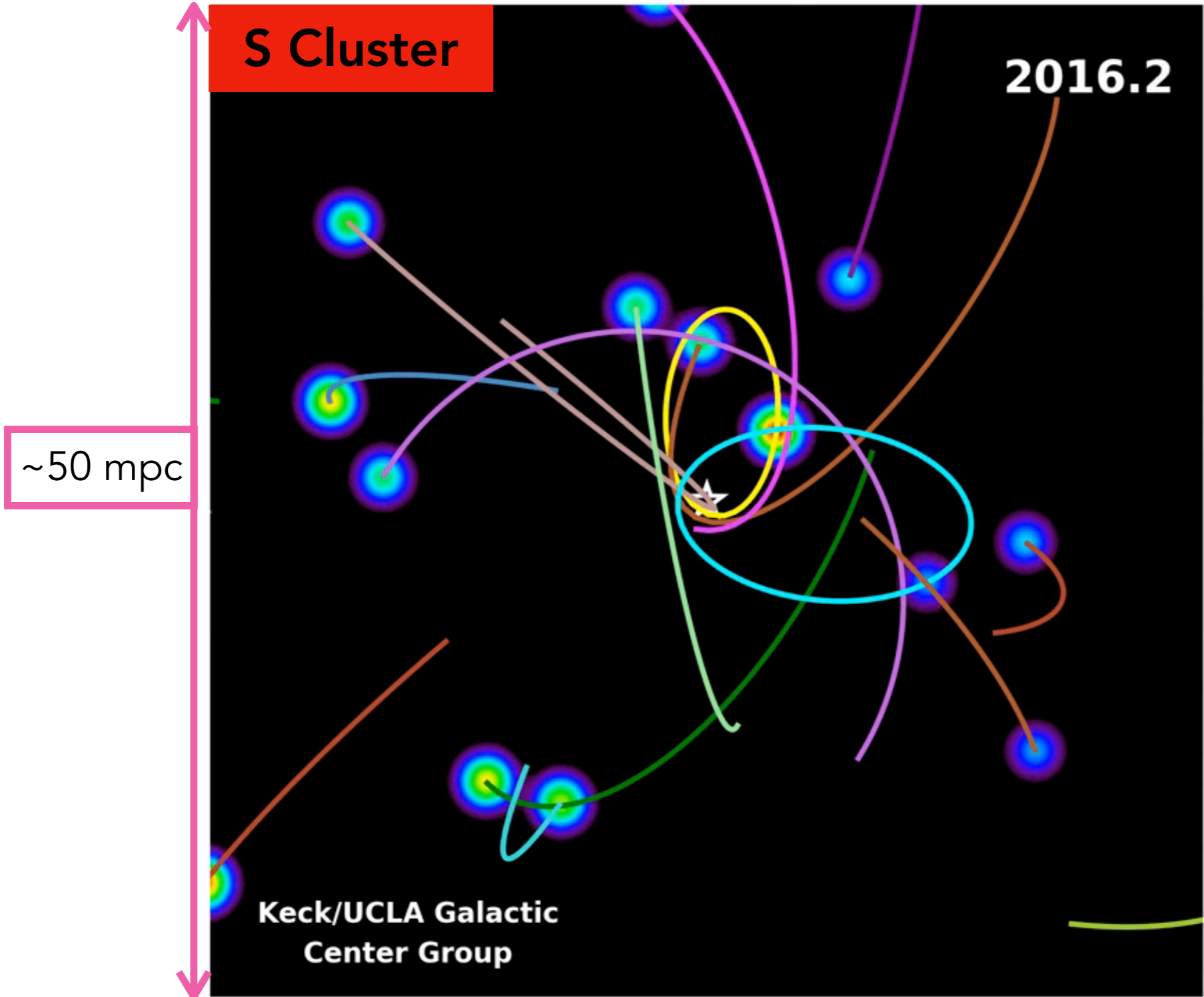


Zoom (x10,000,000)

ESO



# SgrA\*, at the heart of the Milky Way

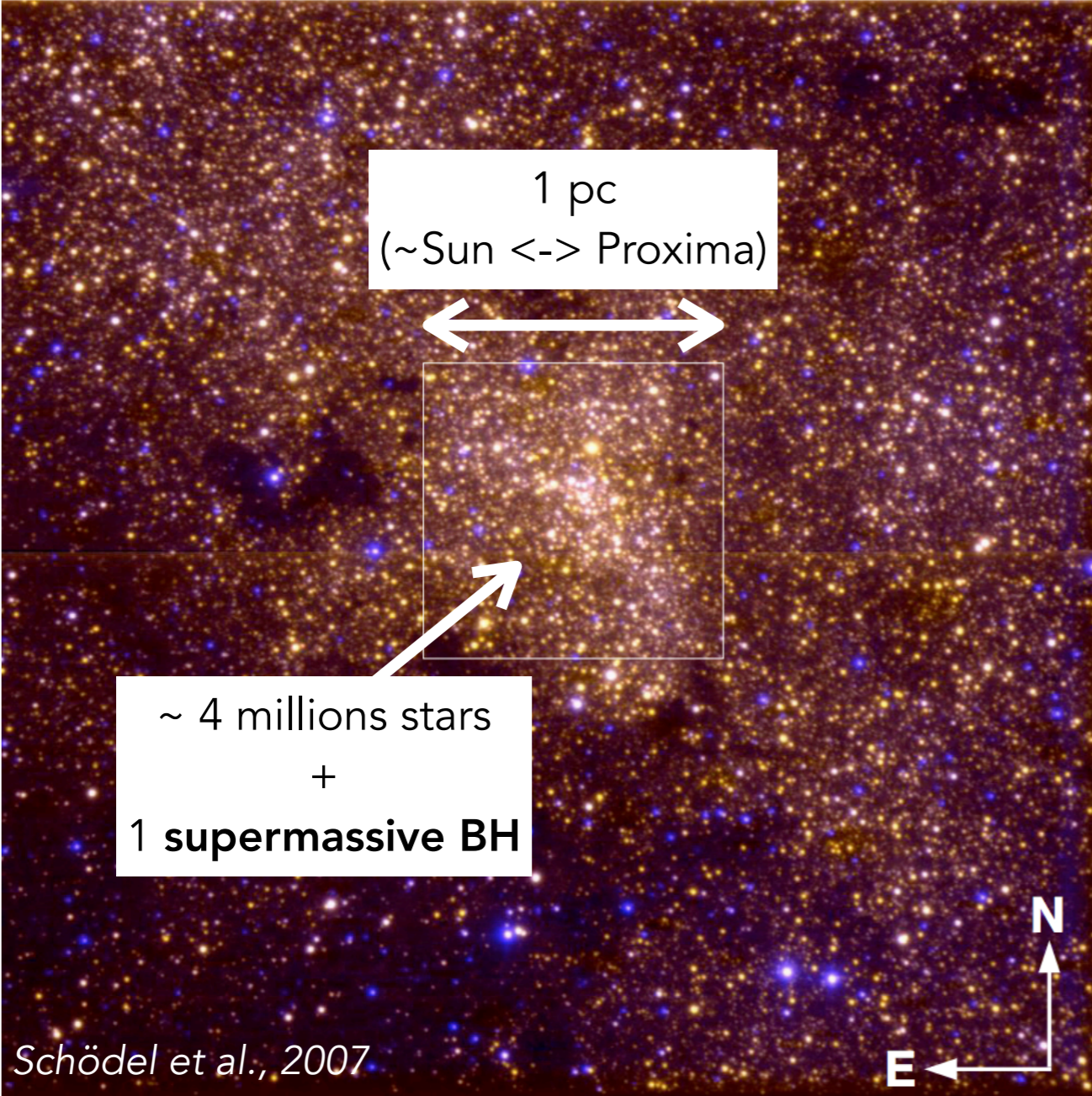


What is the diet of **supermassive black holes**?

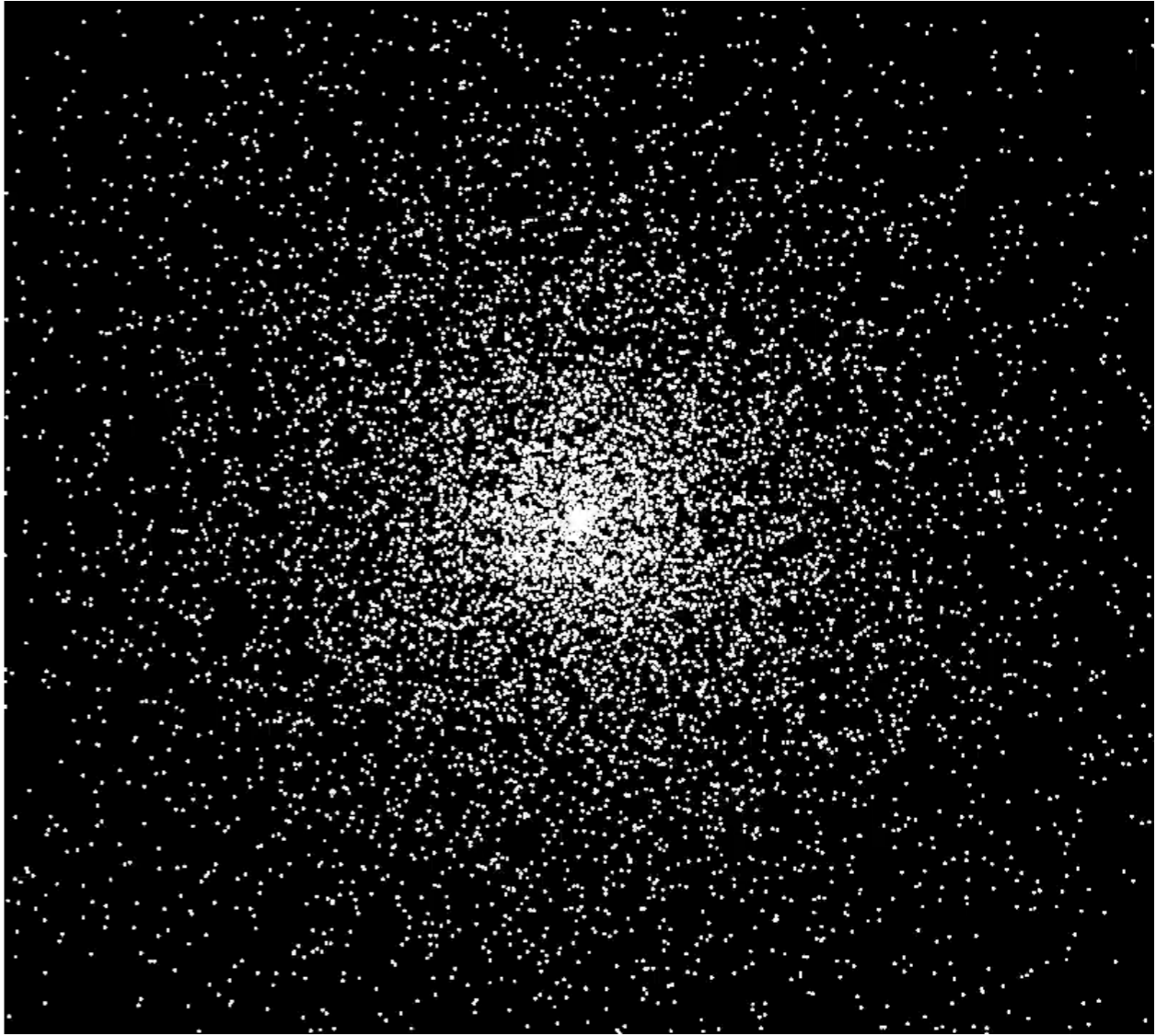


# Extremely dense environment

Behaves like a **gas of stars**



VLT observations



Numerical simulations



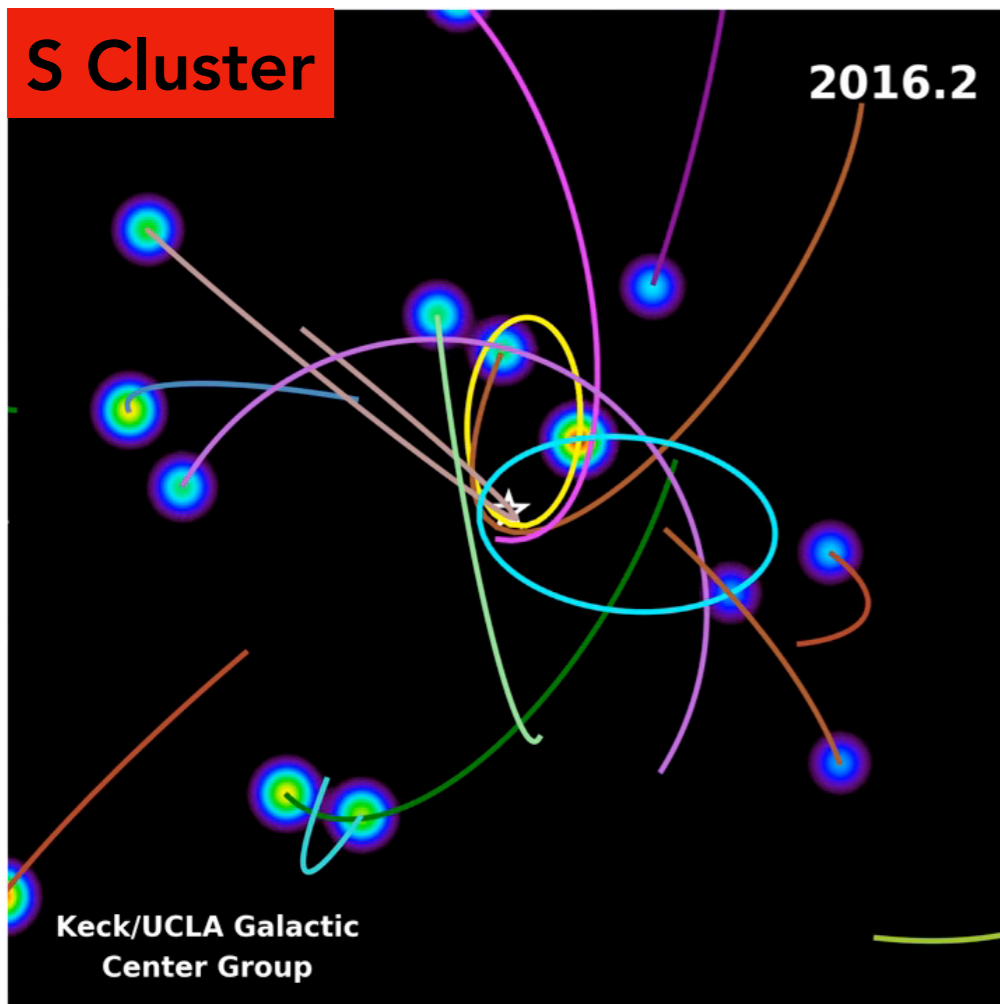
## A simple dynamics?

The central BH is **supermassive**

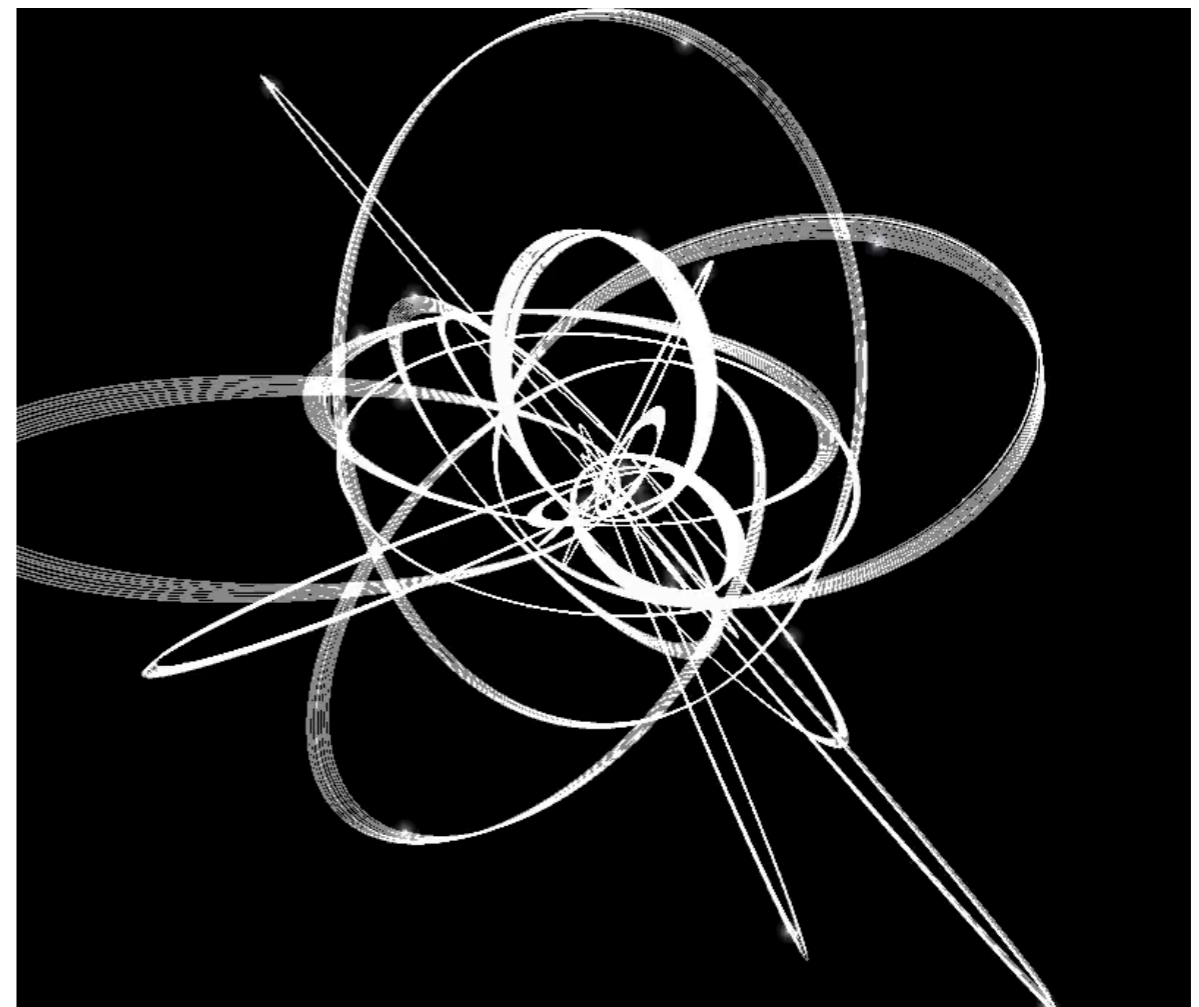
$$M_{\text{SgrA}} \simeq 4,200,000 \times M_{\text{Sun}}$$

vs.

$$M_{\text{Sun}} \simeq 330,000 \times M_{\text{Earth}}$$



Keck observations



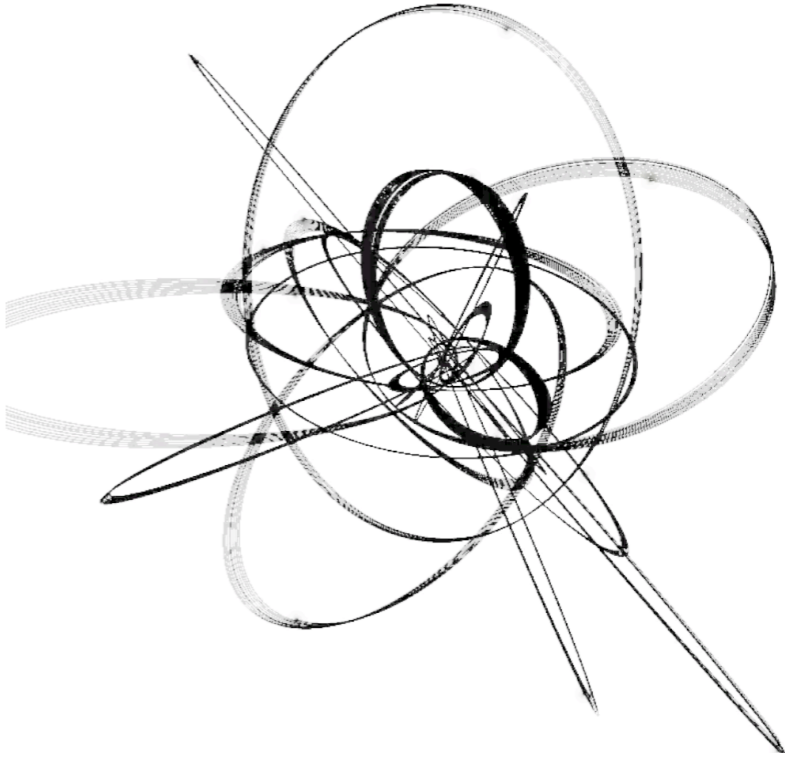
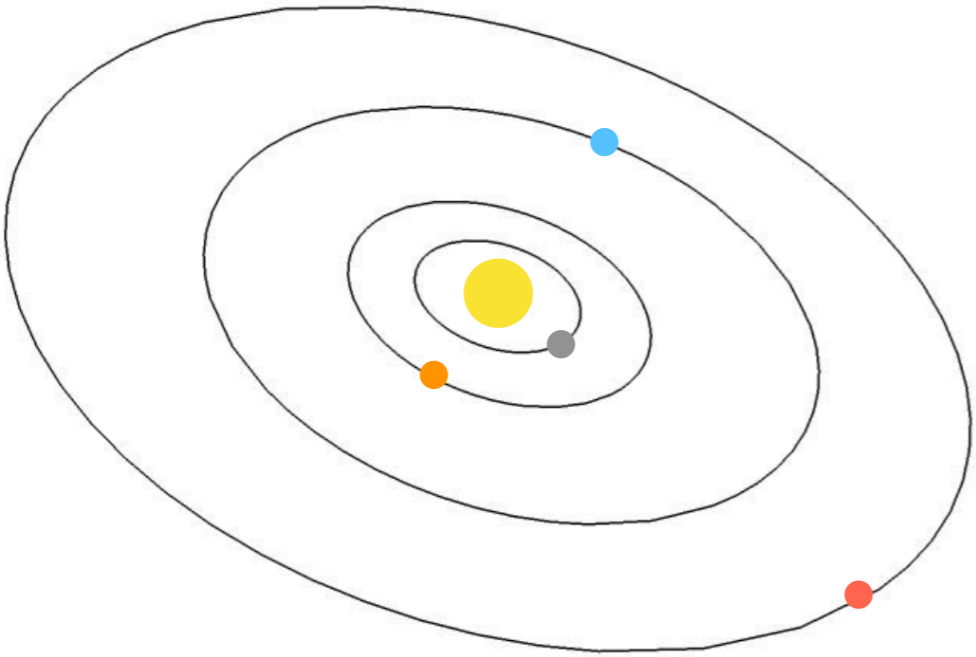
Numerical simulations

Like the Earth around the Sun, stars follow **Keplerian orbits**

# Keplerian systems

Solar system

Galactic centre

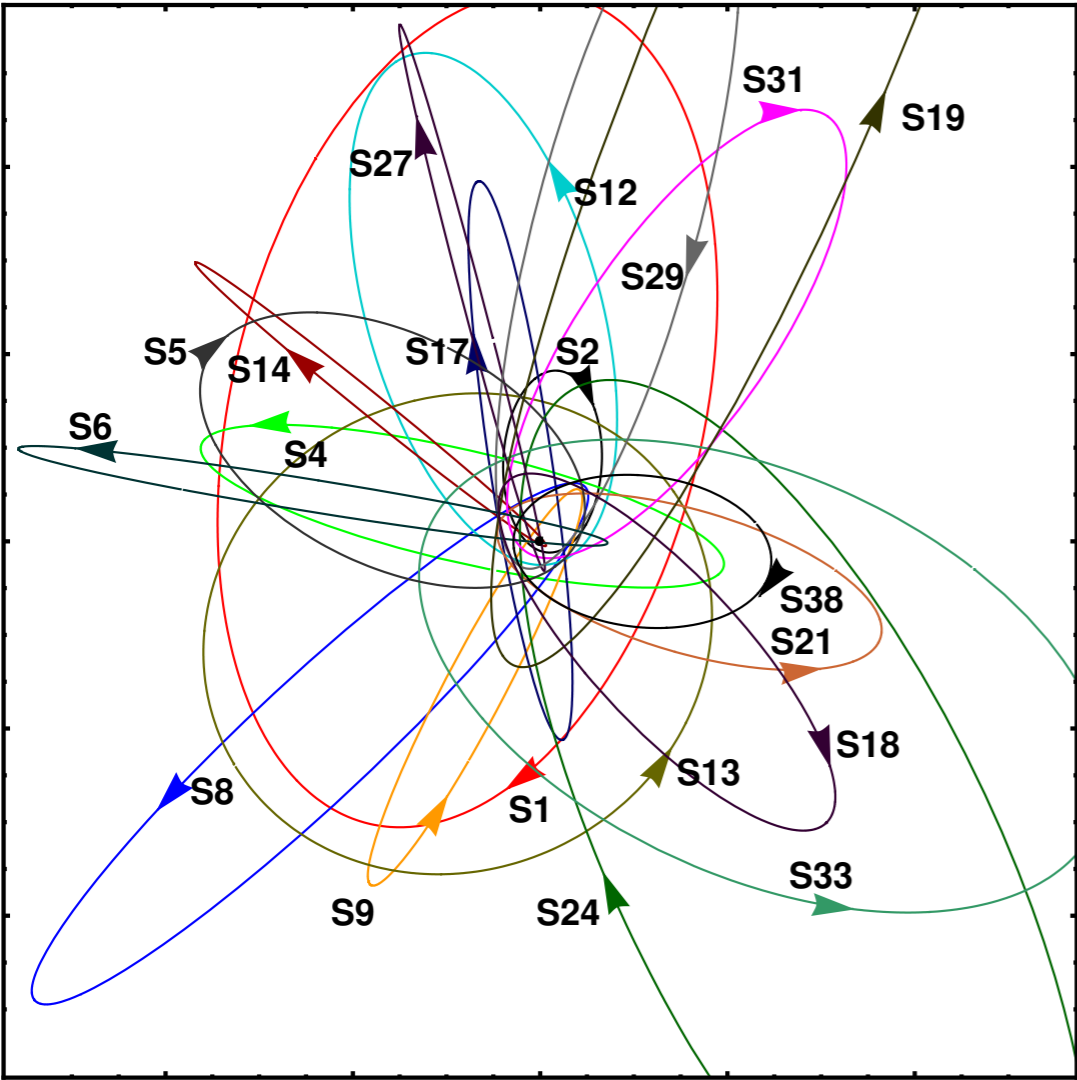


<b>Planets</b>	← Light object →	<b>Stars</b>
<b>Sun</b>	← Heavy object →	<b>Black hole</b>
$N \simeq 10$	← Number of "particles" →	$N \simeq 10^6$
<b>Planar</b> symmetry	← Shape of the system →	<b>Spherical</b> symmetry
<b>Quasi-circular</b> orbits	← Shape of the orbits →	<b>Very eccentric</b> orbits



