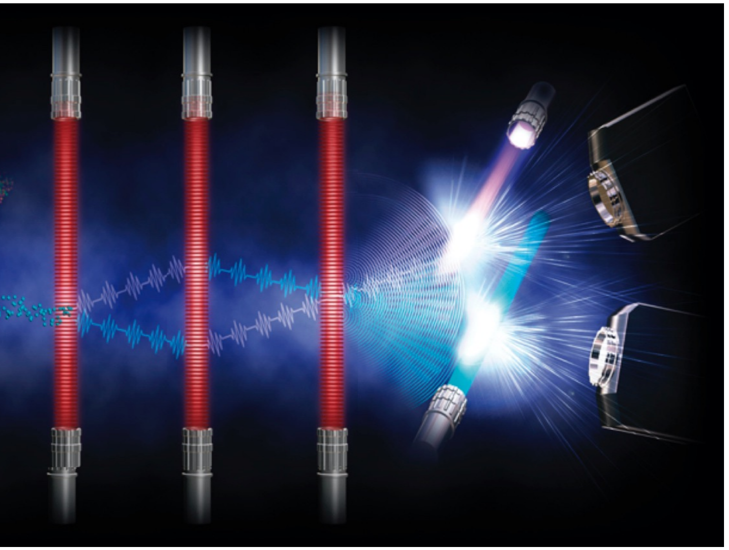
Violation of the equivalence principle induced by oscillating rest mass and transition frequency and its detection in atom interferometers

Jordan Gué, Aurélien Hees, Peter Wolf

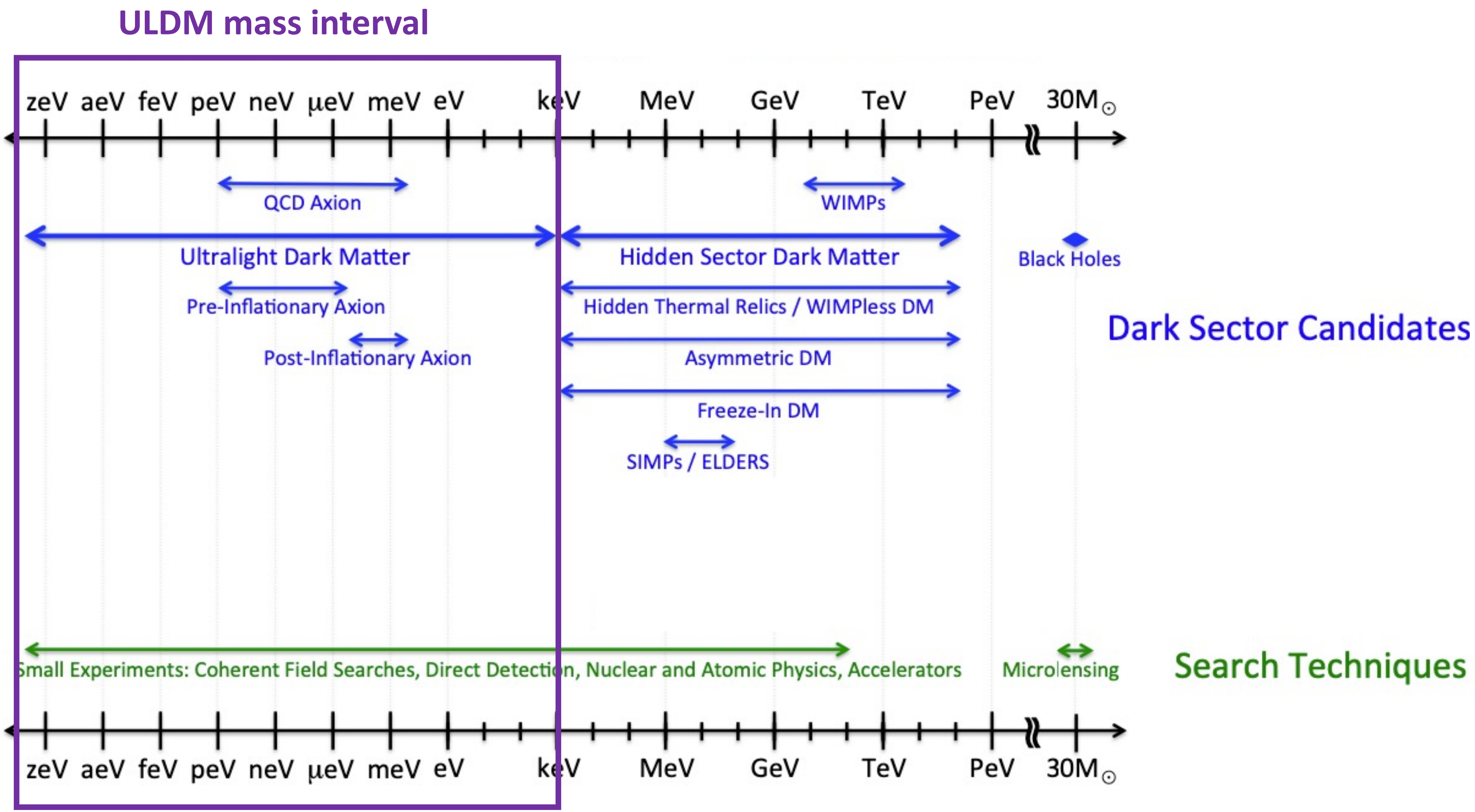
SYRTE, Observatoire de Paris



Credit: European Space Agency

TUG Workshop
October 11th 2023

Classes of DM



From US cosmic vision : new idea for Dark Matter 2017, Arxiv:1707:04951

Ultra Light Dark Matter (ULDM) models

$\sim 0.4\ GeV/cm^3$

Occupation number
in phase space →

$$\frac{n}{n_k} \sim \frac{\rho_{DM}}{(mc^2)^4 v_{max}^3} > \mathbf{1}$$

$\sim 10^{-3}c$

$< 10\ eV$

Inspired from P. Tourrenc et al, Arxiv:quantum-ph/0407187, 2004

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→ULDM with $mc^2 < 10\ eV$ must be bosonic (Pauli exclusion principle)

→Various bosonic ULDM candidates

- Scalar fields (Dilatons,...)
- Pseudo-scalar fields (Axions,...)
- Vector fields (Dark photons,...)

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- When $\boldsymbol{mc^2 \ll eV}$ $\rightarrow n/n_k \gg 1 \rightarrow$ a generic scalar field φ can be treated **classically**, i.e as oscillating solution of the Klein Gordon equation in FRLW expanding universe,

$$\boldsymbol{\varphi = \varphi_0 \cos(\omega_\varphi t)}$$

$\propto \sqrt{\rho_{DM}}$, the local DM energy density

$\hbar\omega_\varphi = m_\varphi c^2$ in DM rest frame

Phenomenology of ULDM (pseudo) scalar fields

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$$\mathcal{L}_\phi \supset \phi\left(\frac{d_e}{e^2}F^{\mu\nu}F_{\mu\nu} - \frac{d_g\beta_3}{2g_s}G^{\mu\nu}G_{\mu\nu} - \sum_{i=e,u,d}\left(d_{m_i} + \gamma_{m_i}d_g\right)m_i\bar{\psi}_i\psi_i\right)$$

T. Damour and T. Donoghue, PRD ***82****, 084033 (2010)*

$$\xrightarrow{\text{yields}} i(1 + d_i\phi), \quad i = \alpha, m_e, \widehat{m}, \delta m, \Lambda_3$$

Rest mass and transition frequency of an atom depend on i

$$m_A, \omega_A = f\left(\alpha, \frac{m_e}{m_p}, \frac{\widehat{m}}{\Lambda_3}, \frac{\delta m}{\Lambda_3}\right) \to f\left(\phi, d_e, d_j - d_g\right), \quad \mathrm{j} = m_e, \widehat{m}, \delta m$$

such that they oscillate as

$$m_A = m_A^0\left(1 + \phi_0\left[Q_M^{\mathrm{A}}\right]_\phi \cos\left(\omega_\phi t + \Phi\right)\right)$$

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Kim and Perez, Arxiv:2205.12988 (2022)

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Rest mass and transition frequency of an atom depend on m_π^2

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These oscillations induce an acceleration on the atom A

$$[\vec{a}_A]_{\overline{UFF}} \propto \omega_{osc}\vec{v}_A\left(Q_M^{\mathrm{A}}+\frac{\hbar\omega_A^0}{m_A^0c^2}Q_\omega^{\mathrm{A}}\right)\sin(\omega_{osc}t+\Phi)\,,\qquad (\omega_{osc}=\omega_\phi \text{ or } 2\omega_a)$$

which violates the UFF since the various $[Q]$ are atom dependent

→ Non-zero differential acceleration measurable in classical test of the UFF (MICROSCOPE) and in atom interferometry

P. Touboul et al. PRL ***129*** *121102 (2022)*

Atom interferometry (AI) : principle

Historically, use of optical interferometer (e.g Michelson for GW detection)

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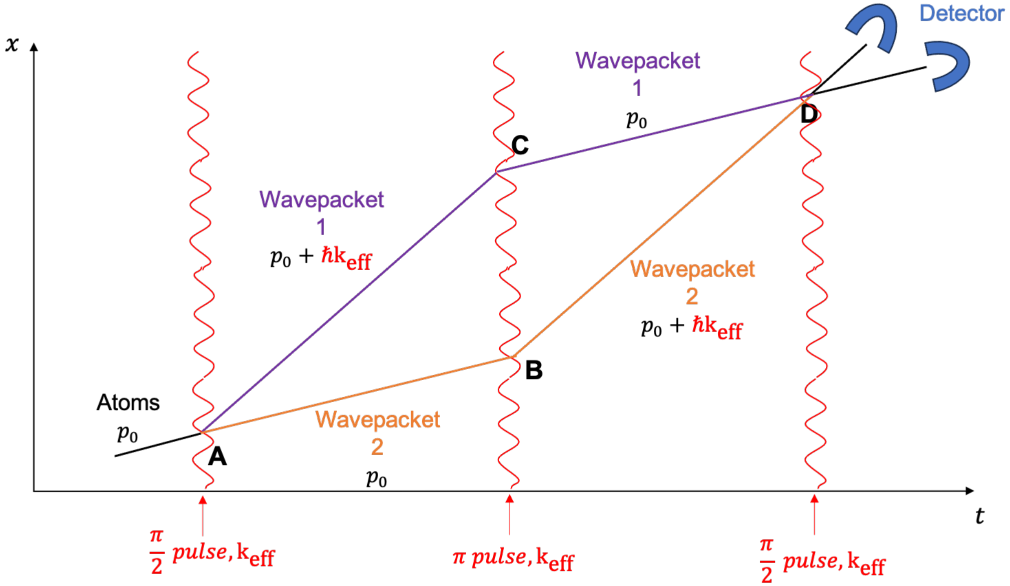

Beam splitters and mirrors replaced by light pulses.

→ $\frac{\pi}{2}$ pulse separates atoms in 2 wavepackets

≡ Beam splitter

→ π pulse inverts state

≡ Mirror

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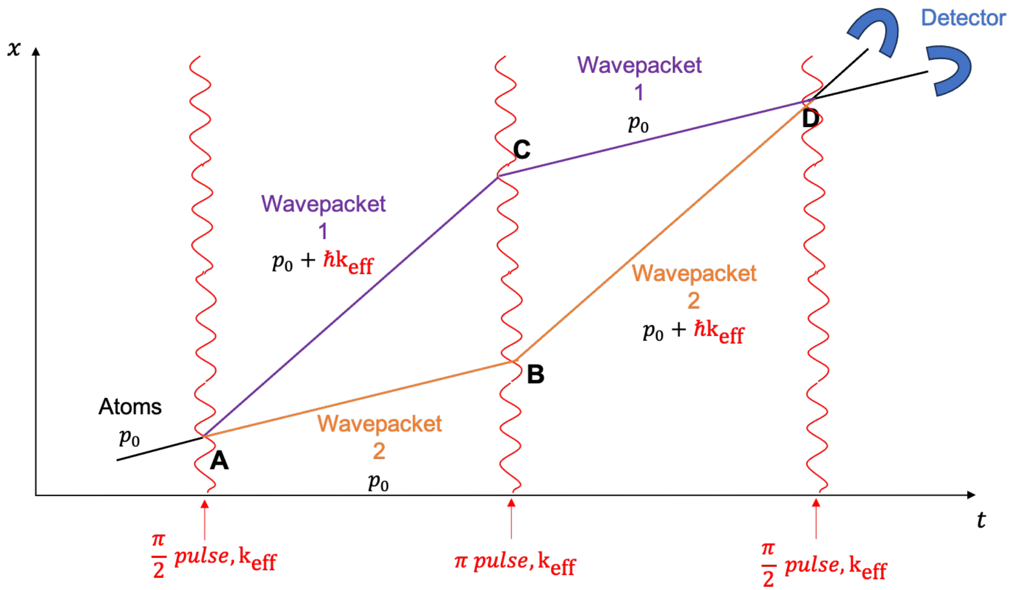

