Non-linear gravitational waves in higher-order scalar-tensor theories

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Introduction

- Nonlinear gravitational wave solutions known in GR (pp-waves, Kundt, Robinson-Trautman, radiative asymptotic Bondi)
- Key property: nonlinear memory effects
- This work: construct such an exact solution for a DHOST theory
- Take advantage of disformal transformations as solution-generating techniques

Scalar-tensor theories

Motivations for modified gravity

Heuristic approach

- Design new tests of GR beyond a null hypothesis check
- EFT of some high energy theory

GR $g_{\mu\nu}$ — Modified gravity $g_{\mu\nu}+?$ — Quantum regime

→ energy

Issues of GR

- Big Bang singularity
- Black hole center singularity
- Cosmic expansion (dark energy)

$\Rightarrow$ Important to look for extensions of GR

$\Rightarrow$ Need to understand properties of gravity in these modified theories

DHOST: principle of construction

- DHOST: Degenerate Higher-Order Scalar-Tensor [Langlois+16]
- Add scalar field ϕ + higher-derivatives to break Lovelock
- Degeneracy conditions to ensure only one additional degree of freedom
- Action contains first and second derivatives of ϕ
- Obtain all possible terms and classify by powers of derivatives

$$\text{DHOST} = \boxed{\text{GR}} \times \boxed{\text{Coupling}} + \boxed{\text{Orders 0 and 1 in } \nabla\nabla\phi} + \boxed{(\nabla\nabla\phi)^2} + \boxed{(\nabla\nabla\phi)^3}$$

Lagrangian building blocks

$$S[g_{\mu\nu},\phi] = \int \mathrm{d}^4x\,\sqrt{-g}\left(F_2 R + P + Q\Box\phi + \sum_{i=1}^{5} A_i L_i^{(2)} + F_3 G^{\mu\nu}\phi_{\mu\nu} + \sum_{i=1}^{10} B_i L_i^{(3)}\right),$$

$$\phi_\mu = \nabla_\mu\phi\,,\quad \phi_{\mu\nu} = \nabla_\mu\nabla_\nu\phi\,,\quad X = \phi_\mu\phi^\mu$$

Quadratic terms

$$L_1^{(2)} = \phi_{\mu\nu}\phi^{\mu\nu}\,,\quad L_2^{(2)} = (\Box\phi)^2$$

$$L_3^{(2)} = \phi^\mu\phi_{\mu\nu}\phi^\nu\Box\phi\,,\quad \ldots$$

Cubic terms

$$L_1^{(3)} = (\Box\phi)^3\,,\quad L_2^{(3)} = \phi_{\mu\nu}\phi^{\mu\nu}\Box\phi$$

$$L_3^{(3)} = \phi_{\mu\nu}\phi^{\nu\rho}\phi_\rho^\nu\,,\quad \ldots$$

All functions depend on ϕ and X

Transformations between theories

Generalization of conformal transformations [Bekenstein+93]

Disformal transformations

$$\tilde{g}_{\mu\nu} = A(\phi, X) g_{\mu\nu} + B(\phi, X)\phi_\mu \phi_\nu \qquad (X = \phi_\mu \phi^\mu)$$

$\longrightarrow$ transforms any DHOST theory into another one

$\longrightarrow$ generate DHOST theories from simpler scalar-tensor theories

Solution-generating technique

Simple scalar-tensor theory
(Einstein-scalar, Brans-Dicke)
Exact solution $g_{\mu\nu}$

Disformal transformation →

Higher-order scalar-tensor theory
(Horndeski, DHOST)
Exact solution $\tilde{g}_{\mu\nu}$

- Already used to construct several solutions [Ben Achour+20; Ben Achour+20; Anson+21; Faraoni+21]
- Modified Petrov type, singularities, horizons... [Ben Achour+21] kept in track by the limit $A = 1$, $B = 0$

