



Winter school analogue gravity/cosmology in Benasque in January 2026!

Experiments on rotating geometries with fluids of light

Quantum Optics Group
Laboratoire Kastler Brossel, Paris

Maxime Jacquet, Kévin Falque, Killian Guerrero,
Elisabeth Giacobino, Alberto Bramati

Theoretical and Experimental GR group
Louisiana State University, USA

Adrià Delhom, Anthony Brady, Paula Calizaya,
Ivan Agullo



TUG 10/10/2023

In a (quantum) fluid

Fluid velocity $\mathbf{v} = (\hbar/m)\nabla\phi$

Speed of sound $c_s \propto \sqrt{\frac{g\rho_0}{m}}$

m – mass
 g – interaction constant
 ρ_0 – density

Wave eq for collective excitations of (super)fluid $\psi = \psi_0 + \epsilon_1\psi_1$

$$-\partial_t\left(\frac{\rho_0}{c_s^2}(\partial_t\rho_1 + \mathbf{v}_0\nabla\rho_1)\right) + \nabla\left(\rho_0\nabla\rho_1 - \frac{\rho_0\mathbf{v}_0}{c_s^2}\partial_t\rho_1 + \mathbf{v}_0\nabla\rho_1\right) = 0$$

Relativistic form of wave eq for collective excitations: $\Delta\rho_1 = \frac{1}{\sqrt{-\eta}}\partial_\mu(\sqrt{-\eta}\eta^{\mu\nu}\partial_\nu\rho_1) = 0$

with $\eta_{\mu\nu} = \begin{pmatrix} -(c_s^2 - \mathbf{v}_0^2) & -v_o^x & -v_o^y \\ -v_o^x & 1 & 0 \\ -v_o^y & 0 & 1 \end{pmatrix}$

Motion of collective excitations in inhomogeneous fluid flow \leftrightarrow scalar field on curved spacetime

Control parameters: $\mathbf{v}_0, \mathbf{c}_s$

In a (quantum) fluid

Fluid velocity $\mathbf{v} = (\hbar/m)\nabla\phi$

Speed of sound $c_s \propto \sqrt{\frac{g\rho_0}{m}}$

m – mass
 g – interaction constant
 ρ_0 – density

Possible geometries with $\eta_{\mu\nu} = \begin{pmatrix} -(c_s^2 - \mathbf{v}_0^2) & -v_o^x & -v_o^y \\ -v_o^x & 1 & 0 \\ -v_o^y & 0 & 1 \end{pmatrix}$

(i) accelerating flow along 1 spatial dimension → static 1D spacetime

Horizon where $v_0 = c_s$

(ii) radially accelerating flow in 2 spatial dimensions → static spherically symmetric 2D spacetime

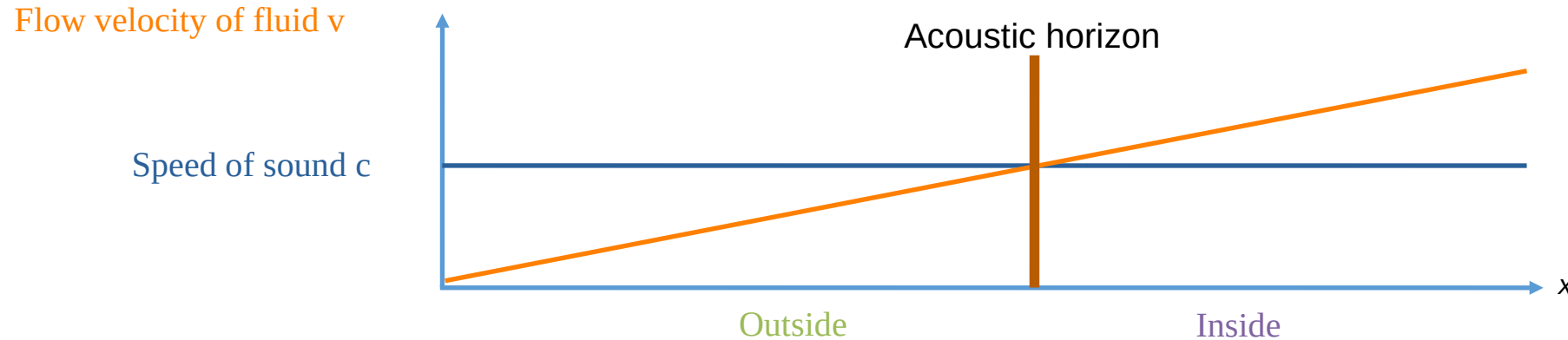
Horizon where $v_r = c_s$

(iii) radially and azimuthally accelerating flow in 2 spatial dimensions → static rotating spacetime

Horizon where $v_r = c_s$

Ergosurface where $|\mathbf{v}_0| = c_s$

Static 1D geometry \leftrightarrow waterfall geometry



Quantised acoustic field:

in: $\phi = \int d\omega (a_\omega f_\omega + a_\omega^\dagger f_\omega^*) \quad a |0\rangle = 0$

out: $\phi = \int d\omega (\bar{a}_\omega F_\omega + \bar{a}_\omega^\dagger F_\omega^*) \quad \bar{a} |\bar{0}\rangle = 0$

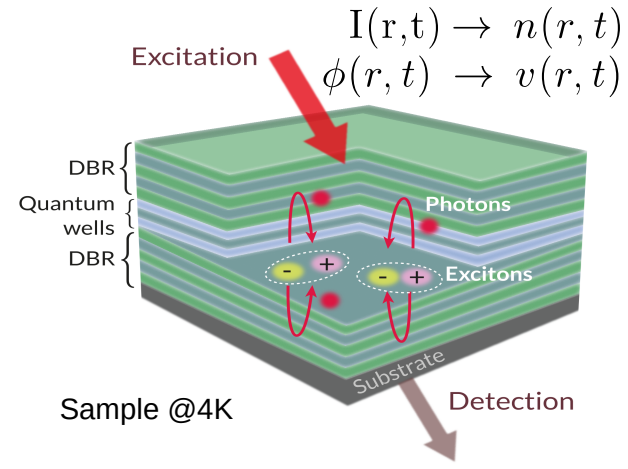
Express out modes in terms of in modes:

$$F_\omega = \int d\omega' (\alpha_{\omega\omega'} f_{\omega'} + \beta_{\omega\omega'} f_{\omega'}^*)$$

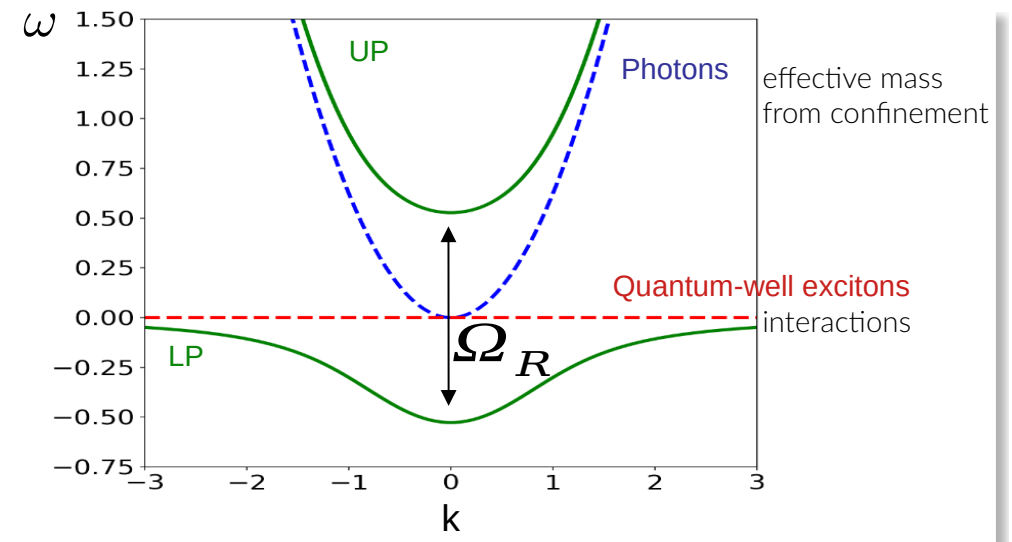
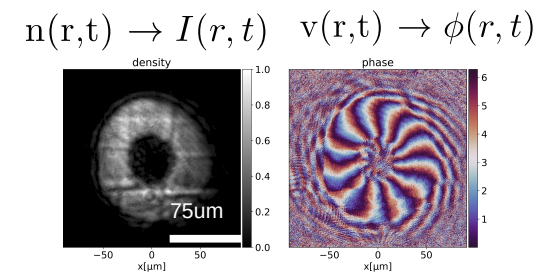
Different speeds on either side of the horizon $\Rightarrow |\bar{0}\rangle \neq |0\rangle \Rightarrow \beta_{\omega\omega'} \neq 0$

