Primordial black holes as dark matter: Interferometric tests of phase transition origin

Iason Baldes

Based on work with María Olalla Olea-Romacho, 2307.11639

CosmoChart
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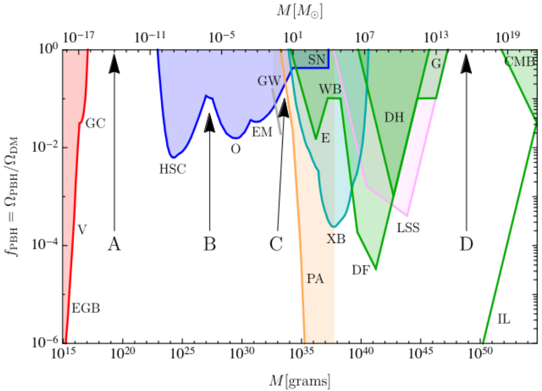
Théorie, Univers et Gravitation
11 October 2023

PBHs as DM



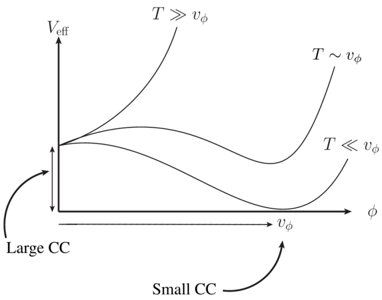
Escrivà, Kühnel, Tada, 2211.05767

$$f_{\rm pbh} \equiv \frac{\rho_{\rm pbh}}{\rho_{\rm DM}} = 1 \qquad\qquad 10^{-16} M_\odot \lesssim M_{\rm PBH} \lesssim 10^{-10} M_\odot$$

PBH Production

- Inflationary overdensities (ultra-slow roll, waterfall...)
- Phase transition at the end of inflation - (see talk by C. Animali)
- Early universe dissipative processes - Flores, Kusenko 2008.12456
- Phase transition starting from radiation dominated epoch - focus here.
- ...

Phase Transition



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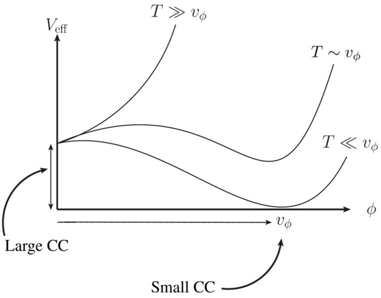

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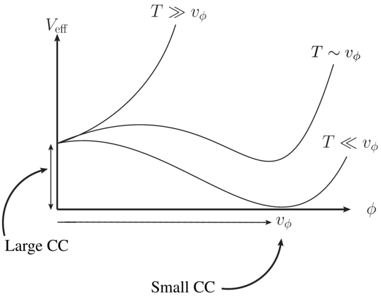

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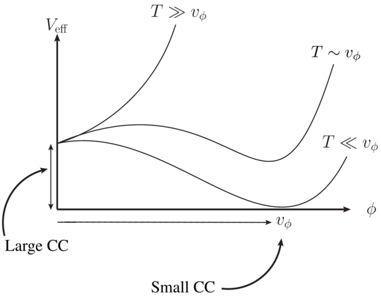

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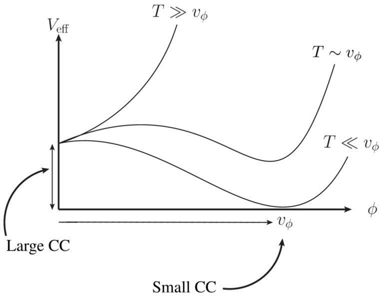

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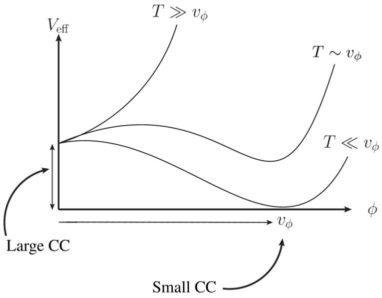

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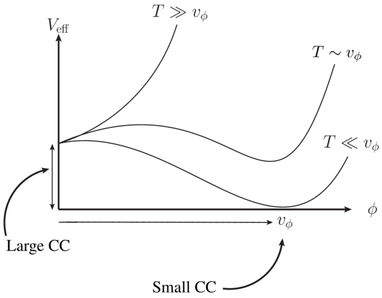

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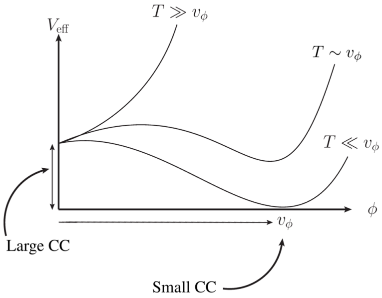