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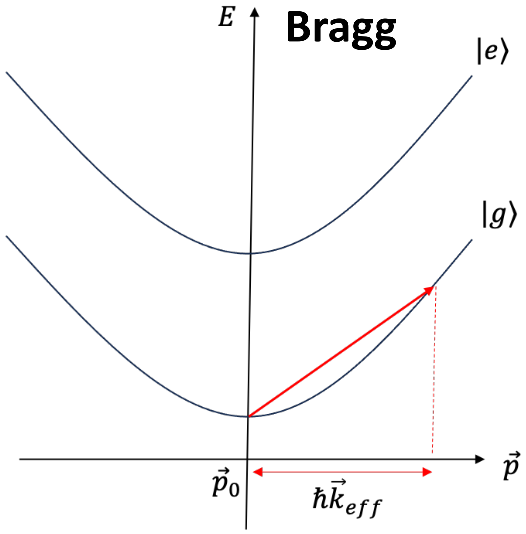

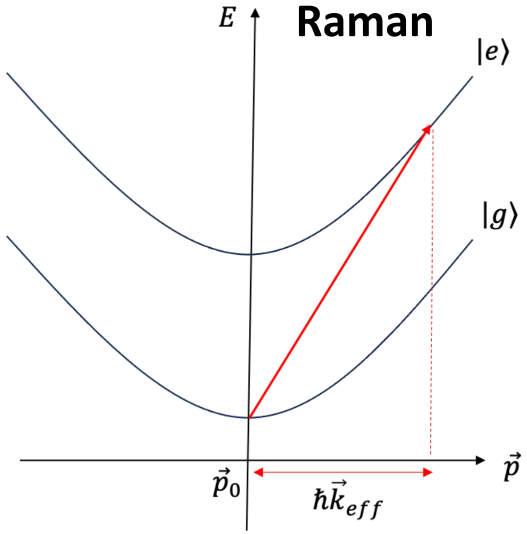

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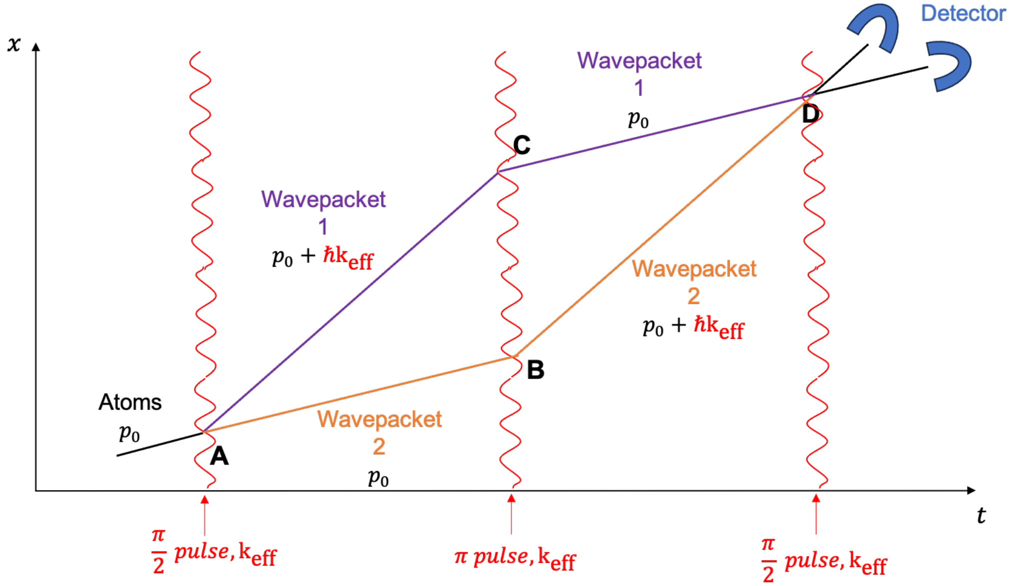

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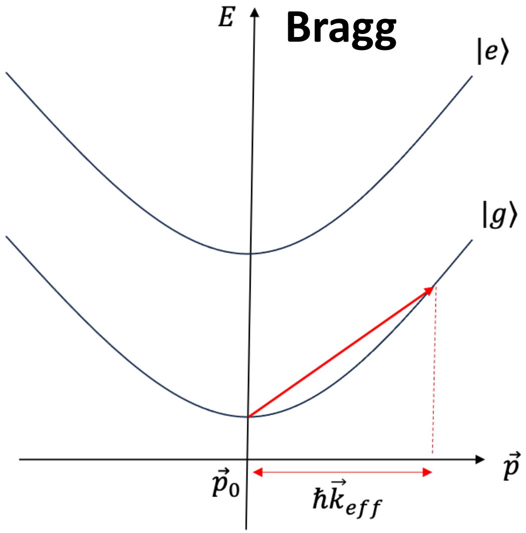
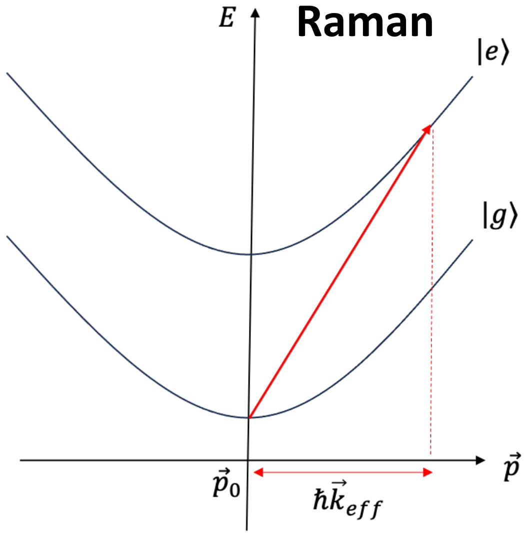


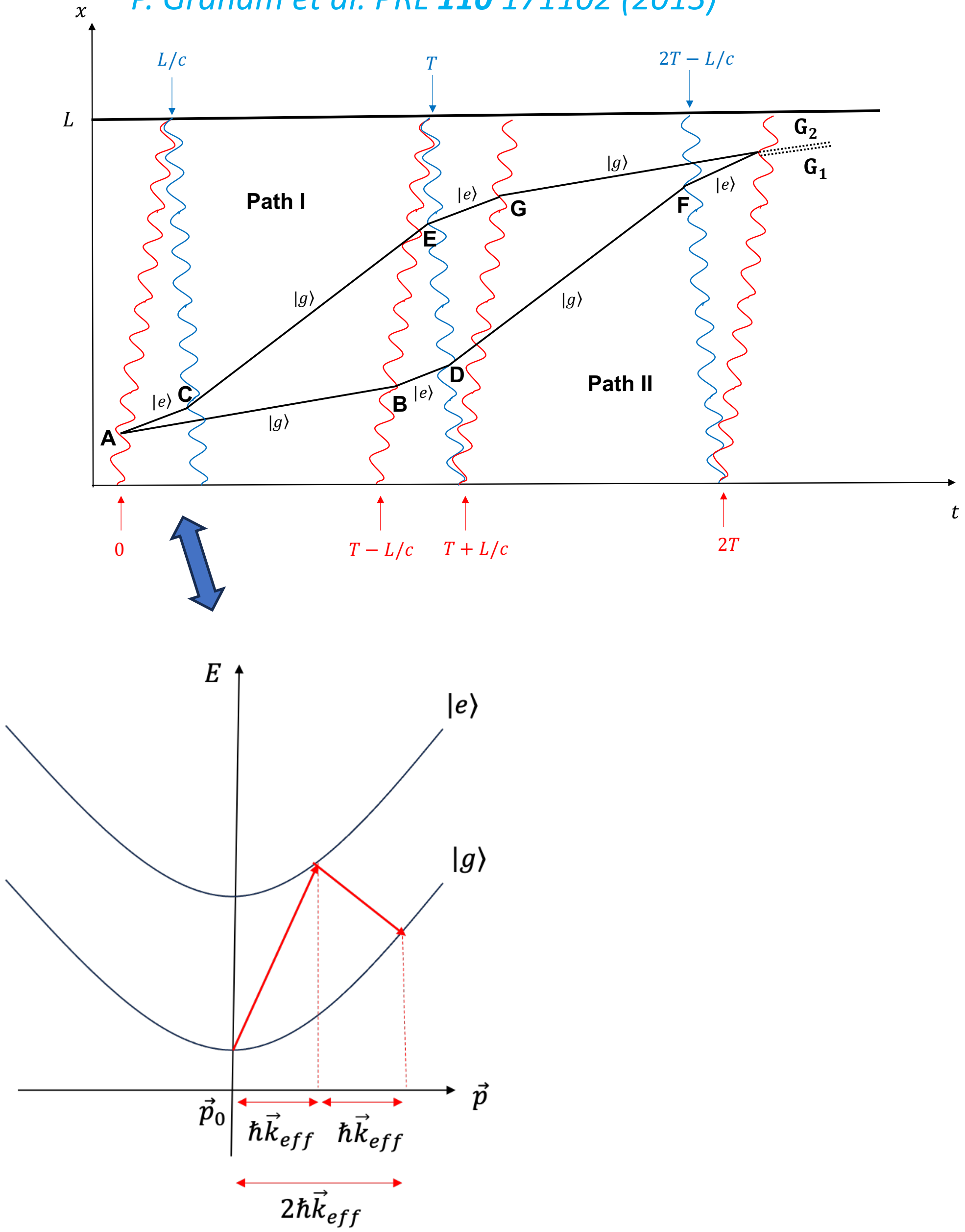


These $\left(\frac{\pi}{2} - \pi - \frac{\pi}{2}\right)$ setup are vastly used, e.g in the Stanford tower experiment or in Wuhan gravimeter experiment

*P. Asenbaum et al. PRL **125** 191101 (2020)*   *Z. Hu et al. PRA **88** 043610 (2020)*

Single photon transition gradiometers : AION-10

*P. Graham et al. PRL **110** 171102 (2013)*



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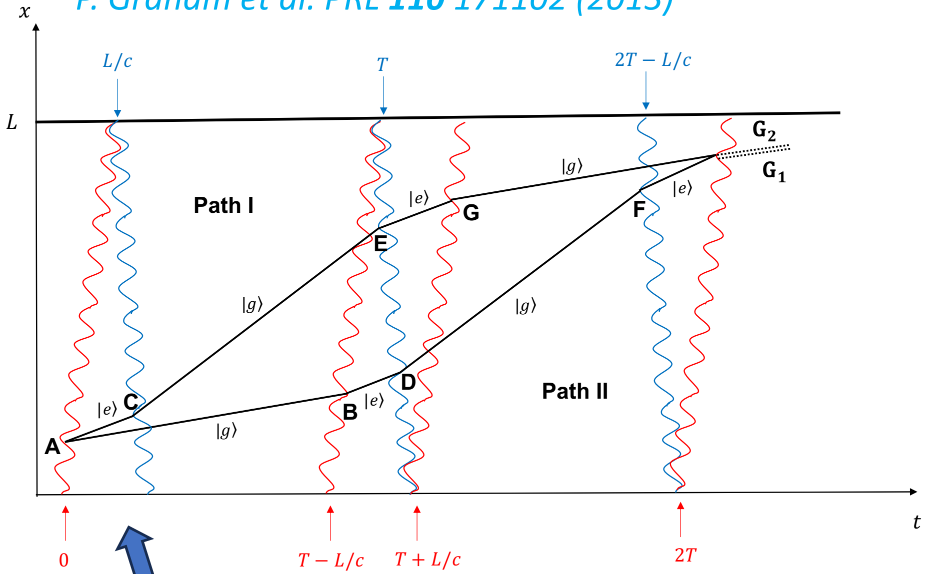
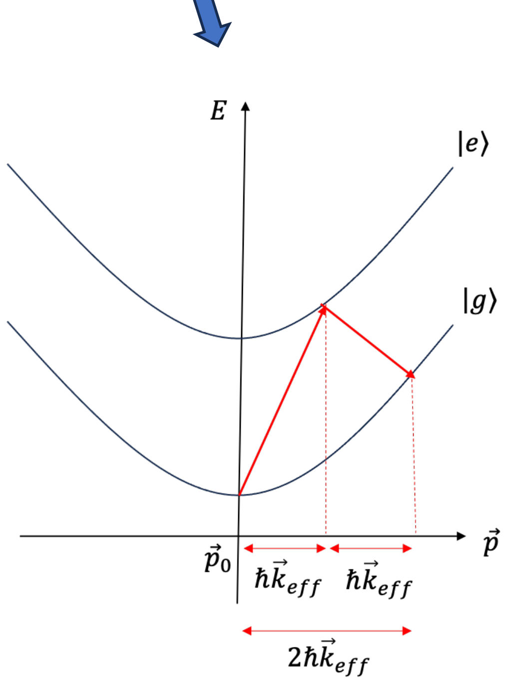
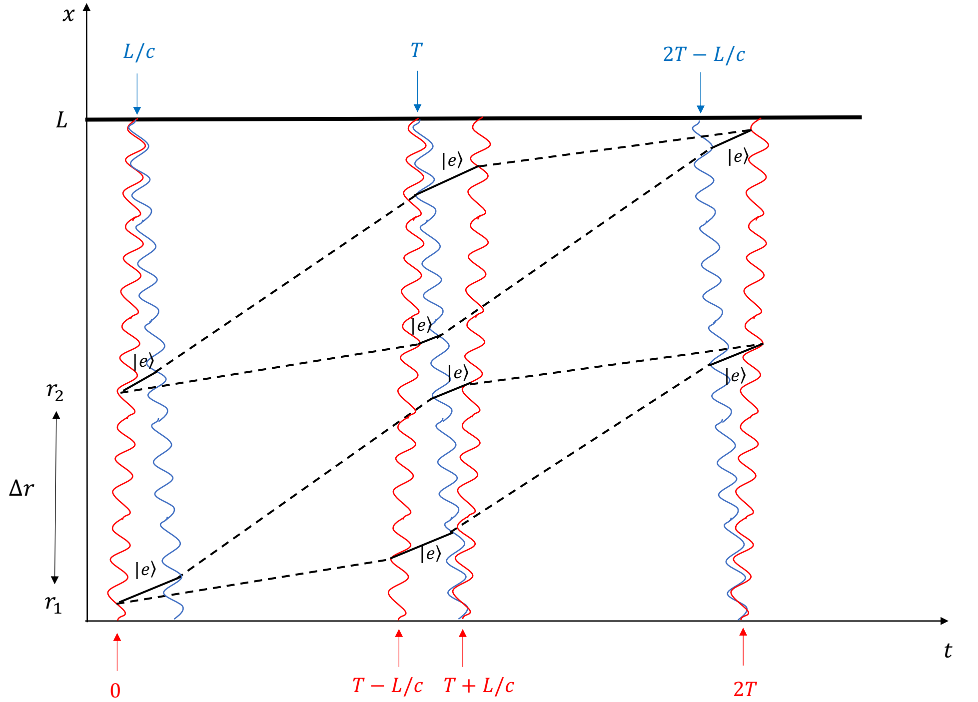




