Numerical investigation of screened scalar-tensor theories in space-based experiments

Hugo Lévy, Joël Bergé, Jean-Philippe Uzan

Workshop TUG 2023, LPENS, Paris

Based on arχiv 2209.07226 & 2310.03769

Outline

I. Introduction to scalar-tensor theories with screening mechanisms

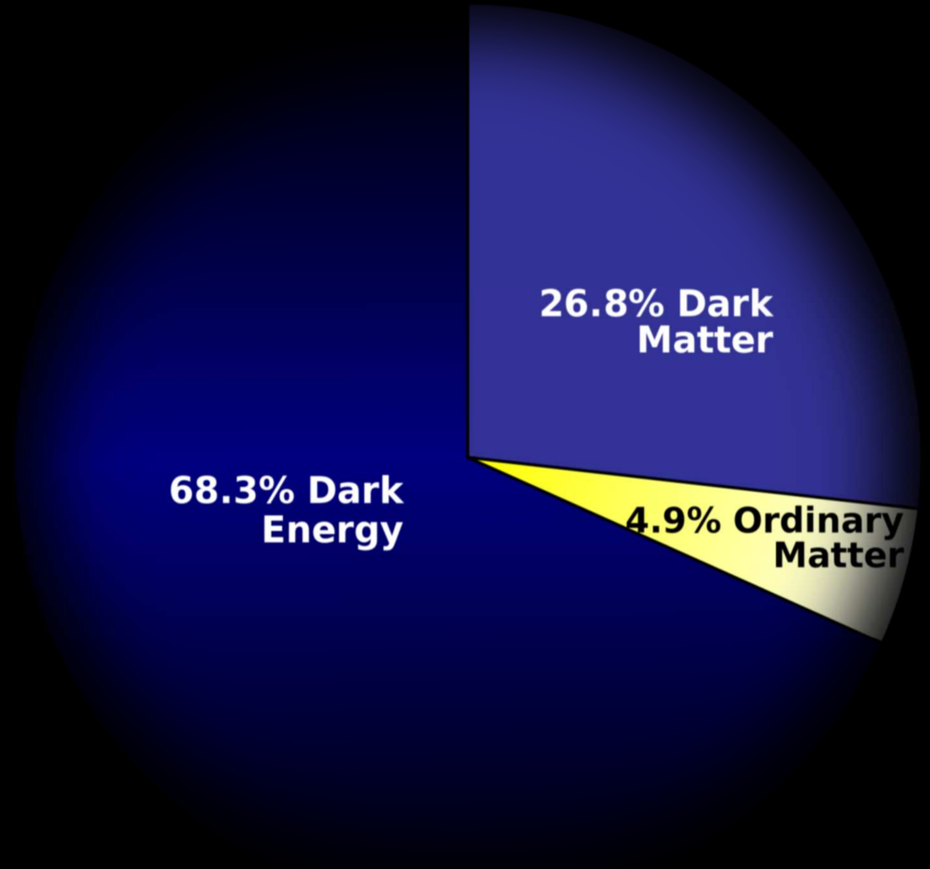
II. Solving nonlinear Klein-Gordon equation on unbounded domains – *femtoscope* code

III. Can we test scalar-tensor models with space geodesy techniques?

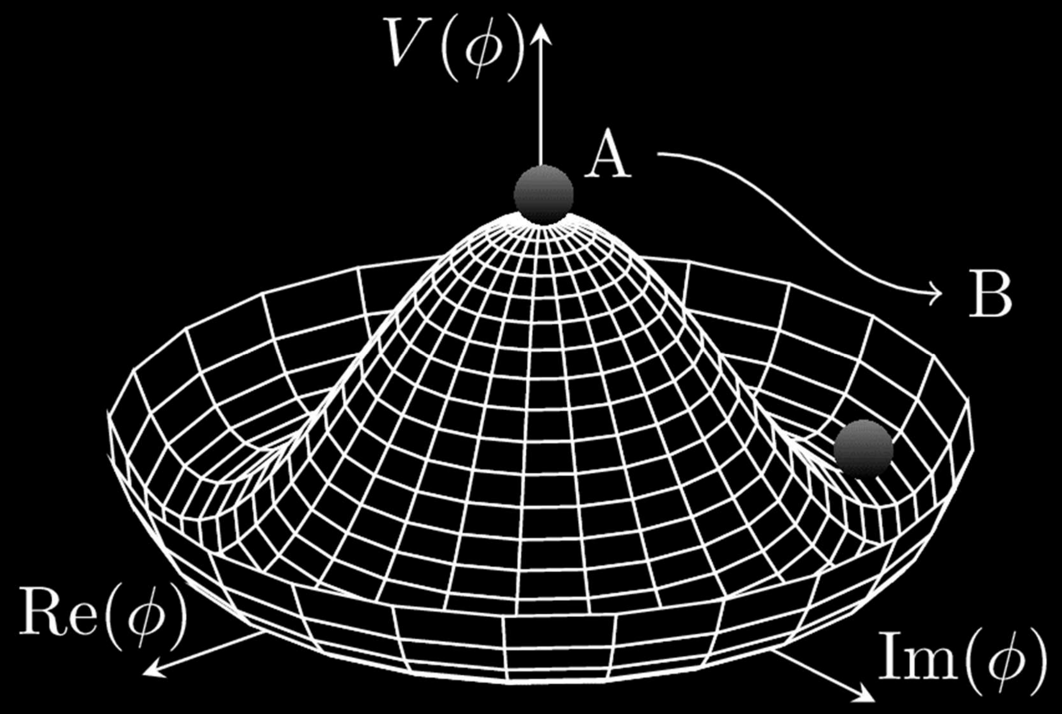
Scalar-Tensor theories

Main motivations for introducing a scalar field in the gravitational sector

1. **The dark sector** [1]
   - Dark Matter
   - Dark Energy
   - + inflation paradigm



2. **A 'true' scalar field exists in nature**
   - Discovery of the Higgs boson in 2012
   - … ?

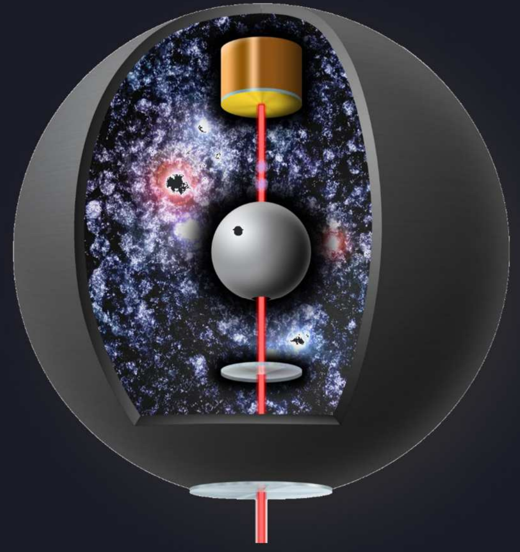


3. **More fundamental theories**
   - String theory as an effective 4-dimensional theory [1]
   - f(R)-theories ≡ scalar-tensor [2]

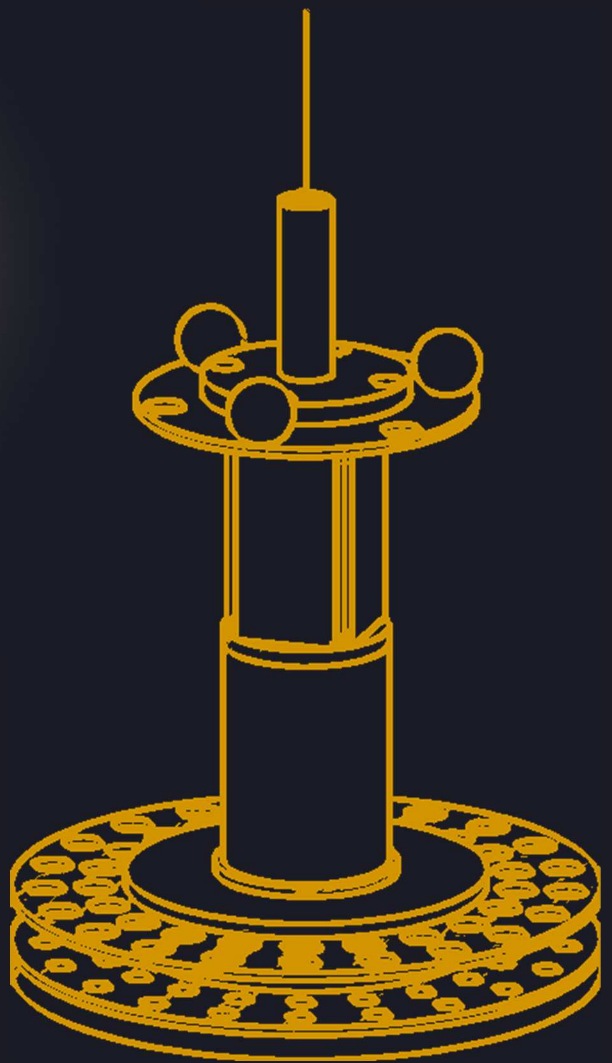
[1] *A. Joyce et al, arXiv:1407.0059*

[2] *J. Velásquez and L. Castañeda, arXiv:1808.05615*

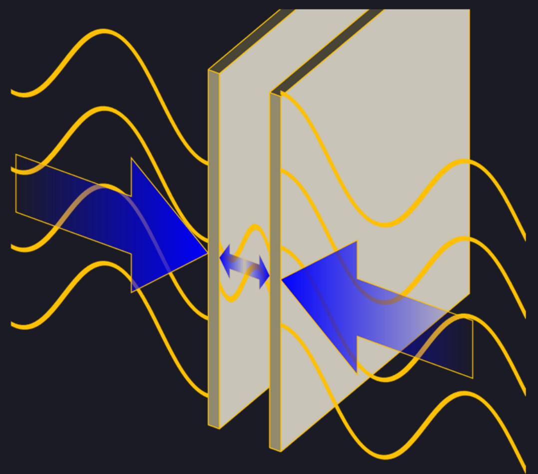
Laboratory Tests



Atom interferometry [3]

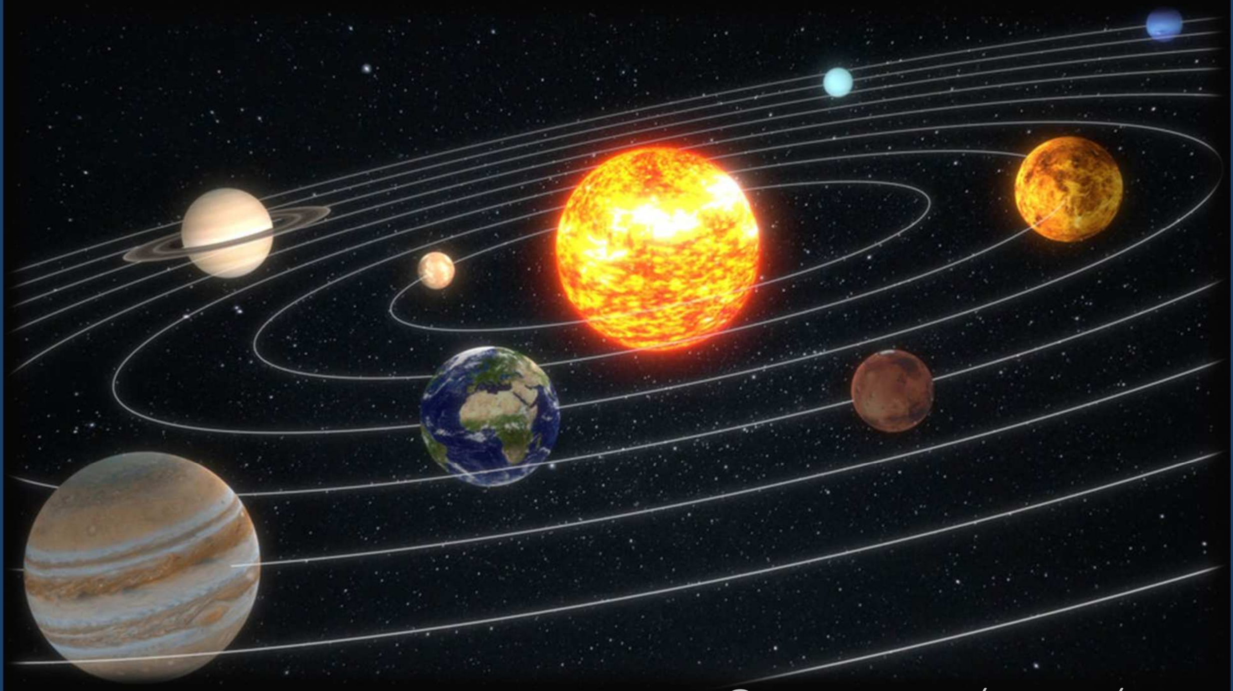


Eöt-Wash torsion pendulum [4]

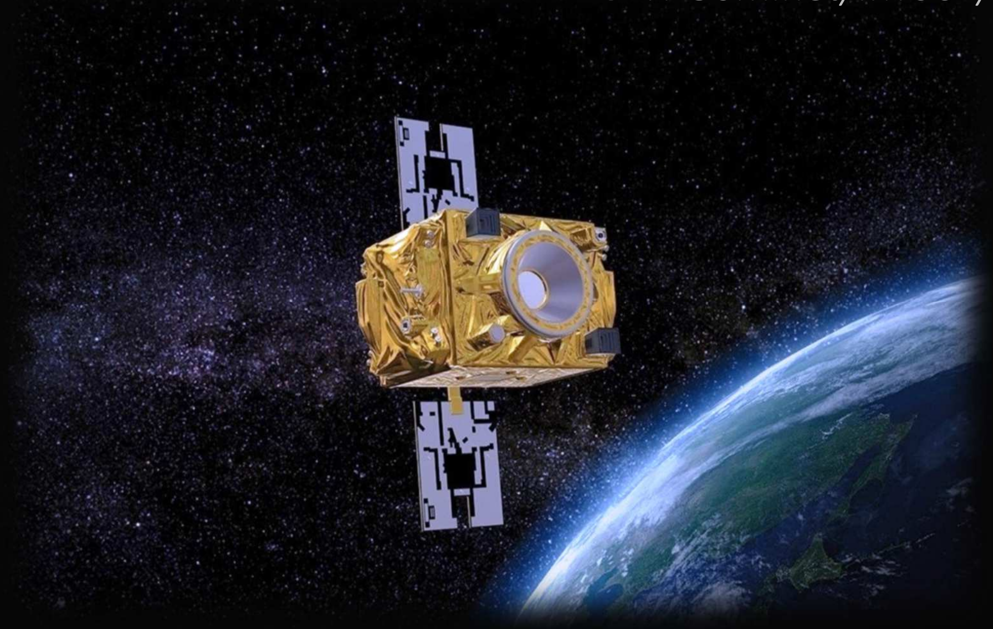


Casimir effect [5]

Solar System Tests



© Y. Gominet/IMCCE/Nasa

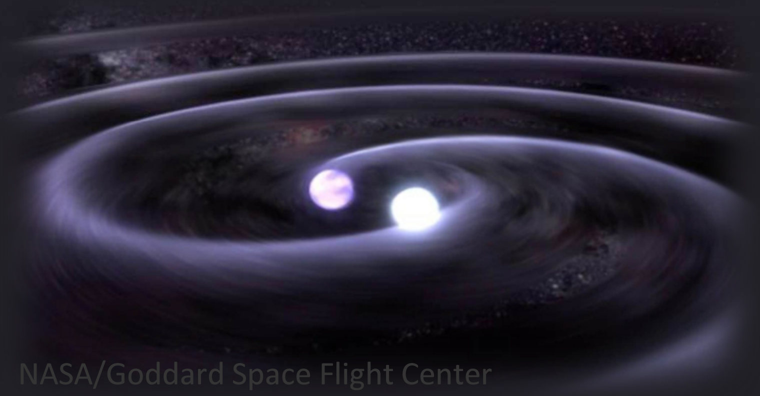


MICROSCOPE [6]

Astrophysical Tests

Galaxy rotation curves [7]

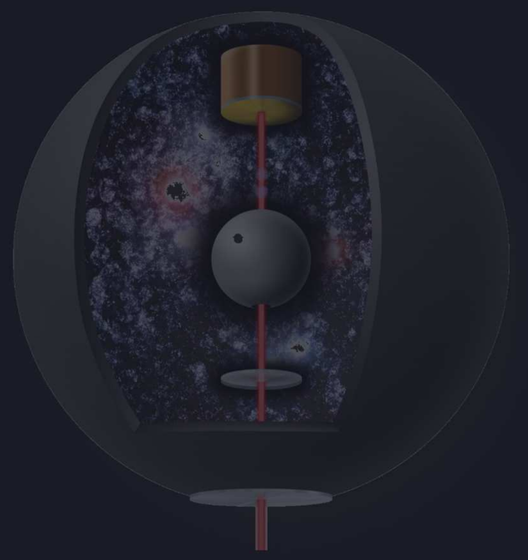
Cluster lensing [8]



Scalar radiation in BNS [9]

Length scales

Laboratory Tests



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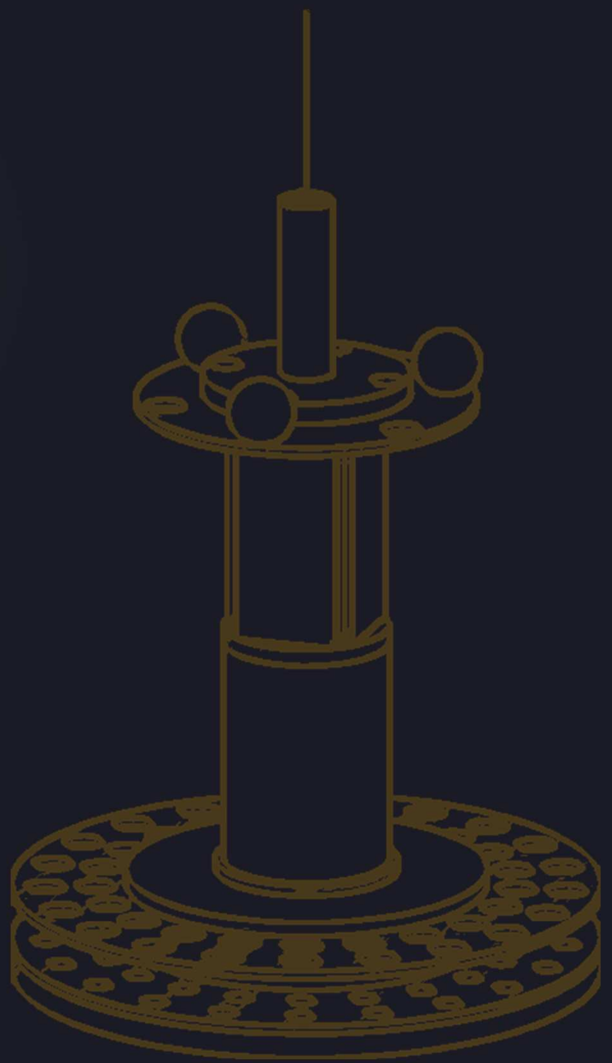

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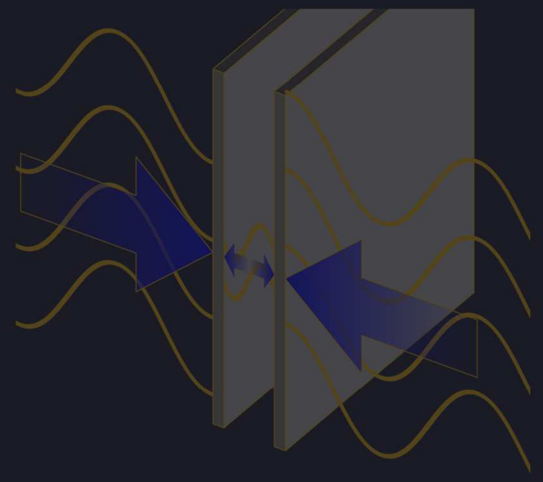

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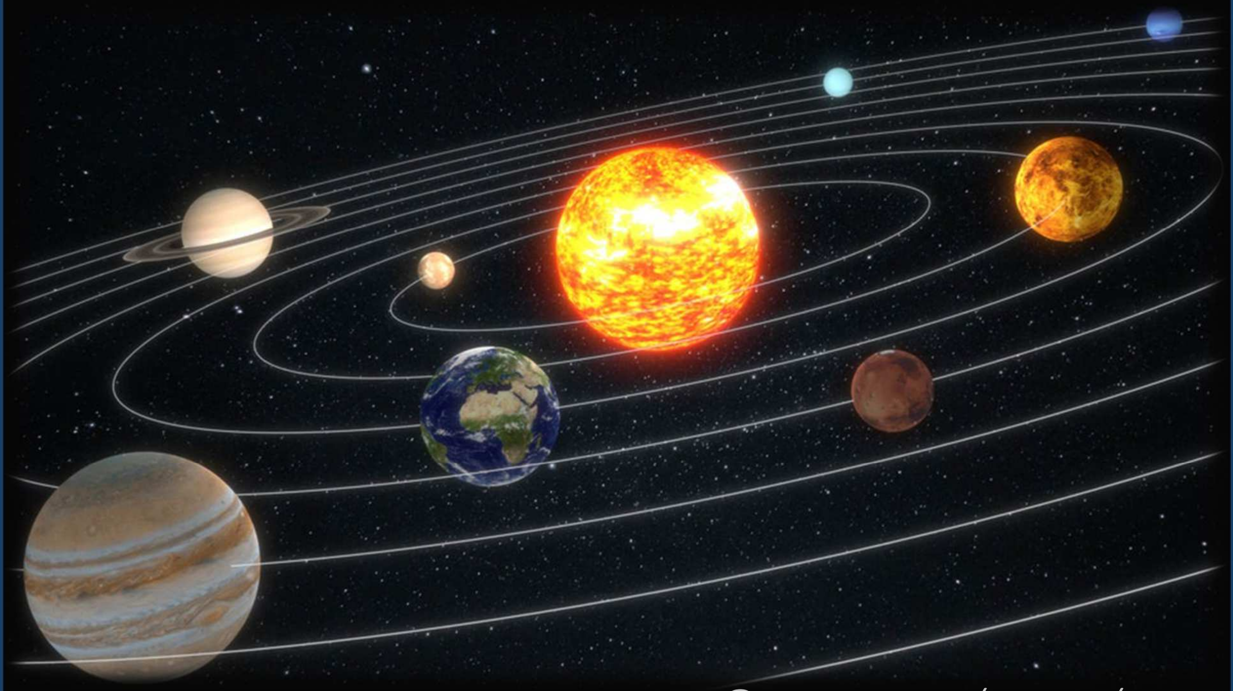

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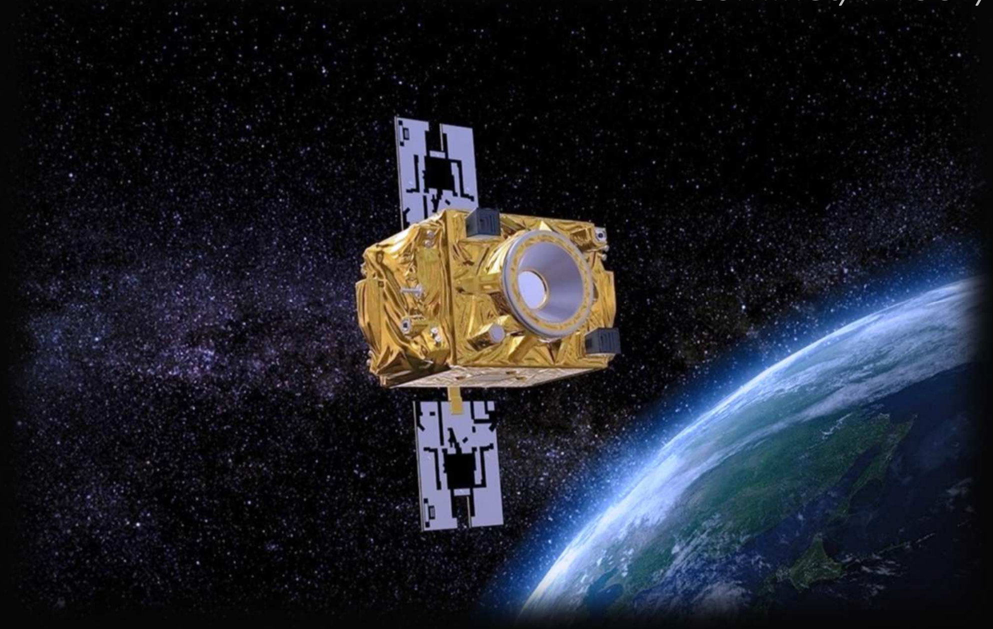

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Length scales

Scalar fields playing *hide-and-seek*

Review of the most popular screening mechanisms [2]		
Classification	Type of Equation	Rule of Thumb
Weak coupling • Symmetron • Damour-Polyakov	$\Box\phi = \frac{\mathrm{d}V_{\mathrm{eff}}}{\mathrm{d}\phi}$	Occurs in regions of high Newtonian potential
Large mass • Chameleon		
Large inertia • K-Mouflage	$\Box\phi + A_1\partial_\mu\left[(\partial\phi)^2\partial^\mu\phi\right] + A_2 T = 0$	Occurs in regions where the gravitational acceleration is large
• Vainshtein	$6\Box\phi + B_1\left[(\Box\phi)^2 - (\partial_\mu\partial_\nu\phi)^2\right] = B_2 T^\mu{}_\mu$	Occurs in regions where spatial curvature is large

Take-home messages:

- Different mechanisms to ‘screen’ scalar fields from local tests of gravity (i.e. recover GR at Solar System scales)
- At the equation level, screening ≡ non-linearity

[2] *A. Joyce et al, Beyond the Cosmological Standard Model, arXiv:1407.0059*

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Numerical considerations

arχiv:2209.07226

Chameleon field equation

Field equation (in the Newtonian limit)	5^{th} force
$\Box\phi = \frac{\beta}{M_{Pl}}\rho - n\frac{\Lambda^{n+4}}{\phi^{n+1}}$ Free parameters: β, n, Λ Mass distribution: $\rho = \rho(\boldsymbol{x}, t)$ Unknown: $\phi = \phi(\boldsymbol{x}, t)$	$\vec{F}_\phi = -m\frac{\beta}{M_{Pl}}\boldsymbol{\nabla}\phi$ Point-mass follows geodesics of the Jordan frame metric ≠ Einstein frame geodesics

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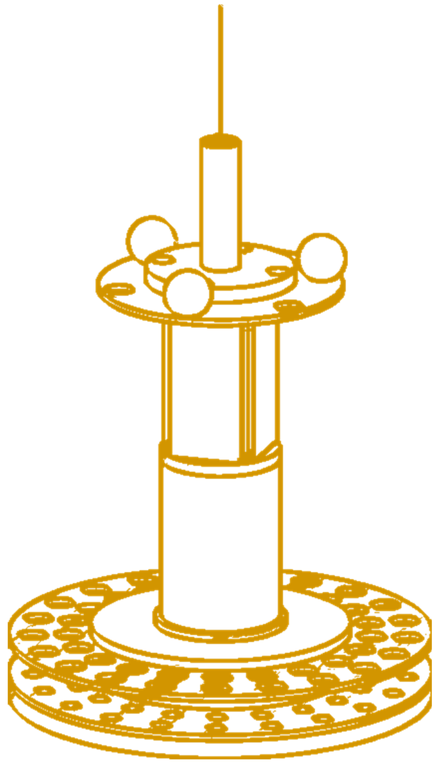
Geometry can be quite complex!

