Some gravitational aspects of scalar field dark matter

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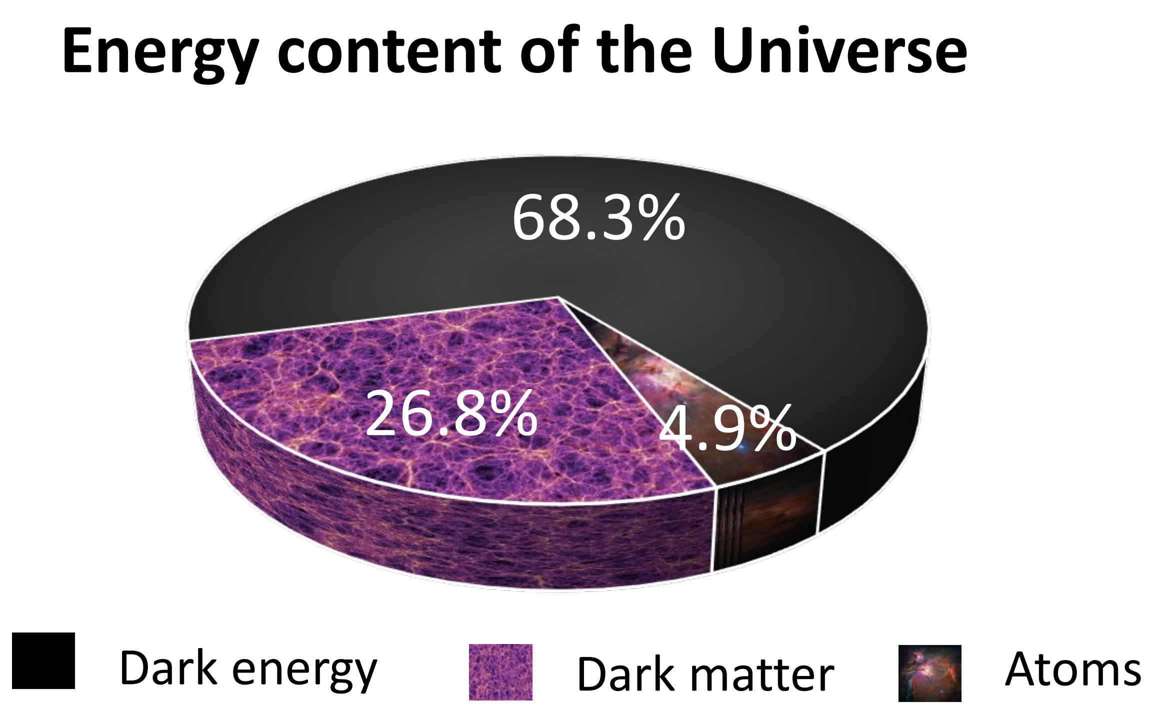
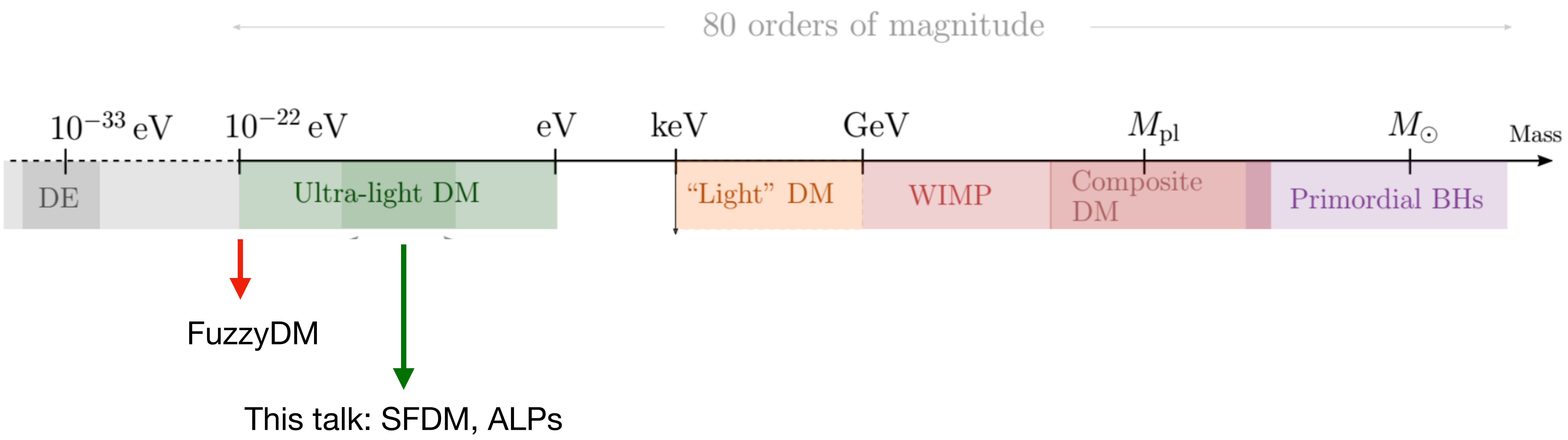
I- DARK MATTER

Known properties of DM:

- 27% of the energy density of the universe
- Cold (non-relativistic)
- Dark: small electromagnetic interactions
- Collisionless / pressureless: small self-interactions or interactions with baryons



However there remains a huge uncertainty on its mass and many scenarios exist, from elementary particles to macroscopic objects:



II- SCALAR-FIELD DARK MATTER

Fuzzy Dark Matter (FDM) + self-interactions

$$S_\phi = \int d^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

$$V(\phi) = \frac{m^2}{2}\phi^2 + V_{\rm I}(\phi) \;\; \text{with} \;\; V_{\rm I}(\phi) = \frac{\lambda_4}{4}\phi^4, \quad \lambda_4 > 0$$

$\rho \propto a^{-3}$

Repulsive self-interaction → Effective pressure

One characteristic density / length-scale: $\rho_a = \dfrac{4m^4}{3\lambda_4}, \quad r_a = \dfrac{1}{\sqrt{4\pi\mathcal{G}\rho_a}}$

Relativistic regime - strong self-interaction

Jeans length - Radius of solitons

Very large occupation numbers: $N \sim \dfrac{\rho}{mp^3} \gg 1 \qquad m \ll 1 \text{ eV}$

De Broglie wavelength: $\lambda_{\rm dB} = \dfrac{2\pi}{mv} \lesssim 1 \text{ kpc} \qquad m \gtrsim 10^{-22} \text{ eV}$

Also, k-essence models: $S_\phi = \int d^4x\sqrt{-g}\left[\Lambda^4 K(X) - \frac{m^2}{2}\phi^2\right] \qquad X = -\frac{1}{2\Lambda^4}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \qquad K(X) = X + K_{\rm I}(X)$

Galaxy-scale dynamics:

Formation of DM halos with a flat core

I- NON_RELATIVISTIC REGIME

From Klein-Gordon eq. to Schrödinger eq.:

$$\phi = \frac{1}{\sqrt{2m}}\left(e^{-imt}\psi + e^{imt}\psi^\star\right) \quad \longrightarrow \quad i\dot{\psi} = -\frac{\nabla^2\psi}{2m} + m(\Phi_\mathrm{N} + \Phi_\mathrm{I})\psi$$

$m \gg \partial$ keep only even terms

$$\nabla^2\Phi_\mathrm{N} = 4\pi\mathcal{G}\rho \qquad \Phi_\mathrm{I} = \frac{m|\psi|^2}{\rho_a}$$

From Schrödinger eq. to Hydrodynamical eqs (Madelung transformation):

$$\psi = \sqrt{\frac{\rho}{m}}e^{is} \qquad \vec{v} = \frac{\nabla s}{m} \quad \longrightarrow \quad \dot{\rho} + \nabla\cdot(\rho\vec{v}) = 0, \qquad \dot{\vec{v}} + (\vec{v}\cdot\nabla)\vec{v} = -\nabla(\Phi_\mathrm{N} + \Phi_\mathrm{I})$$

$$\Phi_\mathrm{I} = \frac{\rho}{\rho_a} \qquad \gamma = 2$$

effective pressure $P_\mathrm{eff} \propto \rho^2$

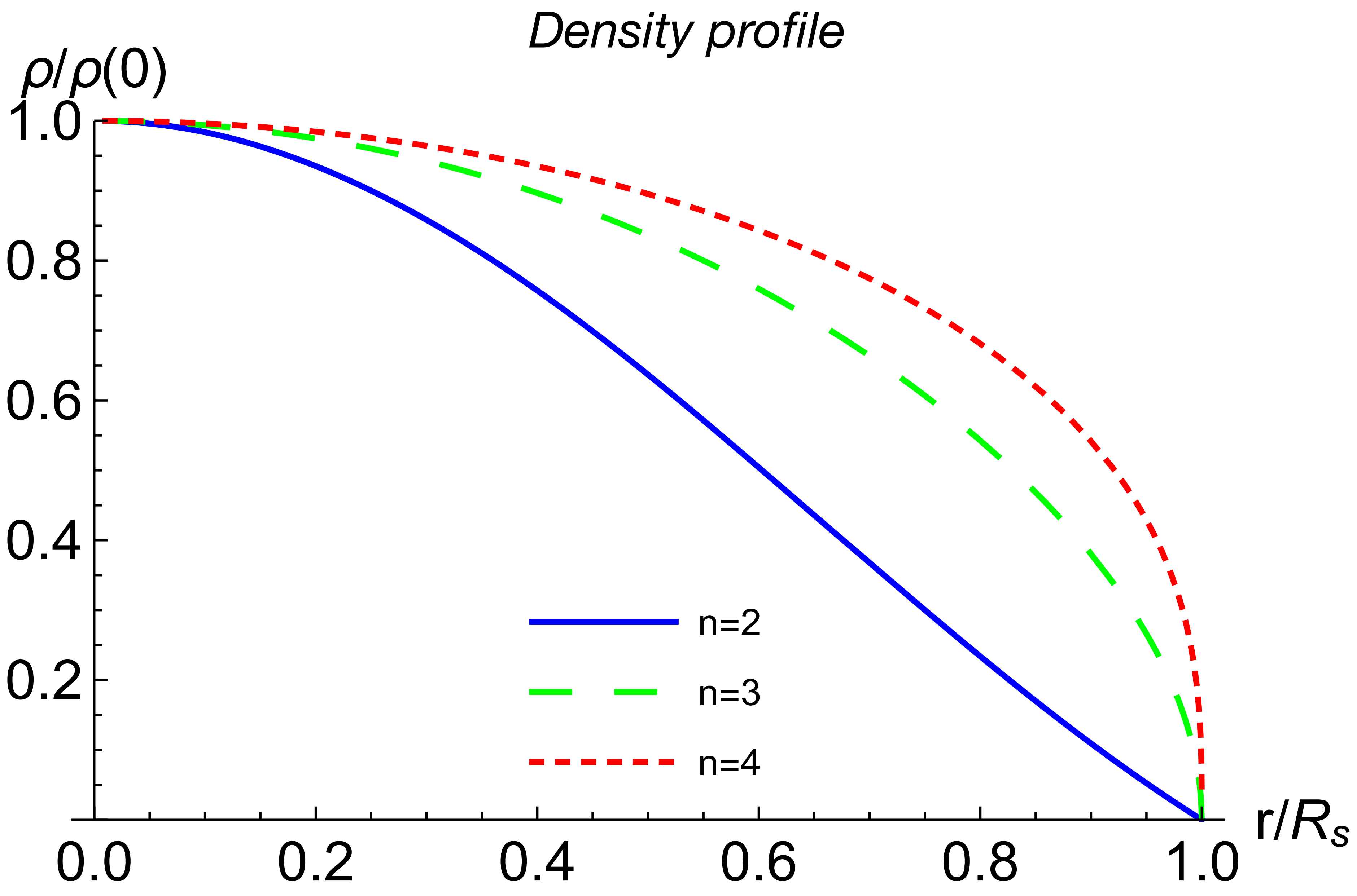
Neglecting « quantum pressure » (which dominates for FDM): $\Phi_\mathrm{Q} = -\frac{\nabla^2\sqrt{\rho}}{2m^2\sqrt{\rho}}$ large-m limit

II- SOLITON (ground state): HYDROSTATIC EQUILIBRIUM

As compared with CDM, the self-interactions allow the formation of hydrostatic equilibrium solutions, with a balance between gravity and the effective pressure:

$$\nabla(\Phi_{\rm N} + \Phi_{\rm I}) = 0 \quad \longrightarrow \quad \rho(r) = \rho_0 \frac{\sin(r/r_a)}{(r/r_a)} \qquad R_{\rm sol} \simeq \pi r_a$$

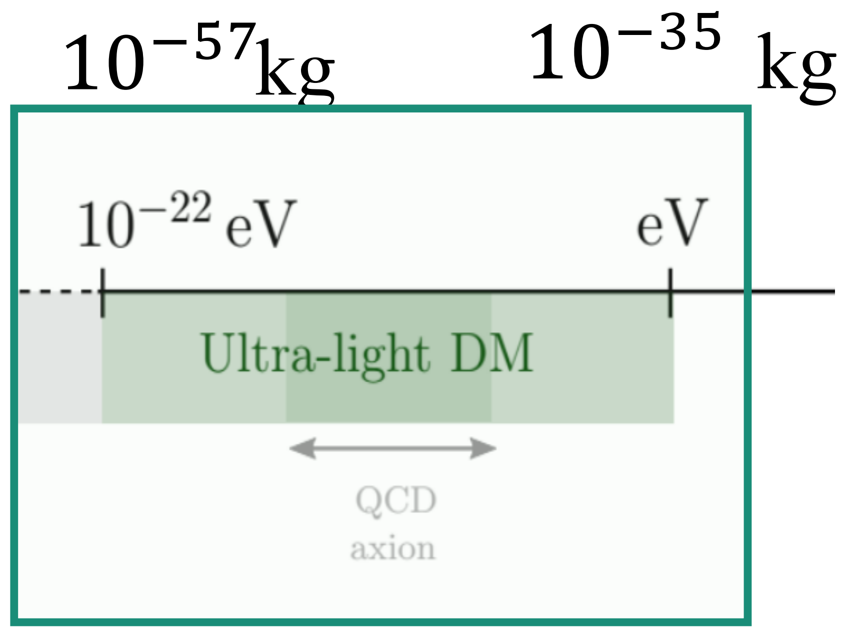
Finite-size halo, called « soliton » or « boson star »



$$V_{\rm I}(\phi) = \Lambda^4 \frac{\lambda_{2n}}{2n} \frac{\phi^{2n}}{\Lambda^{2n}}$$

