Entanglement generation by rotating black holes in thermal baths

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In collaboration with I. Agullo, A. J. Brady, A. Delhom

Goals

- Advertise simple yet powerful tools from the quantum information theory of continuous variable systems and Gaussian states.
- Quantify the amount of entanglement generated in the Hawking process including **thermal environments** and **rotation**.

Main references:

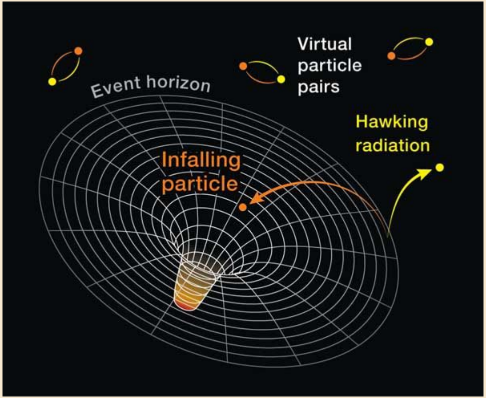
— I. Agullo, A. J. Brady, A. Delhom, and D. Kranas, “Entanglement from rotating black holes in thermal baths”, July 2023, arXiv: 2307.06215 [gr-qc].

Hawking effect

Hawking effect: Spontaneous creation of entangled particle pairs by black hole event horizons.

[S. W. Hawking (1974)]

Hawking process in a nutshell



- Ingredients: Black hole horizon + a quantum field.
- Thermal radiation emitted from the exterior of black holes.
- Hawking temperature:
  $$T_H = \frac{\hbar c^3}{8\pi G k_B M}$$
- Carries a quantum signature: Entanglement

Elements of quantum information theory of Gaussian states

Reference: A. Serafini, *Quantum Continuous Variables: A Primer of Theoretical Methods* (2017)

- Consider a system of N quantum bosonic degrees of freedom (harmonic oscillators): $\hat{\boldsymbol{R}} = (\hat{x}_1, \hat{p}_1, ..., \hat{x}_N, \hat{p}_N)$.

  Commutation relations: $[\hat{x}, \hat{p}] = i\hbar \to [\hat{R}^i, \hat{R}^j] = i\hbar\Omega^{ij}$, $\quad \Omega^{ij} = \bigoplus \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

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- Gaussian state $\hat{\rho}$: Completely characterized by the **first** and **secnond** moments.
  $\rightarrow\ \mu^i \equiv \mathrm{Tr}\left[\hat{\rho}\hat{R}^i\right]$
  $\rightarrow\ \sigma^{ij} \equiv \mathrm{Tr}\left[\hat{\rho}\{(\hat{R}^i - \mu^i), (\hat{R}^j - \mu^j)\}\right]$

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- The properties of the system can be derived in an elegant manner from μ^i and σ^{ij}.
  - $\to \ \sigma^{ij} + i\Omega^{ij} \geq 0$
  - $\to \ \hat{\rho}$: pure iff $\mathrm{eigen}\{\sigma^{ik}\Omega_{kj}\} = \pm i$
  - $\to \ \langle \hat{n} \rangle = \frac{1}{4}\sigma^i{}_i + \frac{1}{2}\mu^i\mu_i - N/2$

Evolution

For quadratic Hamiltonians, Gaussian states evolve to Gaussian states

$$(\boldsymbol{\mu}^{\text{in}}, \boldsymbol{\sigma}^{\text{in}}) \longrightarrow (\boldsymbol{\mu}^{\text{out}}, \boldsymbol{\sigma}^{\text{out}})$$

$$\boldsymbol{\mu}^{\text{out}} = \boldsymbol{S}\boldsymbol{\mu}^{\text{in}}, \quad \boldsymbol{\sigma}^{\text{out}} = \boldsymbol{S}\boldsymbol{\sigma}^{\text{in}}\boldsymbol{S}^{\mathsf{T}}, \quad \boldsymbol{S} \cdot \boldsymbol{\Omega} \cdot \boldsymbol{S}^{\mathsf{T}} = \boldsymbol{\Omega}$$

Forget about Schrödinger equation, infinite by infinite density matrices, etc. The evolution of Gaussian states is implemented by simple matrix multiplications of finitely dimensional matrices.