Mixing of positive and negative frequency waves \Rightarrow mixing of creation and annihilation operators

$$a |\bar{0}\rangle = \sum_{\omega'} \beta_{\omega\omega'} |\bar{1}\rangle > 0$$



Imaging photons leaking out of the cavity



Polaritons= photons dressed with material excitations that live in the cavity plane

Dynamics in the cavity plane described by Gross-Pitaevskii (Nonlinear Schrödinger) equation:

$$i\hbar \frac{\partial \psi}{\partial t} = \left(-\frac{\hbar^2 \nabla^2}{2m_{LP}^*} + gn \right) \psi - \frac{i\hbar\gamma}{2} \psi + P(r,t)$$

Driven-dissipative dynamics \rightarrow Out-of-equilibrium system

- g polariton-polariton interaction constant
- γ losses
- P pump

$$\text{GPE: } i\hbar \frac{\partial \psi}{\partial t} = \left(-\frac{\hbar^2 \nabla^2}{2m_{LP}^*} + gn \right) \psi - \frac{i\hbar\gamma}{2} \psi + P(r, t)$$

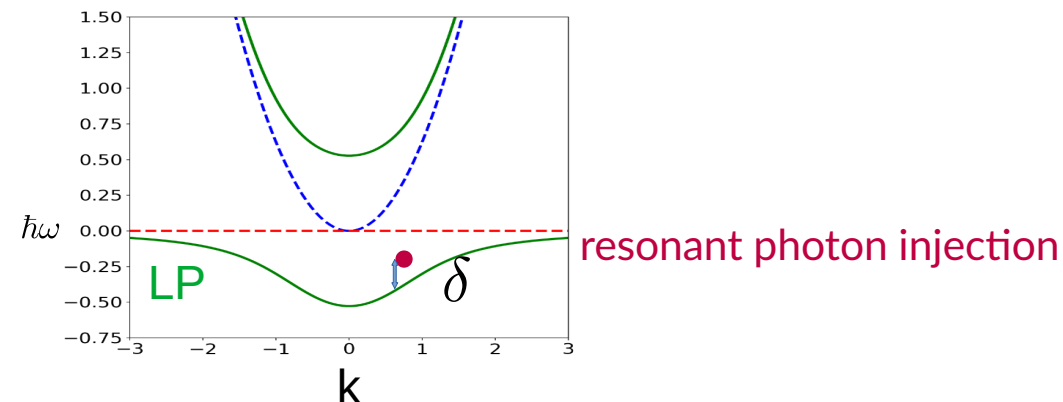
Bogoliubov theory:

1. Linearise GPE around steady-state solution $\psi(r, t) = \psi_0(r, t) + \delta\psi(r, t)$

2. Equation of motion of weak perturbations
$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} \delta\psi(r, t) \\ \delta\psi^*(r, t) \end{pmatrix} = L_{\text{Bog}} \begin{pmatrix} \delta\psi(r, t) \\ \delta\psi^*(r, t) \end{pmatrix}$$

3. Eigenvalues of L_{Bog} == dispersion relation

$$\hbar\omega(k) = \pm \sqrt{\left(\frac{\hbar^2 k^2}{2m} - \delta + 2gn \right)^2 - (gn)^2} - i\gamma$$



$$\text{GPE: } i\hbar \frac{\partial \psi}{\partial t} = \left(-\frac{\hbar^2 \nabla^2}{2m_{LP}^*} + gn \right) \psi - \frac{i\hbar\gamma}{2} \psi + P(r, t)$$

Bogoliubov theory:

1. Linearise GPE around steady-state solution $\psi(r, t) = \psi_0(r, t) + \delta\psi(r, t)$

2. Equation of motion of weak perturbations
$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} \delta\psi(r, t) \\ \delta\psi^*(r, t) \end{pmatrix} = L_{\text{Bog}} \begin{pmatrix} \delta\psi(r, t) \\ \delta\psi^*(r, t) \end{pmatrix}$$

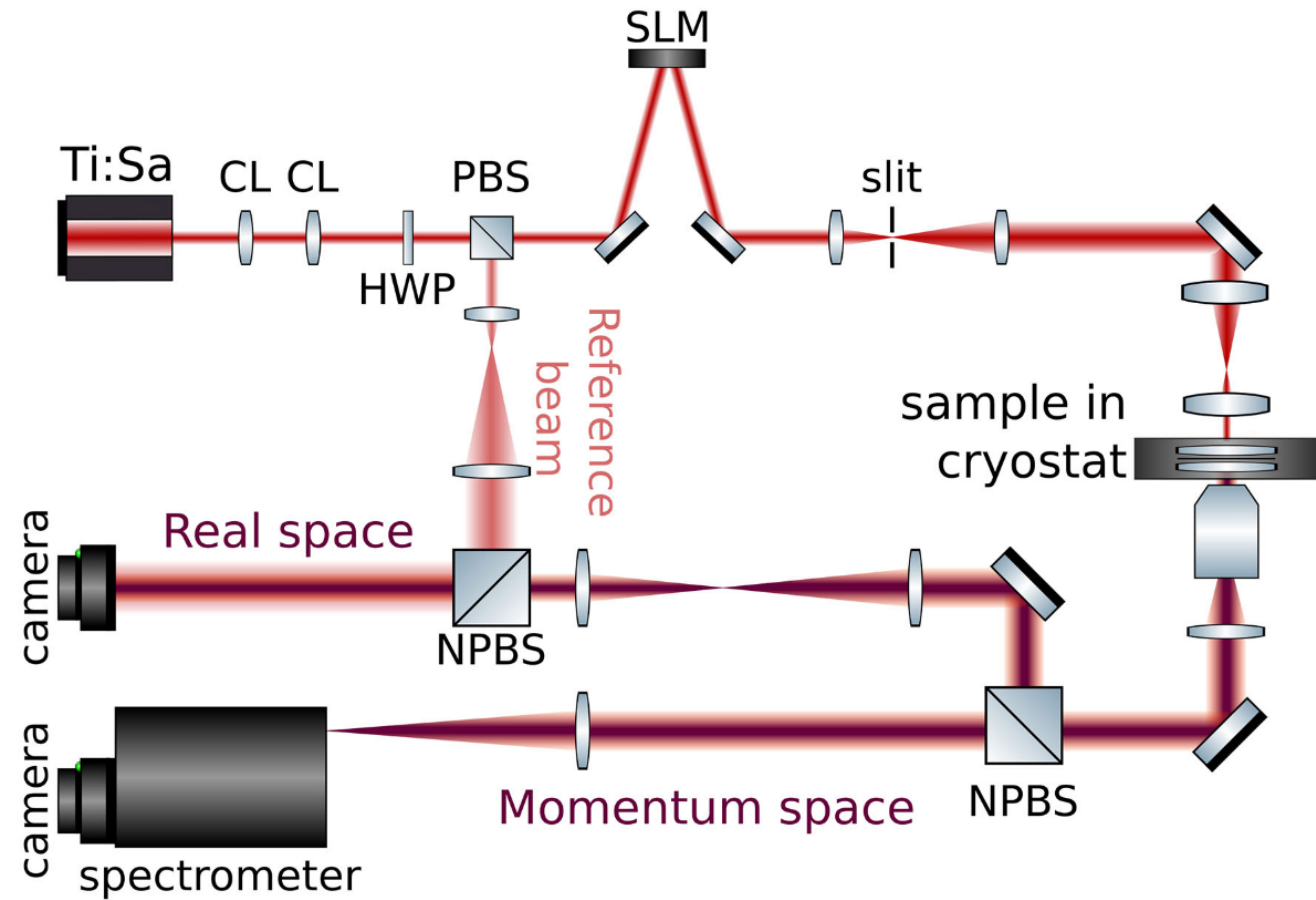
3. Eigenvalues of L_{Bog} == dispersion relation

$$\hbar\omega(k) = \pm \sqrt{\left(\frac{\hbar^2 k^2}{2m} - \delta + 2gn \right)^2 - (gn)^2} - i\gamma$$

$$\delta = gn$$

$$\omega(k) = \pm \sqrt{\frac{\hbar k^2}{2m} \left(\frac{\hbar k^2}{2m} + 2gn \right)}$$