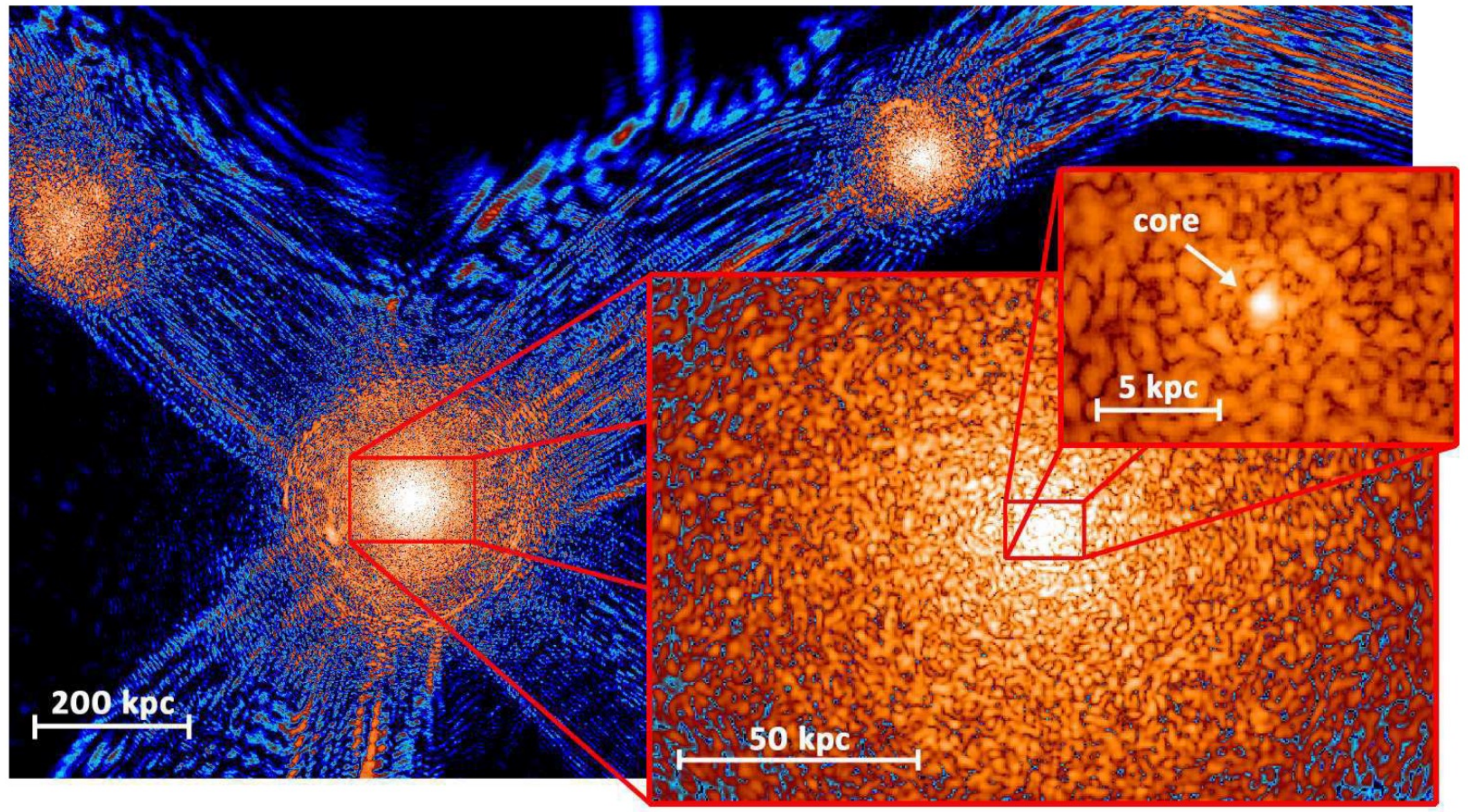
III- SOLITON FORMATION

A) Formation of a FDM soliton inside cosmological halos



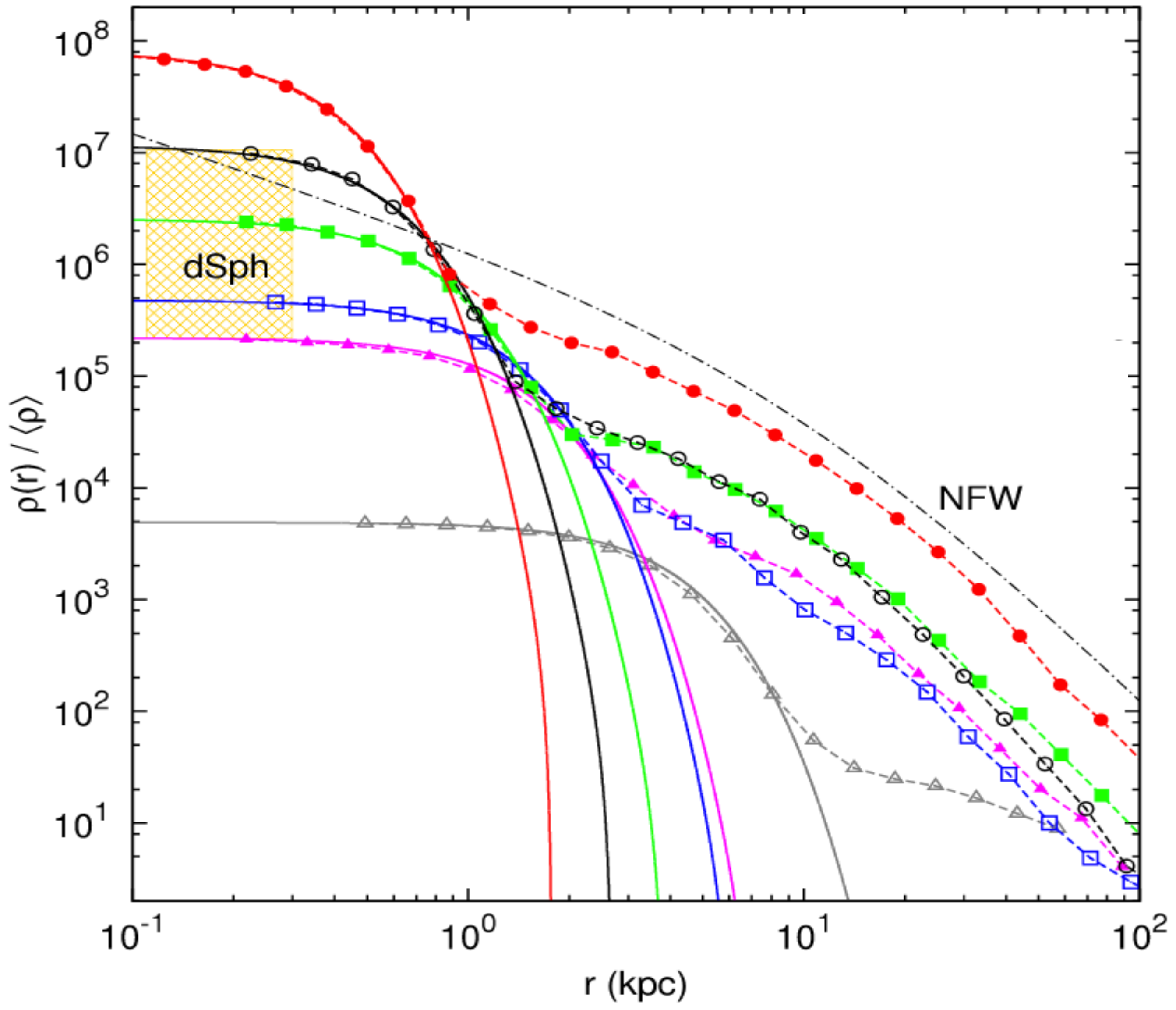
Because of its ultra-light mass → Large de Broglie wavelength, $\lambda_{dB} \sim 1/mv$

- $\lambda_{dB} \sim$ pc — kpc
- Small scales: wavelike behaviour.
- **Solitons**: stable equilibrium configurations → **Flat density profile.**



A slice of density field of ψDM simulation on various scales at z=0.1

Schive, Chiueh, and Broadhurst (2014)



Radial density profiles of haloes formed in the ψDM model

B) Formation of a SFDM soliton inside halos

Initial conditions: halo (+ central soliton): $\psi_{\rm initial} = \psi_{\rm sol} + \psi_{\rm halo}$

$$\rho_{\rm sol}(r) = \rho_{0\rm sol} \frac{\sin(\pi r/R_{\rm sol})}{\pi r/R_{\rm sol}}, \quad \hat{\psi}_{\rm sol}(r) = \sqrt{\rho_{\rm sol}(r)}.$$

Stochastic halo: sum over eigenmodes of the target gravitational potential with random coefficients

$$\psi_{\rm halo}(\vec{x},t) = \sum_{n\ell m} a_{n\ell m} \hat{\psi}_{n\ell m}(\vec{x}) e^{-iE_{n\ell}t/\epsilon} \qquad -\frac{\epsilon^2}{2}\nabla^2\hat{\psi}_E + \bar{\Phi}\hat{\psi}_E = E\hat{\psi}_E$$

$$a_{n\ell m} = a(E_{n\ell}) e^{i\Theta_{n\ell m}} \qquad \bar{\Phi}(r) = \bar{\Phi}_N(r), \quad \nabla^2\bar{\Phi}_N = 4\pi\bar{\rho}$$

random phase

$$\langle \rho_{\rm halo} \rangle = \sum_{n\ell m} a(E_{n\ell})^2 |\hat{\psi}_{n\ell m}|^2 \longrightarrow$$