AION-10
(futuristic gradiometer)



*L. Badurina et al. PRD **105** 023006 (2022)*

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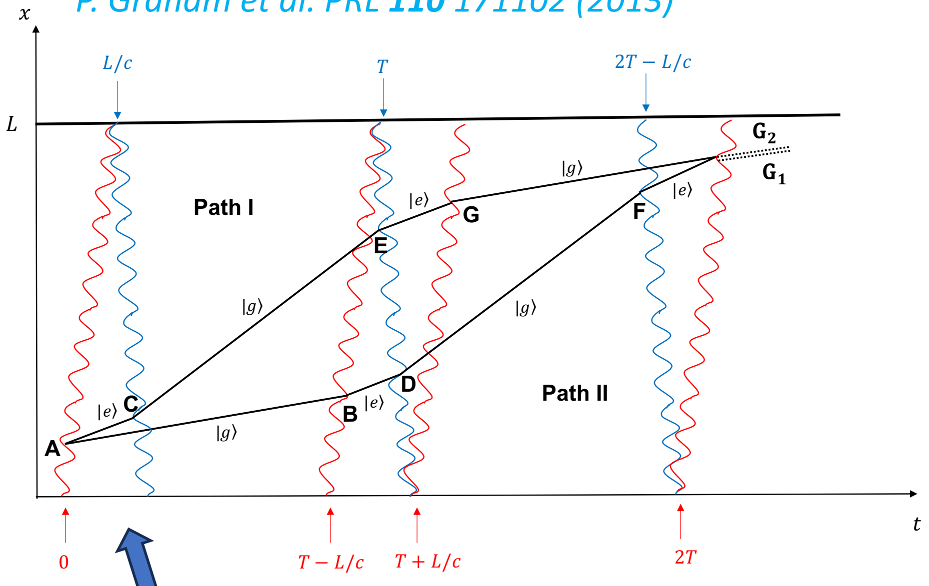

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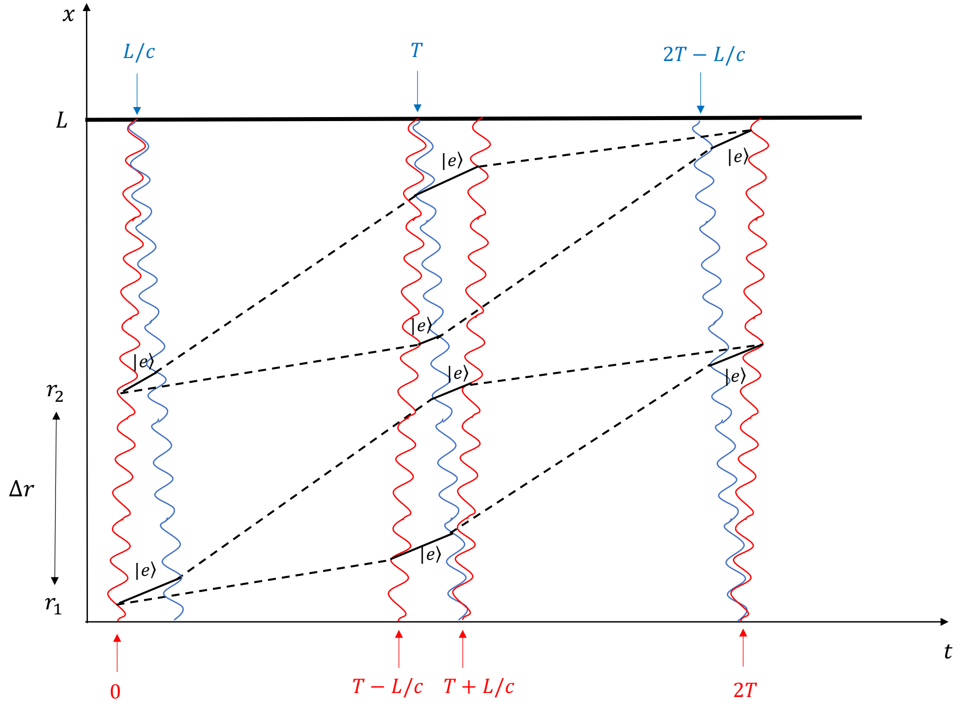

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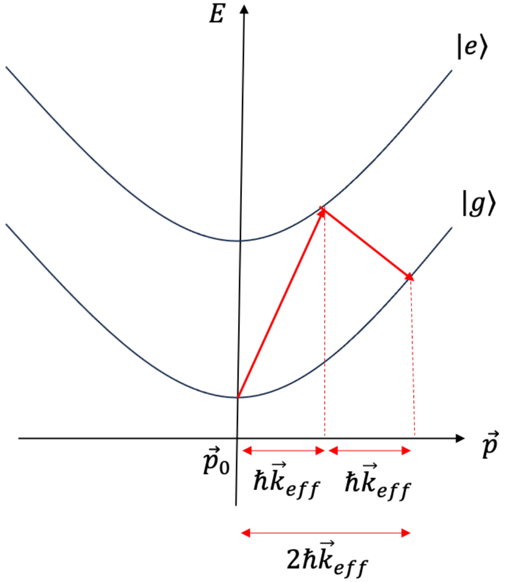

Same atom species undergoes the same AI sequence at different elevation.
Same laser beams interact with both AI → **NO** laser phase noise

$$\begin{cases} \Delta\phi_A^{Grad} \propto \rho_{DM}\left[Q_\omega^{\mathrm{A}}\right]_a \dfrac{\boldsymbol{\Delta r}}{\boldsymbol{c}} \dfrac{\sin^2(\omega_a T)}{\omega_a^2} \\ \Delta\phi_A^{Grad} \propto \sqrt{\rho_{DM}}\left[Q_\omega^{\mathrm{A}}\right]_\phi \dfrac{\boldsymbol{\Delta r}}{\boldsymbol{c}} \dfrac{1}{\omega_\phi} \sin^2\left(\dfrac{\omega_\phi T}{2}\right) \end{cases}$$

AION-10's signal limited by the time the atom lasts in the excited state ($\sim\boldsymbol{\Delta r/c} \ll 1$)

Can we do better ? Modified setup : “SPID"

Use of two isotope species with sequence based on previous AI setup but **at same elevation** → Single Photon transition Isotope Differential “SPID” AI

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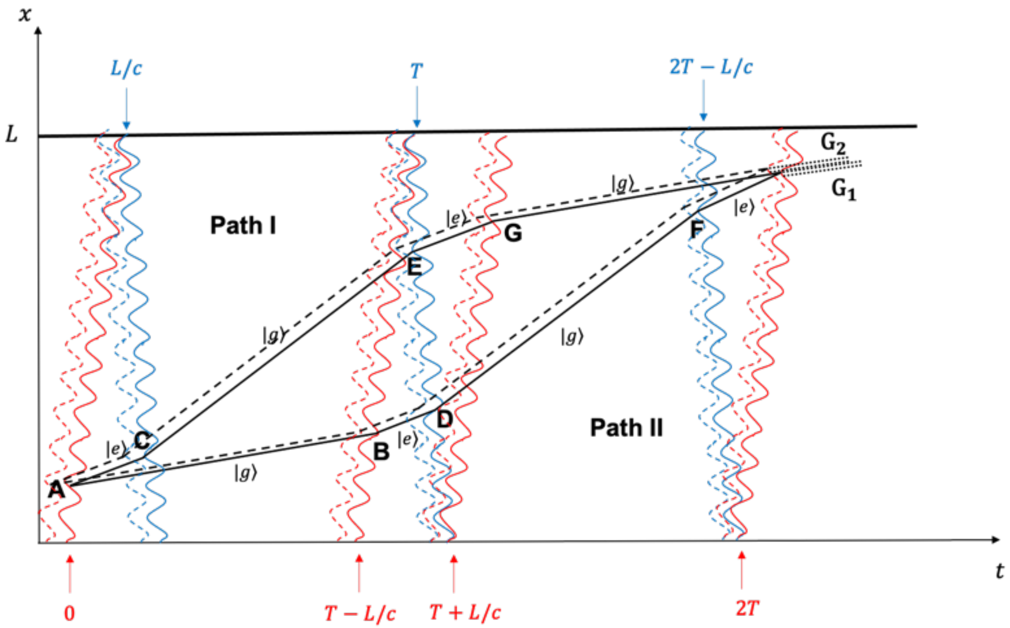

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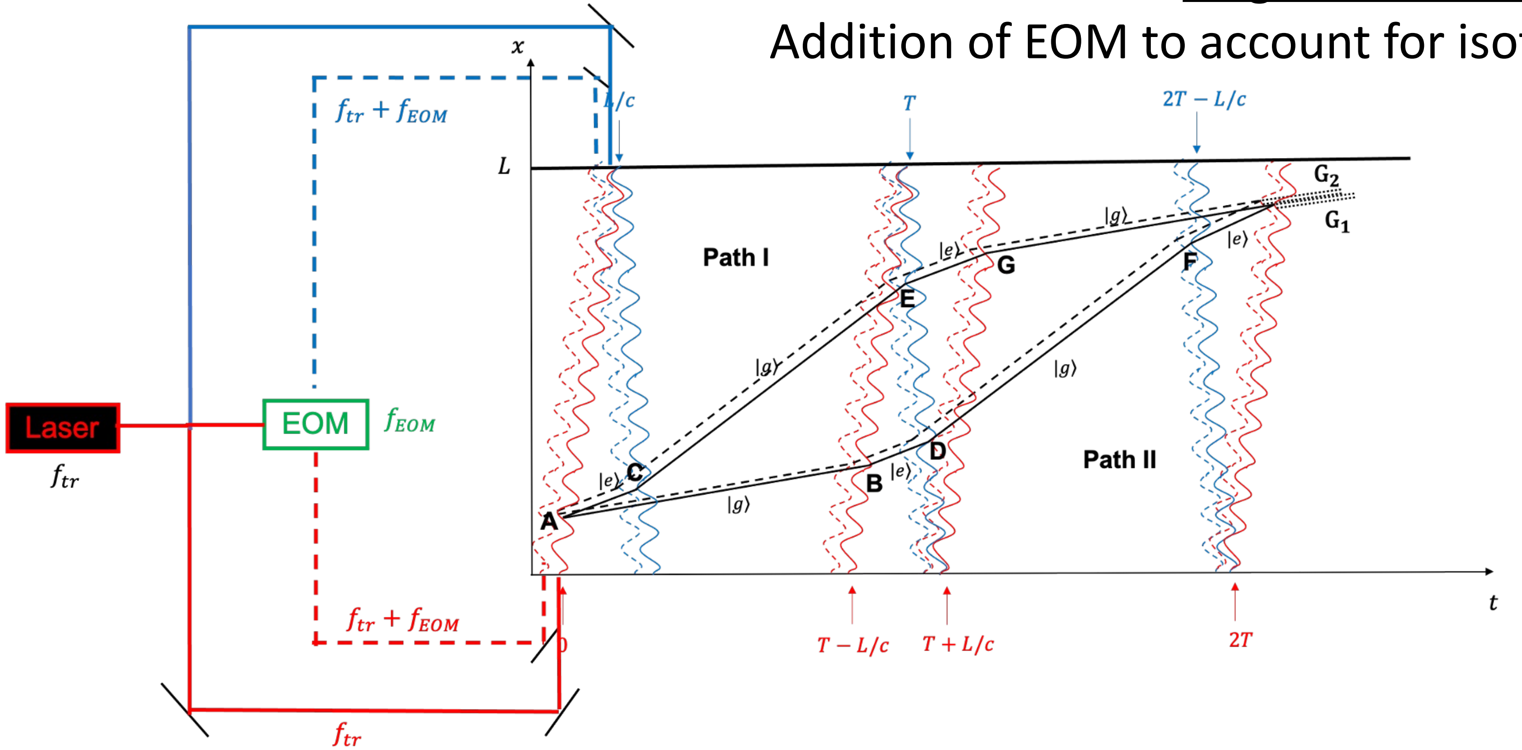
Addition of EOM to account for isotope shift $\Delta\omega \sim 10^{-6}\ \omega_0$