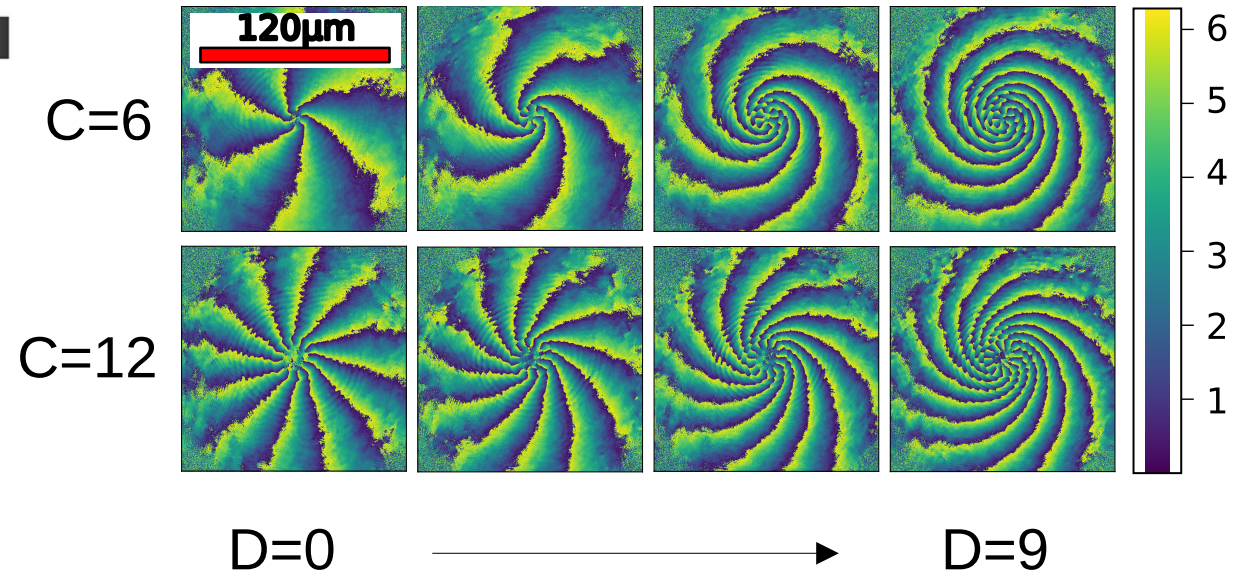
At low k , dispersion is linear \rightarrow excitations are phononic with “speed of sound” $c_s = \sqrt{\hbar gn/m}$



$$\nabla \phi_{SLM} = \frac{C}{r} \mathbf{u}_\theta - \frac{D}{r} \mathbf{u}_r$$

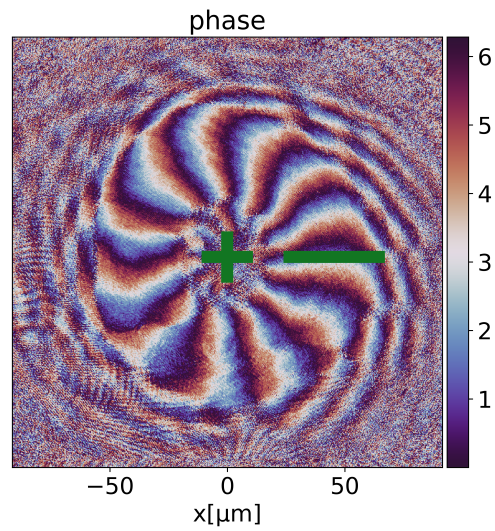
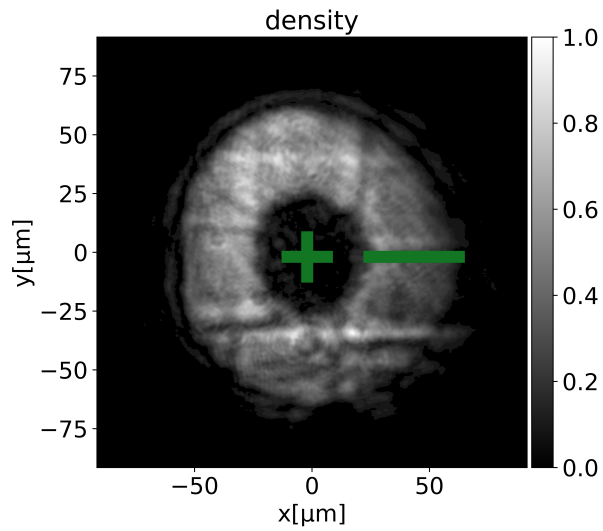


$$\phi_{SLM} = C\theta - D \ln(r)$$

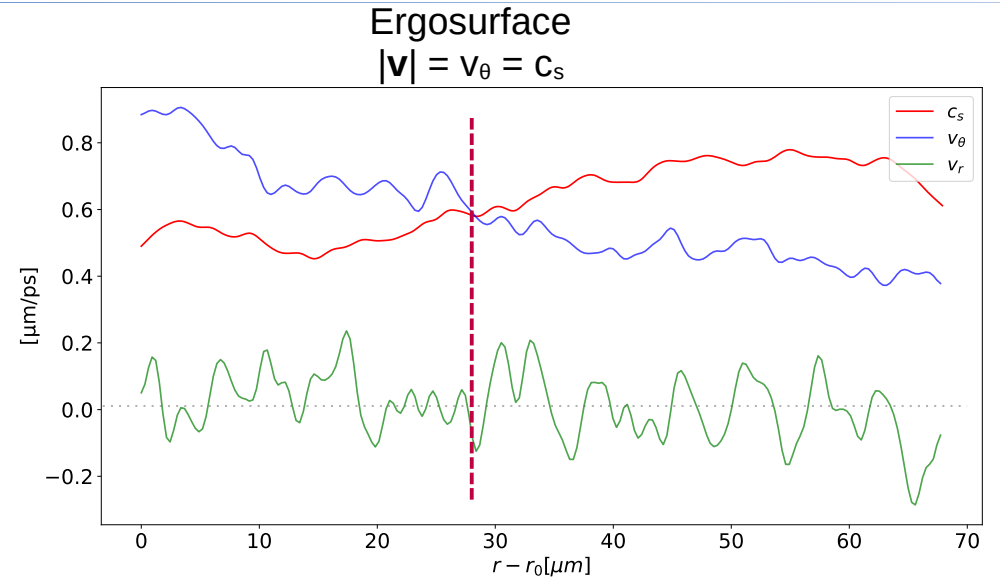




Rotating geometry $D = 0, C = 12$

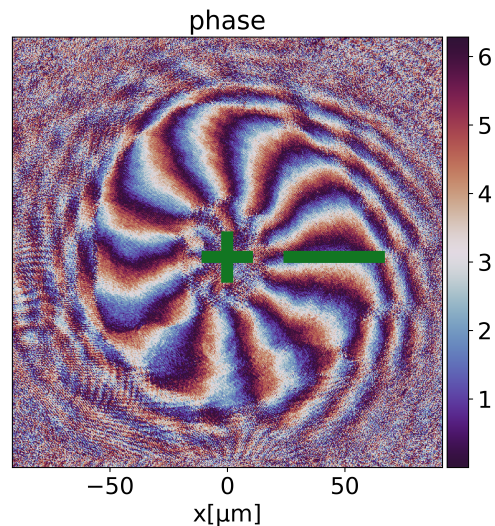
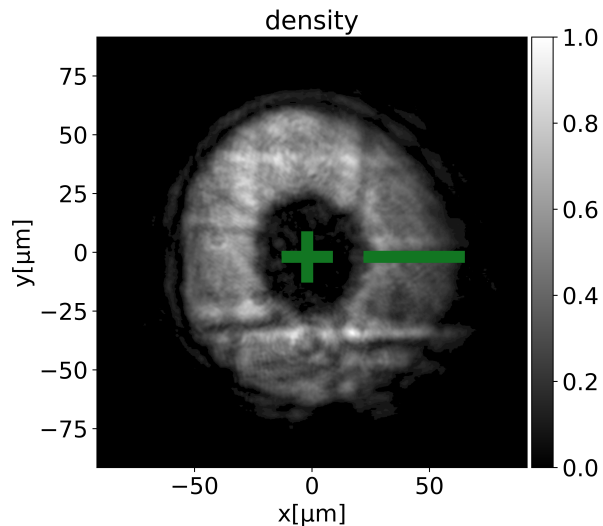