✓ Finite Element Method can deal with complex geometries



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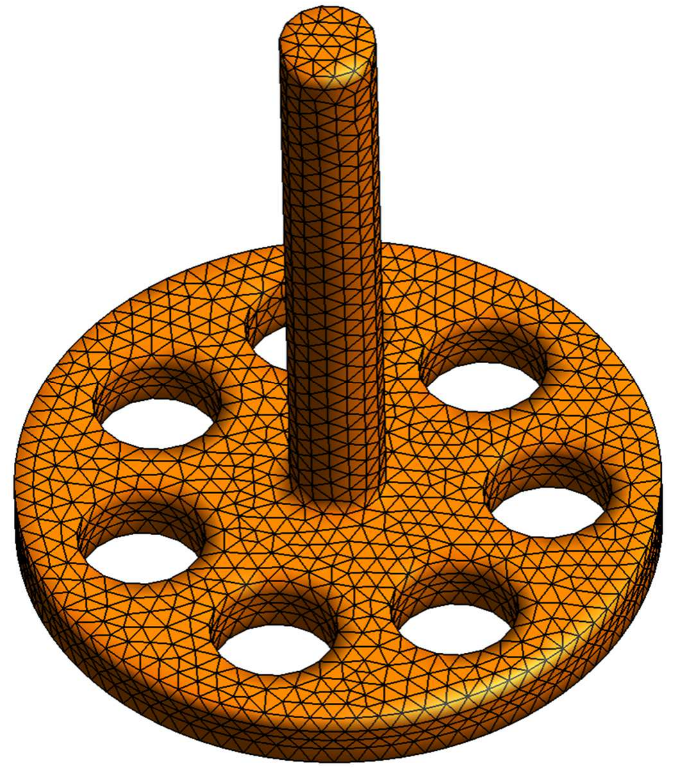

[2] *A. Upadhye, Dark energy fifth forces in torsion pendulum experiments, arXiv:1209.0211*

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Dimensionless & no time-dependence

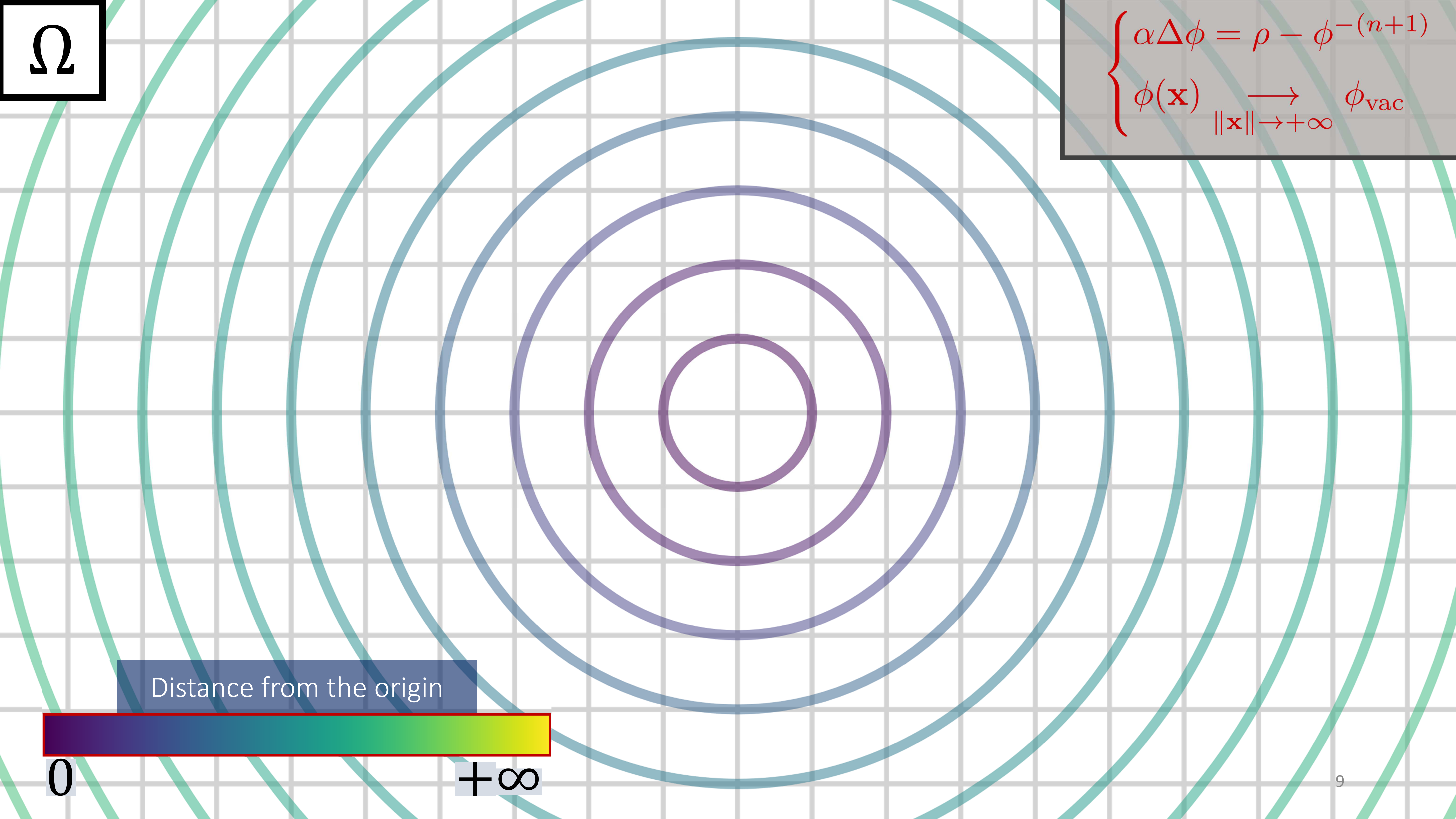
Solve $\alpha\tilde{\Delta}\tilde{\phi} = \tilde{\rho} - \tilde{\phi}^{-(n+1)}$ on $\Omega \subseteq \mathbb{R}^3$

$\beta, n, \Lambda \mapsto \alpha, n$

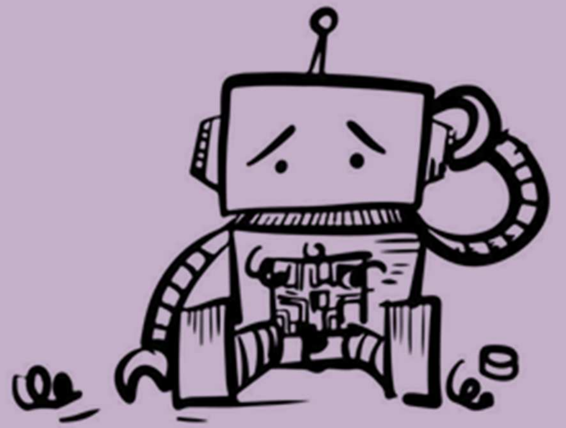
- Dropping time-dependence
- Arbitrary function of $\boldsymbol{x}$ — **Finite Element Method**
- Non-linearity — **Newton's iterations**
- Not necessarily bounded — $\phi(x) \xrightarrow[\|x\|\to+\infty]{} \phi_\infty$

Ω

$$\begin{cases} \alpha \Delta \phi = \rho - \phi^{-(n+1)} \\ \phi(\mathbf{x}) \underset{\|\mathbf{x}\| \to +\infty}{\longrightarrow} \phi_{\text{vac}} \end{cases}$$



Ω



Not possible to mesh a domain of infinite spatial extent...

Ω

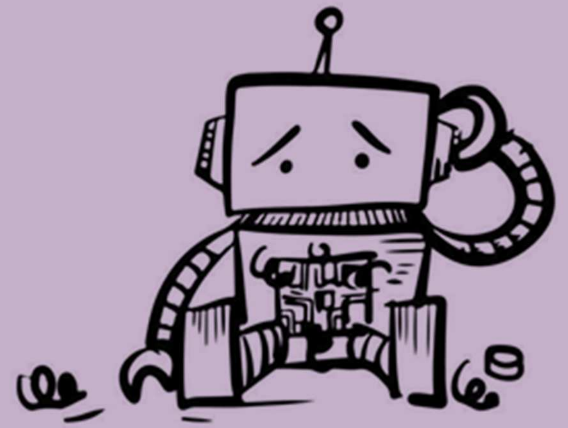
Let's *compactify* space*!

***(i.e. apply a global coordinate transform that will map the whole plane to a bounded domain)**

For instance

$$\mathcal{T}: \mathbb{R}^2 \to \mathbb{R}^2$$

$$\mathbf{x} = (x, y) \mapsto \frac{R_{\mathrm{cut}}}{1 + \|\mathbf{x}\|}(x, y)$$



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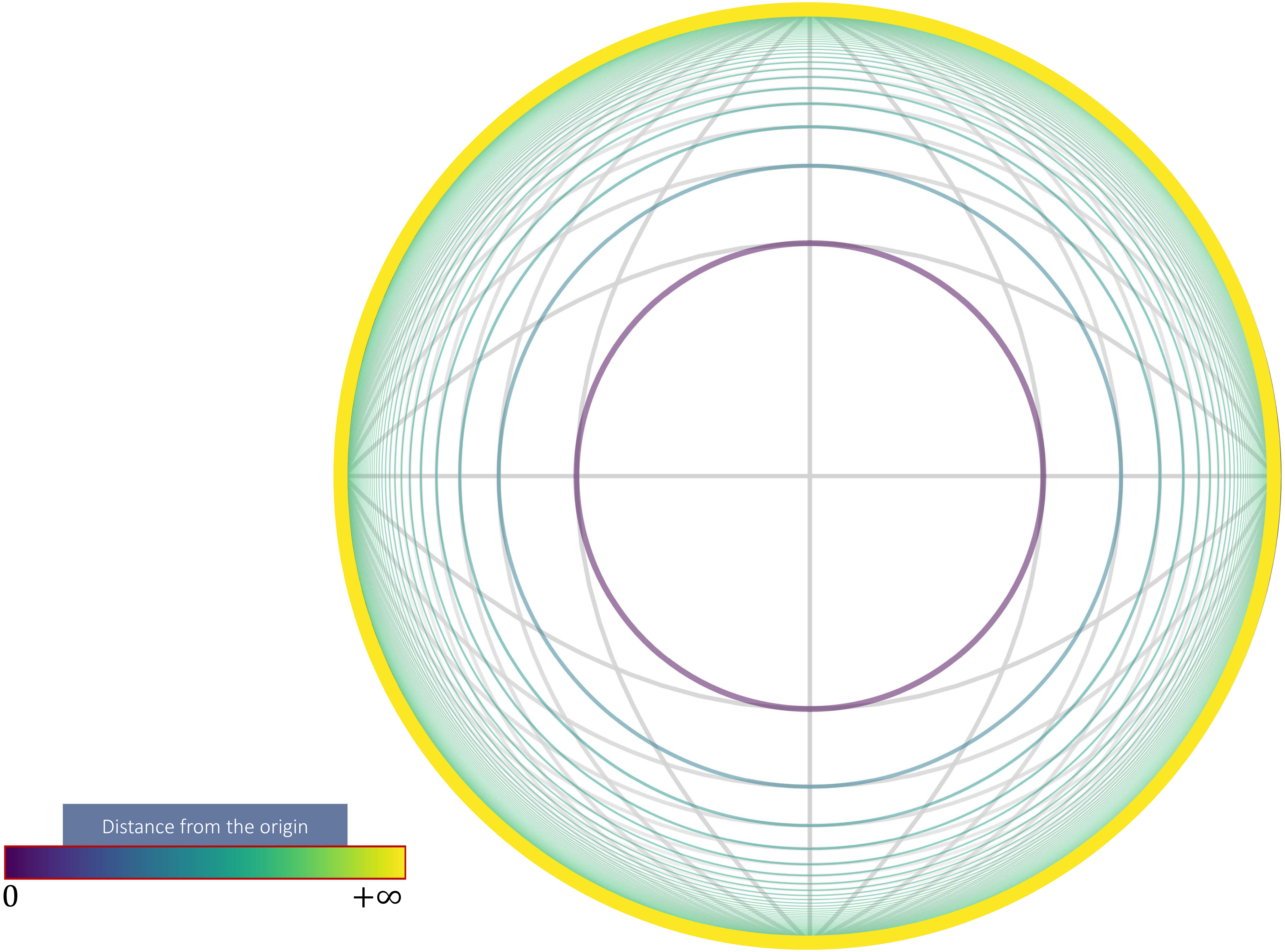
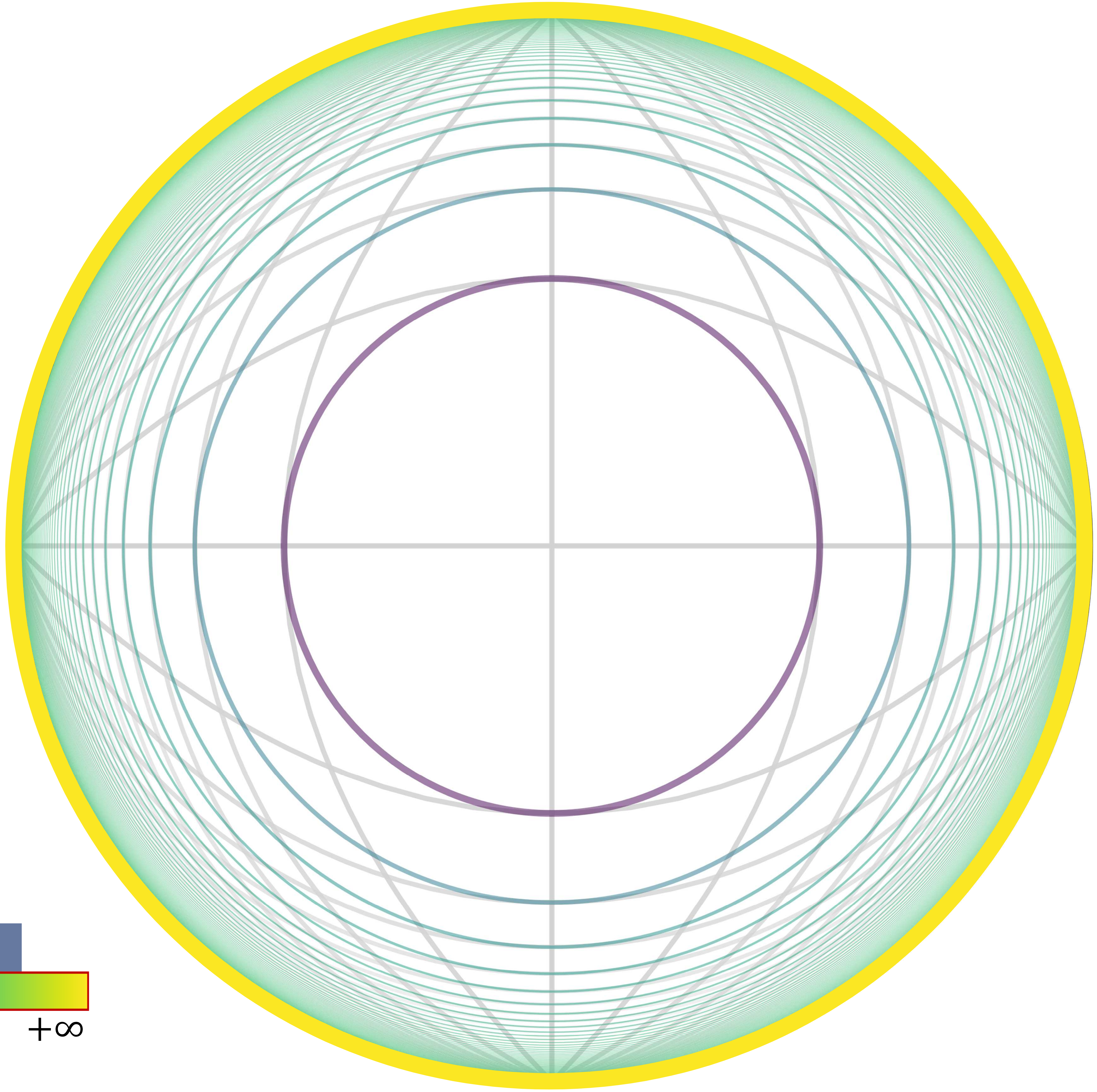
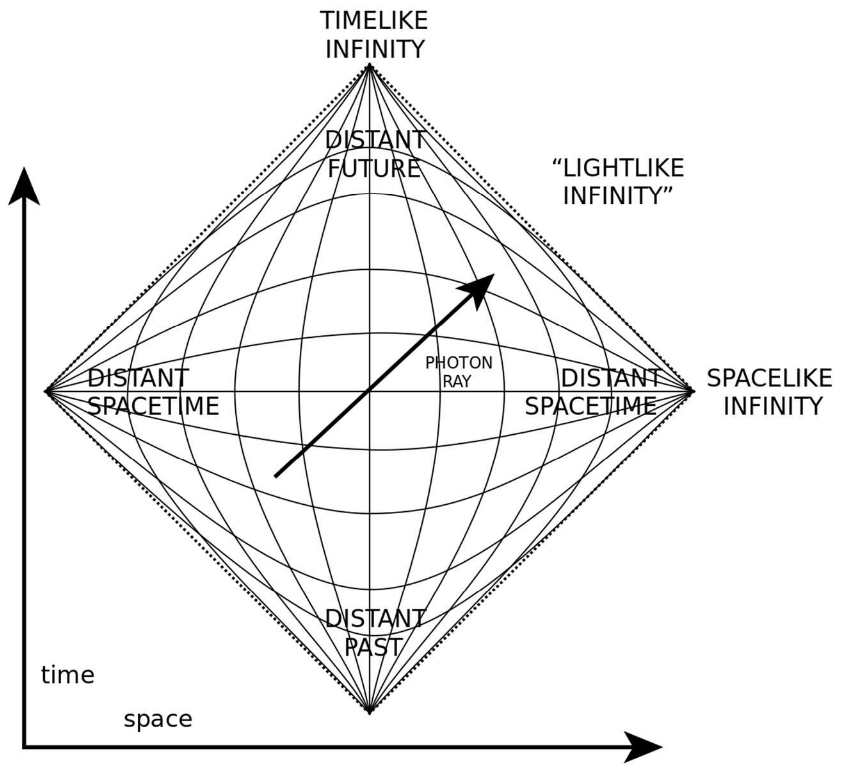
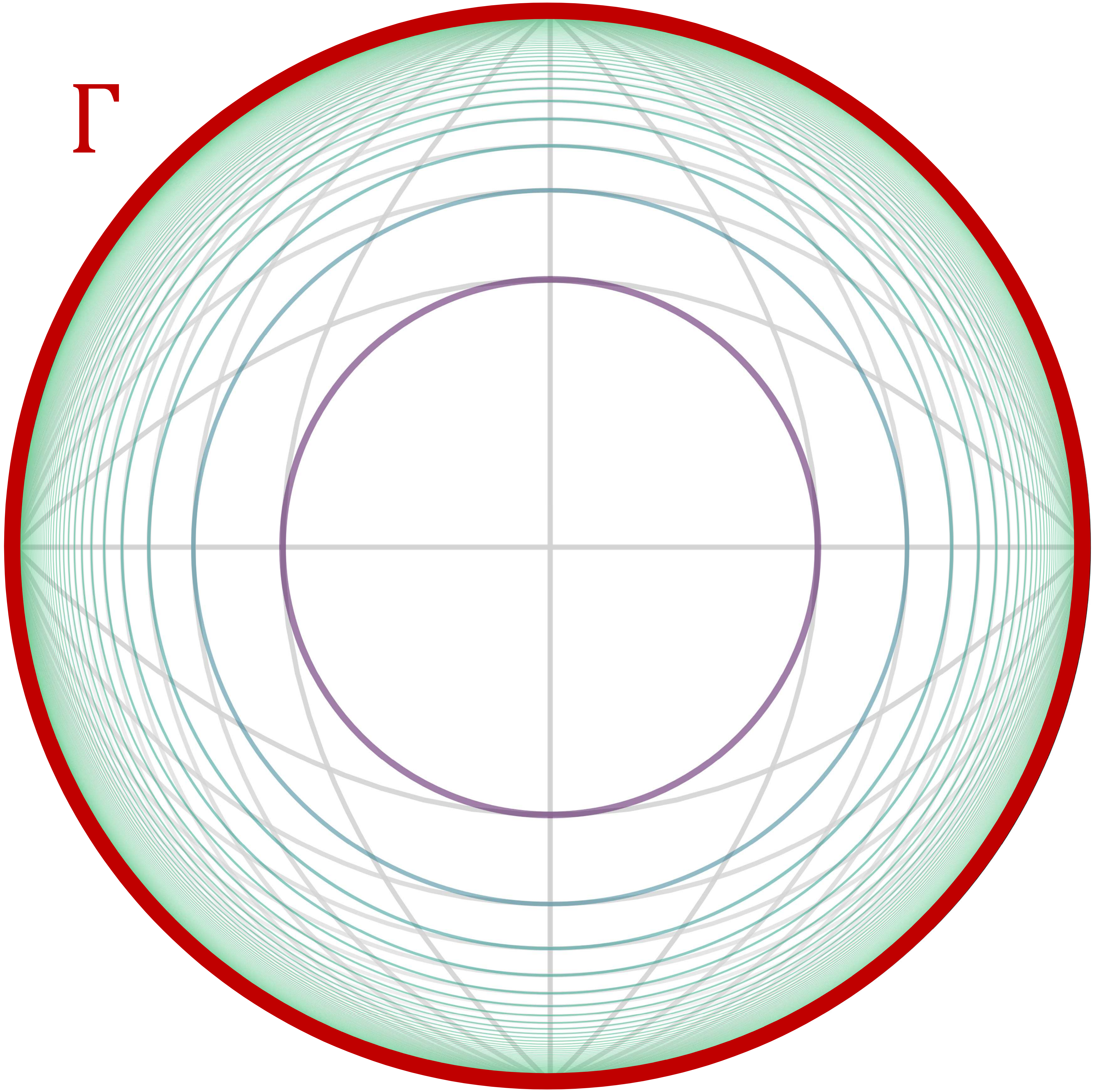
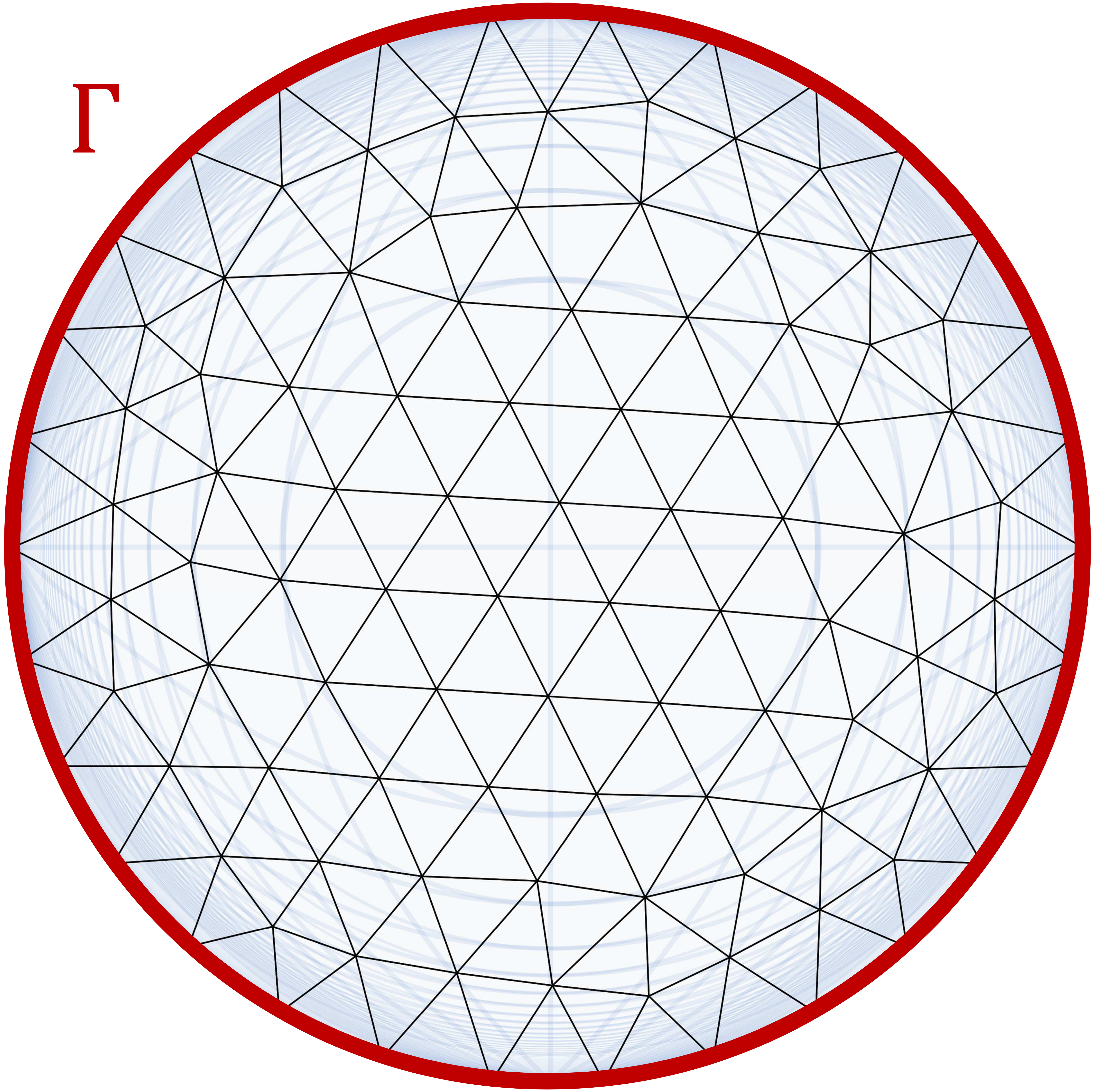