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Possible PBH production

Late patch mechanism - overview

Late patch mechanism for PBH production Liu et al. 2106.05637

- In an average patch the first bubbles nucleate at $\Gamma_{\rm bub} \sim H^4$.
- Some rare patches experience late nucleation.
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Significant PBH production (probability of late patches) with large $\delta \equiv \rho_{\rm rad}/\bar{\rho}_{\rm rad} - 1 \sim 0.45$ for:

$$\beta \equiv \frac{1}{\Gamma_{\rm bub}} \frac{d\Gamma_{\rm bub}}{dt} \approx 8H$$

Gouttenoire/Volansky 2305.04942

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The PBH mass is related to the Hubble scale/mass which is in turn set by the vacuum energy → reheat temperature.

$$M_{\rm PBH} \sim \frac{M_{\rm Pl}^3}{T_{\rm RH}^2} \implies 10~\text{TeV} \lesssim T_{\rm RH} \lesssim 10^4~\text{TeV}$$

GWs from bubble collisions

From previous estimates from the literature: The GW signal $\propto 1/R_{\mathrm{bub}}^2$

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$\Longrightarrow$ detectable GWs. The details we calculated follow.

Details - Example model

Classically scale invariant gauged $B - L$ with $Q_{B-L}(\rho) = -2$

$$\mathcal{L} \supset (D_\mu \rho)^* (D^\mu \rho) - \lambda_\rho |\rho|^4 - \lambda_{\rho h} |\rho|^2 |H|^2 - \lambda_h |H|^4$$

Three right handed neutrinos cancel off anomalies.

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Coleman-Weinberg symmetry breaking

$$V_0(\rho) = \beta_{\lambda_\rho}\frac{\rho^4}{4}\left(\log\left[\frac{\rho}{v_\rho}\right] - \frac{1}{4}\right) \qquad \beta_{\lambda_\rho} \approx \frac{1}{(4\pi)^2}\left(96 g_{B-L}^4 - y_{Ni}^2\right)$$

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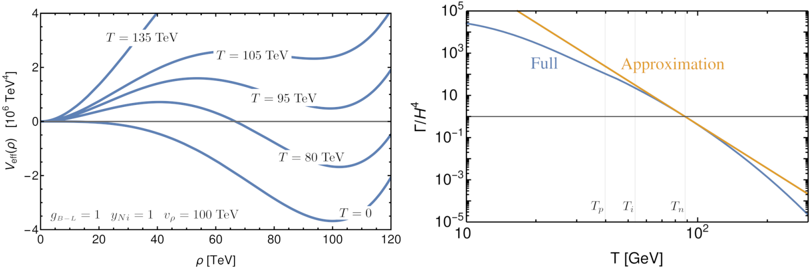
$$\Lambda_{\rm vac} = \frac{\beta_{\lambda_\rho}v_\rho^4}{16}$$

Details - Finite temperature effective potential



Potential barrier from the gauge boson thermal corrections.

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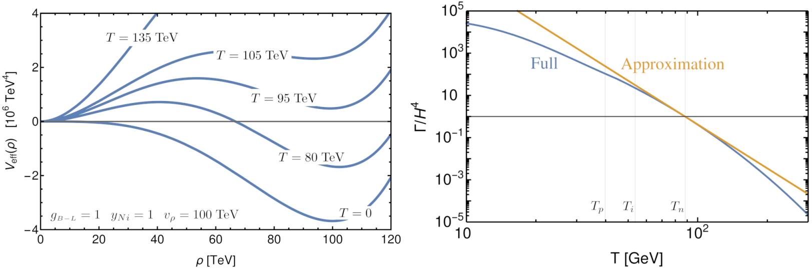

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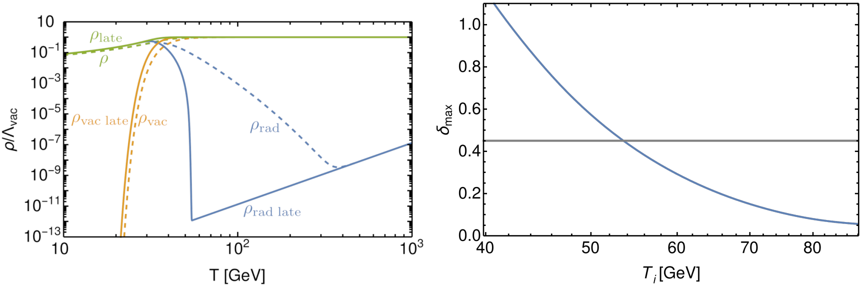
Bubble nucleation rate calculated numerically

$$\Gamma_{\rm bub} \approx \mathrm{Max}\left[\frac{1}{R_c^4}\left(\frac{S_4}{2\pi}\right)^2 e^{-S_4}, \quad T^4\left(\frac{S_3}{2\pi T}\right)^{3/2} e^{-S_3/T}\right]$$