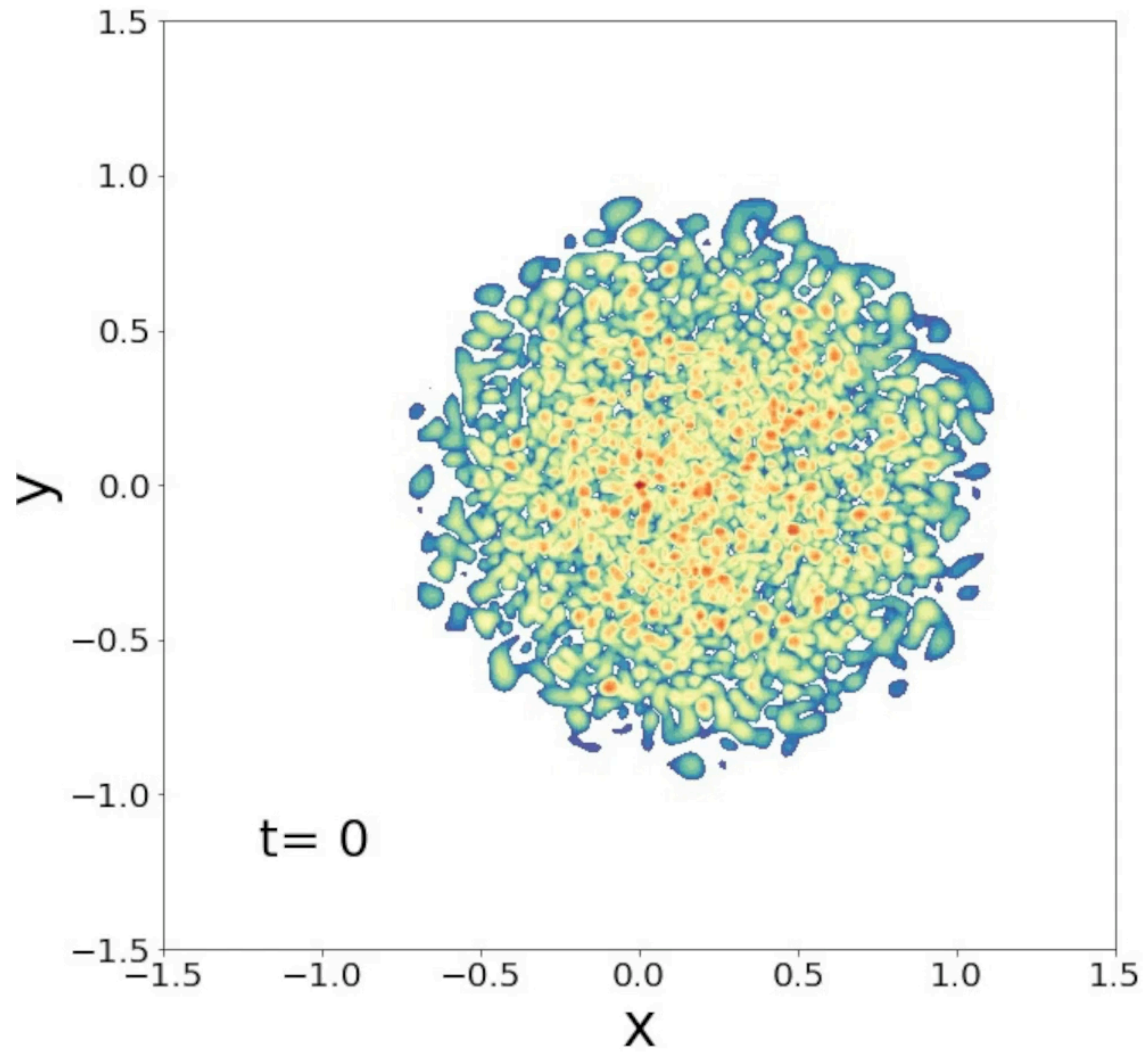
sets $a(E)$

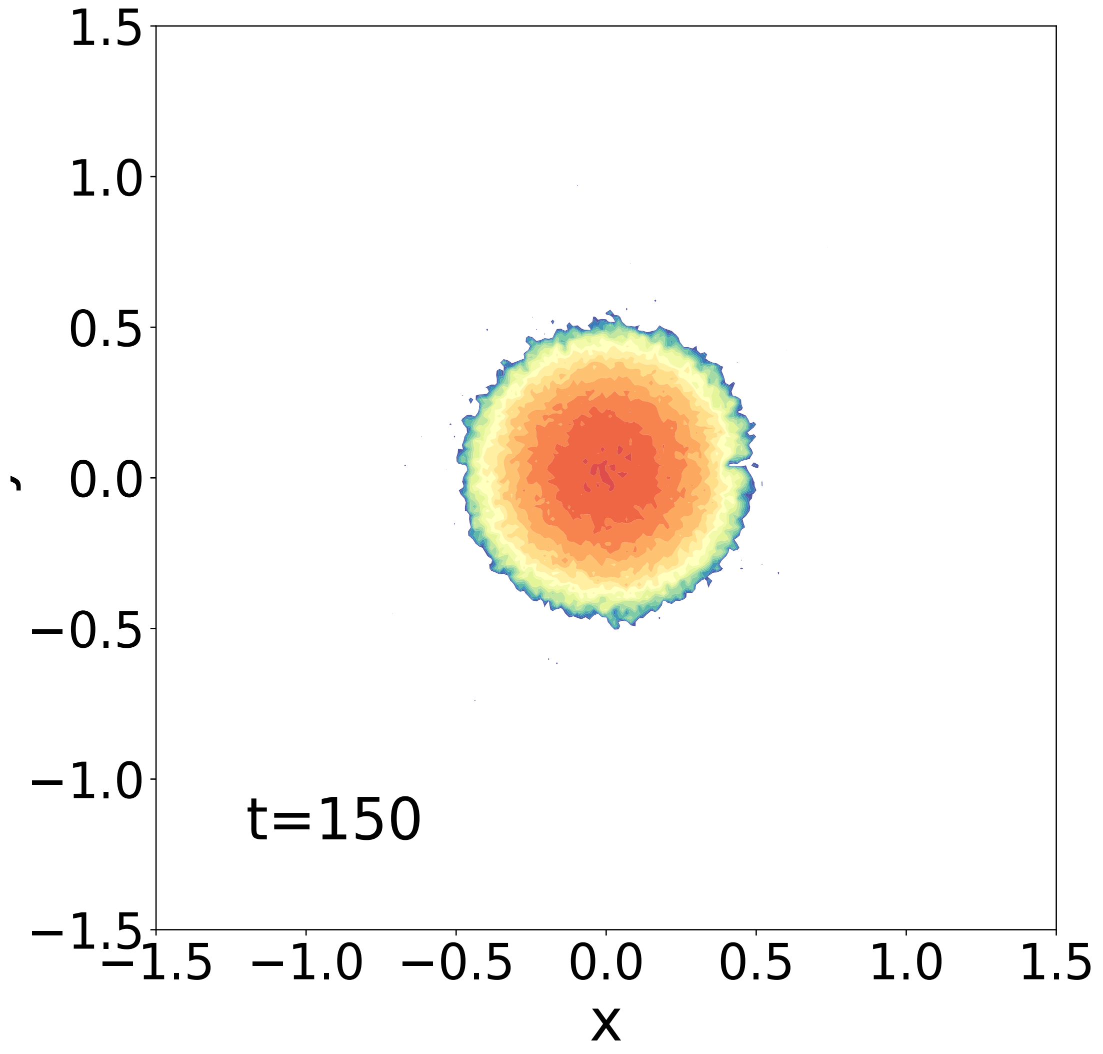
$$a(E)^2 = (2\pi\epsilon)^3 f(E) \qquad f(E) = \frac{1}{2\sqrt{2}\pi^2}\frac{d}{dE}\int_E^0 \frac{d\Phi_N}{\sqrt{\Phi_N - E}} \frac{d\rho_{\rm classical}}{d\Phi_N}$$

(Eddington formula)

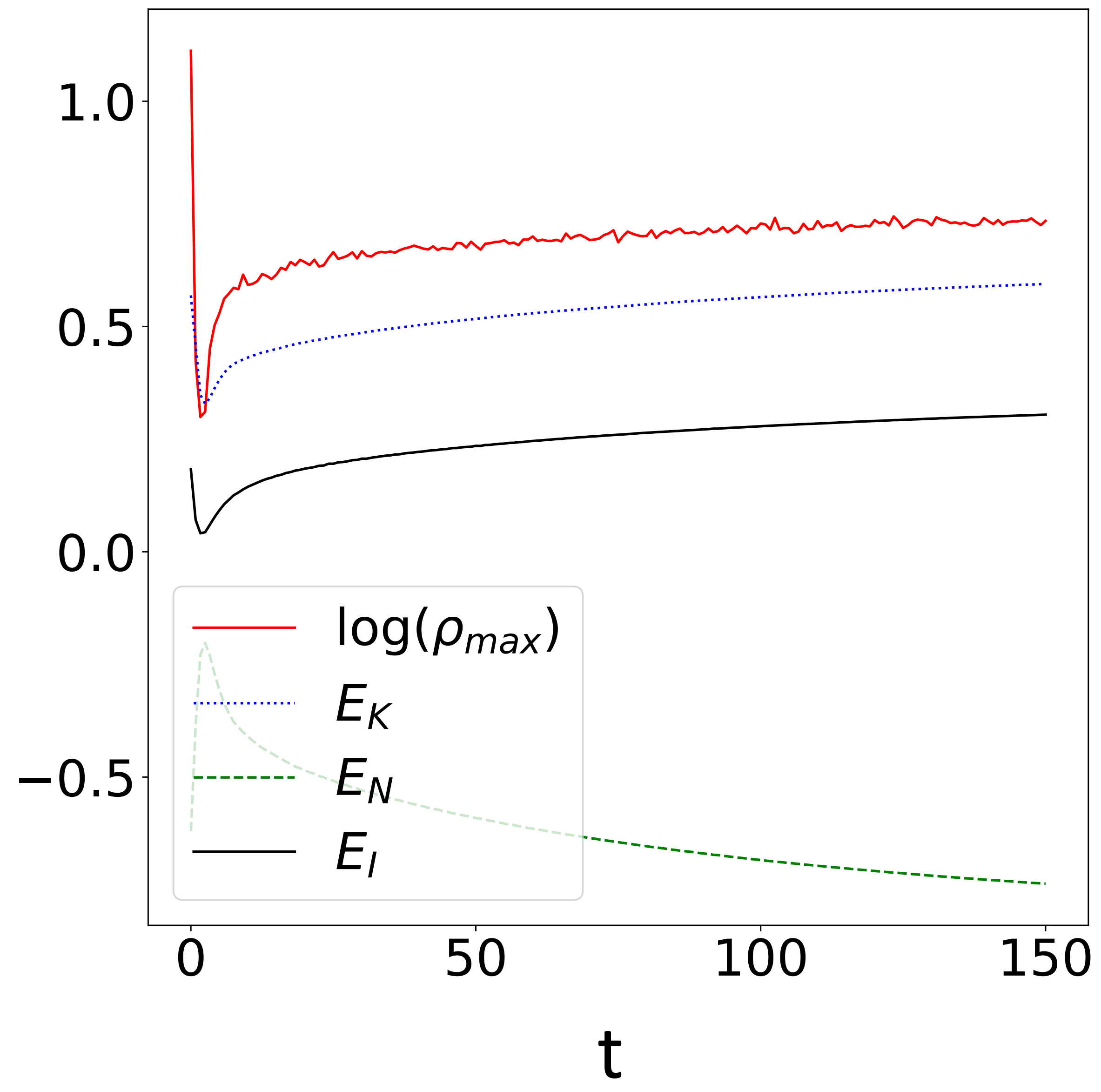
1) Characteristic scale of the same order as the halo size