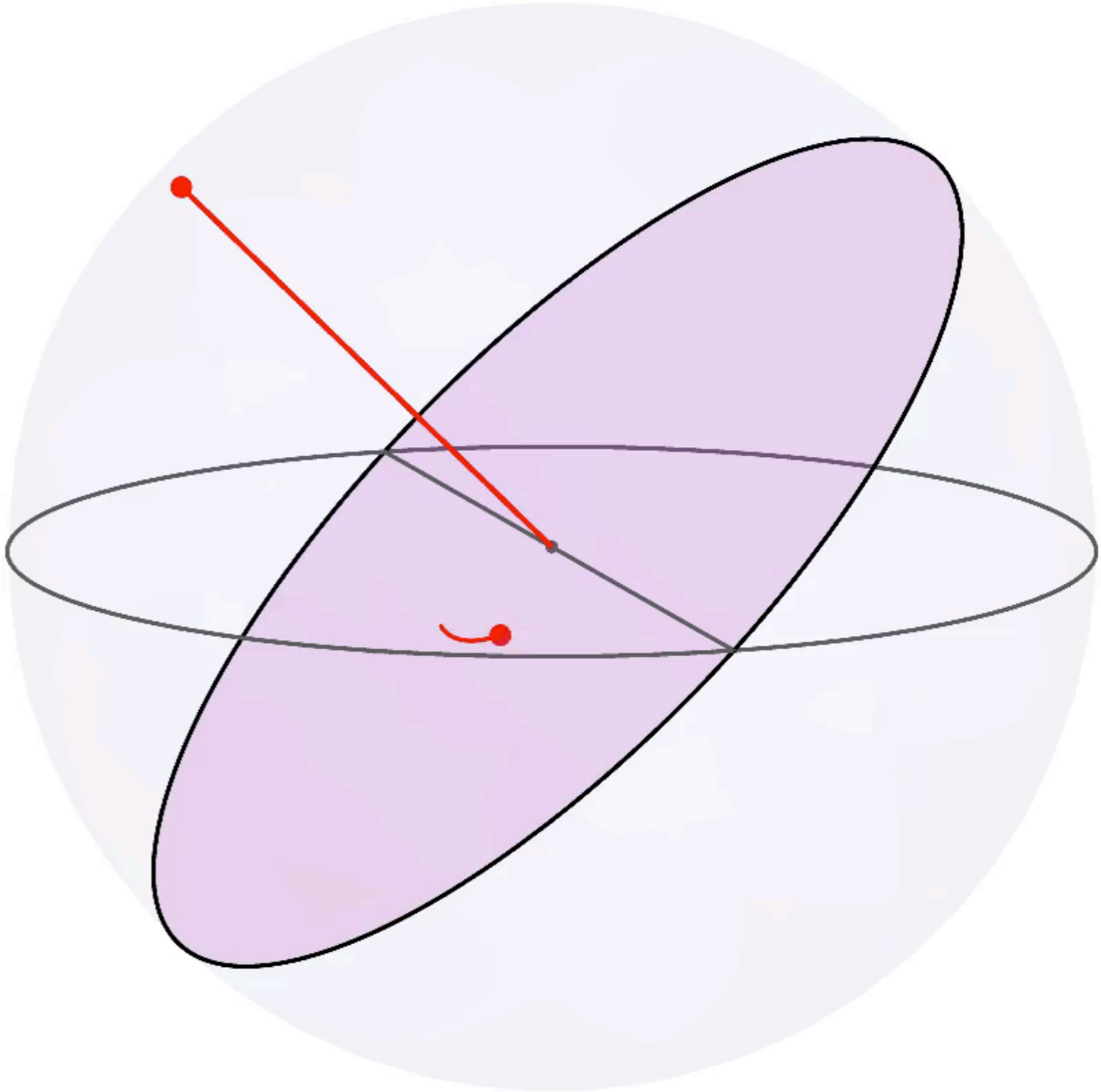
# Keplerian orbits

The BH dominates the stars' dynamics



*Gillessen et al., 2009*

VLT observations



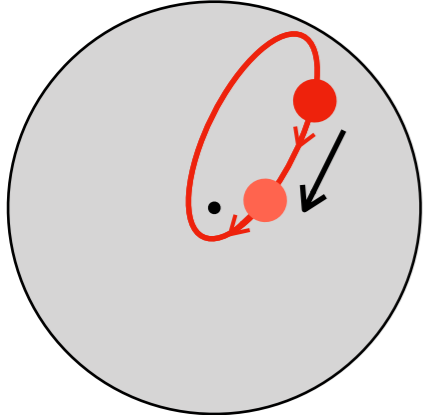
Typical orbit



# What is an orbit?

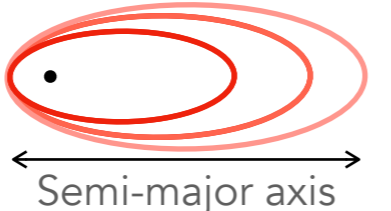
## Describing an orbit

### Position of the star

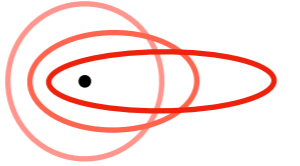


Dynamical motion

### Shape of the orbit

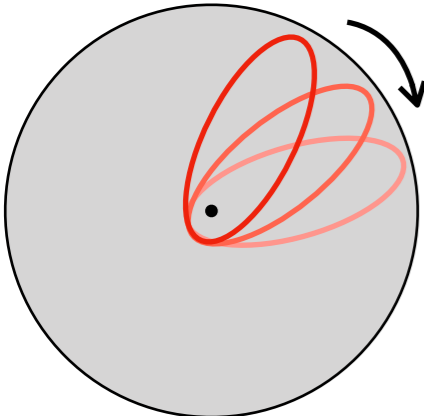


Semi-major axis



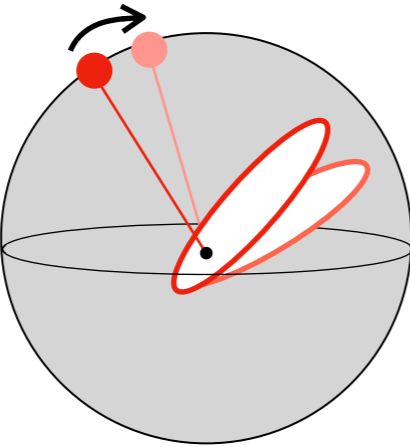
Eccentricity

### Phase of the orbit

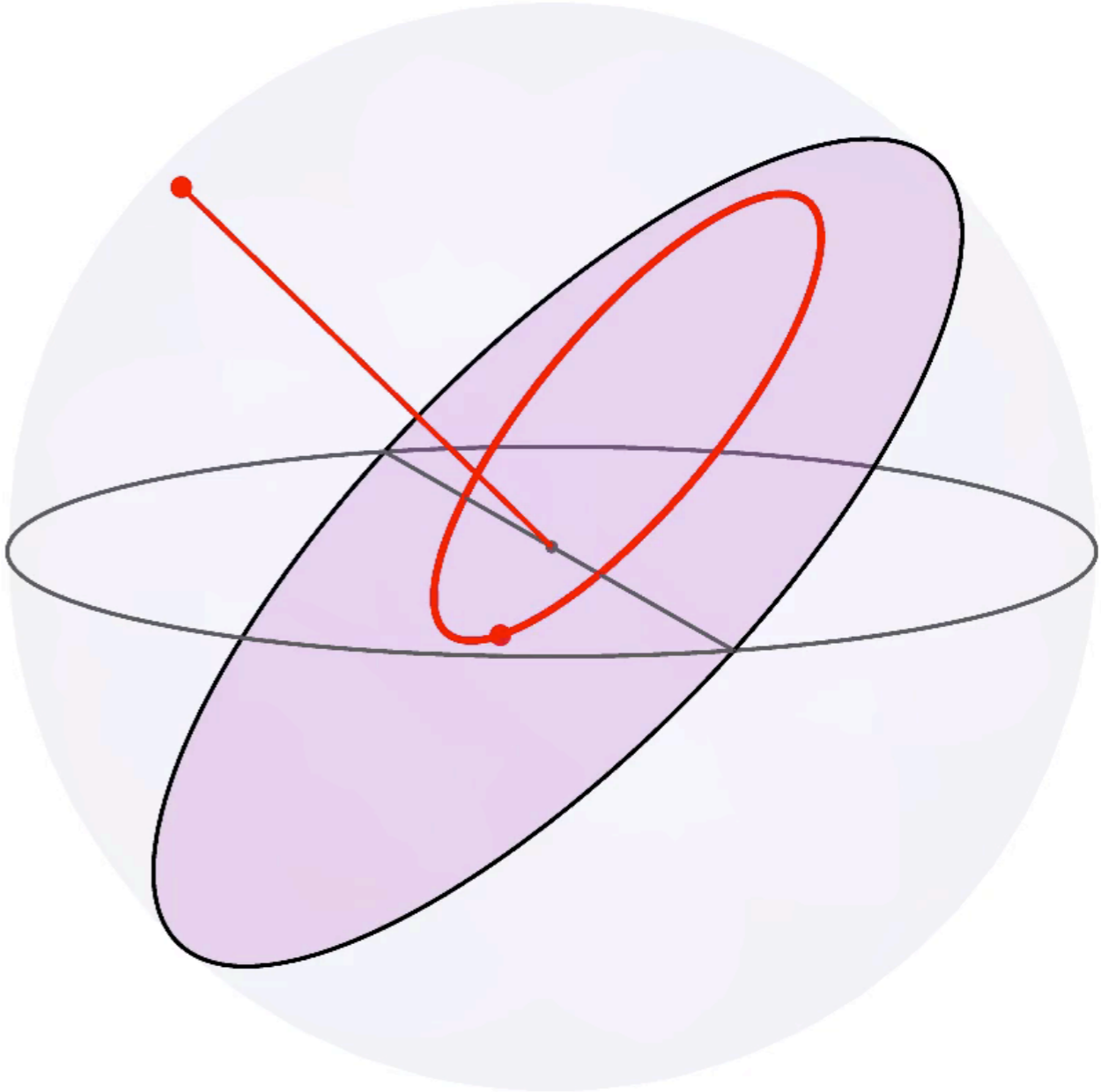


Phase of the pericentre

### Orientation of the orbit



Spatial orientation

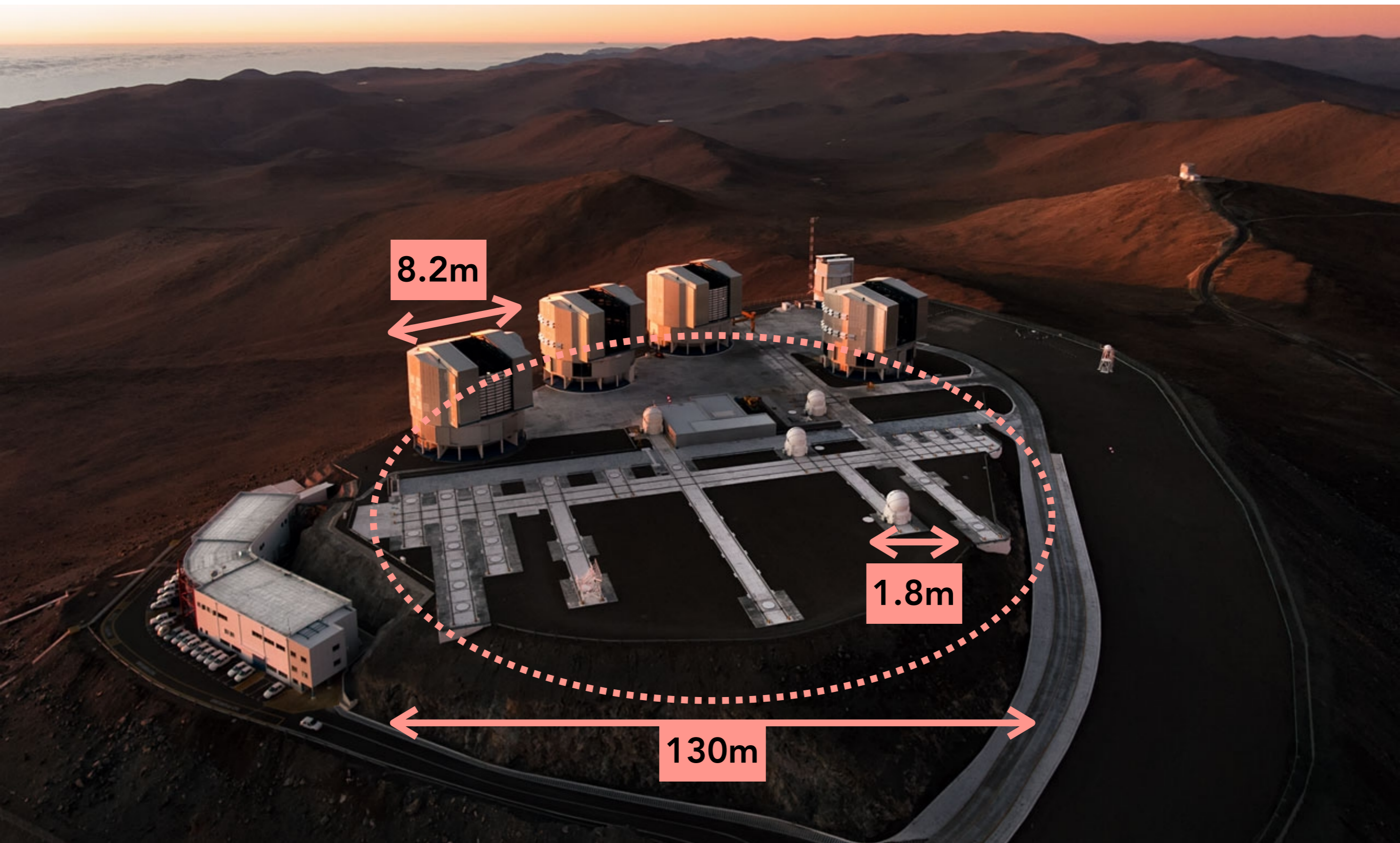


Keplerian orbit

## What is the dynamics of Keplerian orbits?



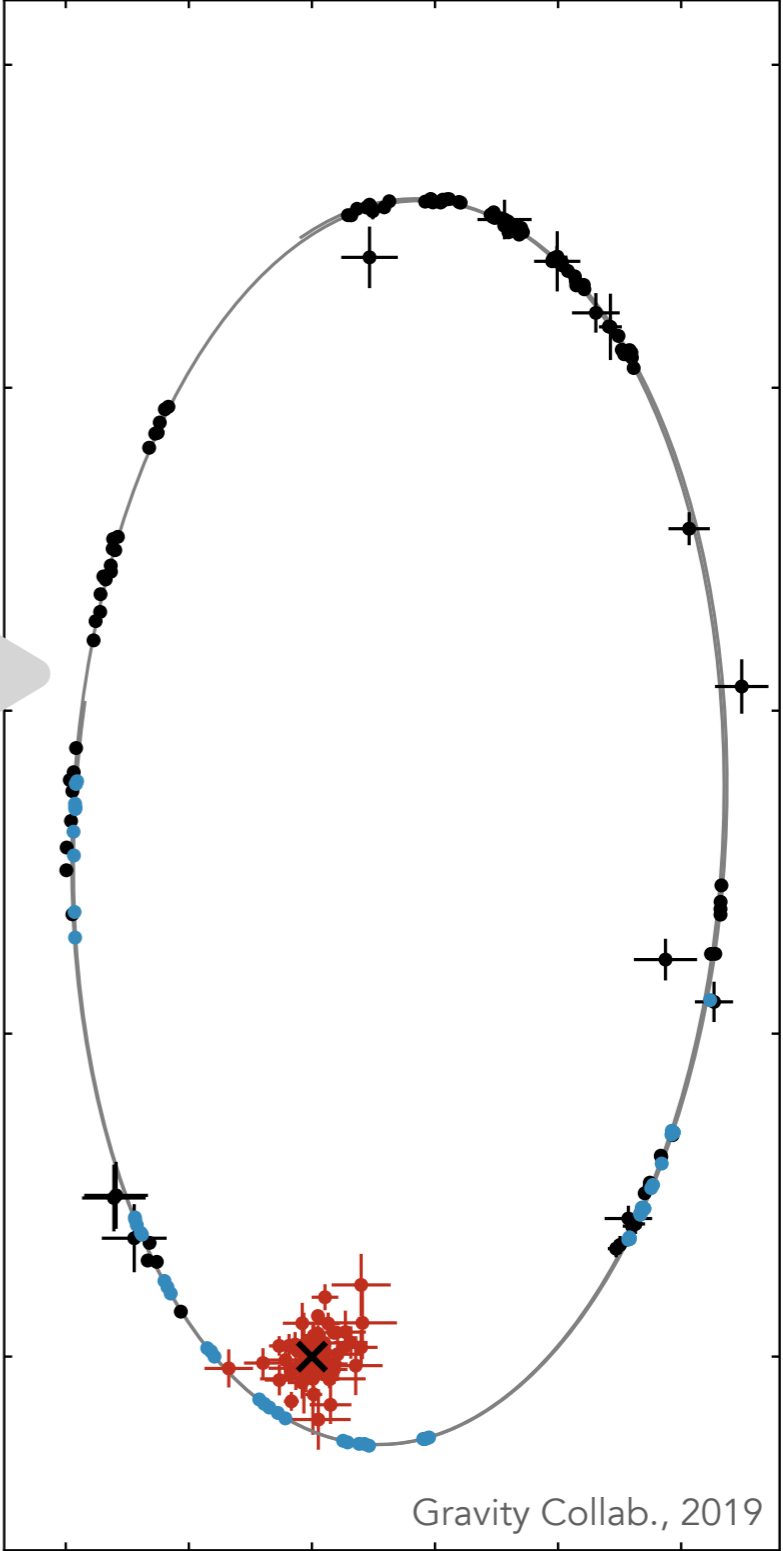
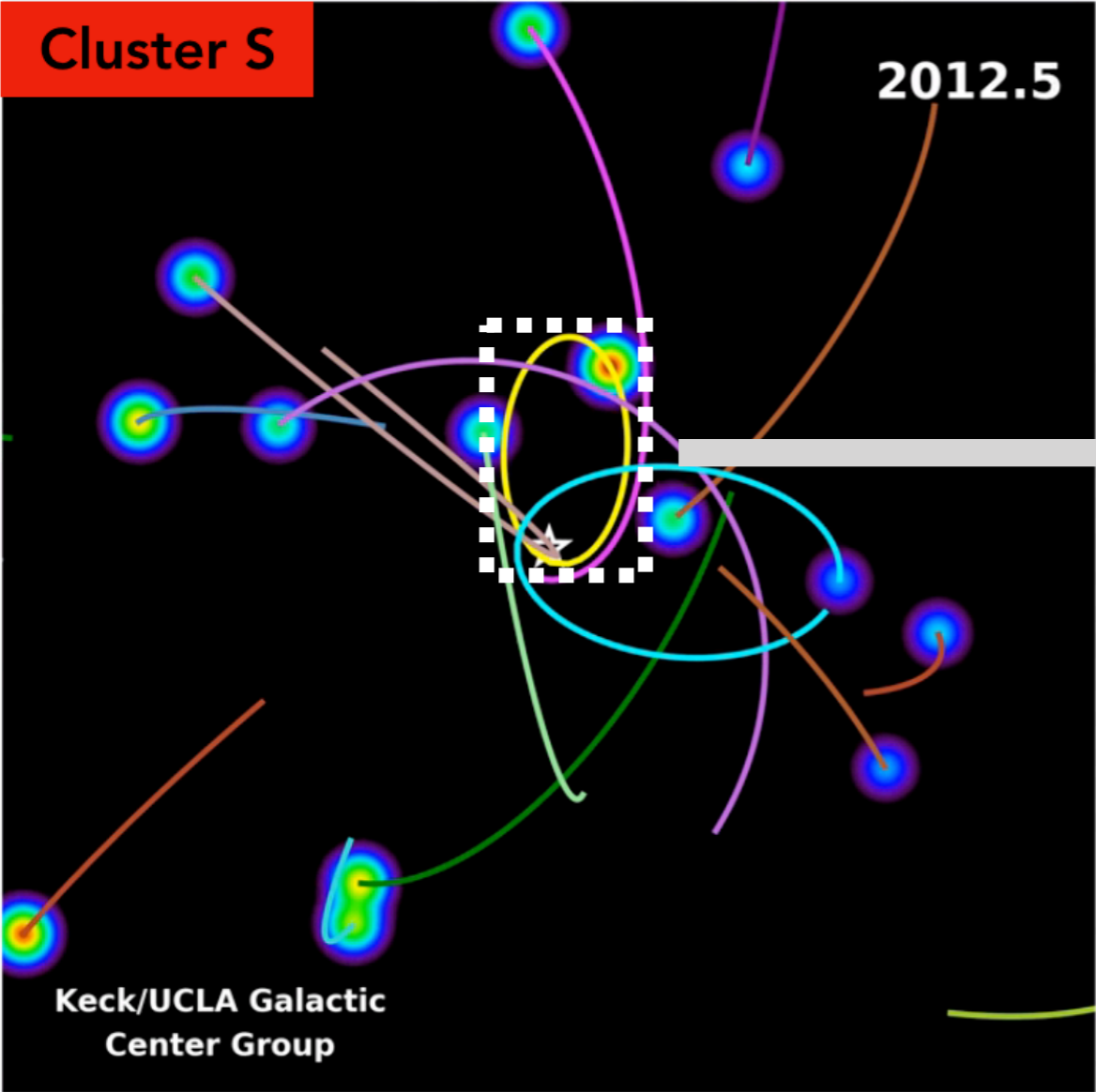
# Gravity Interferometer – VLTI – Chili





# S2's observation

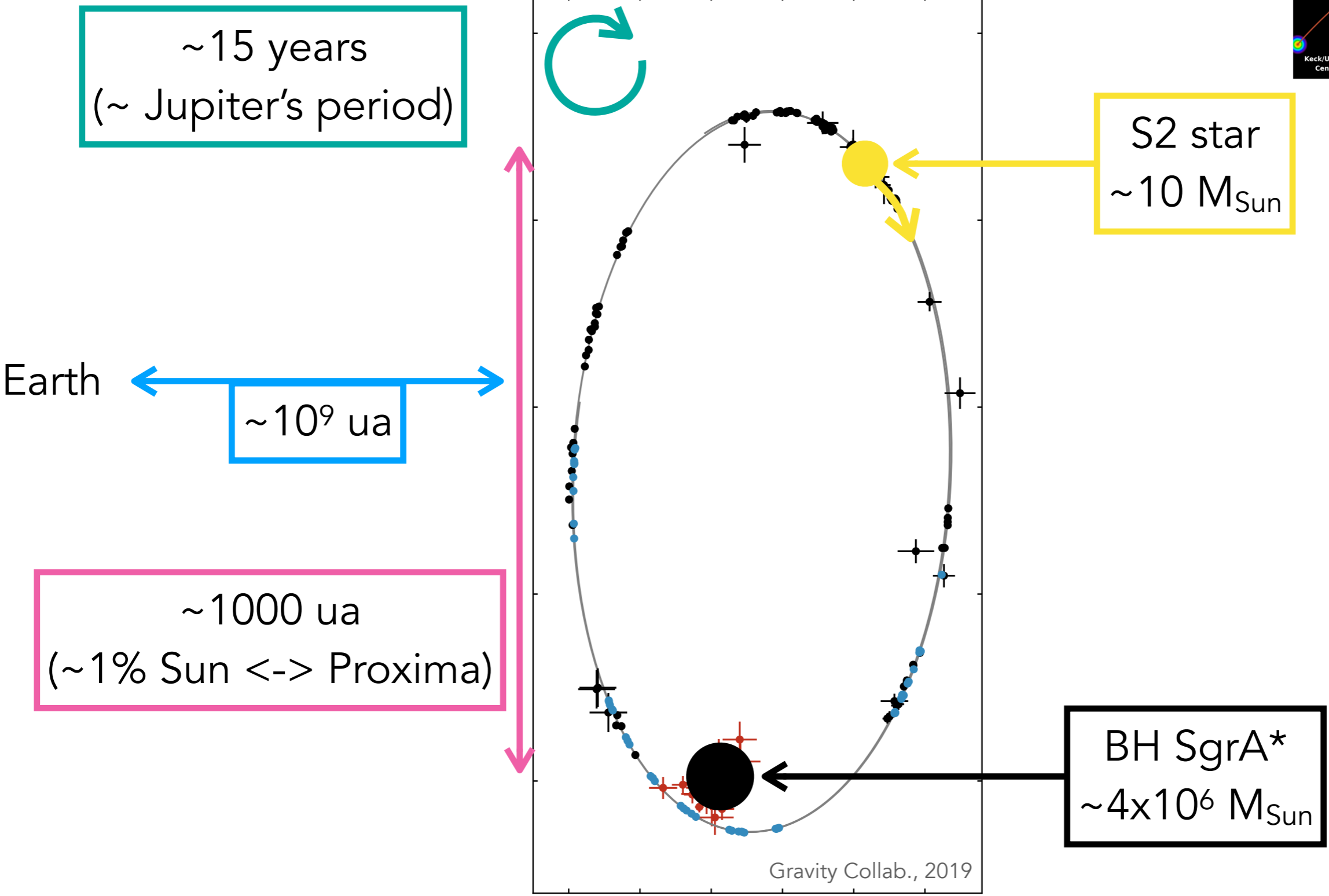
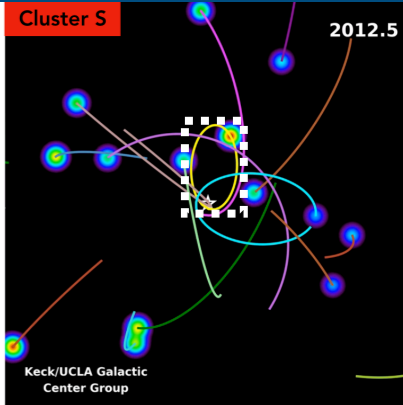
S2's observations (Gravity)



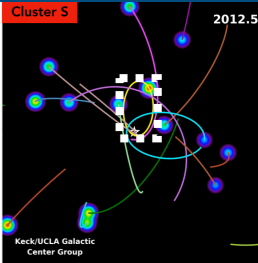
Classical (adaptative) telescope

Interferometric telescope

# S2's observation (Gravity)

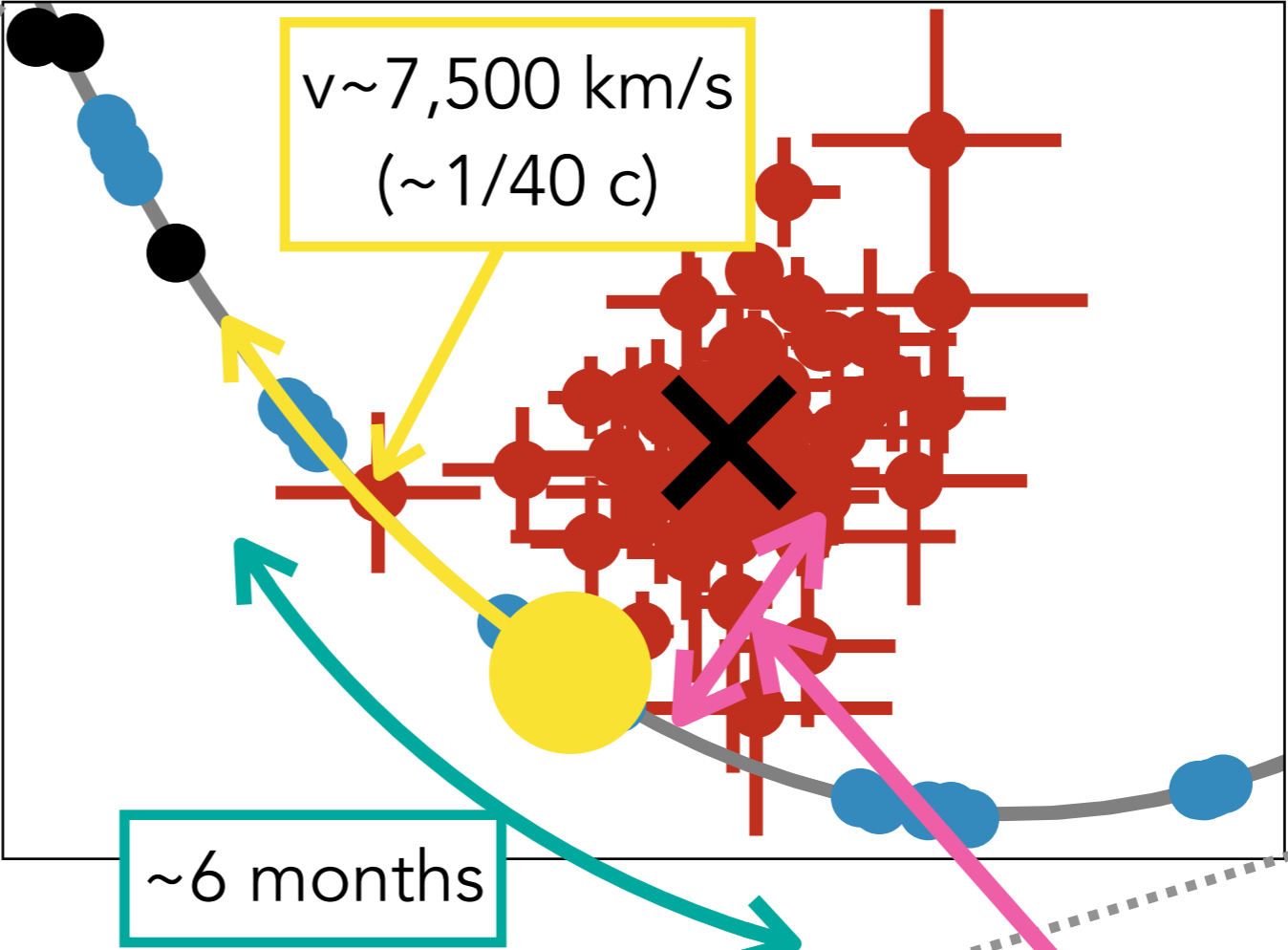
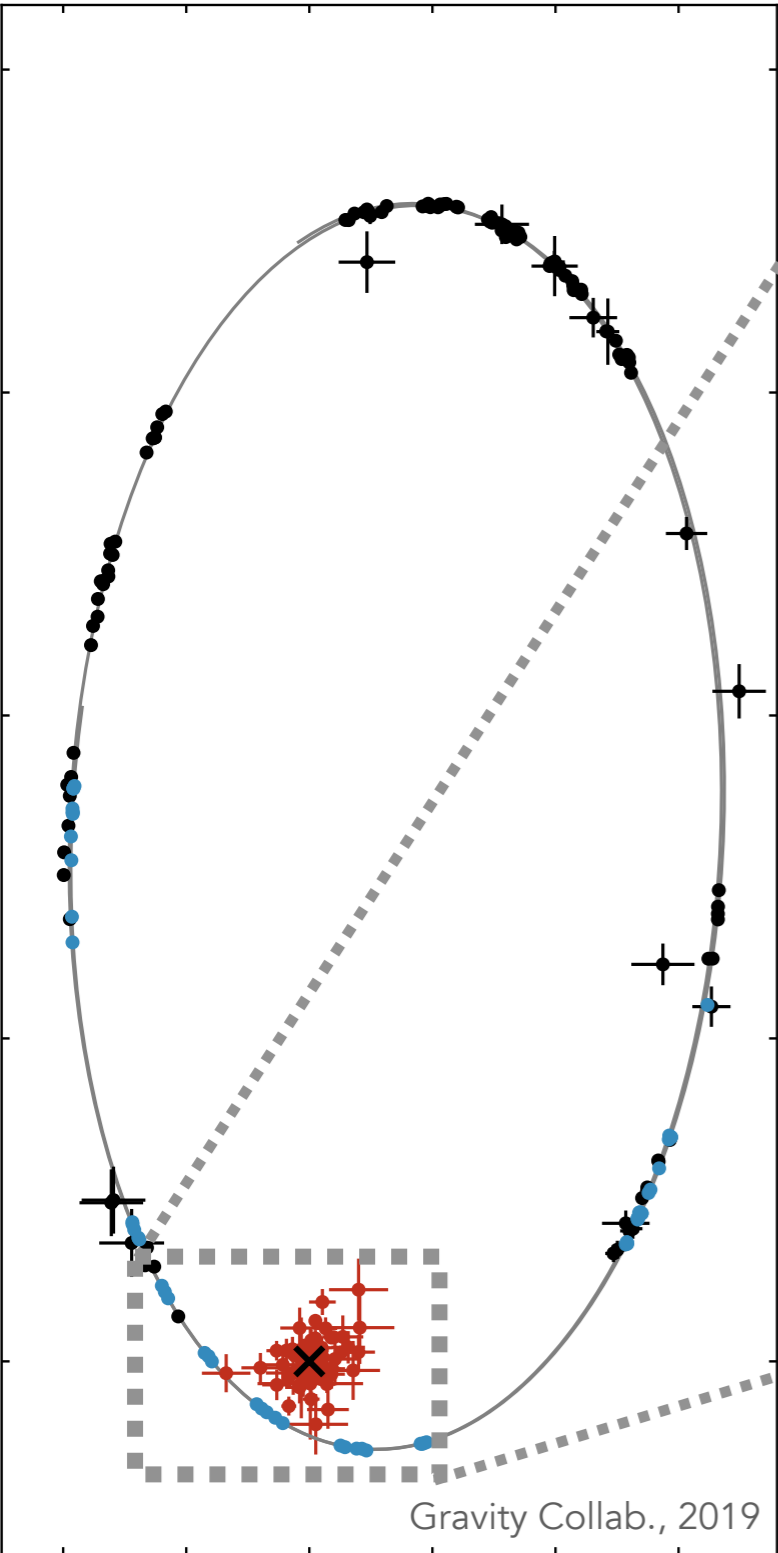






# Pericentre passage

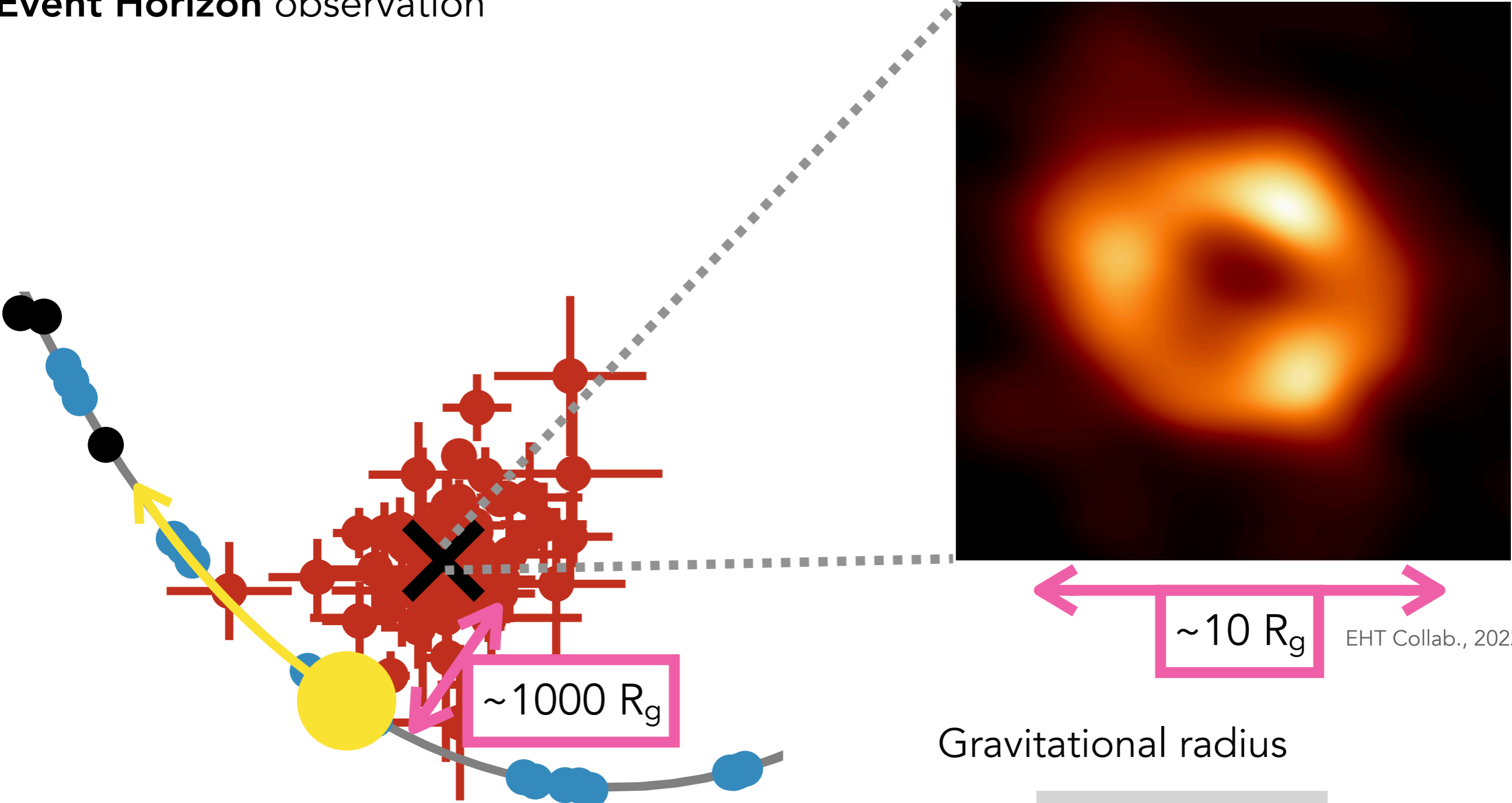
S2's observations (Gravity)



100 ua  
(~ 2x Sun <-> Neptune)

# Black hole's shadow

Event Horizon observation



$\sim 10 R_g$  EHT Collab., 2022

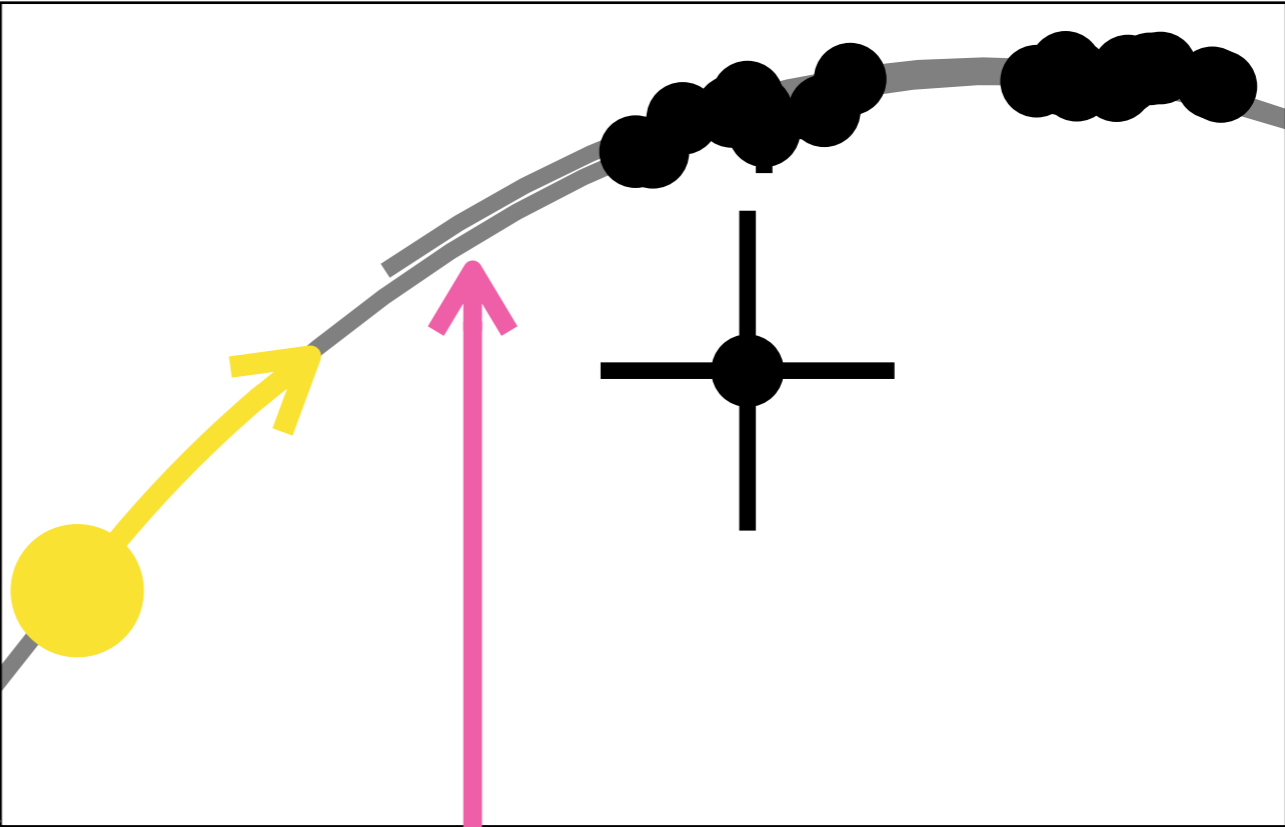
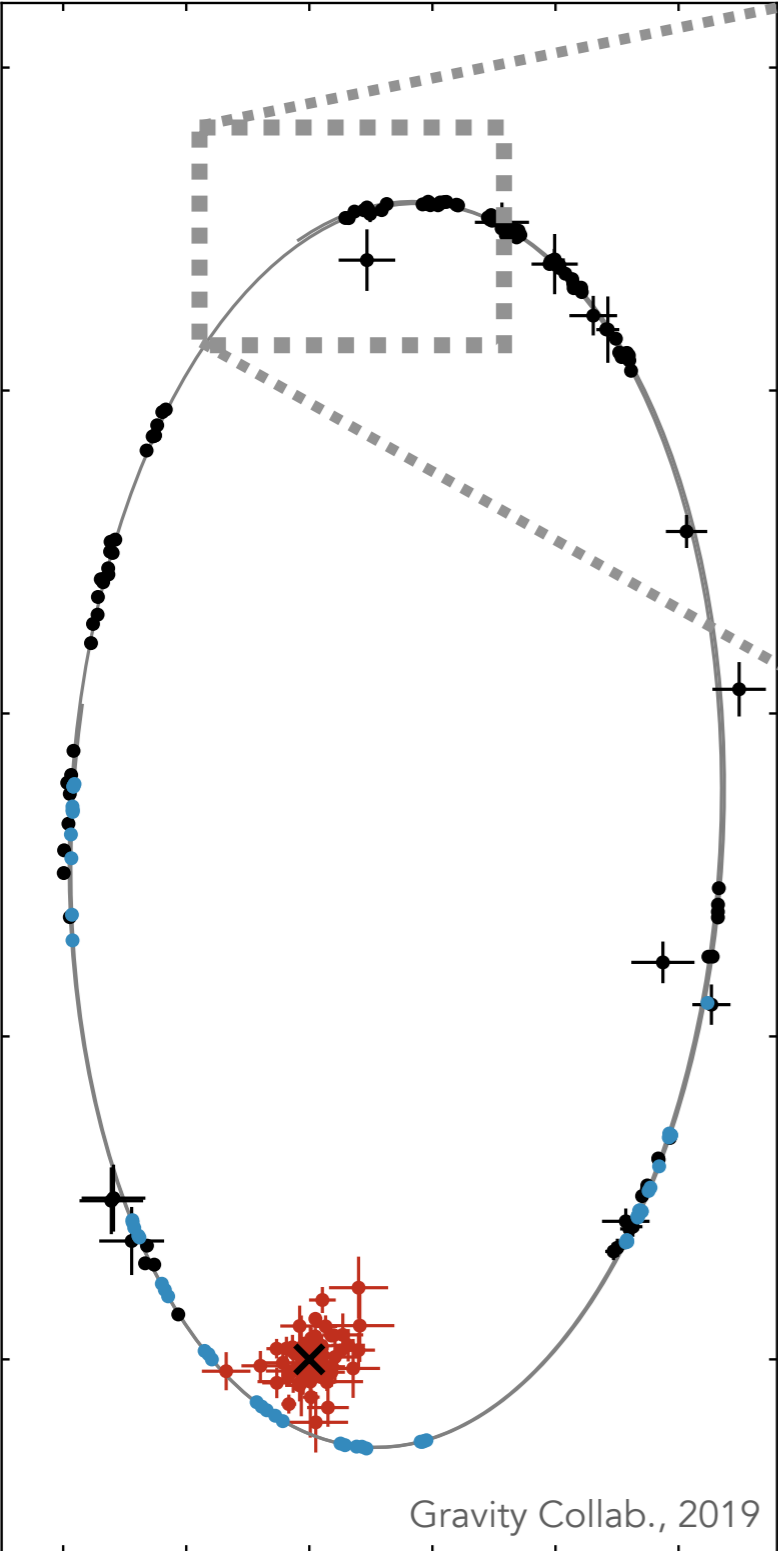
Gravitational radius

$$R_g = \frac{GM_\bullet}{c^2}$$



# Relativistic precession

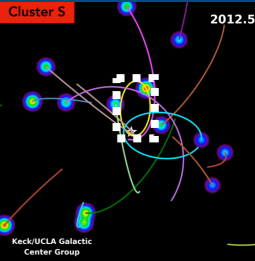
Observation of S2 (Gravity)