Entanglement

Logarithmic Negativity

To quantify entanglement of quantum states, including mixed ones, we will use *Logarithmic Negativity* LN, associated to the PPT criterion.

- Can be used to quantify the entanglement of mixed states.
- Based on the Positivity of Partial Transposition (PPT) criterion.
- For Gaussian states where either subsystem is made of a single degree of freedom, LN is a **faithful** entanglement quantifier.
- Can be computed from $\boldsymbol{\sigma}$.
- Measures entanglement in units of Bell states. For an operational interpretation look at [X. Wang, M. M. Wilde, Phys. Rev. Lett. 125, 040502 (2020)].

From ∞ to 3

- QFT: Infinitely-many degrees of freedom.

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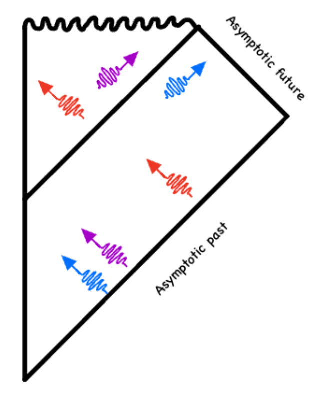
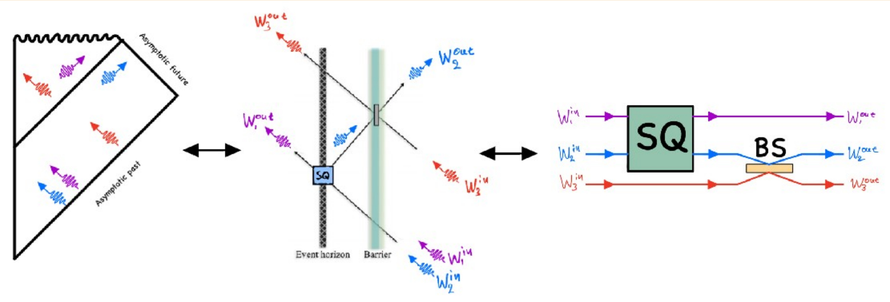
- Wald (1975): Found the progenitors of the Hawking modes $\rightarrow$ evolution diagonalizes to interactions among sets of three modes.



Hawking process as symplectic transformations



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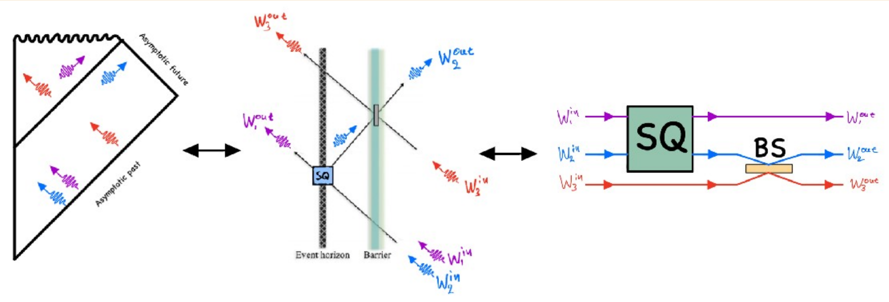

The scattering process at the black hole can be modeled via a **two-mode squeezer** followed by a **beam splitter**.

Squeezer

$$\hat{a}_1^{\text{out}} = \cosh r_\omega \; \hat{a}_1^{\text{in}} + e^{i\phi} \sinh r_\omega \; (\hat{a}_2^{\text{in}})^\dagger$$
$$\hat{a}_2^{\text{up}} = e^{i\phi} \sinh r_\omega \; (\hat{a}_1^{\text{in}})^\dagger + \cosh r_\omega \; \hat{a}_2^{\text{in}}$$

Beam splitter

$$\hat{a}_2^{\text{out}} = T_\omega \; \hat{a}_2^{\text{up}} - R_\omega \; \hat{a}_3^{\text{in}}$$
$$\hat{a}_3^{\text{out}} = R_\omega \; \hat{a}_2^{\text{up}} + T_\omega \; \hat{a}_3^{\text{in}}$$