Distance from the origin
0
+∞

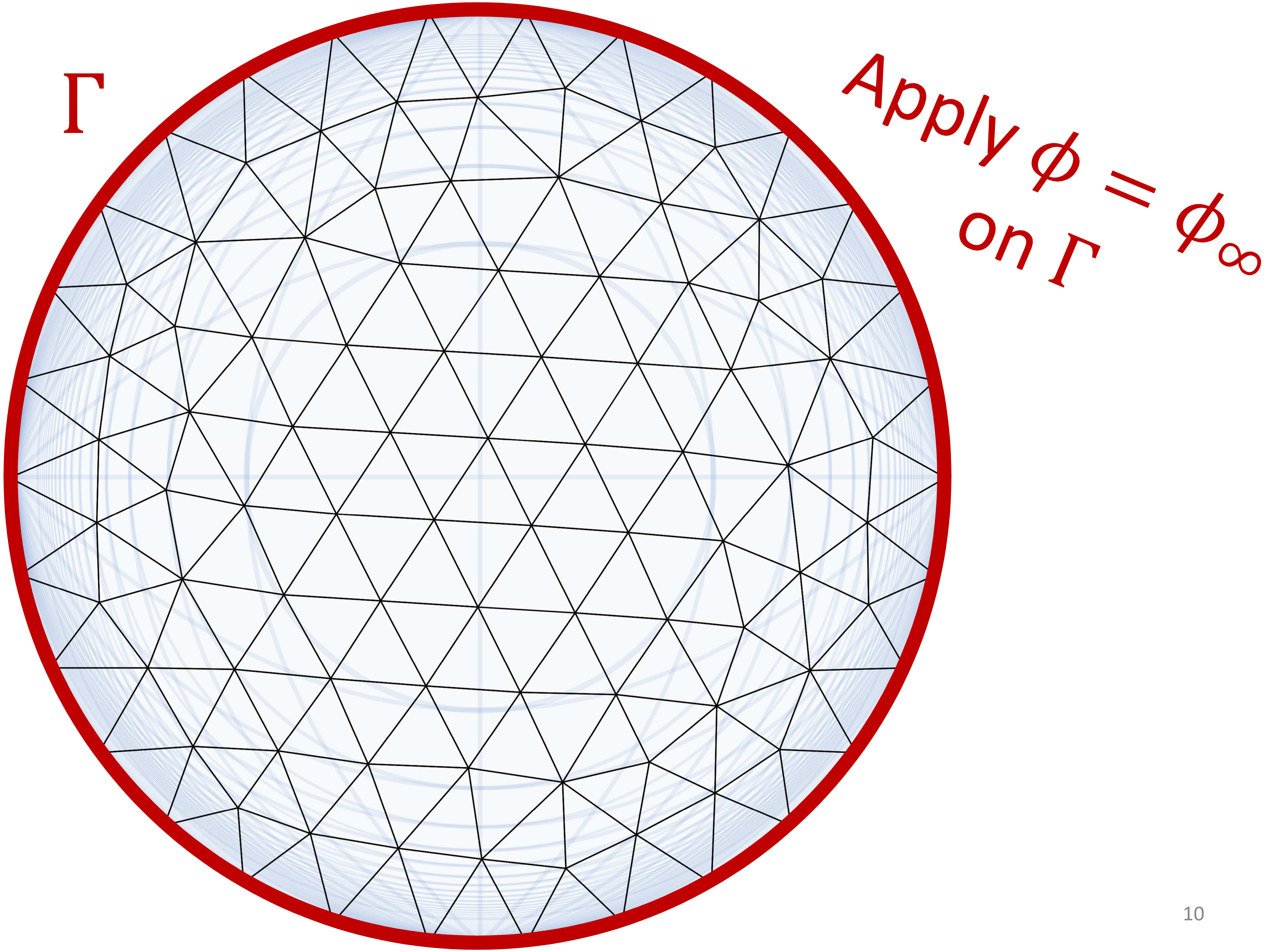


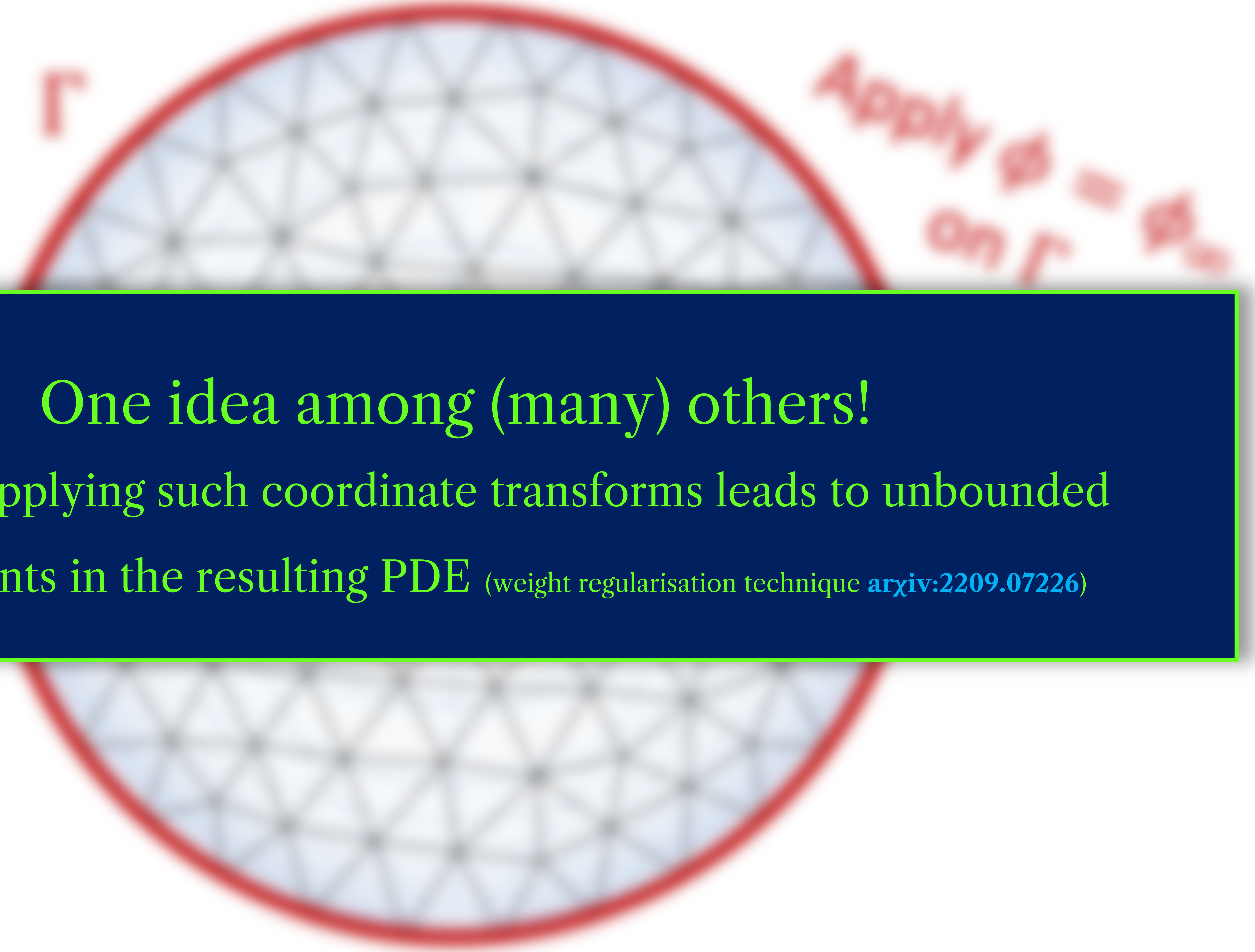


Penrose diagram


Γ


Γ


Γ
Apply $\phi = \phi_\infty$ on Γ



One idea among (many) others!

Caveat: Applying such coordinate transforms leads to unbounded coefficients in the resulting PDE (weight regularisation technique arχiv:2209.07226)

Inspired by
Grosch and Orszag (1977)
Zenginoglu (2011)
Chernogorova et al. (2016)
Boulmezaoud (2005)

Programming *(low level code)*

Physics *(high level code)*

Study Scripts (`from femtoscope import ...`)

- Custom nonlinear solver with line-search
- Implementation of 3 techniques to handle asymptotic boundary conditions
- 1D, 2D and 3D Finite Element Method

femtoscope

- Poisson *Class*
  (+ analytical & semi-analytical solutions available)
- Chameleon *Class*
  (+ few analytical approximations)

Third-party Python librairies

Gmsh

meshio

sfepy

NumPy

Application to space geodesy*

arχiv:2310.03769

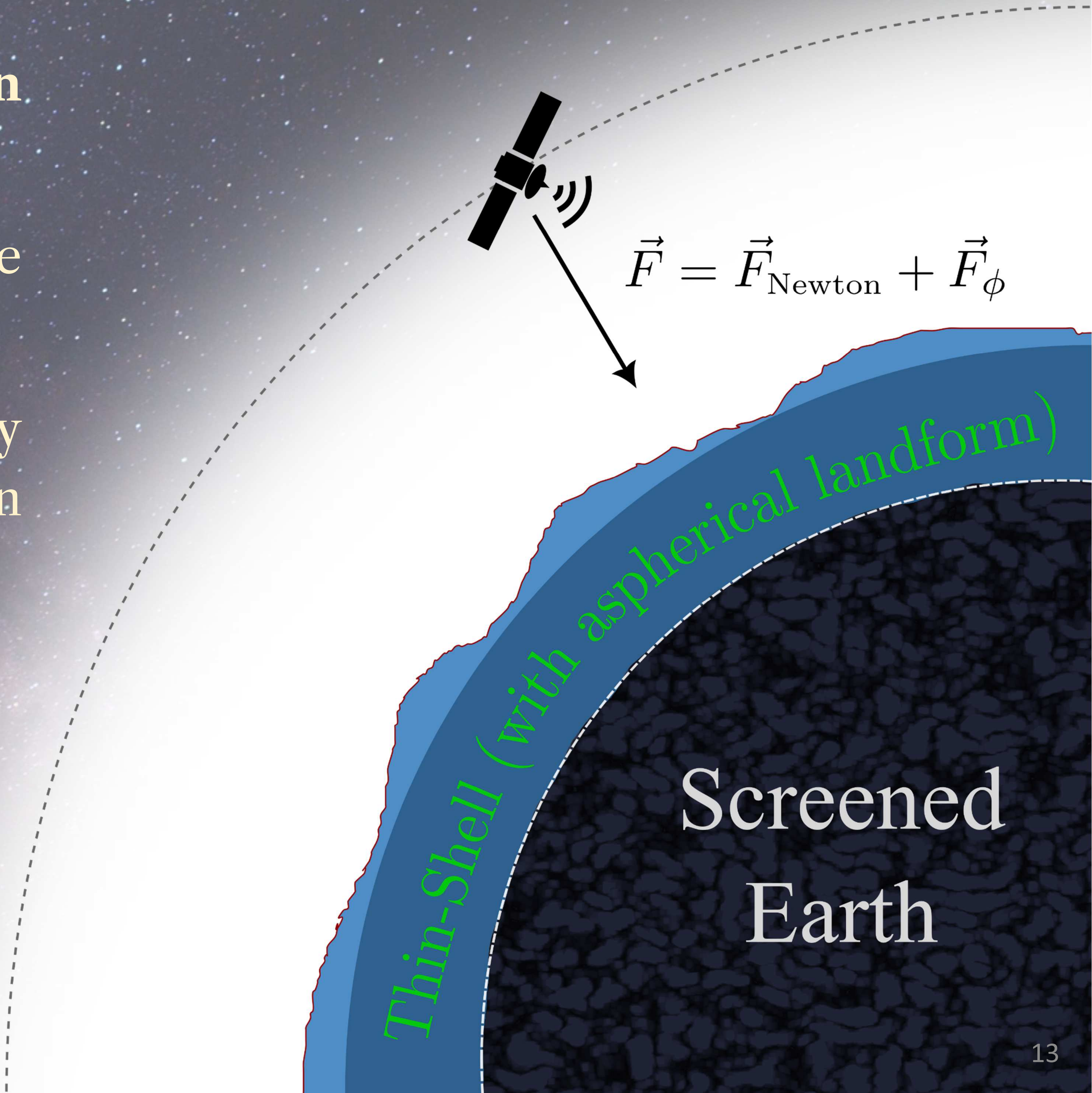
All computations are performed with

femtoscope

**space geodesy: Space geodesy is a scientific discipline that involves precise measurements and analysis of the Earth's shape, gravitational field, and the dynamic behavior of its surface using satellite-based technologies.*

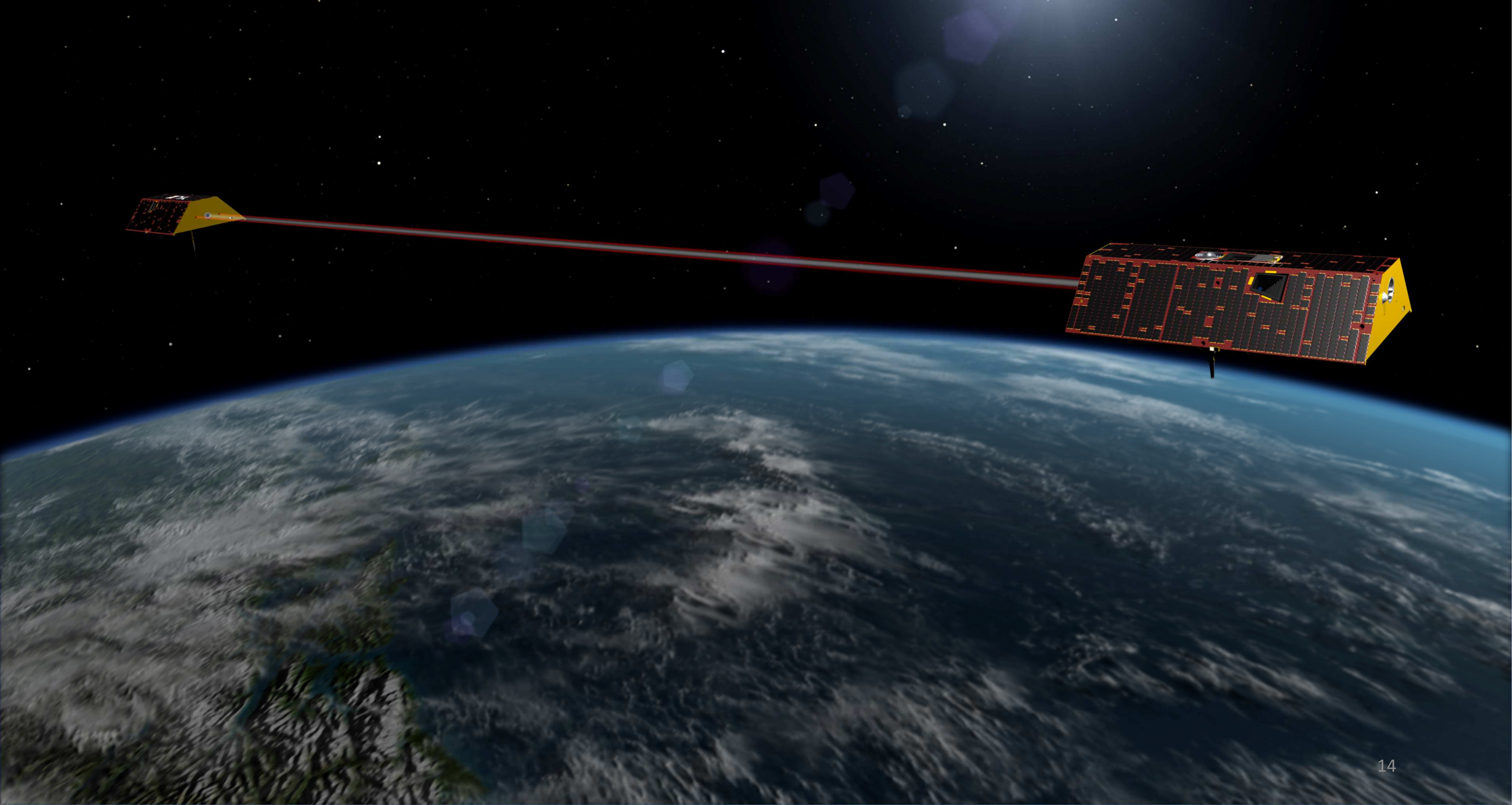
Motivations

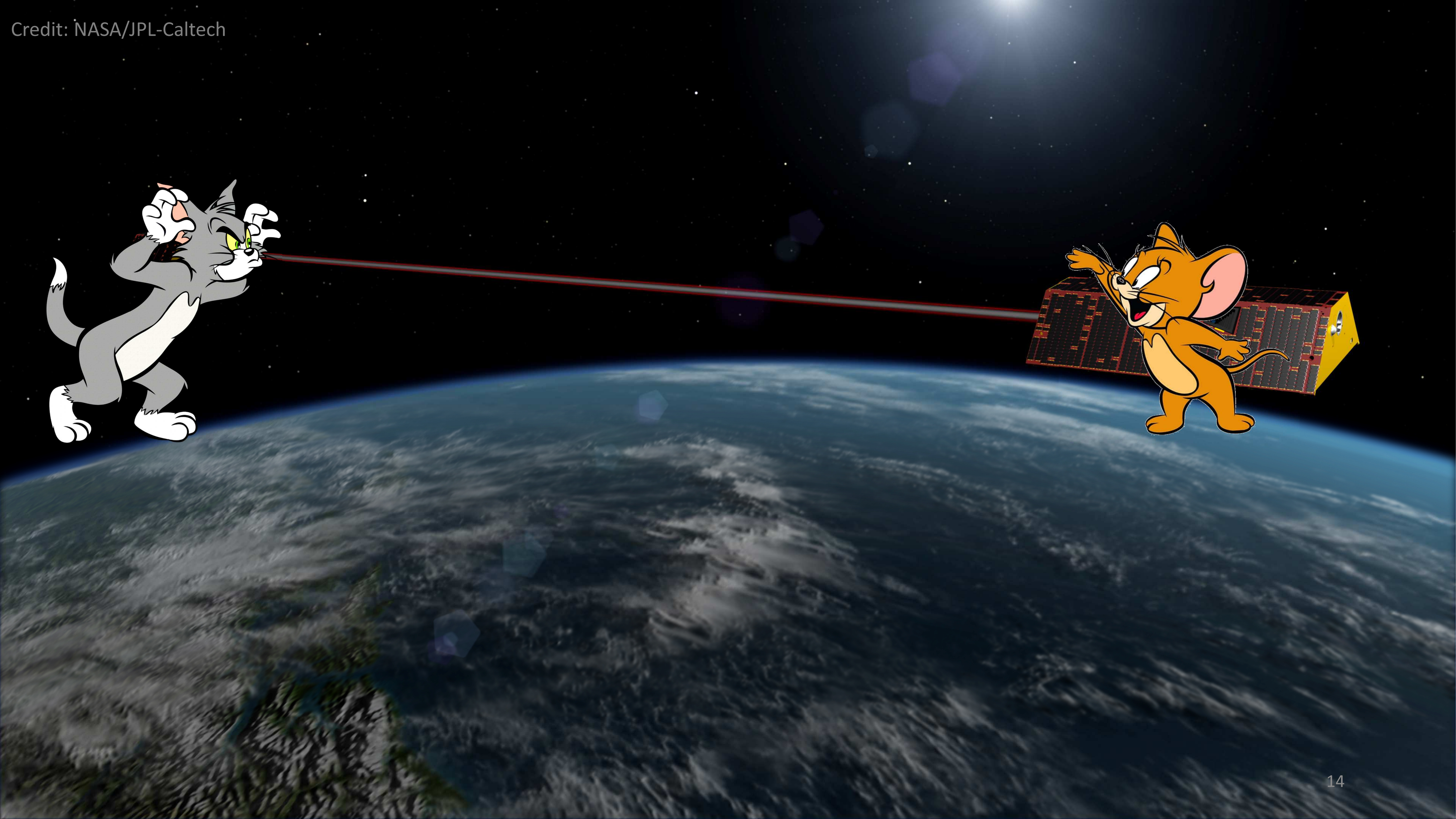
- A satellite in orbit is subject to both **Newtonian attraction** and **fifth force**
- Strong **impact of the local landform** on the scalar field in the **screened regime**
- **Mountain** ≡ deviation from spherical symmetry + analogy with the **'lightning-rod effect'** in chameleon and symmetron models [10]
- Can a satellite flying over a mountain **distinguish** between **Newtonian gravity** and **chameleon gravity**?