Details - Friedmann equations



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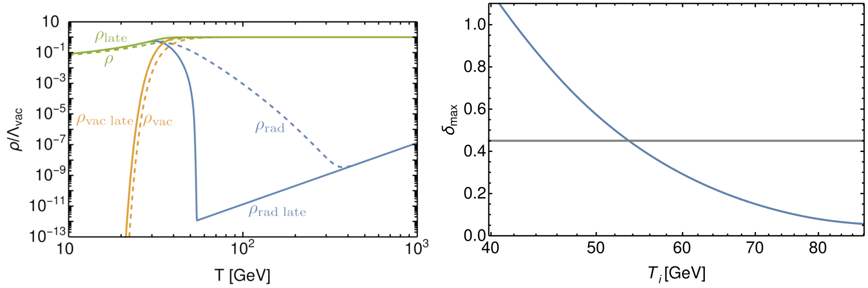

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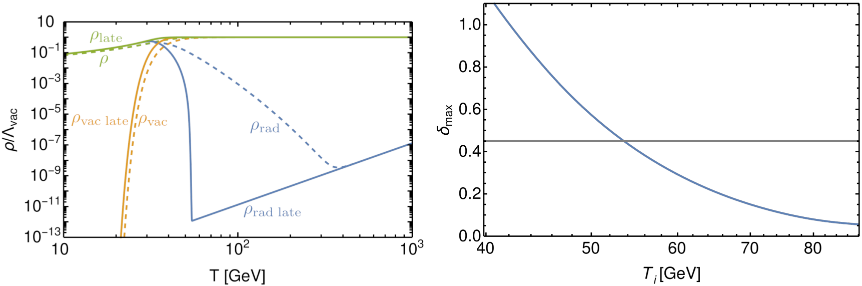

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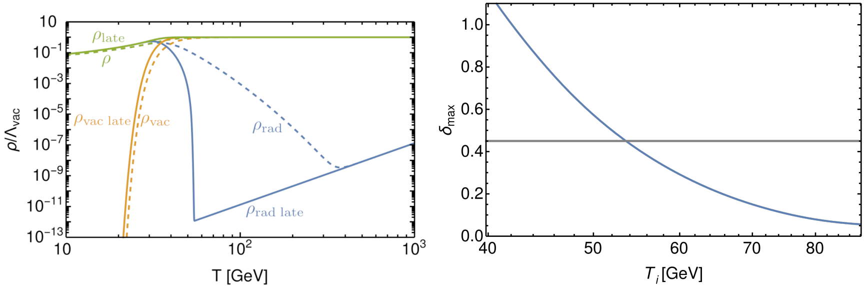

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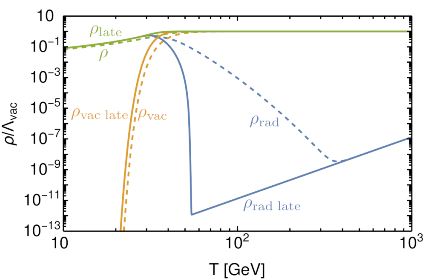

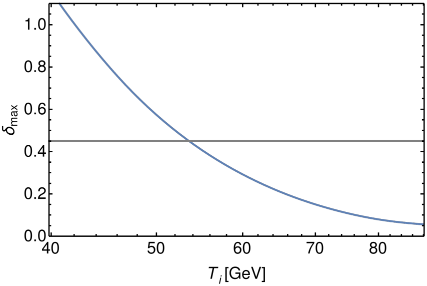

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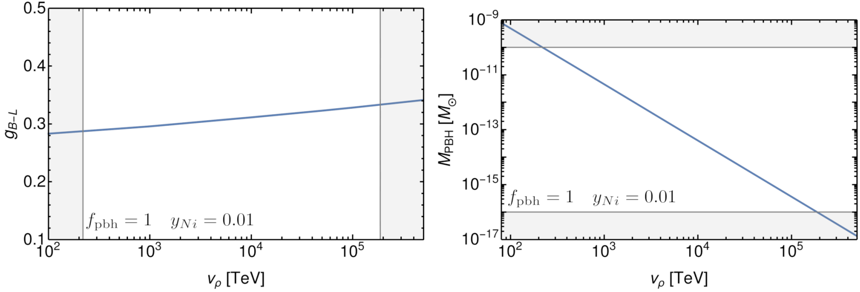
- Solve for the background and late patches.
- Find what initial late first nucleation temperature, T_i, gives $\delta_{\rm max} = 0.45$.
- Use this to calculate the PBH formation probability.
- The monochromatic PBH mass approximation is used:

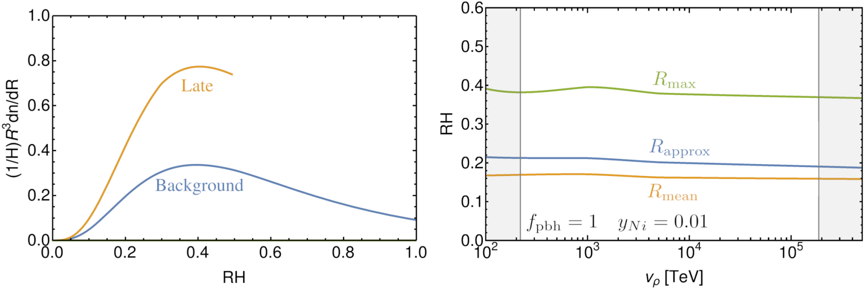
$$M_{\rm PBH} = \frac{4\pi \rho_{\rm radlate} c_s^3}{3 H_{\rm late}^3}$$

Details - PBH as DM



- The PBH abundance $f_{\rm pbh}$ and $M_{\rm pbh}$ then depend on the gauge coupling $g_{\rm B-L}$ and v_ρ (assuming small y_N).
- We numerically find where $f_{\rm pbh} = 1$.
- The macroscopic phase transition properties are then used to estimate $\Omega_{\rm GW}$.

Details - Bubble size



- The bubble size at percolation is a crucial input to Ω_{GW}.

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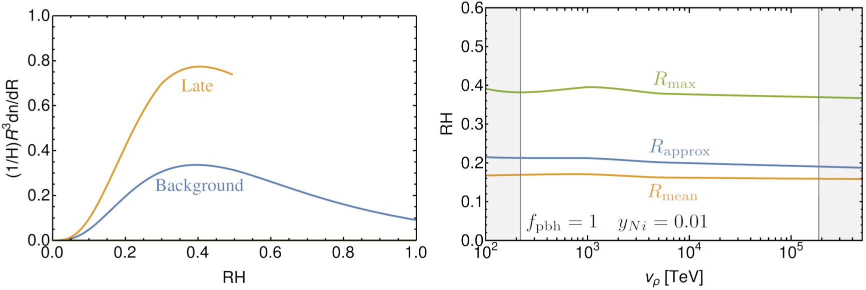

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$$R_{\rm approx} = \frac{\pi^{1/3}}{\beta(T_n)}$$

appears appropriate for close-to-conformal potentials.

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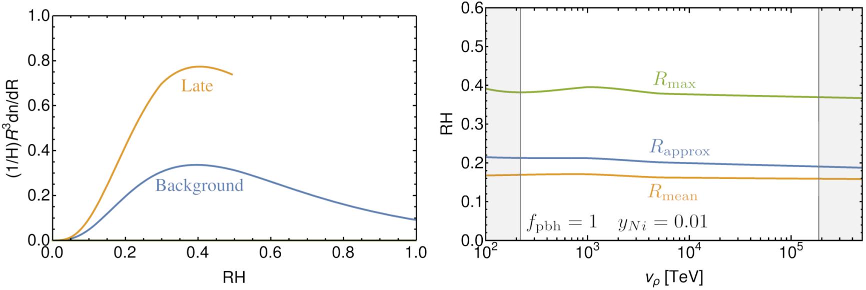

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  appears appropriate for close-to-conformal potentials.
- The use of $R_{\rm max}$ would give a larger signal.

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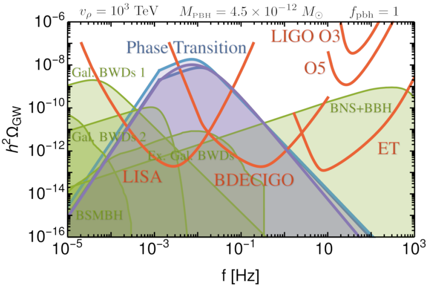

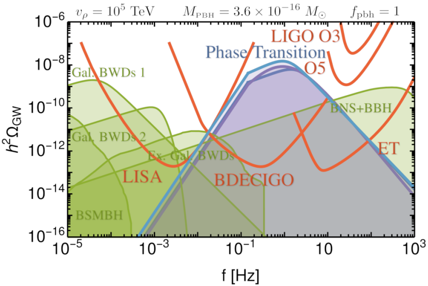

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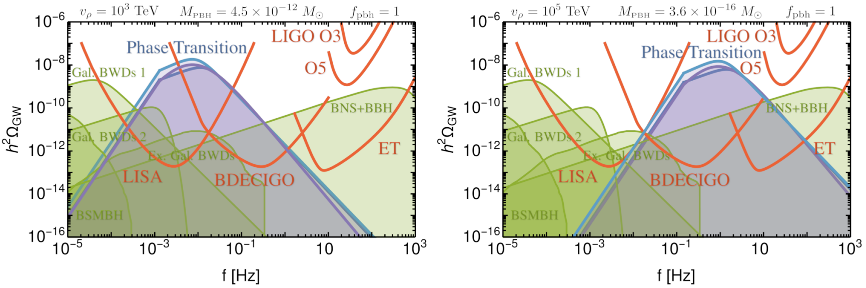