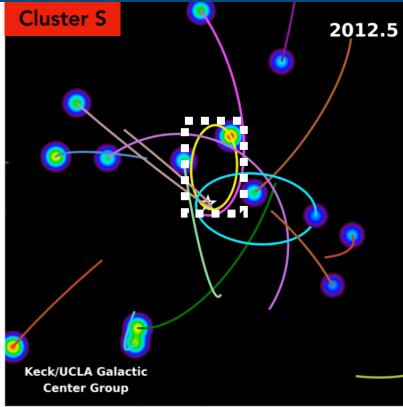
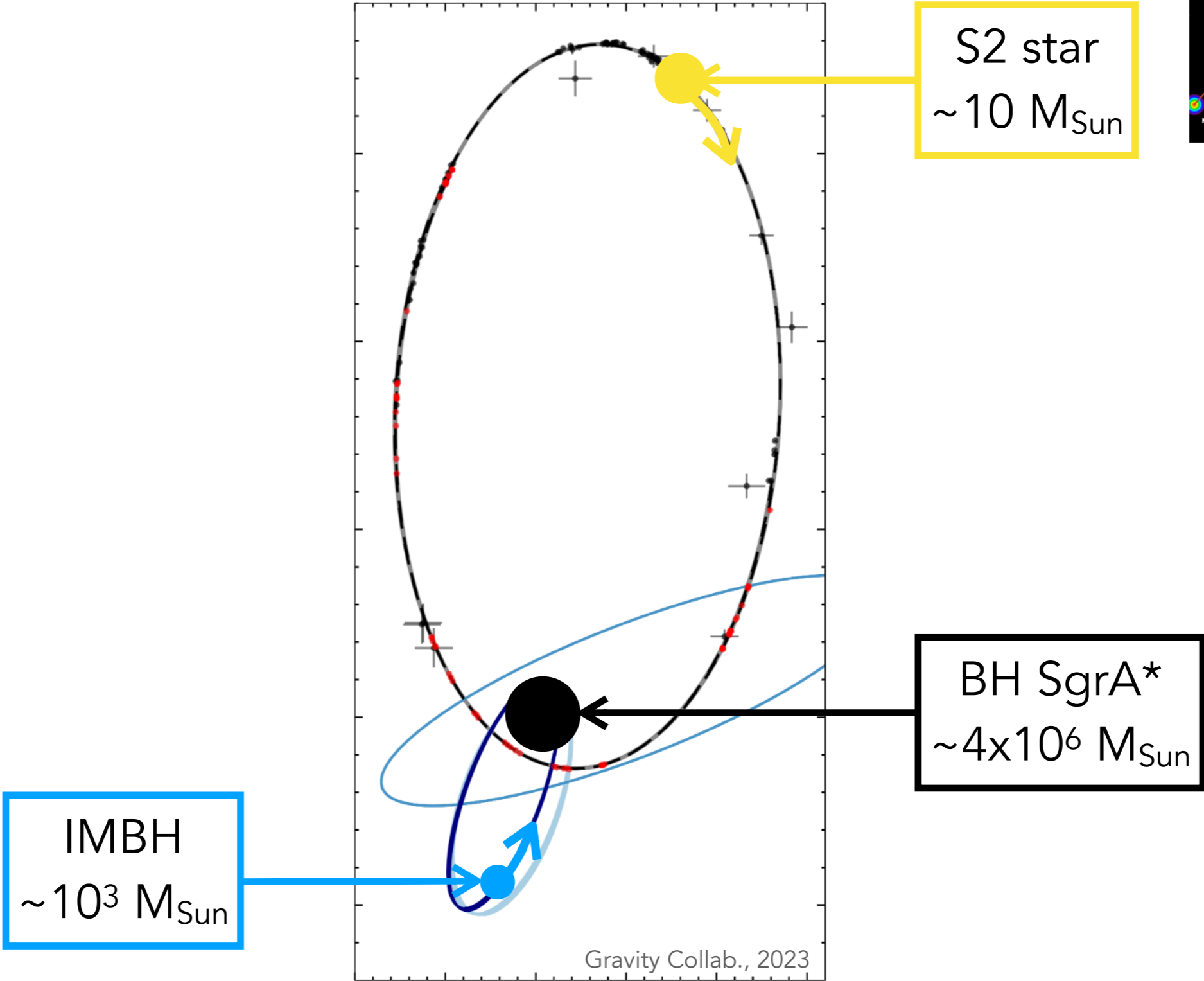
**Precession** of the orbit  
 12' (=0.2°) per orbit

- S2 arrived early:  
 + Orbit is **non-closed**  
 + **Prograde** precession

→ **Relativistic effects**



# What about an IMBH?



An inner IMBH hard to reconcile with **S2's orbit**

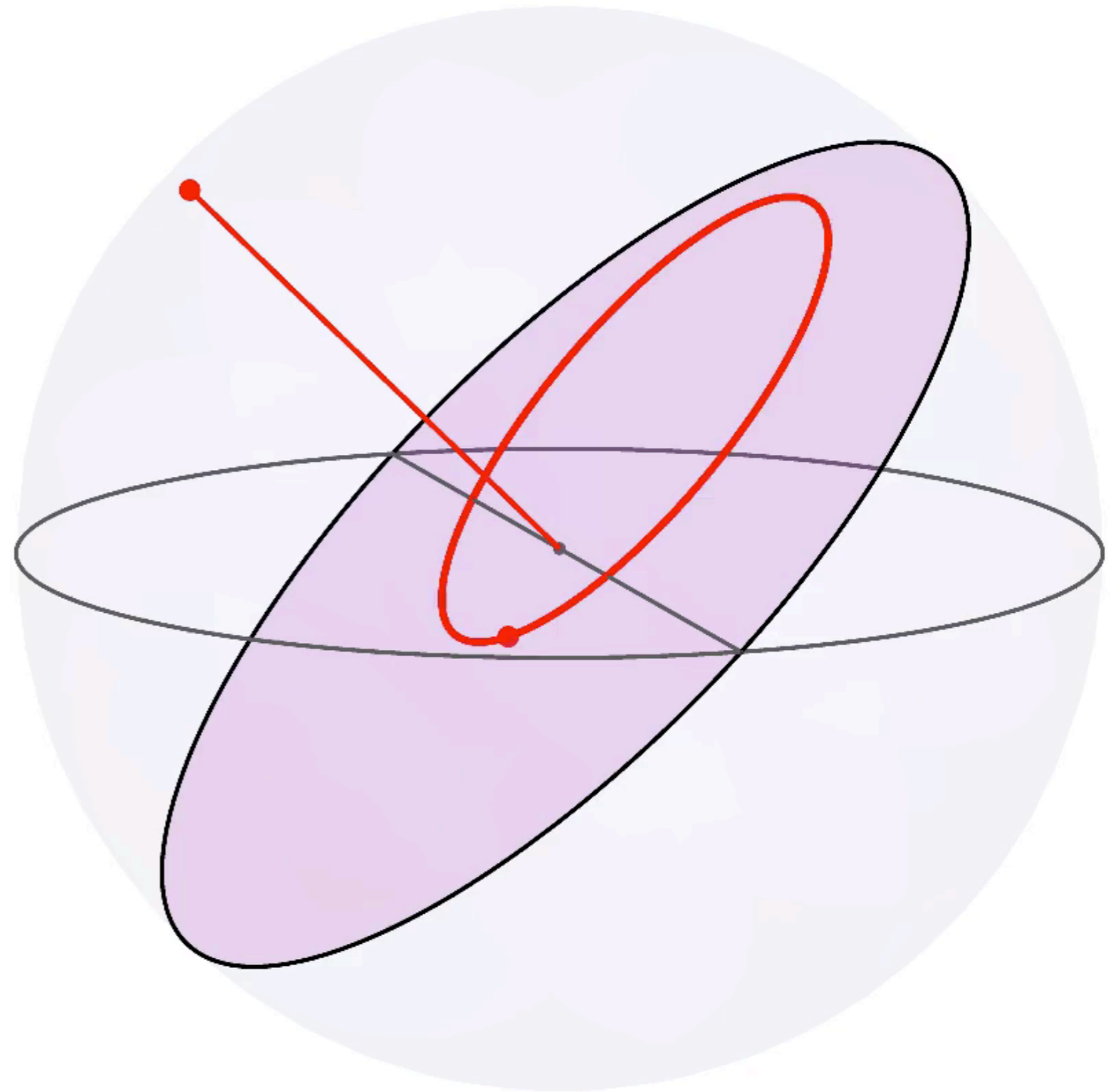


## Pericentre precession

Origins of the **precession**:

- + **Relativistic** effects from the BH
- + **Perturbations** from other stars

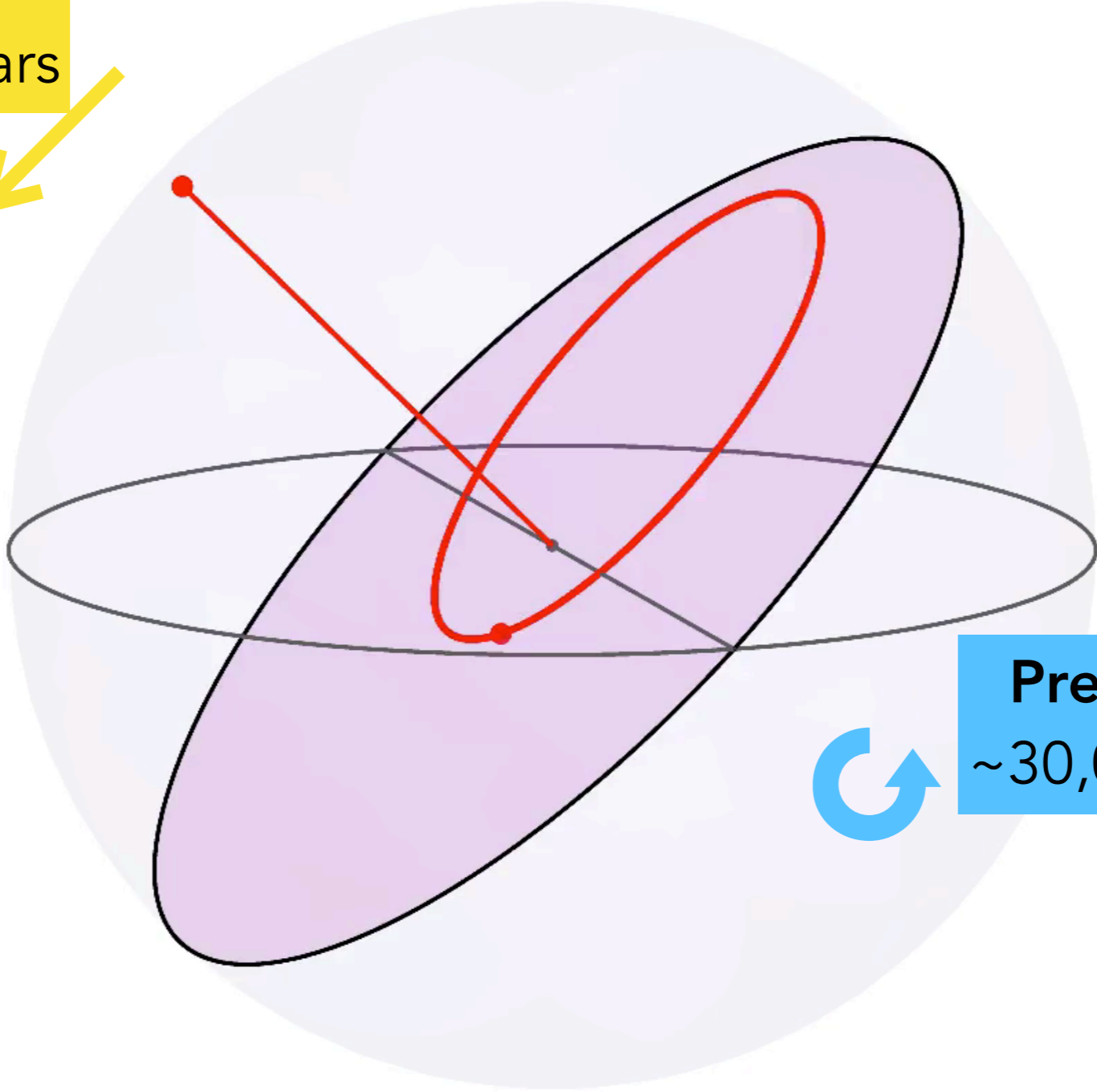
~30,000 years  
for S2



Orbits **precess** in their planes

# Orbits also change in orientations

**Orientation**  
~1,000,000 years



**Precession**  
~30,000 years

Two timescales:

**Precession**



**Orientation**

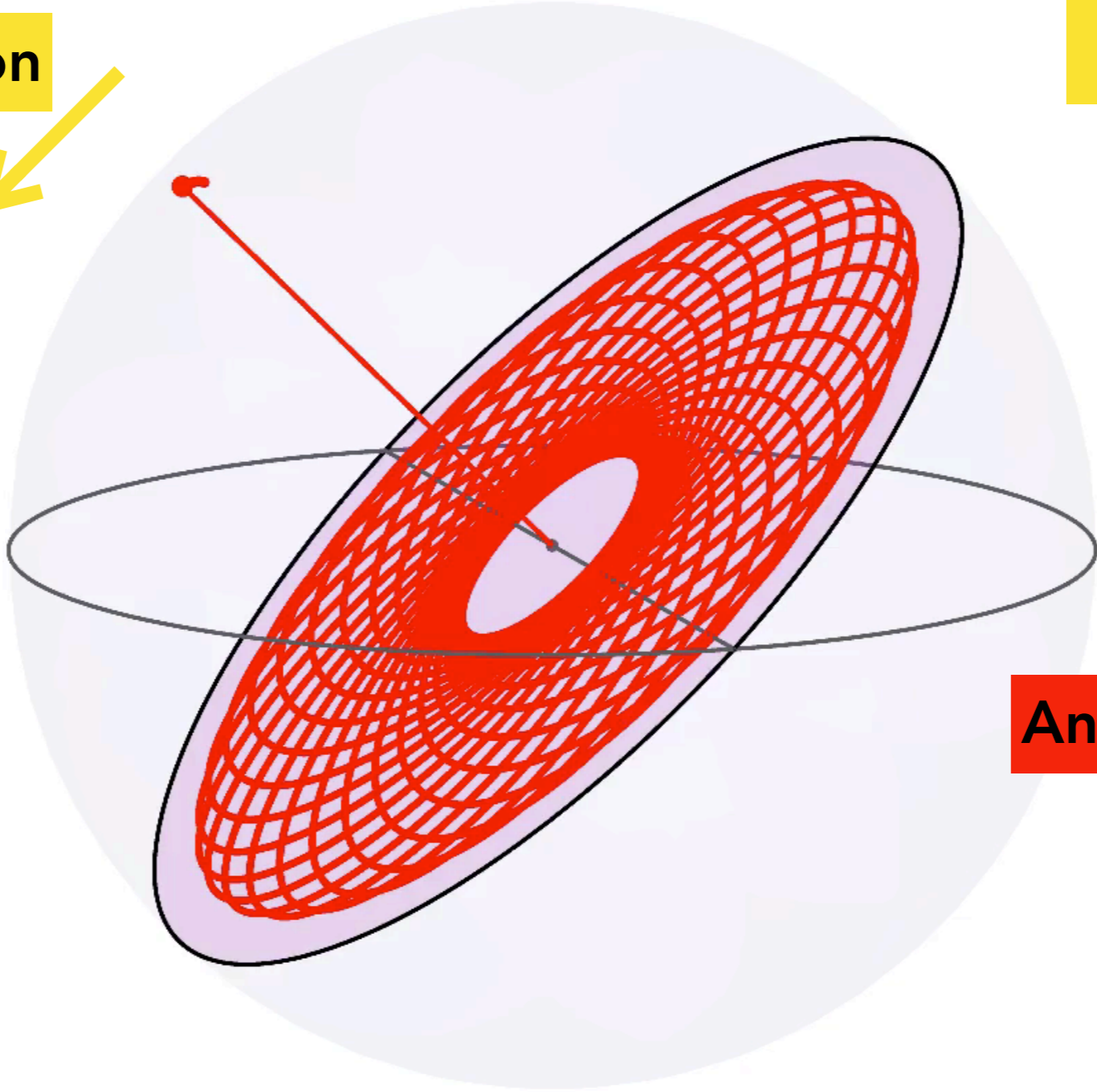


# Stellar orientations

Orientation



Typical timescale  
~1,000,000 years

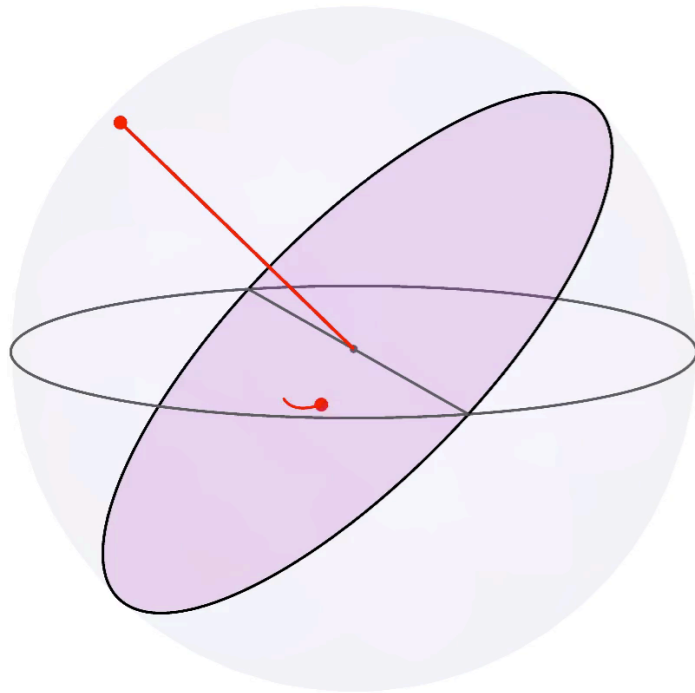


Annuli

After a full precession, **ellipses** become **annuli**

# Stellar dynamics

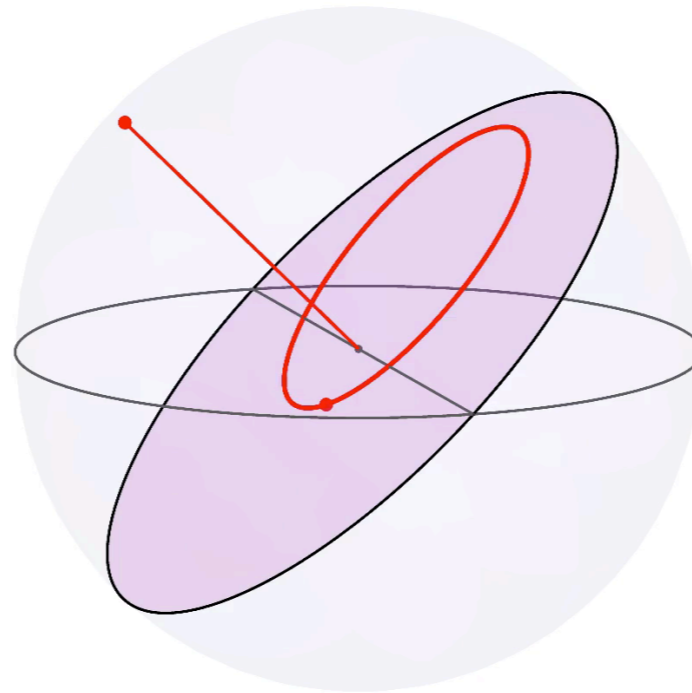
Stars



~10 years

**Orbital** motion

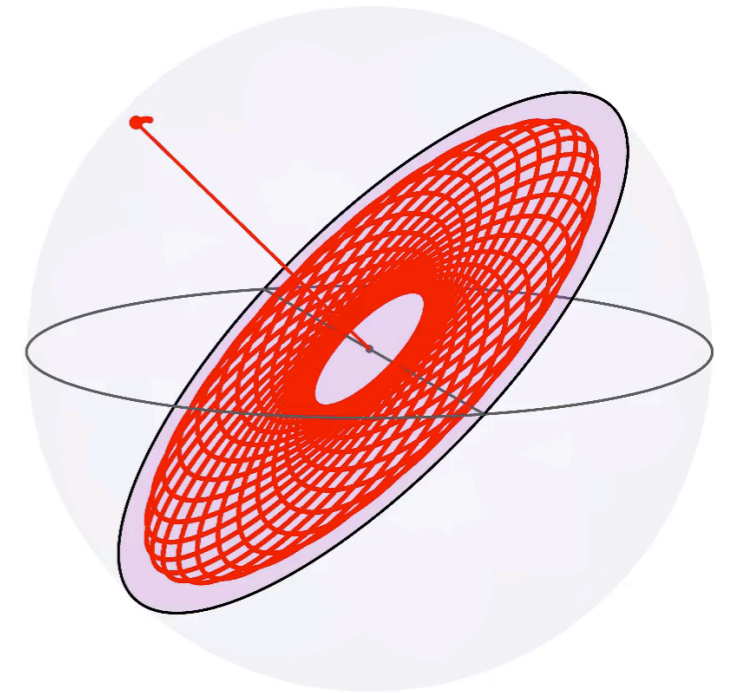
Ellipses



30,000 years

**Pericentre** precession

Annuli



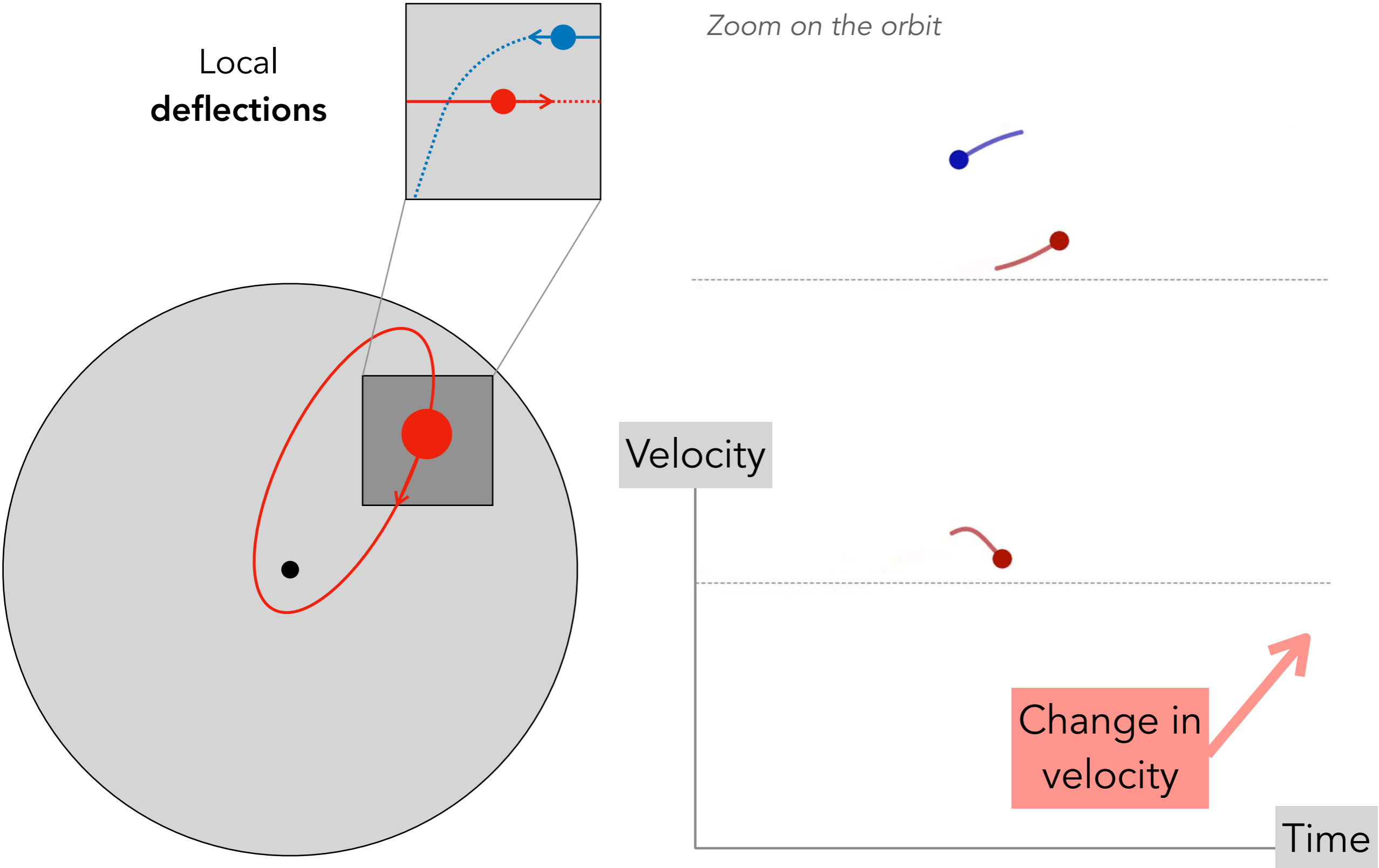
~1,000,000 years

**Orientation** precession

SgrA\* is 10 Gyr orld. We can wait longer

# Stellar energy

Orbital distortions sourced by instantaneous **kicks and deflections**

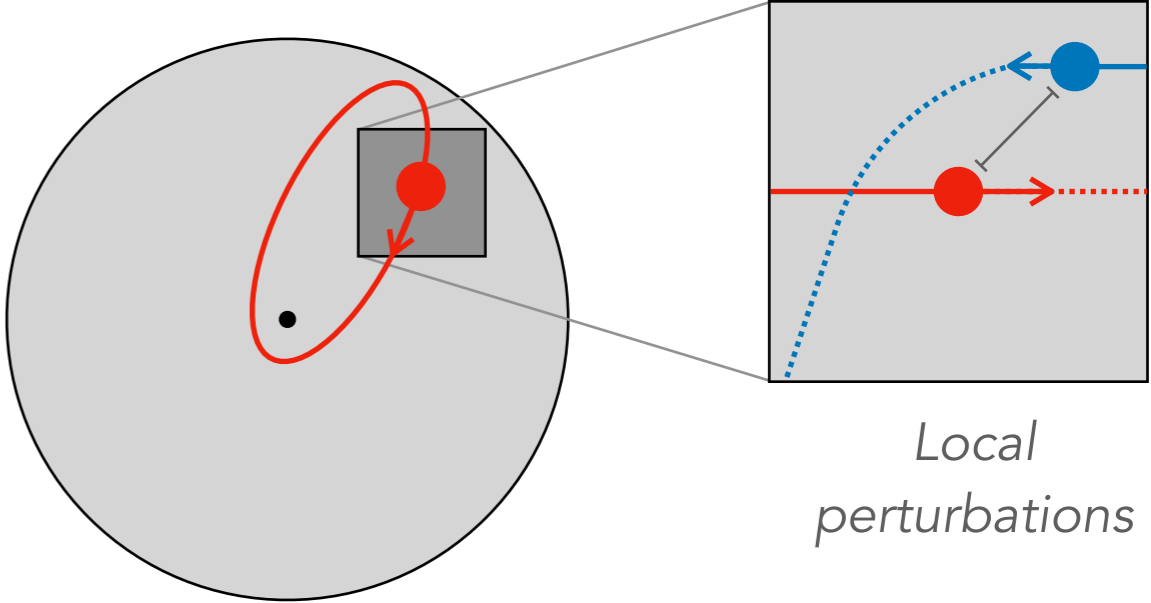
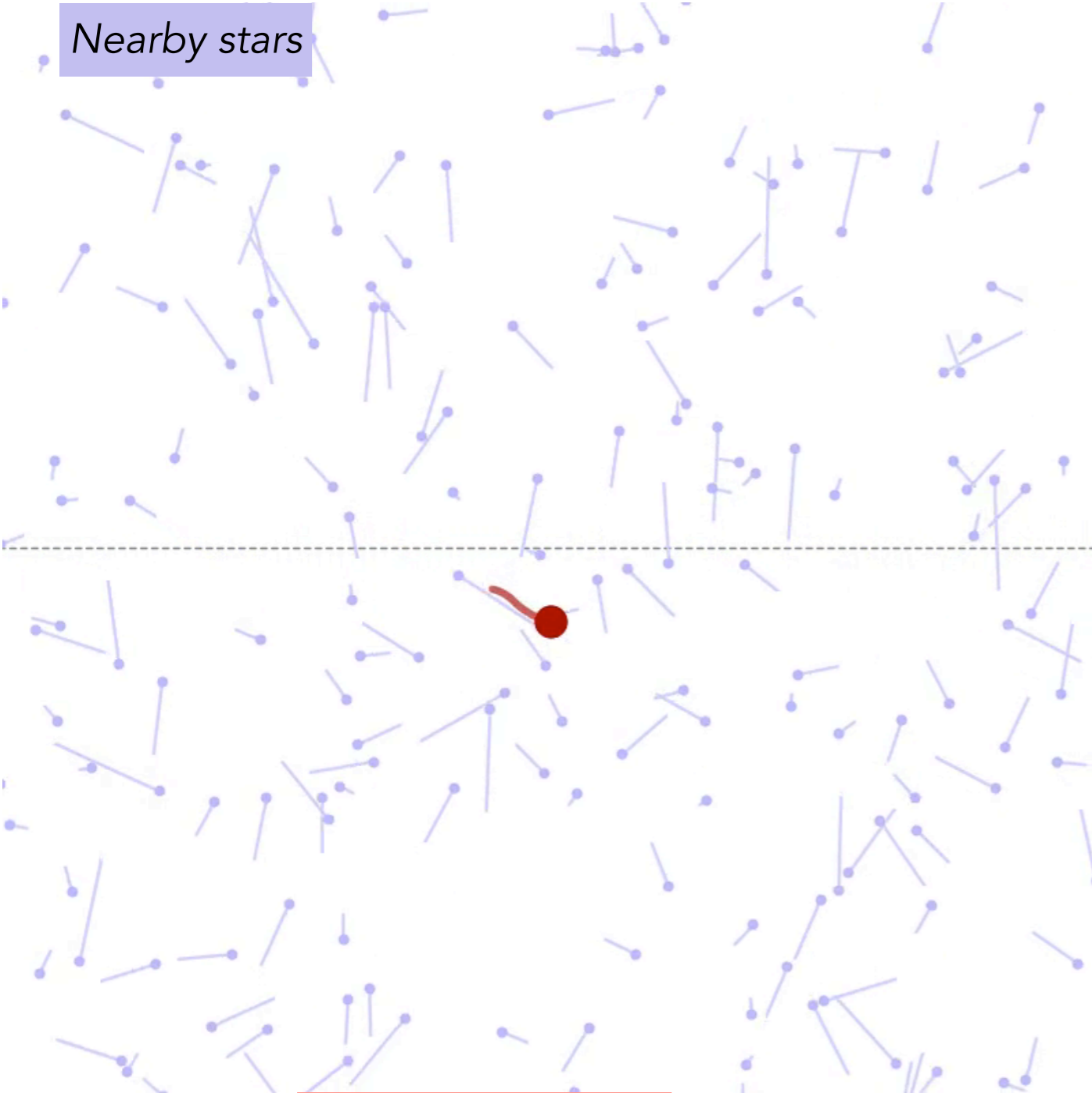




# Deflections

The star has a lot of **close neighbours**

Nearby stars



Local perturbations

Series of deflections

Velocity

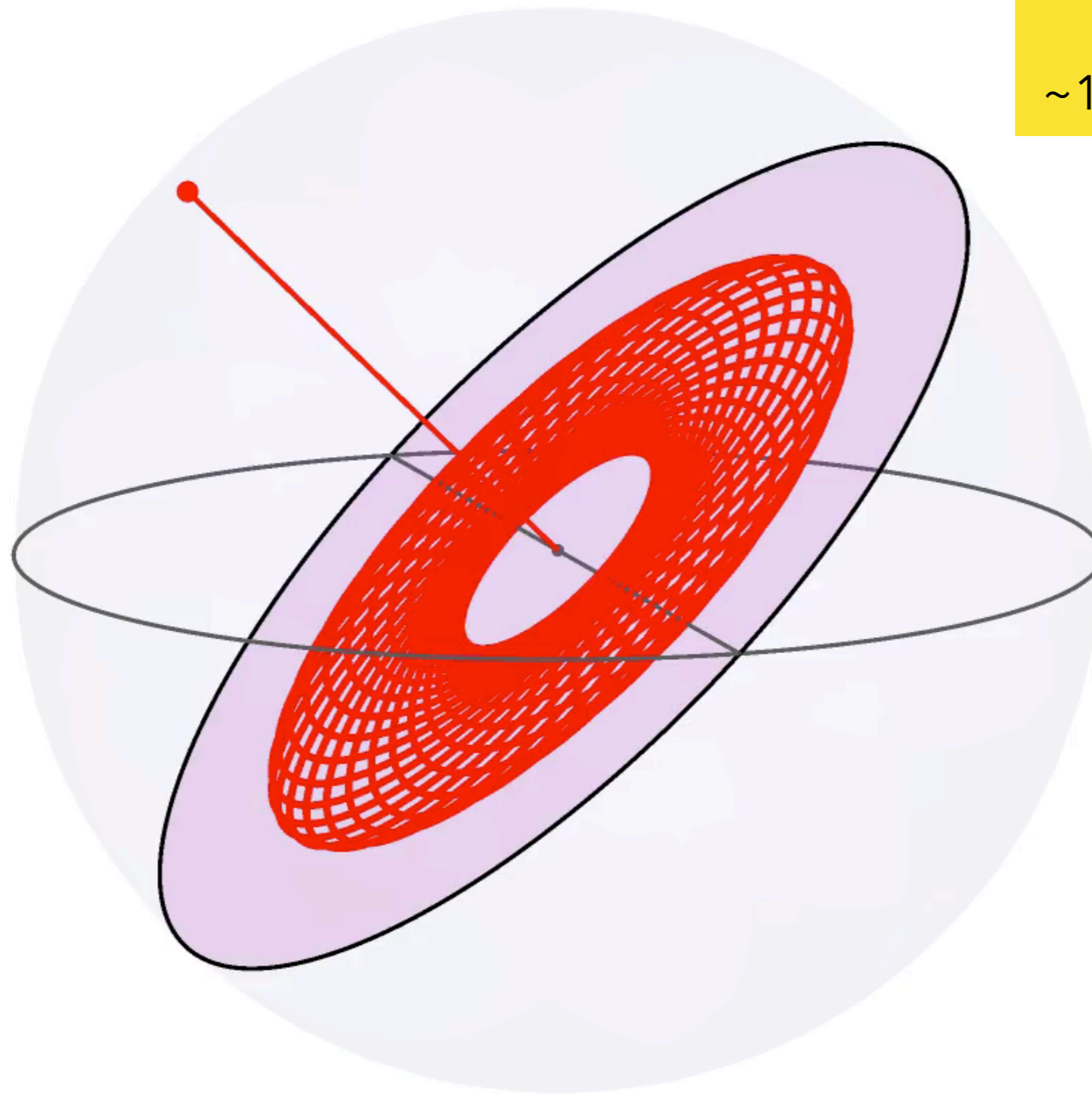


Time

Random walk

## Stellar energy

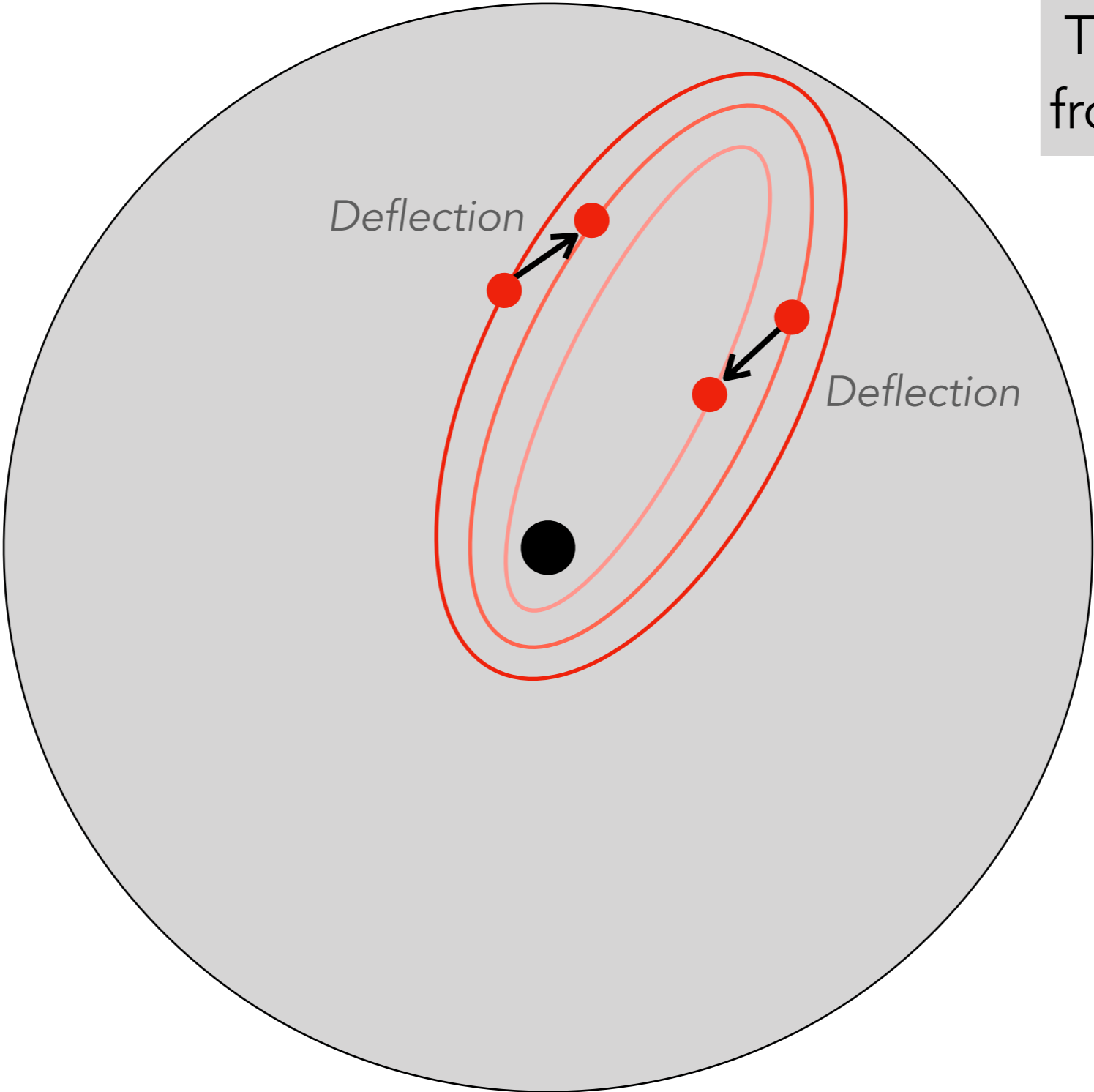
Typical timescale  
~1,000,000,000 years