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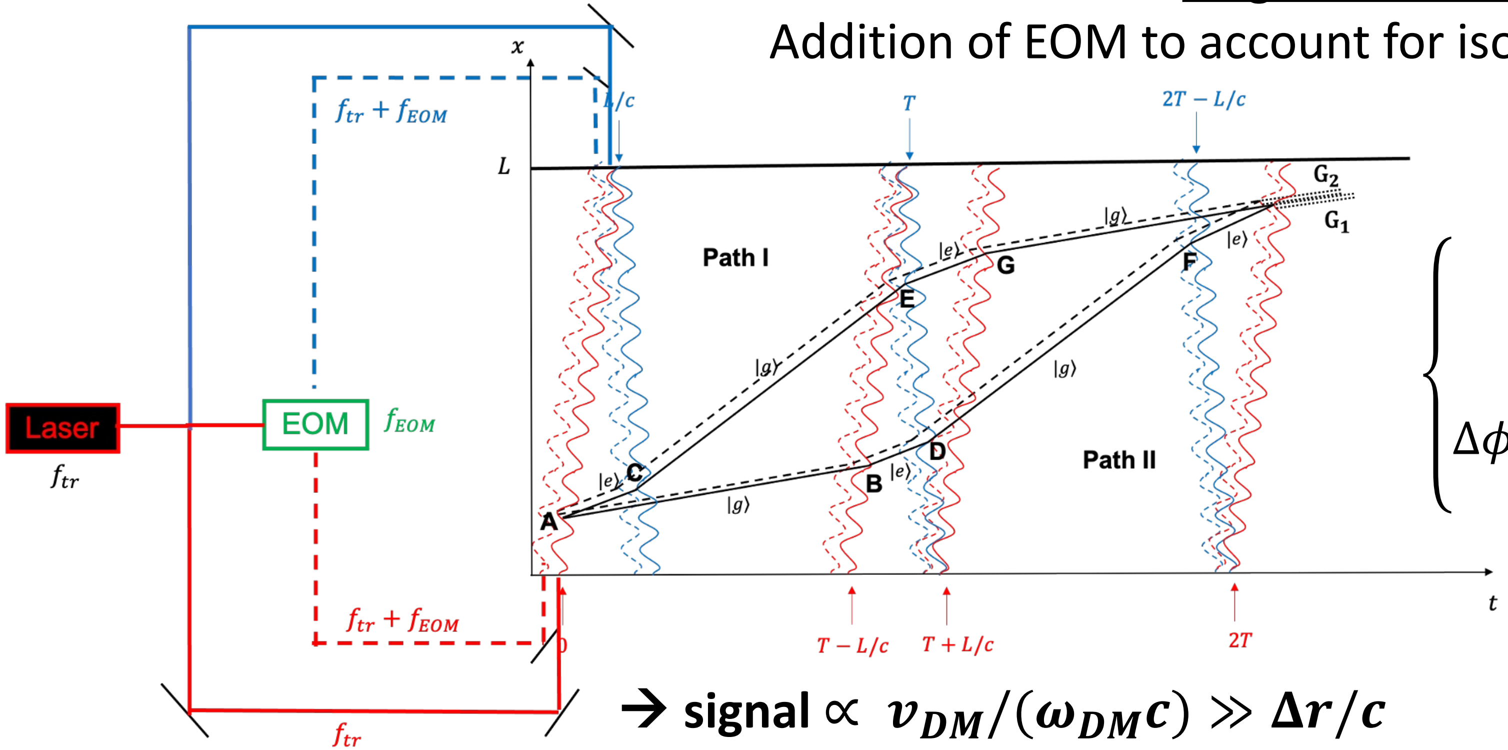


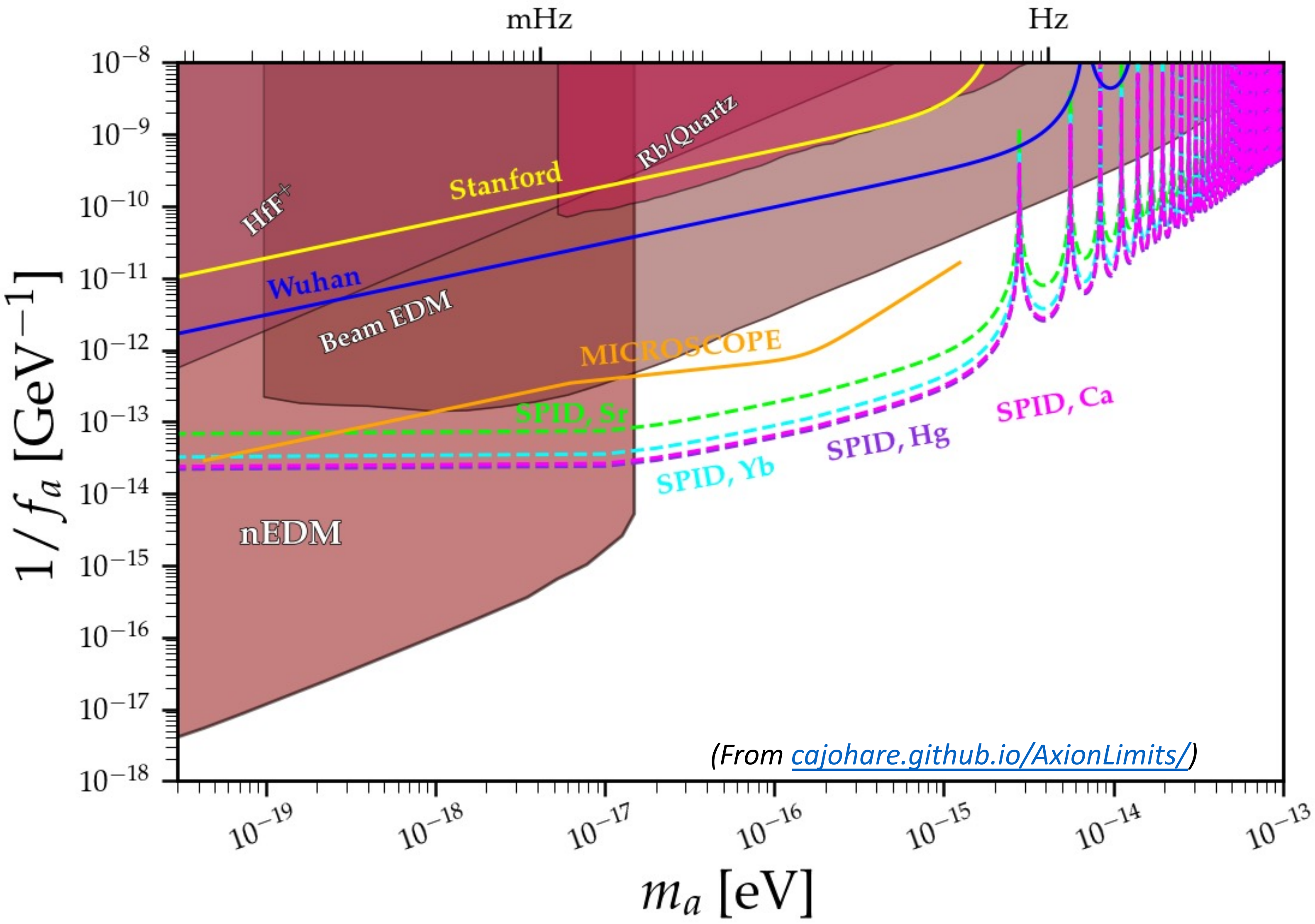
$$\begin{cases} \Delta\phi_{AB}^{SPID} \propto \dfrac{\rho_{DM} v_{DM}}{\omega_a\, c}\left(\left[Q_M^{\mathrm{A}}\right]_a - \left[Q_M^{\mathrm{B}}\right]_a\right)\dfrac{\sin^2(\omega_a T)}{\omega_a^2} \\ \Delta\phi_{AB}^{SPID} \propto \dfrac{\sqrt{\rho_{DM}}\ v_{DM}}{\omega_\phi\, c}\dfrac{1}{\omega_\phi}\left(\left[Q_M^{\mathrm{A}}\right]_\phi - \left[Q_M^{\mathrm{B}}\right]_\phi\right)\sin^2\left(\dfrac{\omega_\phi T}{2}\right) \end{cases}$$

→ **signal** ∝ $\boldsymbol{v_{DM}/(\omega_{DM} c) \gg \Delta r/c}$
However, EOM comes with an additional noise…

In principle, this setup can be realized using any isotope pair of neutral atoms with stable optical transition, e.g $\boldsymbol{({}^{88}_{38}Sr, {}^{86}_{38}Sr); ({}^{40}_{20}Ca, {}^{44}_{20}Ca); ({}^{171}_{70}Yb, {}^{176}_{70}Yb); ({}^{196}_{80}Hg, {}^{202}_{80}Hg)}$

→ With the same experimental parameters as AION-10, is SPID more sensitive to axion and dilaton couplings ?

Results on axion-gluon coupling



(From cajohare.github.io/AxionLimits/)

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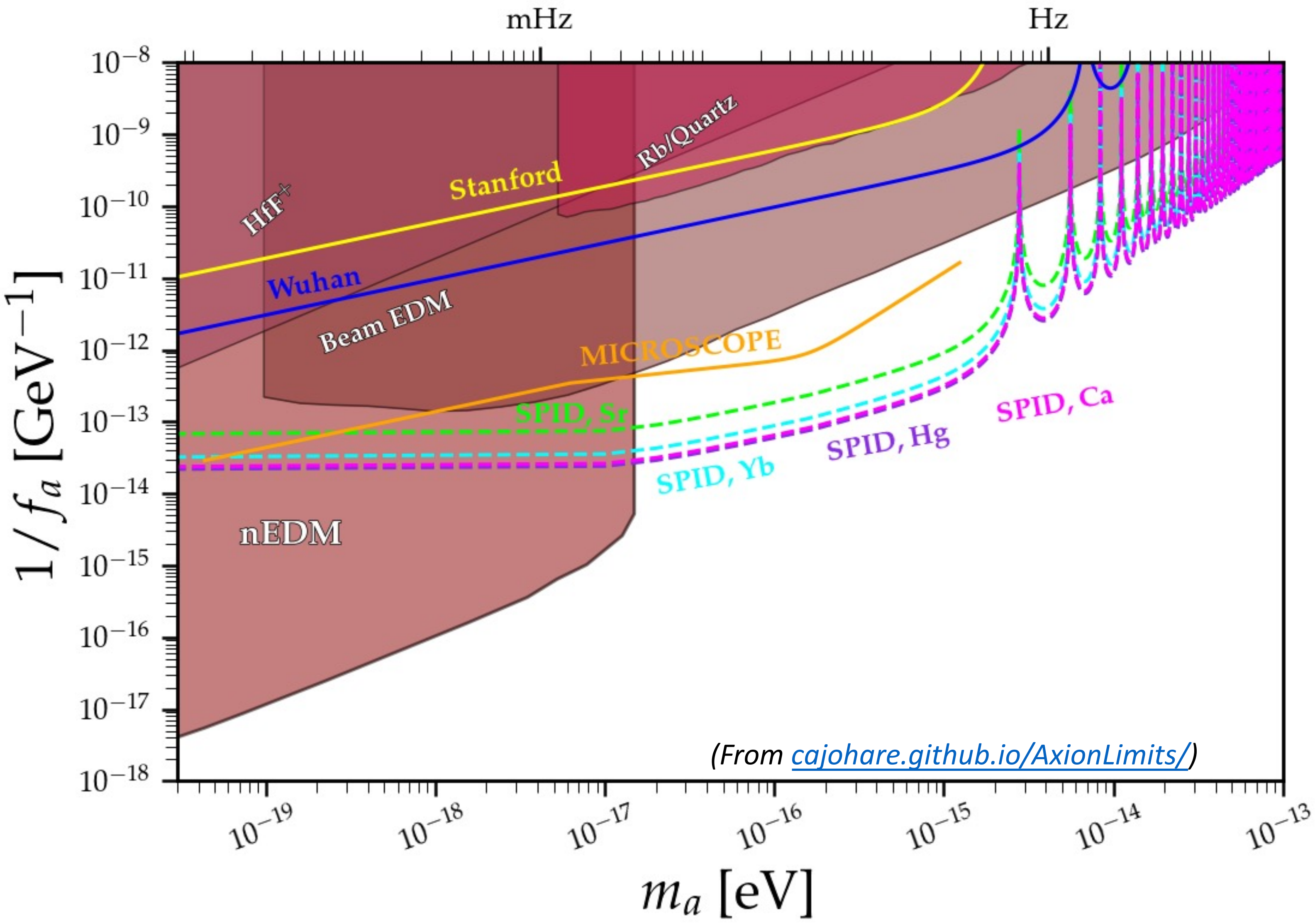