Density slice

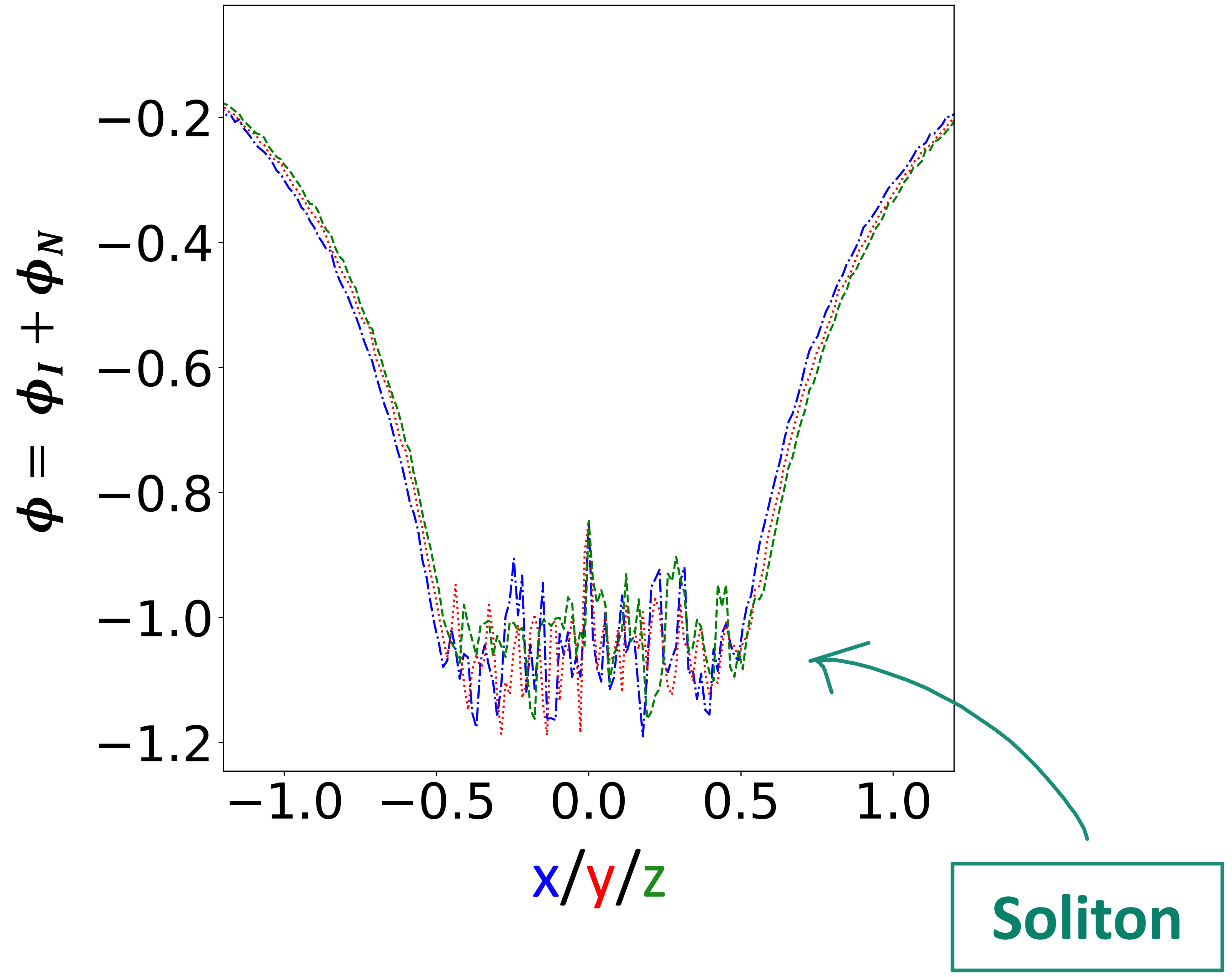




$\rho_{max}, E_K, E_I, E_N$



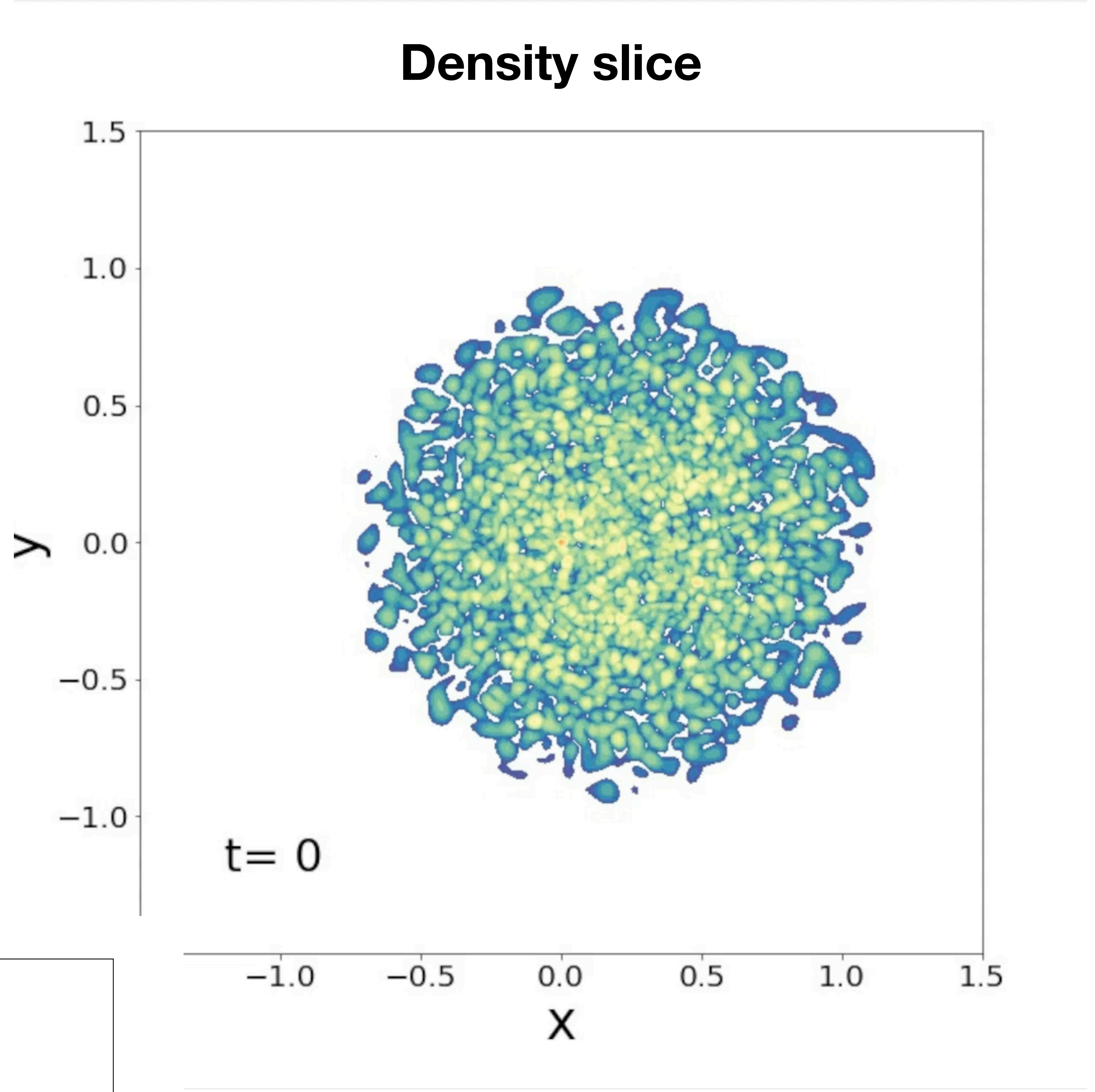
Potential at t=150

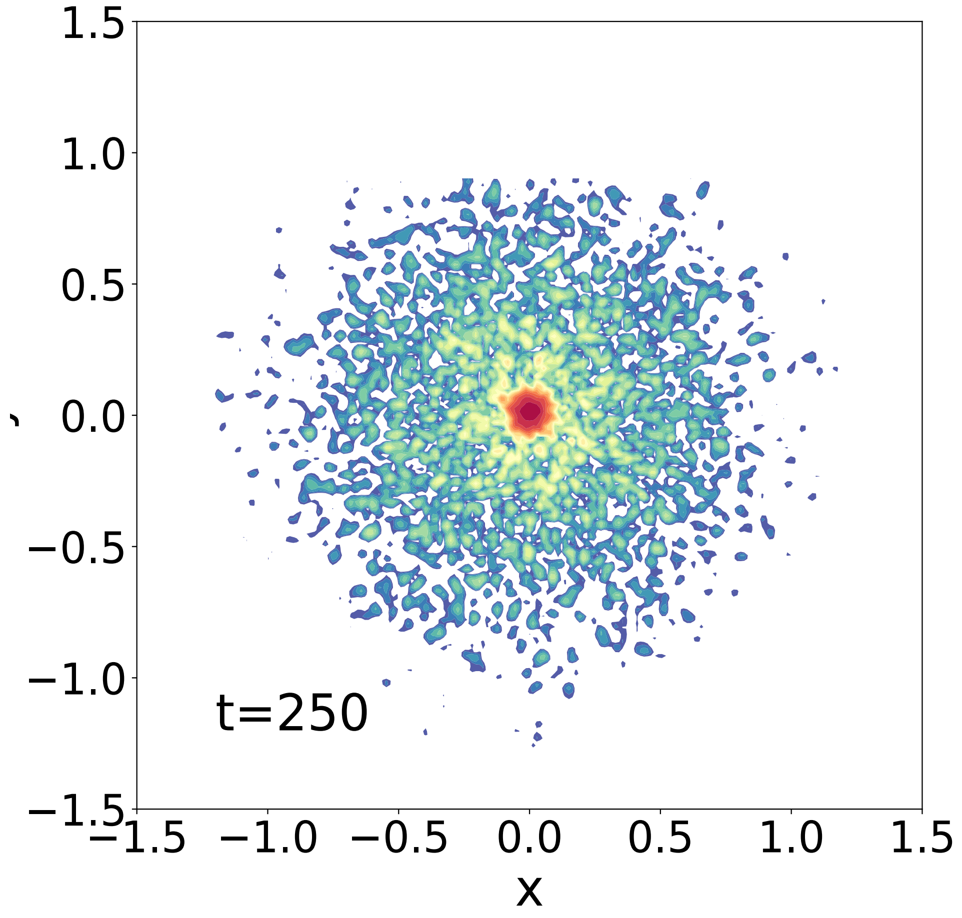


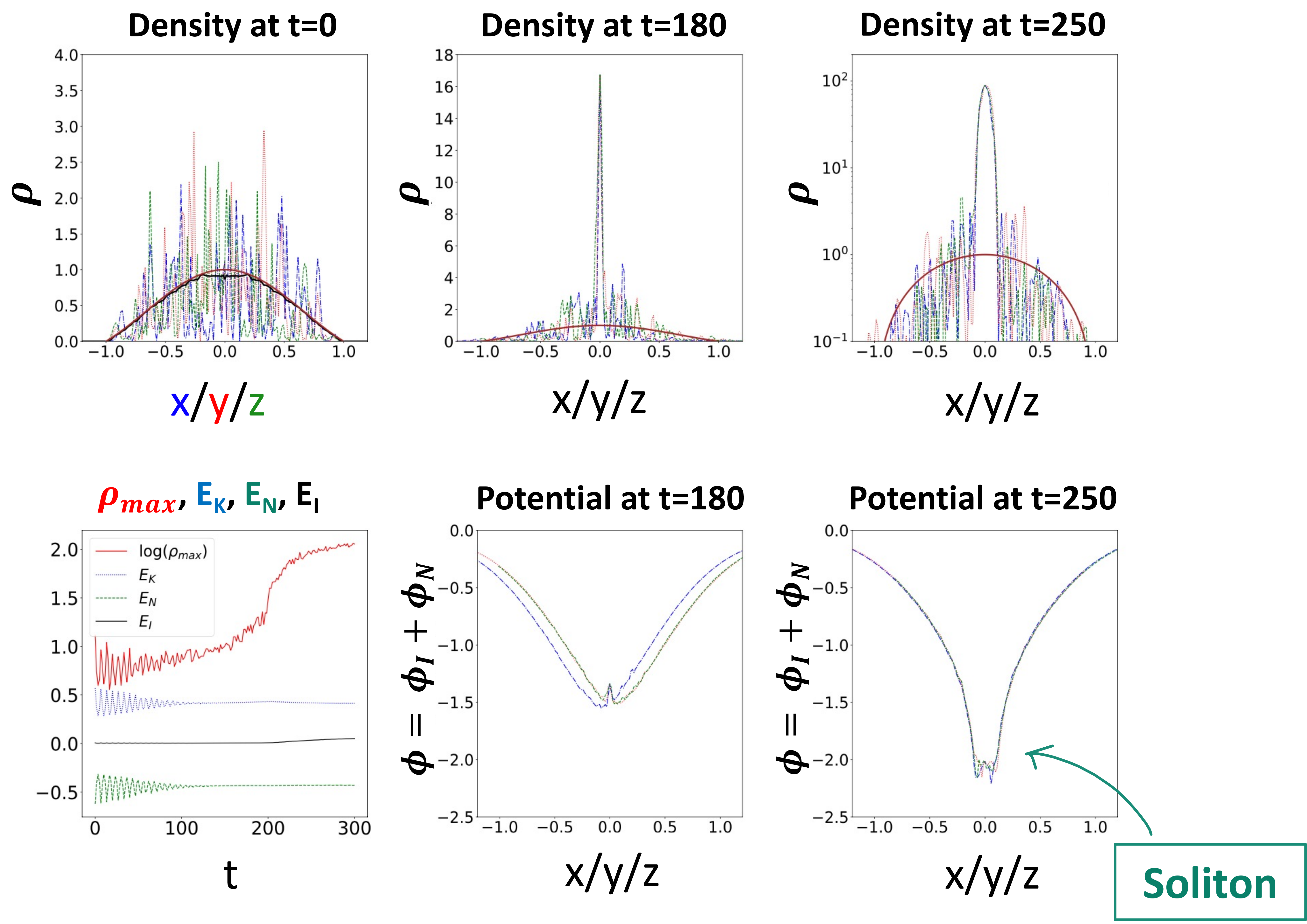
$$\Phi_N + \Phi_I = \alpha$$

- At t ~ 8, the soliton is formed with R_{sol} = 0.5 and contains about 40% of the total mass.
- The system reaches a quasi-stationary state.
- Afterwards, ρ_{max} and the energies only show a slow evolution.

2) Characteristic scale much smaller than the halo size





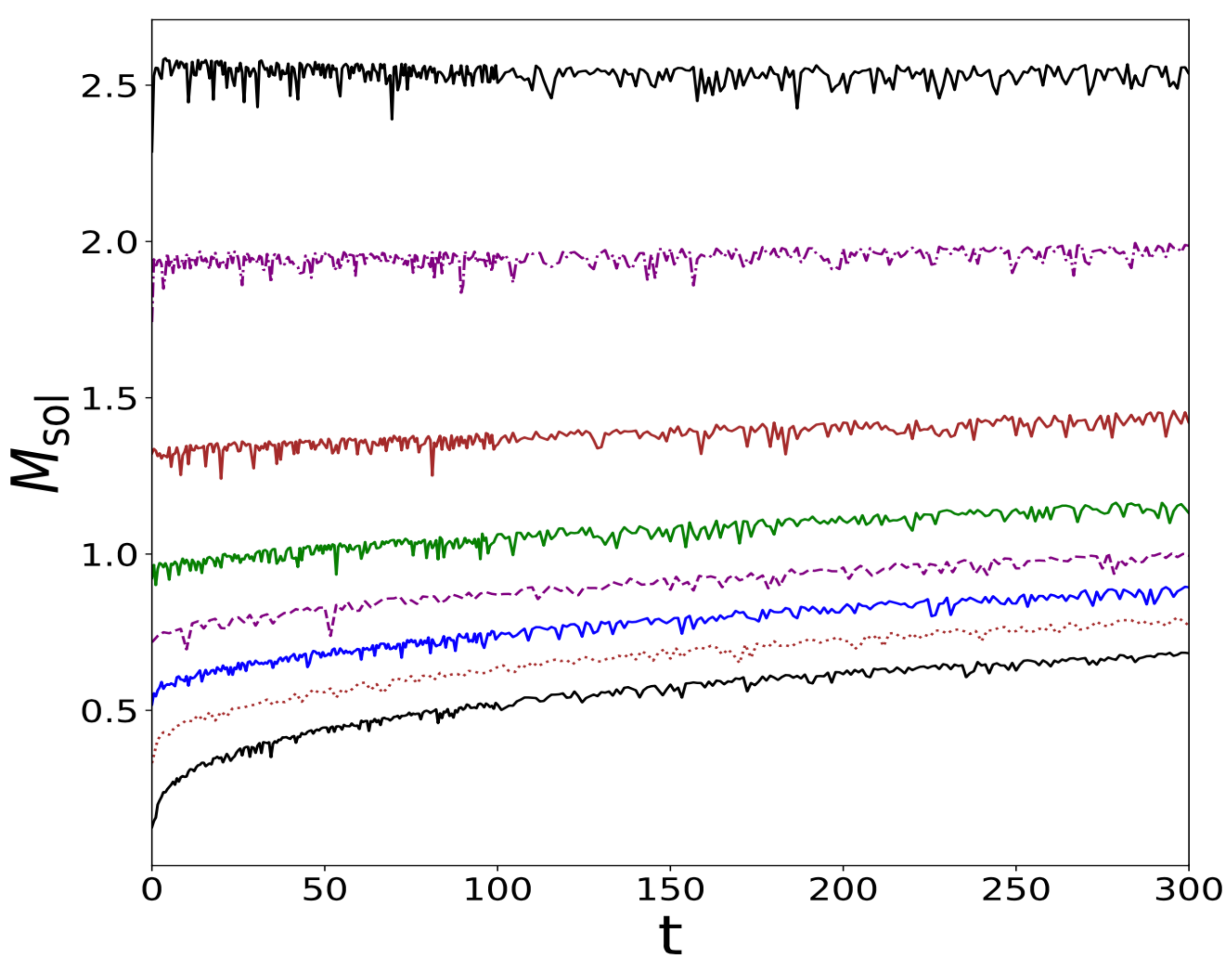


- By t ~ 100, the halo relaxes to a quasi-stationary state.
- At t ~180, FDM peak.
- At t ~ 200, self-interacting soliton forms, R_{sol} = 0.1 .

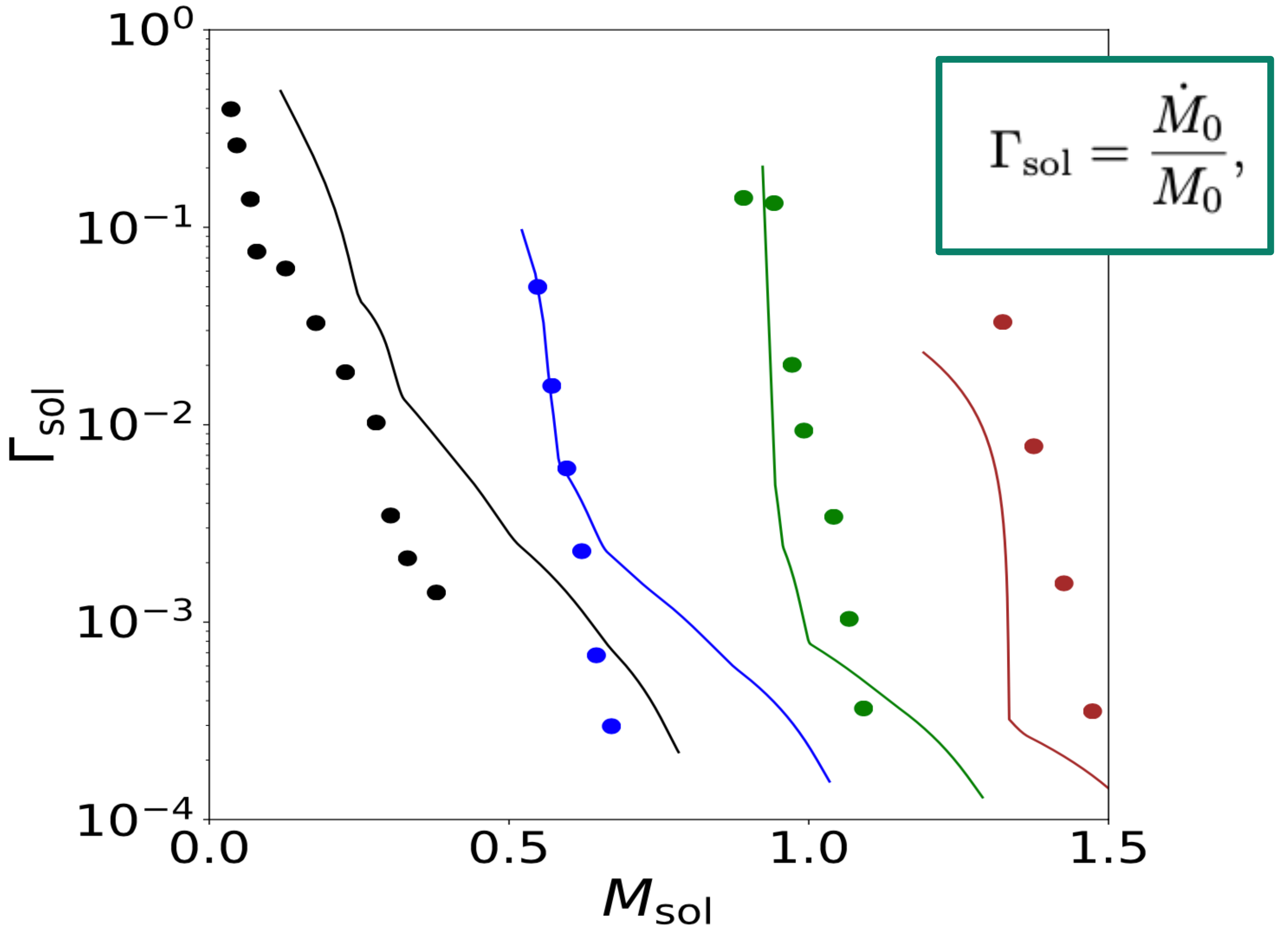
Transition from a FDM phase to a self-interacting phase.