**Deflections** drive a slow change in the Keplerian energy

# Deflections

The star “jumps”  
from orbit to orbit

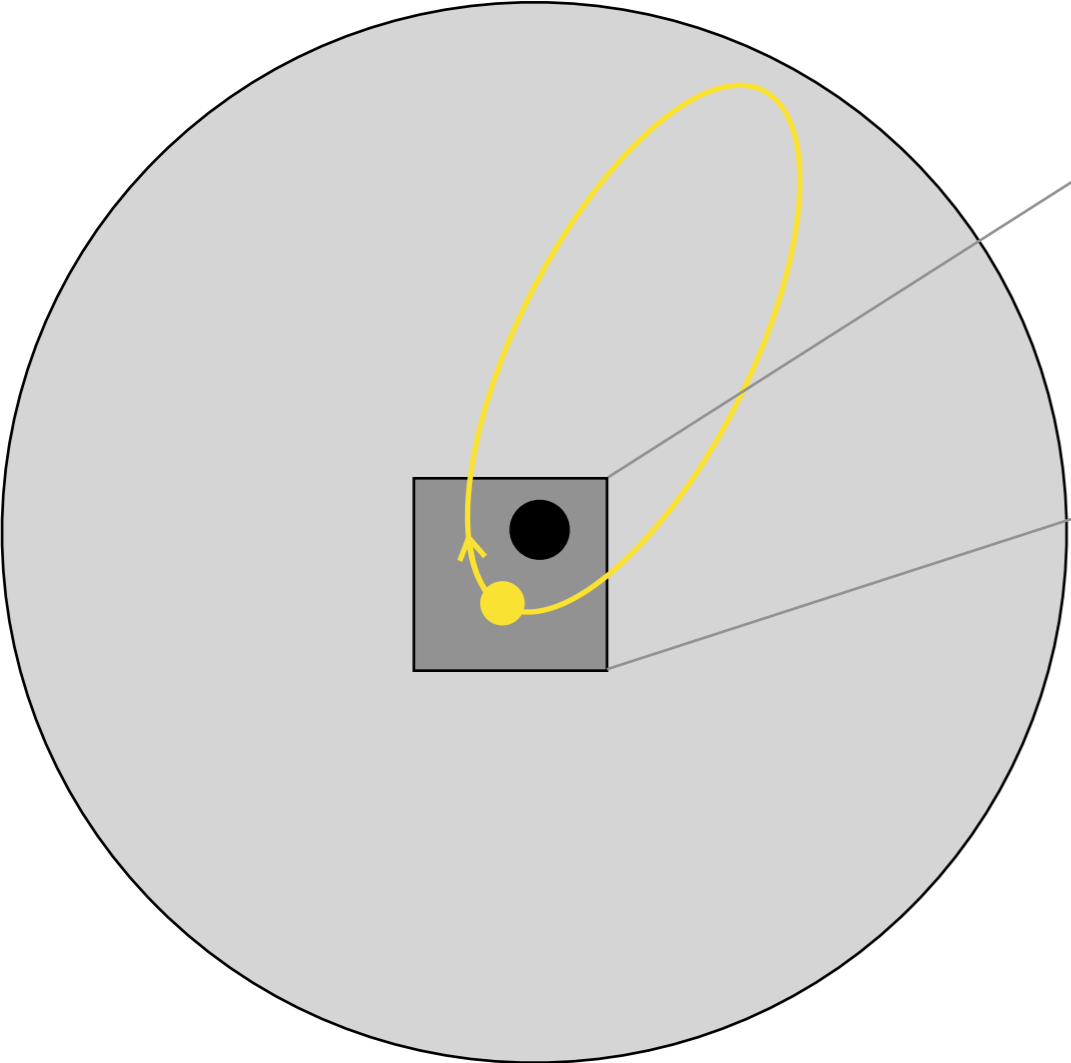


Deflections allow the star to get even closer to the **central BH**

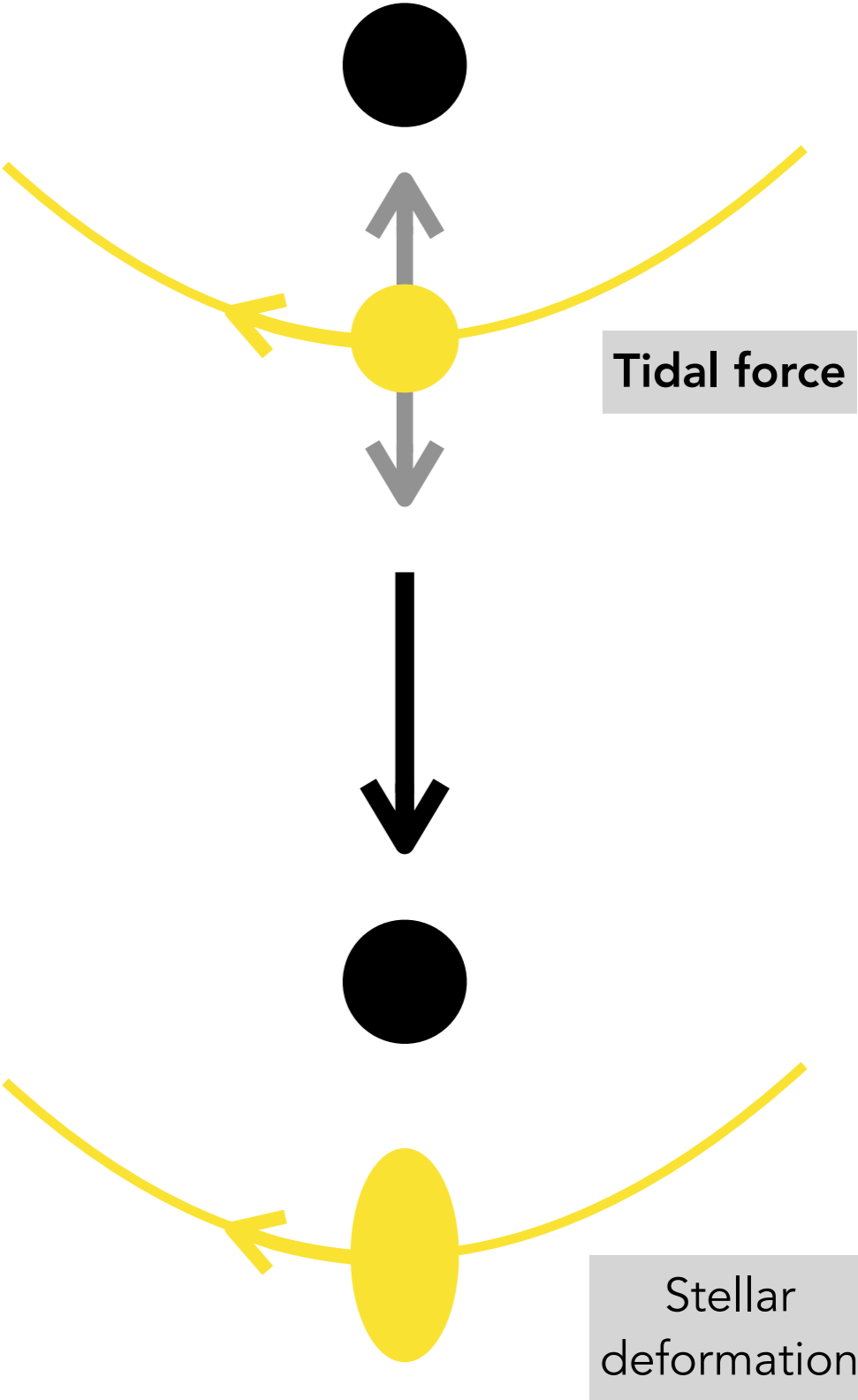


# The neighborhood of the BH

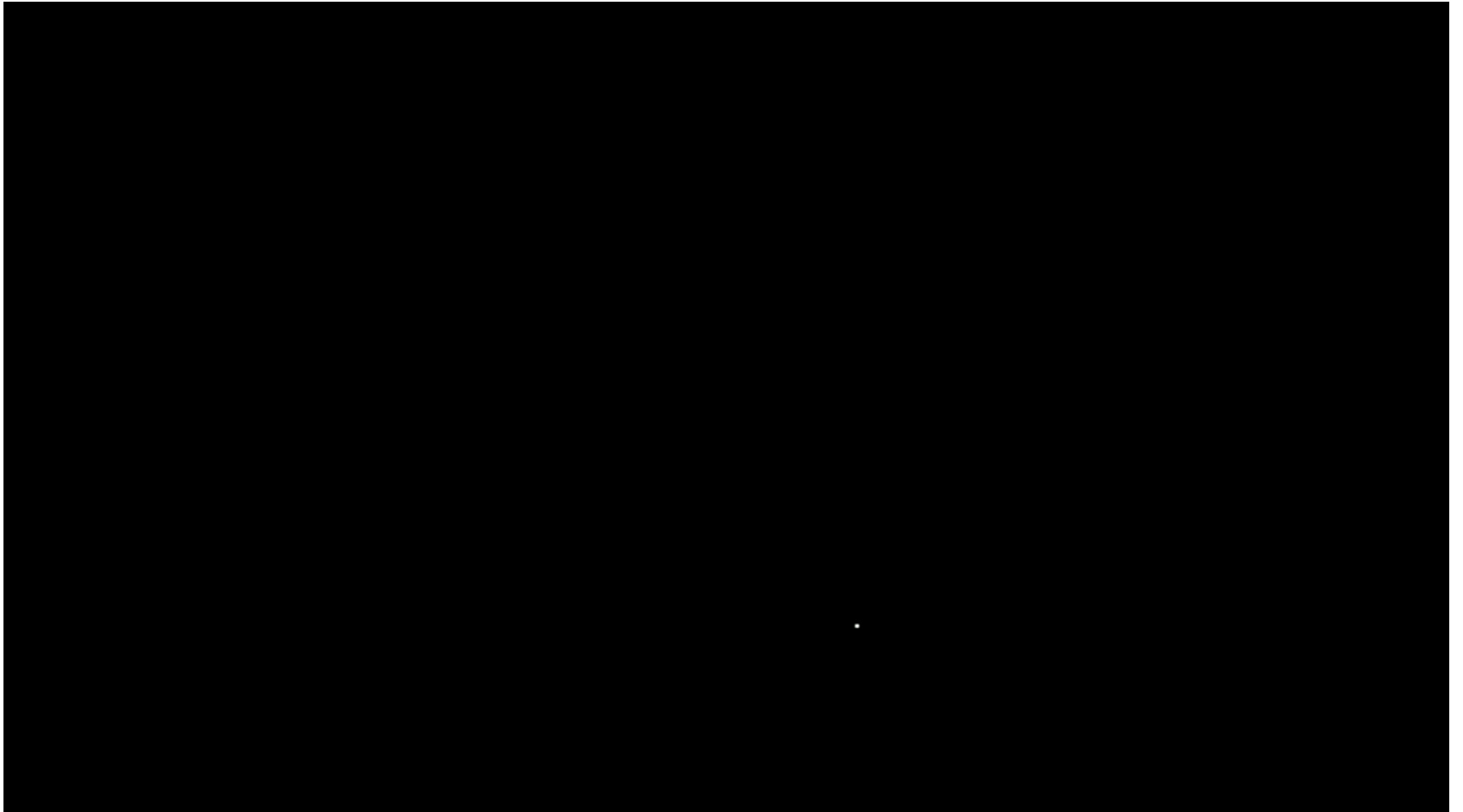
What does a star feel close to the BH?



Very close to the BH,  
the star is no more a point



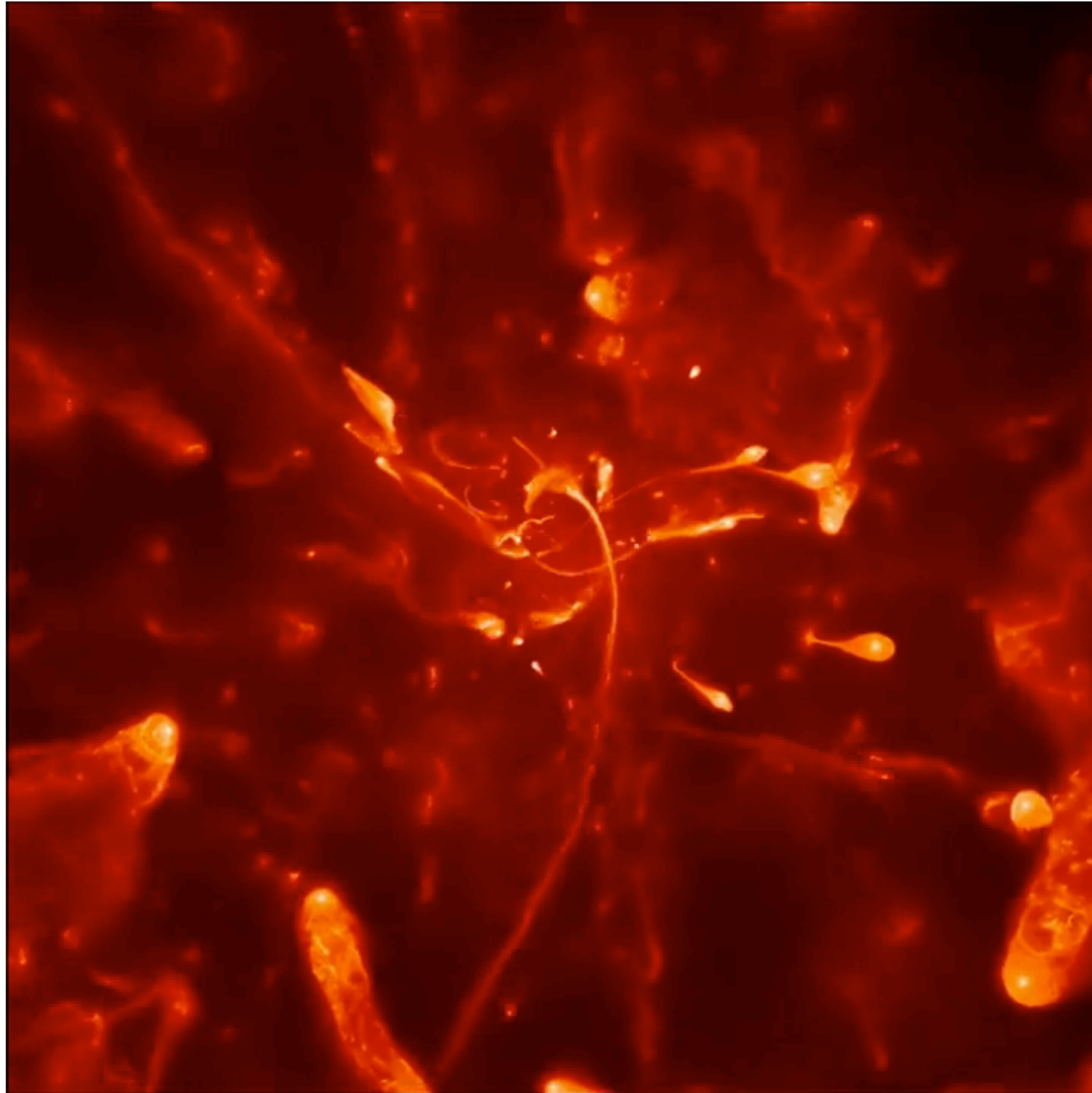
# Tidal disruption events



*J. Guillochon*

Stars get **disrupted** by the BH's force field

## Stellar winds



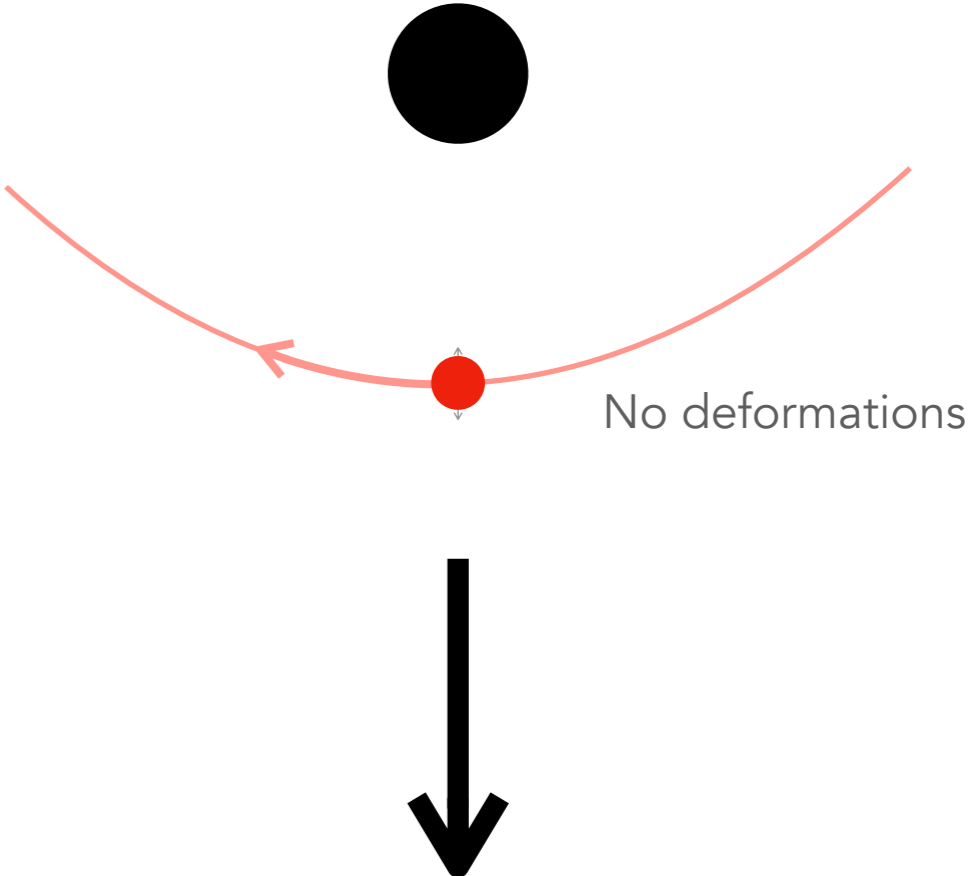
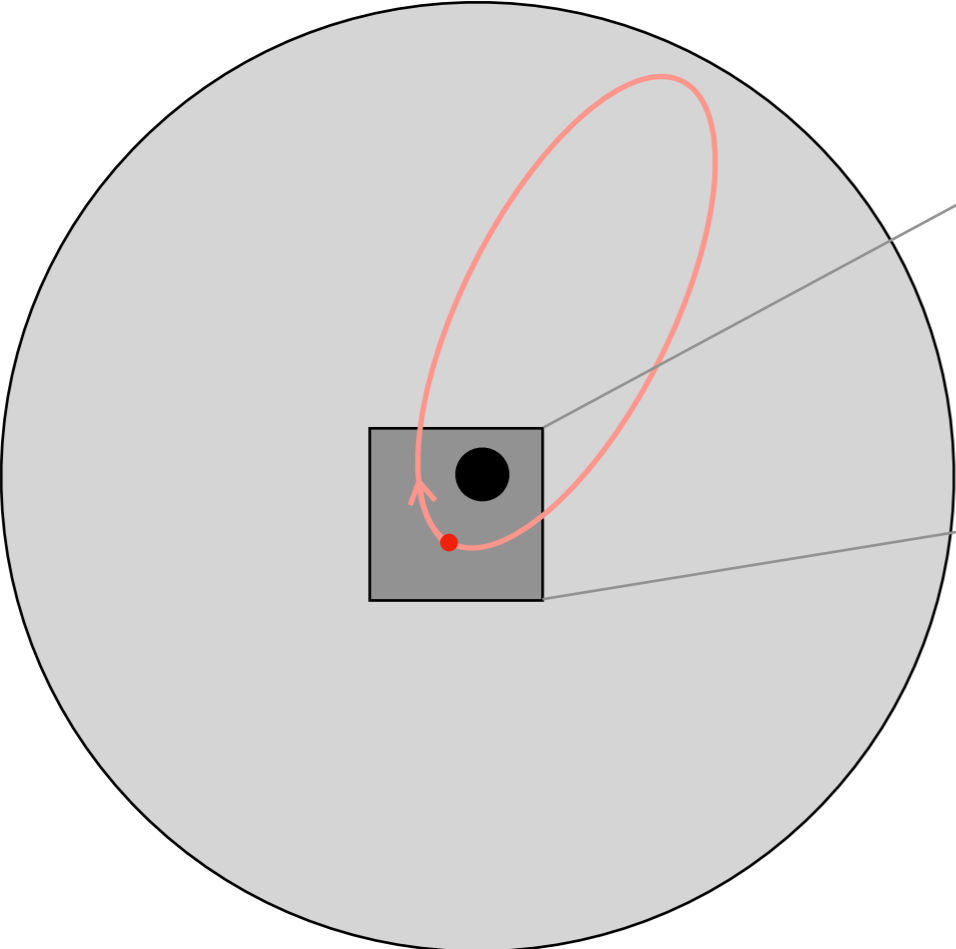
*N. Luetzgendorf*

Gas is **striped** from stars close to the BH



# Even closer to the BH

What does a **compact object** feel close to the BH?



Compact objects (neutron stars, BHs) are **really small**

Sun



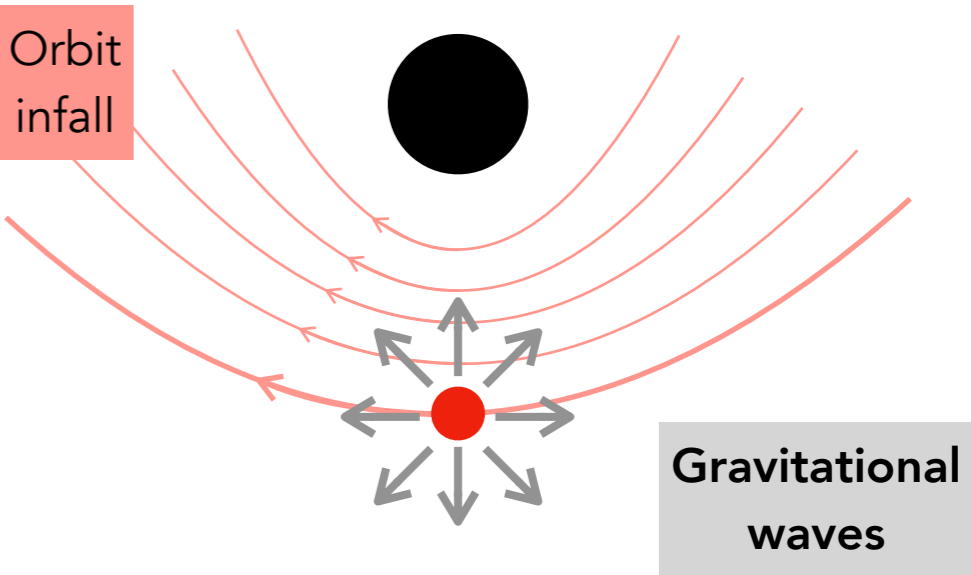
1,400,000 km

Same mass

Neutron star



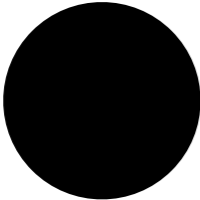
30 km



# Extreme mass ratio inspirals

Typical time  
~a few months

**Supermassive BH**



~4,000,000  $M_{\text{Sun}}$

**Neutron star**



~2  $M_{\text{Sun}}$



**Gravitational waves**



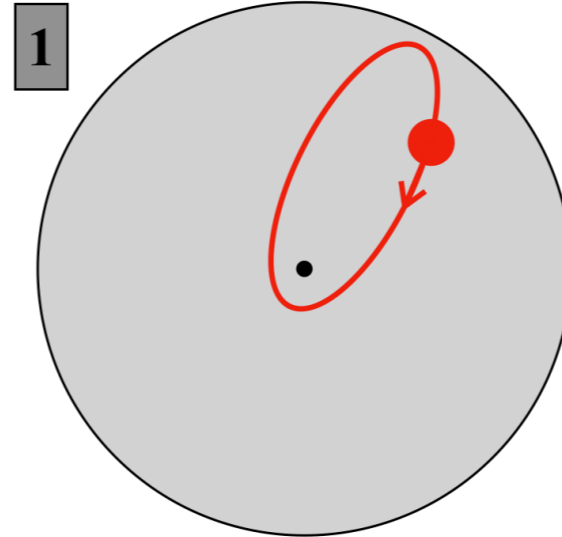
The emission of gravitational waves leads to an **unavoidable infall**

# Timescales are highly hierarchical

## 1. Dynamical time

*Fast orbital motion induced by the BH*

$$\frac{dM}{dt} = \Omega_{\text{Kep}}$$





# Timescales are highly hierarchical

## 1. Dynamical time

Fast orbital motion induced by the BH

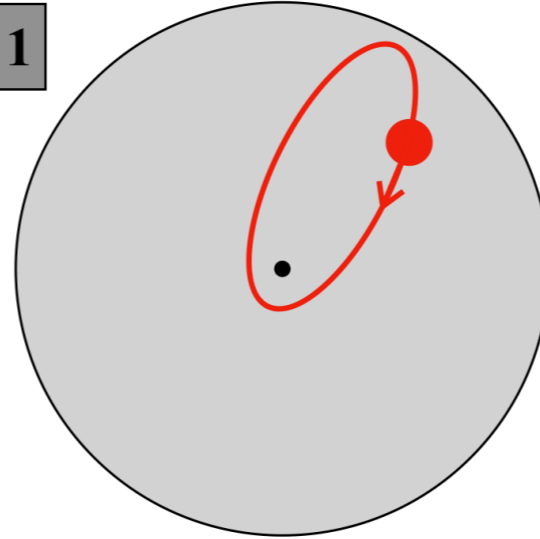
$$\frac{dM}{dt} = \Omega_{\text{Kep}}$$

## 2. Precession time

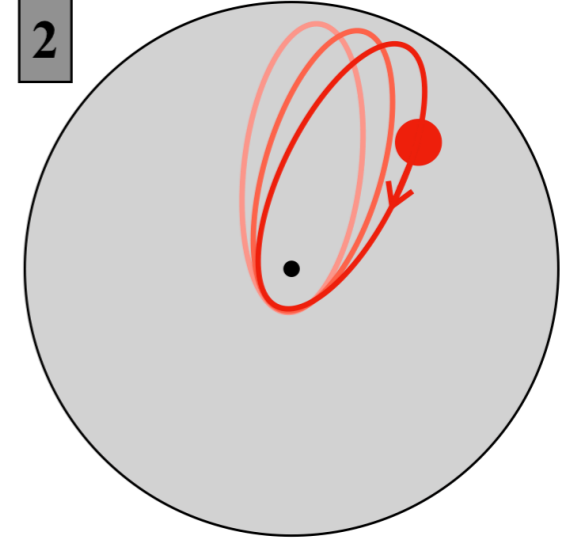
In-plane precession (mass + relativity)

$$\frac{d\omega}{dt} = \Omega_p$$

1



2



# Timescales are highly hierarchical

## 1. Dynamical time

Fast orbital motion induced by the BH

$$\frac{dM}{dt} = \Omega_{\text{Kep}}$$

## 2. Precession time

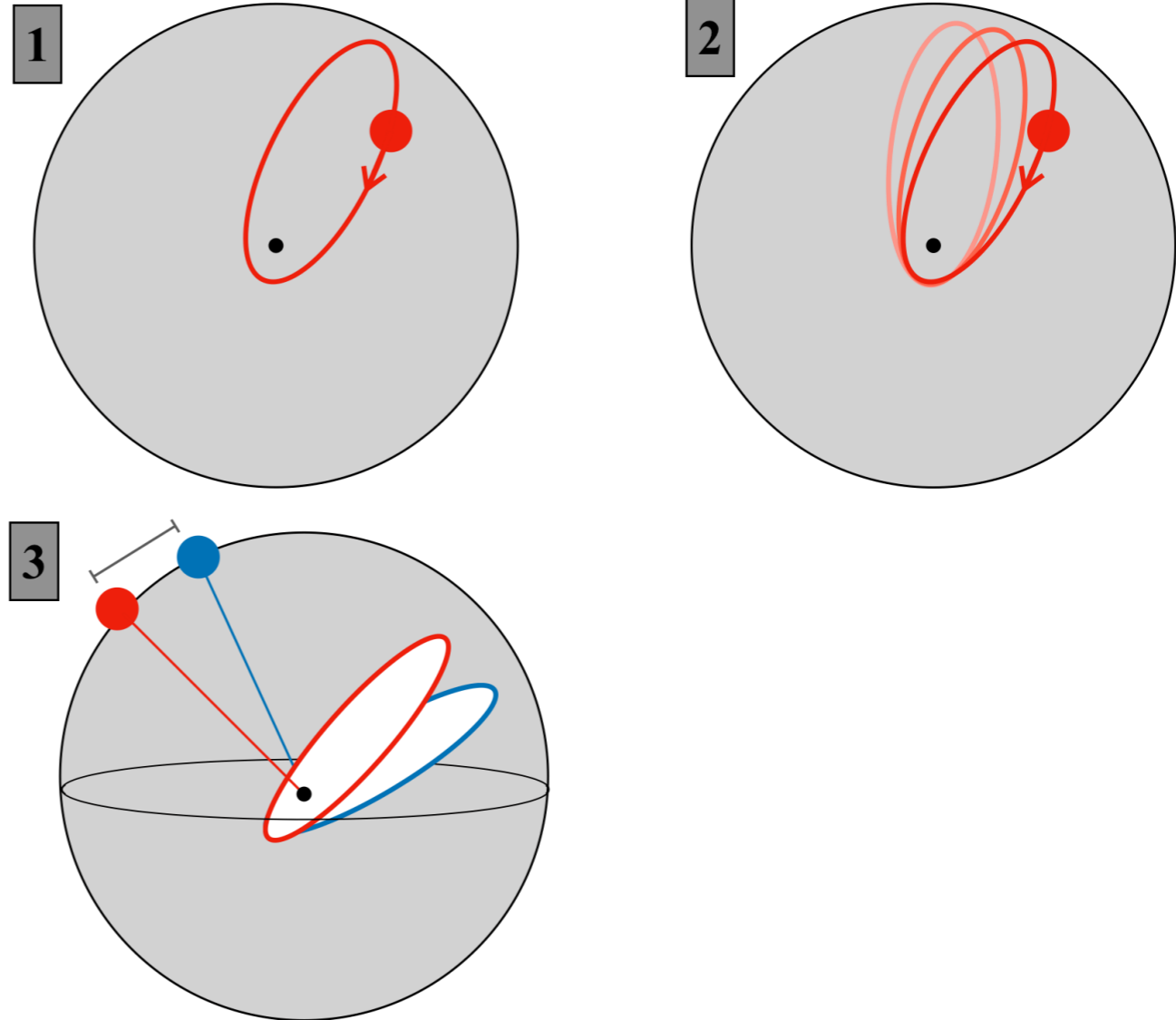
In-plane precession (mass + relativity)

$$\frac{d\omega}{dt} = \Omega_p$$

## 3. Vector Resonant Relaxation

Non-spherical torque coupling

$$\frac{d\hat{\mathbf{L}}}{dt} = \eta(\hat{\mathbf{L}}, t)$$



# Timescales are highly hierarchical

## 1. Dynamical time

Fast orbital motion induced by the BH

$$\frac{dM}{dt} = \Omega_{\text{Kep}}$$

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In-plane precession (mass + relativity)

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## 3. Vector Resonant Relaxation