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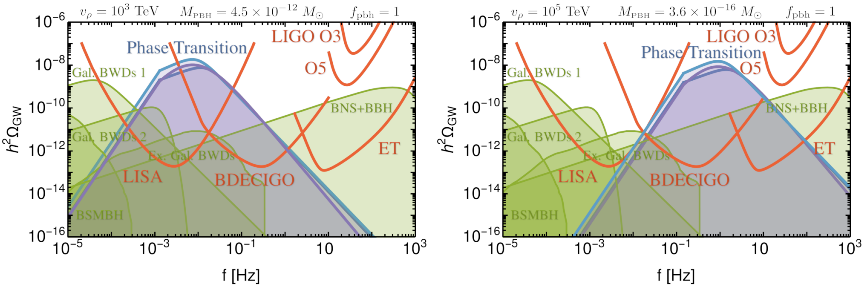

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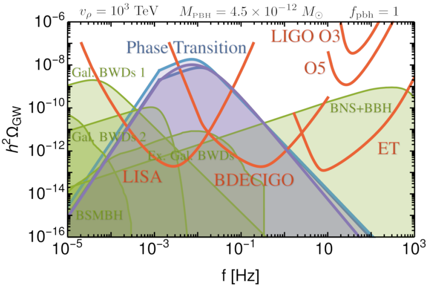

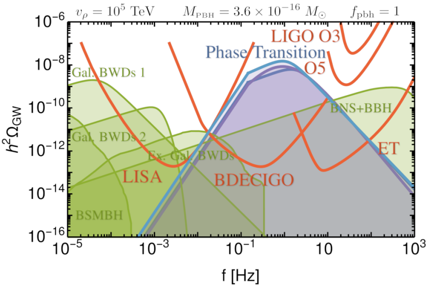

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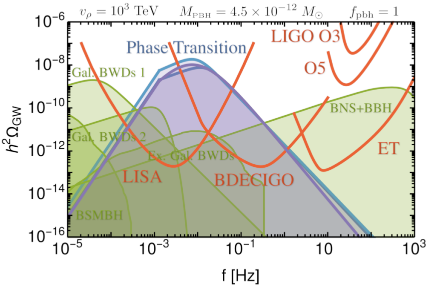

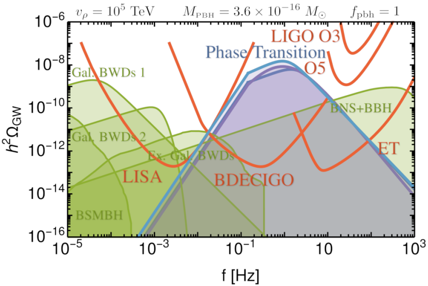

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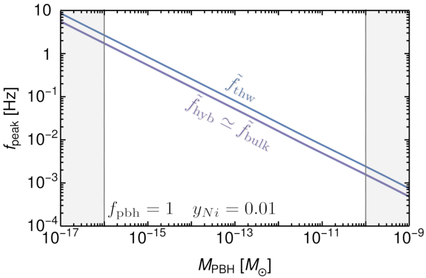
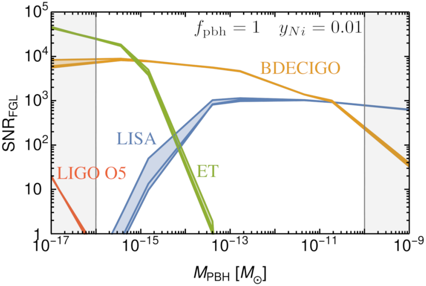
These all return similar estimates. Detectable above astro foregrounds.





$$\mathrm{SNR}_{\mathrm{FGL}} = \sqrt{t_{\mathrm{obs}} \int \left(\frac{\mathrm{Max}[0, \Omega_{\mathrm{GW}}(\nu) - \Omega_{\mathrm{FG}}(\nu)]^2}{\Omega_{\mathrm{sens}}^2 + 2\Omega_{\mathrm{GW}}\Omega_{\mathrm{sens}} + 2\Omega_{\mathrm{GW}}^2} \right) df}$$

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- Not yet clear regarding distinguishing $\Omega_{\rm GW}$ from different PBH formation mechanisms.
- Strong signal does not prove $f_{\rm pbh} = 1$ as the quantity is very sensitive to β/H. However, a large $\Omega_{\rm GW}$ would anyway be welcomed.

Conclusions

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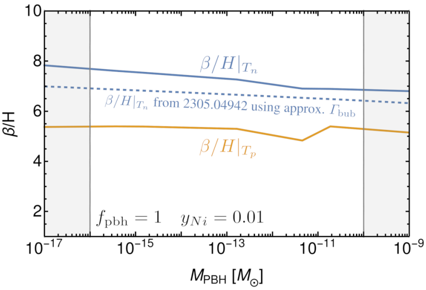
- Supercooled phase transition can lead to PBH formation.
- The required phase transition implies large Ω_{GW} from bubble collisions.
- The PBH mass is related to the Hubble scale.
- The allowed PBH window maps onto GW frequencies covered by upcoming detectors.
- Very promising way of getting indirect evidence of PBH DM or ruling it out from a late patch mechanism.