$$c_s \propto \sqrt{gn}$$
$$v_{fluid} \propto \nabla\phi$$



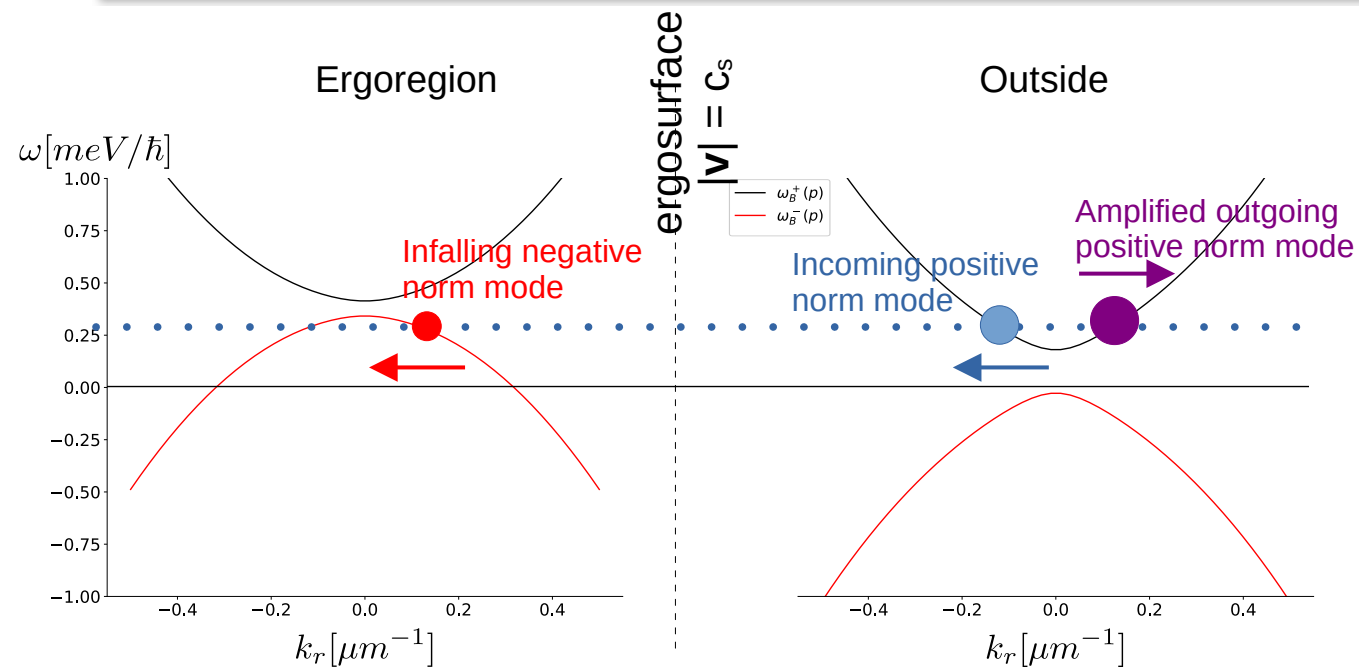
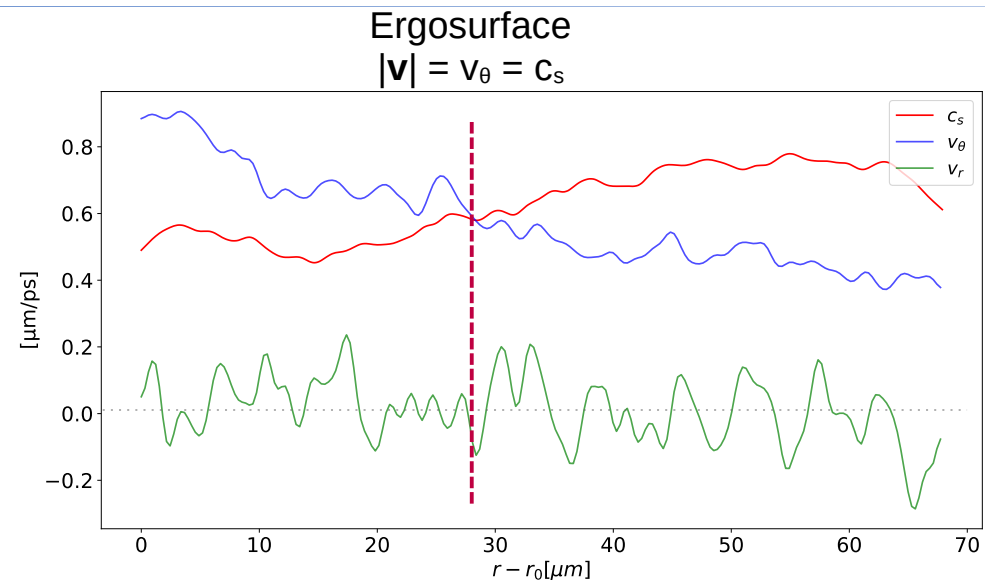


Rotating geometry $D = 0, C = 12$



$$c_s \propto \sqrt{gn}$$

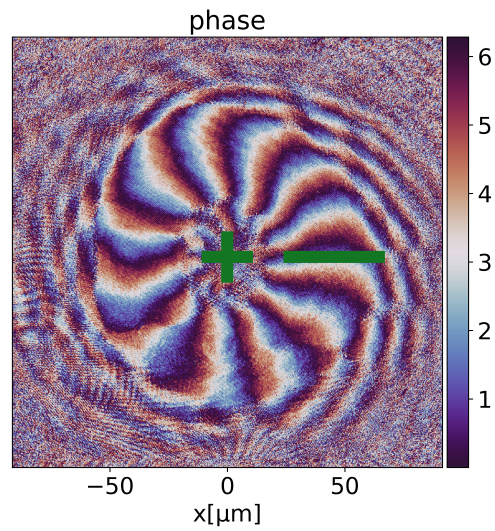
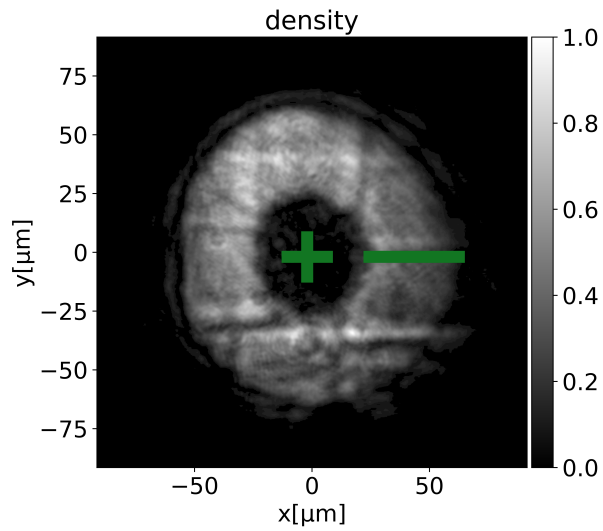
$$v_{fluid} \propto \nabla \phi$$



$$Q(\phi) = i \int_0^\infty \int_0^{2\pi} d\theta \frac{r}{c^2} (\phi^* \partial_t \phi - \partial_t \phi^* \phi)$$