C) Dependence of the soliton mass on the formation history

Growth with time of the soliton $M_{sol}(t)$



Growth rate as a function of M_{sol}



- **The soliton always grows**, with a growth rate that decreases with time.
- The numerical simulations suggest that the central soliton can slowly grow until it makes a large fraction of the total mass of the system, of the order of **40%**.
- There is **no clear sign of a scaling regime**, as the growth rate still depends on the initial conditions at late times.
- **Our ansatz underestimate Γsol**, which remains positive but steadily decreasing in the numerical simulations.

Probably no well-defined halo-mass/soliton mass relation

Kinetic theory → Eq. similar to 4-wave systems for the soliton (more complicated for higher states)

$$\dot{M}_0 = \frac{\pi}{\epsilon} \sum_{123} M_0 M_1 M_2 M_3 \, \delta_D(\omega_{01}^{23}) \, (V_{02;13} + V_{03;12})^2 \left(\frac{1}{M_0} + \frac{1}{M_1} - \frac{1}{M_2} - \frac{1}{M_3} \right)$$

BH dynamics inside DM solitons:

Accretion, dynamical friction, and GW

I- RADIAL INFALL ONTO A BH

Bondi problem $1 < \gamma < 5/3$

Here: $\gamma = 2$

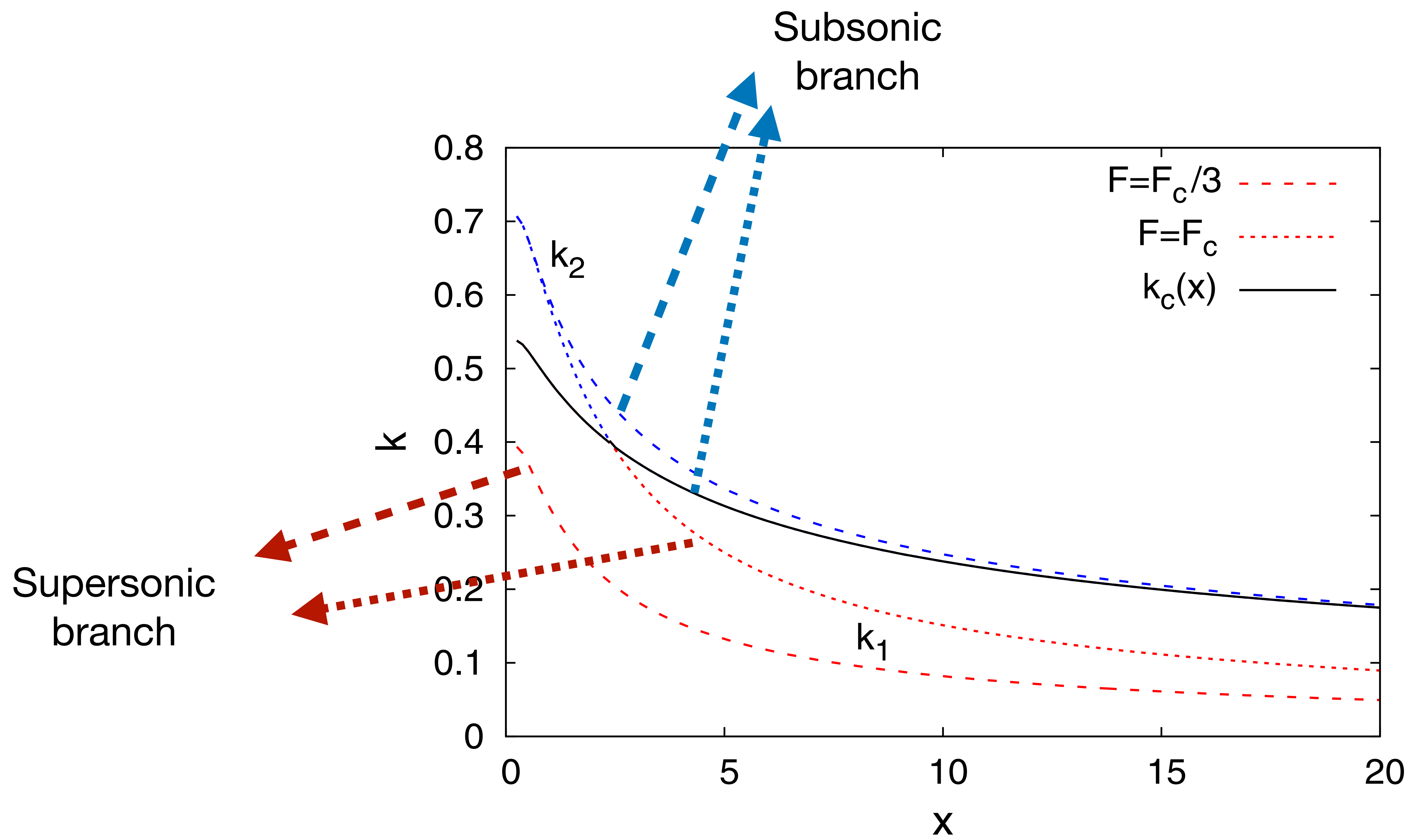
Relativistic close to BH horizon → Klein-Gordon

$$\frac{\partial^2 \phi}{\partial t^2} - \sqrt{\frac{f}{h^3}} \frac{1}{r^2} \frac{\partial}{\partial r}\left[\sqrt{f h} r^2 \frac{\partial \phi}{\partial r}\right] + f m^2 \phi + f \lambda_4 \phi^3 = 0.$$

$$\phi = \phi_0(r) \mathrm{cn}[\omega(r) t - \mathbf{K}(r) \beta(r), k(r)]$$

$$m \gg \nabla$$

As for Bondi problem, there is a critical flux where there is a unique transsonic solution



$$\dot{M}_{\mathrm{Bondi-Hoyle}} = \frac{2\pi \rho_0 \mathcal{G}^2 M_{\mathrm{BH}}^2}{(c_s^2 + v_0^2)^{3/2}}$$

$$\dot{m}_{\max} = 3\pi F_\star \rho_a r_s^2 c = \frac{12\pi F_\star \rho_0 \mathcal{G}^2 m_{\mathrm{BH}}^2}{c_s^2 c}$$

relativistic,
much smaller than Bondi

II- LARGE-DISTANCE DOMAIN

Far from the BH: hydrodynamical equations of an isentropic gas of effective adiabatic index $\gamma = 2$

Continuity eq. + Euler eq.

Potential flow $\vec{v} = \nabla\beta$

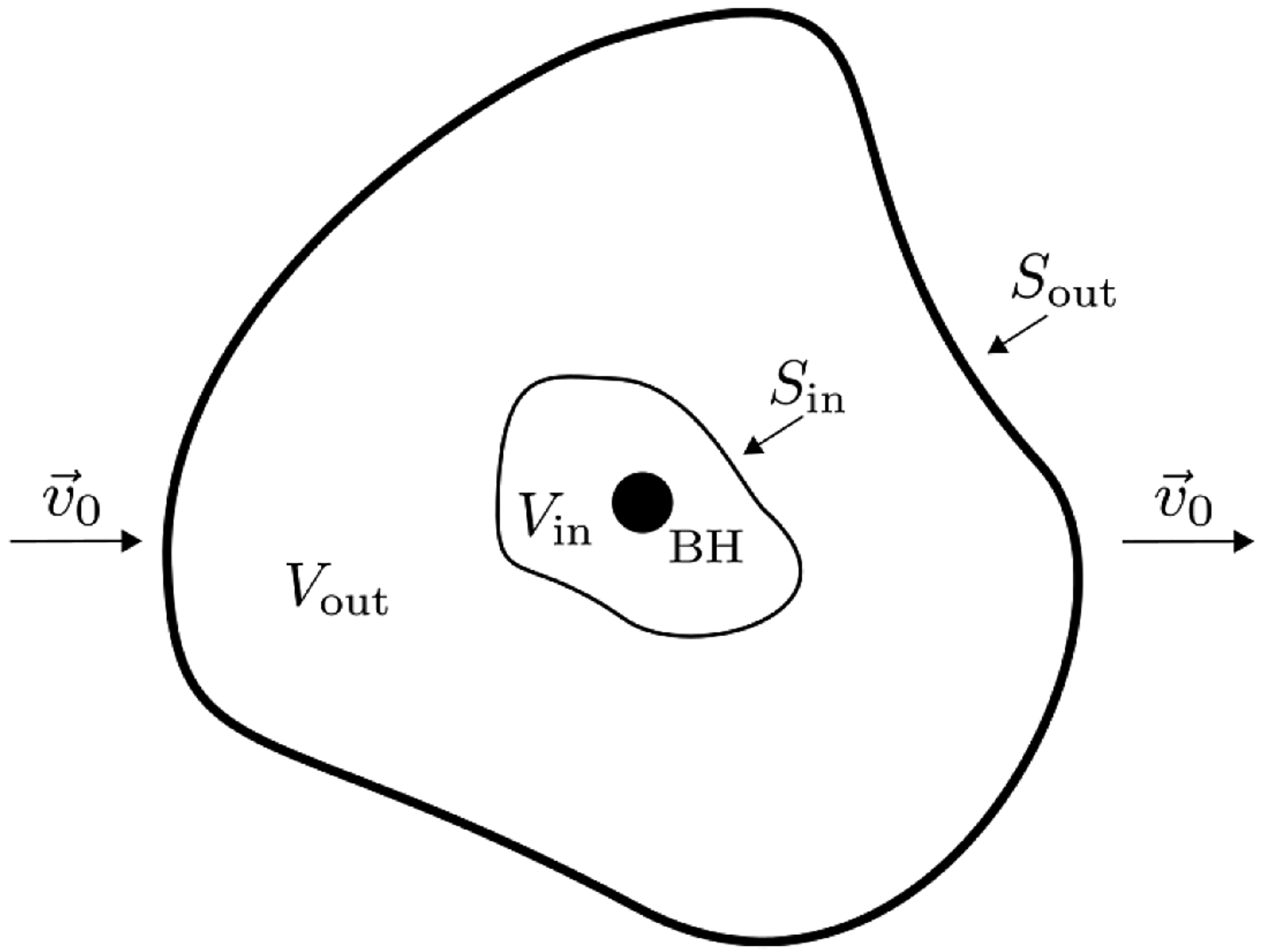
Bernoulli eq.: $$\frac{v^2}{2} - \frac{1}{2r} + \frac{\rho}{2} = \text{constant}$$

BH gravity

effective pressure

Isentropic potential flow eq.: $$\hat{\nabla}\cdot\left[\left(\hat{\rho}_0 + \frac{1}{\hat{r}} + v_0^2 - (\hat{\nabla}\hat{\beta})^2\right)\hat{\nabla}\hat{\beta}\right] = 0.$$

Steady state, in the BH frame



Conservation of mass and momentum allow us to obtain the mass and momentum flux through any arbitrarily distant surface:

Allows us to obtain analytical results from large-distance expansions

Linear deviation from uniform flow: $$\frac{\partial^2 \delta\beta}{\partial x^2} + \frac{\partial^2 \delta\beta}{\partial y^2} + \left(1 - \frac{v_0^2}{c_{s0}^2}\right) \frac{\partial^2 \delta\beta}{\partial z^2} = \frac{v_0 z}{\rho_0 r^3}$$

subsonic / supersonic regimes

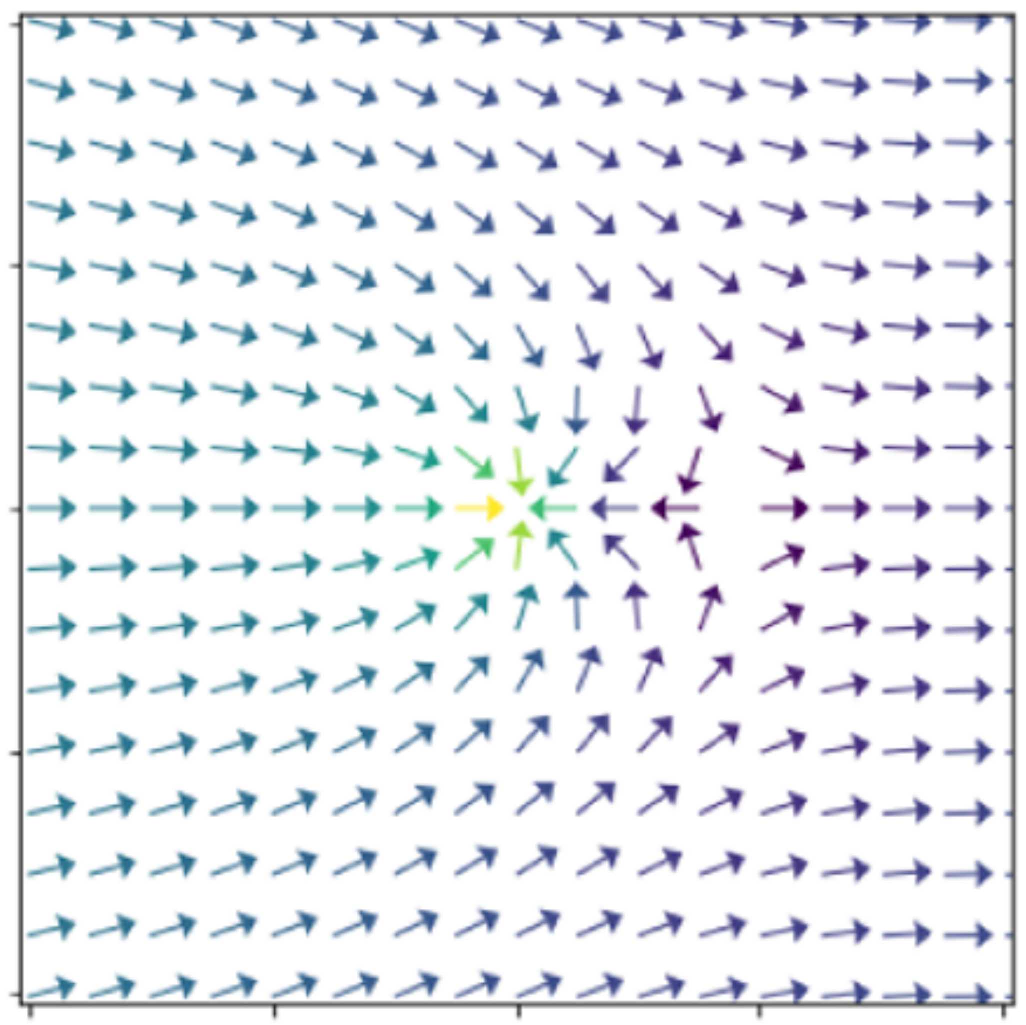
III- SUBSONIC REGIME

Exact analytical results using a large-distance expansion: $\hat{\beta} = \hat{\beta}_{-1} + \hat{\beta}_0 + \hat{\beta}_1 + \ldots, \quad \text{with} \quad \hat{\beta}_n \sim \hat{r}^{-n}$

$$\rho_{\text{even}} = \rho_0 + \frac{\mathcal{G} M_{\text{BH}} \rho_0}{c_s \sqrt{(c_s^2 - v_0^2) r^2 + v_0^2 z^2}} + \ldots, \qquad \rho_{\text{odd}} = \frac{4 B \rho_0 \mathcal{G}^2 M_{\text{BH}}^2 v_0 c_s z}{[(c_s^2 - v_0^2) r^2 + v_0^2 z^2]^{3/2}} + \ldots$$