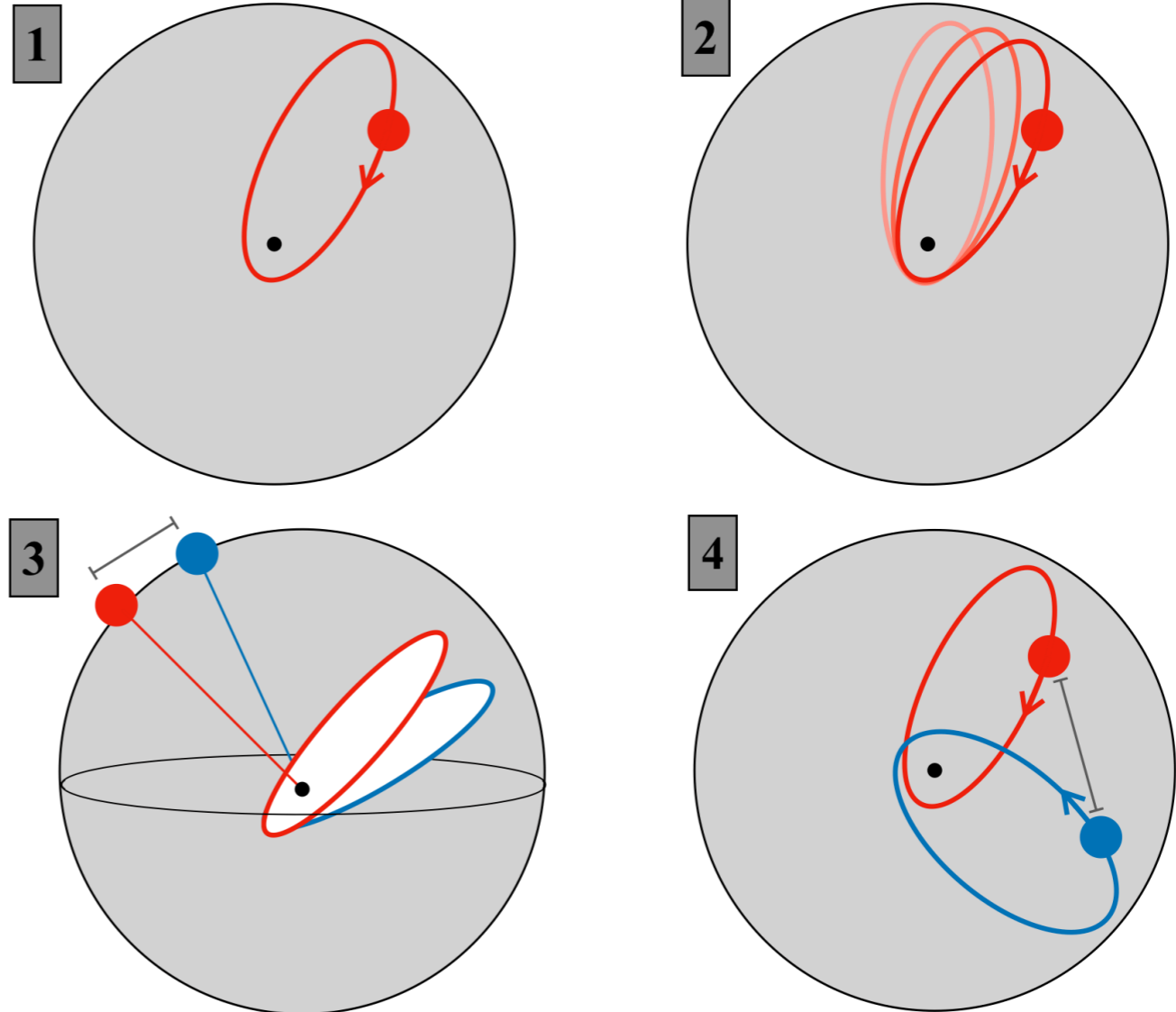
Non-spherical torque coupling

$$\frac{d\hat{\mathbf{L}}}{dt} = \eta(\hat{\mathbf{L}}, t)$$

## 4. Scalar Resonant Relaxation

Resonant coupling on precessions

$$\frac{de}{dt} = \eta(e, t)$$



# Timescales are highly hierarchical

## 1. Dynamical time

Fast orbital motion induced by the BH

$$\frac{dM}{dt} = \Omega_{\text{Kep}}$$

## 2. Precession time

In-plane precession (mass + relativity)

$$\frac{d\omega}{dt} = \Omega_p$$

## 3. Vector Resonant Relaxation

Non-spherical torque coupling

$$\frac{d\hat{\mathbf{L}}}{dt} = \eta(\hat{\mathbf{L}}, t)$$

## 4. Scalar Resonant Relaxation

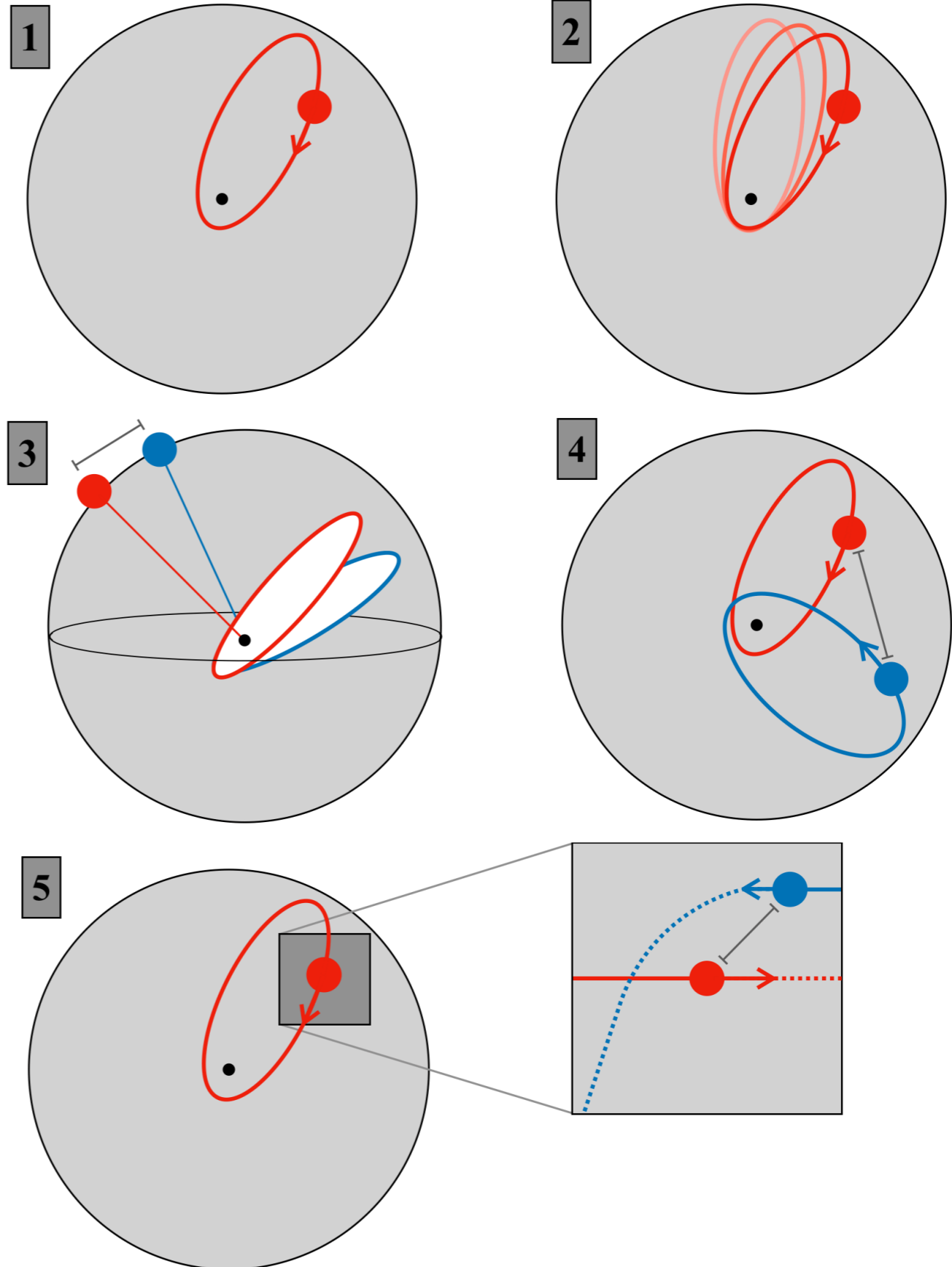
Resonant coupling on precessions

$$\frac{de}{dt} = \eta(e, t)$$

## 5. Non-Resonant Relaxation

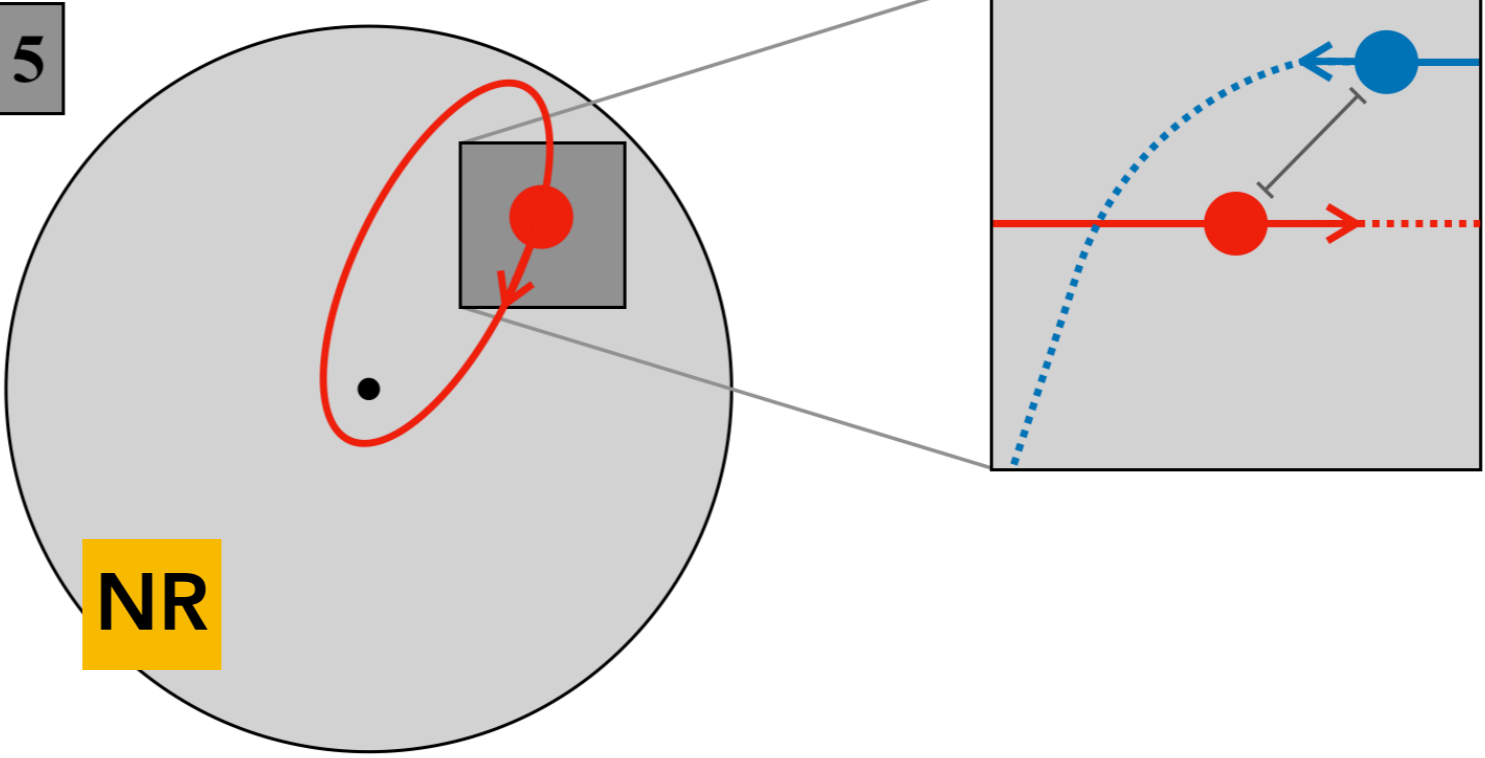
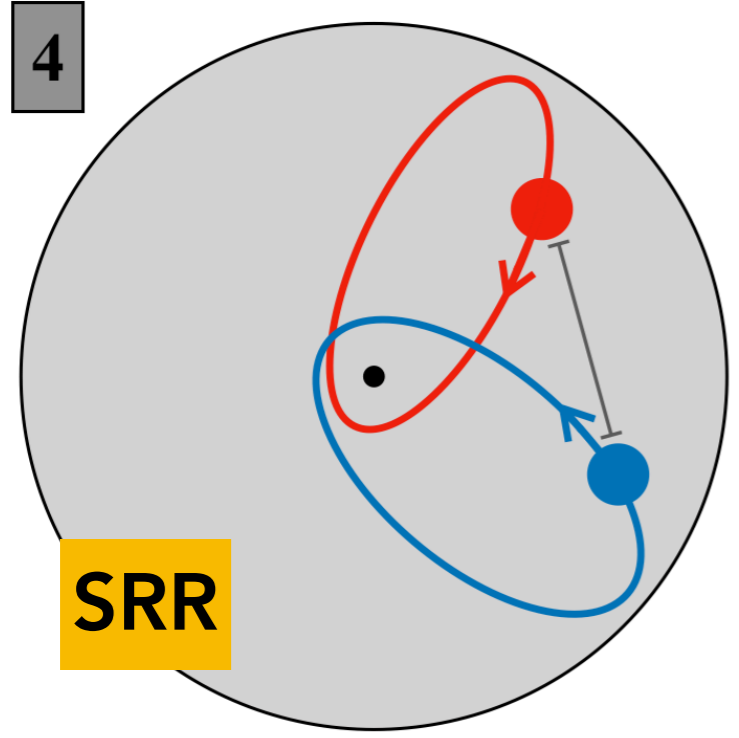
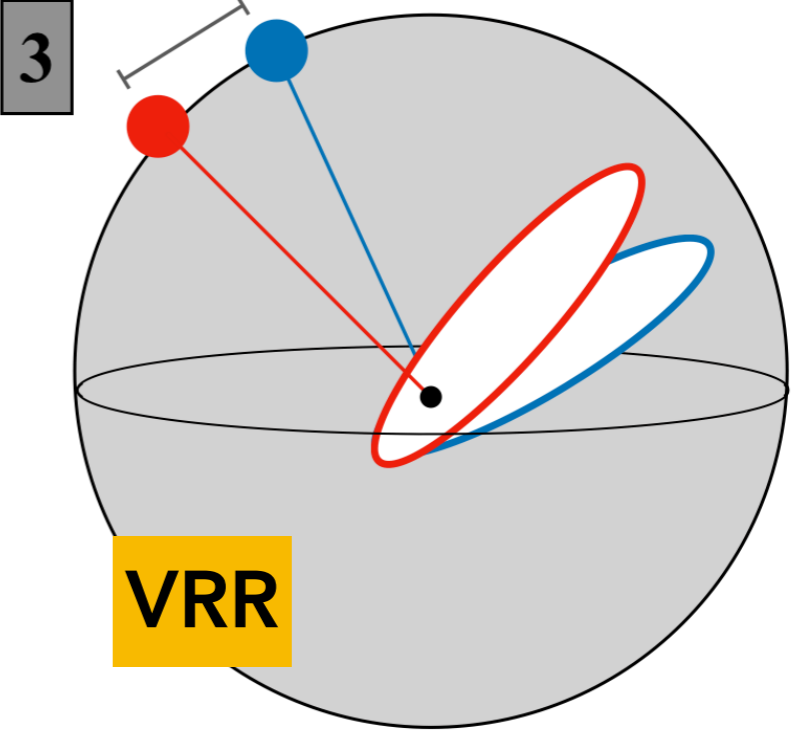
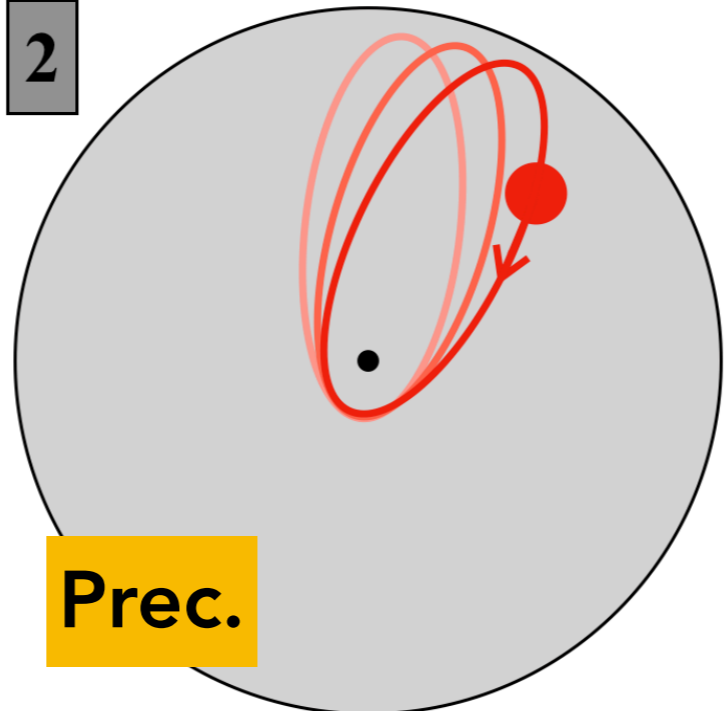
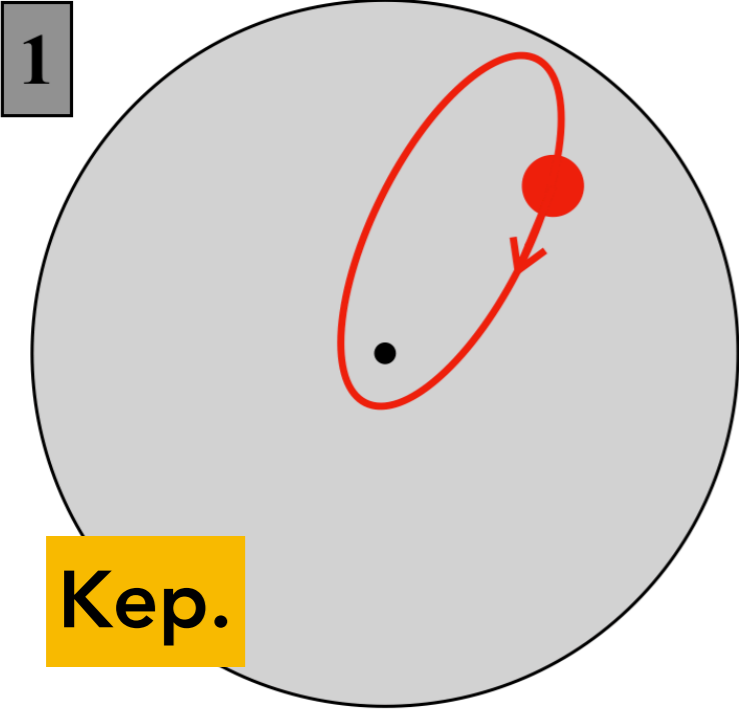
Local two-body encounters

$$\frac{da}{dt} = \eta(a, t)$$



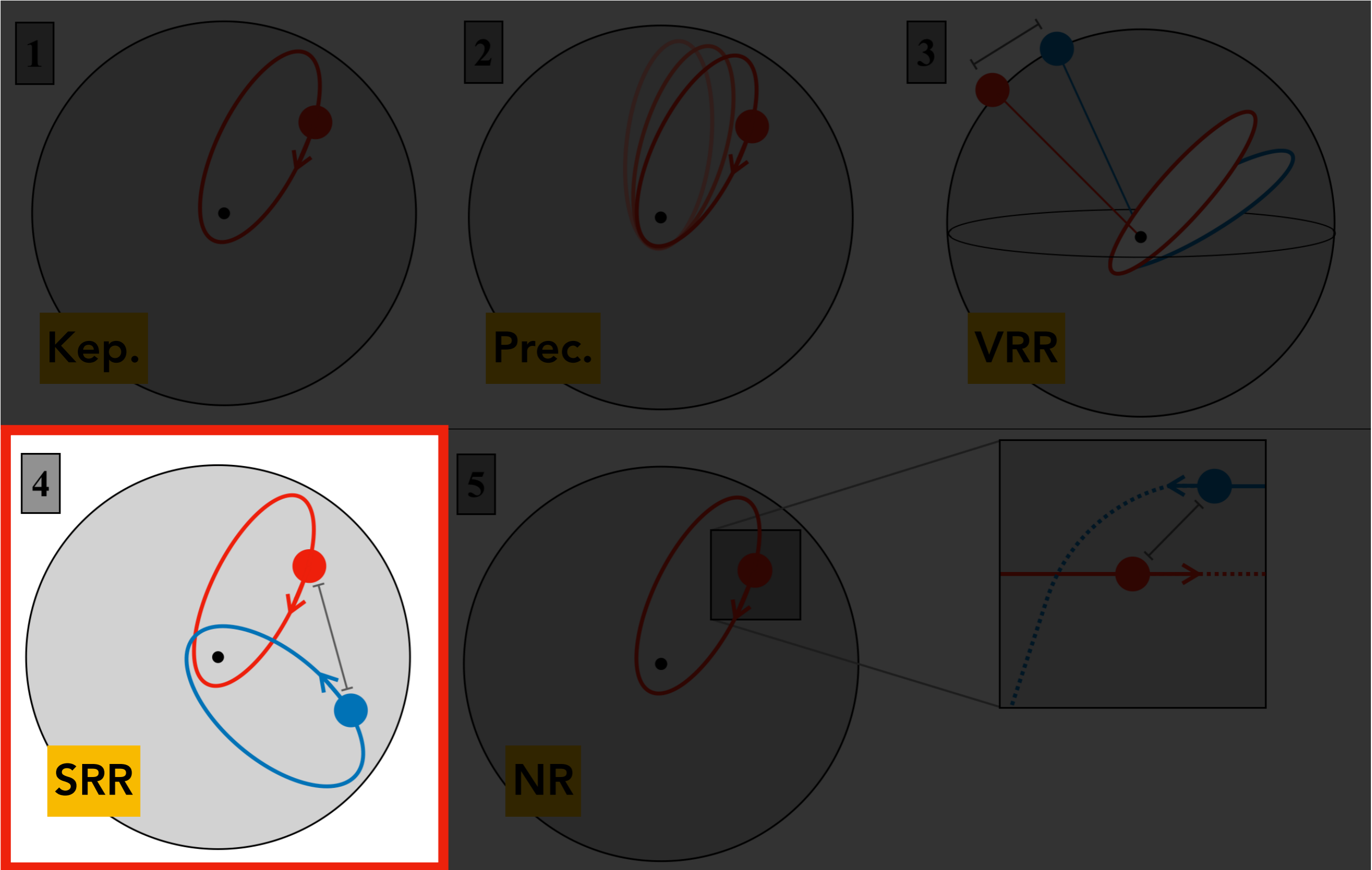


# A wealth of dynamical processes



An extremely **hierarchical system**

# Scalar Resonant Relaxation

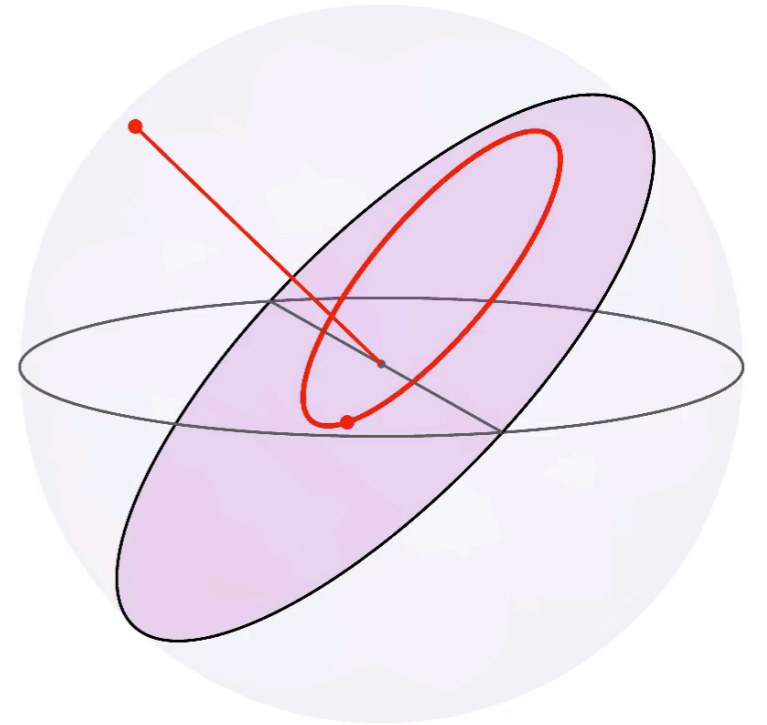
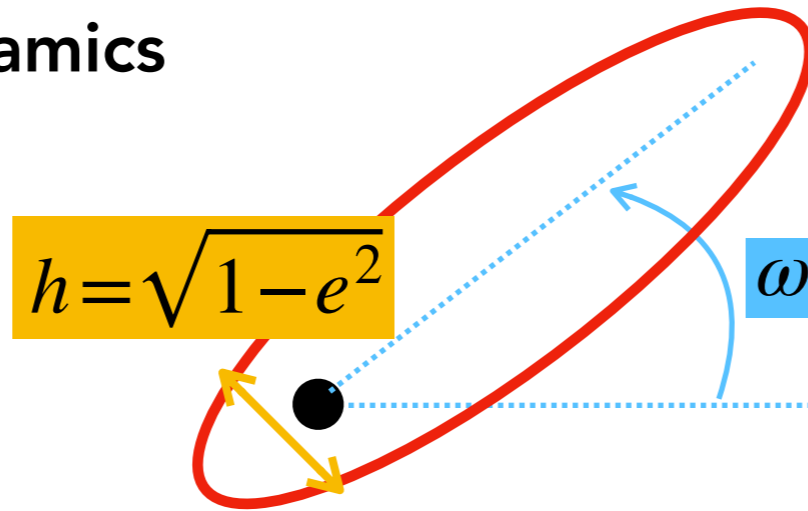


The (resonant) dynamics of **eccentricities**

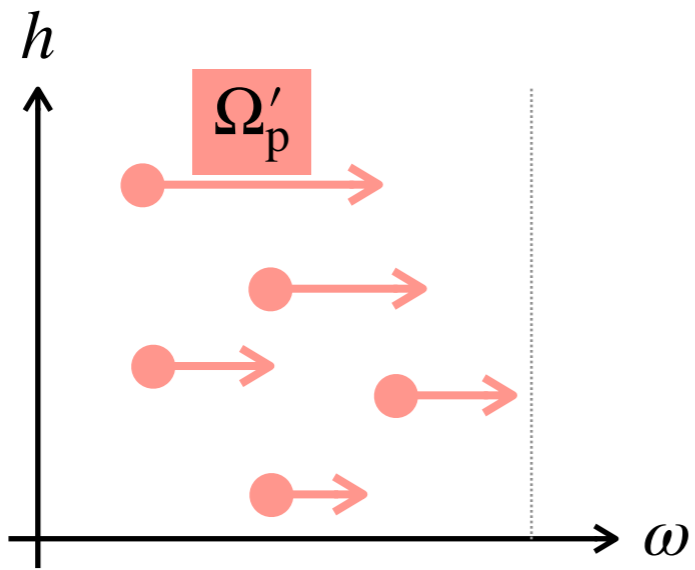
# Scalar Resonant Relaxation

A simple **unperturbed** dynamics

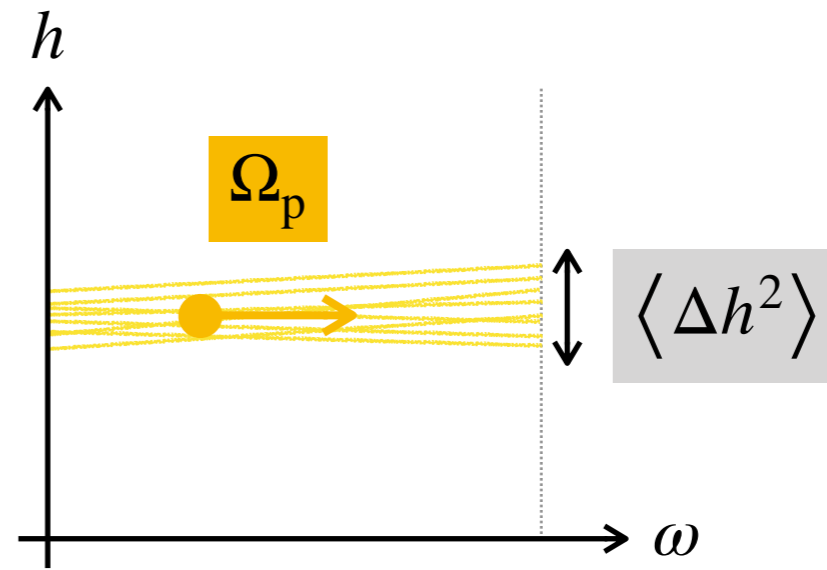
$$\begin{cases} \dot{\omega} \simeq \Omega_p(h) \\ \dot{h} \simeq \eta(t, \omega, h) \end{cases}$$