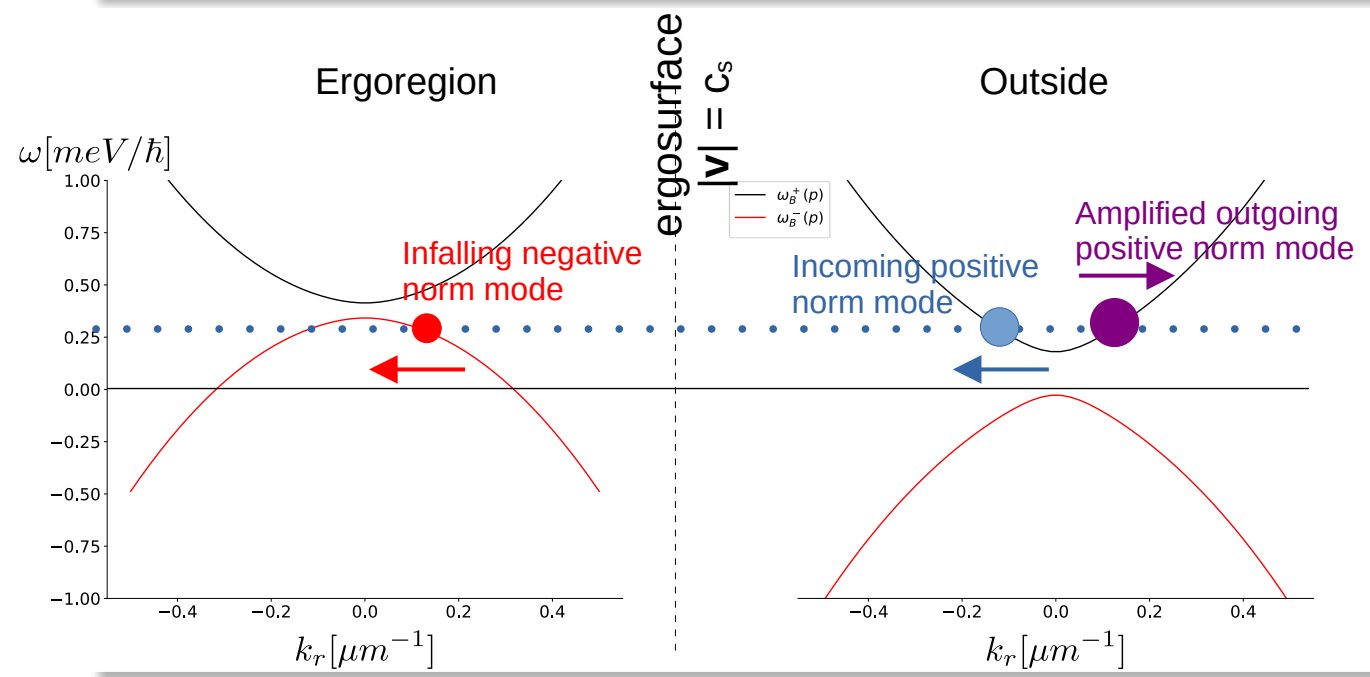
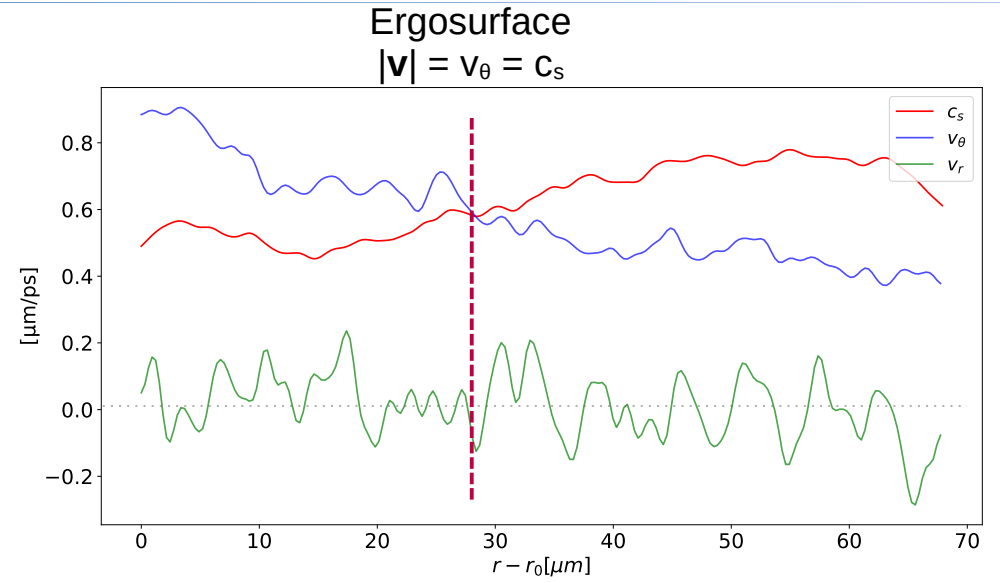


Rotating geometry $D = 0, C = 12$



$$c_s \propto \sqrt{gn}$$

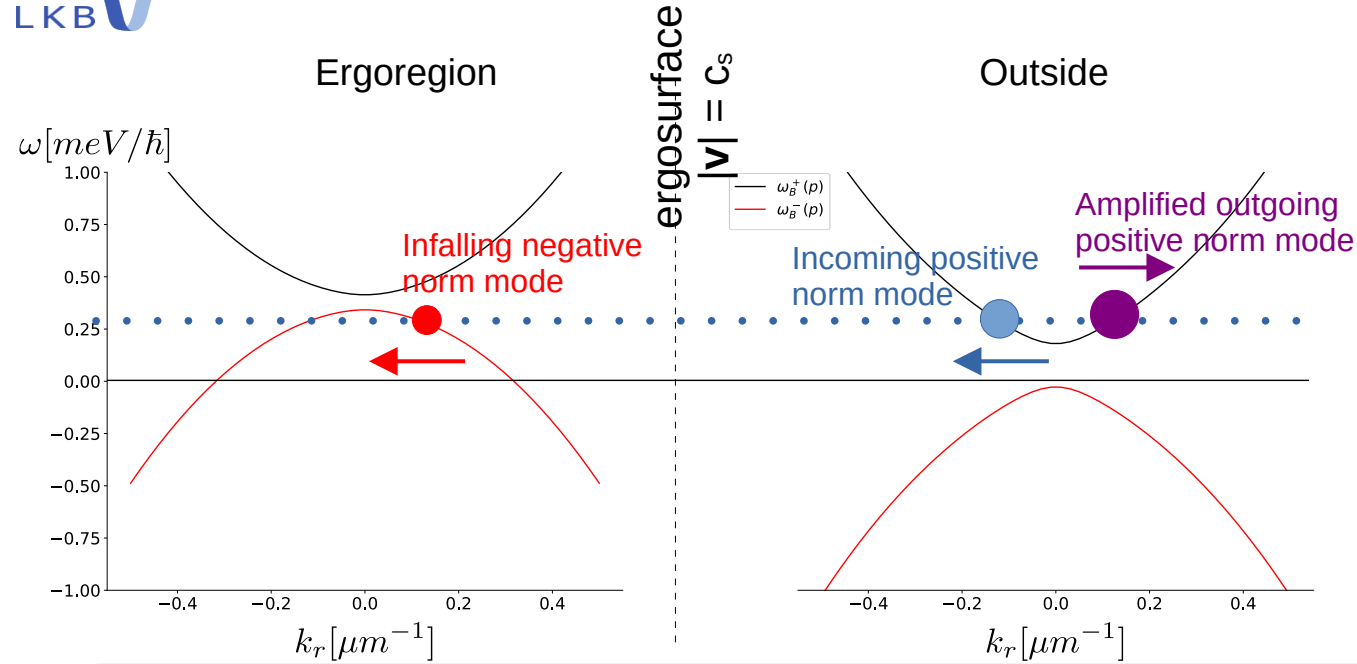
$$v_{fluid} \propto \nabla\phi$$



Stable ergosurface without a horizon?

$$v_g = 1 \mu m \cdot ps^{-1} \quad \tau = 15 ps$$

Entanglement in rotating geometry



$$[(\partial_t - \vec{v}\vec{\nabla})^2 - c^2\Delta]\Phi(t, r, \theta) = 0$$

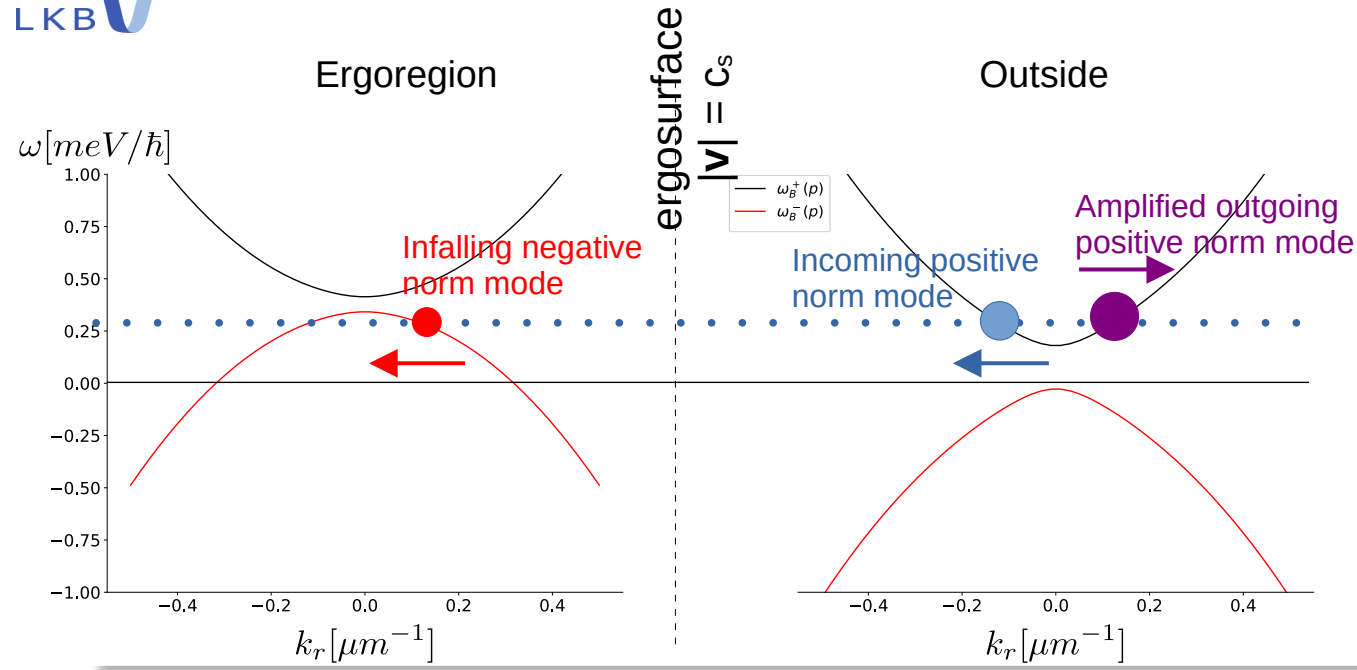
$$\Phi_{\omega, l}(t, r, \theta) = e^{-i\omega t} e^{il\theta} \phi_{\omega l}(r)$$