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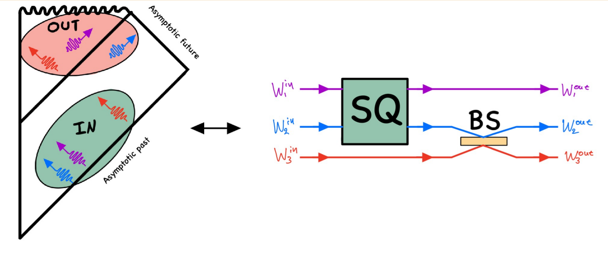

$$\begin{pmatrix} \hat{a}_1^{\text{out}} \\ (\hat{a}_1^{\text{out}})^\dagger \\ \hat{a}_2^{\text{out}} \\ (\hat{a}_2^{\text{out}})^\dagger \\ \hat{a}_3^{\text{out}} \\ (\hat{a}_3^{\text{out}})^\dagger \end{pmatrix} = \begin{pmatrix} \cosh r_\omega & 0 & e^{i\phi}\sinh r_\omega & 0 & 0 & 0 \\ 0 & \cosh r_\omega & 0 & e^{-i\phi}\sinh r_\omega & 0 & 0 \\ 0 & e^{i\phi}T_\omega \sinh r & T_\omega \cosh r & 0 & -R_\omega & 0 \\ e^{-i\phi}T_\omega \sinh r_\omega & 0 & 0 & T_\omega \cosh r_\omega & 0 & -R_\omega \\ 0 & e^{i\phi}R_\omega \sinh r_\omega & R_\omega \cosh r_\omega & 0 & T_\omega & 0 \\ e^{-i\phi}R_\omega \sinh r_\omega & 0 & 0 & R_\omega \cosh r_\omega & 0 & T_\omega \end{pmatrix} \begin{pmatrix} \hat{a}_1^{\text{in}} \\ (\hat{a}_1^{\text{in}})^\dagger \\ \hat{a}_2^{\text{in}} \\ (\hat{a}_2^{\text{in}})^\dagger \\ \hat{a}_3^{\text{in}} \\ (\hat{a}_3^{\text{in}})^\dagger \end{pmatrix}$$

Hawking process as symplectic transformations

Number of emitted quanta:

$$\langle 0 | \left(\hat{a}_2^{\text{out}}\right)^\dagger \hat{a}_2^{\text{out}} |0\rangle_{\text{in}} = T_\omega \sinh^2 r_\omega = T_\omega \left(e^{\hbar\omega/k_{\text{B}} T_{\text{H}}} - 1\right)^{-1}$$

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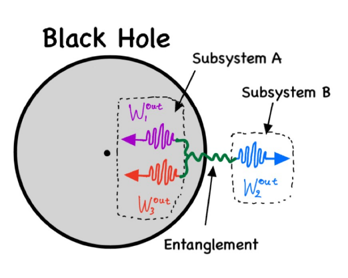
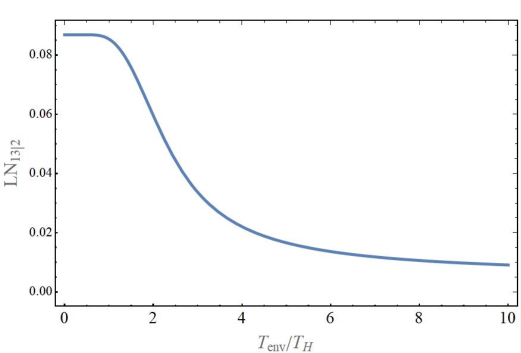
Entanglement produced by black holes

Let us compute the entanglement between Hawking radiation and the modes falling inside the black hole using Logarithmic Negativity. Page did the computation using entanglement entropy [Page (1993), (2013)].



Entanglement for BHs in a thermal bath



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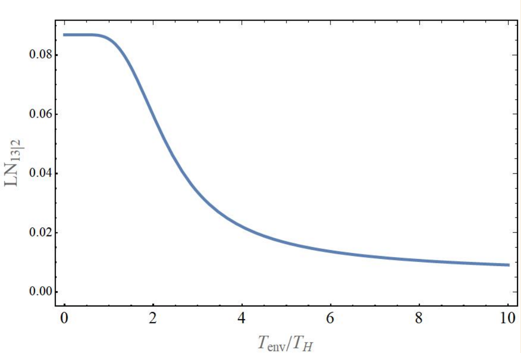