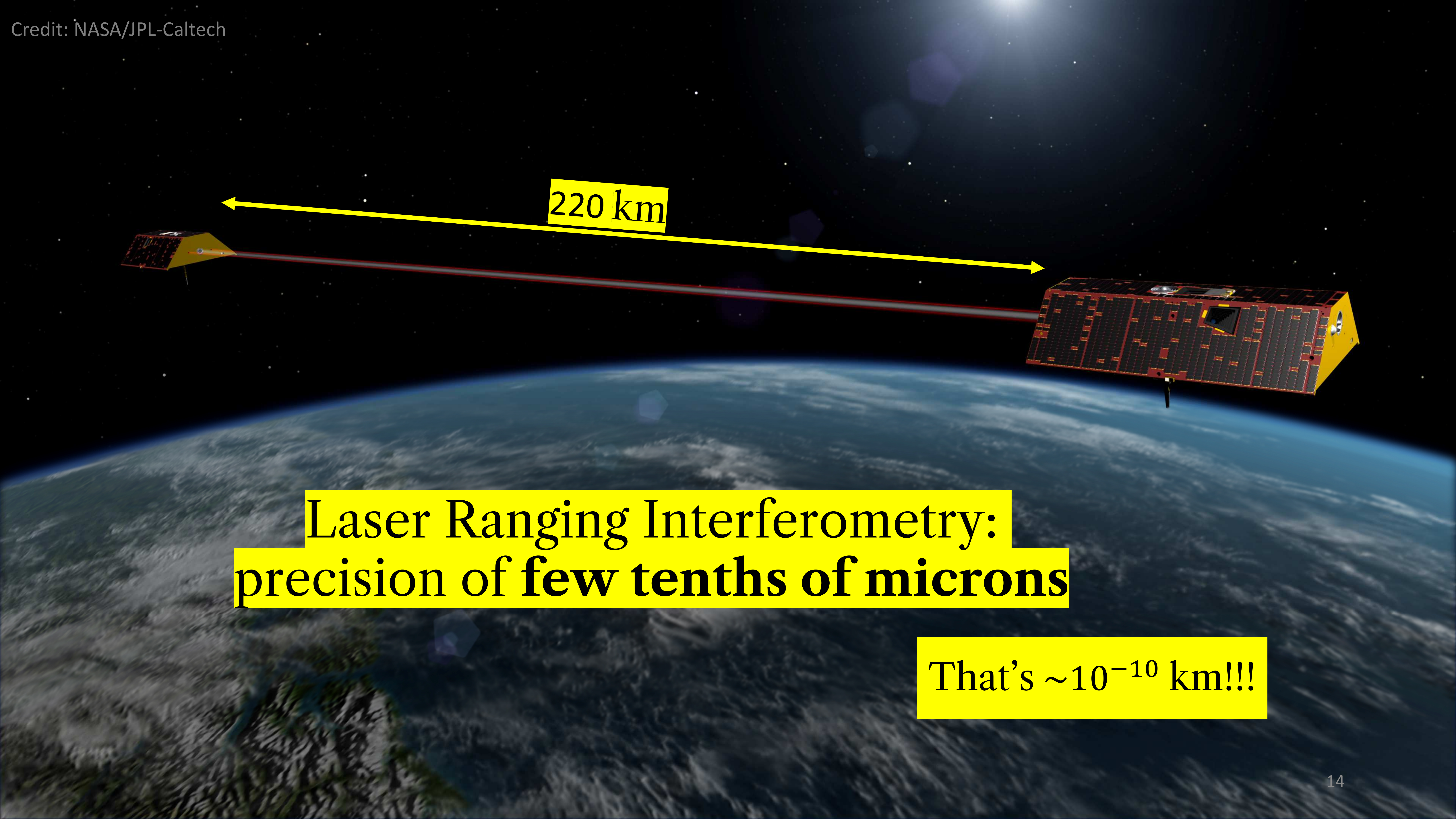


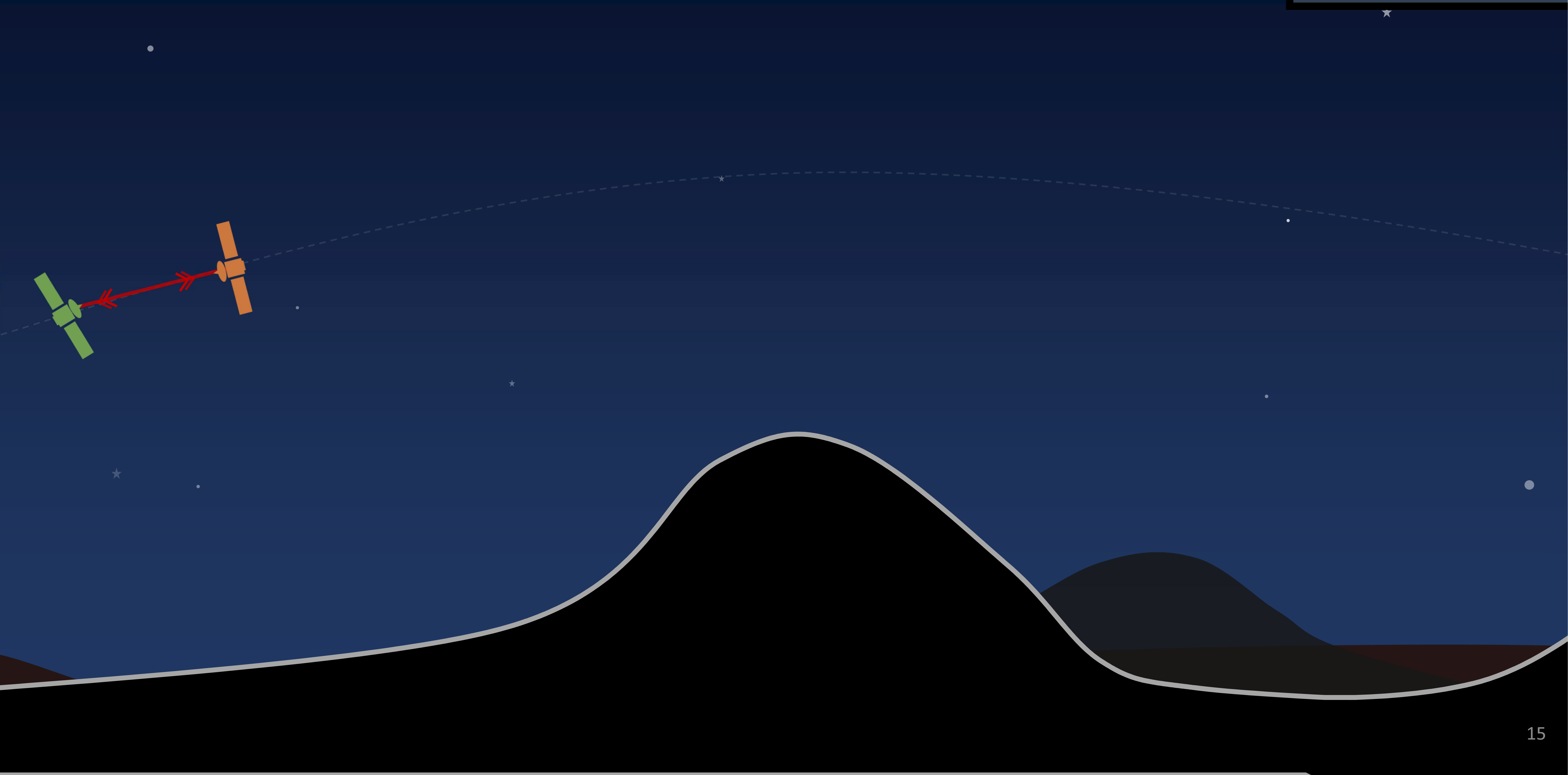
[10] K. Jones-Smith and F. Ferrer, *Detecting Chameleon Dark Energy via an Electrostatic Analogy*, Phys. Rev. Lett. 108, 221101 – Published 29 May 2012

Credit: NASA/JPL-Caltech

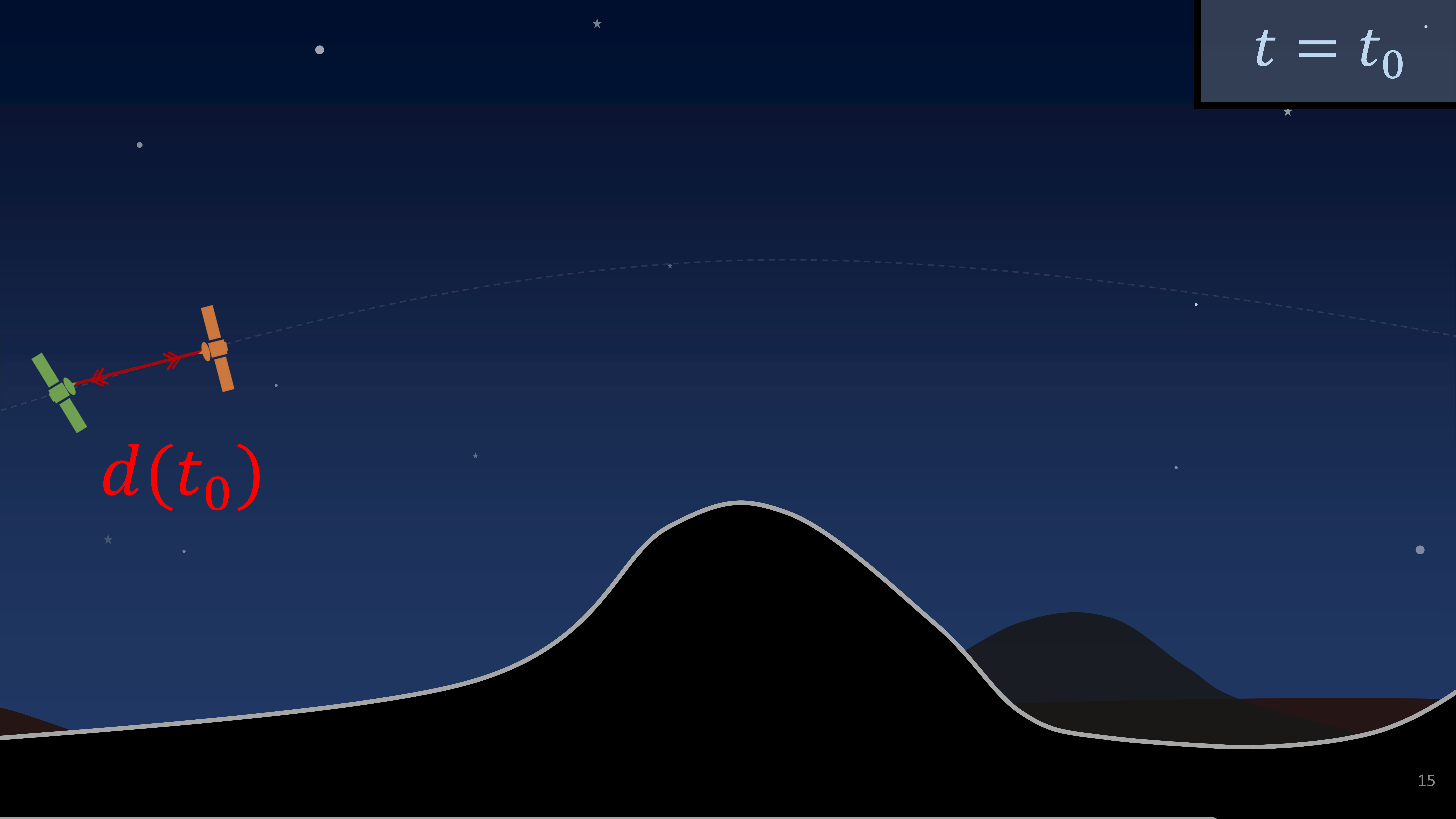


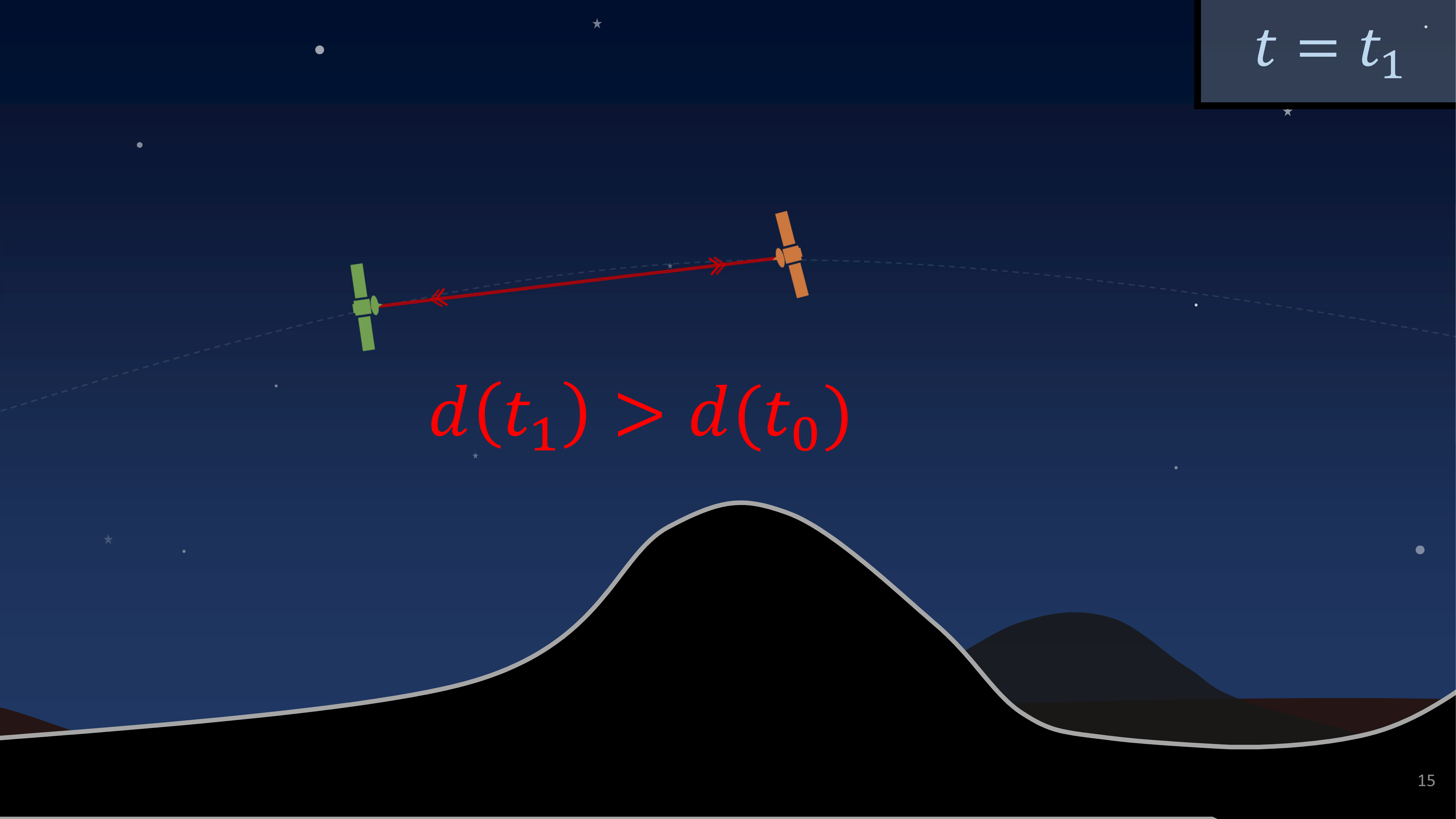

Credit: NASA/JPL-Caltech


Credit: NASA/JPL-Caltech
220 km
Laser Ranging Interferometry:
precision of **few tenths of microns**
That's $\sim 10^{-10}$ km!!!