$$\left(\frac{1}{r} \frac{d}{dr} \left(r \frac{d}{dr}\right) + V_{\omega l}(r)\right) \phi_{\omega l}(r) = 0$$

$$(W_r^{in}, W_l^{in}) \rightarrow (W_r^{out}, W_r^{out})$$



Entanglement in rotating geometry

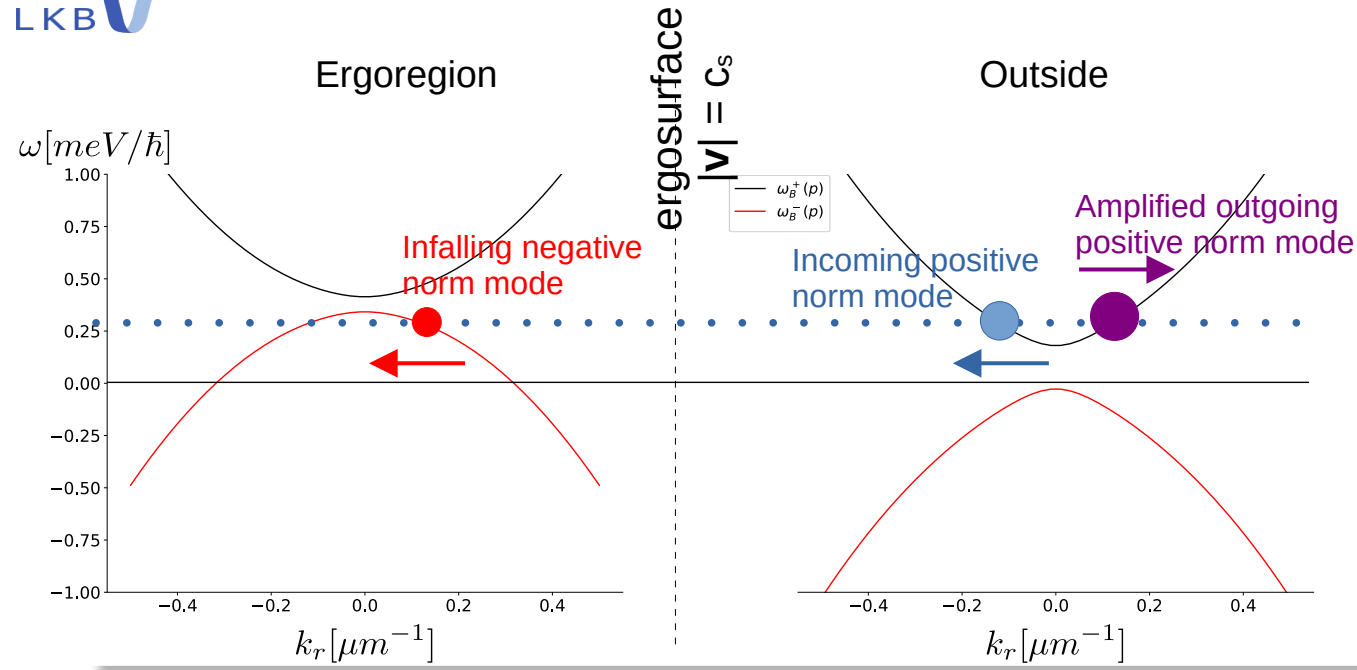


$$(W_r^{in}, W_l^{in}) \rightarrow (W_r^{out}, W_r^{out})$$

$$B_{\omega l} = \begin{pmatrix} T_{\omega l} & r_{\omega l} \\ R_{\omega l} & t_{\omega l} \end{pmatrix}$$



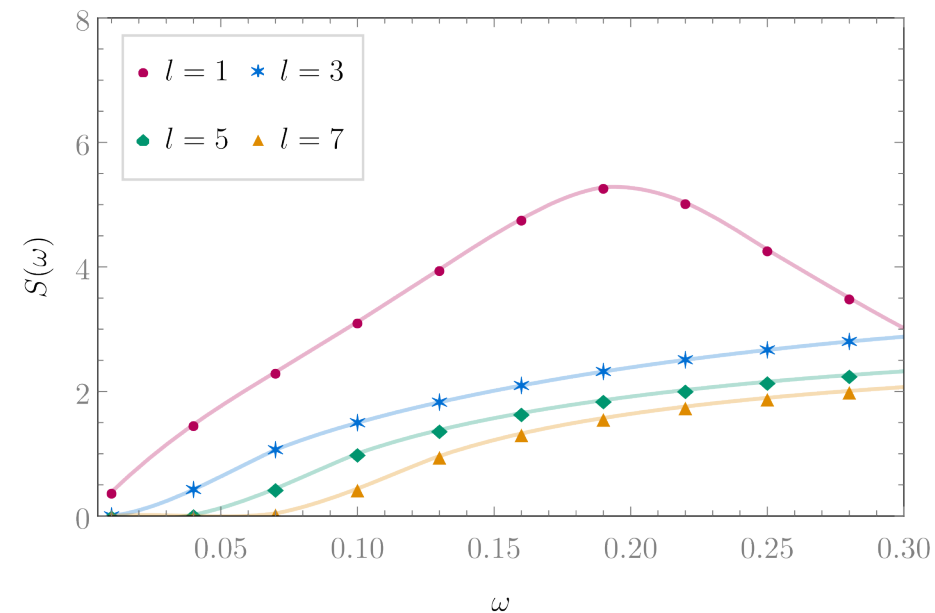
Entanglement in rotating geometry



$$(W_r^{in}, W_l^{in}) \rightarrow (W_r^{out}, W_r^{out})$$

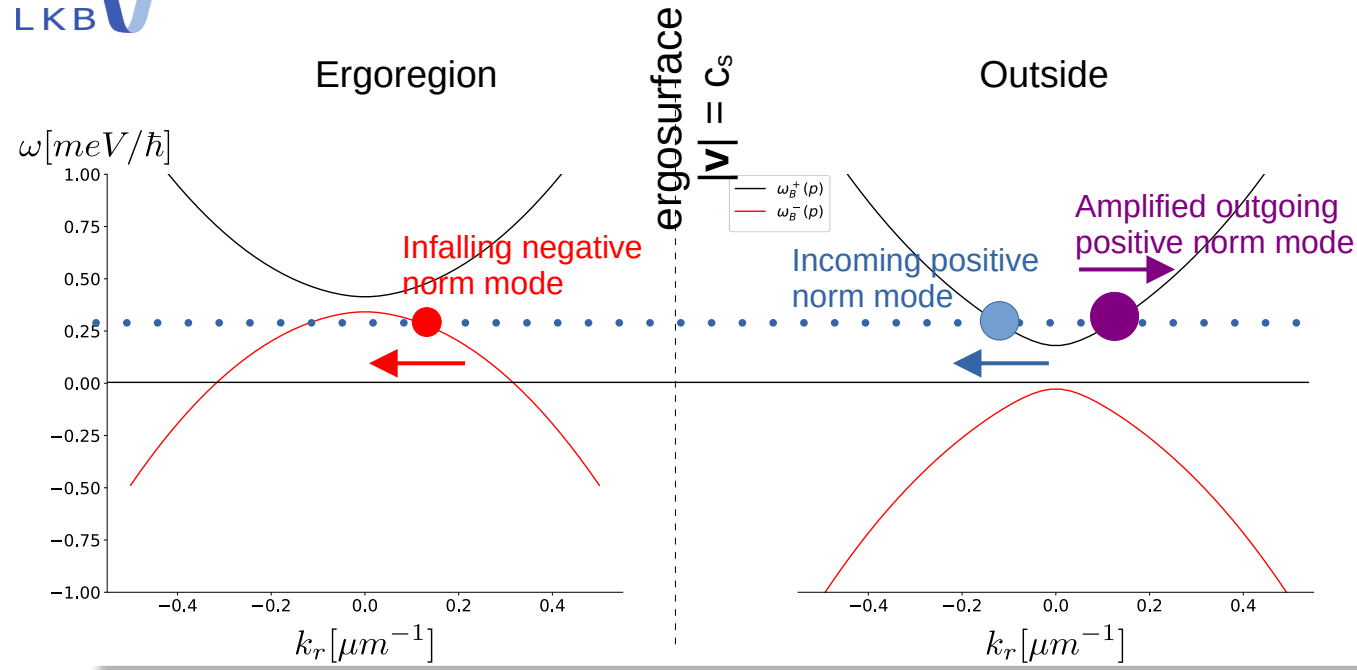
$$B_{\omega l} = \begin{pmatrix} T_{\omega l} & r_{\omega l} \\ R_{\omega l} & t_{\omega l} \end{pmatrix}$$