1 remaining integration constant *B*

Velocity field (v)



Conservation of mass: *B* in terms of $\dot{m}_{\text{BH}}$

Conservation of momentum: $$F_z = \frac{dp_z}{dt} = -\int_{S_{\text{out}}} \overrightarrow{dS} \cdot \rho \vec{v} v_z - \int_{S_{\text{out}}} \overrightarrow{dS} \cdot P \vec{e}_z$$

Accretion drag force, no dynamical friction

(d'Alembert paradox)

IV- SUPERSONIC REGIME

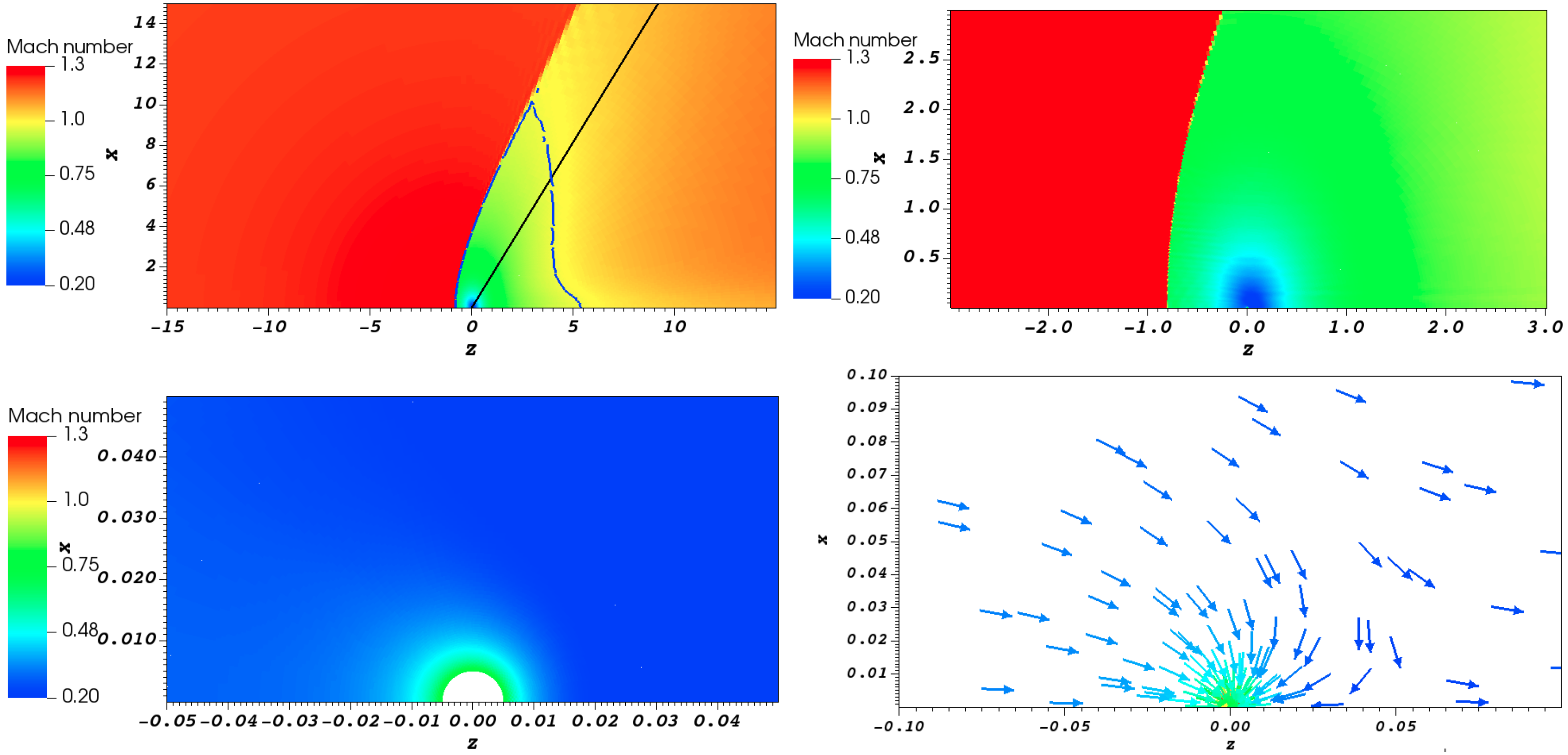
A) Moderate Mach numbers

$$v_0 < \frac{c_{s0}^{2/3}}{(3F_\star)^{1/3}} : \quad \dot{M}_{\rm BH} = \frac{12\pi F_\star \rho_0 \mathcal{G}^2 M_{\rm BH}^2}{c_{s0}^2}$$

Max. radial accretion rate

Shock front upstream of the BH, radial accretion close to the BH

3 maps of the Mach number (3 zoom-in onto the BH) and 1 map of the velocity field



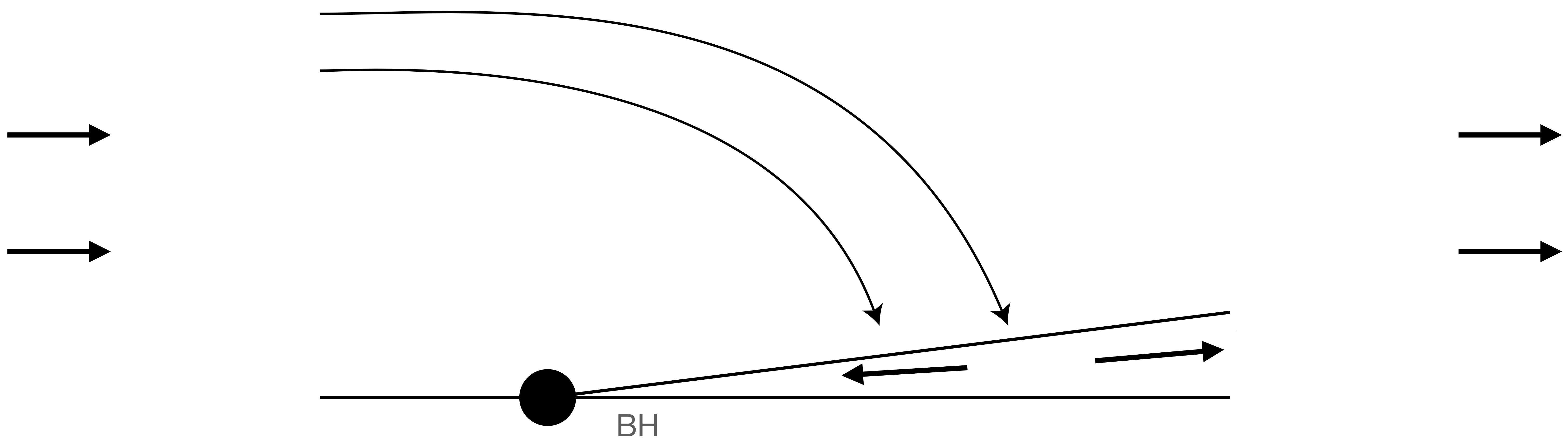
B) High Mach numbers

$$v_0 > \frac{c_{s0}^{2/3}}{(3F_\star)^{1/3}} : \quad \dot{M}_{\rm BH} = \frac{4\pi\rho_0\mathcal{G}^2M_{\rm BH}^2}{v_0^3}$$

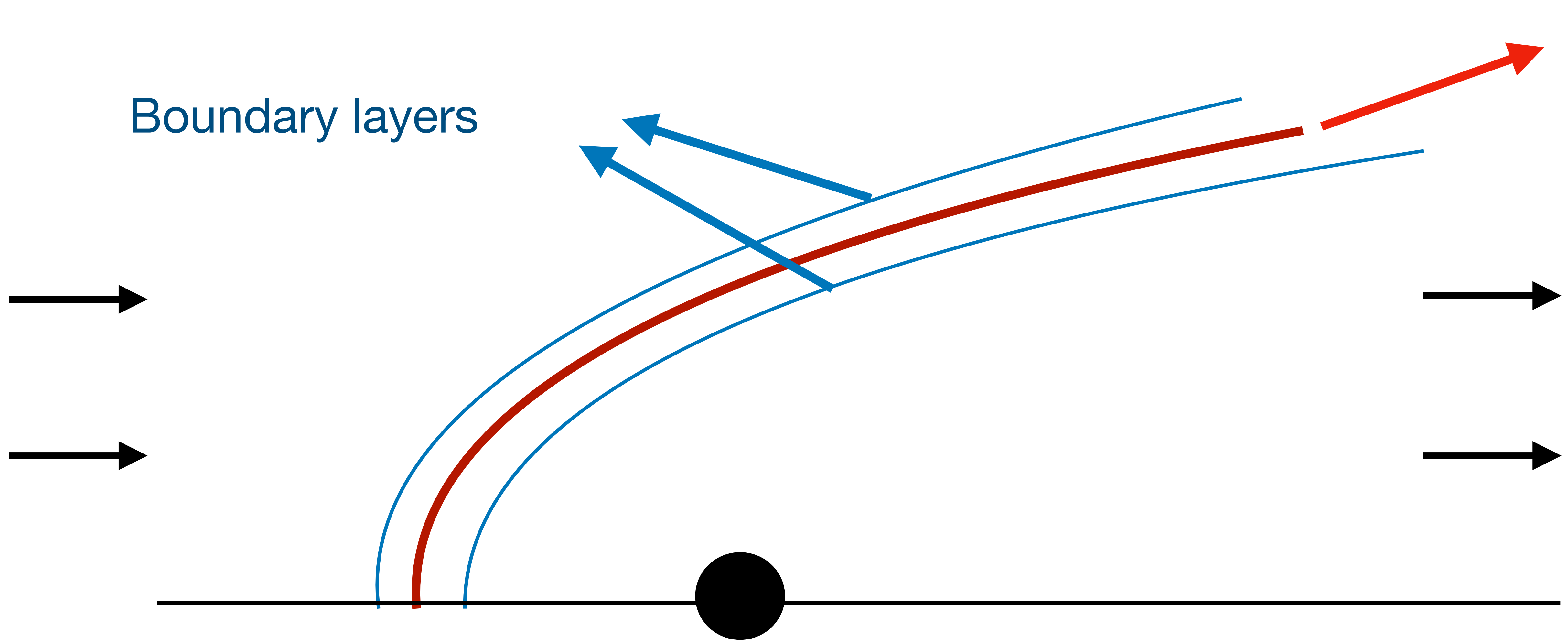
BHL accretion rate

Bondi-Hoyle-Lyttleton accretion mode   Edgar (2004)

BH frame: incoming dark matter fluid, accretion column at the rear



C) Exact analytical results using large-distance expansions:



$$u = \cos\theta$$

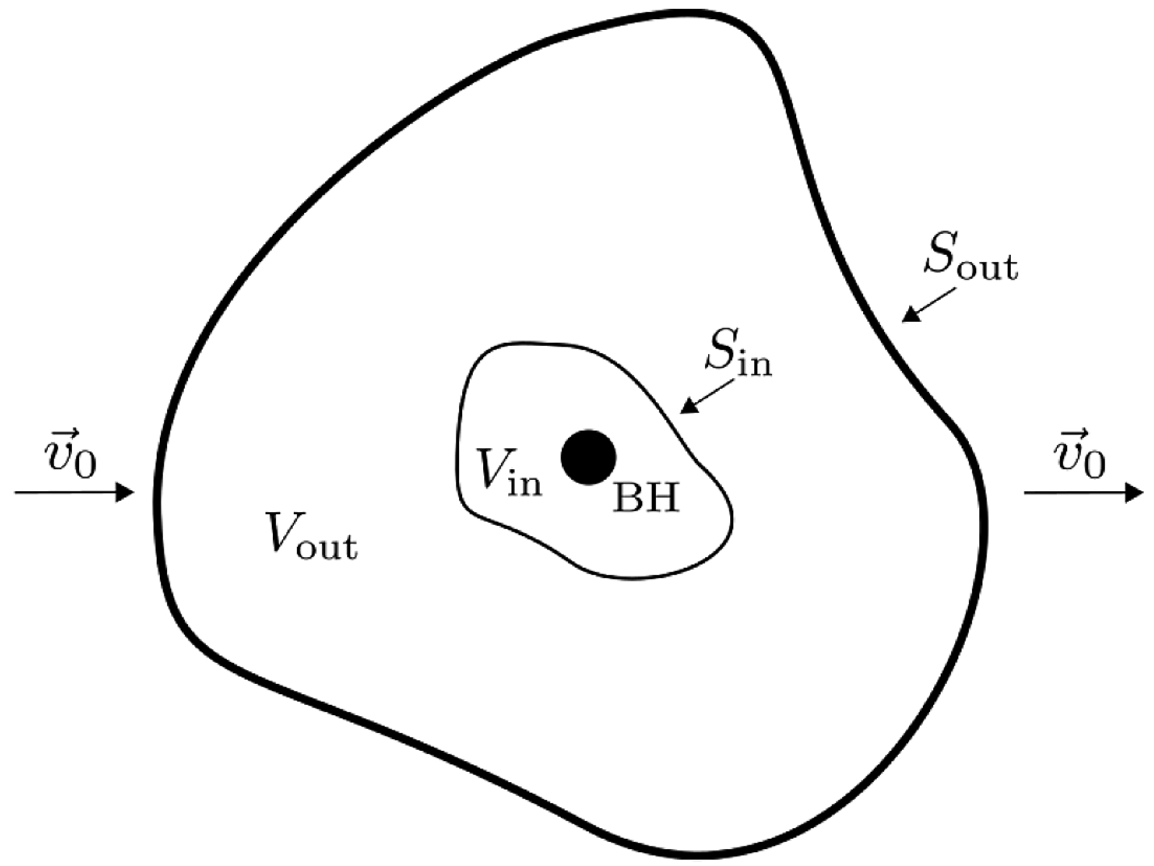
In the bulk, upstream: $$\hat{\beta} = v_0 r u + a\ln(r) + f_0(u) + \frac{f_1(u)}{r} + \dots$$