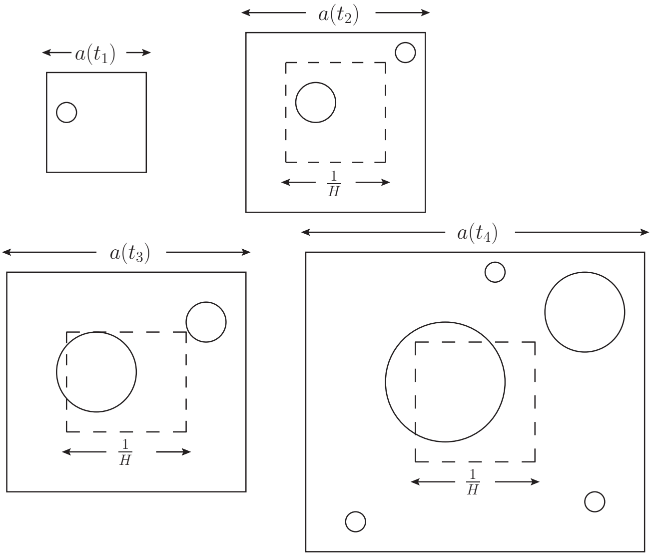


- Similarly the use of the approximate nucleation rate common in the literature

$$\Gamma_{\rm bub}(t) = H^4(t_n)e^{\beta(t-t_n)}$$

is shown to be appropriate for close-to-conformal potentials.

Completion of the Phase Transtion



If nucleation rate is low, we can form bubbles which never meet.

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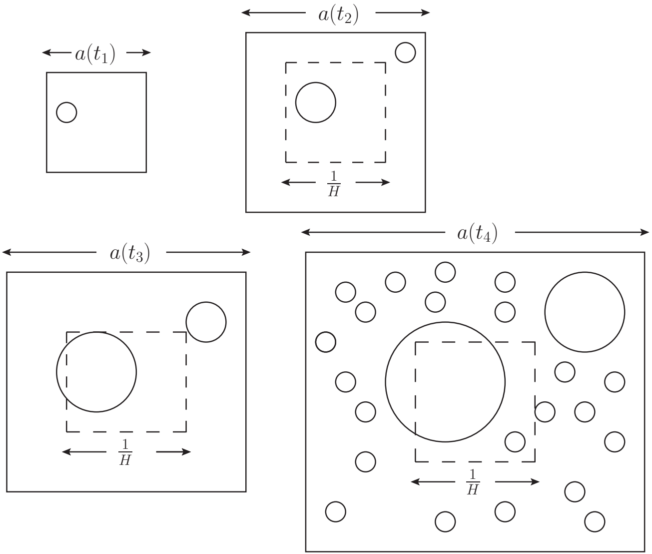

If nucleation grows enough, sufficient bubbles to meet will nucleate.

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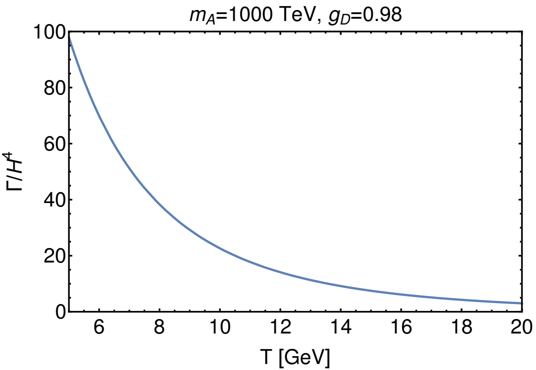
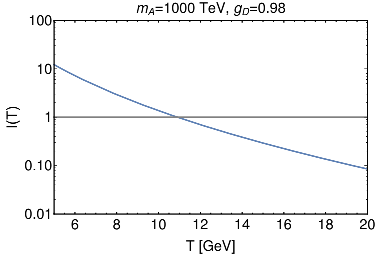
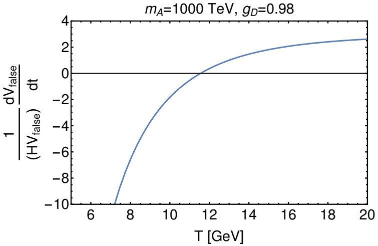


In the classically scale invariant potential we have a slow transition but an exponentially growing nucleation rate.