Vacuum only at input $|0\rangle$





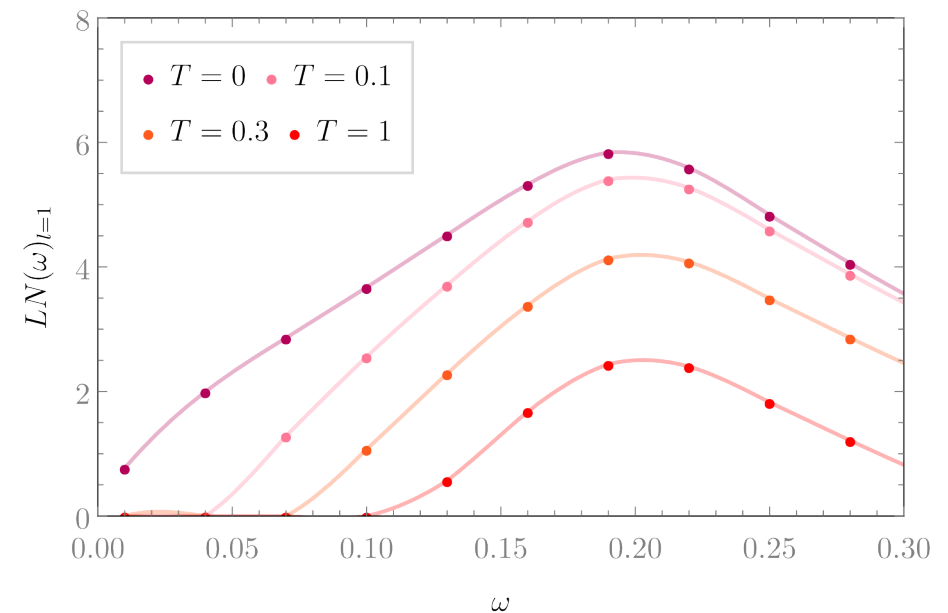
Entanglement in rotating geometry



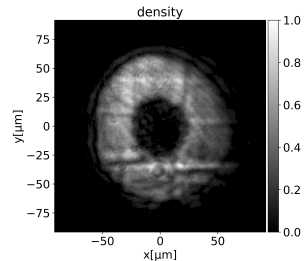
$$(W_r^{in}, W_l^{in}) \rightarrow (W_r^{out}, W_r^{out})$$

$$B_{\omega l} = \begin{pmatrix} T_{\omega l} & r_{\omega l} \\ R_{\omega l} & t_{\omega l} \end{pmatrix}$$

Vacuum + thermal $|0\rangle \otimes |Th\rangle$
noise at input

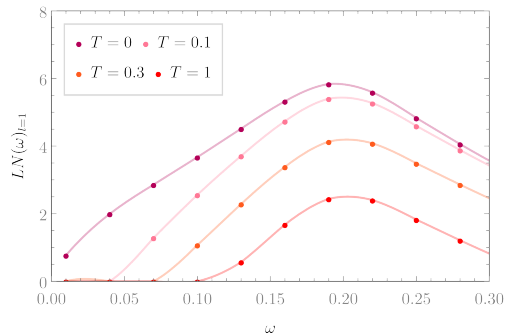


Where do we go from here?



Experiments with polaritons

- Measure Hawking radiation and rotational superradiance independently from one another
- Measure interplay between the two effects → modification of correlations?

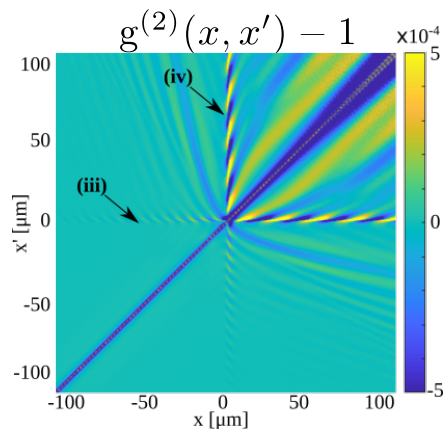


All optical experiments

- Measure phase and density → access full field statistics and dynamics
- High resolution spectroscopy in 1 and 2D with and without rotation
- Homodyne detection to enhance signal strength and measure quantum correlations
- Enhance strength of emission and degree of entanglement by probing with squeezed state

F Claude *et al* PRL **129** 103601 2022,
PRB **107** 174507 2023

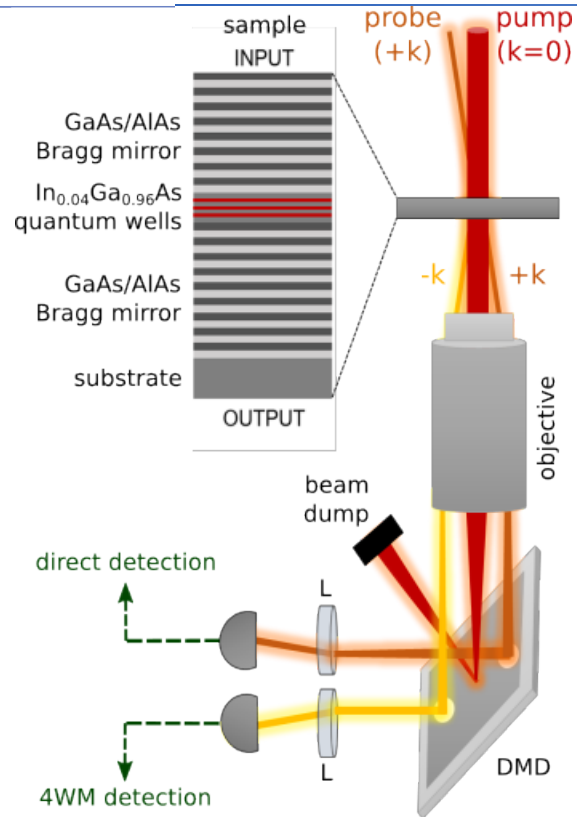
I Agullo *et al* PRL **128** 091301 2022



Numerical simulations

- New effect of quantum fields predicted: vacuum excitation of quasi-normal mode of acoustic field
- Good experimental configuration to observe strong correlations