**Phase-space** dynamics



Background cluster



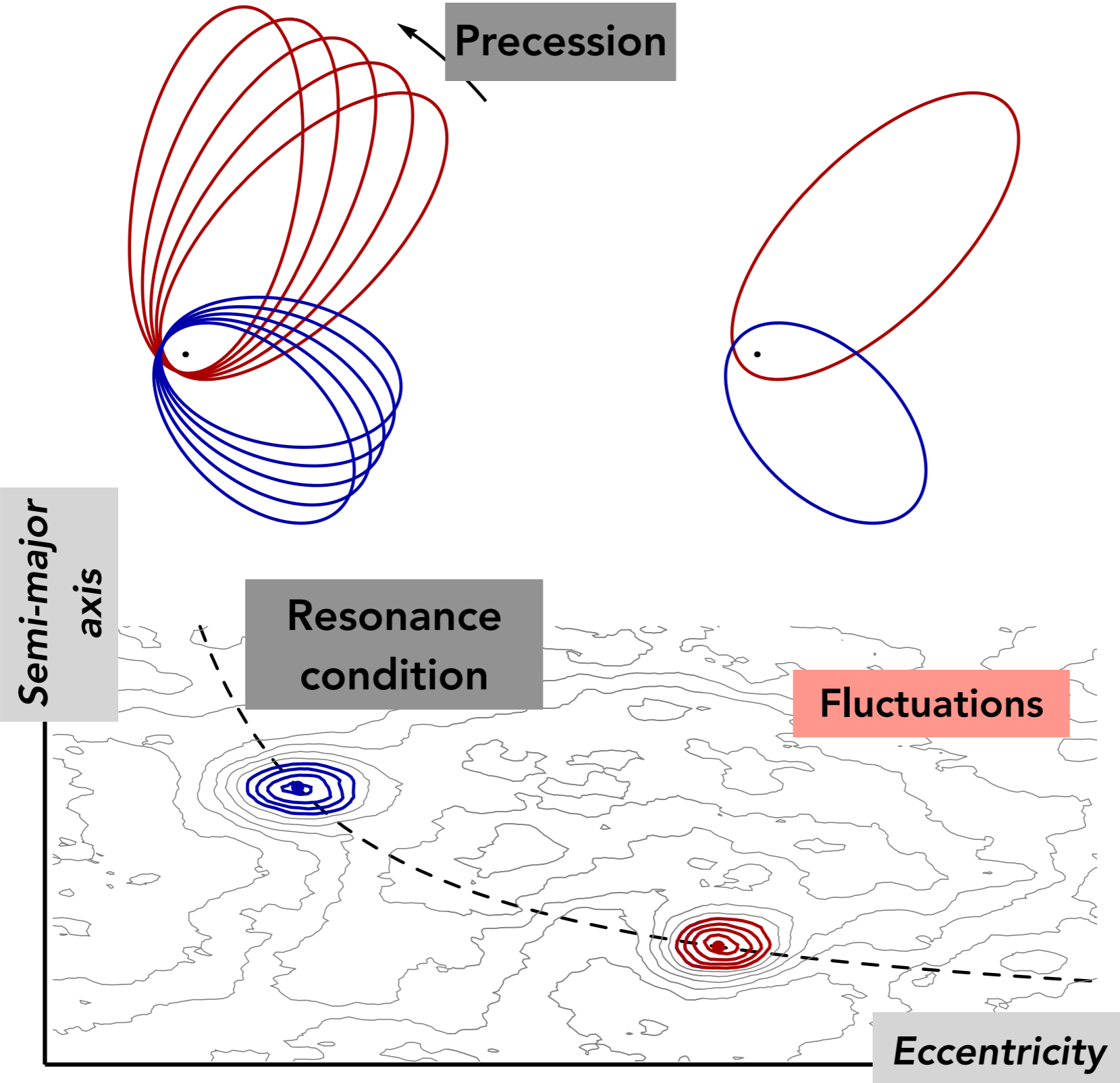
Test particle

Relaxation occurs at **resonance**

$$k \Omega_p(a, h) = k' \Omega_p(a', h')$$

*Balescu-Lenard equation*

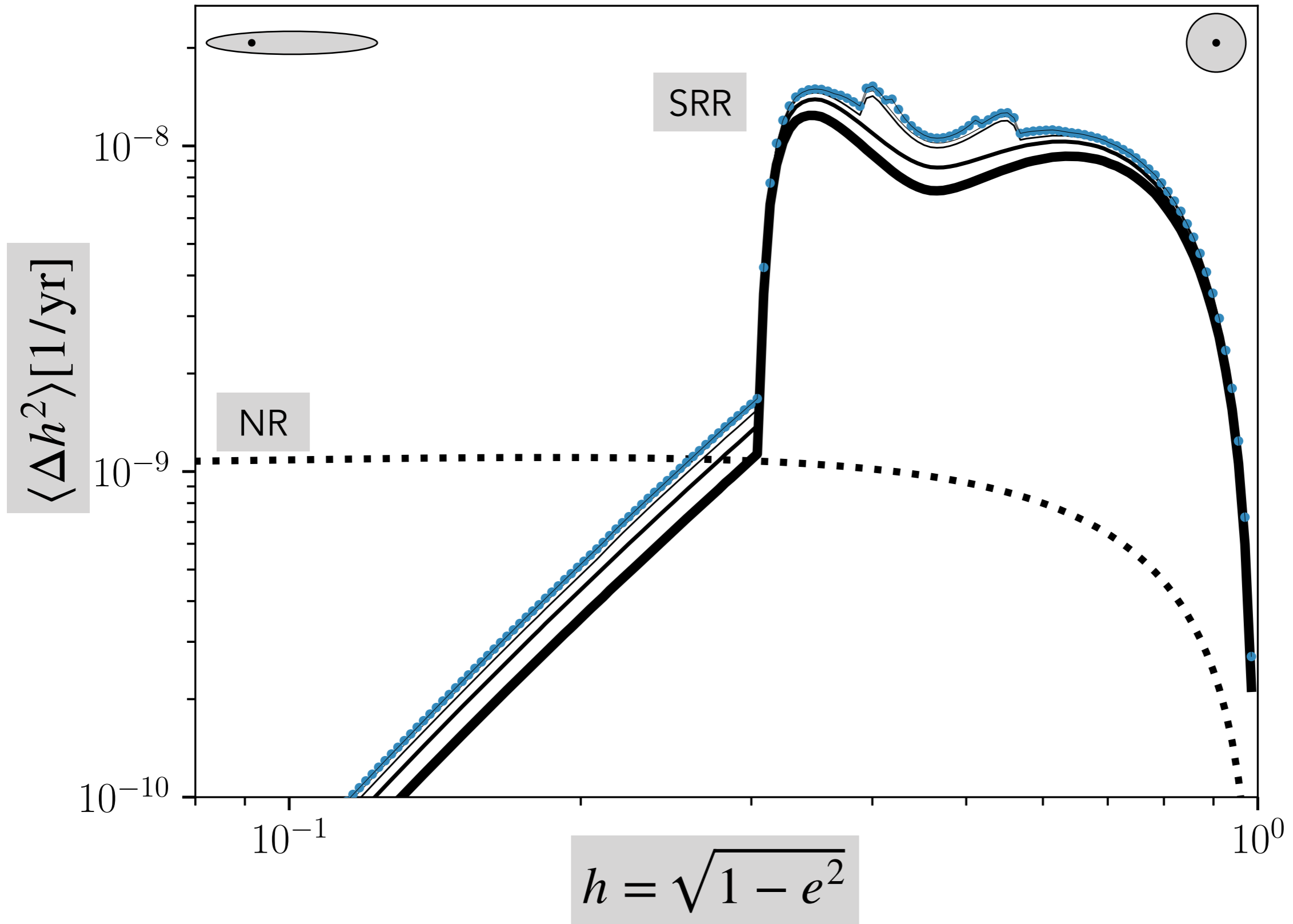
# Non-local resonances



*Non-local resonances  
between wires*

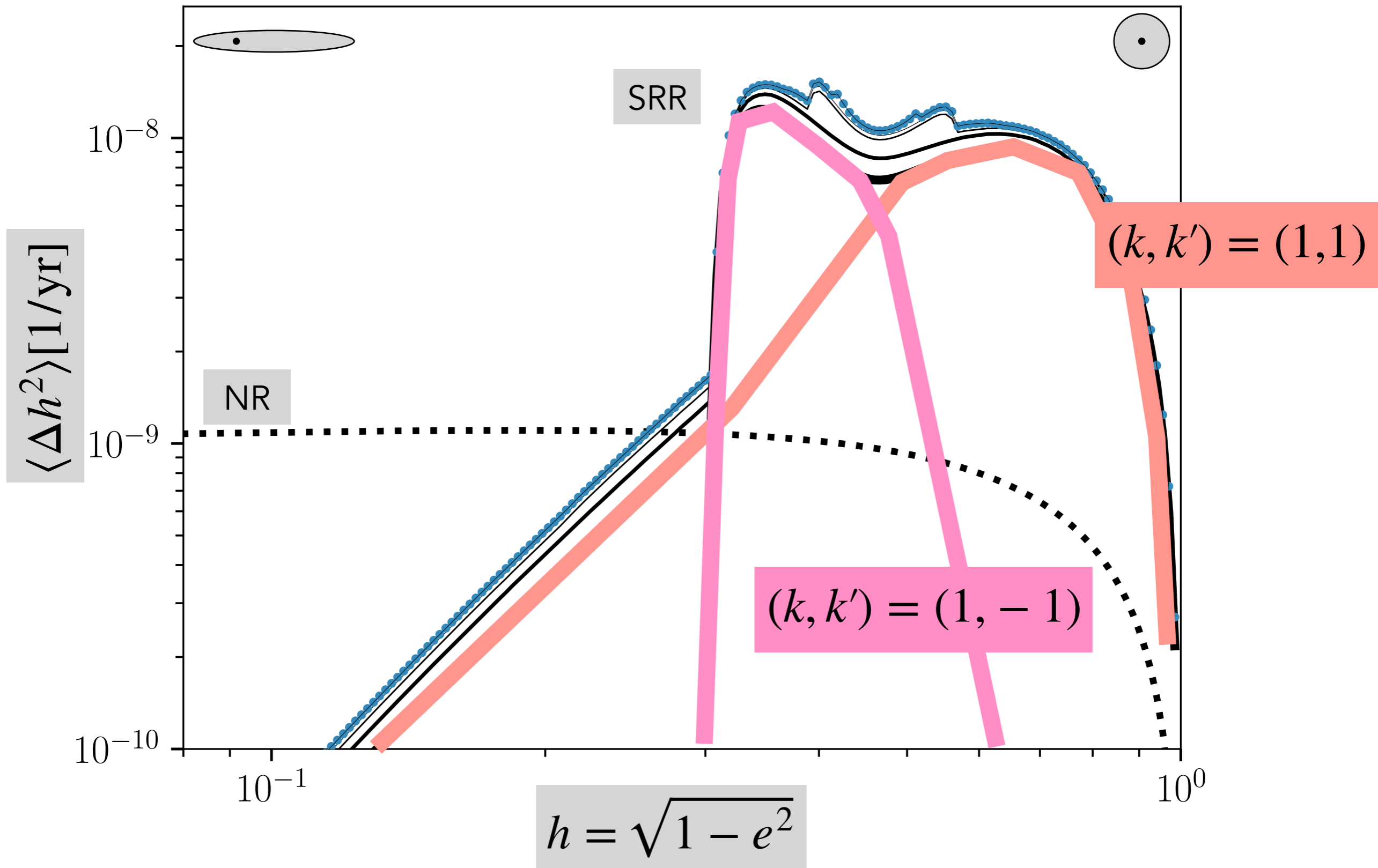


# The diffusion coefficients in eccentricity



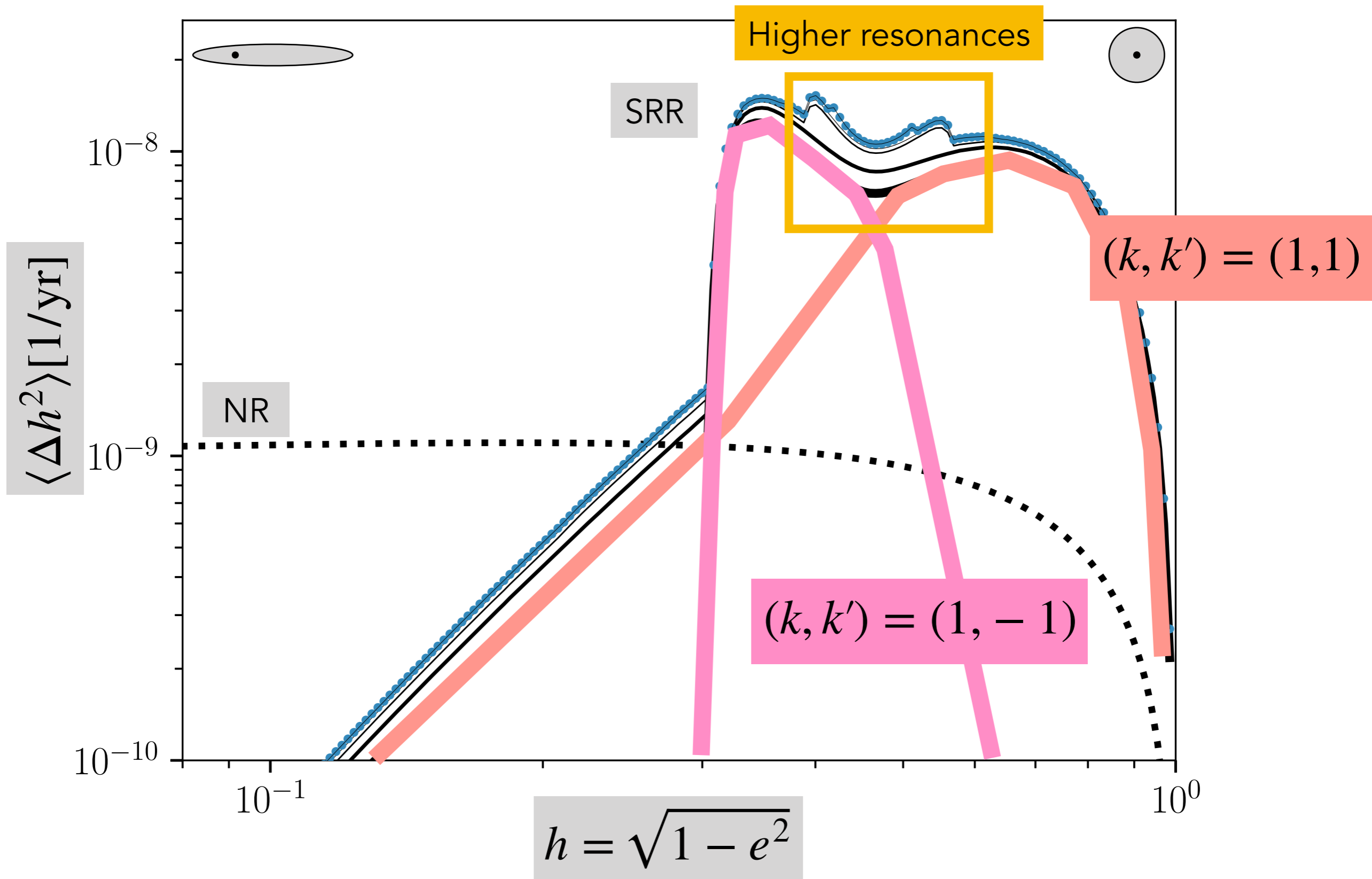
# Non-local resonances

$$\delta_D[k \Omega_p(a, h) - k' \Omega_p(a', h')]$$

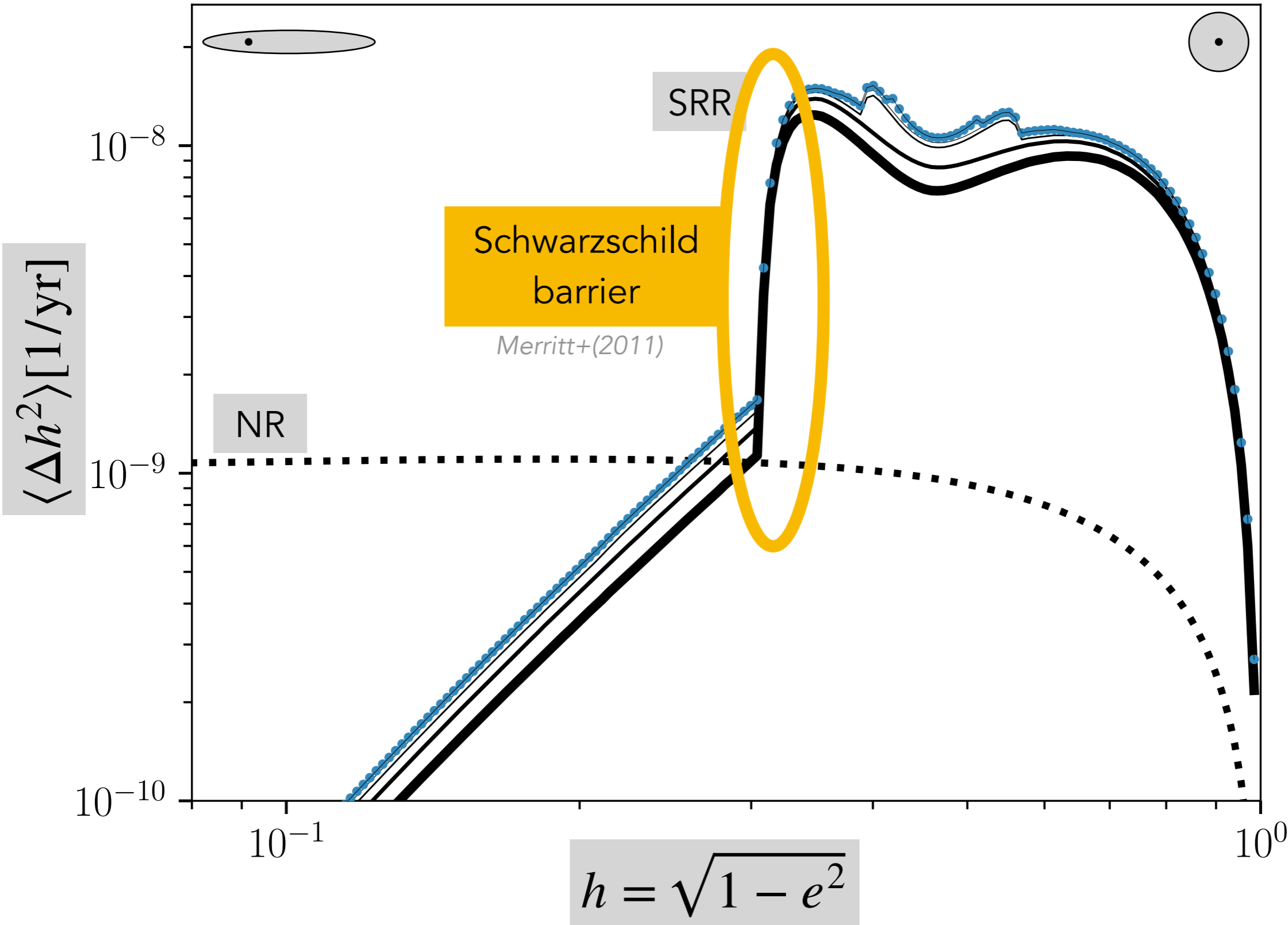


# Non-local resonances

$$\delta_D[k \Omega_p(a, h) - k' \Omega_p(a', h')]$$



# The diffusion coefficients in eccentricity



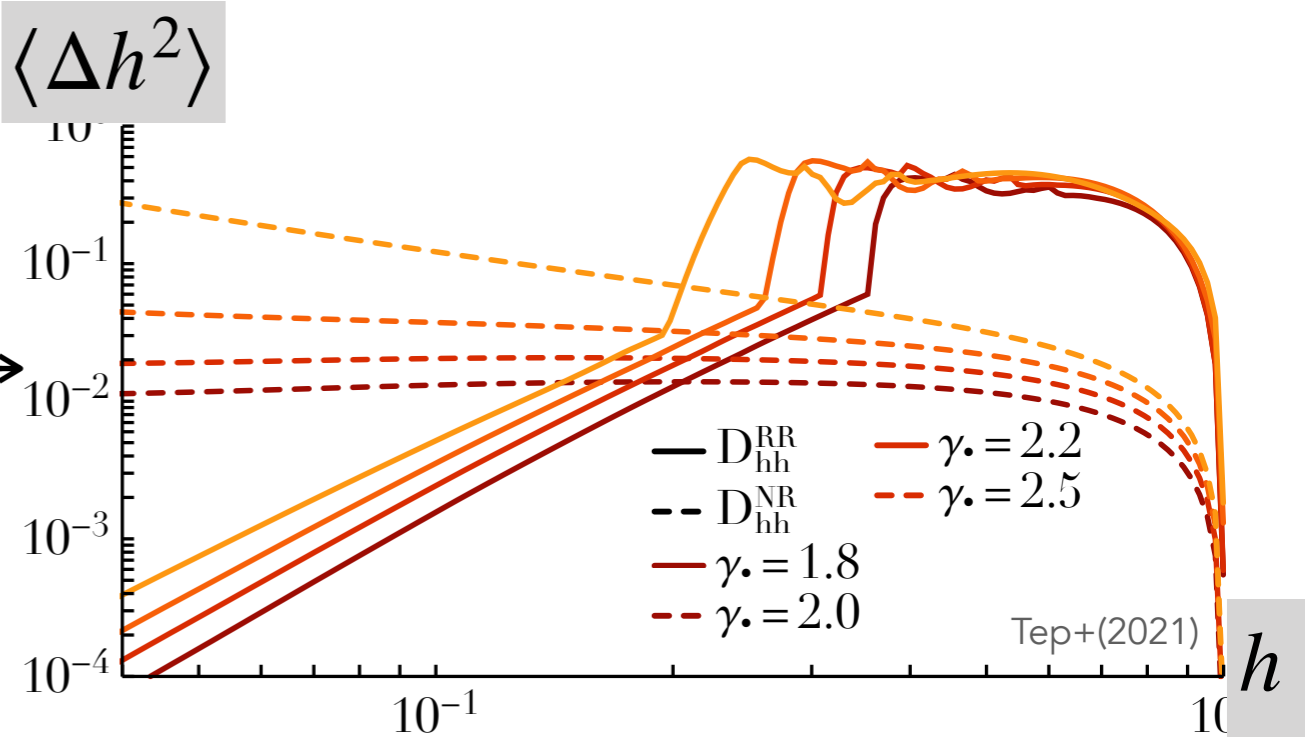


# SRR around SgrA\*

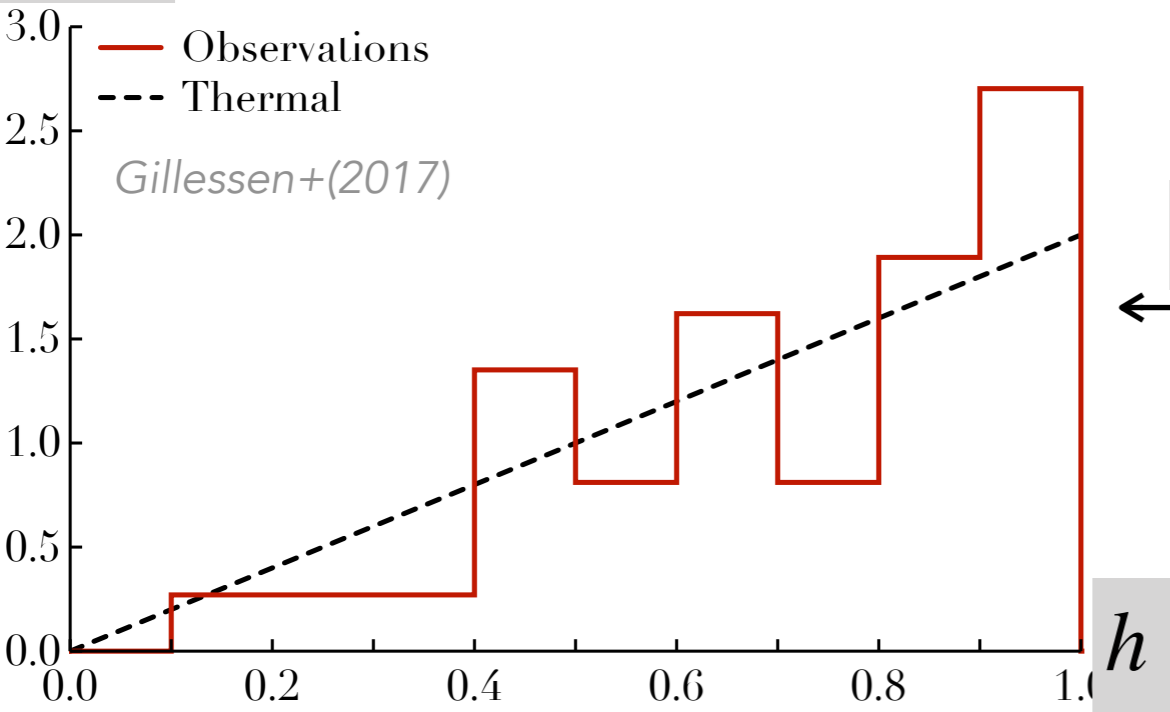
## Model

- Old stars  
*(unresolved but relaxed)*
- **IMBHs**  
*(strong source of Poisson noise)*
- S-stars ICs  
*(Tidal disruption vs disc formation)*

## Kinetic theory

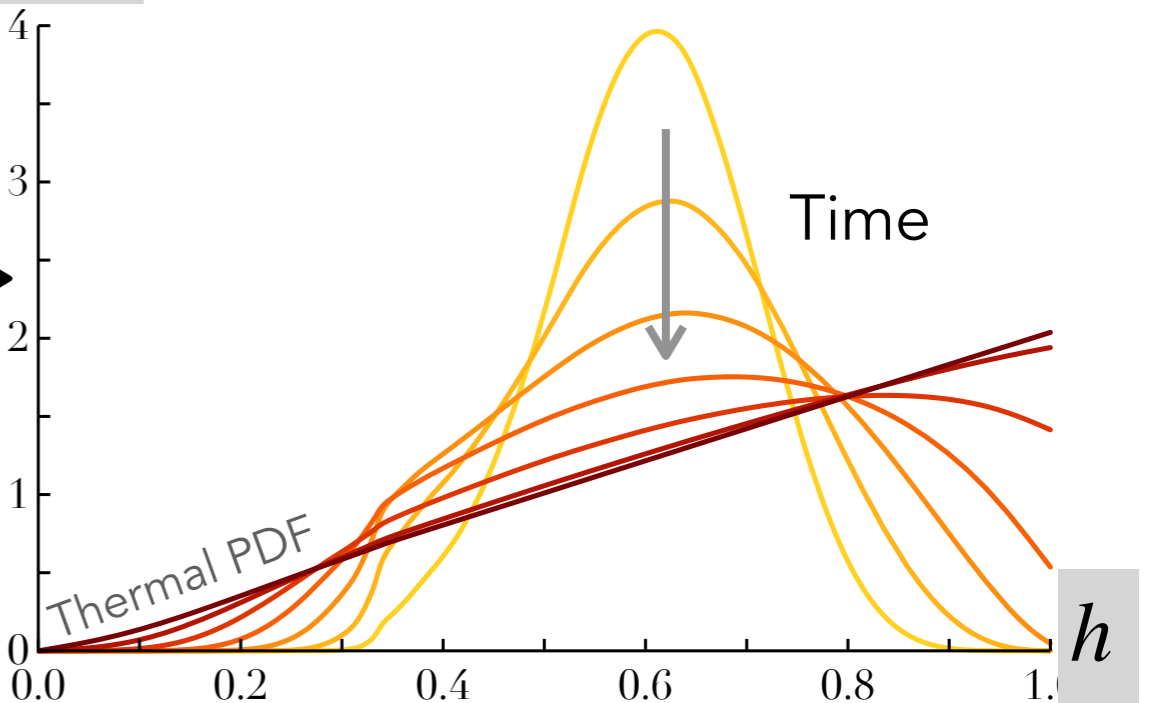


## $P(h)$



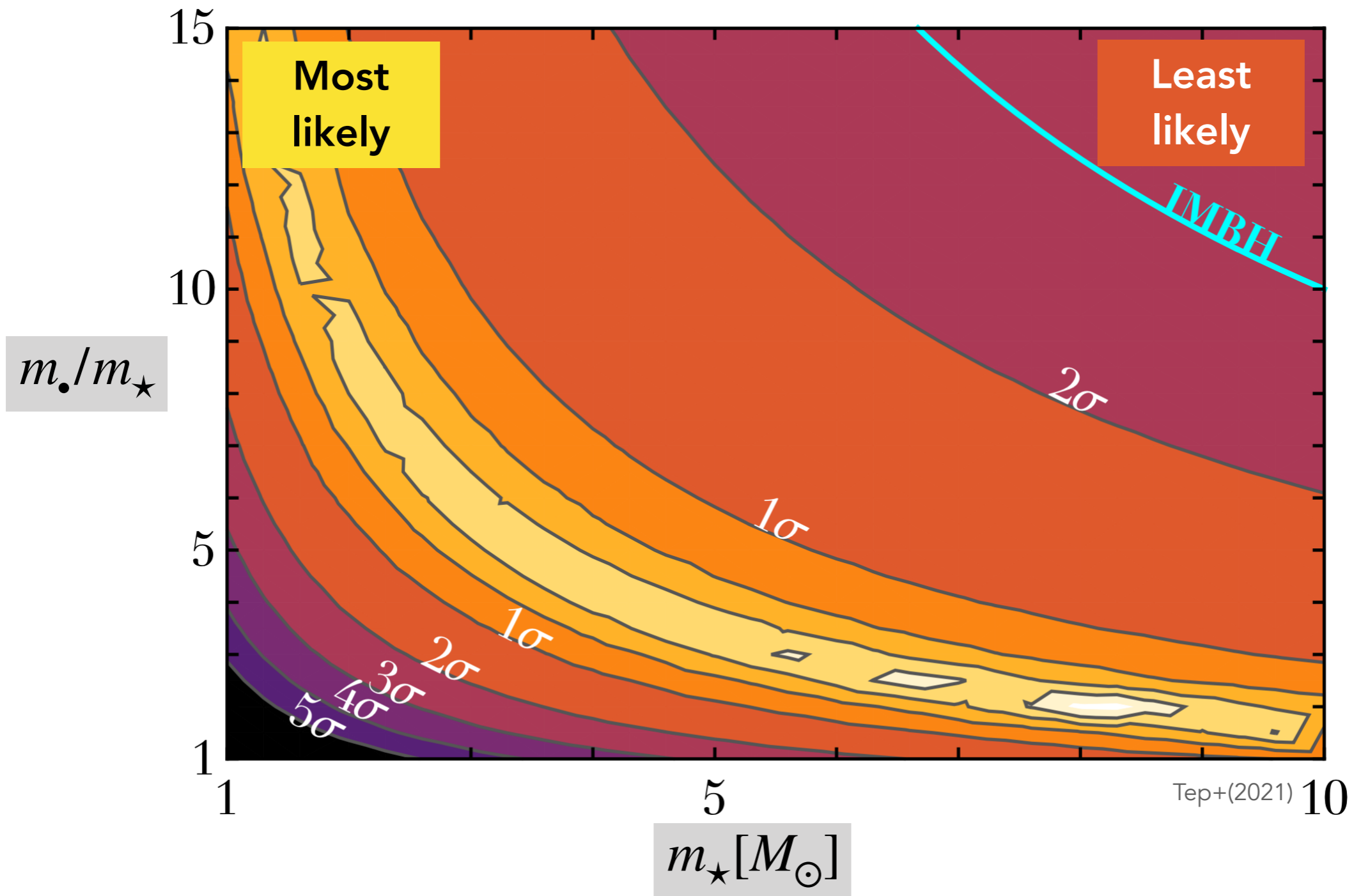
## Relaxation

## $P(h)$



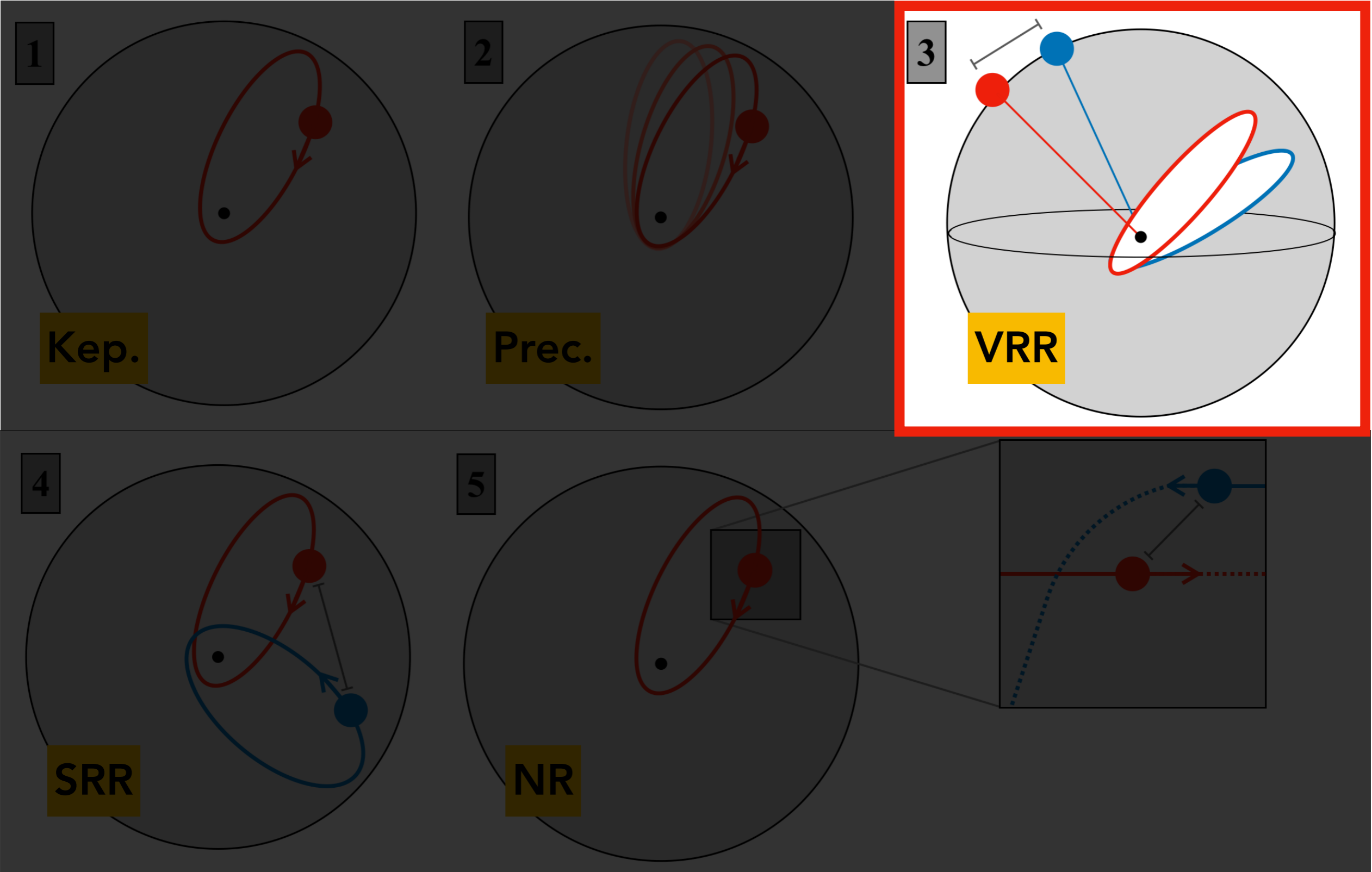
# Constraining IMBHs from SRR

2-population model (stars+IMBHs)



IMBHs population hard to reconcile with the **eccentricity distribution**

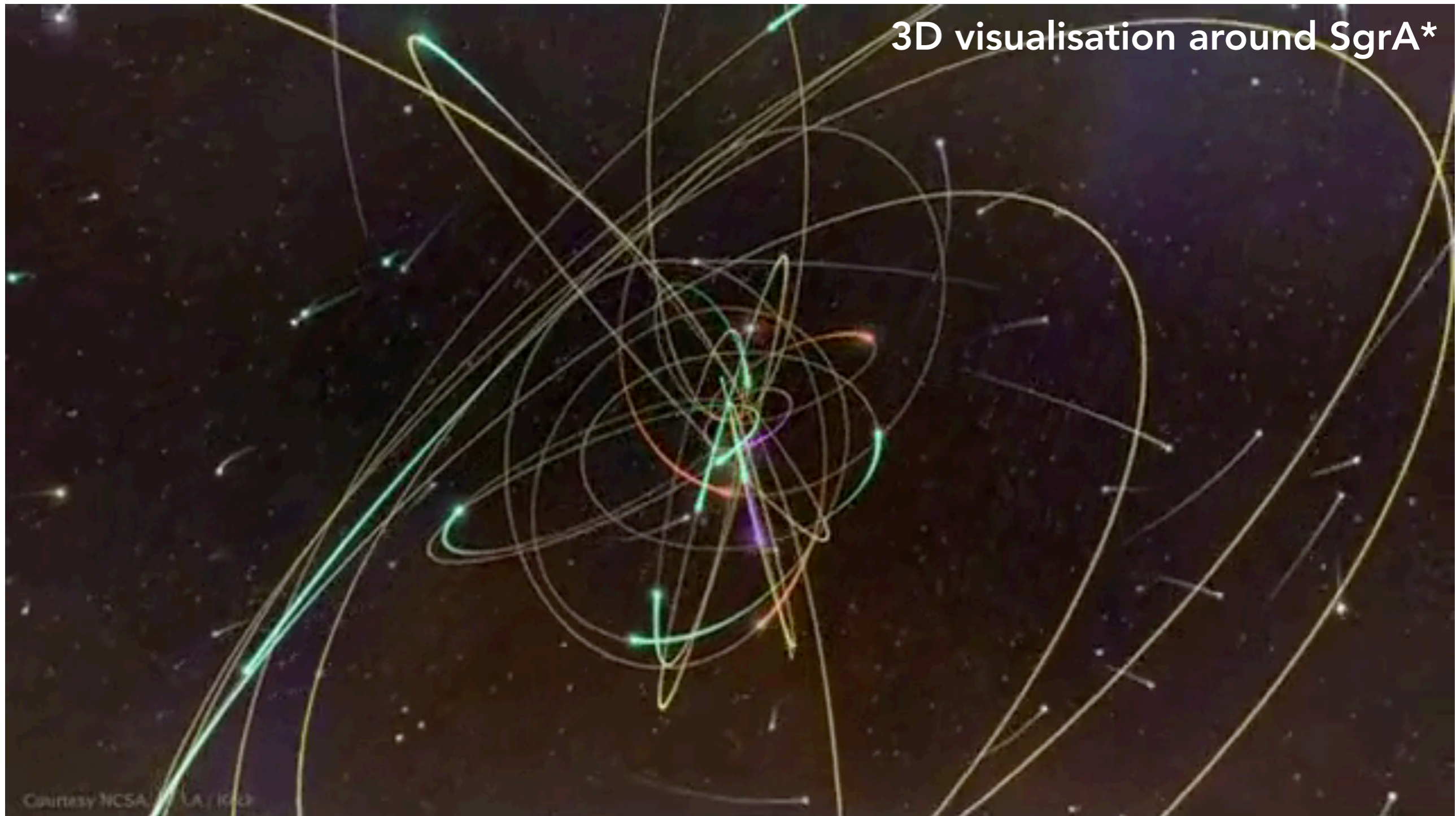
# Vector Resonant Relaxation



The coherent dynamics of **orientations**

## Stellar orientations

Orbits are in **all directions**



How do stars change of **orientations**?

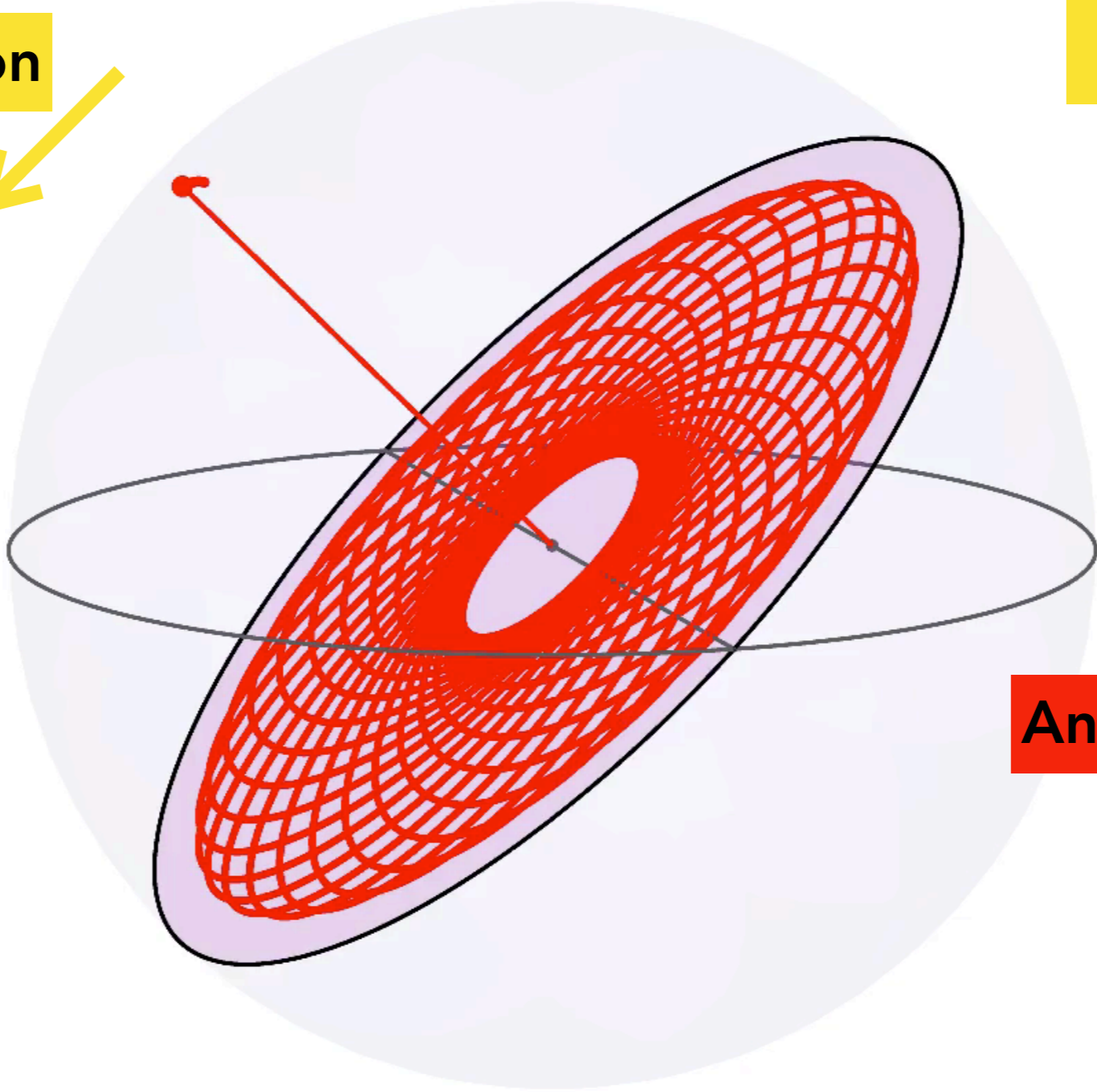


# Stellar orientations

Orientation



Typical timescale  
~1,000,000 years



Annuli

After a full precession, **ellipses** become **annuli**

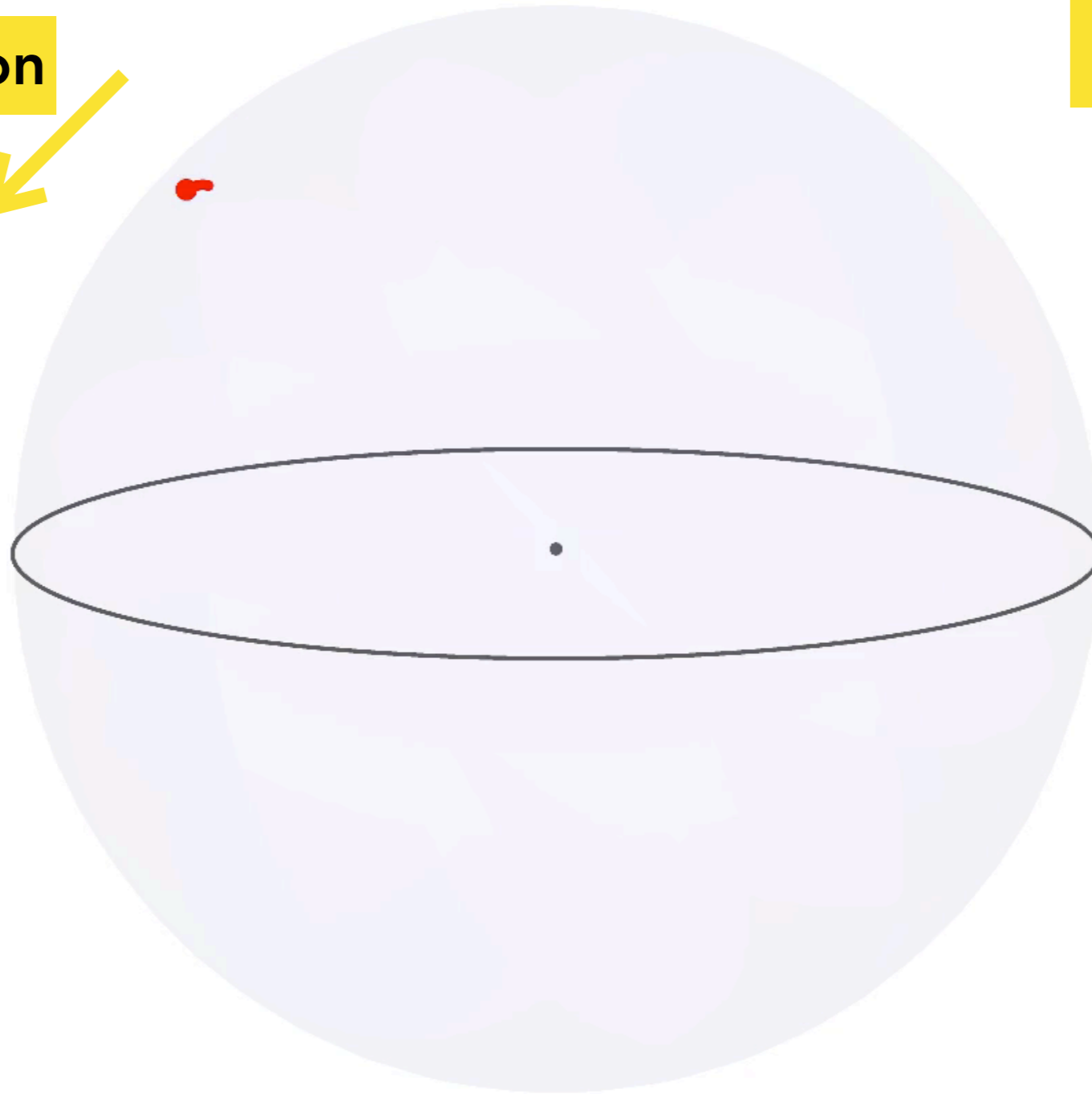
## Orbital orientations

Orientation

$\hat{\mathbf{L}}$

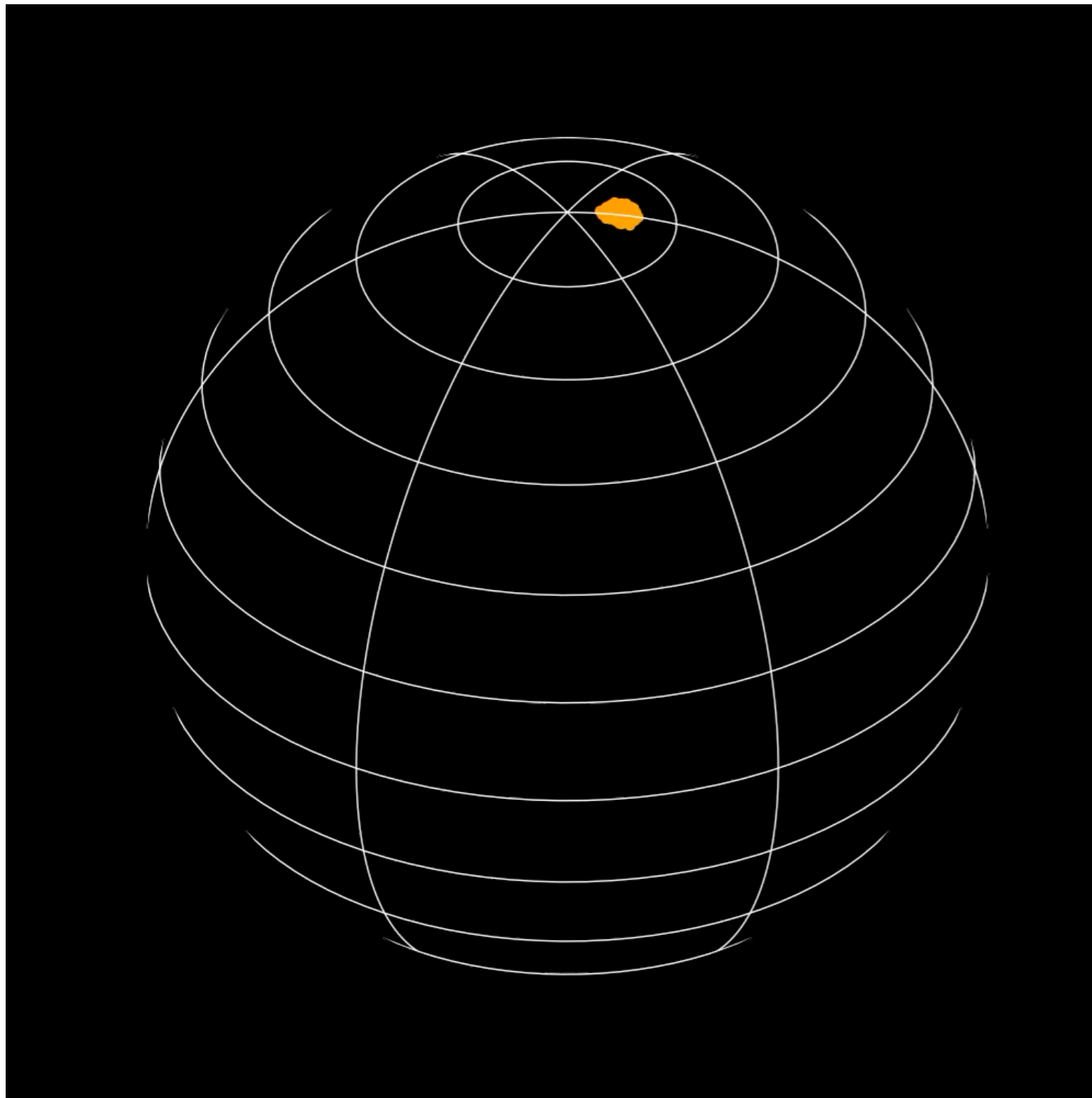
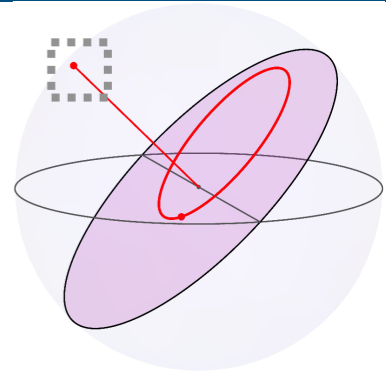