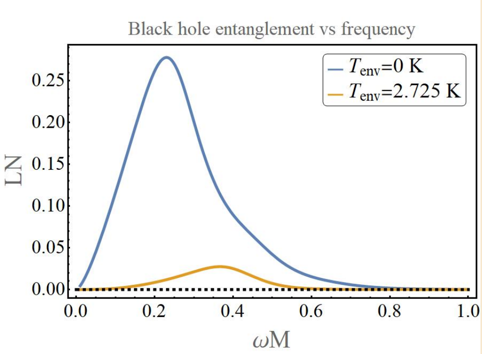

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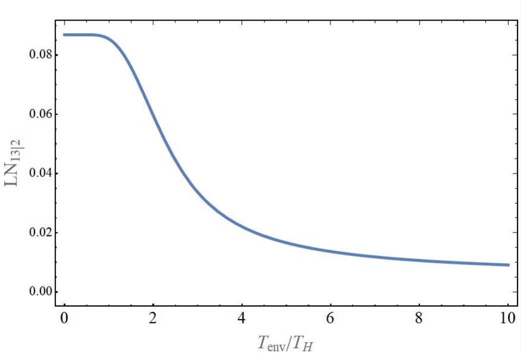
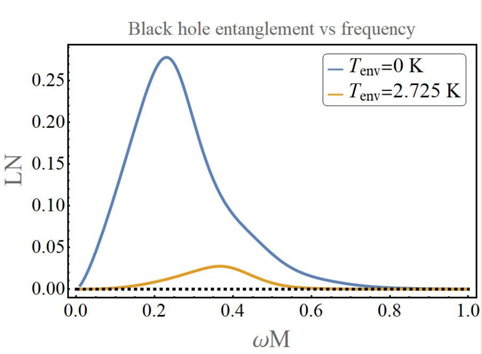
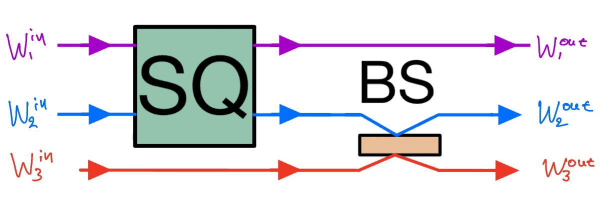




Main message: Thermal baths (mixed input quantum states) reduce the amount of entanglement produced in the Hawking process.

Rotation

NSRM: $\omega > m\Omega$



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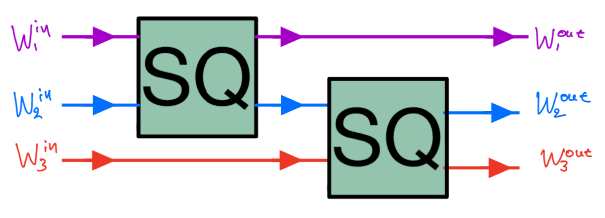
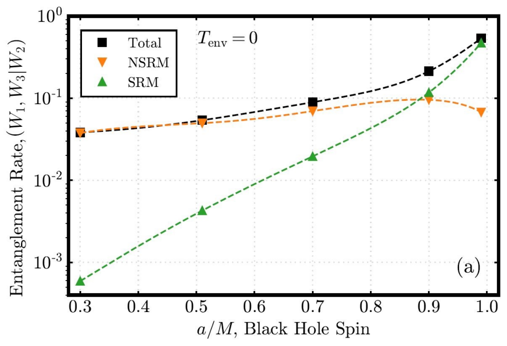


Entanglement from rotation



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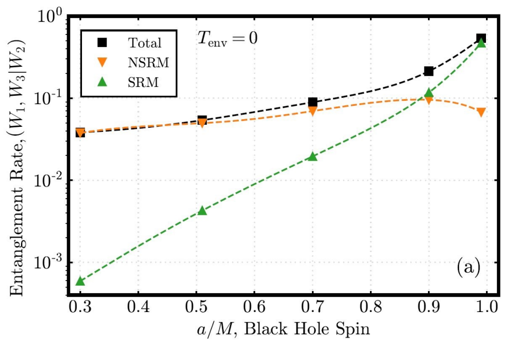