Jacquet *et al.* PRL **130** 2023, EPJD **76** 2022

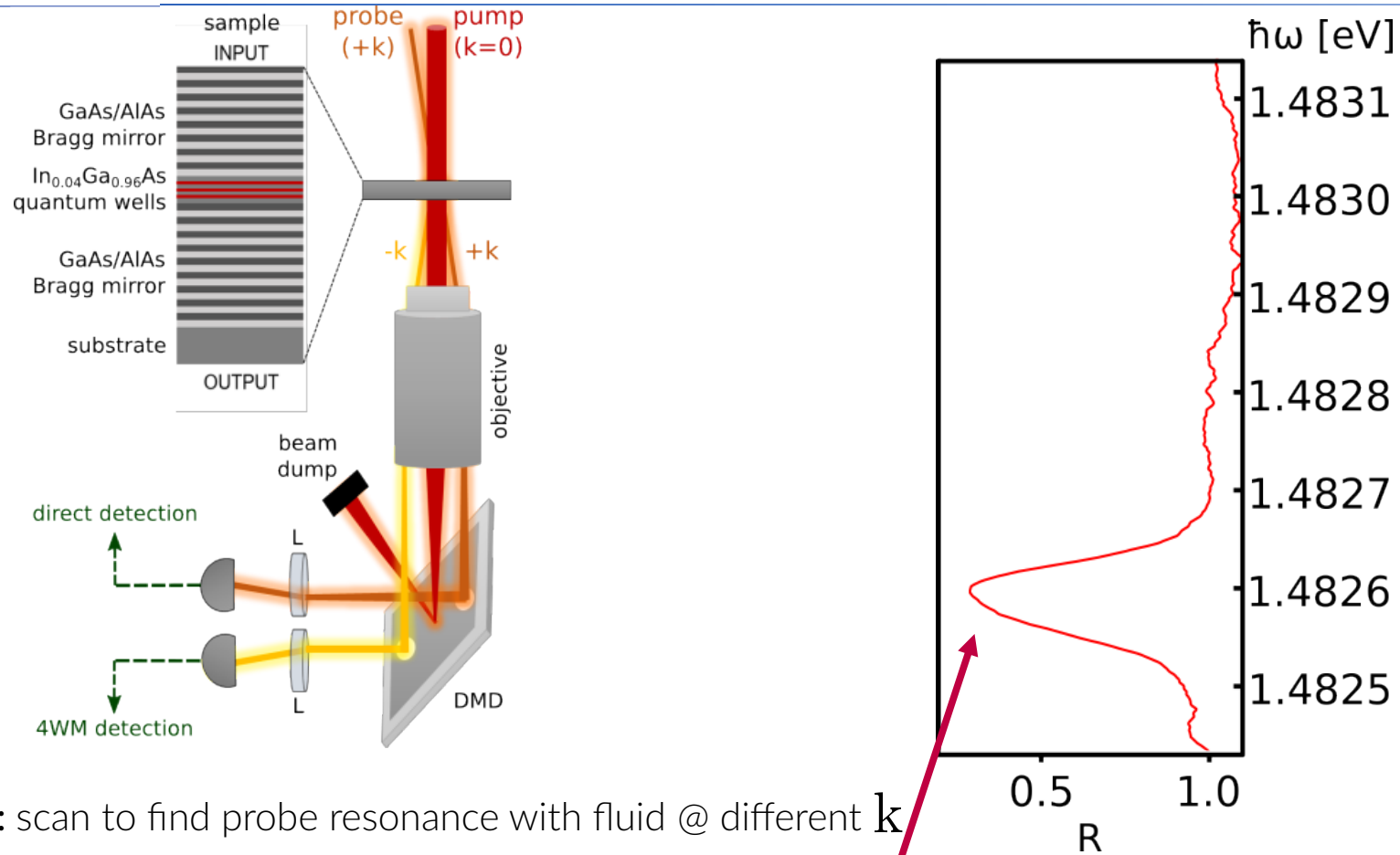


Probe: scan to find probe resonance with fluid @ different \mathbf{k}
 energy scan: $\sim 100\text{GHz}$

Detection: probe resonates with fluid \rightarrow transmission = dip in reflectivity

ω resolution fixed by the probe laser linewidth ($< 250\text{kHz}$)

\mathbf{k} resolution fixed by the k -space filtering ($0.02\mu\text{m}^{-1}$)

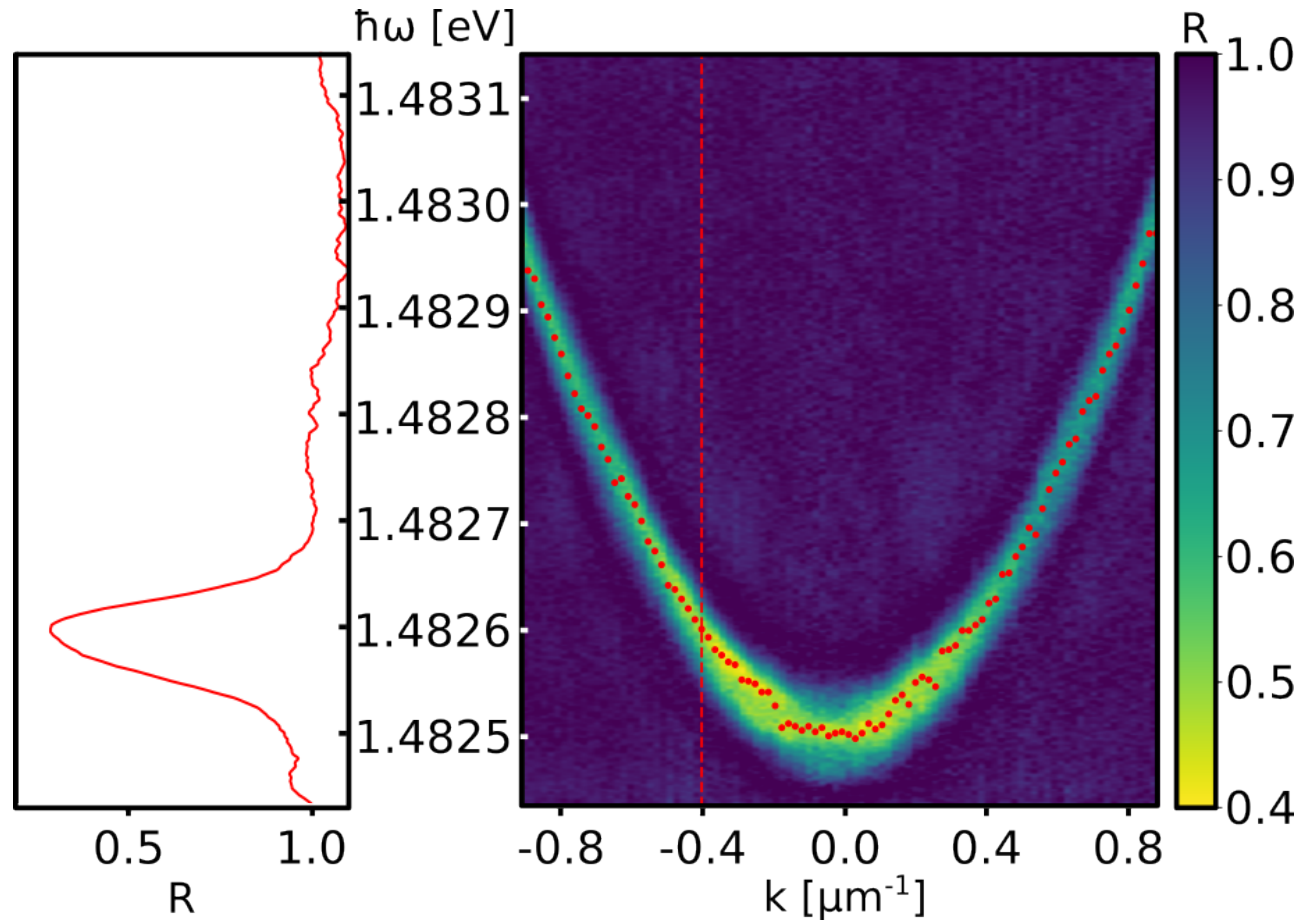
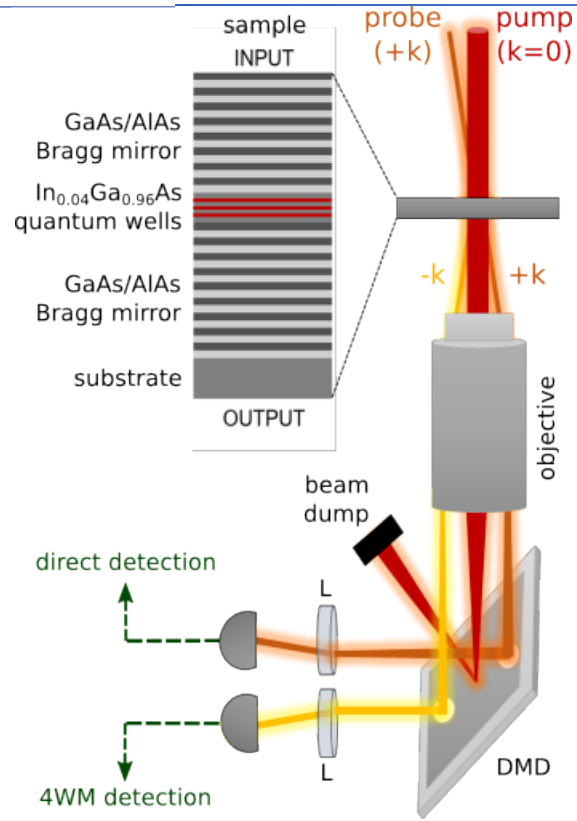


Reflectivity of probe @ k

Probe: scan to find probe resonance with fluid @ different k
 energy scan: $\sim 100\text{GHz}$

Detection: probe resonates with fluid \rightarrow transmission = dip in reflectivity

- ω resolution fixed by the probe laser linewidth ($< 250\text{kHz}$)
- k resolution fixed by the k -space filtering ($0.02\mu\text{m}^{-1}$)



Probe: scan to find probe resonance with fluid @ different k
 energy scan: $\sim 100\text{GHz}$

Detection: probe resonates with fluid \rightarrow transmission = dip in reflectivity

Coherent probe spectroscopy: Reflectivity map of probe vs (k, ω) \leftrightarrow spectrum of collective excitations

- ω resolution fixed by the probe laser linewidth ($< 250\text{kHz}$)
- k resolution fixed by the k -space filtering ($0.02\mu\text{m}^{-1}$)