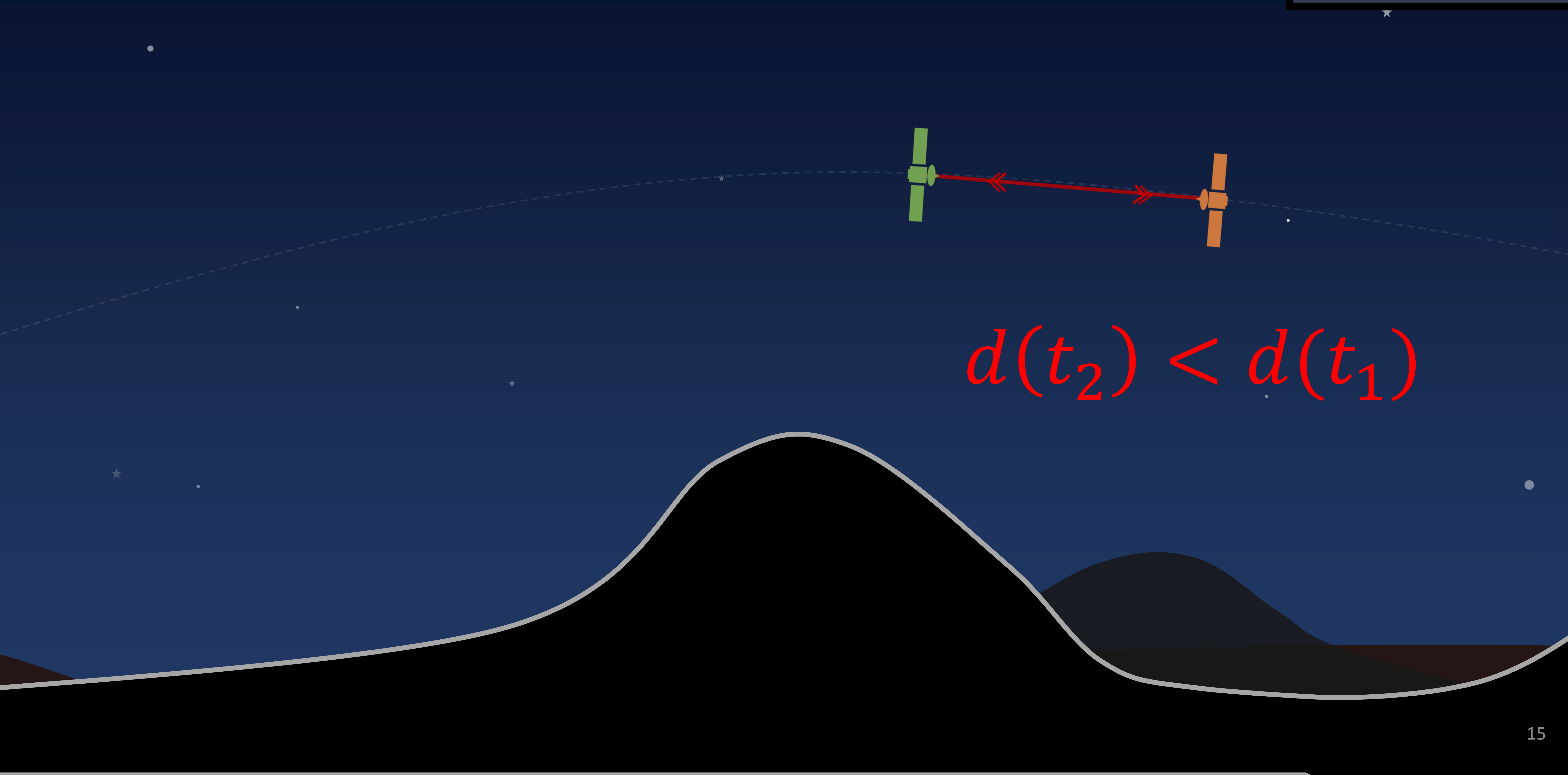


$t = t_0$


$t = t_0$
$d(t_0)$

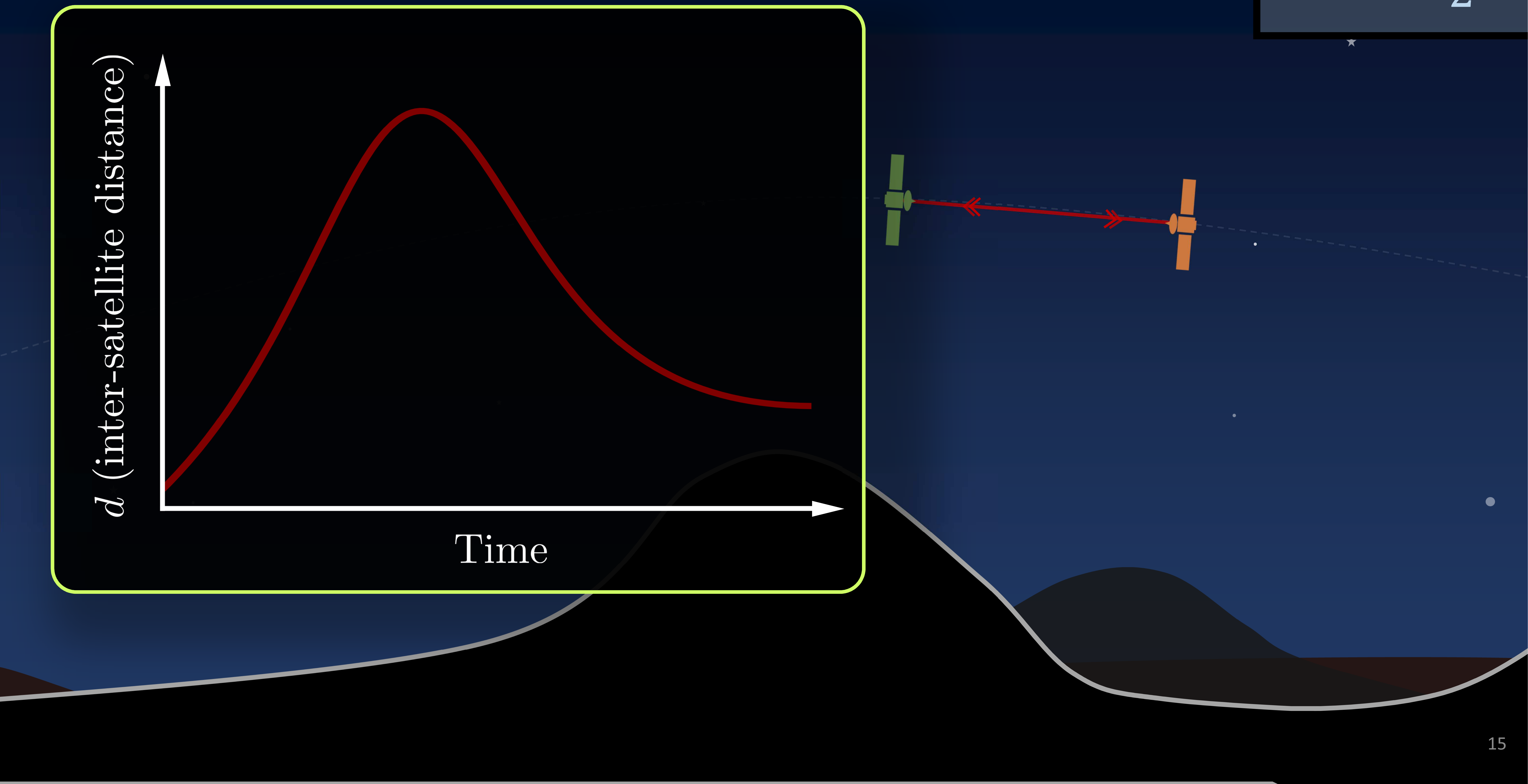

$t = t_1$
$d(t_1) > d(t_0)$

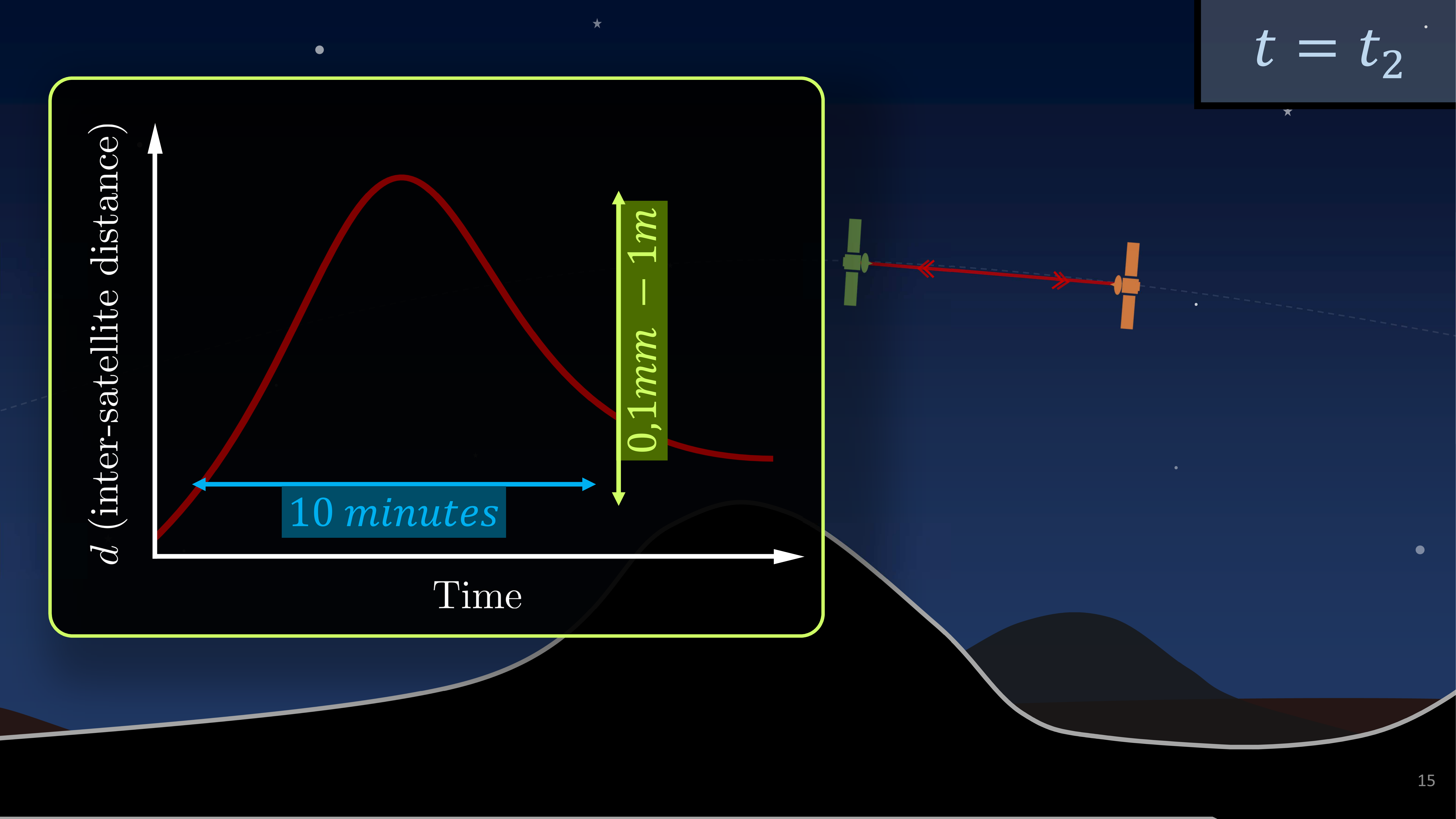
$t = t_2$



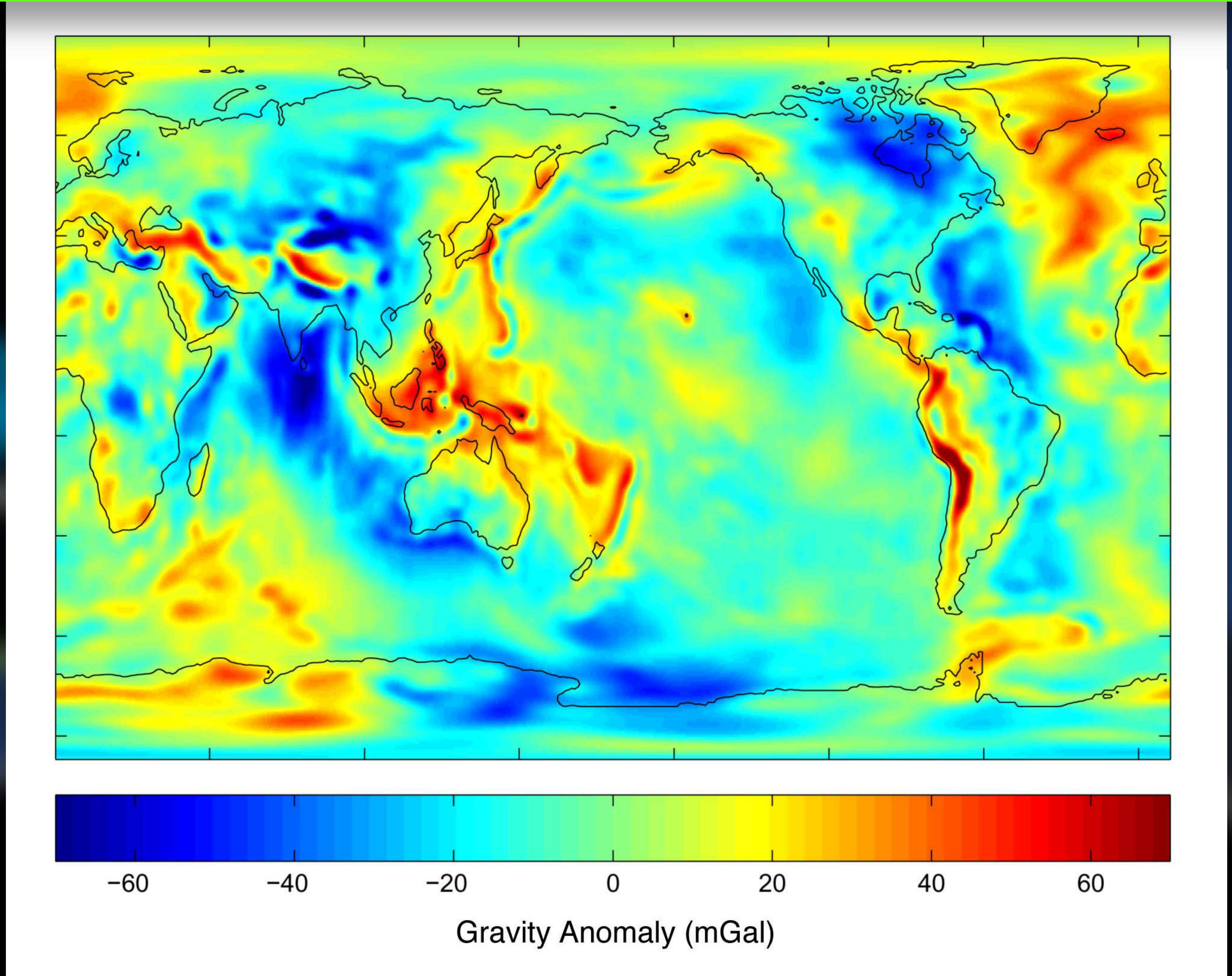
$$d(t_2) < d(t_1)$$

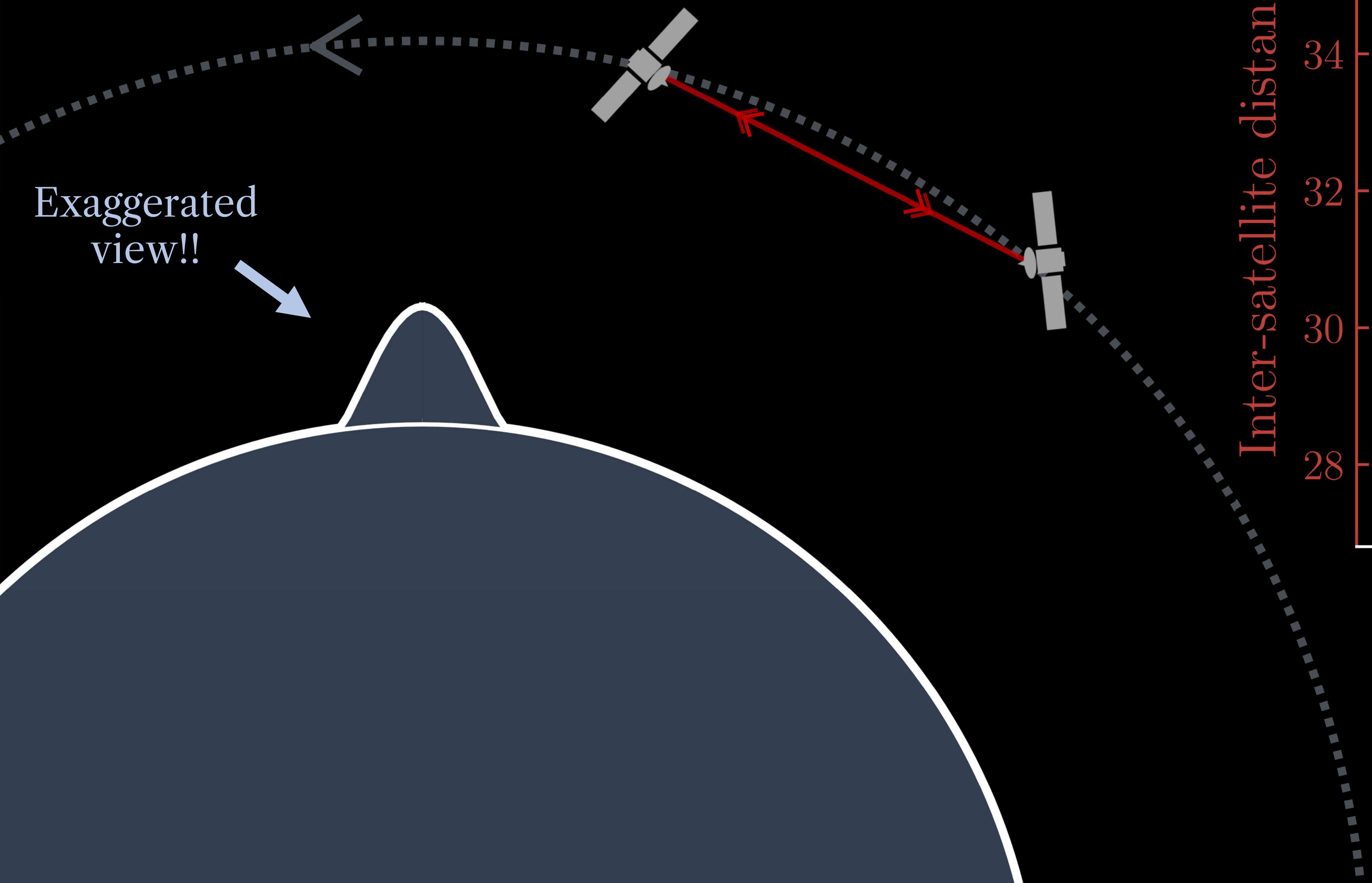
$t = t_2$



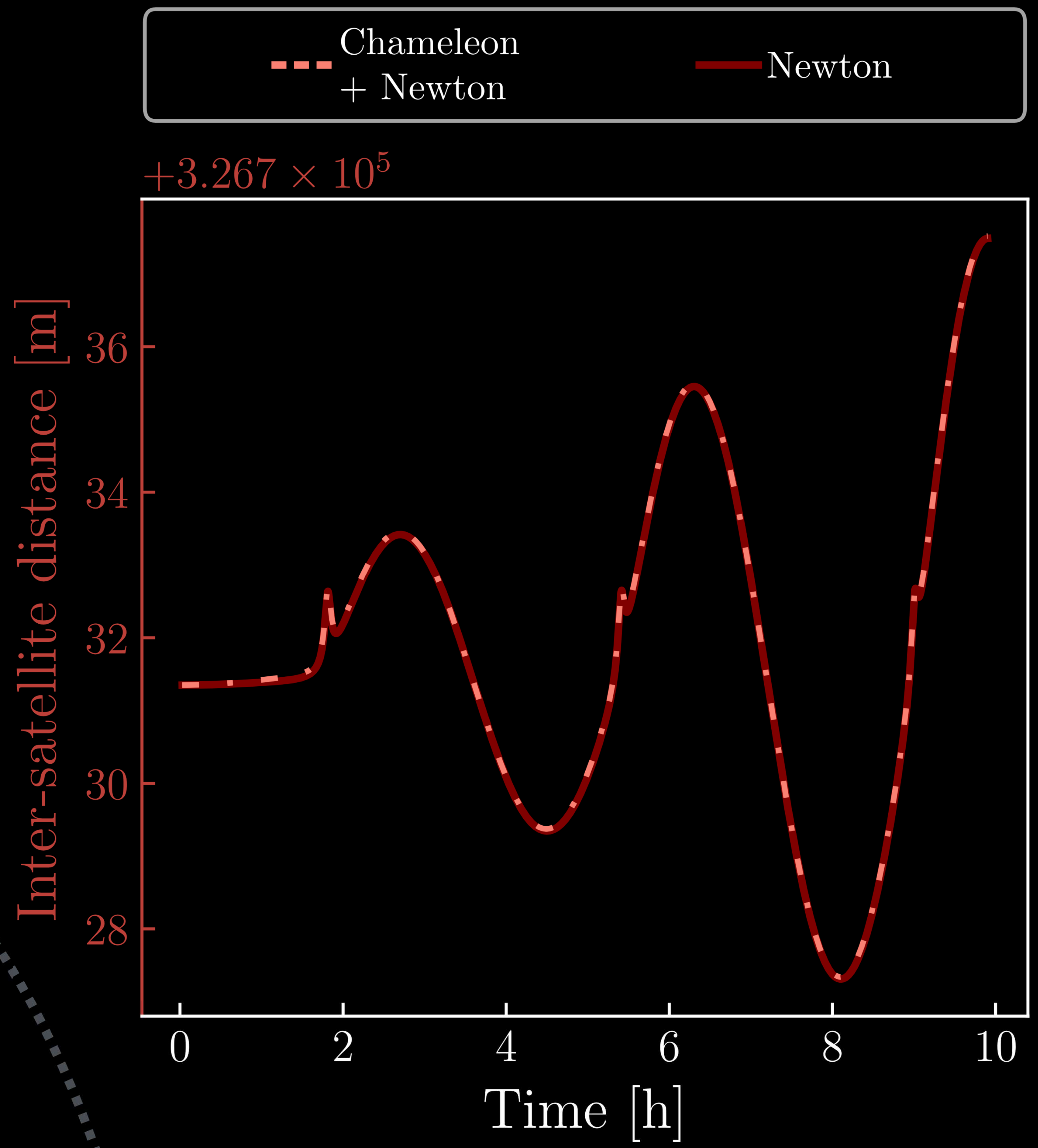

$t = t_2$
d (inter-satellite distance)
Time
10 *minutes*
0,1*mm* – 1*m*

Interpreting the measurement in the framework of Newtonian gravity gives us a gravity map

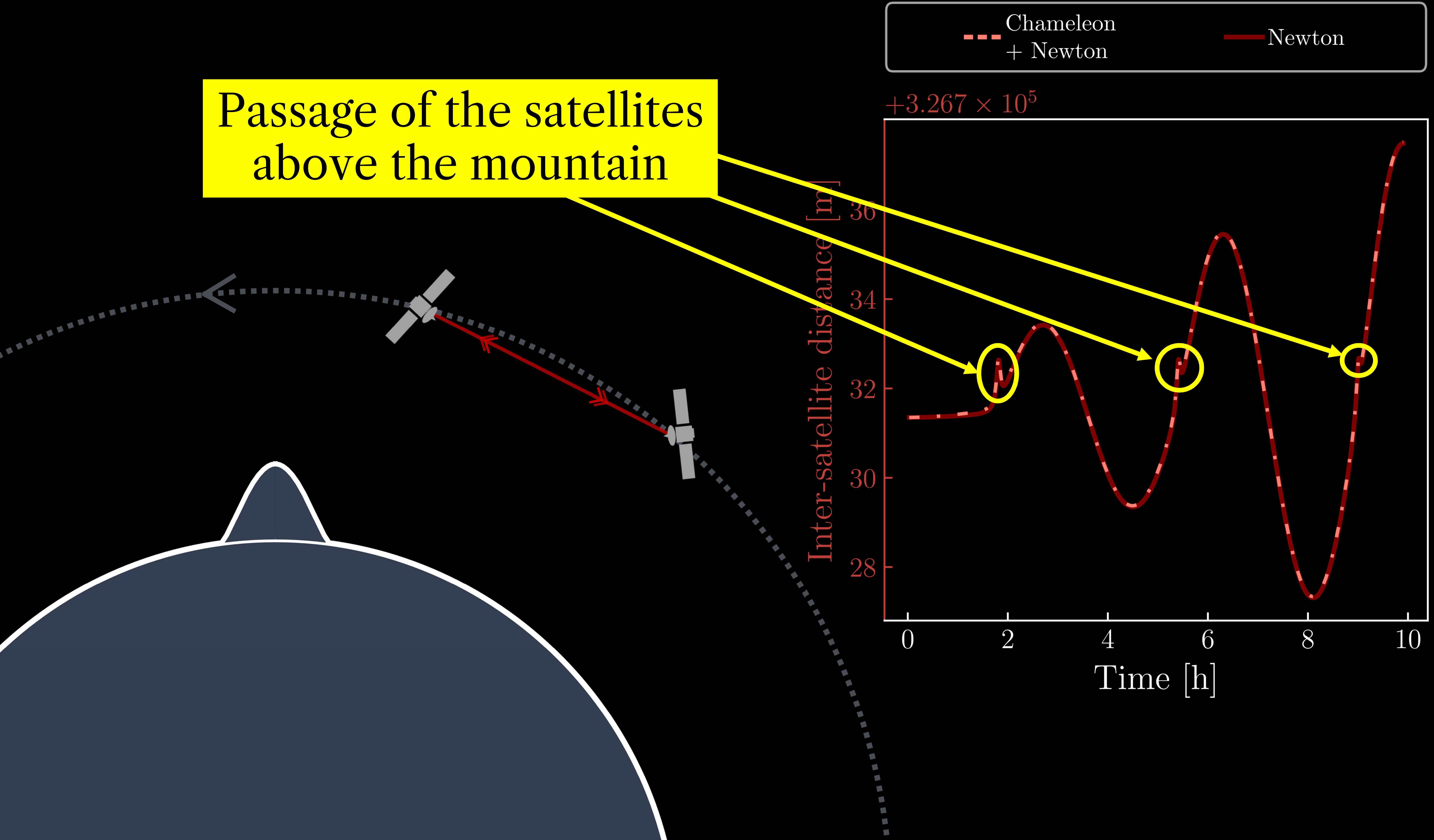


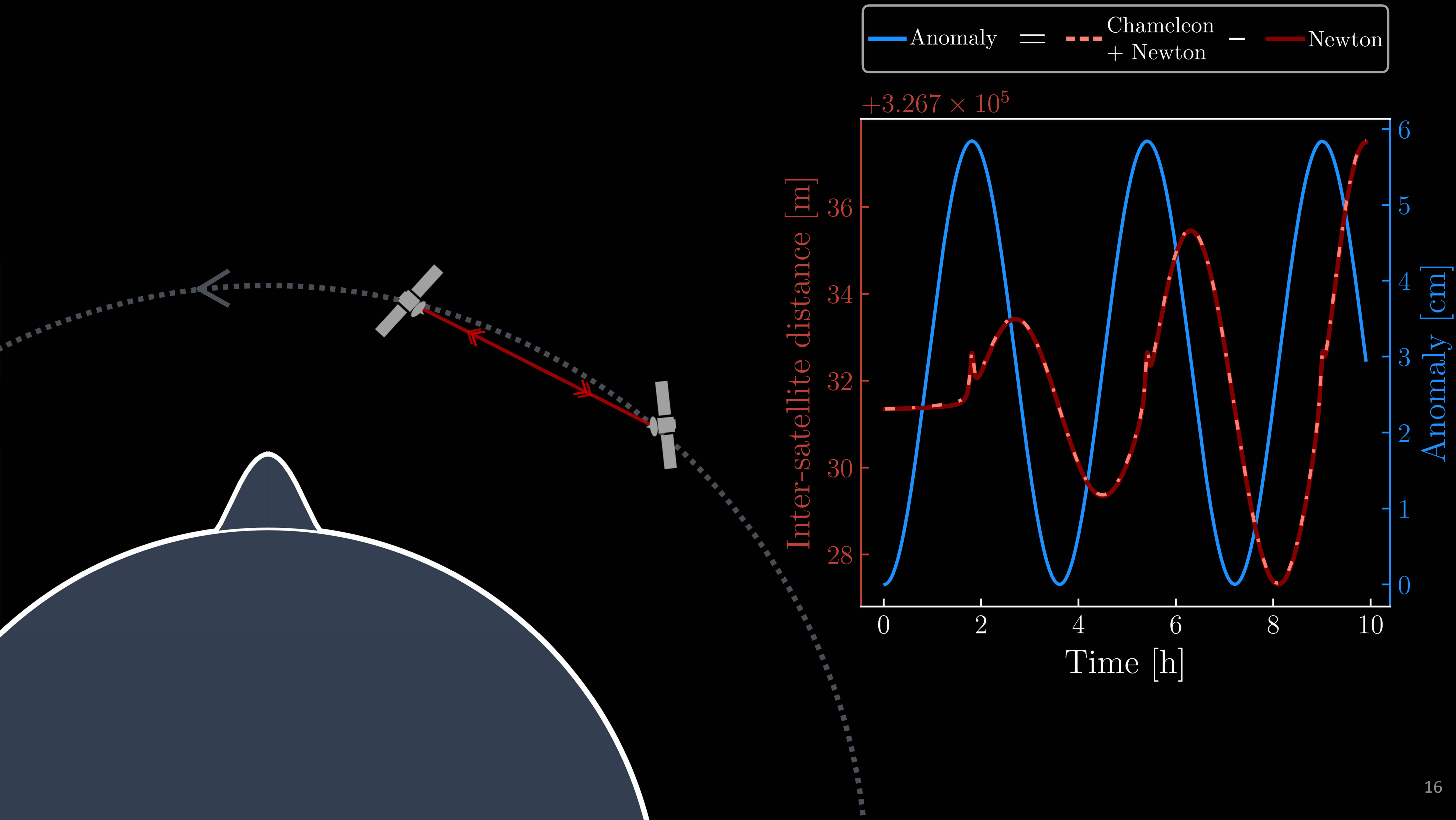


Exaggerated view!!



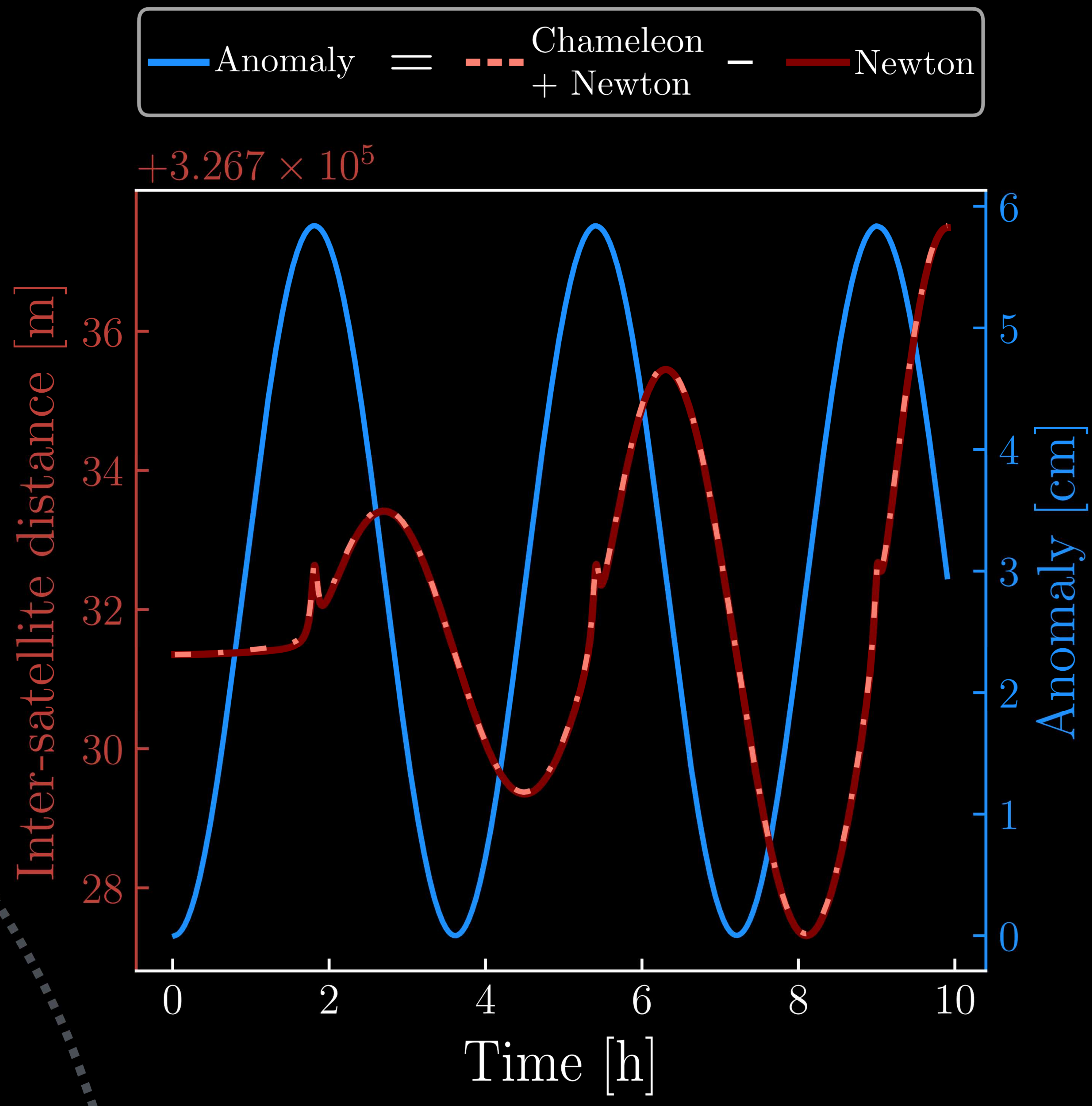
Chameleon + Newton
Newton
+3.267 × 10^5
Inter-satellite distance [m]
36
34
32
30
28
0
2
4
6
8
10
Time [h]


Passage of the satellites above the mountain
Chameleon + Newton
Newton
+3.267 × 10^5
Inter-satellite distance [m]
36
34
32
30
28
0
2
4
6
8
10
Time [h]

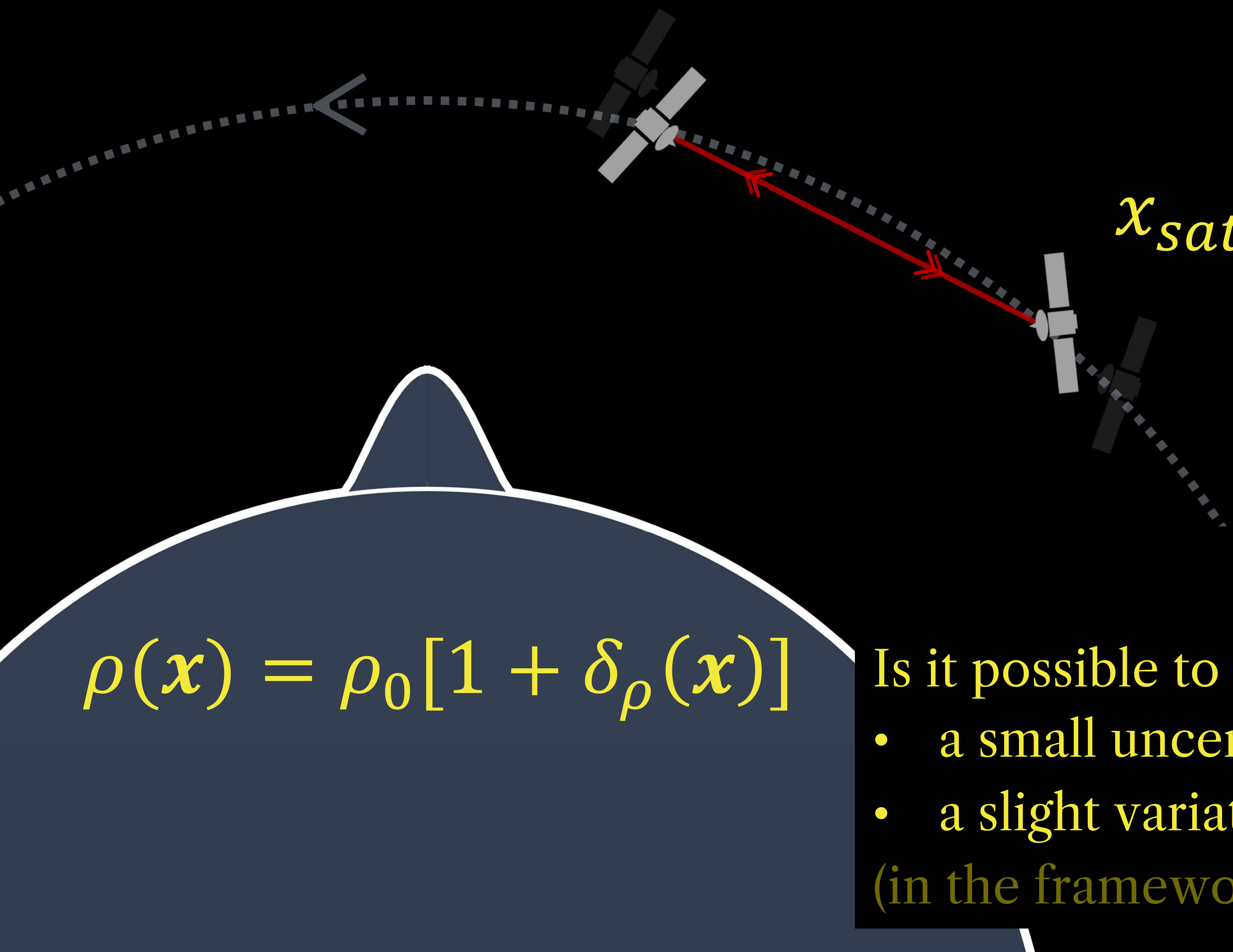

Anomaly
Chameleon + Newton
Newton
+3.267 × 10^5
Inter-satellite distance [m]
Anomaly [cm]
Time [h]
0 2 4 6 8 10
28 30 32 34 36
0 1 2 3 4 5 6

The anomaly is well within the range of GRACE-FO precision!

$$\mathcal{O}(1cm) \gg \mathcal{O}(10^{-5}cm)$$



Sources of degeneracy



$$x_{sat}(t_0) = x_0[1 + \delta_x]$$

$$\rho(\boldsymbol{x}) = \rho_0[1 + \delta_\rho(\boldsymbol{x})]$$

Questions

Is it possible to absorb the chameleon anomaly in

- a small uncertainty in the satellite's initial state vector δ_x?
- a slight variation in the {Earth + Mountain} density δ_ρ?