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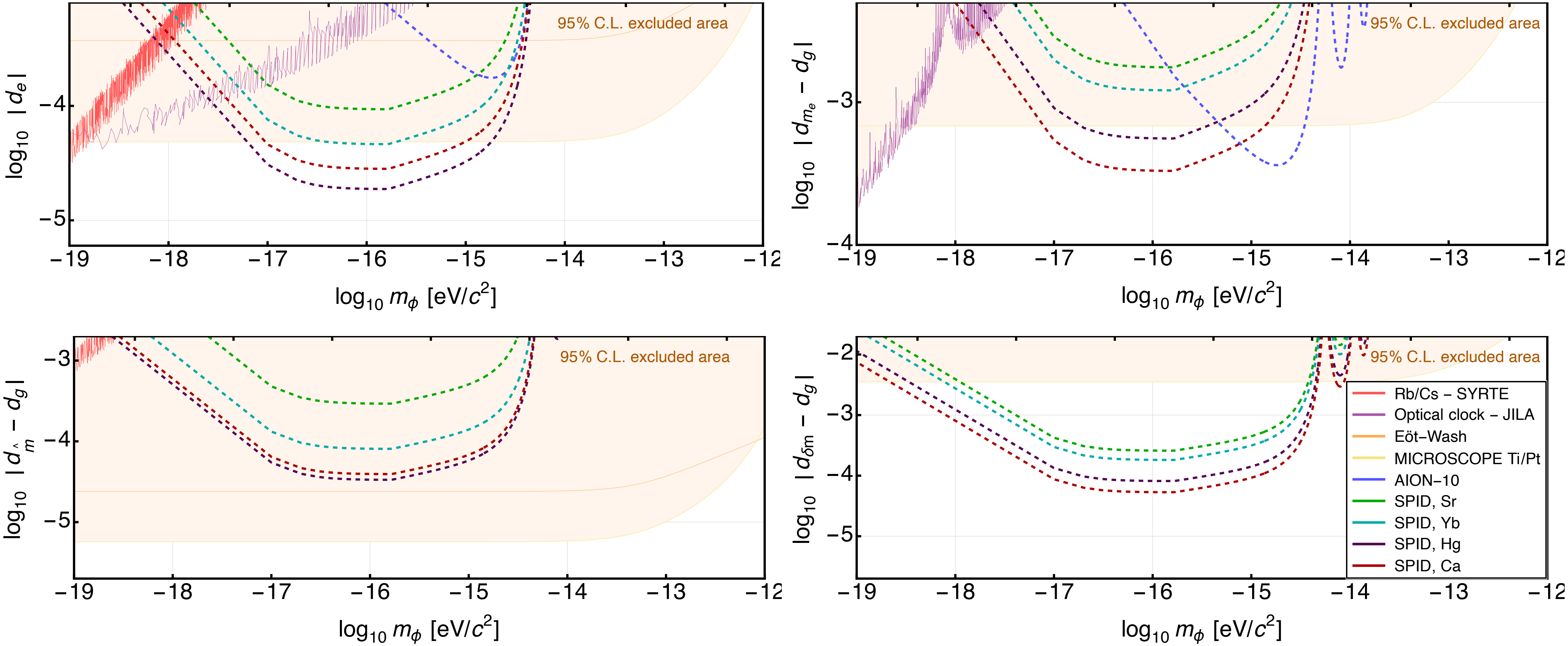
→ MICROSCOPE and SPID experiment would provide the best laboratory constraints on the axion-gluon coupling over multiple orders of magnitude of mass

Results on dilaton couplings



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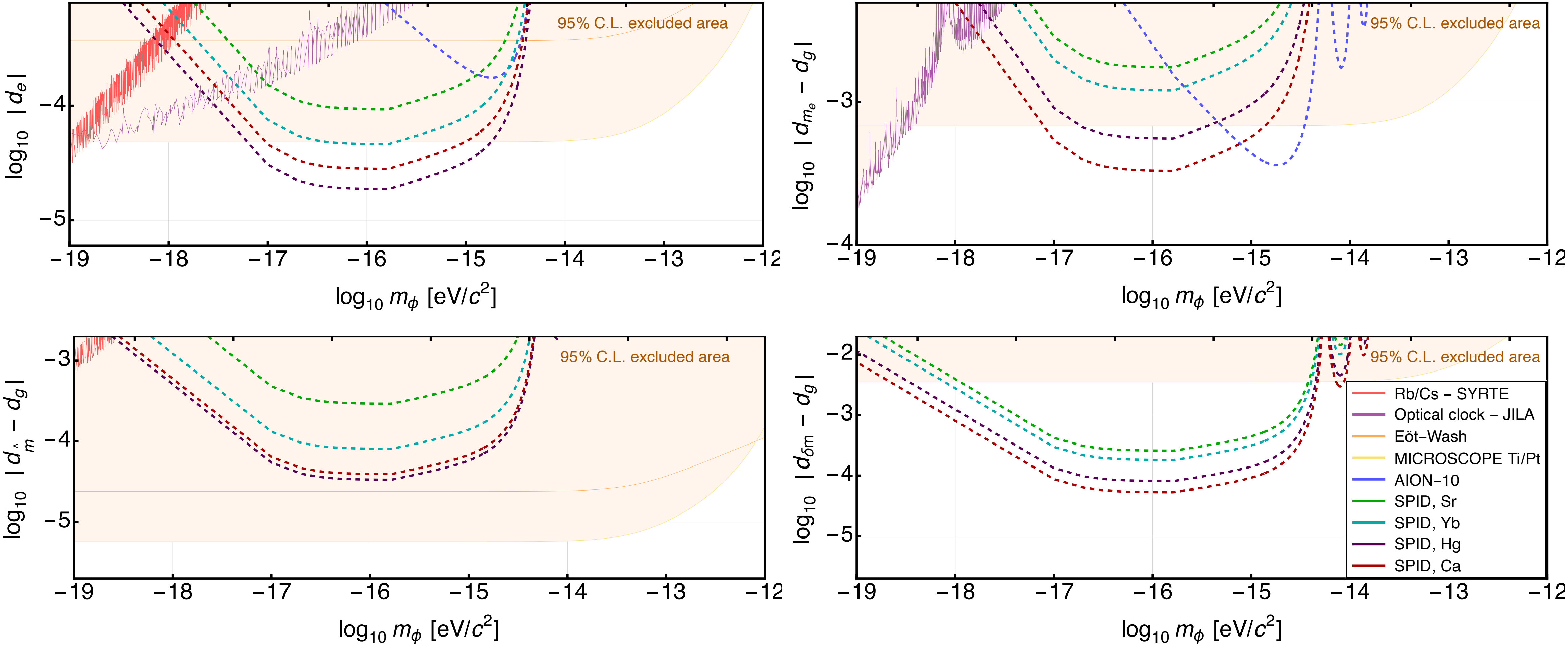

$$\phi(r,t) = \phi_0 \cos(\omega_\phi t) + \left[Q_M^C\right]_\phi \frac{GM_C}{rc^2} e^{-r/\lambda_\phi}$$

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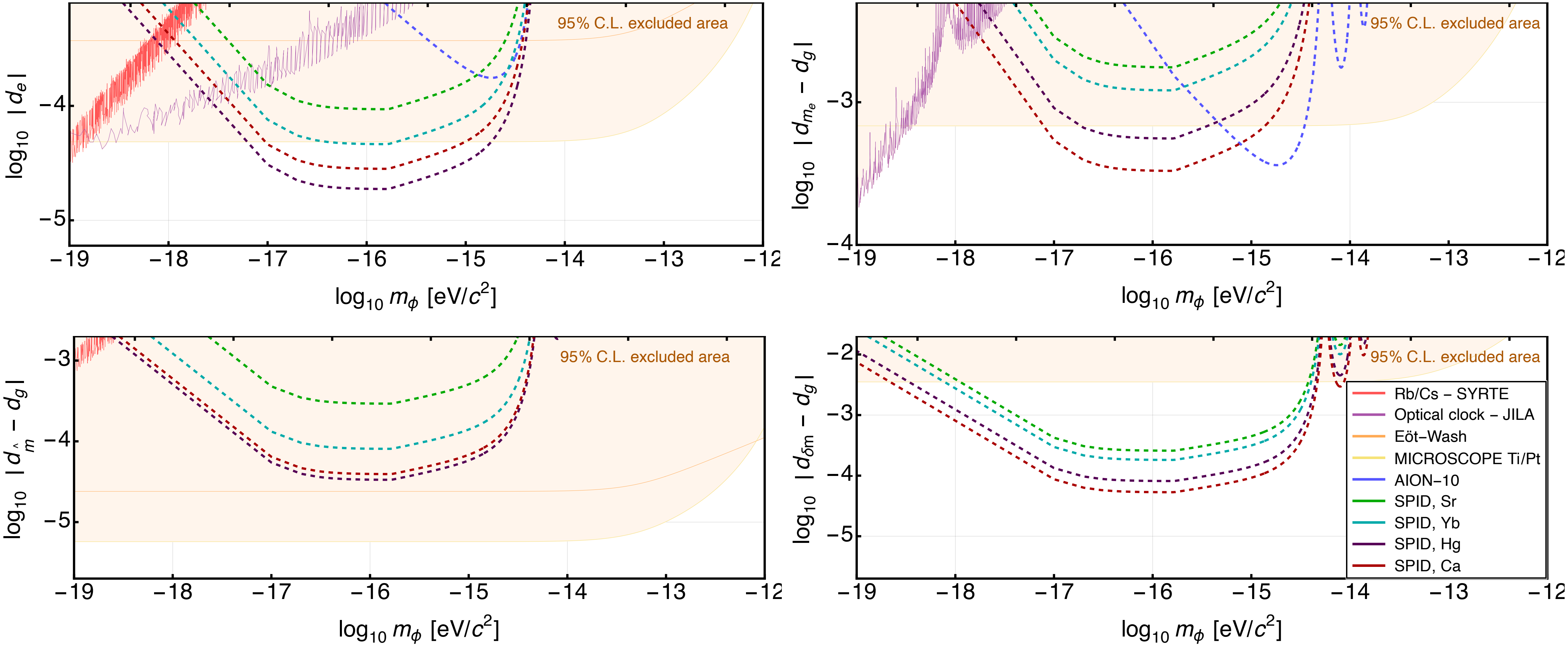

Low sensitivity of MICROSCOPE (from oscillatory behavior of the field), Stanford and Wuhan

Compared to AION-10, the proposed experiment SPID would be overall more sensitive to dilaton couplings.

Conclusion

- Some of most promising ULDM candidates, dilaton and axion fields, produce oscillating rest mass and transition frequency of atoms.
- Those oscillations produce violation of UFF and could be detected in various types of atom interferometers
- Based on futuristic proposal of AION-10 gradiometer, proposition of variation AI setup, which is overall more sensitive to those couplings than AION-10.

In addition, this new setup would test the UFF with a very interesting constraint on Eötvös parameter, i.e.

$$\eta \sim 10^{-16}$$

Thank you for your attention !

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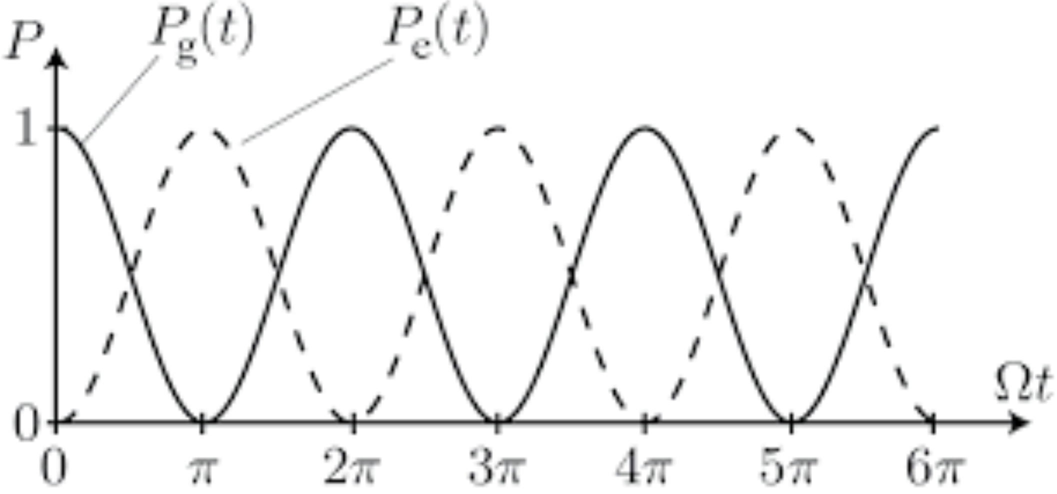
with INP and IN2P3 co-funded by CNES.