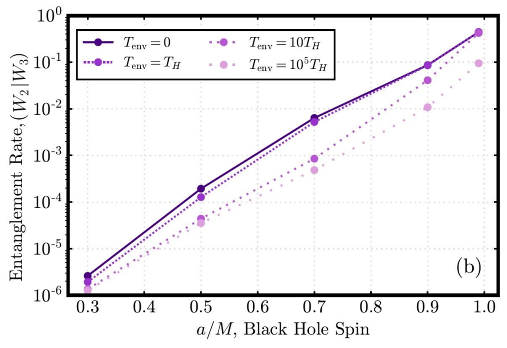

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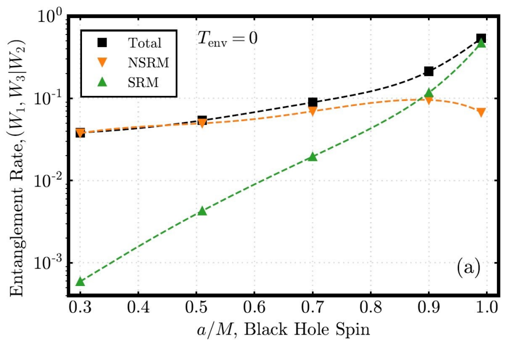

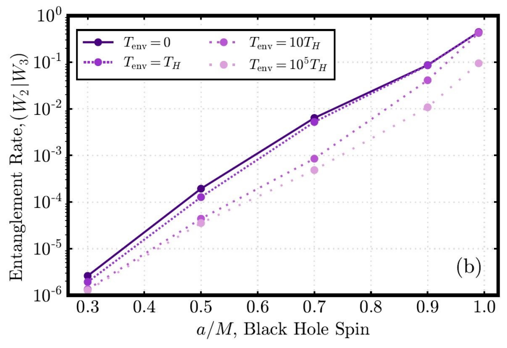

Main message: The ergoregion amplifies the amount of entanglement produced by black holes and, for high-spinning black holes, it becomes the dominant source of entalgement.

Take-home messages

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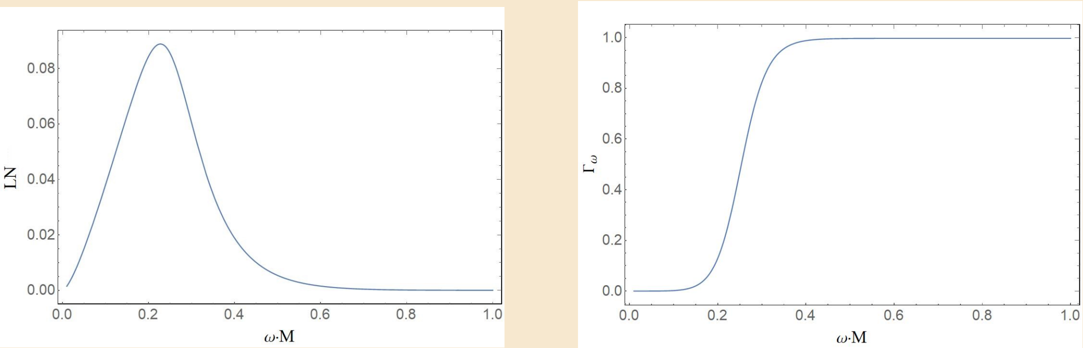
- We found that thermal baths **decrease** significantly the amount of generated entanglement while rotation **increases** entanglement.
- Our results extend Page's calculation and open a new avenue for studying information-related topics in a more realistic framework.
- Our tools for quantifying entanglement generated by horizons and ergoregions are generic and can be applied to other systems, e.g. analogue gravity setups (see Maxime's talk).

Additional Slides

One of the main contributions of this work is the incorporation of quantum information tools of Gaussian states into the physics of field theory to reformulate the Hawking process in a simple yet efficient manner.

Extend Page's calculation [Page 2013] to a more realistic scenario by adding **rotation** and **thermal envirinments**.

Entanglement in the Hawking effect



- At low ω, $\Gamma_\omega \to 0$: the gravitational barrier becomes fully reflective $\to$ No Hawking quanta escape.
- At high ω, $\Gamma_\omega \to 1$: the gravitational barrier becomes fully transparent $\to$ All Hawking quanta escape.
- The number of Hawking quanta produced at the horizon decreases monotonically with ω (since it follows a Bose-Einstein distribution).
- The competition of the last two functional forms results in a maximum value of LN at $\omega = 0.228\, M^{-1}$.