Main idea: apply a disformal transformation to a GW solution

Robinson-Trautman: from GR to DHOST

Metric element

Solution of Einstein-scalar theory [Tahamtan+15; Tahamtan+16]

$$S = \int \mathrm{d}^4 x \sqrt{-g} \left(R - \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi \right)$$

wave pulse

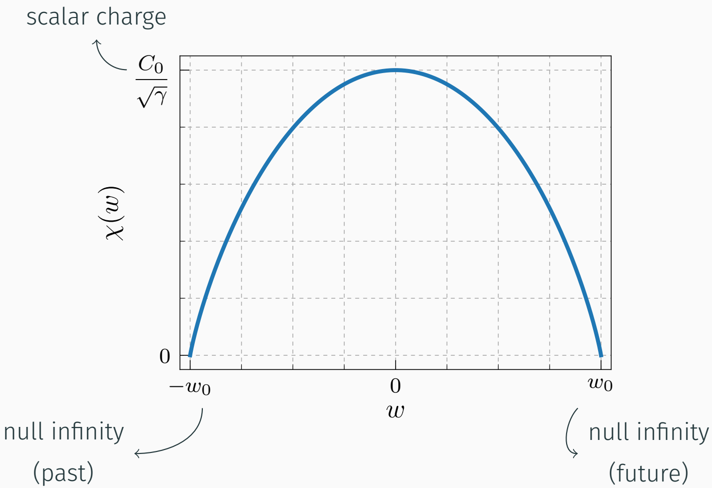
$$\mathrm{d}s^2 = -K(x,y)\,\mathrm{d}w^2 - 2\,\mathrm{d}w\,\mathrm{d}\rho + \frac{\rho^2 - \chi(w)^2}{P(x,y)^2} (\mathrm{d}x^2 + \mathrm{d}y^2)$$

lightlike coordinate

$$\phi = \frac{1}{\sqrt{2}} \log \left(\frac{\rho - \chi(w)}{\rho + \chi(w)} \right)$$

- Wave propagation towards outgoing ρ
- No spherical symmetry
- Presence of an apparent horizon
- Fully nonlinear solution

Representing the wave pulse



- Longitudinal waves of Gaussian curvature
- Generated by the scalar field
- Recover empty spacetime at null infinity (no mass)

Disformal transformation

Disformal transformation with $C = 1$ and $D = \text{cst}$:

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + B_0 \phi_\mu \phi_\nu$$

Resulting theory

DHOST with F_2, A_1, A_2 and A_4

$$F_2 = \frac{1}{\sqrt{1 - B_0 \tilde{X}}} \qquad A_1 = -A_2 = \frac{B_0}{\sqrt{1 - B_0 \tilde{X}}} \qquad A_4 = -\frac{B_0^2}{\sqrt{1 - B_0 \tilde{X}}}$$

$\longrightarrow$ in the following, study properties of the solution at $\mathcal{O}(B_0^3)$

Waves in the transformed solution

Geodesic deviation along a null geodesic

$+$: direction of the null geodesic, $-$: null transverse direction

$$\frac{\mathrm{d}^2\xi_a}{\mathrm{d}t^2} = A_{ab}\xi^b \quad \text{with} \quad A_{ab} = R^{+}{}_{a-b}$$

$$A_{ab} = \begin{pmatrix} a_0 + B_0 a_1 + B_0^2 a_2 & 0 \\ 0 & a_0 + B_0 a_1 + B_0^2 a_2 \end{pmatrix} + B_0^2 \begin{pmatrix} +b_2 & c_2 \\ c_2 & -b_2 \end{pmatrix} + \mathcal{O}(B_0^3)$$

$a_0 + B_0 a_1 + B_0^2 a_2$ ↓ scalar waves; $c_2 \rightarrow A_\times$; $-b_2 \rightarrow A_+$

⟶ the disformal transformation sources *tensorial* gravitational waves

Conclusion

- Start from a **seed solution** for nonlinear wave propagation in Einstein-scalar
- Make use of **disformal transformation** to obtain a DHOST solution
- Effect of disformal transformation: **generate tensor modes** of propagation and **generate shear** from scalar field with radial dependance only
- Full nonlinear dynamics of wave propagation: investigate **memory effect**

Thank you for your attention!