In the bulk, downstream: $$\hat{\beta} = v_0 r u + a\ln(r) + f_0(u) + \frac{f_1(u) + g_1(u)\ln(r)}{r} + \dots$$

In the boundary layers: $$\hat{\beta} = v_0 \hat{r} u - \frac{1}{2v_0}\ln[\hat{r}(1-u_c)] + \frac{F_1(U)}{\hat{r}^{1/3}} + \frac{F_2(U)}{\hat{r}^{2/3}} + \frac{F_3(U) + \mathcal{F}_3(U)\ln\hat{r}}{\hat{r}} + \dots$$

$$U = \hat{r}^{2/3}[u - u_s(\hat{r})]$$

D) Dynamical friction



Again, use conservation of mass and momentum:

$$F_z = \dot{M}_{\rm BH} v_0 + \frac{8\pi\rho_0 \mathcal{G}^2 M_{\rm BH}^2}{3v_0^2} \ln\left(\frac{r_a}{r_{\rm UV}}\right)$$

Accretion drag

Dynamical friction

2/3 smaller than Chandrasekhar's expression

UV cutoff greater than b_min and set by the self-interactions: $r_{\rm UV} = 6\sqrt{\frac{2}{e}}\frac{\mathcal{G}m_{\rm BH}}{c_s^2}\left(\frac{c_s}{v_{\rm BH}}\right)^{3/2}$

$$r_{\rm UV} \simeq \sqrt{\frac{18}{e}} r_{\rm sg} \mathcal{M}_0^{-3/2} \qquad r_{\rm sg} = \frac{r_s}{c_{s0}^2}, \quad c_{s0}^2 = \frac{\rho_0}{\rho_a}$$

V- GRAVITATIONAL WAVES EMITTED BY A BH BINARY INSIDE A SCALAR CLOUD

A) Additional forces on the BHs due to the dark matter environment

Gravity of the dark matter cloud:

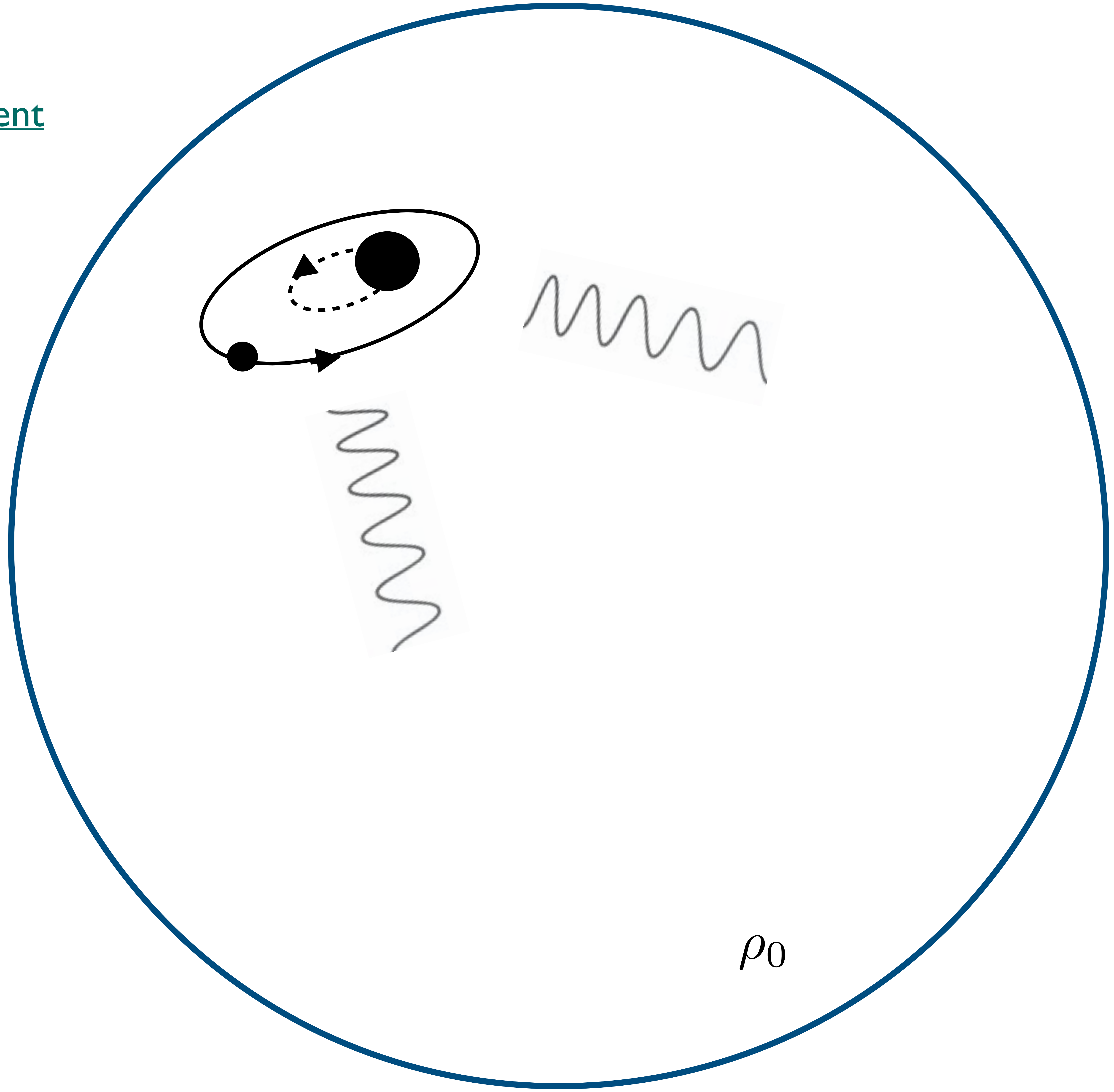
$$m_{\rm BH}\dot{\mathbf{v}}_{\rm BH}|_{\rm halo} = -\frac{4\pi}{3}\mathcal{G}m_{\rm BH}\rho_0(\mathbf{x}-\mathbf{x}_0)$$

Accretion drag:

$$m_{\rm BH}\dot{\mathbf{v}}_{\rm BH}|_{\rm acc} = -\dot{m}_{\rm BH}\mathbf{v}_{\rm BH}$$

Dynamical friction:

$$m_{\rm BH}\dot{\mathbf{v}}_{\rm BH}|_{\rm df} = -\frac{8\pi\mathcal{G}^2 m_{\rm BH}^2\rho_0}{3v_{\rm BH}^3}\ln\left(\frac{r_{\rm IR}}{r_{\rm UV}}\right)\mathbf{v}_{\rm BH}$$



B) Decay of the orbital radius

$$\langle \dot{a} \rangle = \langle \dot{a} \rangle_{\rm acc} + \langle \dot{a} \rangle_{\rm df} + \langle \dot{a} \rangle_{\rm gw}$$

$$\langle \dot{a} \rangle_{\rm gw} = -\frac{64\nu\mathcal{G}^3 m^3}{5c^5 a^3}\left(1 - \frac{4\pi\rho_0 a^3}{3m}\right)$$

Correction due to the halo bulk gravity

$$\langle \dot{a} \rangle_{\rm acc} = -aA_{\rm acc} - a\left(\frac{a}{\mathcal{G}m}\right)^{3/2} B_{\rm acc}$$

Accretion drag

$$\langle \dot{a} \rangle_{\rm df} = -a\left(\frac{a}{\mathcal{G}m}\right)^{3/2}\left[B_{\rm df} + C_{\rm df}\ln\left(\sqrt{\frac{\mathcal{G}m}{a}}\frac{1}{c_s}\right)\right]$$

Dynamical friction

C) Phase of the gravitational waveform

GW frequency: $$\mathfrak{f} = \frac{1}{\pi}\sqrt{\frac{\mathcal{G}m}{a^3}}\left(1 + \frac{2\pi\rho_0 a^3}{3m}\right)$$

Frequency drift: $$\dot{\mathfrak{f}} = \frac{1}{\pi}\sqrt{\frac{\mathcal{G}m}{a^3}}\left(\frac{\dot{m}}{2m} - \frac{3\dot{a}}{2a}\right) + \mathcal{G}\rho_0\left(\frac{a^3}{\mathcal{G}m}\right)^{1/2}\frac{\dot{a}}{a}$$

Phase: $\Phi(t) = 2\pi\int d\mathfrak{f}\,(\mathfrak{f}/\dot{\mathfrak{f}})$ Time: $t = \int d\mathfrak{f}\,(1/\dot{\mathfrak{f}})$

Fourier transform of the GW signal: $\tilde{h}(f) = \mathcal{A}(f)e^{i\Psi(f)}$

Phase: $$\Psi(f) = 2\pi f t_c - \Phi_c - \frac{\pi}{4} + \Psi_{\rm gw} + \Psi_{\rm halo} + \Psi_{\rm acc} + \Psi_{\rm df}$$

DM corrections

$$\Psi_{\rm gw} = \frac{3}{128}\left(\frac{\pi\mathcal{G}\mathcal{M}f}{c^3}\right)^{-5/3}\left[1 + \frac{20}{9}\left(\frac{743}{336} + \frac{11}{4}\nu\right)\left(\frac{\pi\mathcal{G}mf}{c^3}\right)^{2/3}\right]$$ O + 1 PN

$\Psi_{\rm halo}$ -3 PN $\Psi_{\rm acc}$ -4.5 / -5.5 PN $\Psi_{\rm df}$ -5.5 PN

D) Region in the parameter space that can be detected

ρ_0 halo bulk density

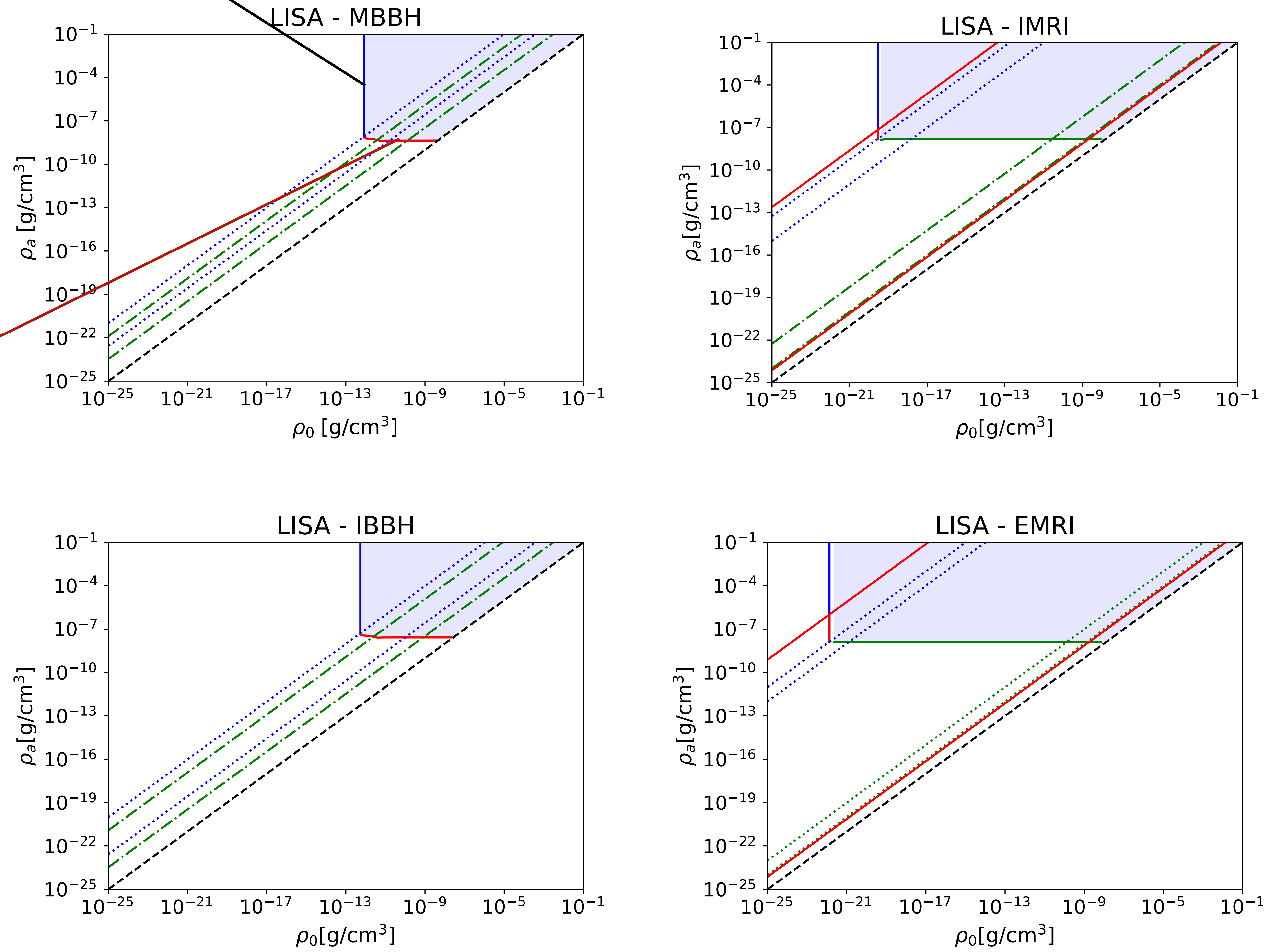
$$\rho_a = \frac{4m^4}{3\lambda_4}$$

LISA:

BHL accretion mode

Plane (ρ_0, ρ_a)