Sometimes, it is more illuminating to write down expressions in terms of annihilation and creation operators. Let us, therefore define the vector $\hat{\boldsymbol{A}} = (\hat{a}_1, \hat{a}_1^\dagger, ...\hat{a}_N, \hat{a}_N^\dagger)$.

$$\hat{a}_l = \frac{1}{\sqrt{2}}\left(\hat{x}_l + i\hat{p}_l\right), \qquad \hat{a}_l^\dagger = \frac{1}{\sqrt{2}}\left(\hat{x}_l - i\hat{p}_l\right), \quad l = 1,..,N$$

We can jump between $\hat{\boldsymbol{A}}$ and $\hat{\boldsymbol{R}}$ via

$$\hat{\boldsymbol{A}} = \boldsymbol{U}\hat{\boldsymbol{R}}, \quad \boldsymbol{U} = \bigoplus_{k=1}^{N} \begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix}$$

$$\hat{\boldsymbol{R}} = \boldsymbol{V}\hat{\boldsymbol{A}}, \quad \boldsymbol{V} = \boldsymbol{U}^{-1} = \bigoplus_{k=1}^{N} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix}$$

Examples of Gaussian States

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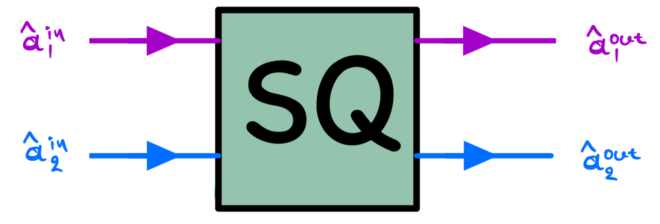
→ Thermal state: $\boldsymbol{\mu} = 0_{2N}$, $\boldsymbol{\sigma} = \oplus_i^N (2\bar{n}_i + 1)\boldsymbol{I}_2$

Two-mode squeezing



$$\hat{a}_1^{\text{out}} = \cosh r\,\hat{a}_1^{\text{in}} + e^{i\varphi}\sinh r\,\hat{a}_2^{\dagger\,\text{in}},$$
$$\hat{a}_2^{\text{out}} = e^{i\varphi}\sinh r\,\hat{a}_1^{\dagger\,\text{in}} + \cosh r\,\hat{a}_2^{\text{in}}$$

Two-mode squeezing for vacuum input

- **State before squeezing**:

$$\boldsymbol{\mu}^{\text{in}} = (0, 0, 0, 0), \quad \sigma^{\text{in}} = I_4$$

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$$\boldsymbol{S}_{2\text{sq}} = \begin{pmatrix} \cosh r & 0 & \cos\phi \sinh r & \sin\phi \sinh r \\ 0 & \cosh r & \sin\phi \sinh r & -\cos\phi \sinh r \\ \cos\phi \sinh r & \sin\phi \sinh r & \cosh r & 0 \\ \sin\phi \sinh r & -\cos\phi \sinh r & 0 & \cosh r \end{pmatrix}$$

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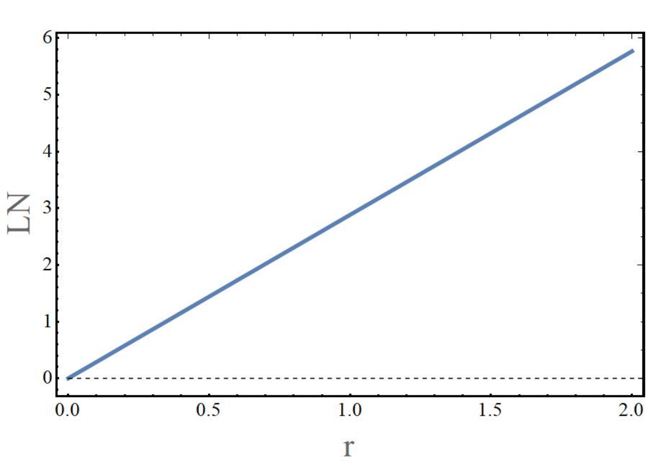
- **Entanglement:**

$$LN(r) = \max\{0, -\log_2 e^{-2r}\} = \frac{2}{\ln 2} r \simeq 2.89\, r$$

Two-mode squeezing for vacuum input

Entanglement vs squeezing amplitude



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The state is entangled only if $r > \frac{1}{2}\ln(2n+1)$

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The production of entangled quanta in the Hawking effect is a two-mode squeezing process.