Typical timescale  
 $\sim 1,000,000$  years



One orientation becomes a single point on the **unit sphere**

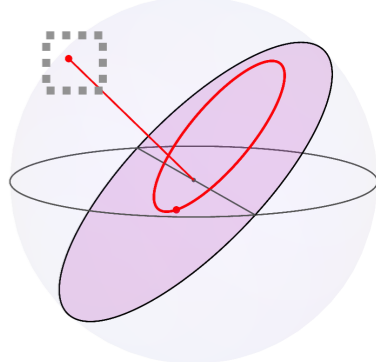
# Vector Resonant Relaxation



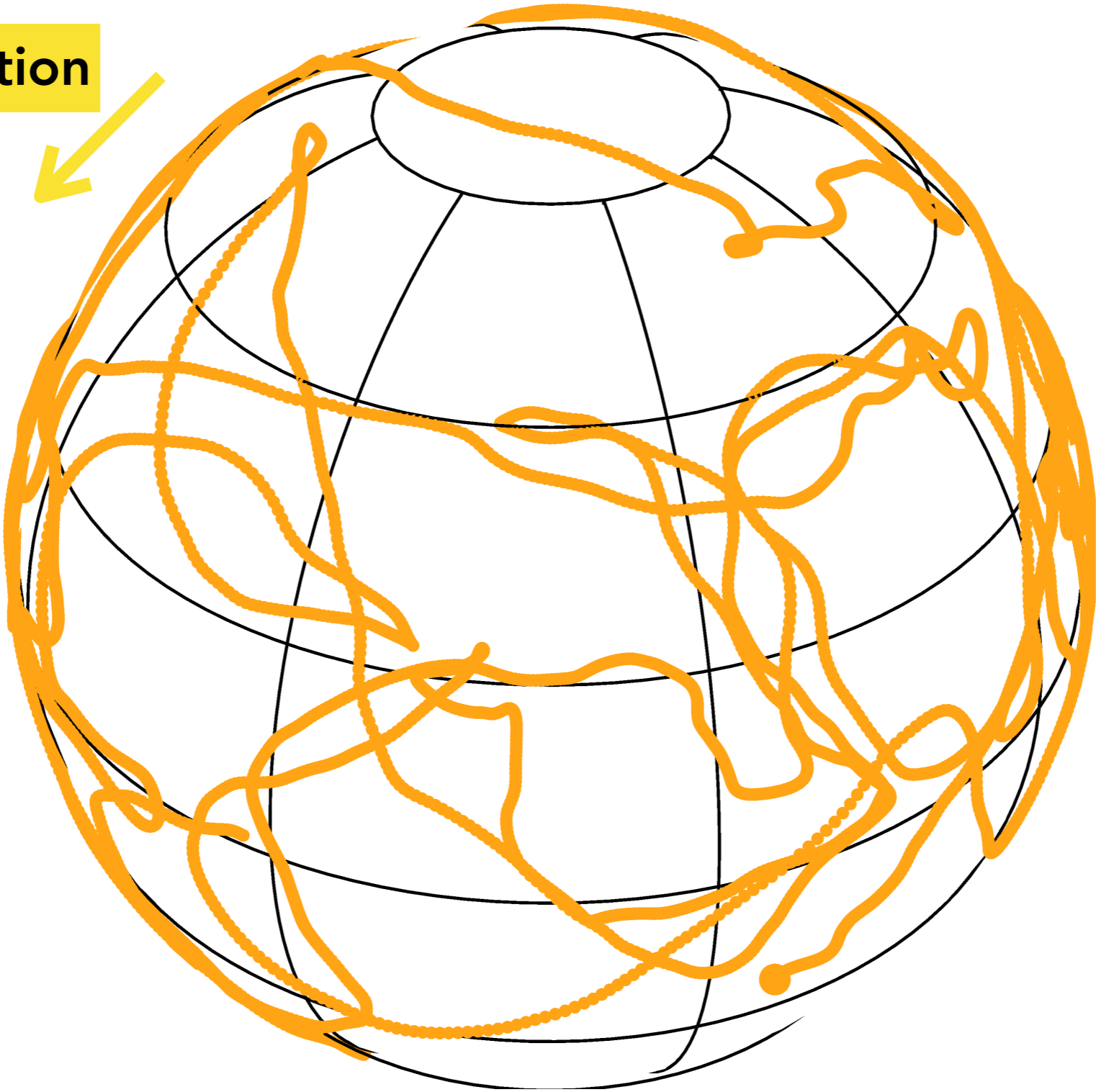
- + Motion coherent on large scales
  - **Long-range interacting system**
- + Motion smooth on short times
  - **Time-correlated noise**
- + Particles have "preferred friends"
  - **Parametric coupling** ( $a, e$ )
- + System in statistical equilibrium
  - **Time stationarity** ( $t - t'$ )
  - **Rotation invariance** ( $\hat{\mathbf{L}} \cdot \hat{\mathbf{L}}'$ )

This is an intrinsically  
turbulent dynamics

# Typical evolution of an orientation



Orientation

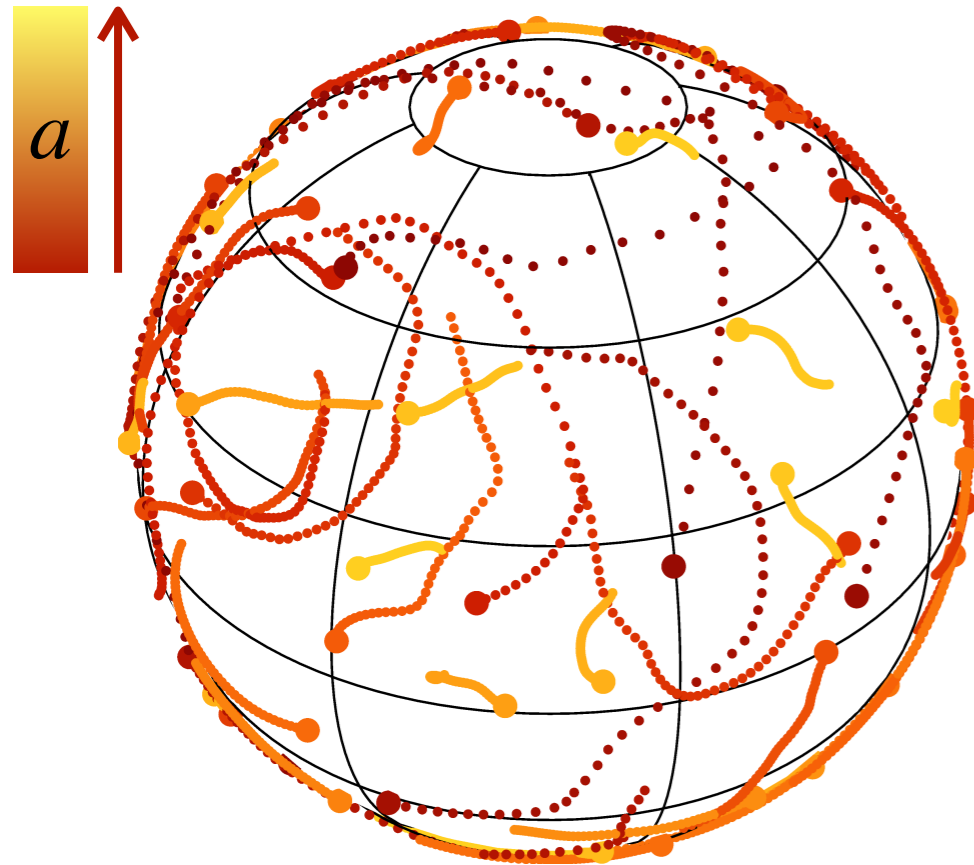


Typical timescale  
~1,000,000 years

Stellar orientations follow a **correlated random walk**

# Self-consistency requirement

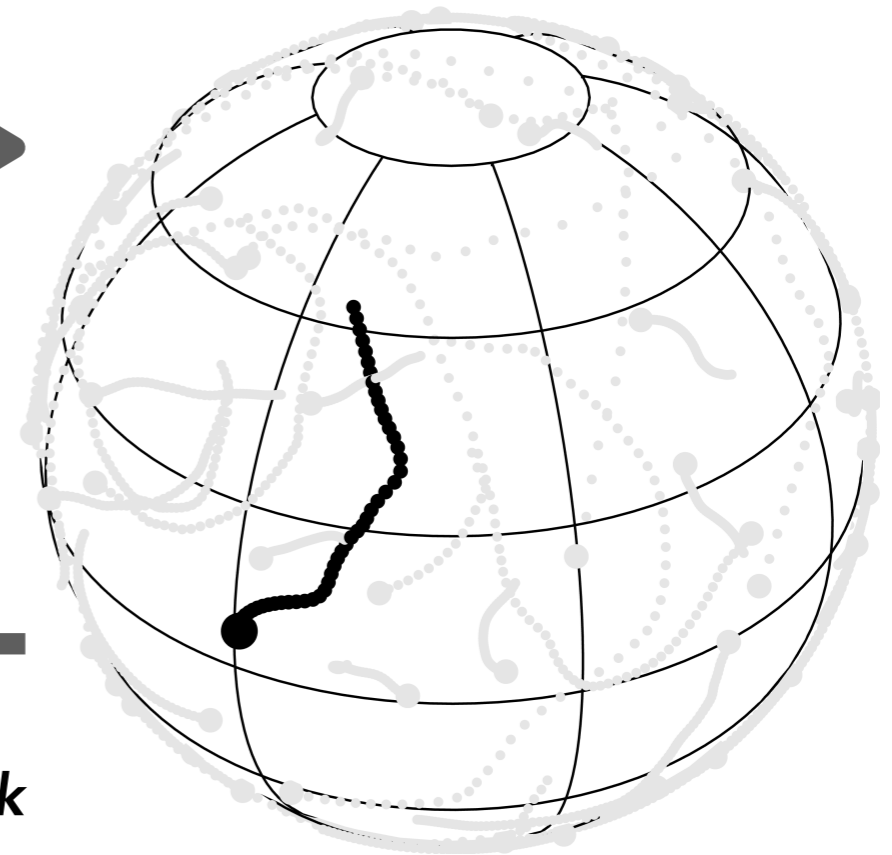
Background particles



Imposes a noisy  
(correlated) **potential**

Test particle

Undergoes a  
(correlated) **random walk**

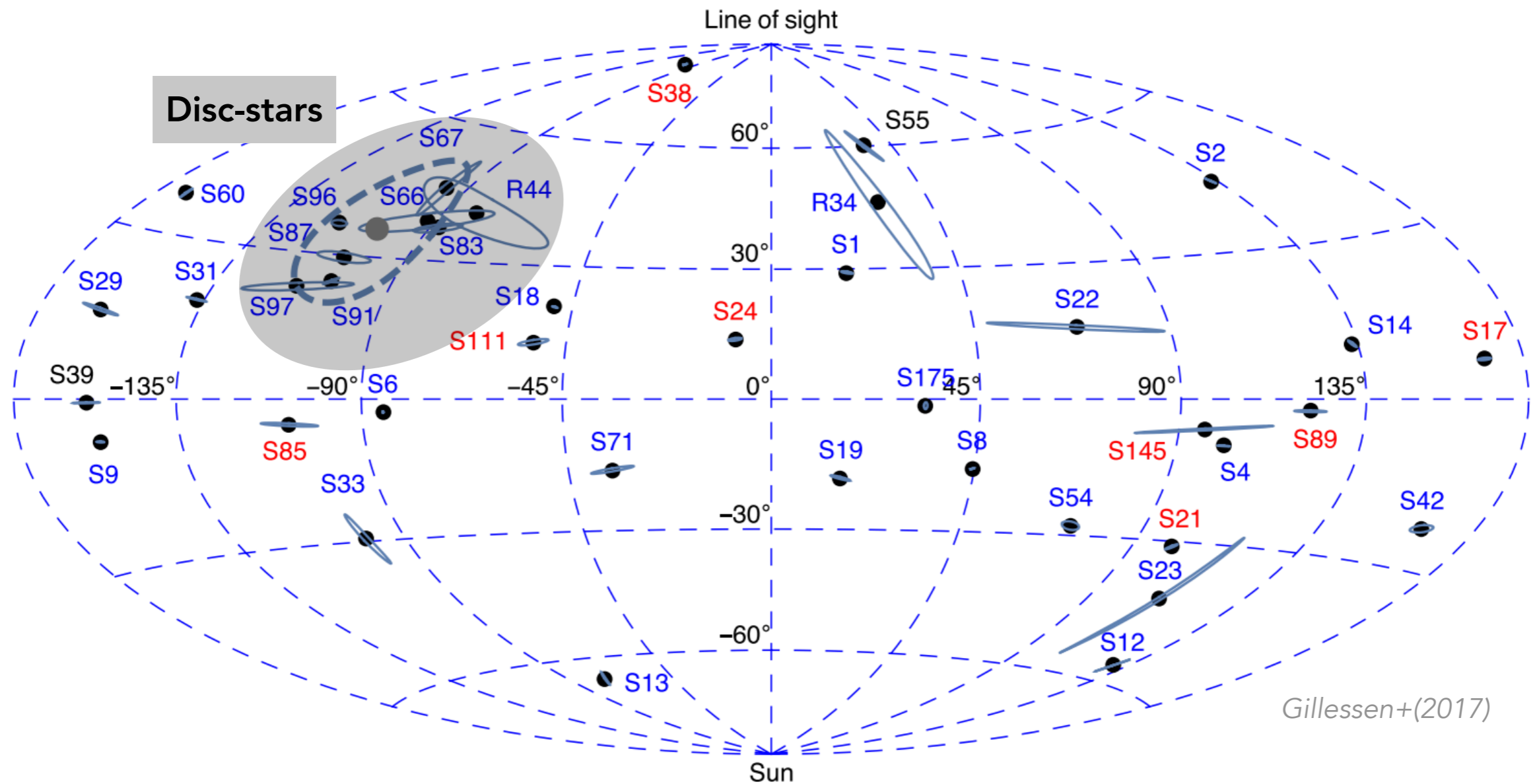


$$\hat{C}_{\text{bath}} = \left\langle \eta(\hat{\mathbf{L}}, t) \eta(\hat{\mathbf{L}}', t') \right\rangle$$

$$\hat{C}_{\text{test}} = \left\langle \hat{\mathbf{L}}_{\text{test}}(t) \cdot \hat{\mathbf{L}}_{\text{test}}(0) \right\rangle$$

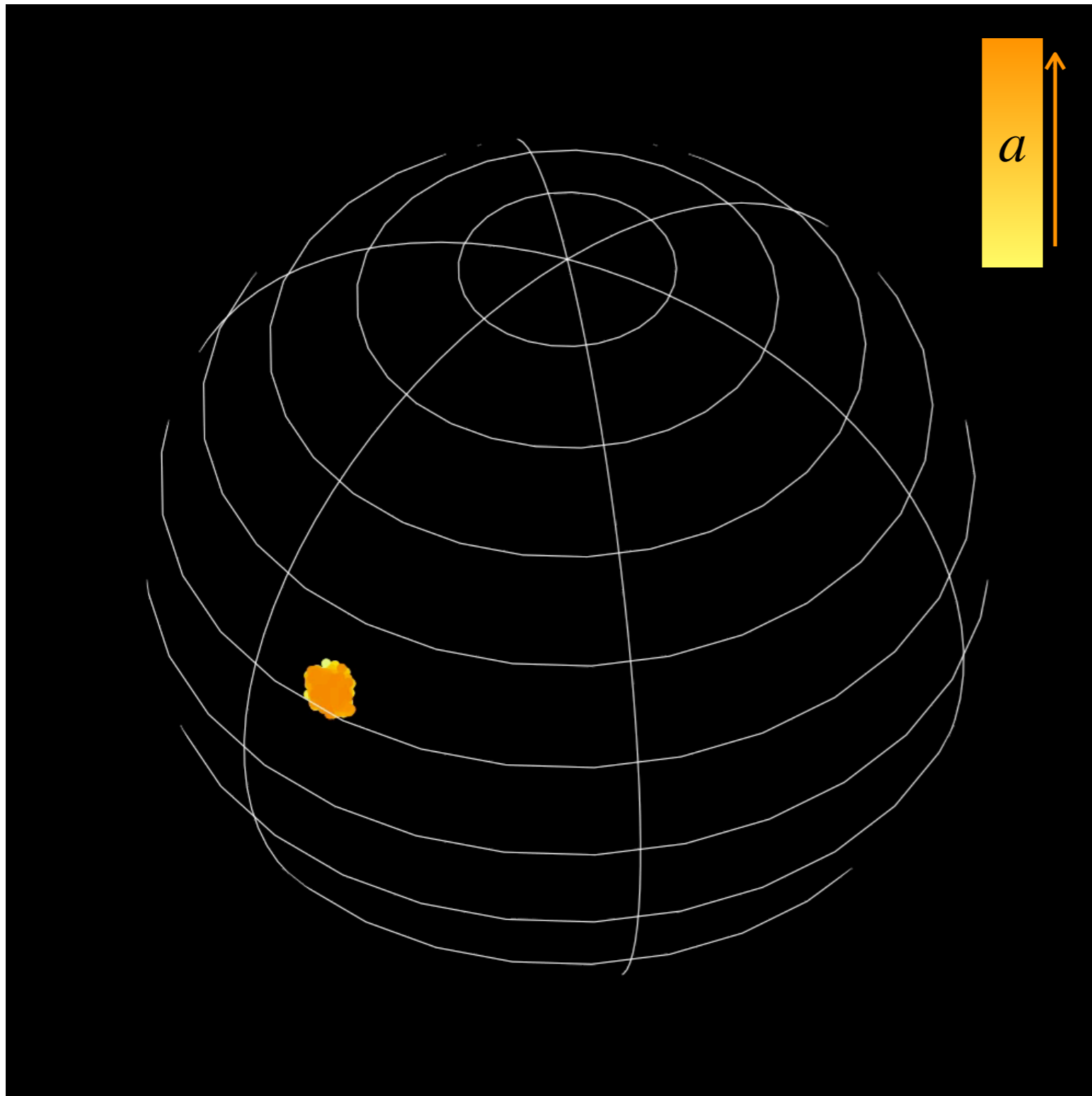


# Vector Resonant Relaxation can affect the disc-stars



How long should these stars stay “neighbors”?  
 Are they **young** enough?

# Vector Resonant Relaxation can randomize disc stars



+ How “neighbors” get separated

$$\frac{d\hat{\mathbf{L}}_i}{dt} = \eta(\hat{\mathbf{L}}_i, t)$$

+ Evolution sourced by a **shared, spatially-extended** and **time-correlated** noise

$$\begin{aligned} & \langle \eta(a_i, \hat{\mathbf{L}}_i, t) \eta(a_j, \hat{\mathbf{L}}_j, t') \rangle \\ & = C(a_i, a_j, \hat{\mathbf{L}}_i \cdot \hat{\mathbf{L}}_j, t - t') \end{aligned}$$

+ Two joint sources of **separation**

- **Parametric** separation

$$a_i \neq a_j$$

- **Angular** separation

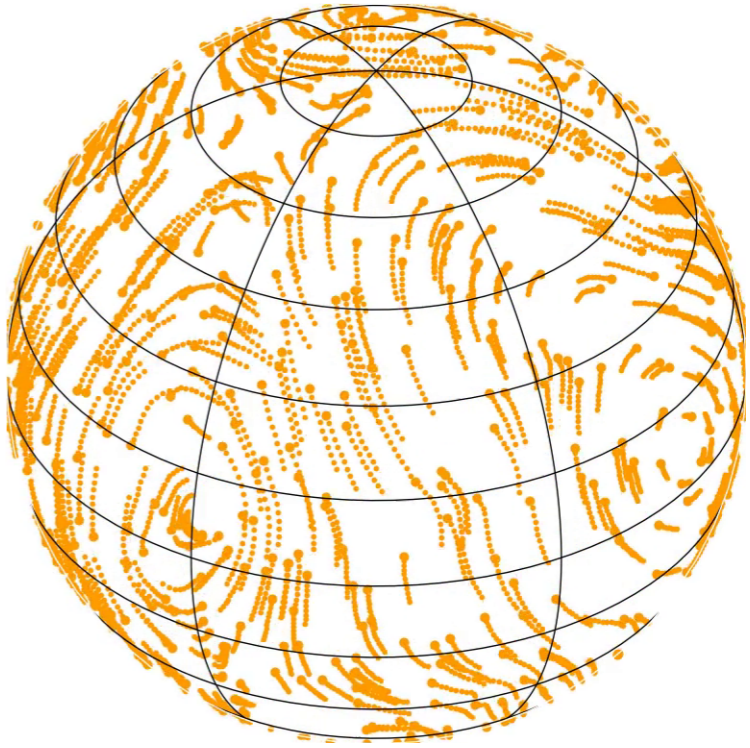
$$\hat{\mathbf{L}}_i \neq \hat{\mathbf{L}}_j$$

# VRR around SgrA\*

## Model

- Old stars  
*(unresolved but relaxed)*
- **IMBHs**  
*(strong source of Poisson noise)*
- S-stars disc ICs  
*(initial angular dispersion)*

Kinetic theory

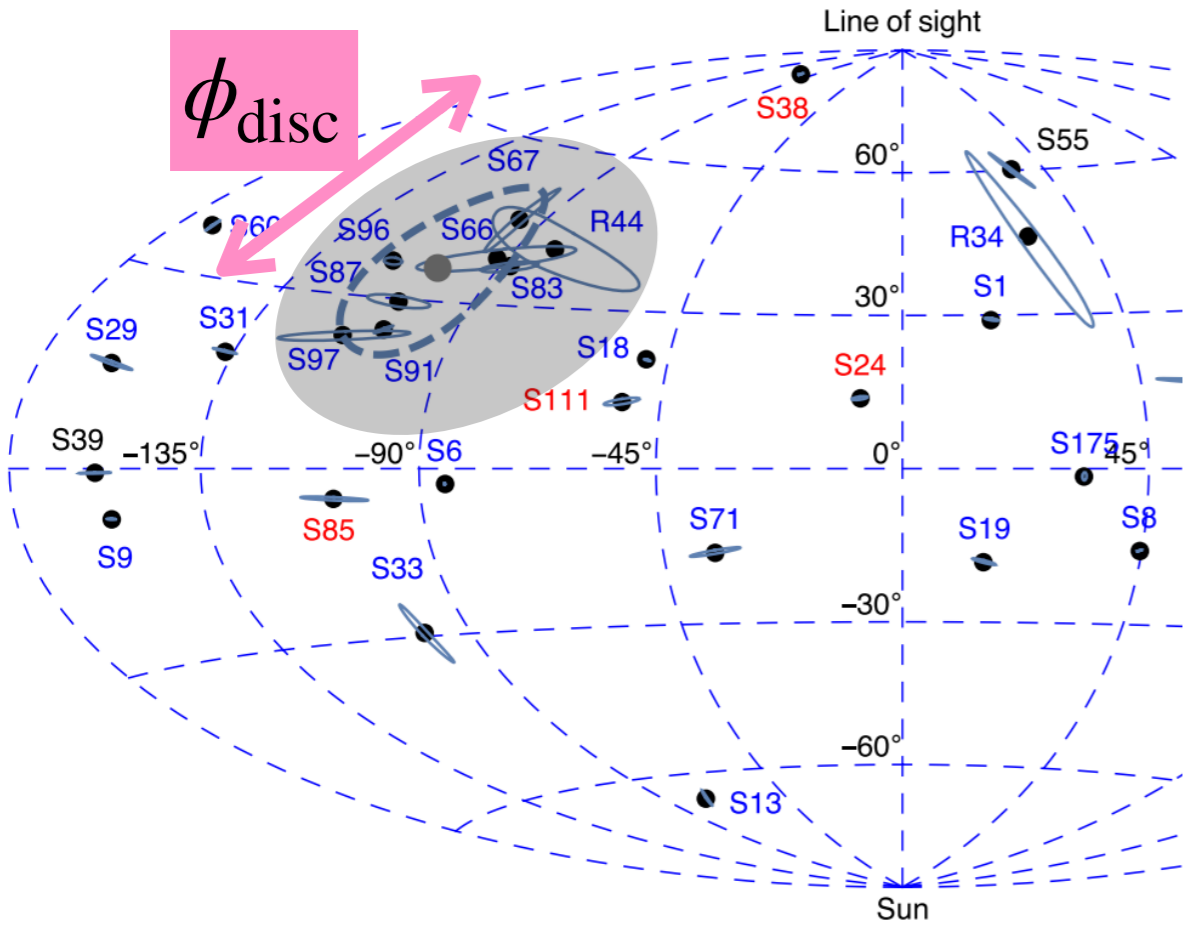
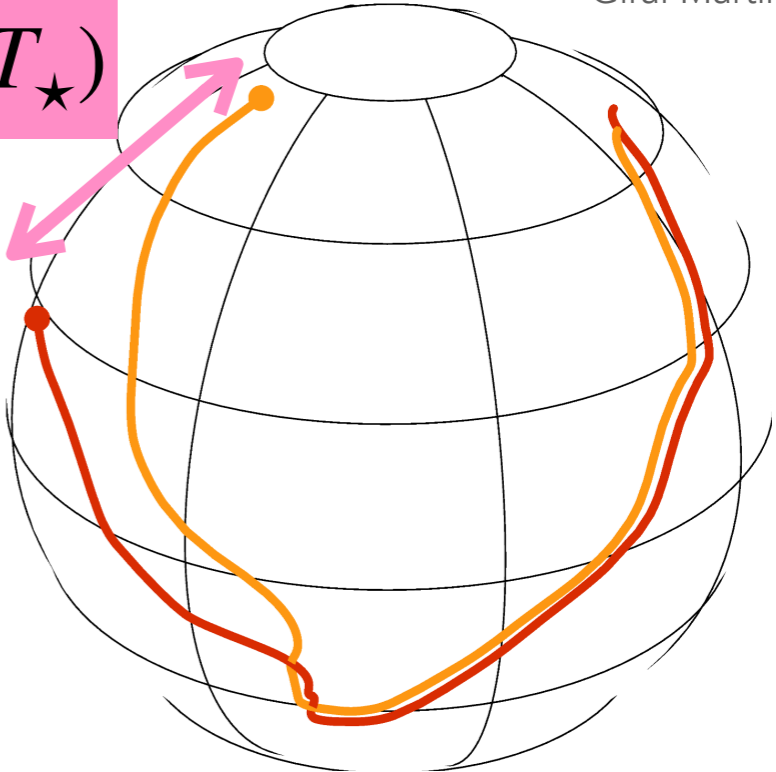


Dilution

Giral Martinez+(2020)

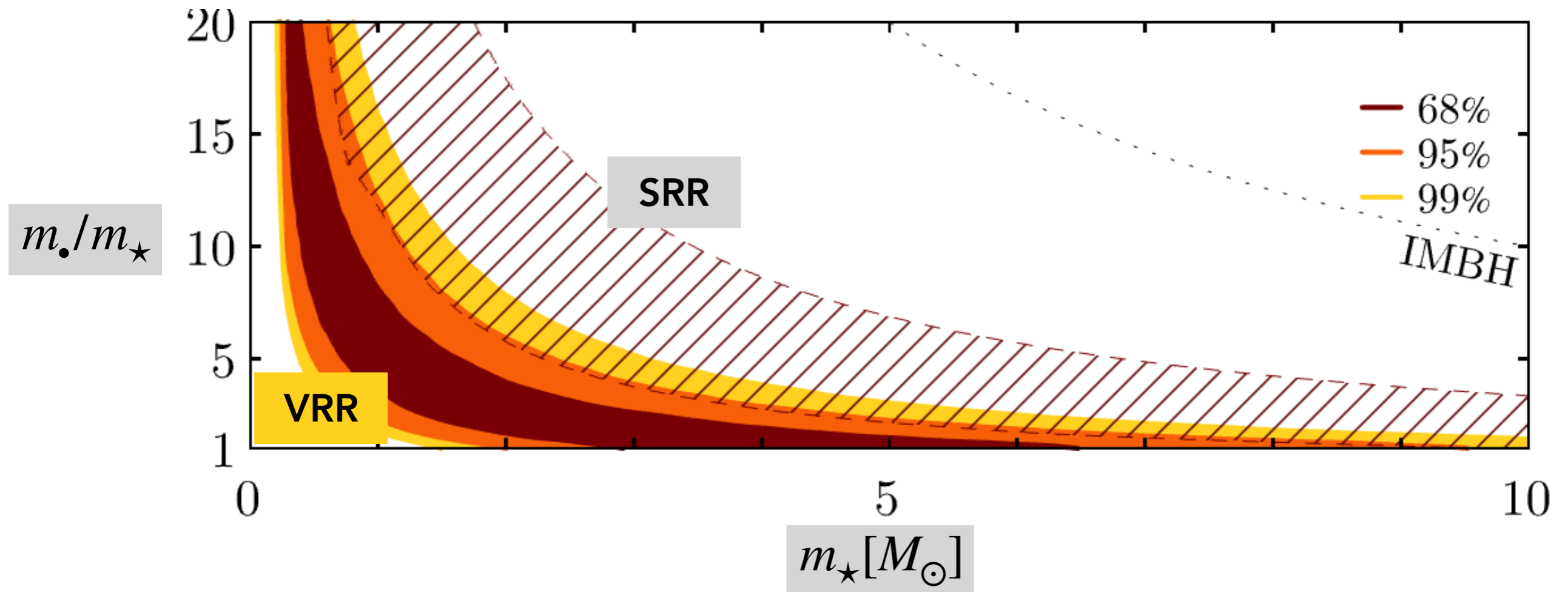
$\phi(T_\star)$

Likelihood



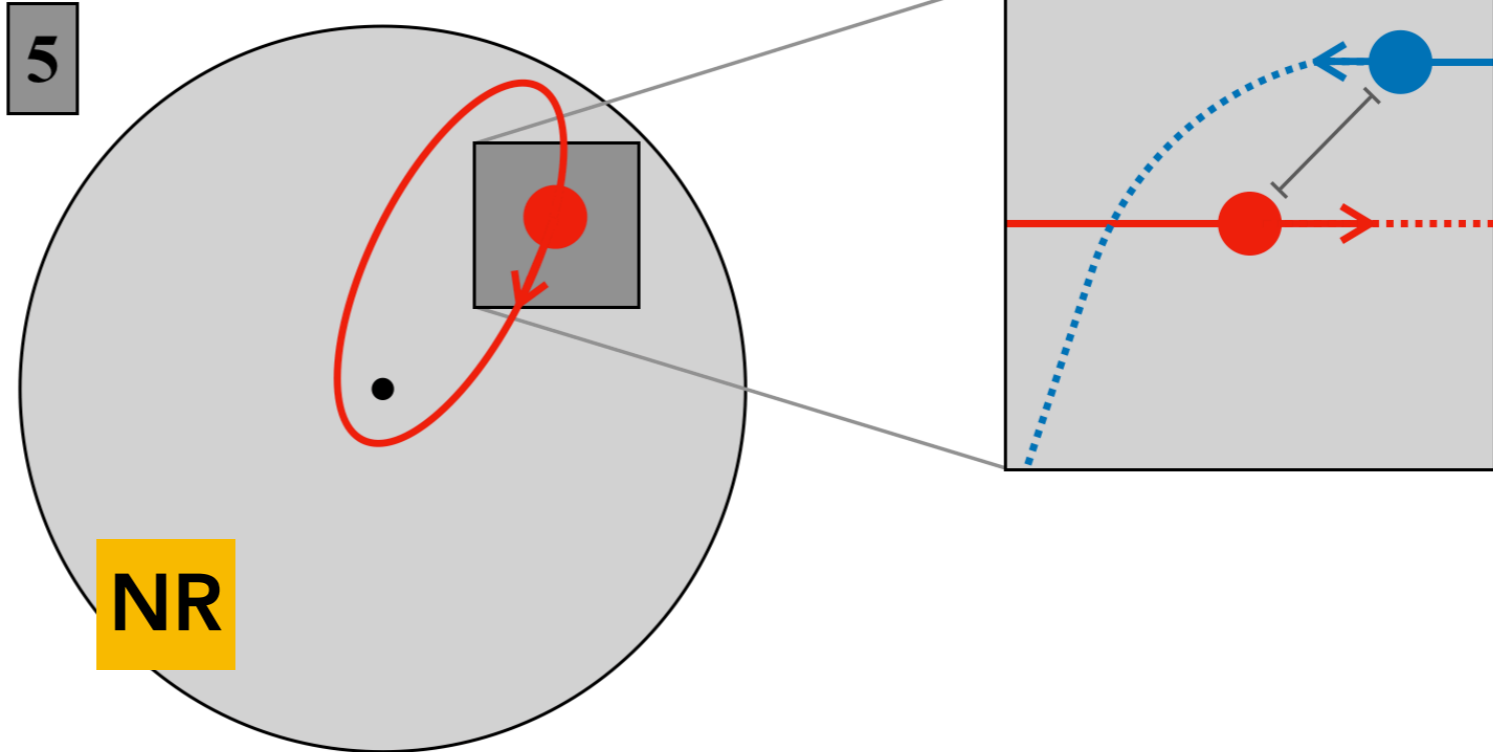
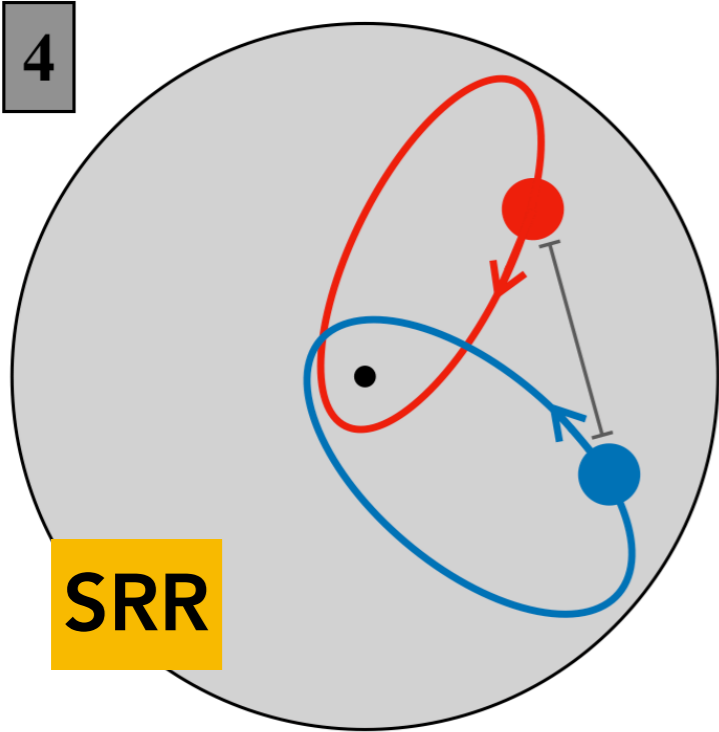
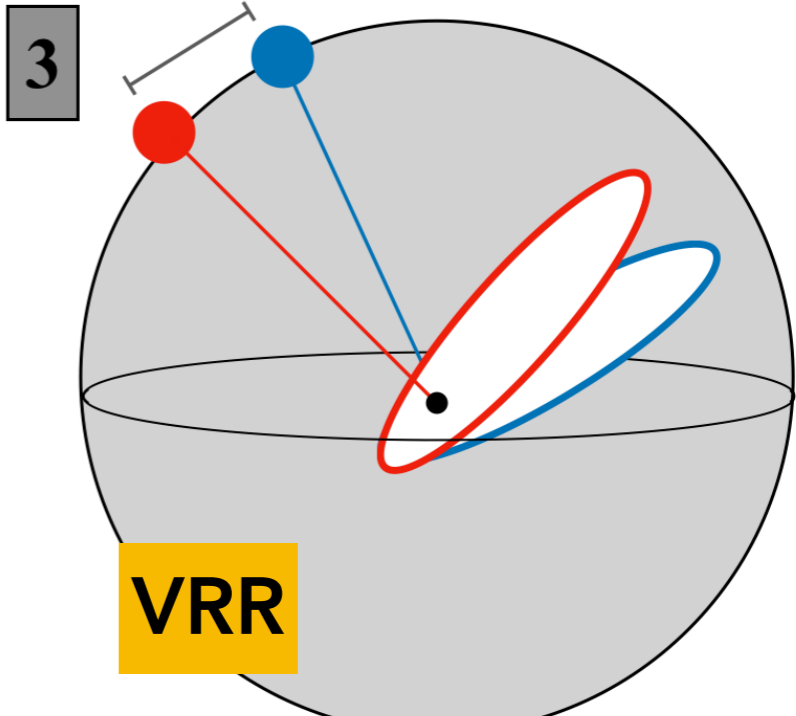
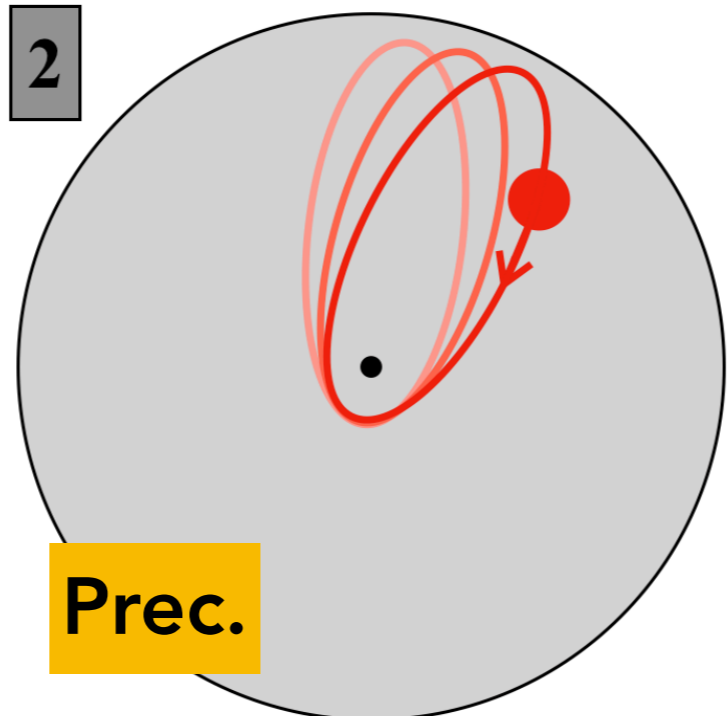
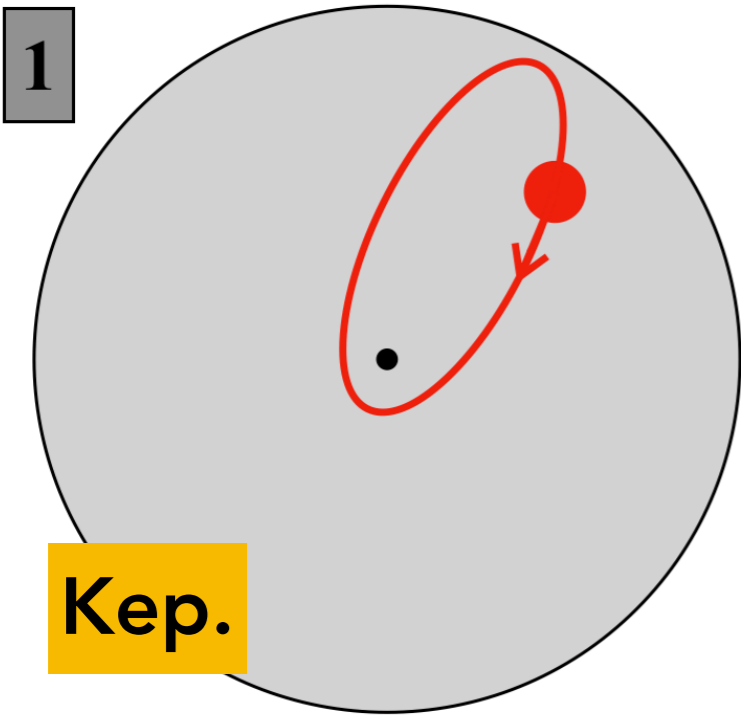
## Constraining IMBHs with VRR