(in the framework of Newtonian gravity)

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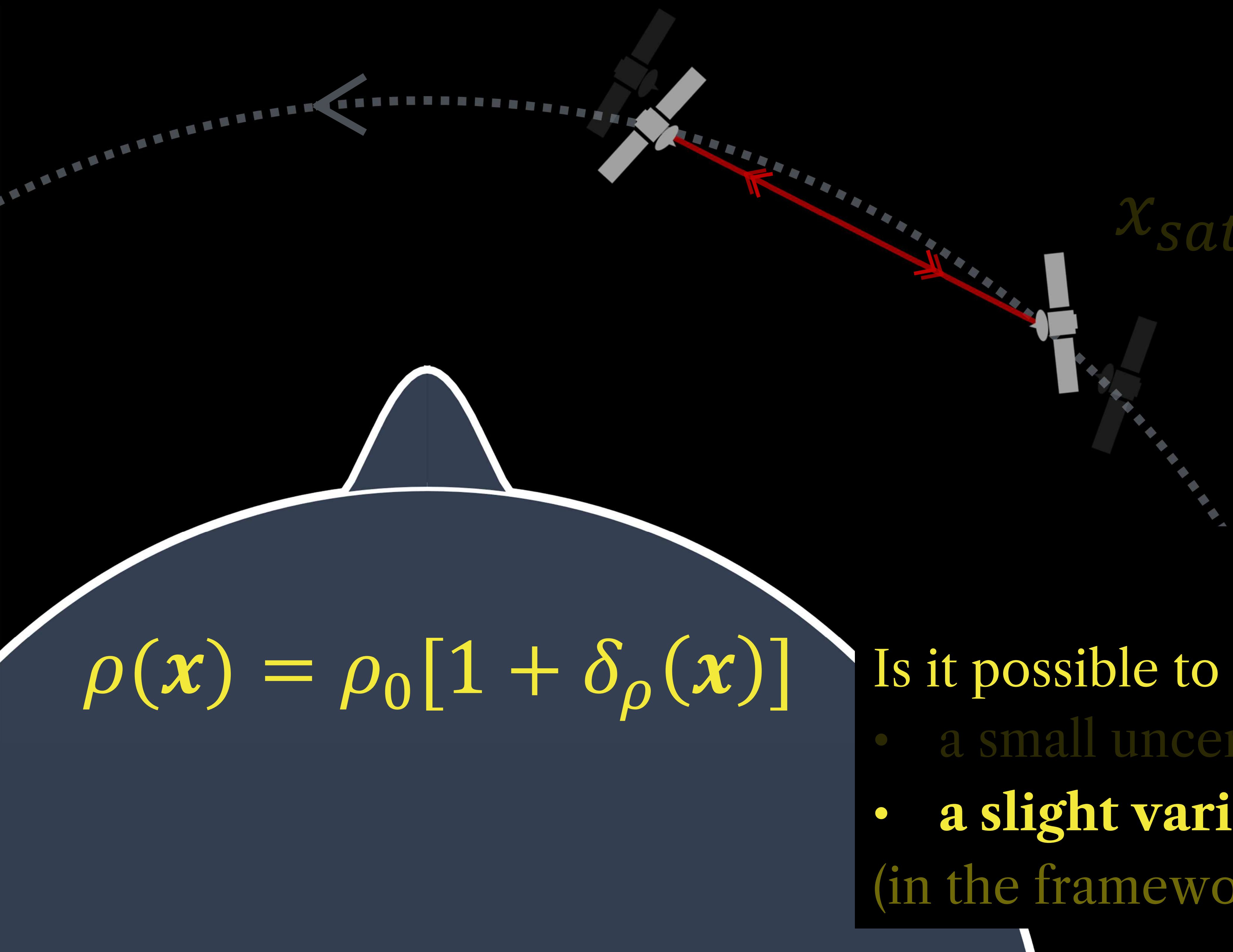

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(in the framework of Newtonian gravity)

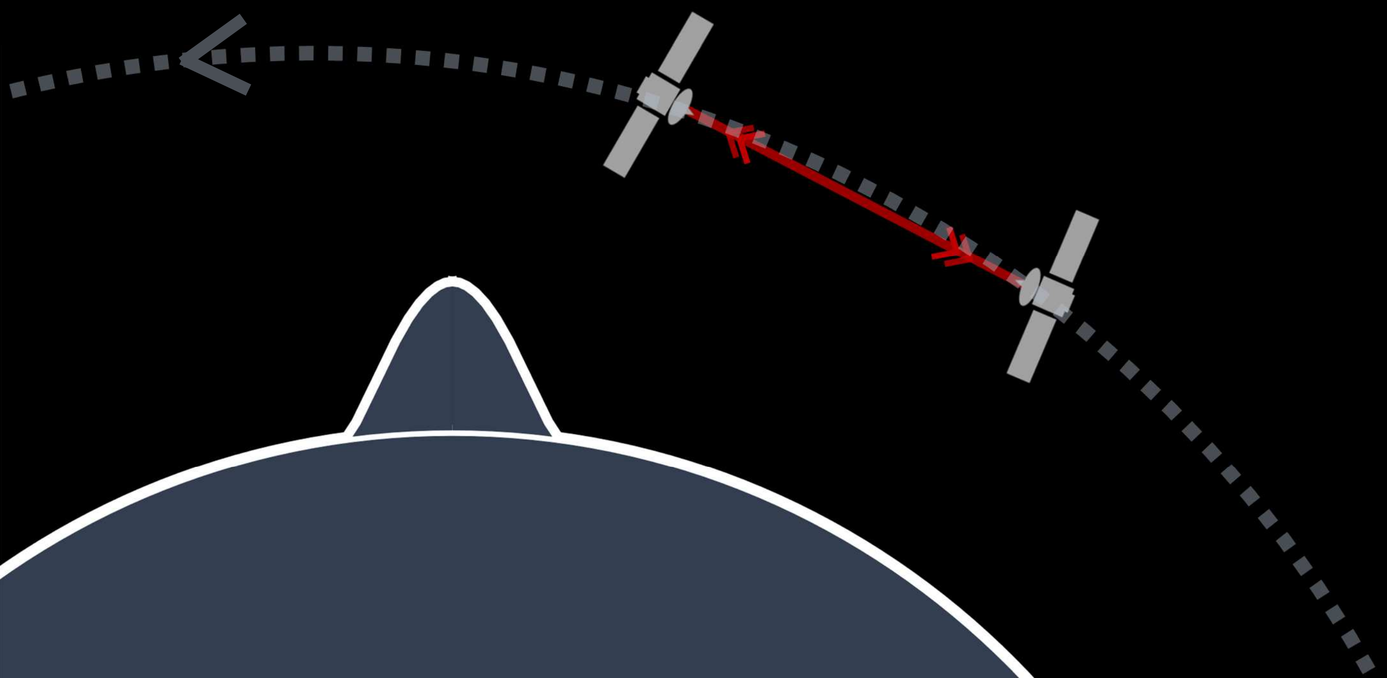
Modified Gravity

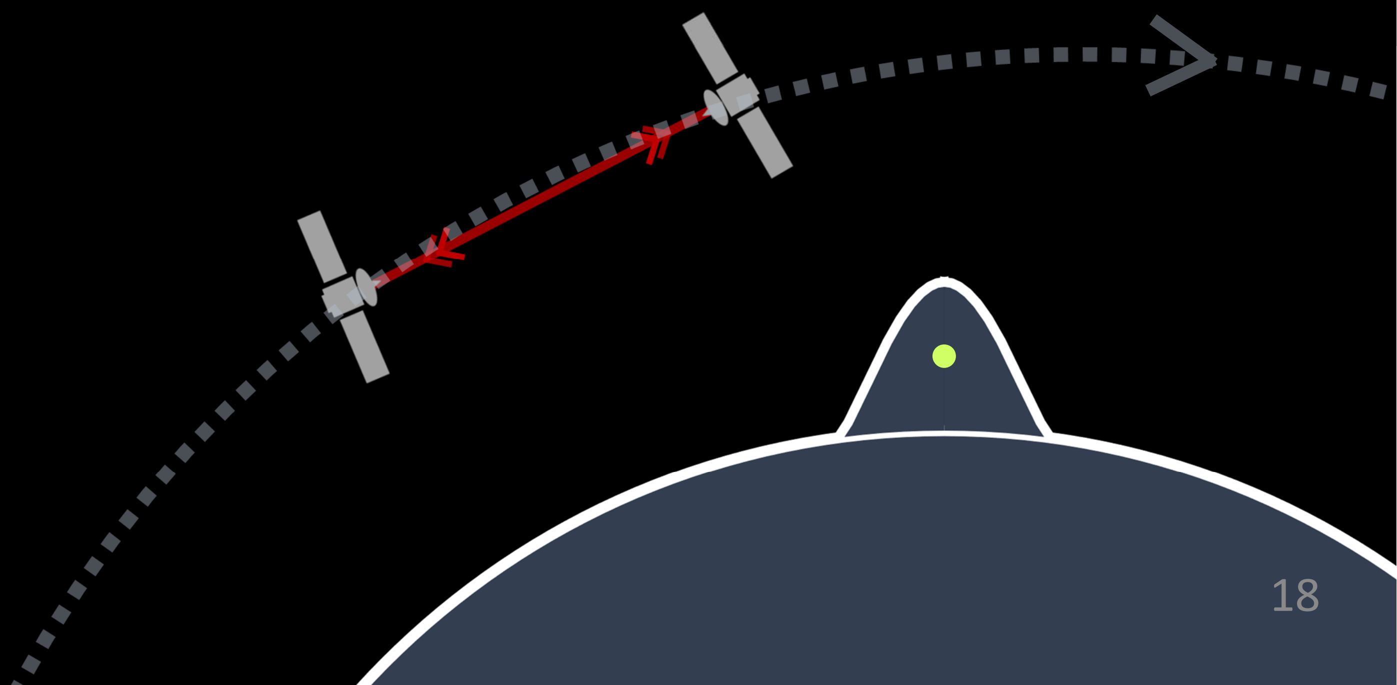
(Newtonian + chameleon accelerations)

vs

Newtonian Gravity

with extra point-mass

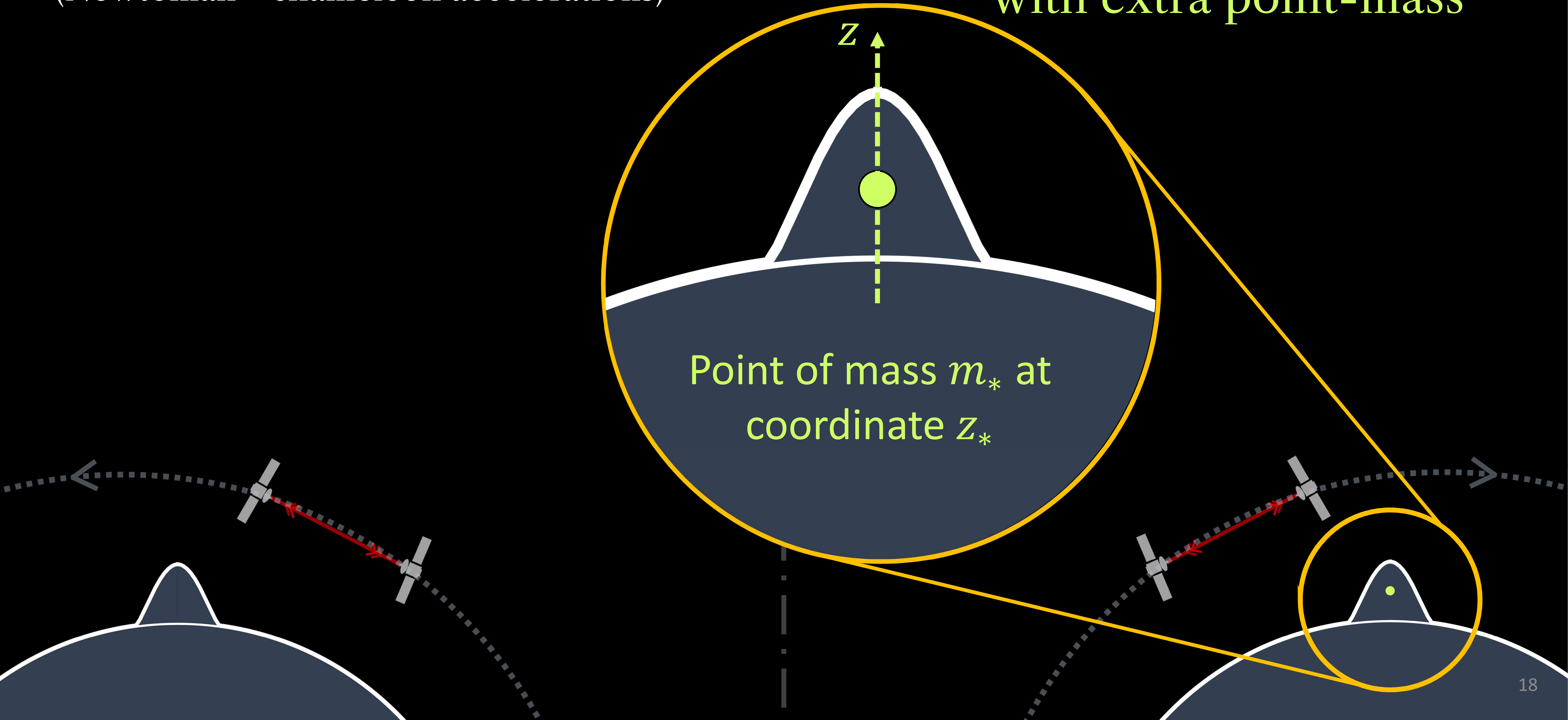




Modified Gravity vs Newtonian Gravity

(Newtonian + chameleon accelerations)

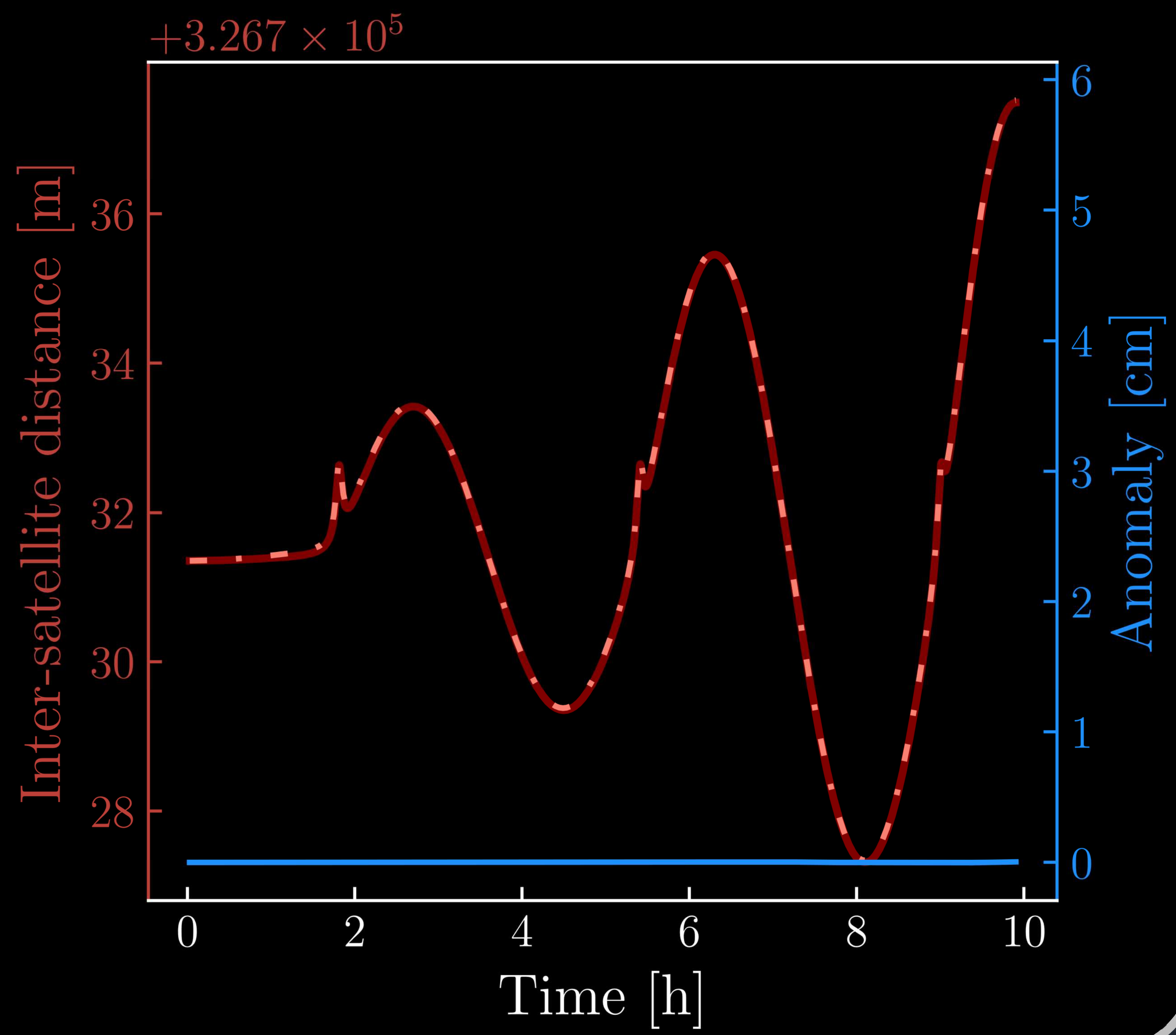
with extra point-mass



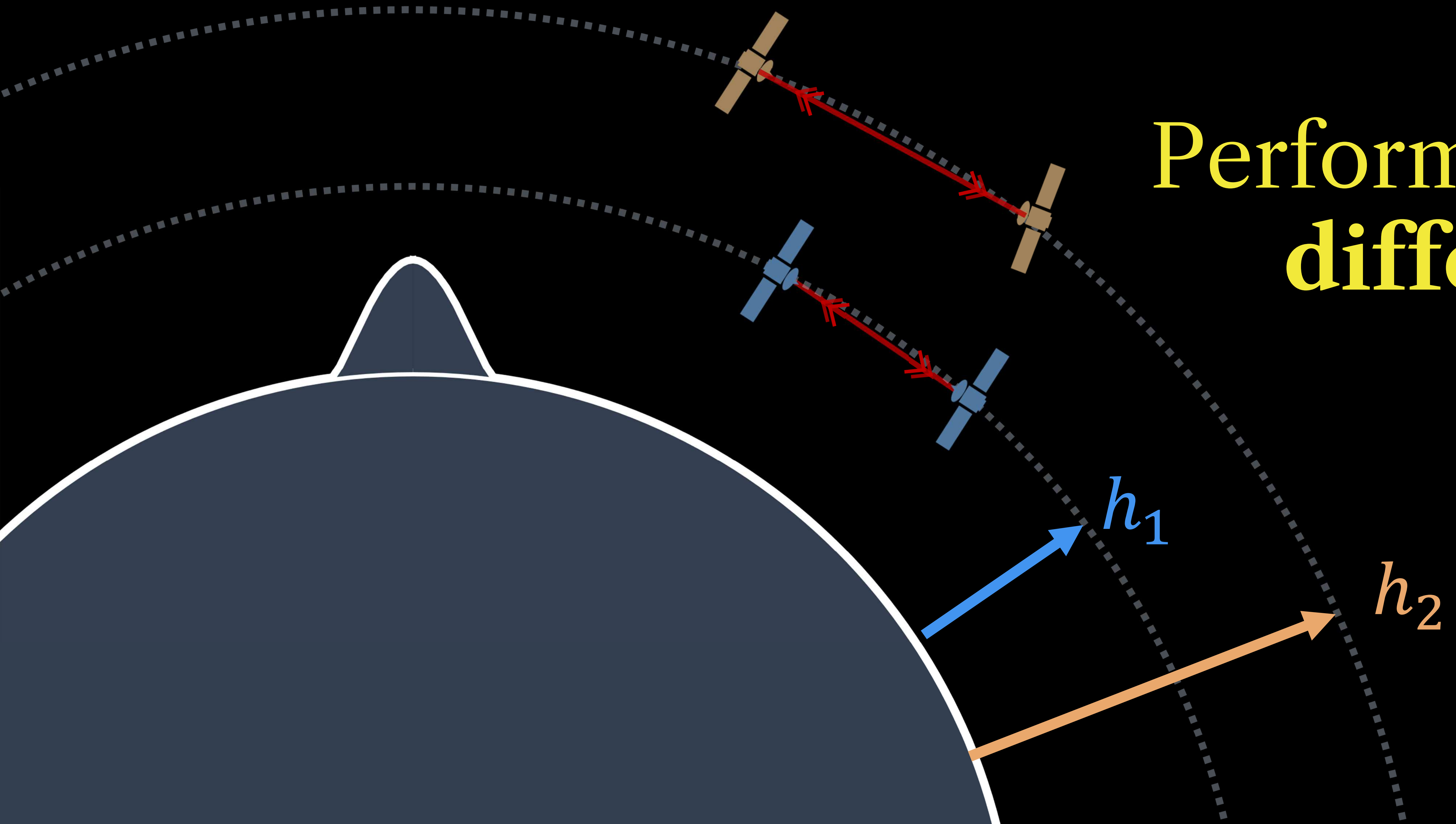
= 0,001% the mass of the initial mountain

Anomaly

$\sim \mathcal{O}(10^{-4} cm)$



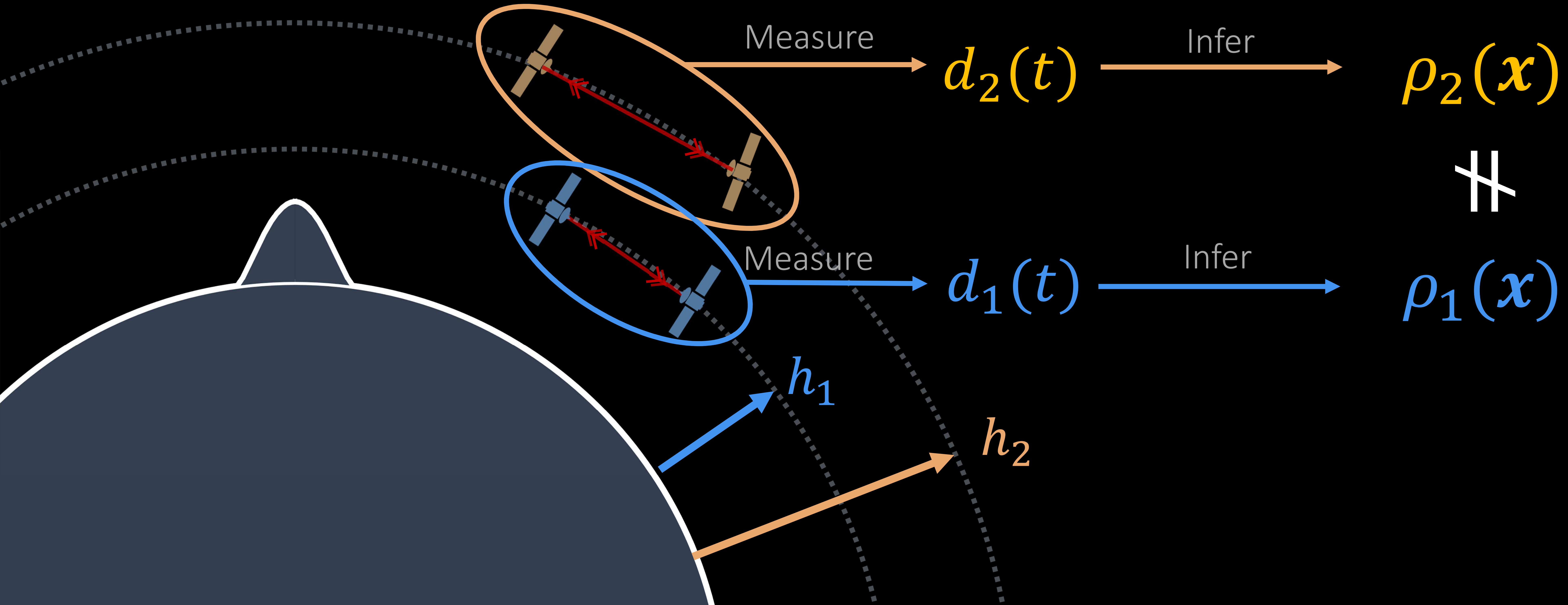
How to lift the degeneracy?