GRAM

Back-up : Rabi oscillations (Raman AI)

The light field couples the states $|g, \vec{p}_0\rangle$ and $|e, \vec{p}_0 + \hbar\vec{k}_{\text{eff}}\rangle$ → Rabi oscillations between the two states.

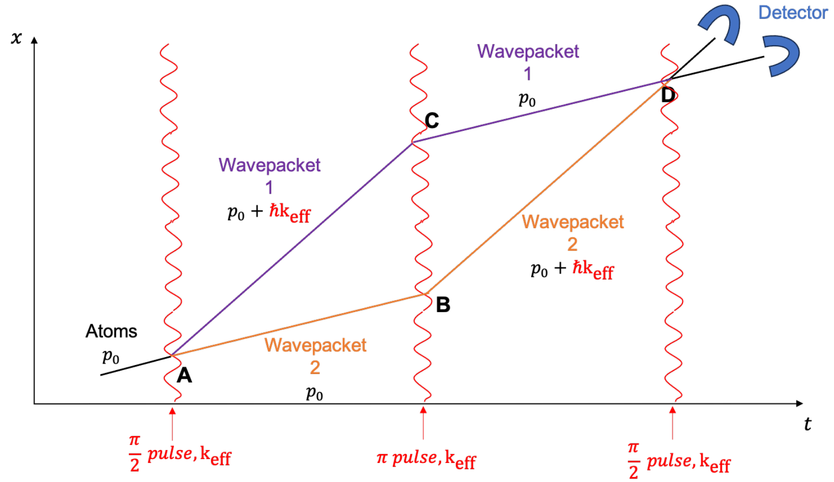


Due to interaction with light, the atom undergoes rabi oscillation between ground (g) and excited (e) states.

For a given coupling between atom and light (i.e electric field strength E_0) and Rabi frequency Ω (which depends on E_0), the final state of the atom depends on the time of interaction τ

→ $\Omega\tau = \pi/2$: half of the atoms are transferred into the excited state

→ $\Omega\tau = \pi$: the full ensemble change state



→ $\frac{\pi}{2}$ pulse separates the states : $|g, \vec{p}_0\rangle \rightarrow \frac{1}{\sqrt{2}}\left(|g, \vec{p}_0\rangle + |e, \vec{p}_0 + \hbar\vec{k}_{\text{eff}}\rangle\right)$

→ π pulse inverts state : $|g, \vec{p}_0\rangle \rightarrow |e, \vec{p}_0 + \hbar\vec{k}_{\text{eff}}\rangle$ and $|e, \vec{p}_0 + \hbar\vec{k}_{\text{eff}}\rangle \rightarrow |g, \vec{p}_0\rangle$

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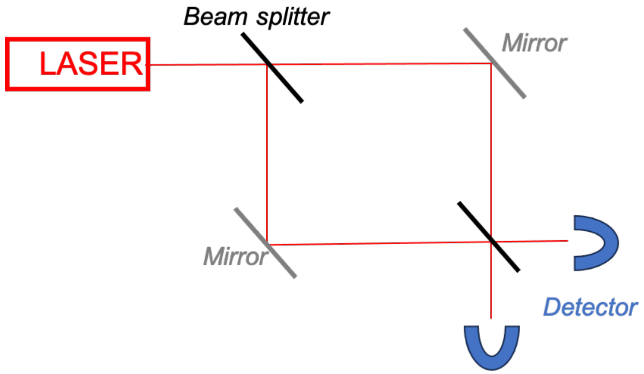

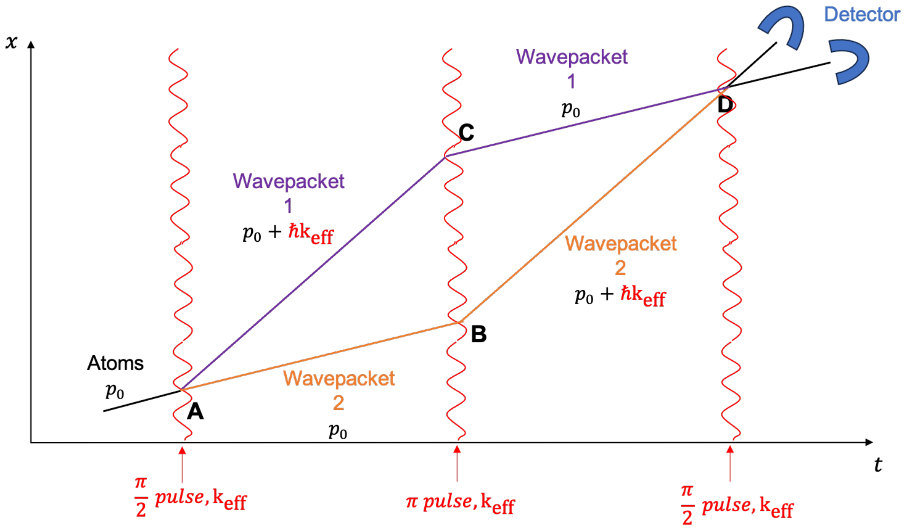

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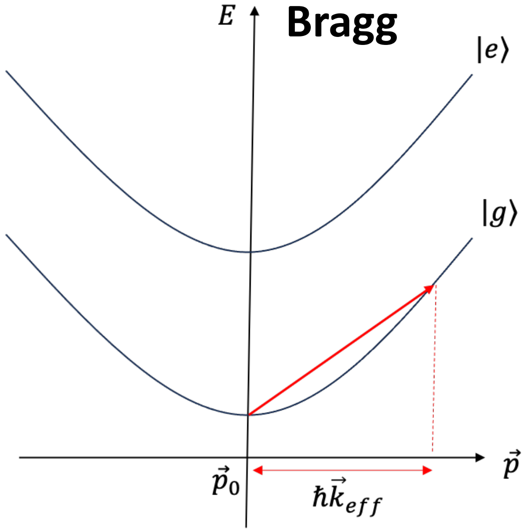
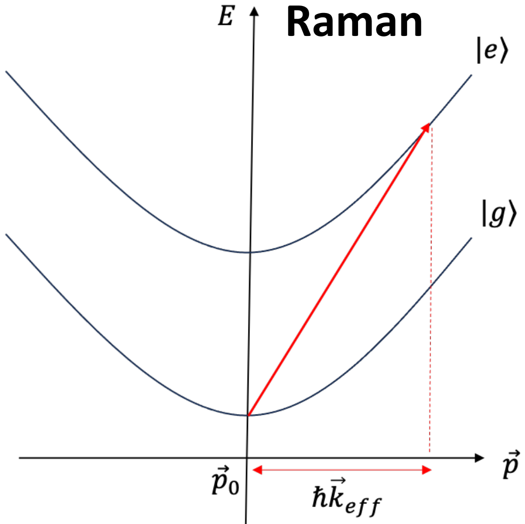
*M. Cadoret et al. EPJST, **172** 121-136 (2009)*





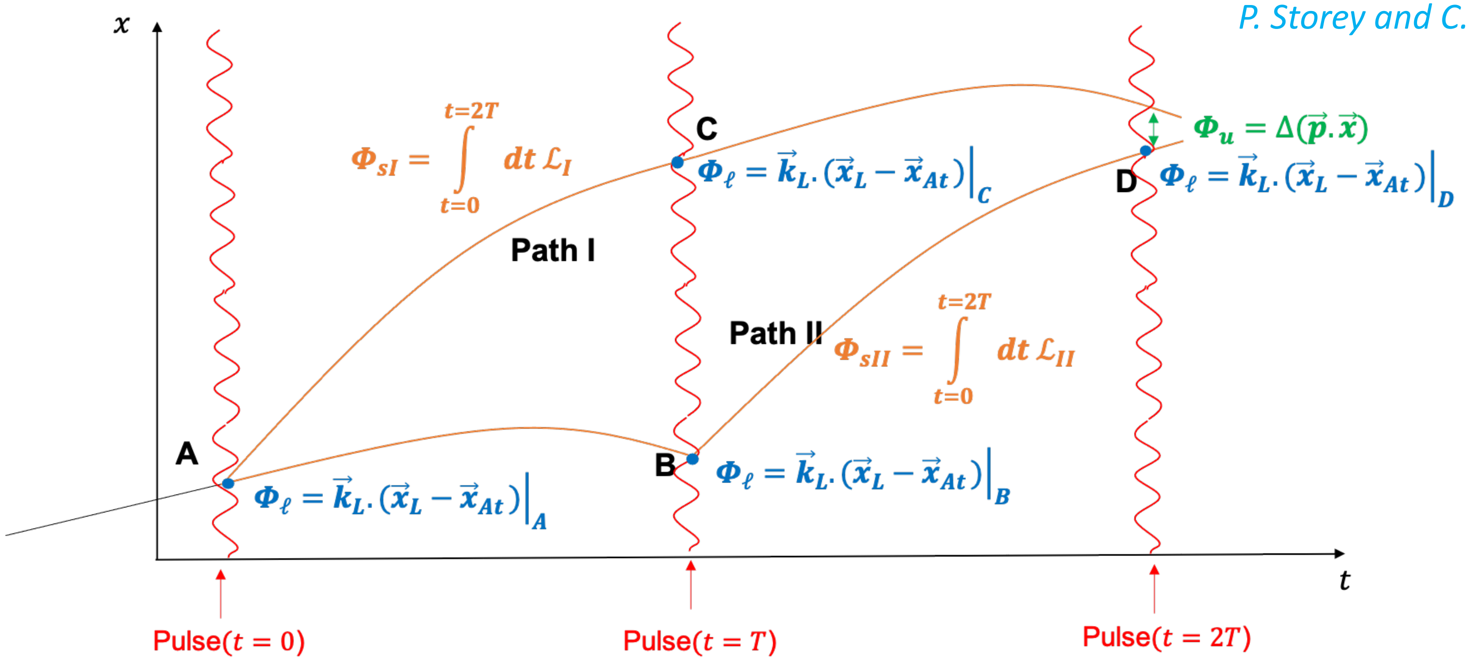
- At the detection step, check for state population

$$\int dS_{det}\,|\Psi_I(T_d,\vec{x}_d)+\Psi_{II}(T_d,\vec{x}_d)|^2$$
$$\propto 1+\cos(\Phi_I-\Phi_{II})$$
$$= 1+\cos\Delta\Phi$$