Completion of the Phase Transtion

We can explicitly check the volume of false vacuum decreases and the bubbles will percolate.





$$P(T) \equiv e^{-I(T)} \lesssim 1/e \implies I(T) = \frac{4\pi}{3}\int_{t_c}^{t} dt'\Gamma(t')a(t')^3 r(t,t')^3 \gtrsim 1$$

$$\frac{1}{H\mathcal{V}_{\rm false}}\frac{d\mathcal{V}_{\rm false}}{dt} = 3 + T\frac{dI}{dT} \lesssim -1.$$

Also see Ellis, Lewicki, No 1809.08242 Ellis, Lewicki, No, Vaskonen 1903.09642

Friedmann equations

The Friedmann equations are given by

$$H_{\rm bkg}^2 = \frac{8\pi}{3}\frac{\rho_{\rm vac}+\rho_{\rm rad}}{M_{\rm Pl}^2},$$
$$\frac{d\rho_{\rm rad}}{dt} = -4H_{\rm bkg}\rho_{\rm rad} - \frac{d\rho_{\rm vac}}{dt},$$

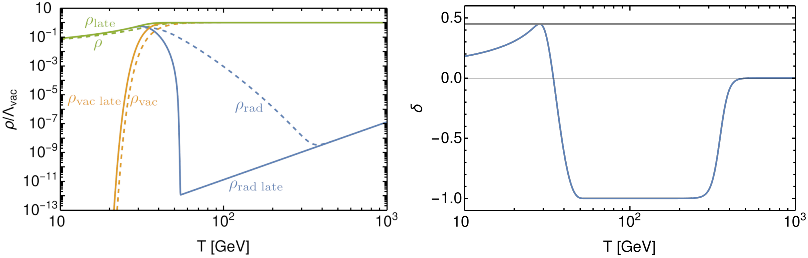
where $H_{\rm bkg}$ denotes the Hubble rate of the average background patch.

Percolation

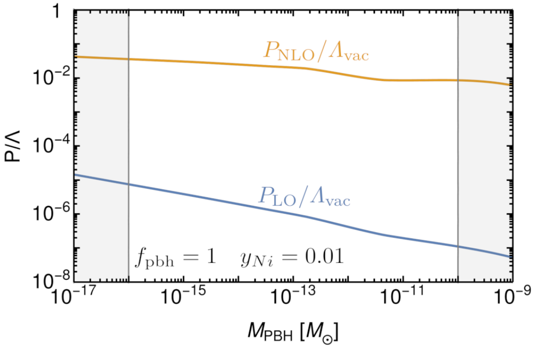
$$I = \frac{4\pi}{3} \int_{t_c}^{t} dt' \Gamma_{\rm bub}(t') a^3(t') r^3(t,t')$$
$$= \frac{4\pi}{3} \int_{T}^{T_c} dT' \frac{\Gamma_{\rm bub}(T')}{T'^4 H(T')} \left(\int_{T}^{T'} \frac{d\tilde{T}}{H(\tilde{T})} \right)^3 .$$

$$\rho_{\rm vac} = \Lambda_{\rm vac} e^{-I}.$$

Percolation when $I(T) > 1$ and $d \log \mathcal{V}_{\rm false}/dt < -H$.



ρ/Λ_{vac}
ρ_{late}
ρ
$\rho_{vac\ late}$
ρ_{vac}
ρ_{rad}
$\rho_{rad\ late}$
T [GeV]
δ
T [GeV]

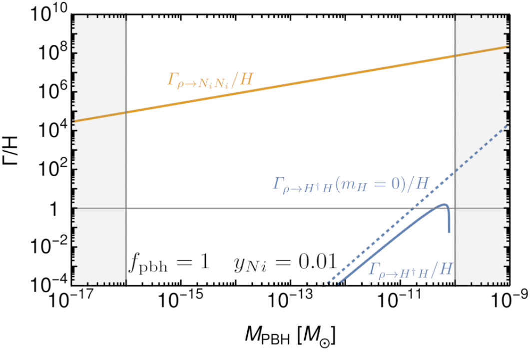


$$P_{\rm LO} \approx \frac{T_p^2}{24}\left(3M_{Z'}^2 + m_\rho^2 + M_{Ni}^2\right),$$

$$P_{\rm NLO} \approx \frac{\kappa\zeta(3)(Q_{B-L}^{\rm eff})^2\alpha_{B-L}\gamma M_{Z'} T_p^3}{\pi^3} \log\left(\frac{v_\rho}{T_p}\right),$$

Soft gauge boson production does not stop the wall accelerating.