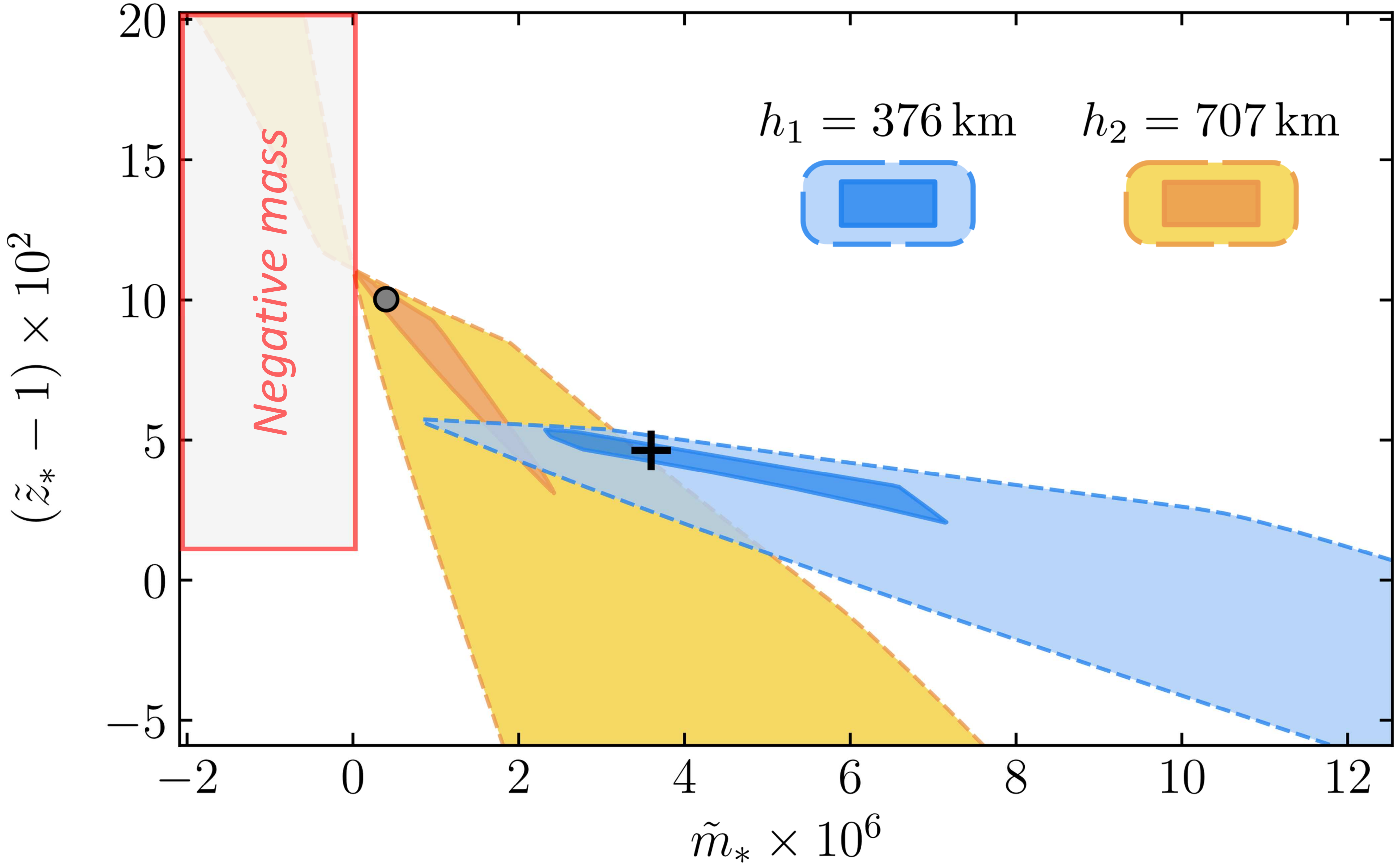
Idea:

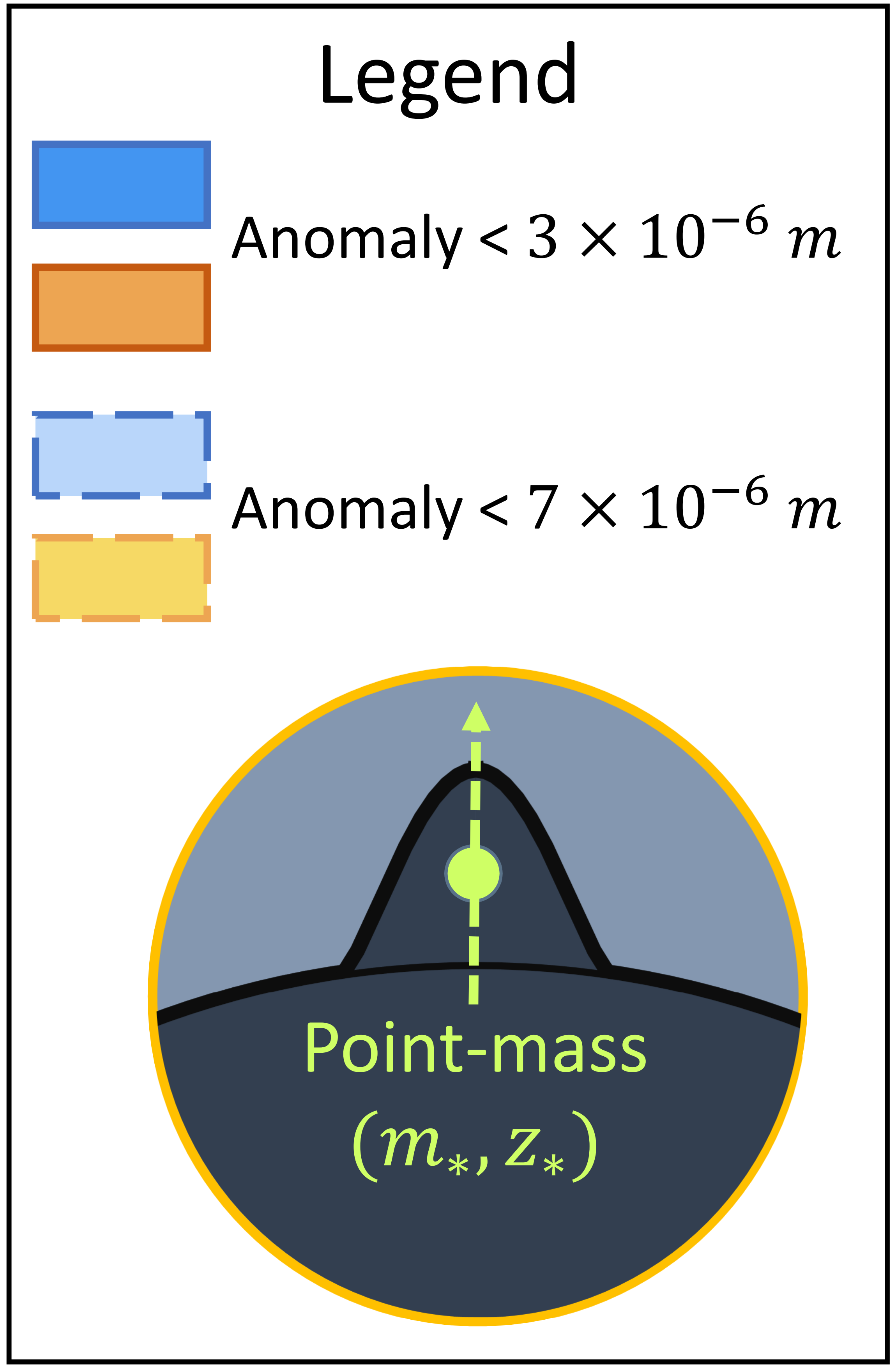
Perform the experiment at **different altitudes**

How to lift the degeneracy?



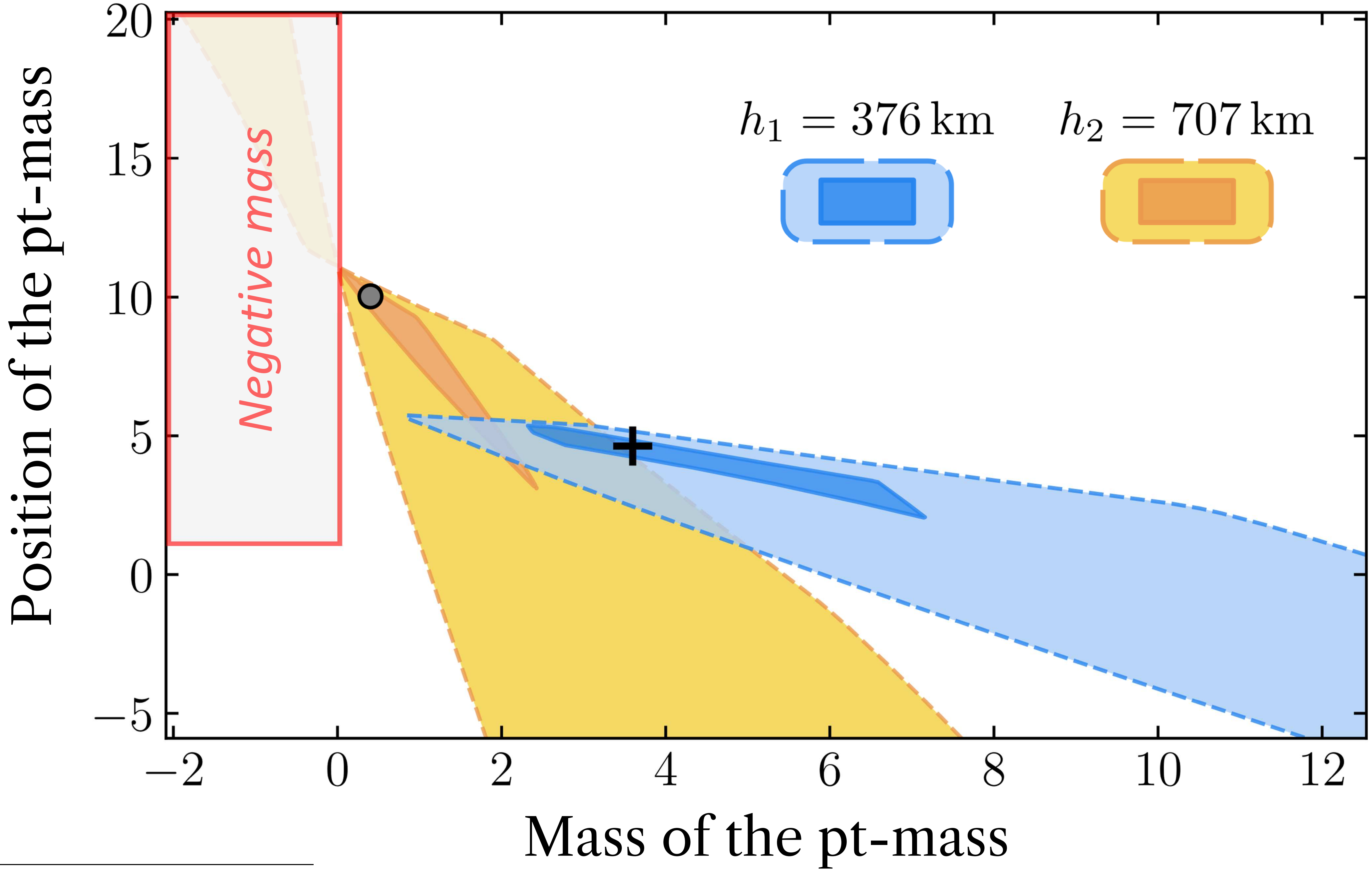
Tensions come in...

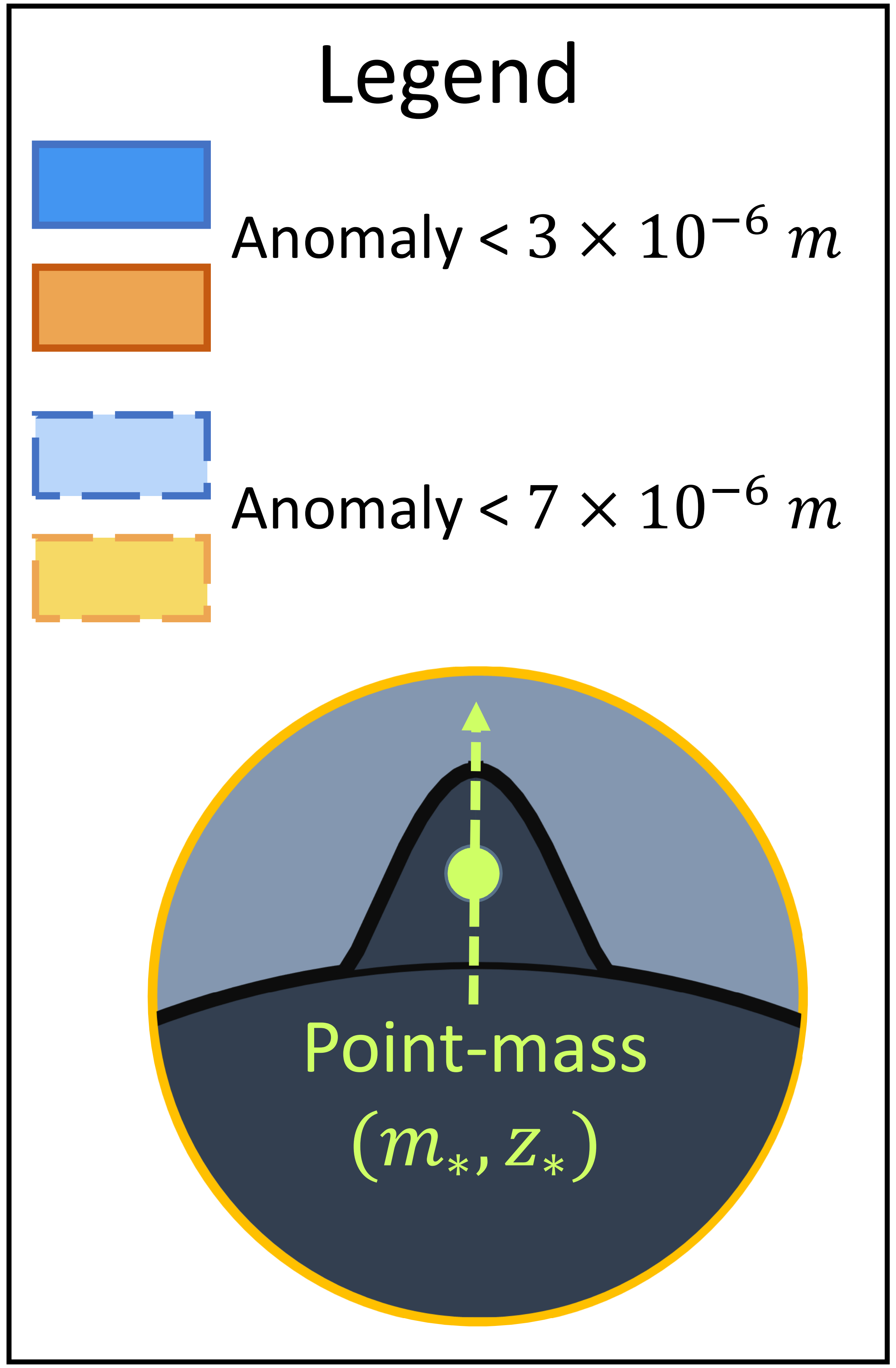




H Lévy et al, arχiv 2310.03769

Tensions come in...





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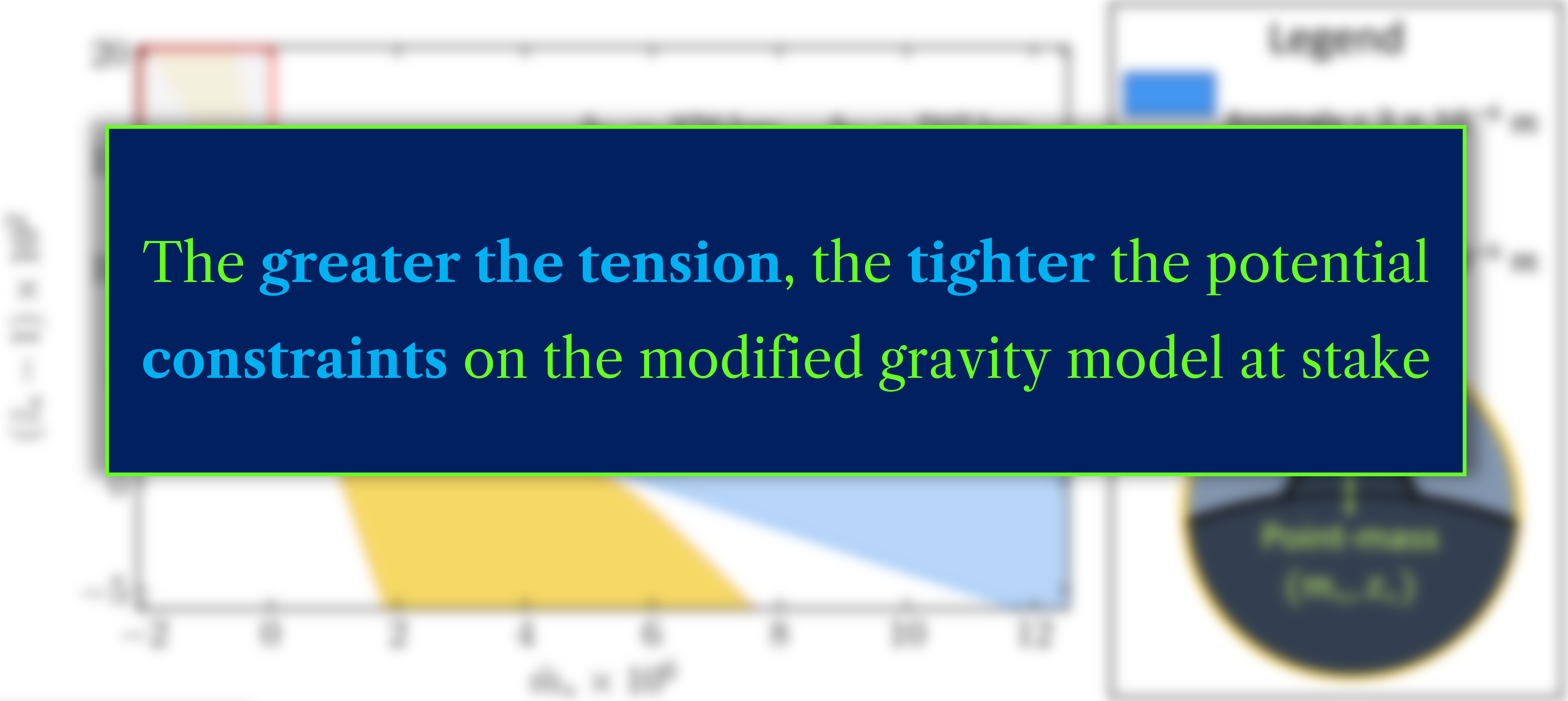
Tensions come in...



The **greater the tension**, the **tighter** the potential **constraints** on the modified gravity model at stake

Conclusion

- *femtoscope*: solve semi-linear elliptic PDE using the Finite Element Method on unbounded domains (general purpose code)

- Application to scalar-tensor theories of gravity:

Linear Poisson equation

$$\begin{cases} \Delta\Phi = 4\pi G\rho(\mathbf{x}) \\ \Phi(\mathbf{x}) \underset{\|\mathbf{x}\|\to+\infty}{\longrightarrow} 0 \end{cases}$$

Nonlinear Klein-Gordon equation

$$\begin{cases} \Delta\phi = \mathrm{d}V_{\mathrm{eff}}/\mathrm{d}\phi \\ \phi(\mathbf{x}) \underset{\|\mathbf{x}\|\to+\infty}{\longrightarrow} \phi \end{cases}$$

$$\mathbf{a} \sim -\boldsymbol{\nabla}(\Phi + \phi)$$

Gravitational Acceleration

- Application to space geodesy: focus on the GRACE-FO configuration
- Can we detect / put constraints on the chameleon model in this context?

With little to **no** uncertainties

Competitive constraints can be derived

With uncertainties

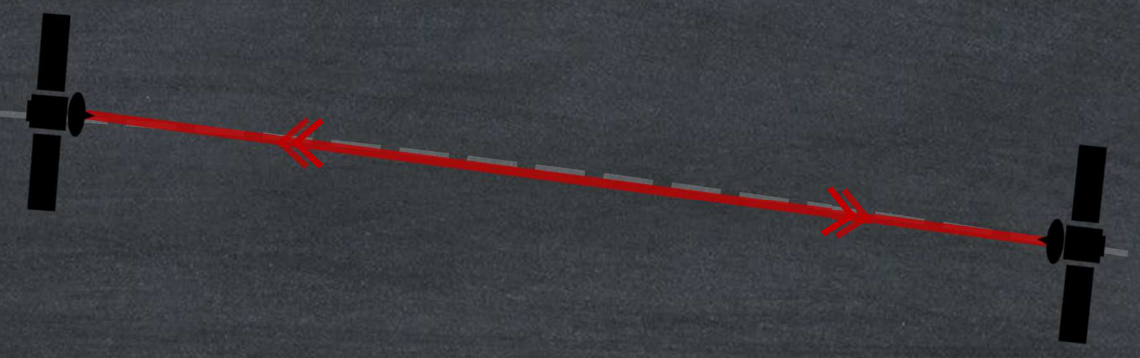
Mass distribution inside the Earth

Satellite state vector

- Dramatically improve our knowledge of the Earth density (unlikely)

- Use ≠ altitudes (taking advantage of the fact that $a_\phi \not\propto 1/r^2$)

- In comparison, lab experiments offer a more controlled environment



hugo.levy@onera.fr

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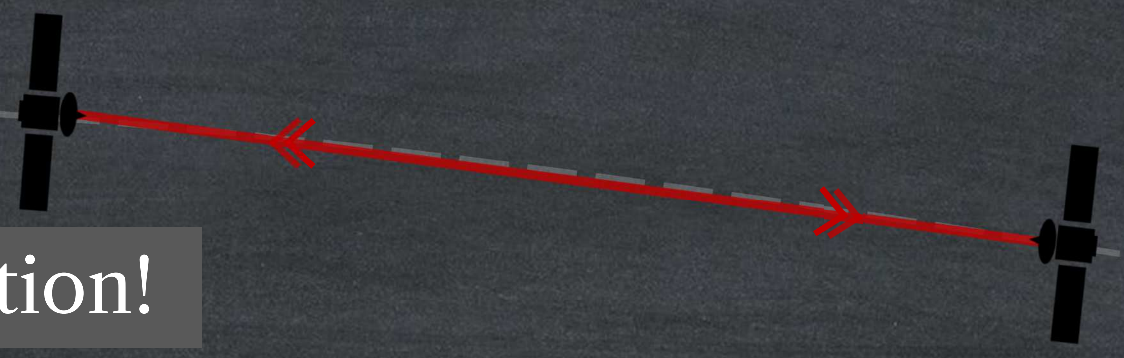

Thanks for your attention!