Back-up : phase contributions

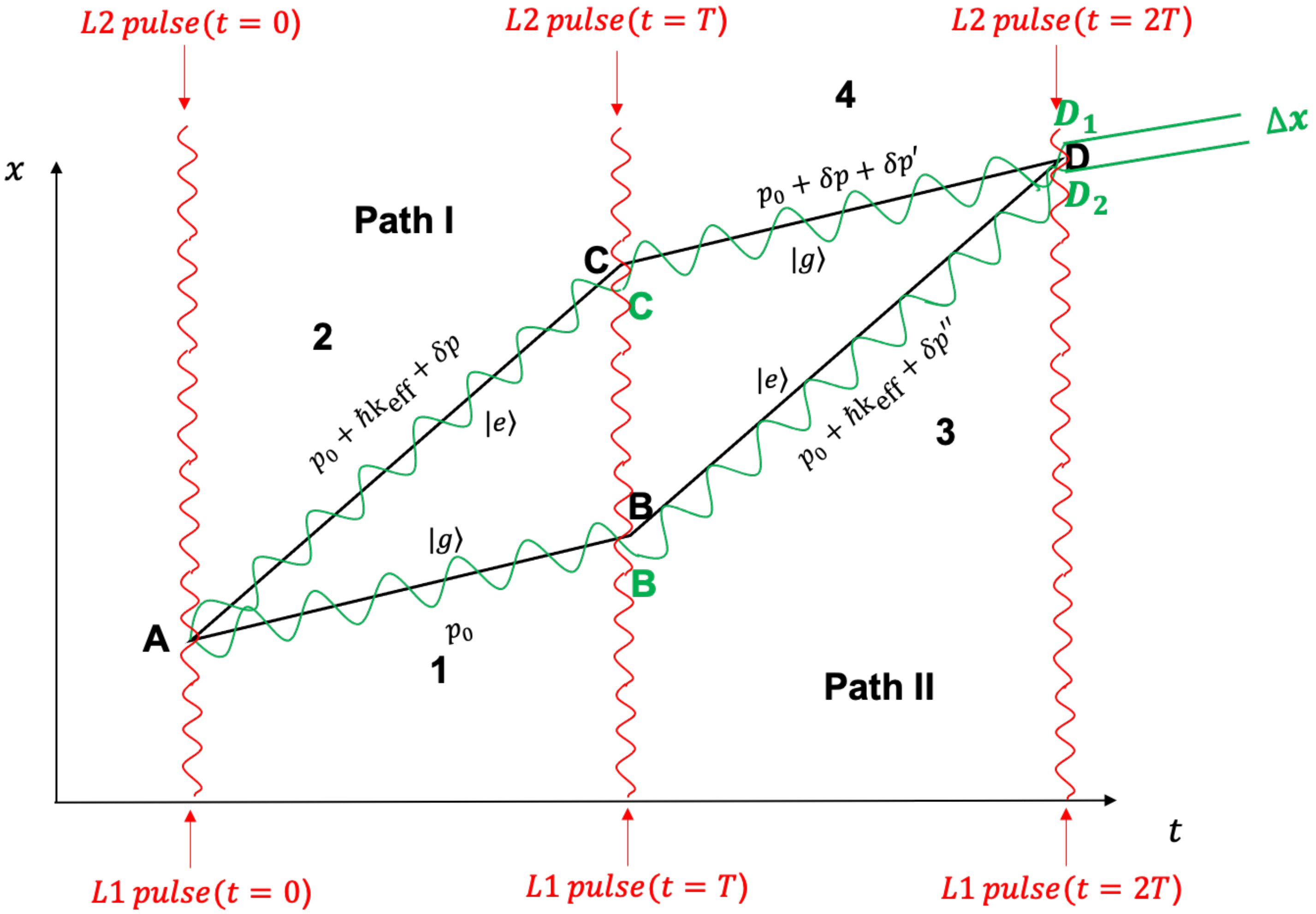
Feynman path integral method to compute phase shift between the 2 wavepackets.
This method works only for lagrangian at most quadratic in the position and in the velocity
→ **we must make the calculations in the galactic frame (no $\cos(\omega t - \vec{k}.\vec{x})$ term)**

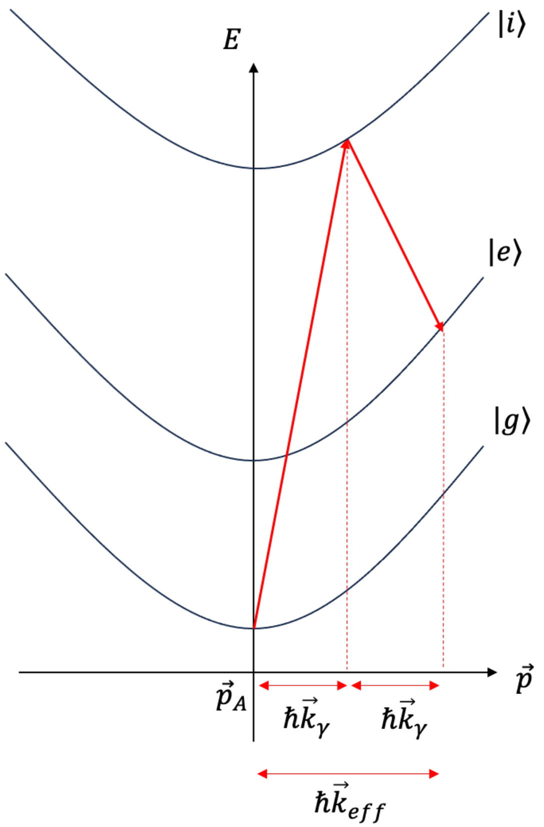
P. Storey and C. Cohen-Tannoudji, JP2 ***4*** *11 (1994)*



- Propagation phase Φ_s : phase accumulated by wavepackets along the trajectory
- Laser phase Φ_ℓ : phase factors of laser, to be calculated on light-matter interaction vertices
- Separation phase Φ_u : spatial incoincidence between the two output wavepackets

Multi-photon Raman/Bragg AI

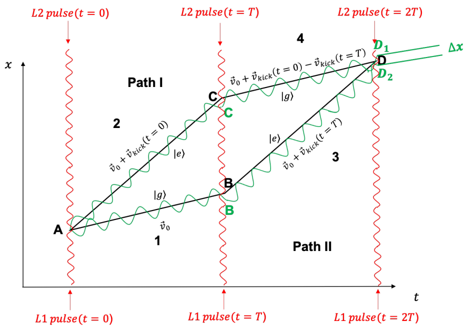




The oscillating acceleration implies a modification of the atom’s equations of motion
→ the atom’s trajectory oscillates in the interferometer.

Back-up : Phase shifts for different AI config.

- Multi-photon transition AI



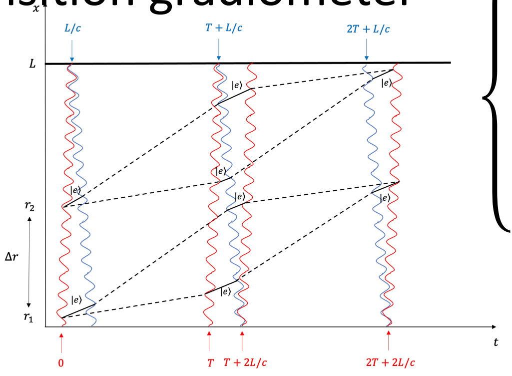
Reference mirror mass charge

Bragg AI

$$\begin{cases} \Delta\phi_A^{Raman} \propto \dfrac{\rho_{DM}}{\omega_a^3}\left(v_{DM}k_{\rm eff}\left(\left[Q_M^{\rm A}\right]_a - \left[Q_M^{\rm M}\right]_a\right) - \omega_A^0\cancel{\left[Q_\omega^{\rm A}\right]_a}\right)\sin^2(\omega_a T) \\ \Delta\phi_A^{Raman} \propto \dfrac{\sqrt{\rho_{DM}}}{\omega_\phi^2}\left(v_{DM}k_{\rm eff}\left(\left[Q_M^{\rm A}\right]_\phi - \left[Q_M^{\rm M}\right]_\phi\right) - \omega_A^0\cancel{\left[Q_\omega^{\rm A}\right]_\phi}\right)\sin^2\left(\dfrac{\omega_\phi T}{2}\right) \end{cases}$$

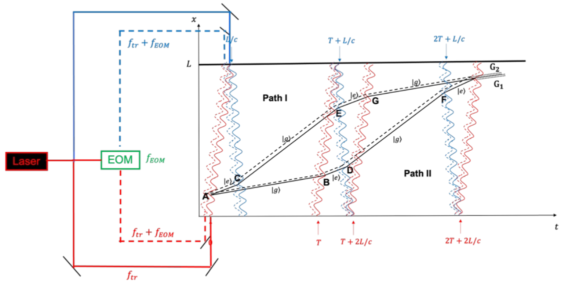
Number of “Large Momentum Transfer” (LMT) kicks

- Single-photon transition gradiometer



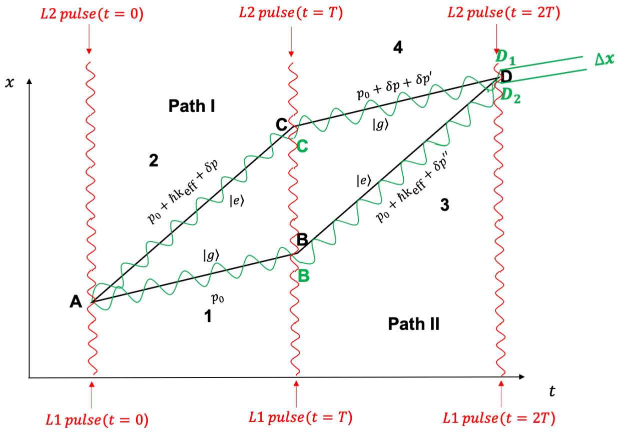
$$\begin{cases} \Delta\phi_A^{Grad} \propto \dfrac{n\,\Delta r\,\rho_{DM}}{\omega_a^2}\left[Q_\omega^{\rm A}\right]_a \sin^2(\omega_a T) \\ \Delta\phi_A^{Grad} \propto \dfrac{n\,\Delta r\sqrt{\rho_{DM}}}{\omega_\phi}\left[Q_\omega^{\rm A}\right]_\phi \sin^2\left(\dfrac{\omega_\phi T}{2}\right) \end{cases}$$

- “SPID” proposal



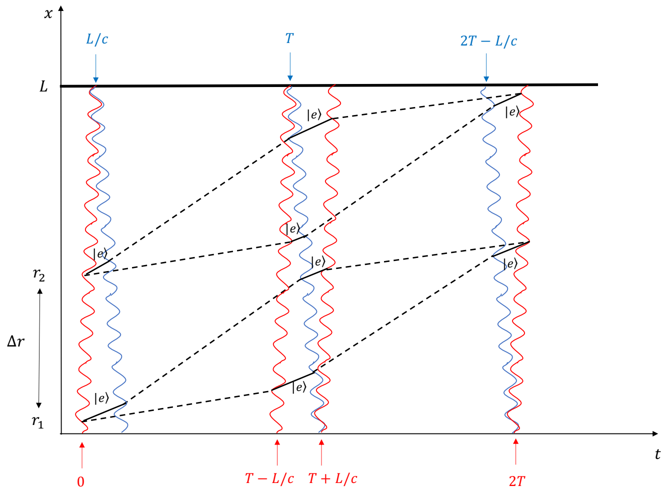
$$\begin{cases} \Delta\phi_{AB}^{SPID} \propto \dfrac{n\,\omega_0\,\rho_{DM} v_{DM}}{\omega_a^3}\left(\left[Q_M^{\rm A}\right]_a - \left[Q_M^{\rm B}\right]_a\right)\sin^2(\omega_a T) \\ \Delta\phi_{AB}^{SPID} \propto \dfrac{n\omega_0\,\sqrt{\rho_{DM}}\,v_{DM}}{\omega_a^2}\left(\left[Q_M^{\rm A}\right]_\phi - \left[Q_M^{\rm B}\right]_\phi\right)\sin^2\left(\dfrac{\omega_\phi T}{2}\right) \end{cases}$$

Set of experiments



- **MICROSCOPE** : (Existing) classical test of UFF to $\eta = \frac{\Delta a}{a} \sim 10^{-15}$ using $^{48}_{22}Ti$ and $^{195}_{78}Pt$
  *P. Touboul et al. PRL **129** 121102 (2022)*

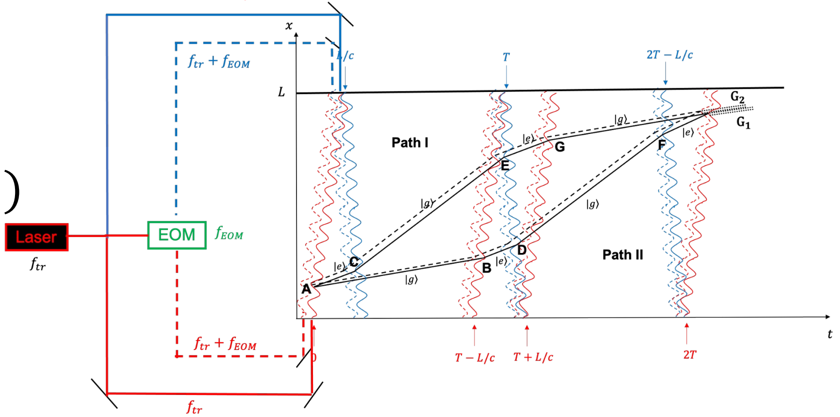
- **Stanford tower** : (Existing) differential Raman Multi-Photon AI test of UFF to $\eta \sim 10^{-12}$ using $^{87}_{37}Rb$ and $^{85}_{37}Rb$
  $(S_\phi \sim 10^{-2}\ \mathrm{rad}^2/\mathrm{s})$,
  *P. Asenbaum et al. PRL **125** 191101 (2020)*