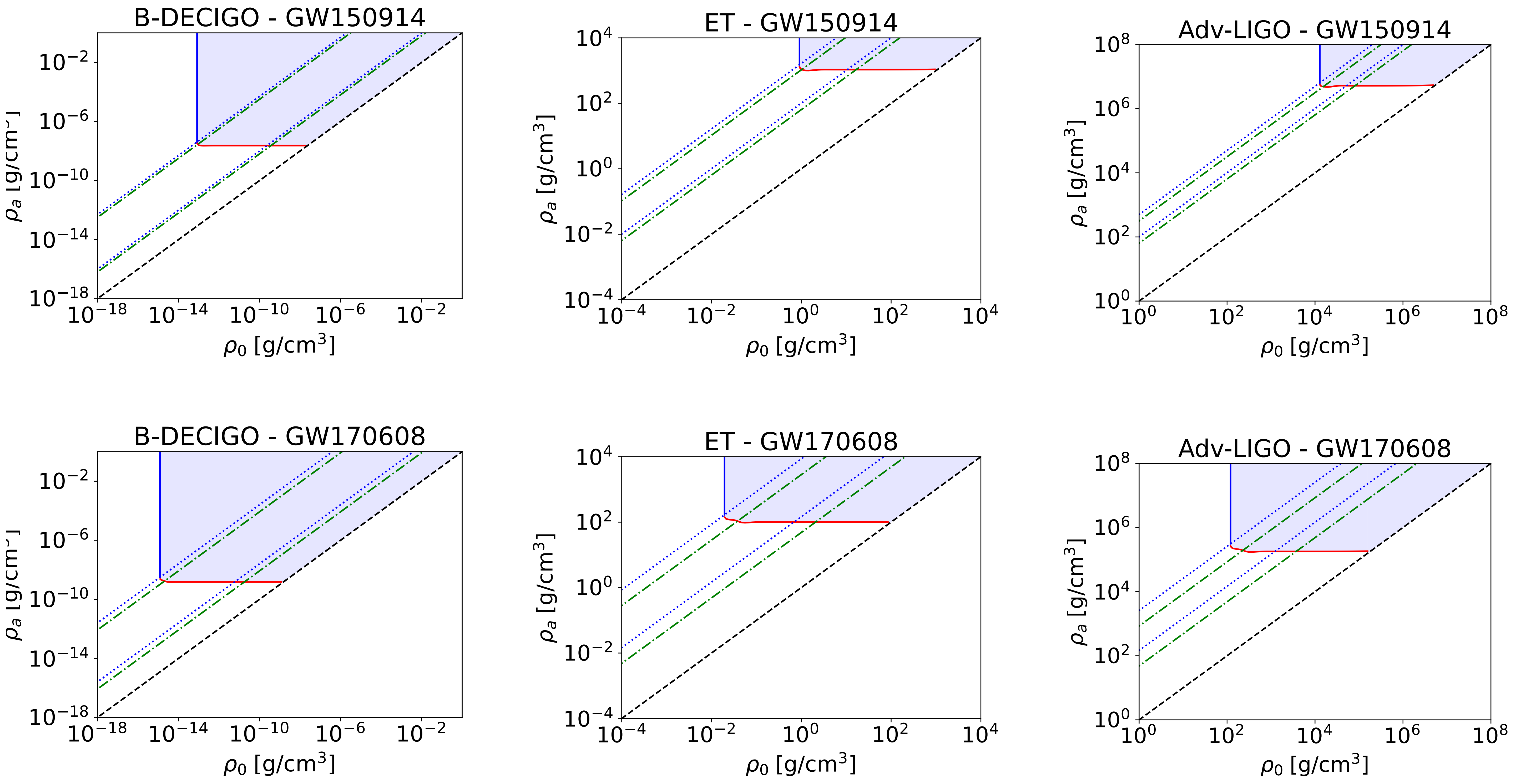
Max. radial accretion mode



$$\frac{\rho_a}{\rho_0} = \frac{c^2}{c_s^2} \geq 1$$

Event \ Properties	m_1 ($M_\odot$)	m_2 ($M_\odot$)	χ_1	χ_2	$\chi_{\rm eff}$
MBBH	10^6	5×10^5	0.9	0.8	0.87
IBBH	10^4	5×10^3	0.3	0.4	0.33
IMRI	10^4	10	0.8	0.5	0.80
EMRI	10^5	10	0.8	0.5	0.80

$$1\ M_\odot/\mathrm{pc}^3 = 6.7\times10^{-23}\ \mathrm{g/cm}^3$$



ρ_0 halo bulk density

$$\rho_a = \frac{4m^4}{3\lambda_4}$$

Event \ Detector	LISA	B-DECIGO	ET	Adv-LIGO
MBBH	$\rho_0 > 8 \times 10^{-13}$ g/cm^3 $\rho_a > 5 \times 10^{-9}$ g/cm^3	× ×	× ×	× ×
IBBH	$\rho_0 > 5 \times 10^{-13}$ g/cm^3 $\rho_a > 3 \times 10^{-8}$ g/cm^3	× ×	× ×	× ×
IMRI	$\rho_0 > 3 \times 10^{-20}$ g/cm^3 $\rho_a > 2 \times 10^{-8}$ g/cm^3	× ×	× ×	× ×
EMRI	$\rho_0 > 10^{-22}$ g/cm^3 $\rho_a > 10^{-8}$ g/cm^3	× ×	× ×	× ×
GW150914	× ×	$\rho_0 > 8 \times 10^{-14}$ g/cm^3 $\rho_a > 2 \times 10^{-8}$ g/cm^3	$\rho_0 > 0.9$ g/cm^3 $\rho_a > 10^3$ g/cm^3	$\rho_0 > 10^4$ g/cm^3 $\rho_a > 5 \times 10^6$ g/cm^3
GW170608	× ×	$\rho_0 > 10^{-15}$ g/cm^3 $\rho_a > 2 \times 10^{-9}$ g/cm^3	$\rho_0 > 0.02$ g/cm^3 $\rho_a > 101$ g/cm^3	$\rho_0 > 120$ g/cm^3 $\rho_a > 2 \times 10^5$ g/cm^3

Critical density:

$$\rho_c \sim 10^{-29}\mathrm{g/cm}^3 \sim 10^{-7} M_\odot/\mathrm{pc}^3$$

Solar neighborhood:

$$\rho_{\mathrm{DM}} \sim 1\ M_\odot/\mathrm{pc}^3 \sim 7 \times 10^{-23}\ \mathrm{g/cm}^3$$

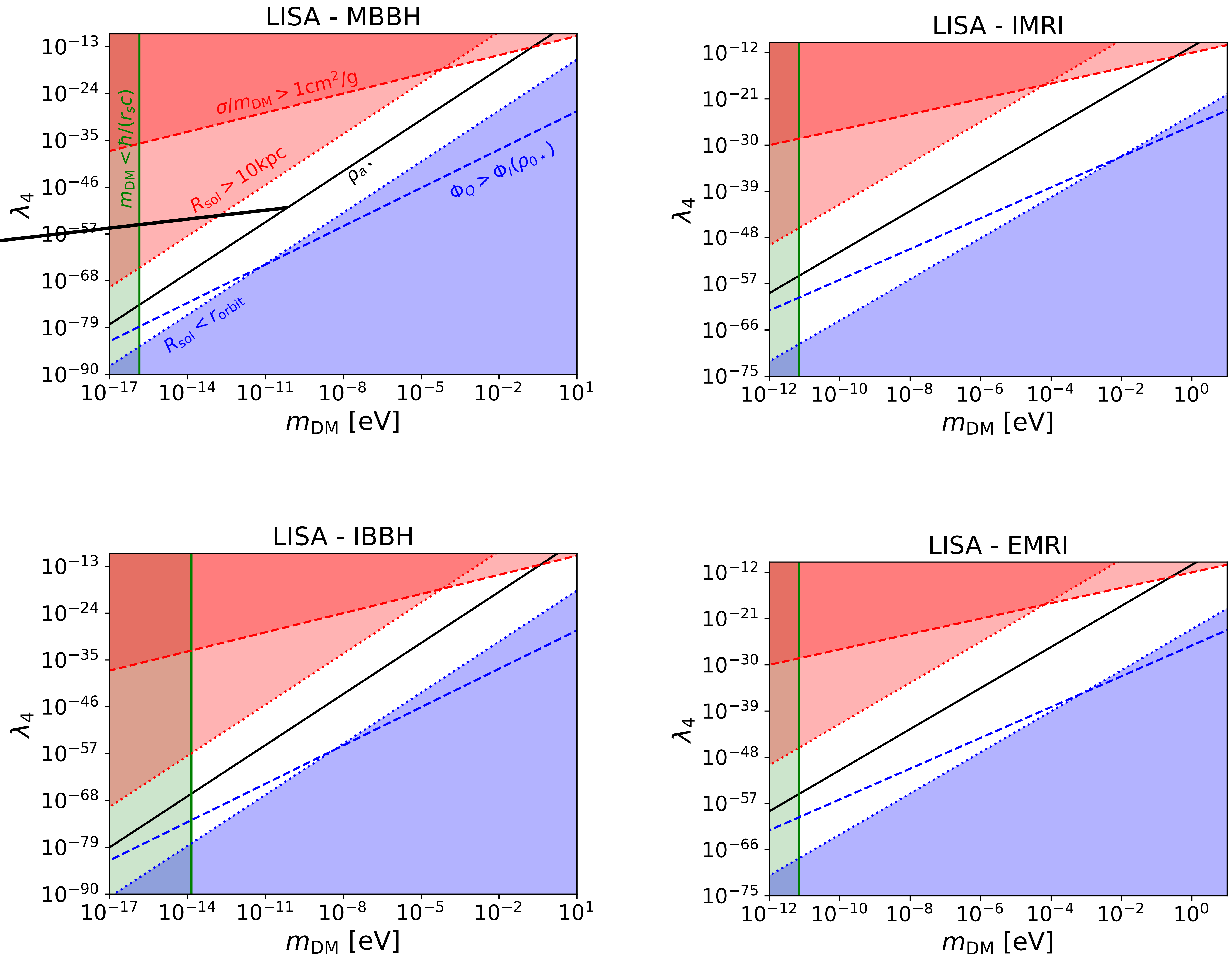
Baryonic density in thick disks:

$$\rho_{\mathrm{b}} \lesssim 10^{-7}\mathrm{g/cm}^3$$

E) Region in the parameter space that can be detected

Plane $(m_{\rm DM}, \lambda_4)$

Models with coupling below this line can be detected



F) Region in the parameter space that can be detected

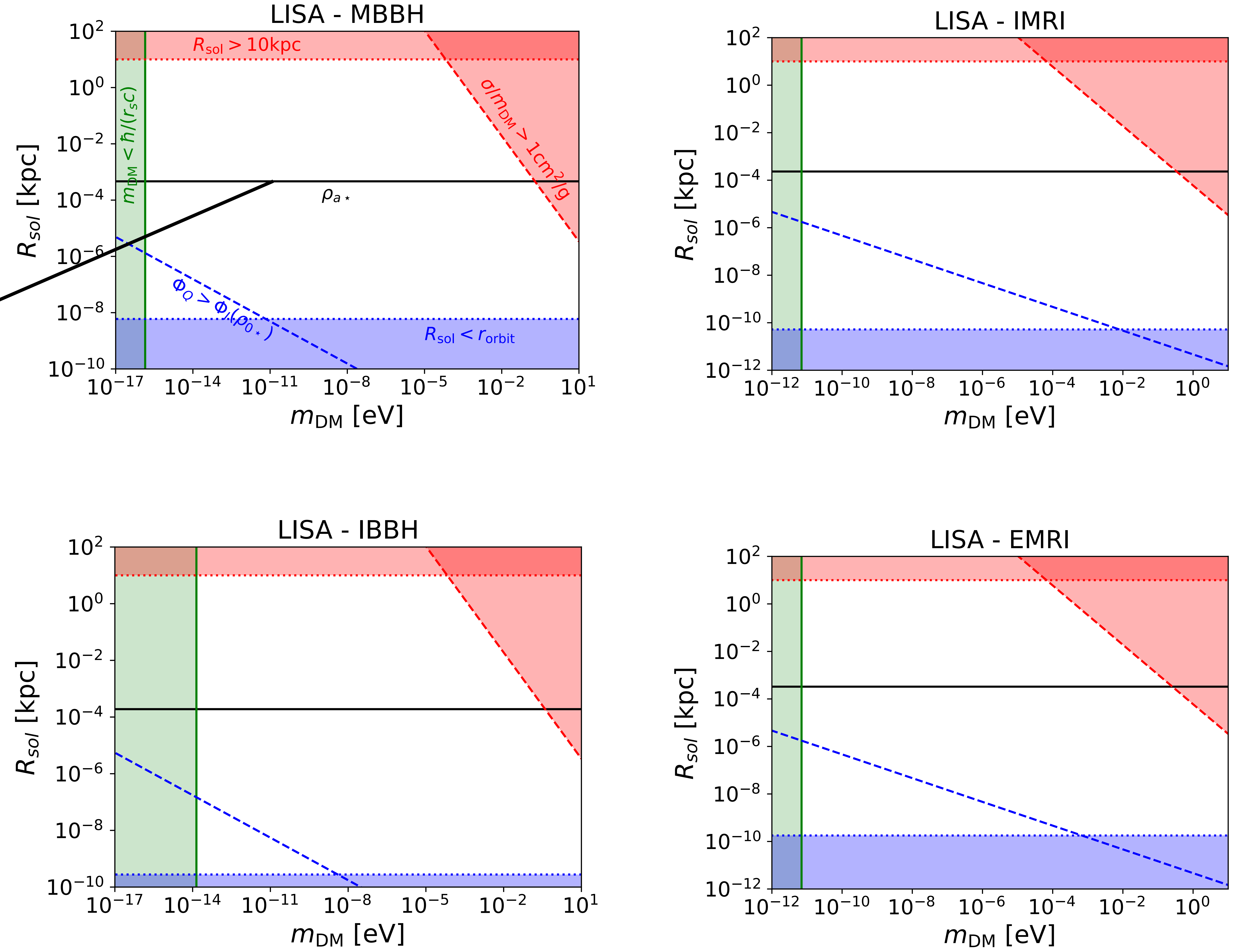
Plane $(m_{\rm DM}, R_{\rm sol})$

$$R_{\rm sol} = \pi\sqrt{\frac{3\lambda_4}{2}}\frac{M_{\rm Pl}}{m^2}$$

$$R_{\rm sol} = \sqrt{\frac{\pi}{4\mathcal{G}\rho_a}}$$

Radius of the scalar cloud (soliton)

Models with soliton radius below this line can be detected



CONCLUSIONS

- Scalar dark matter models with self-interactions allow detailed analysis in the large scalar-mass limit
- Hydrodynamical picture in the non-relativistic regime (but does not always hold: mapping can be singular)

- Solitons (flat cores) appear at the center of virialized halos
- They do not seem to converge to a scaling regime $\longrightarrow$ expect of large diversity of profiles
- Transitions between different regime could take place for some models

- Radial accretion onto a BH similar to Bondi problem, with unique transsonic solution, but much smaller accretion rate, self-regulated by a bottleneck in the relativistic regime

- Such a dark matter environment could be detected by LISA and B-DECIGO, if it contains BH binaries.
- They would see scalar clouds that are smaller than 0.1 pc: difficult to detect by other probes

Other topics: vorticity, gravitational atoms (superradiance),

THANK YOU FOR YOUR ATTENTION !