2-population model (stars+IMBHs)



IMBHs population hard to reconcile with the **disc's survival**

# A wealth of dynamical processes



An extremely **hierarchical system**

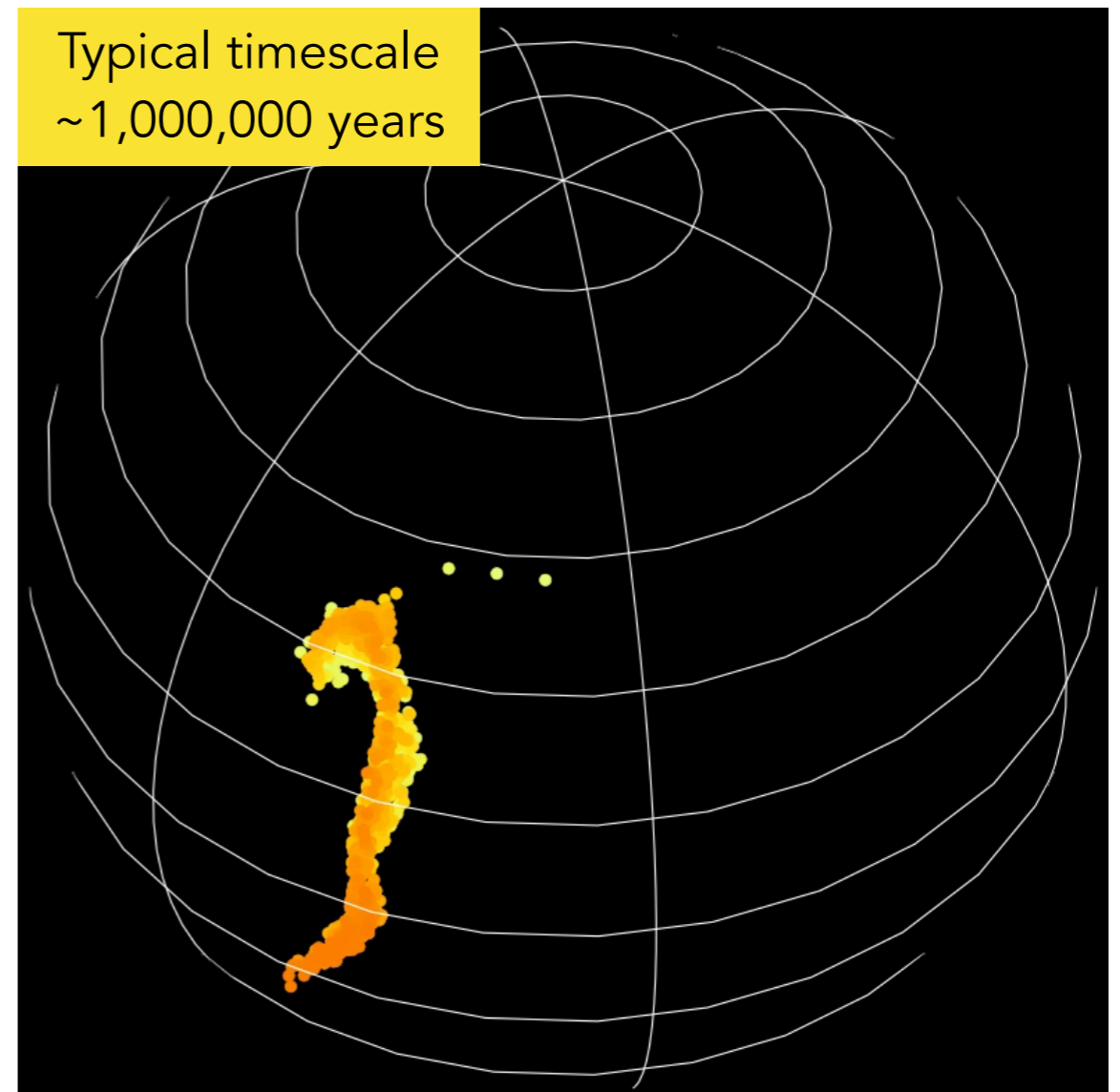


# Dynamics of galactic nuclei

Studying stellar dynamics is studying “ink diffusion”



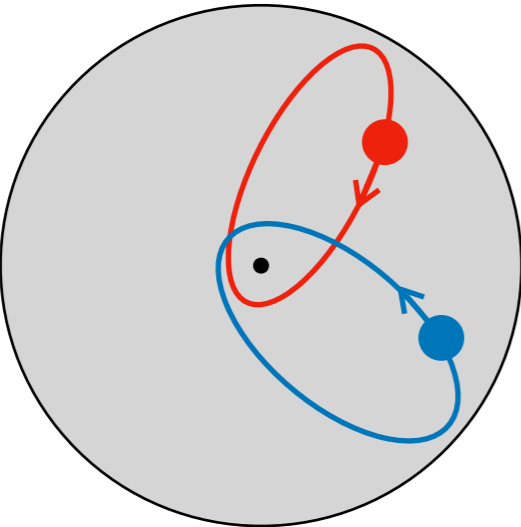
Ink in water



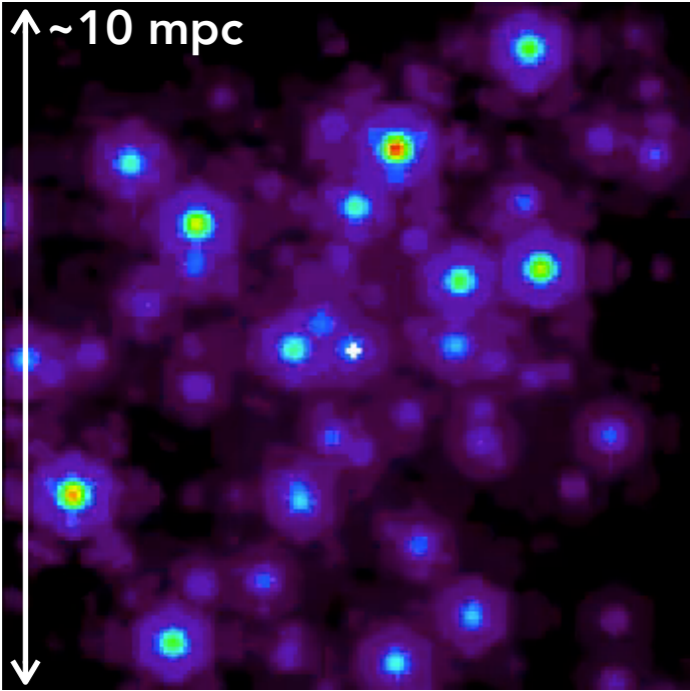
Stellar orientations

# The future of galactic nuclei

New stellar orbits



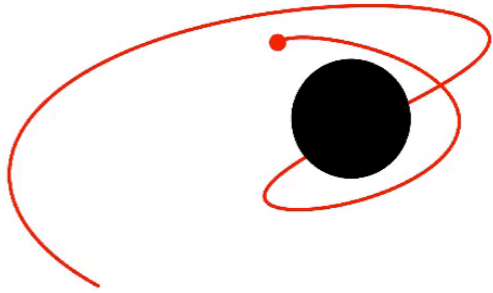
TMT and ELT



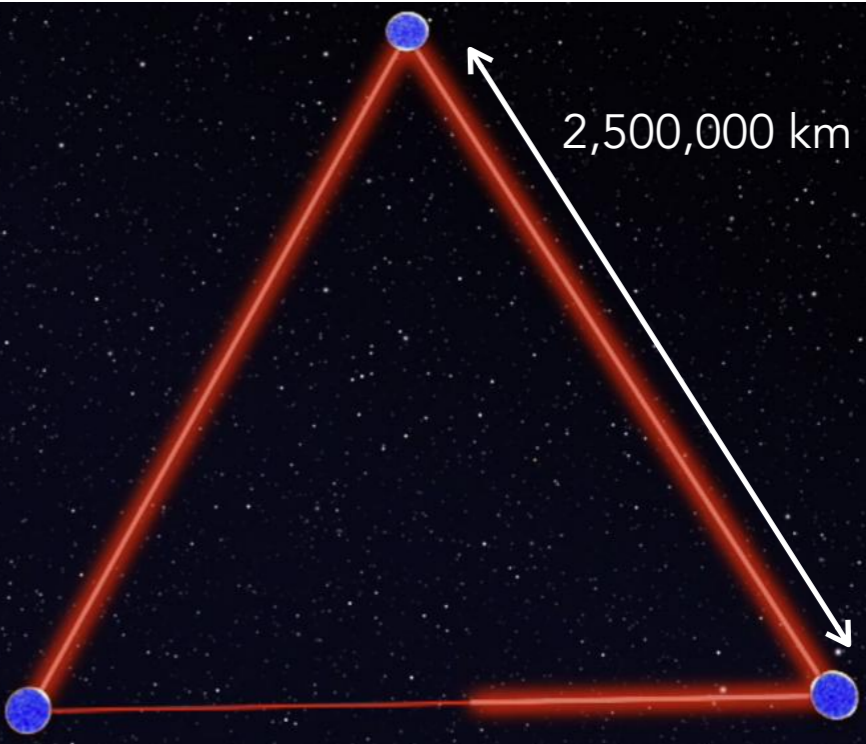
Expected observations

UCLA

Infall of compact objects



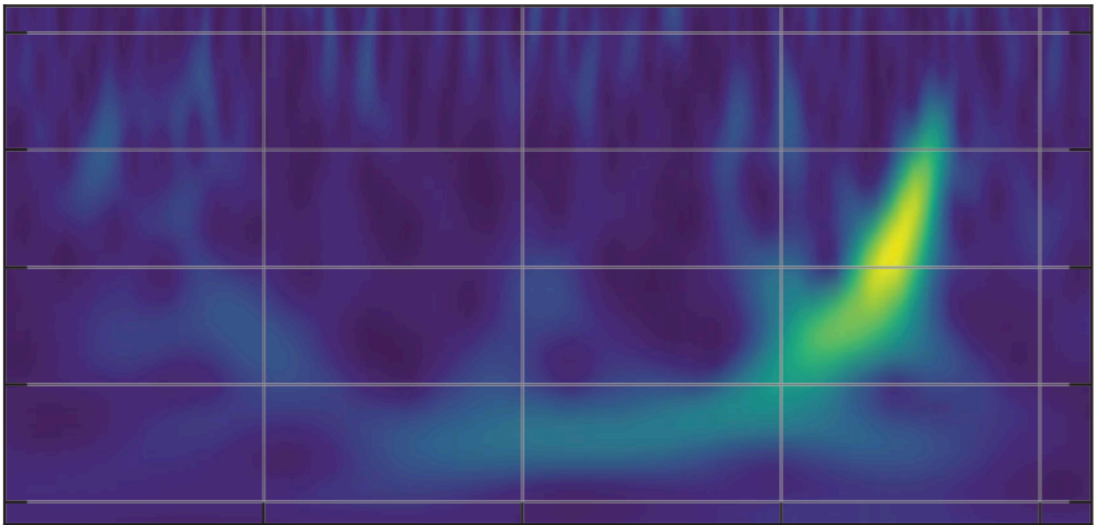
LISA spatial interferometer



# Galactic nuclei are exciting

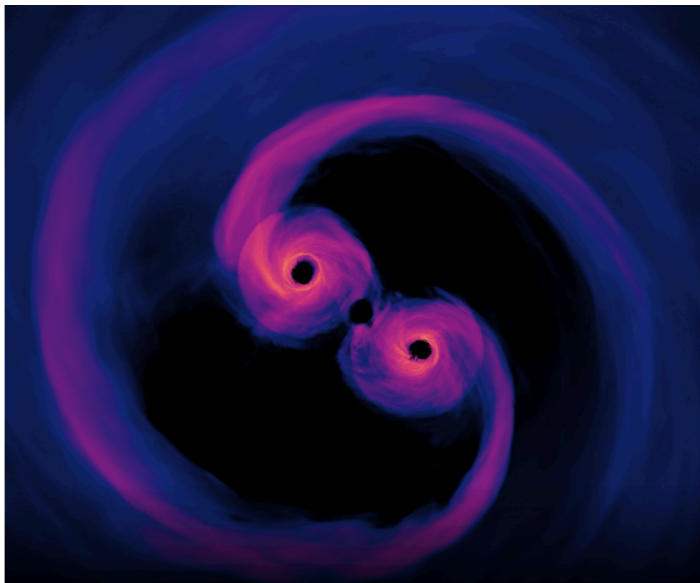
## Gravitational waves

LIGO+(2015)



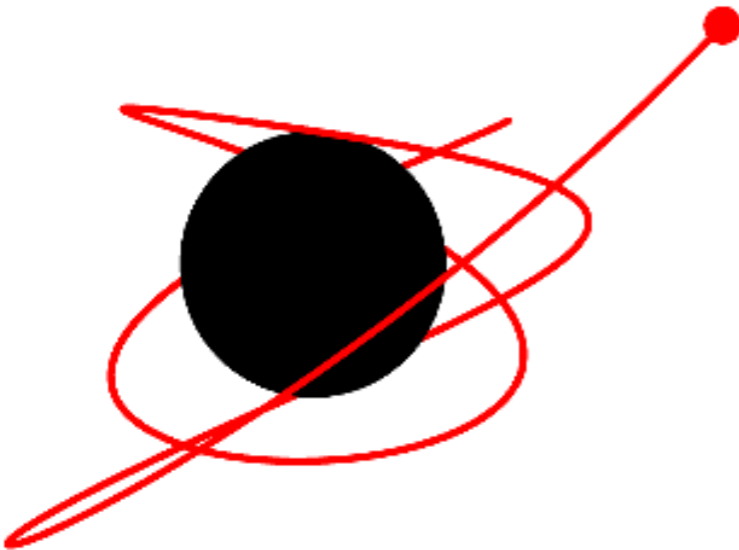
## SMBH merger

Pulsar Timing Array



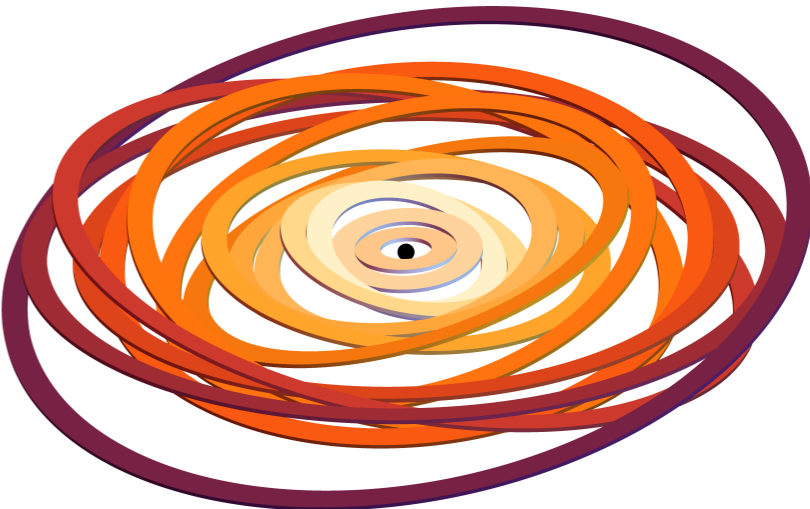
## EMRIs

LISA



## Discs of IMBHs

Szölgvény+(2018)

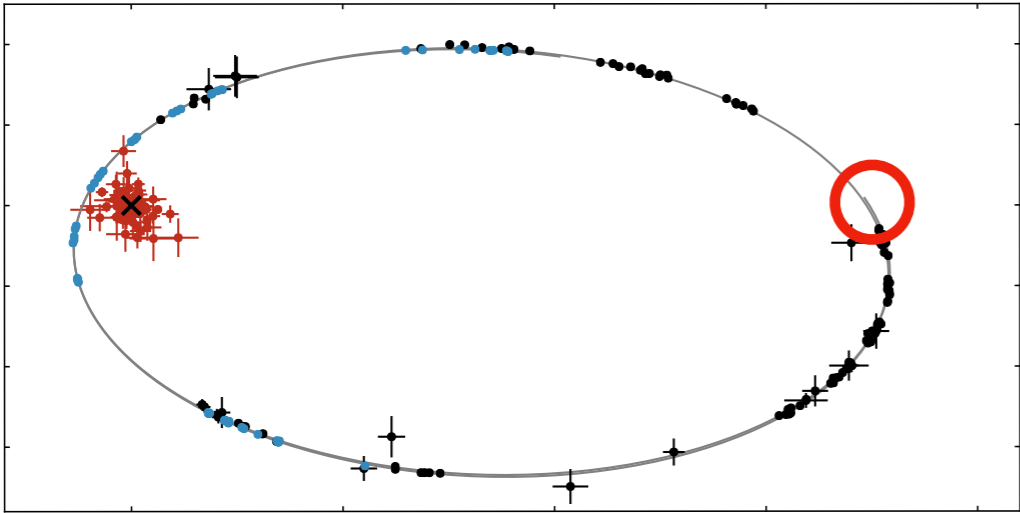




# SgrA\* is exciting

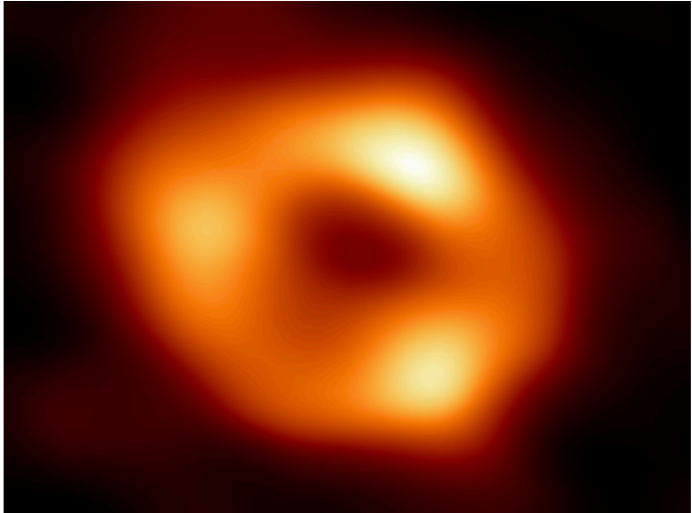
S2's relativistic precession

*Gravity+(2020)*



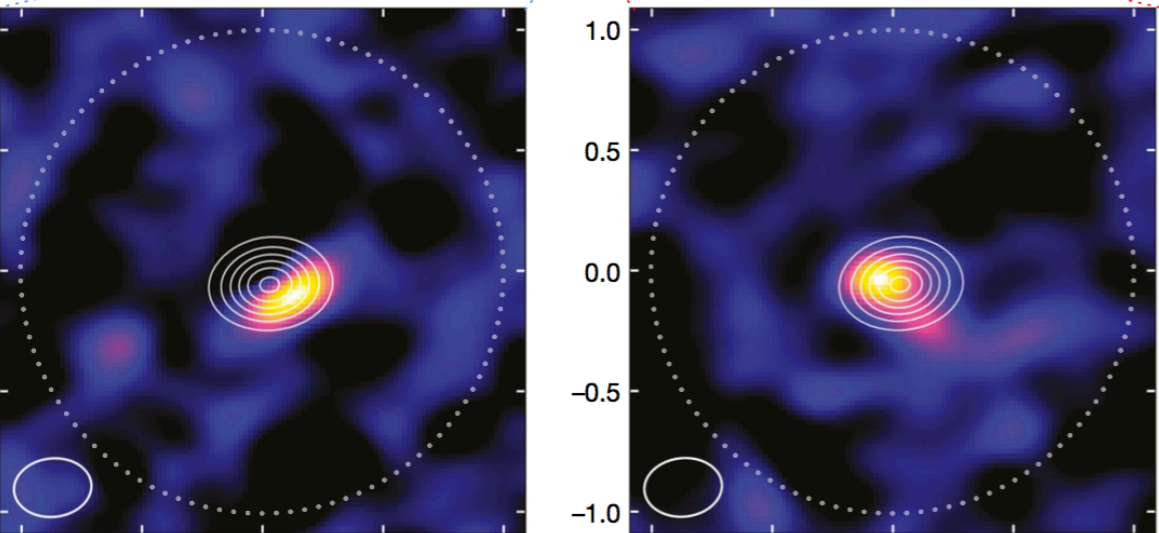
Event Horizon

*EHT+(2022)*



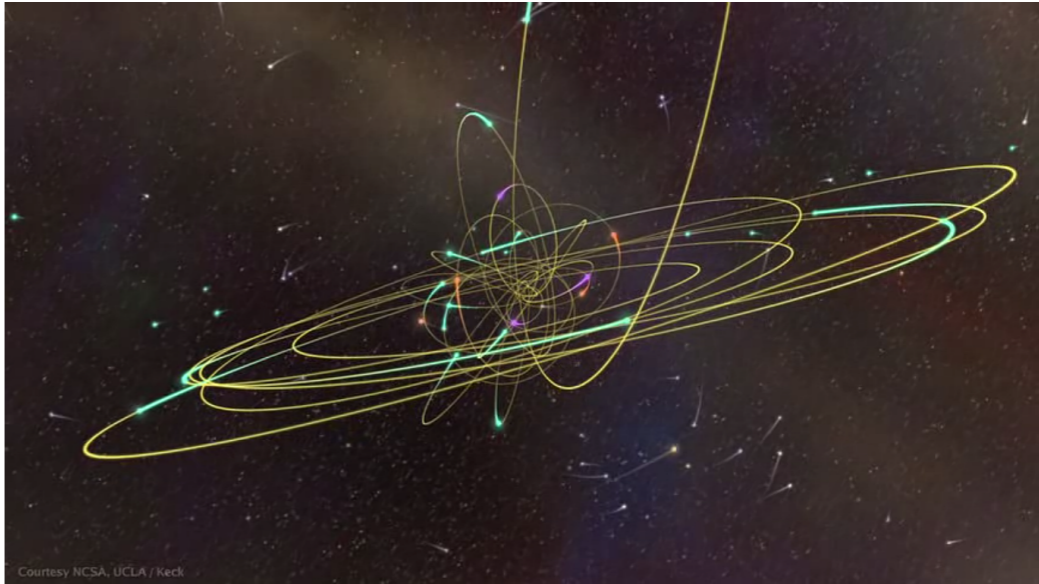
Cold accretion disc

*Murchikova+(2019)*



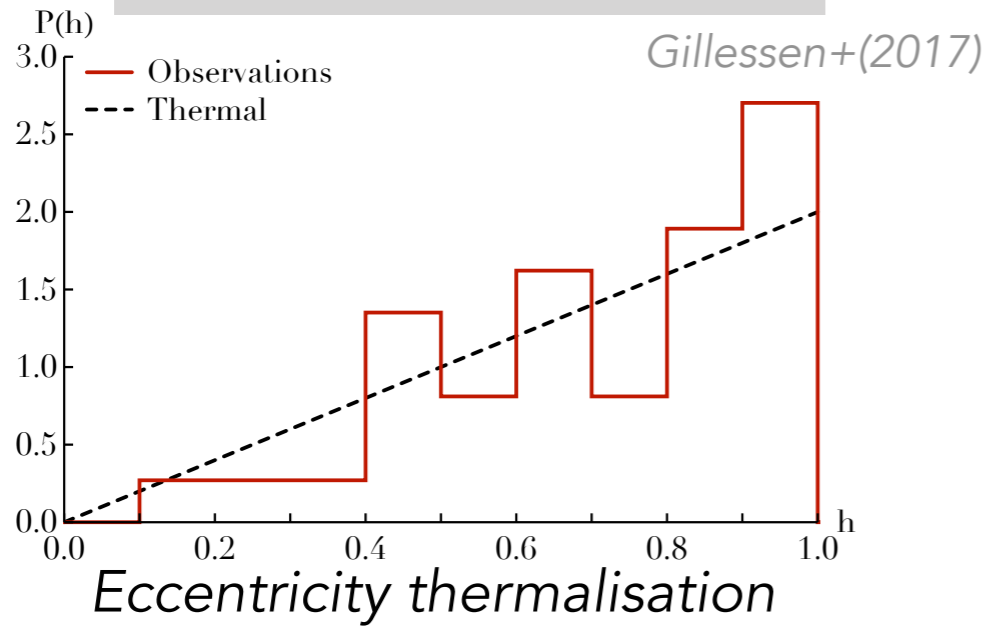
Clockwise stellar disc

*Keck*



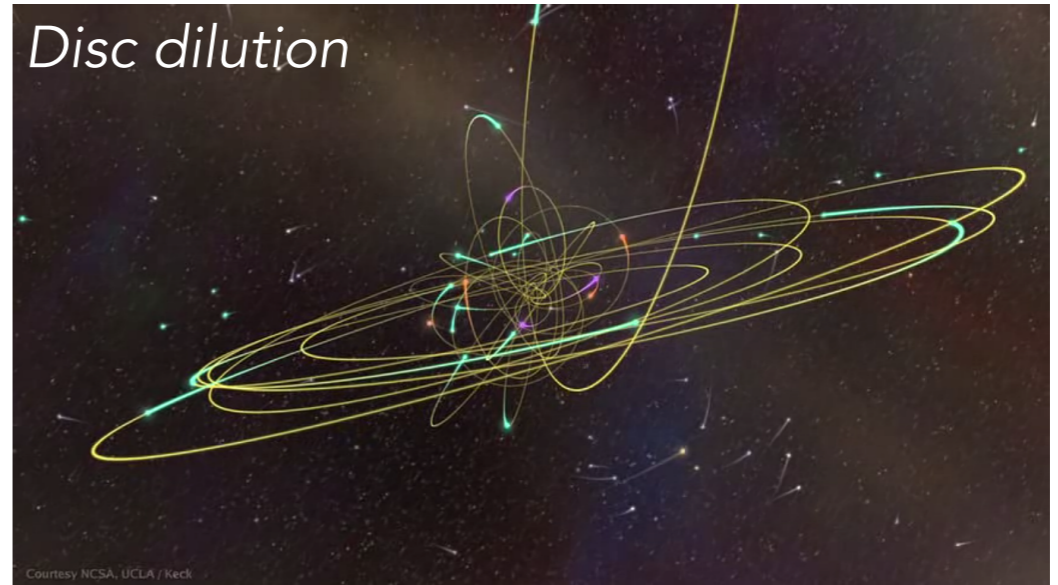
# Next steps – SgrA\* & Observations

## SRR & Eccentricity

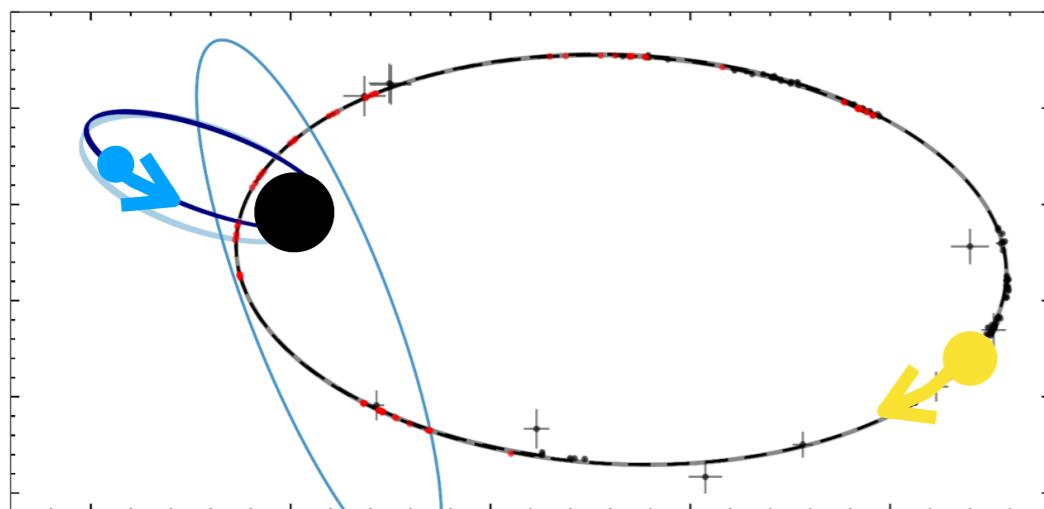


## VRR & Stellar Discs

VLT, Keck



## S2's kinematics *Gravity+(2023)*



*Local perturbations?*

## Future observations

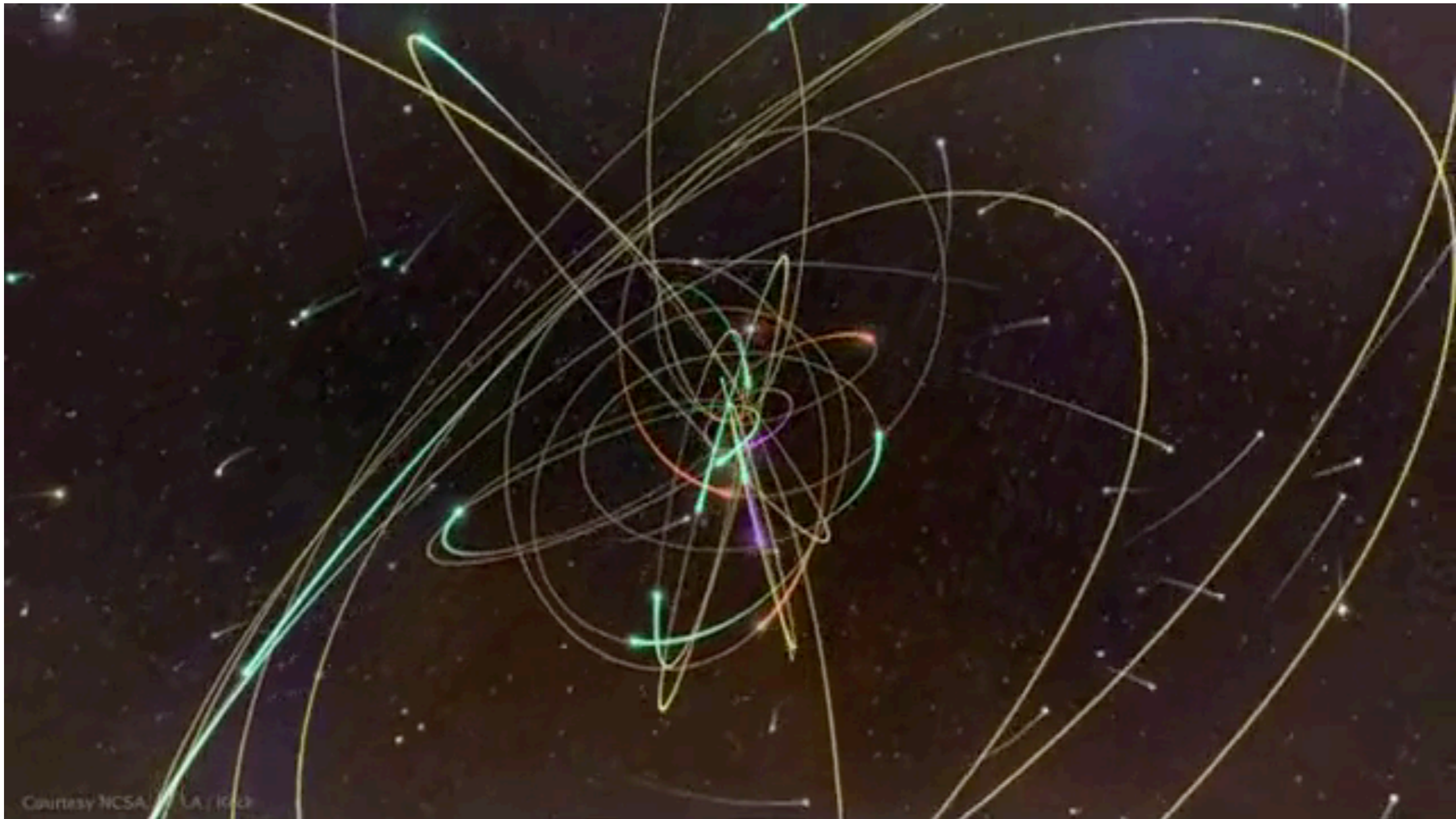
$$P = P(a, h, \hat{\mathbf{h}})$$

*Full PDF statistics*



# Galactic nuclei

Visualisation of **SgrA\***



UCLA

Galactic nuclei, a fantastic “astrophysical lab”

**Dense** (1,000,000x more than around the Sun)

**Relativistic** (BH 4,000,000x heavier than the Sun)

**Far away** (10,000,000x smaller than the Moon in the sky)

**Noisy** (Great source of gravitational waves)