Separability and entanglement

Let us consider a composite system that can be split into two subsystems A and B. Let $\hat{\rho}_A \in \mathcal{D}(\mathcal{H}_A)$ and $\hat{\rho}_B \in \mathcal{D}(\mathcal{H}_B)$ be the density operators describing A and B, respectively. The composite system is characterized by $\hat{\rho} \in \mathcal{D}(\mathcal{H}_A \bigotimes \mathcal{H}_B)$.

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The quantum state $\hat{\rho}$ is said to be **separable** if and only if it can be written as

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where $\hat{\rho}_{A,j} \in \mathcal{D}(\mathcal{H}_A)$, $\hat{\rho}_{B,j} \in \mathcal{D}(\mathcal{B}_A)$, $0 \leq p_j \leq 1$ for $\forall\ j = 1, .., m$ and $\sum_{j=1}^{m} p_j = 1$.

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A quantum state $\hat{\rho}$ is said to be **entangled** if it is not separable.

Entanglement: von Neumann entropy

Standard entanglement quantifier: **entanglement entropy** $\rightarrow$ von Neumann entropy of one of the subsystems.

Let $\hat{\rho}_A = \mathrm{Tr}_B[\hat{\rho_{AB}}]$ be the state describing subsystem A. The entanglement entropy is given by

$$E[\hat{\rho}_A] = -\mathrm{Tr}[\hat{\rho}_A \log_2(\hat{\rho}_A)] = -\Sigma_j \, \lambda_{A,j} \log_2(\lambda_{A,j}), \quad \lambda_{A,j} \equiv \mathrm{eigen}\{\hat{\rho}_A\}$$

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For Gaussian states

$$E = \sum_{j=1}^{n} h(\nu_i^A), \quad h(\nu_i^A) = \frac{\nu_i^A + 1}{2} \log_2\left(\frac{\nu_i^A + 1}{2}\right) - \frac{\nu_i^A - 1}{2} \log_2\left(\frac{\nu_i^A - 1}{2}\right),$$

where $\{\nu_i^A\}$, for $i = 1, ..., N$, is the set of **symplectic eigenvalues** of $\boldsymbol{\sigma}_A$, i.e. $|\mathrm{eigen}\{\boldsymbol{\Omega}\boldsymbol{\sigma}_A\}|$.

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- But if the total state $\hat{\rho}$ is mixed, $E[\hat{\rho}_A]$ could be positive even if $\hat{\rho}_{AB} = \hat{\rho}_A \otimes \hat{\rho}_B$.
- von Neumann entropy **cannot** be used to quantify entanglement in **mixed** states.

PPT criterion

To study the entanglement of quantum states, including mixed ones, we will use the well-known **positivity of the partial transposition (PPT)** criterion [A. Peres (1996), P. Horodecki (1997)].

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$$\hat{\rho}_{AB} = \sum_{i,j,k,\ell} p_{i,j,k,\ell} |i\rangle \langle j|_A \otimes |k\rangle \langle \ell|_B .$$

The partial transposition with respect to B is given by

$$\hat{\rho}^{\mathrm{PT}} = \mathcal{I}_A \otimes T_B(\hat{\rho}_{AB}) = \sum_{i,j,k,\ell} p_{i,j,k,\ell} |i\rangle \langle j|_A \otimes |\ell\rangle \langle k|_B = \sum_{i,j,k,\ell} p_{i,j,\ell,k} |i\rangle \langle j|_A \otimes |k\rangle \langle \ell|_B .$$

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Let $\{\lambda_i^{\mathrm{PT}}\}$ be the set of eigenvalues of $\hat{\rho}^{\mathrm{PT}}$.

- If $\hat{\rho}_{AB}$ is **separable**, then $\lambda_i^{\mathrm{PT}} > 0 \; \forall \; i$.
- If $\exists \; \lambda_i^{\mathrm{PT}} < 0$, then $\hat{\rho}_{AB}$ is **entangled**.