The condensate decays rapidly.



$$\Gamma_{\rho\to N_i N_i} = \frac{y_{Ni}^2 m_\rho}{32\pi}\left(1-\frac{2M_{Ni}^2}{m_\rho^2}\right)\mathrm{Re}\left[\sqrt{1-\frac{4M_{Ni}^2}{m_\rho^2}}\right].$$

$$\Gamma_{\rho\to H^\dagger H} = \frac{\lambda_{\rho h}^2 v_\rho^2}{8\pi m_\rho}\mathrm{Re}\left[\sqrt{1-\frac{4m_H^2}{m_\rho^2}}\right].$$

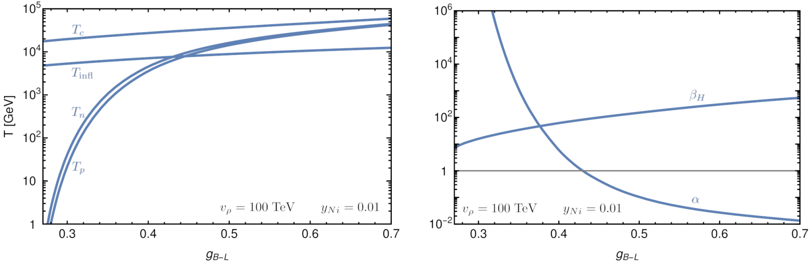
Potential

$$V_{\mathrm{eff}}(\rho, T) = V_0(\rho) + V_T(\rho, T) + V_{\mathrm{daisy}}(\rho, T)$$

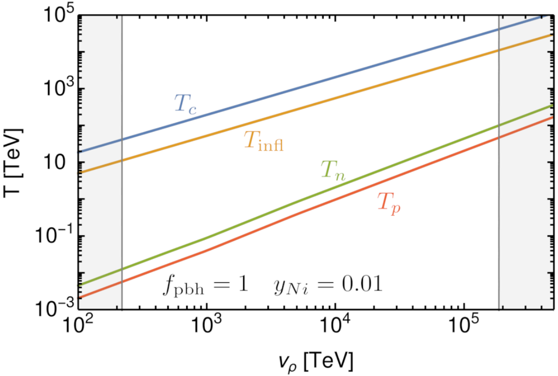
$$V_T(\rho, T) = \frac{T^4}{2\pi^2}\left(3J_B\left[\frac{M_{Z'}^2}{T^2}\right] + 2J_F\left[\frac{M_{Ni}^2}{T^2}\right]\right)$$

$$V_{\mathrm{daisy}}(\rho, T) = \frac{T}{12\pi}\left(M_{Z'}^3 - [M_{Z'}^2 + \Pi_{\mathrm{Z'}}]^{3/2}\right)$$

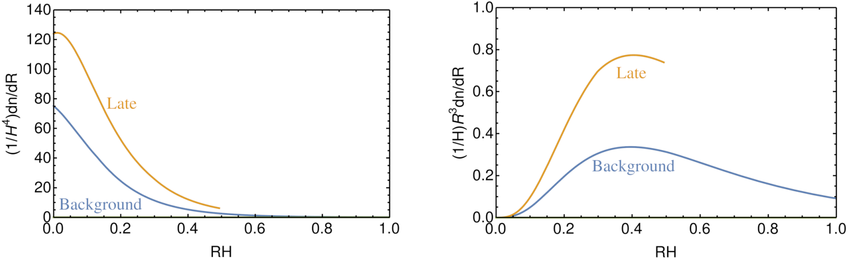
$$\Lambda_{\mathrm{vac}} = \frac{\beta_{\lambda_\rho} v_\rho^4}{16}$$



Small gauge couplings result in supercooling.



10^5
10^3
10
10^{-1}
10^{-3}
T [TeV]
T_c
$T_{\rm infl}$
T_n
T_p
$f_{\rm pbh} = 1 \quad y_{Ni} = 0.01$
10^2
10^3
10^4
10^5
v_ρ [TeV]


140
120
100
80
60
40
20
0
0.0
0.2
0.4
0.6
0.8
1.0
RH
(1/H^4)dn/dR
Late
Background
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
RH
(1/H)R^3dn/dR
Late
Background

$$
\begin{aligned}
P_{\rm coll} &= {\rm Exp}\left[-\int_{t_c}^{t_i} dt' \Gamma_{\rm bub}(t') a_{\rm late}(t')^3 V_{\rm coll}\right] \\
&= {\rm Exp}\left[-\int_{T_i}^{T_c} \frac{dT' \Gamma_{\rm bub}(T')}{T' H(T')} a_{\rm late}(T')^3 V_{\rm coll}\right]
\end{aligned}
$$

The volume factor is

$$
\begin{aligned}
V_{\rm coll} &= \frac{4\pi}{3}\left[\frac{1}{a_{\rm late}(t_{\delta{\rm max}}) H_{\rm late}(t_{\delta{\rm max}})} + r(t_{\delta{\rm max}}, t')\right]^3 \\
&= \frac{4\pi}{3}\left[\frac{1}{a_{\rm late}(T_{\delta{\rm max}}) H_{\rm late}(T_{\delta{\rm max}})} + \int_{T_{\delta{\rm max}}}^{T'} \frac{d\tilde{T}}{\tilde{T} H(\tilde{T}) a_{\rm bkg}(\tilde{T})}\right]^3 .
\end{aligned}
$$