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Backup Slides

FEM in a nutshell (example on Poisson's equation)

strong form

Find $u : \Omega \to \mathbb{R}$

$$\begin{cases} -\Delta u = f & \text{in } \Omega \\ u = 0 & \text{on } \Gamma := \partial\Omega \end{cases} \quad (1)$$

FEM in a nutshell (example on Poisson's equation)

strong form

Find $u : \Omega \to \mathbb{R}$

$$\begin{cases} -\Delta u = f & \text{in } \Omega \\ u = 0 & \text{on } \Gamma := \partial\Omega \end{cases} \tag{1}$$

FEM Recipe

1. Multiply Eq. (1) by a test function v
2. Integrate over Ω
3. Perform an integration by parts

FEM in a nutshell (example on Poisson's equation)

strong form

Find $u : \Omega \to \mathbb{R}$

$$\begin{cases} -\Delta u = f & \text{in } \Omega \\ u = 0 & \text{on } \Gamma := \partial\Omega \end{cases}$$

Find $u \in V := H_0^1(\Omega)$ $\quad \forall v \in V, \underbrace{\int_\Omega \nabla u \cdot \nabla v \,\mathrm{d}x}_{a(u,v)} = \underbrace{\int_\Omega f v \,\mathrm{d}x}_{l(v)}$

FEM in a nutshell (example on Poisson's equation)

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Find $u : \Omega \to \mathbb{R}$ $\quad \begin{cases} -\Delta u = f & \text{in } \Omega \\ u = 0 & \text{on } \Gamma := \partial\Omega \end{cases}$

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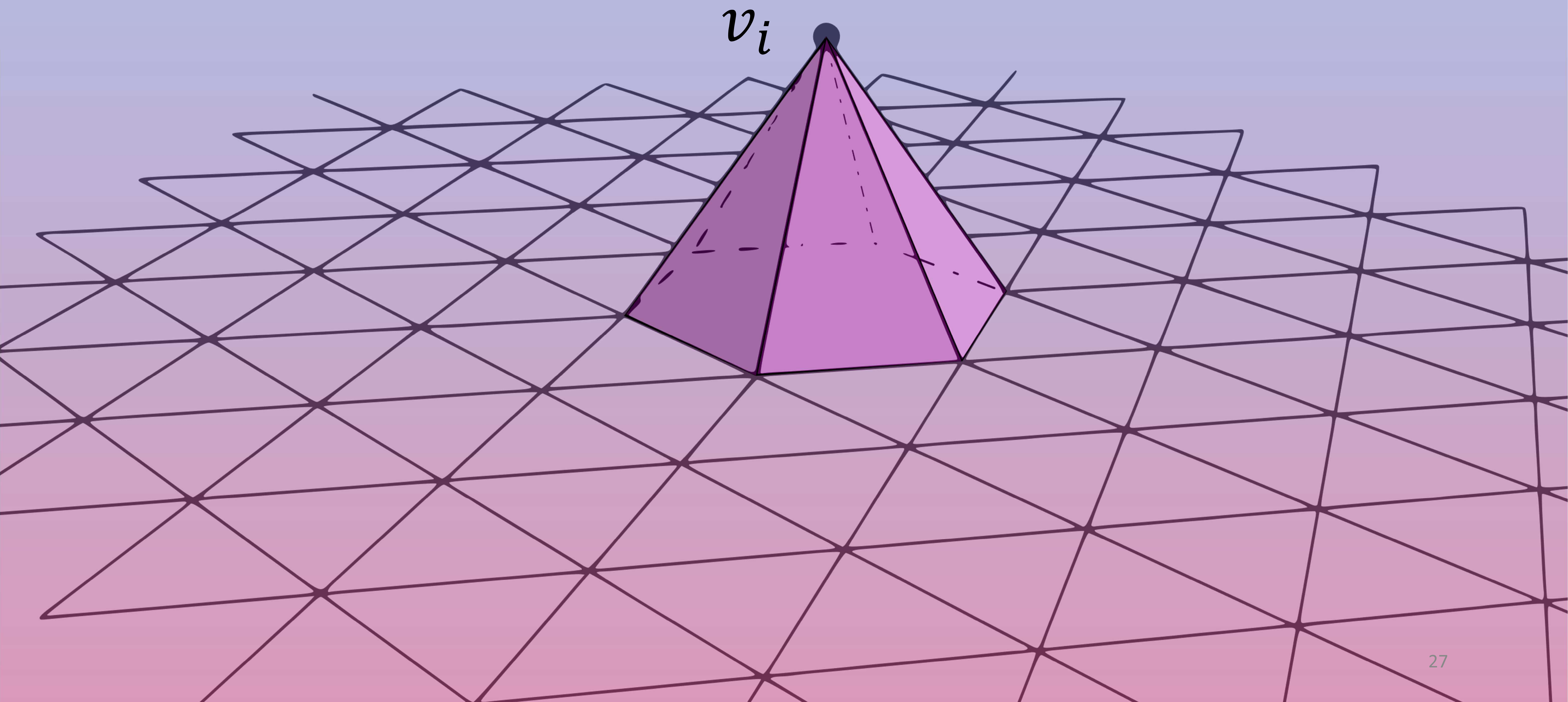
4. Look for u in a finite-dimensional subspace $V^h \subset V$ (e.g. space of piecewise polynomial functions)

The landscape of basis functions



FEM in a nutshell (example on Poisson's equation)

strong form

Find $u : \Omega \to \mathbb{R}$

$$\begin{cases} -\Delta u = f & \text{in } \Omega \\ u = 0 & \text{on } \Gamma := \partial\Omega \end{cases}$$

weak formulation

Find $u \in V := H_0^1(\Omega)$

$$\forall v \in V, \underbrace{\int_\Omega \nabla u \cdot \nabla v \, \mathrm{d}x}_{a(u,v)} = \underbrace{\int_\Omega f v \, \mathrm{d}x}_{l(v)}$$

continuous

Find $u_h \in V_h \subset V$

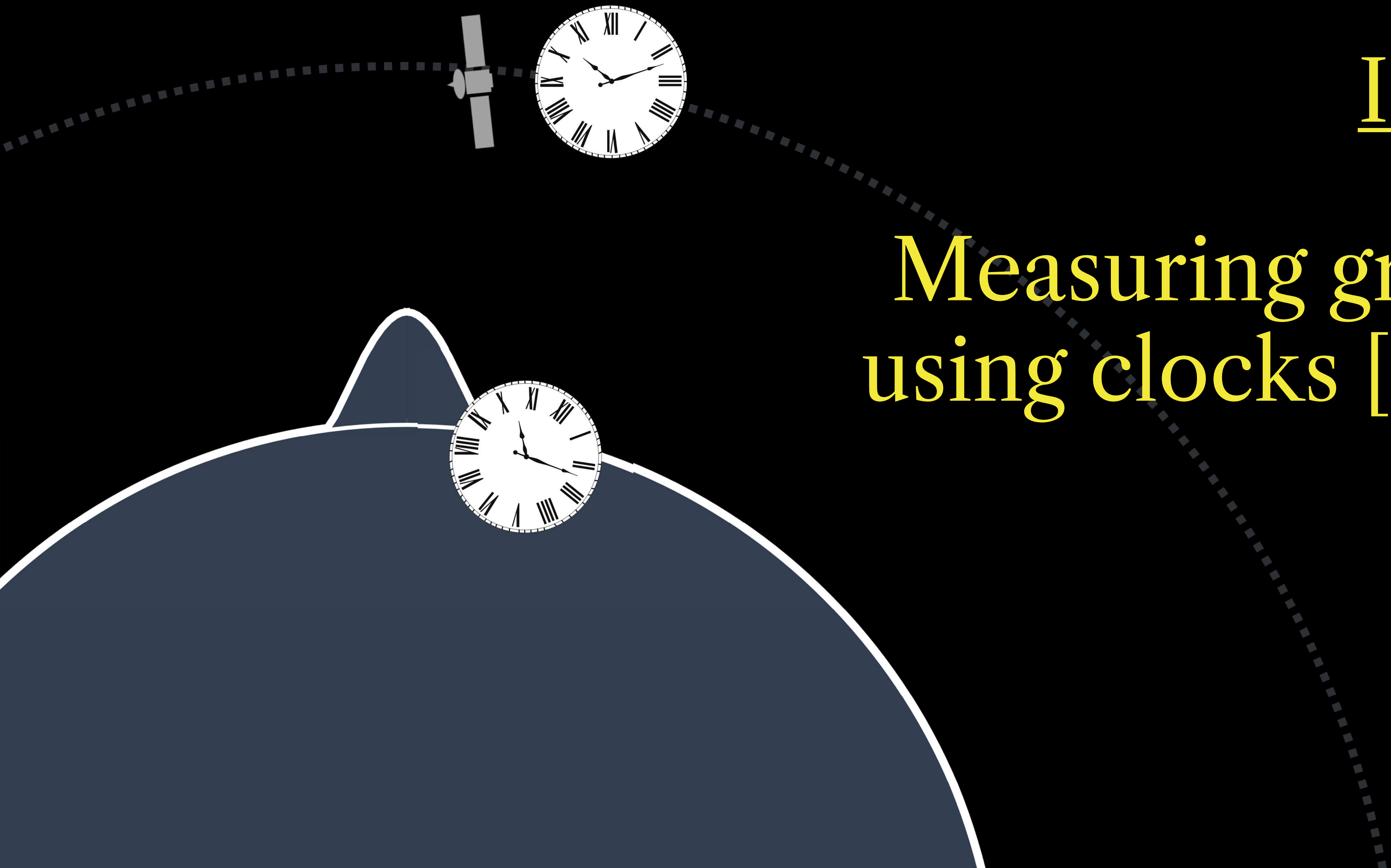
$$\forall i \in [\![1, N]\!] \; a(u_h, v_i) = l(v_i)$$

Solve $A\,U = L$

$$\begin{cases} A_{ij} = a(v_j, v_i) \\ L_i = l(v_i) \end{cases}$$

discrete

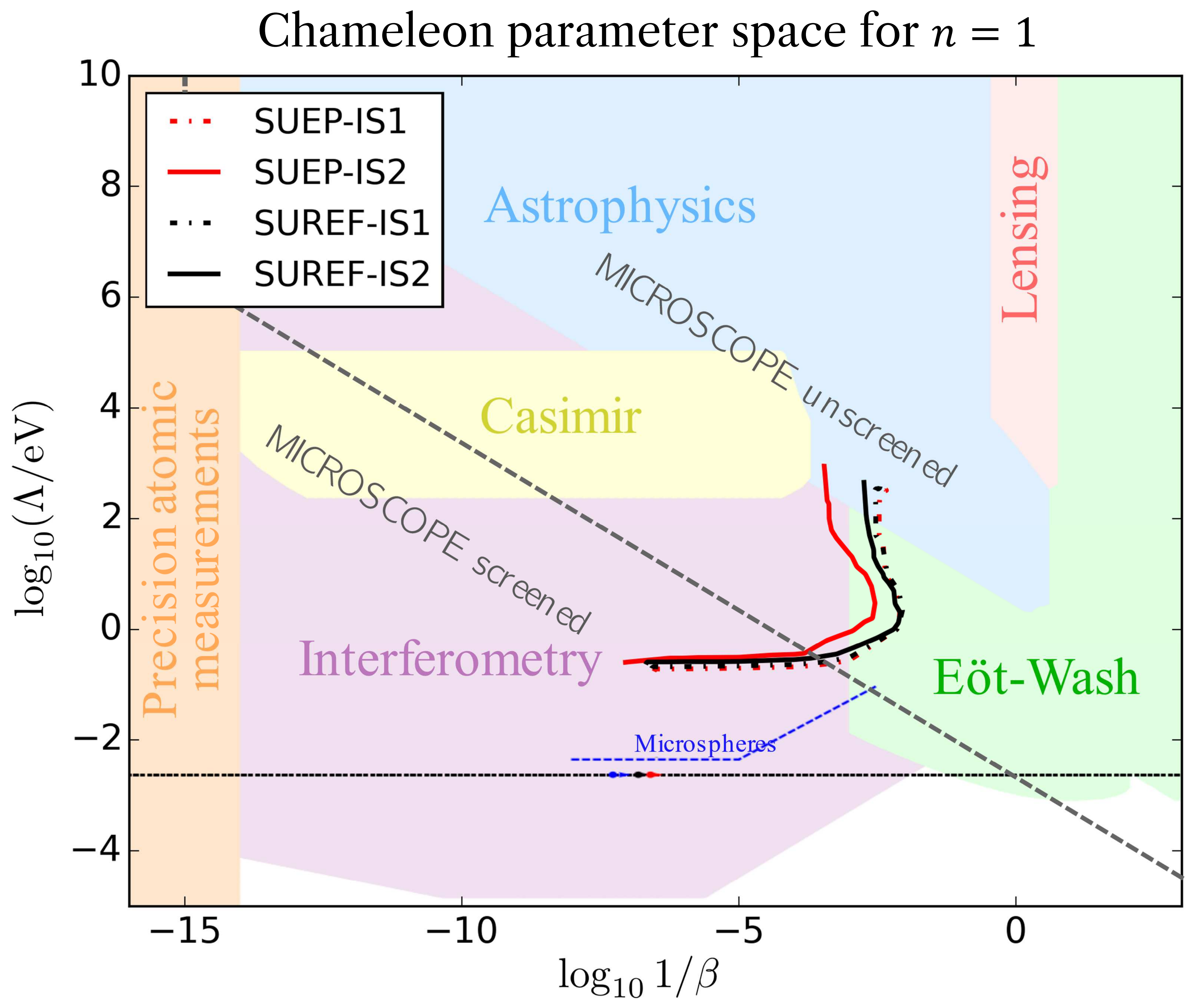
How to lift the degeneracy?



Idea n°2

Measuring gravitational redshift using clocks [under investigation]

Chameleon constraints from MICROSCOPE



M. Pernot-Borràs et al. (2019)

Chameleon constraints from MICROSCOPE

MICROSCOPE can do:

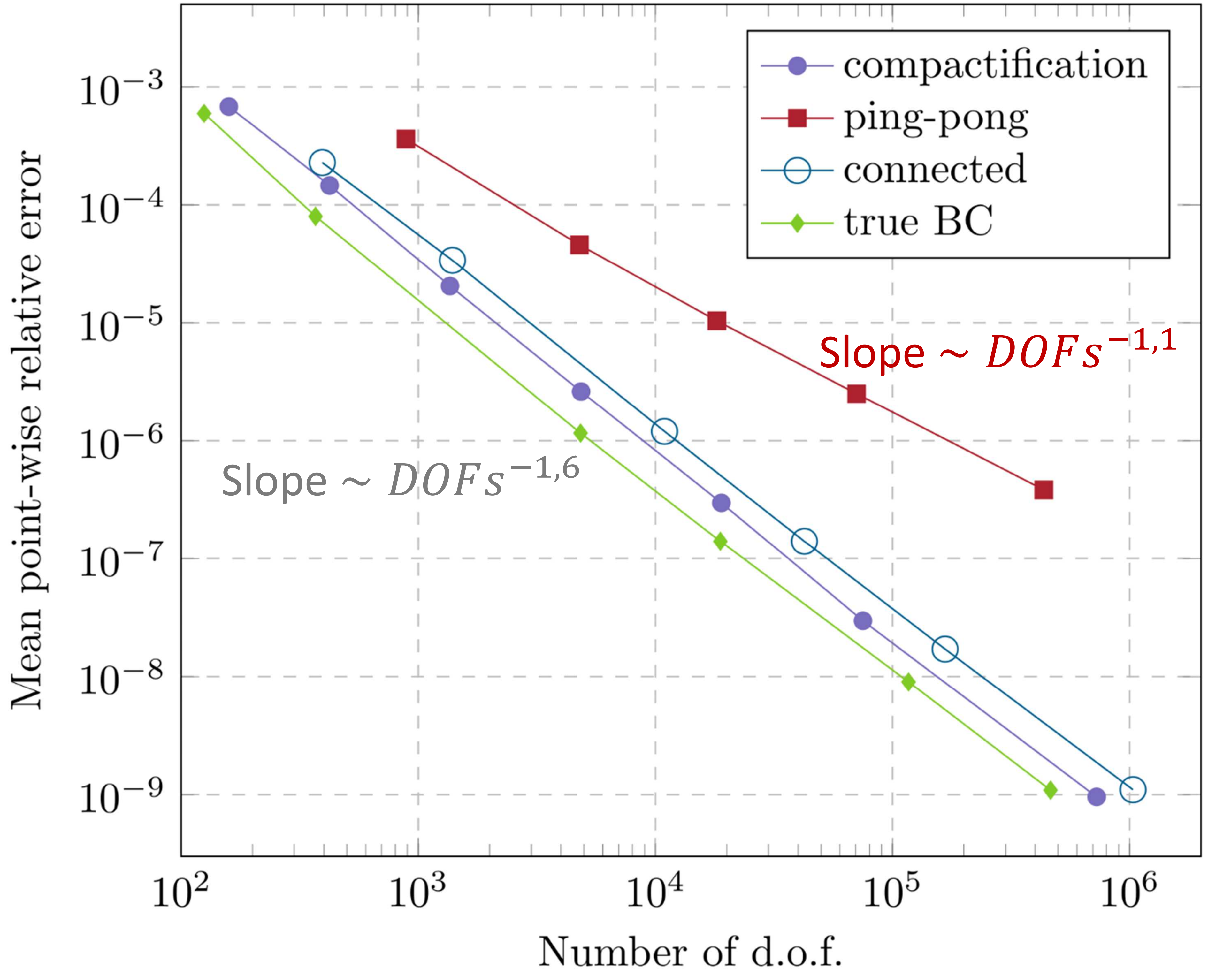
- weak equivalence principle [state of the art]
- generic long-range Yukawa 5^{th}-force [state of the art]
- light dilaton [competitive]
- Lorentz invariance [state of the art]

MICROSCOPE cannot do:

- generic short-range Yukawa 5^{th}-force [not competitive]
- chameleon 5^{th}-force [not competitive]

MICROSCOPE was not designed for testing short-ranged modified gravity theories. Recent work challenges the claim on the ability of space experiments to detect chameleon-sourced violations of the WEP sourced by the Earth [2, 3].

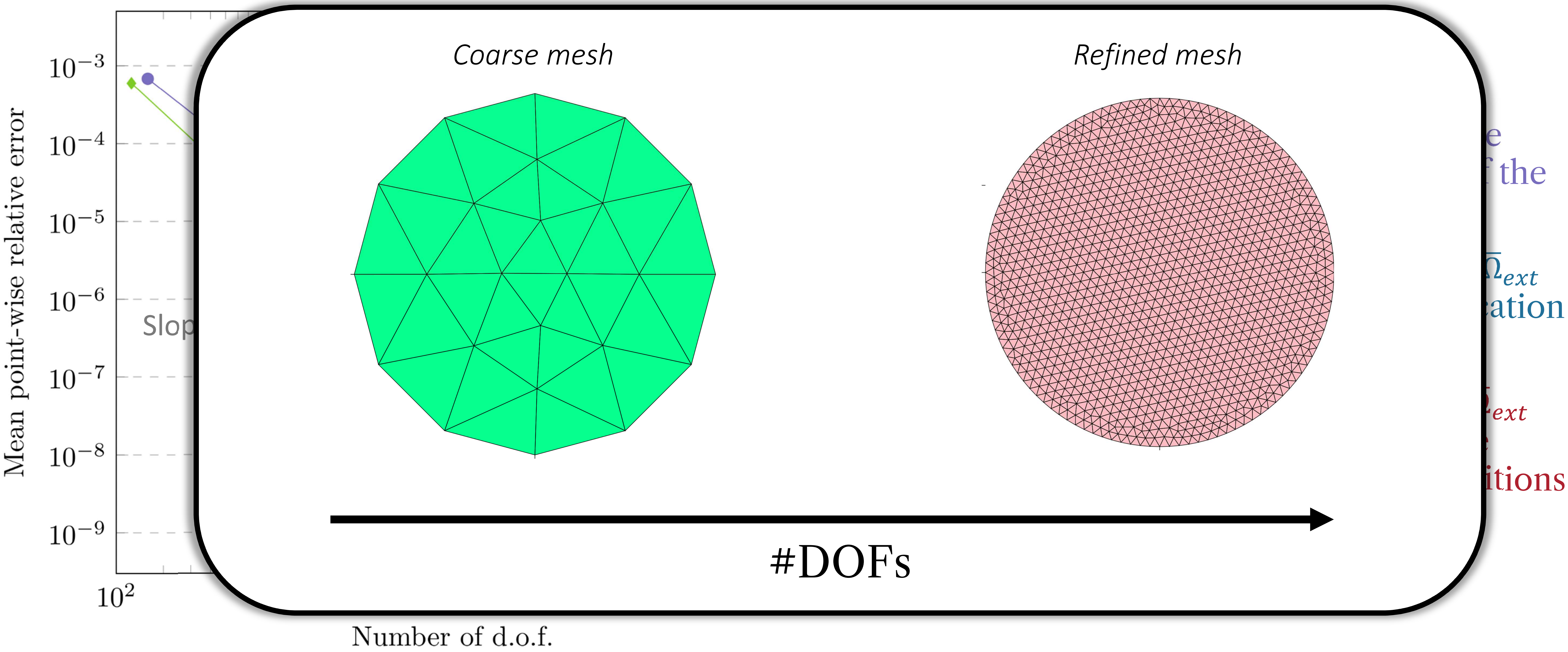
Convergence Analysis (FEM)



Implemented techniques

- Compactification : $\Omega \to \widetilde{\Omega}$ (global coordinate transform) + BC applied at the boundary of the compactified domain.
- "Connected" : domain splitting $\overline{\Omega} = \overline{\Omega}_{int} \cup \overline{\Omega}_{ext}$ and Kelvin inversion $\Omega_{ext} \to \widetilde{\Omega}_{ext}$ + identification of the boundary DOFs $\partial\Omega_{int} \equiv \partial\widetilde{\Omega}_{ext}$.
- "ping-pong" : domain splitting $\overline{\Omega} = \overline{\Omega}_{int} \cup \overline{\Omega}_{ext}$ and Kelvin inversion $\Omega_{ext} \to \widetilde{\Omega}_{ext}$ + iterative method with DtN / NtD transmission conditions at the boundary

Convergence Analysis (FEM)



Building blocks of Scalar-Tensor theories

Metric Tensor

$g_{\mu\nu}$

Einstein-Hilbert action in General Relativity

$$S_{EH} = \frac{M_{Pl}^2}{2} \int d^4x \sqrt{-g}\, R + \int d^4x \sqrt{-g}\, L_m\left(g_{\mu\nu}, \psi_m^{(i)}\right)$$

Building blocks of Scalar-Tensor theories

Metric Tensor
$g_{\mu\nu}$

+

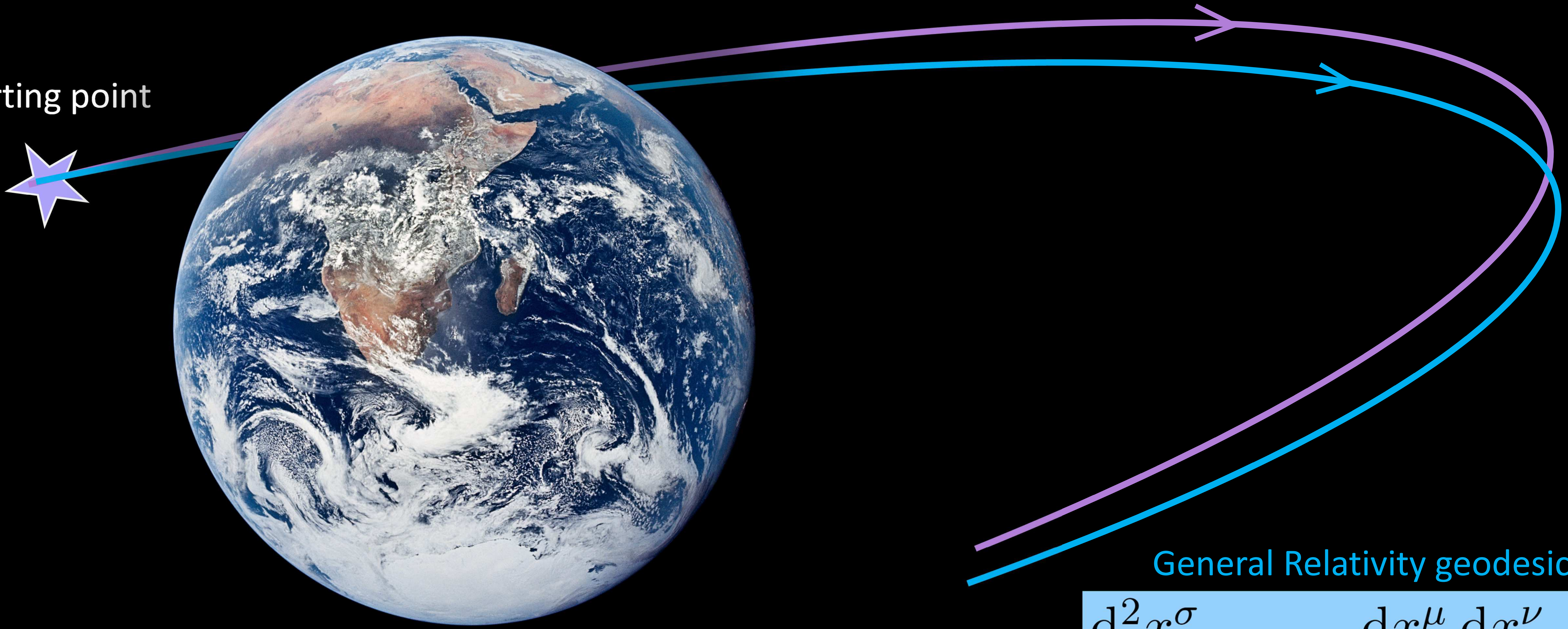
Scalar Field
ϕ

Modified action with a scalar field (example)

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{Pl}^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] + \int d^4x \sqrt{-\tilde{g}} \, L_m \left(\Omega^2(\phi) g_{\mu\nu}, \psi_m^{(i)} \right)$$

Modified Gravity geodesics

$$u^\mu \nabla_\mu u^\rho = -\frac{\partial \ln\Omega}{\partial \phi} \perp^{\mu\rho} \partial_\mu \phi$$



General Relativity geodesics

$$\frac{\mathrm{d}^2 x^\sigma}{\mathrm{d}s^2} + \Gamma^\sigma_{\mu\nu} \frac{\mathrm{d}x^\mu}{\mathrm{d}s} \frac{\mathrm{d}x^\nu}{\mathrm{d}s} = 0$$

$$\mathbf{F}_\phi = -m \frac{\partial \ln \Omega}{\partial \phi} \boldsymbol{\nabla} \phi$$

Modified Gravity geodesics

$$u^\mu \nabla_\mu u^\rho = -\frac{\partial \ln \Omega}{\partial \phi} \perp^{\mu\rho} \partial_\mu \phi$$

General Relativity geodesics

$$\frac{\mathrm{d}^2 x^\sigma}{\mathrm{d}s^2} + \Gamma^\sigma_{\mu\nu} \frac{\mathrm{d}x^\mu}{\mathrm{d}s} \frac{\mathrm{d}x^\nu}{\mathrm{d}s} = 0$$

Derivation can be found in M. Pernot-Borràs 2020, PhD thesis