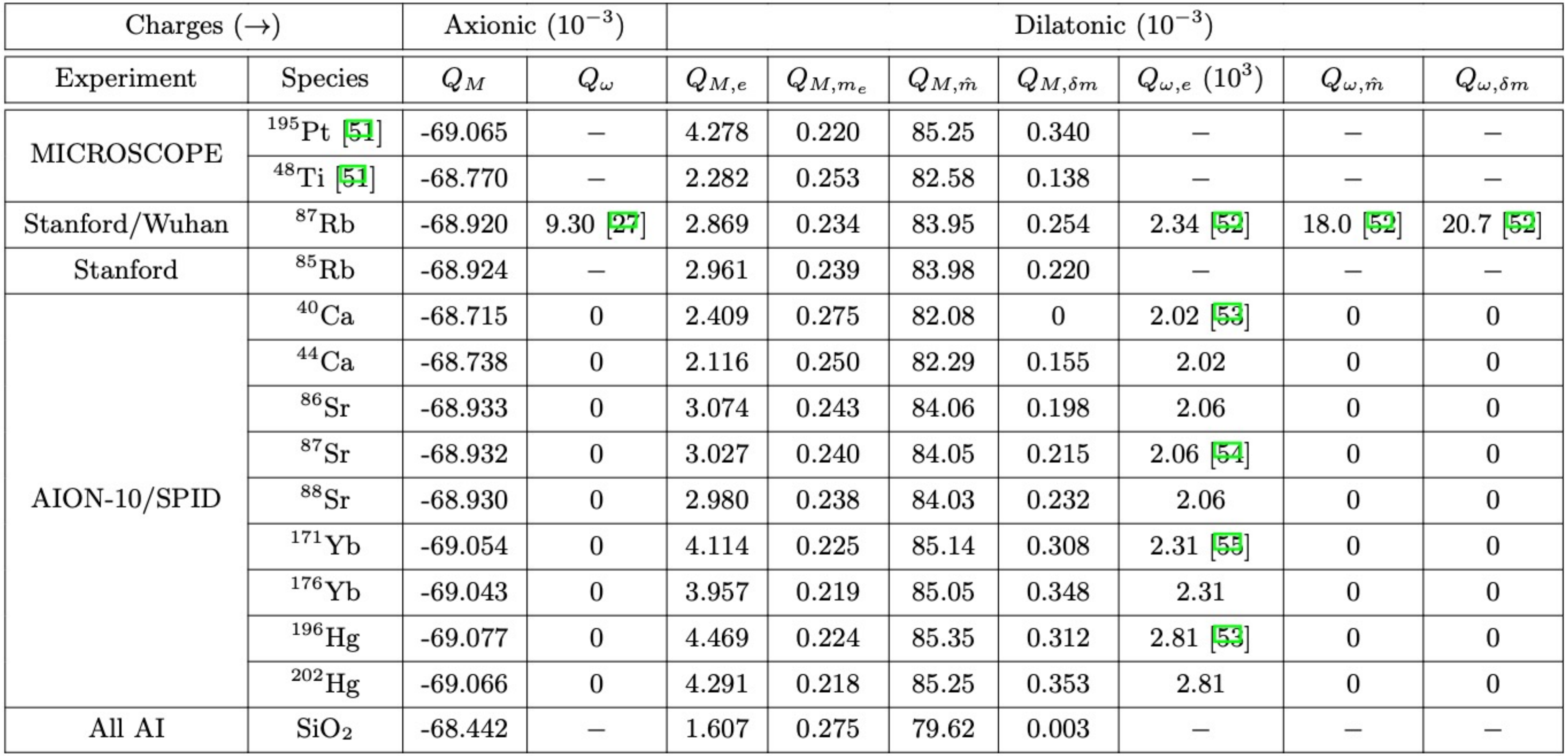
- **Wuhan gravimeter** : (Existing) single Raman Multi-Photon AI using $^{87}_{37}Rb$
  $(S_\phi \sim 10^{-3}\ \mathrm{rad}^2/\mathrm{s})$
  *Z. Hu et al. PRA **88** 043610 (2020)*

- **AION-10** : (Future) Gradiometer using $^{87}_{38}Sr$
  $(S_\phi = 10^{-8}\ \mathrm{rad}^2/\mathrm{s})$ *L. Badurina et al. PRD **105** 023006 (2022)*

- **New setup** : SPID using $(^{88}_{38}Sr, ^{86}_{38}Sr)$; $(^{40}_{20}Ca, ^{44}_{20}Ca)$; $(^{171}_{70}Yb, ^{176}_{70}Yb)$; $(^{196}_{80}Hg, ^{202}_{80}Hg)$
  $(S_\phi = S_\phi^{AION} + S_\phi^{EOM})$



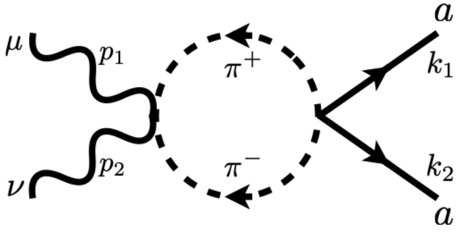
Back-up : Dilatonic/axionic charges for various atoms/transitions



Charges (→)		Axionic (10^{-3})		Dilatonic (10^{-3})						
Experiment	Species	Q_M	Q_ω	$Q_{M,e}$	Q_{M,m_e}	$Q_{M,\hat{m}}$	$Q_{M,\delta m}$	$Q_{\omega,e}$ (10^3)	$Q_{\omega,\hat{m}}$	$Q_{\omega,\delta m}$
MICROSCOPE	^{195}Pt [51]	-69.065	–	4.278	0.220	85.25	0.340	–	–	–
	^{48}Ti [51]	-68.770	–	2.282	0.253	82.58	0.138	–	–	–
Stanford/Wuhan	^{87}Rb	-68.920	9.30 [27]	2.869	0.234	83.95	0.254	2.34 [52]	18.0 [52]	20.7 [52]
Stanford	^{85}Rb	-68.924	–	2.961	0.239	83.98	0.220	–	–	–
AION-10/SPID	^{40}Ca	-68.715	0	2.409	0.275	82.08	0	2.02 [53]	0	0
	^{44}Ca	-68.738	0	2.116	0.250	82.29	0.155	2.02	0	0
	^{86}Sr	-68.933	0	3.074	0.243	84.06	0.198	2.06	0	0
	^{87}Sr	-68.932	0	3.027	0.240	84.05	0.215	2.06 [54]	0	0
	^{88}Sr	-68.930	0	2.980	0.238	84.03	0.232	2.06	0	0
	^{171}Yb	-69.054	0	4.114	0.225	85.14	0.308	2.31 [55]	0	0
	^{176}Yb	-69.043	0	3.957	0.219	85.05	0.348	2.31	0	0
	^{196}Hg	-69.077	0	4.469	0.224	85.35	0.312	2.81 [53]	0	0
	^{202}Hg	-69.066	0	4.291	0.218	85.25	0.353	2.81	0	0
All AI	SiO_2	-68.442	–	1.607	0.275	79.62	0.003	–	–	–

Back-up : AION sensitivity to $1/f_a$

At leading order, optical transition frequency does not depend on m_π^2 → AION-10 independent of a^2, f_a^{-2}

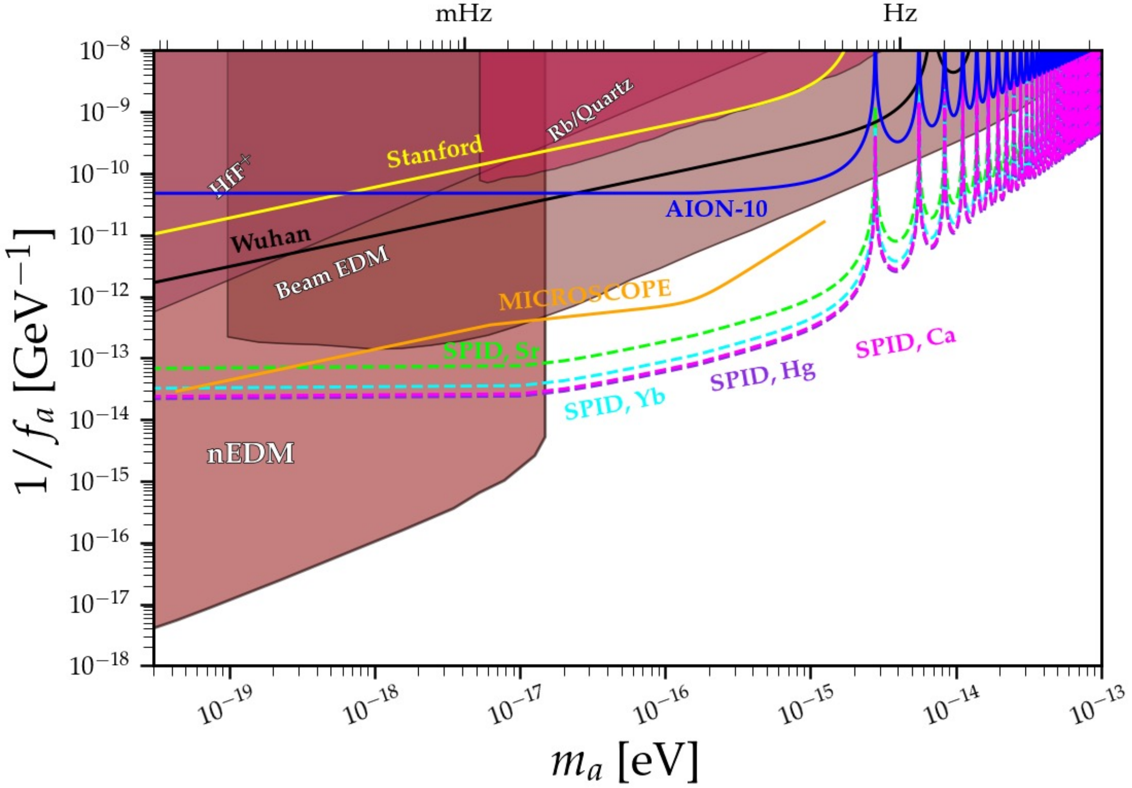


But recent paper showed that the axion-gluon coupling leads to axion-photon coupling at loop level *C. Beadle et al. Arxiv:2307.10362*

→ Any experiment sensitive to variation of fine structure constant α would be sensitive to $1/f_a$ coupling

→ This correction is negligible for experiments sensitive to variation of rest mass and hyperfine transition frequency ($\sim 10^{-6}$ and $\sim 10^{-4}$ smaller respectively)

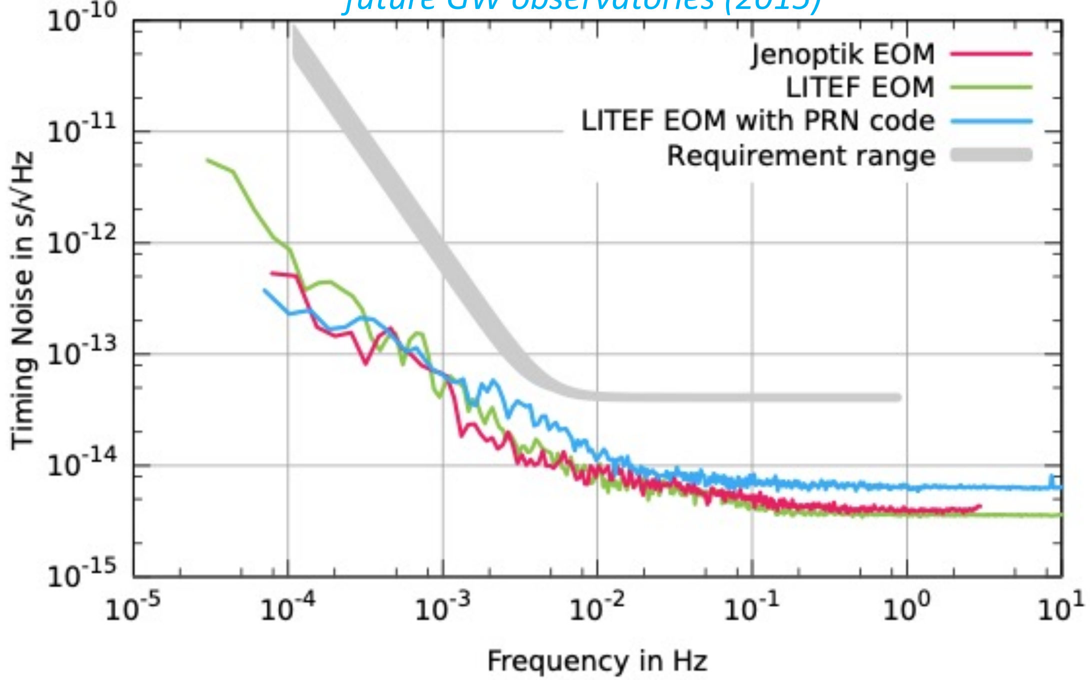
→ AION-10 becomes sensitive to $1/f_a$
(but sensitivity suppressed by α^2)



Inter-Spacecraft Frequency Distribution

Back-up : Phase noise from EOM

S. Barke, Ph.D Thesis, Inter-Spacecraft frequency distribution for future GW observatories (2015)



→ Noise contribution of *Jeoptik* EOM

\+ Noise from the 1 GHz modulation source

*X. Xie et al, Nature Photonics **11** 44 (2017)*

*J. Hartnett et al, Applied Physics Letters **100** 183501 (2012)*

$$\rightarrow \boldsymbol{S_{\phi}^{EOM}(f) \sim 10^{-13}\left(\frac{f}{Hz}\right)^{-2}\ \mathrm{rad^2/Hz}}$$

which dominates at very low frequency, compared to gradiometer noise in AION-10 ($S_\phi = 10^{-8}\ \mathrm{rad^2/s}$)