PPT for Gaussian states

- For Gaussian states, all statements about correlations, separability, and entanglement can be extracted solely from the covariance matrix $\boldsymbol{\sigma}$.
- The operation of partial transposition of a system of $M + K = N$ d.o.f.s, partitioned as $(M - \text{d.o.f.s}|K - \text{d.o.f.s})$, is implemented by

$$\boldsymbol{\sigma}^{\text{PT}} = \mathbf{T}\boldsymbol{\sigma}\mathbf{T}, \quad \mathbf{T} = \mathbf{I}_{2M} \bigoplus \boldsymbol{\Sigma}_{2K}, \quad \boldsymbol{\Sigma}_{2K} = \bigoplus_{i=1}^{K} \boldsymbol{\sigma}_z$$

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Let $\{\nu_i^{\text{PT}}\}$ be the set of symplectic eigenvalues of σ^{PT}.

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Two-mode squeezing for vacuum input

- **Logarithmic Negativity**

$$LN(r) = \max\{0, -\log_2 \nu_{\min}^{\mathrm{PT}}\} = \frac{2}{\ln 2} r \simeq 2.89\, r$$

- **Entanglement entropy**

$$E = \frac{\nu^A + 1}{2} \log_2 \left(\frac{\nu^A + 1}{2} \right) - \frac{\nu^A - 1}{2} \log_2 \left(\frac{\nu^A - 1}{2} \right),$$

where $\nu^A = \cosh 2r$.

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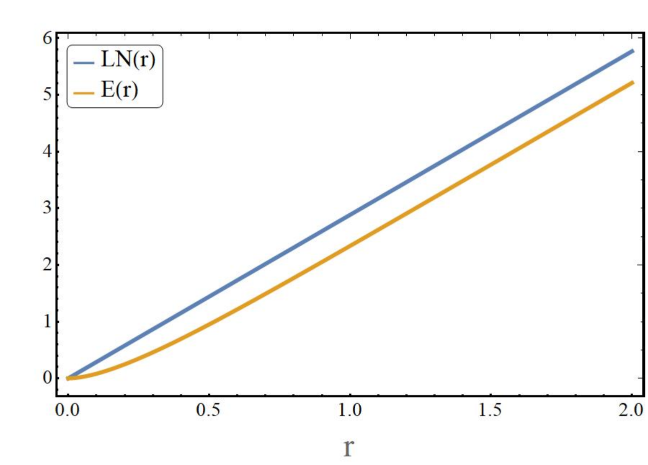

Both E and LN increase monotonically with r and capture the entanglement produced by the squeezing.

Logarithmic Negativity

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— For the particular case where of Gaussian systems partitioned as $(1-\text{d.o.f.s}|M-\text{d.o.f.s})$ (which are most of the situations we are interested in), LN is given by

$$LN = \max\{0, -\log_2 \nu_{\mathrm{min}}^{\mathrm{PT}}\},$$

where $\nu_{\mathrm{min}}^{\mathrm{PT}}$ is the lowest symplectic eigenvalue of σ^{PT}.

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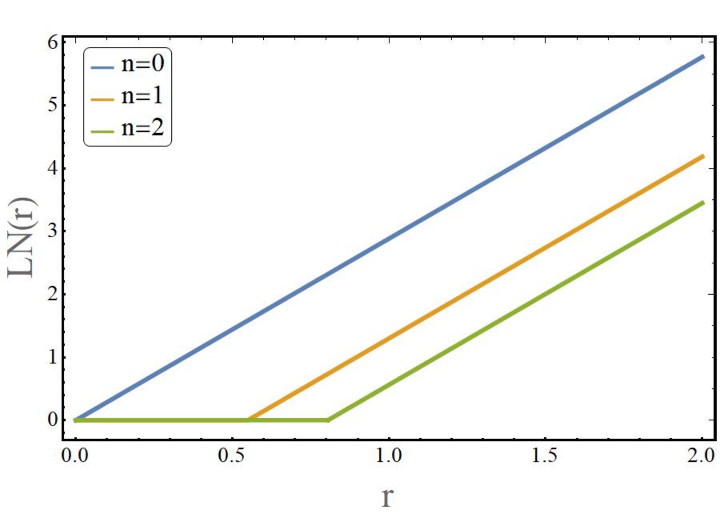

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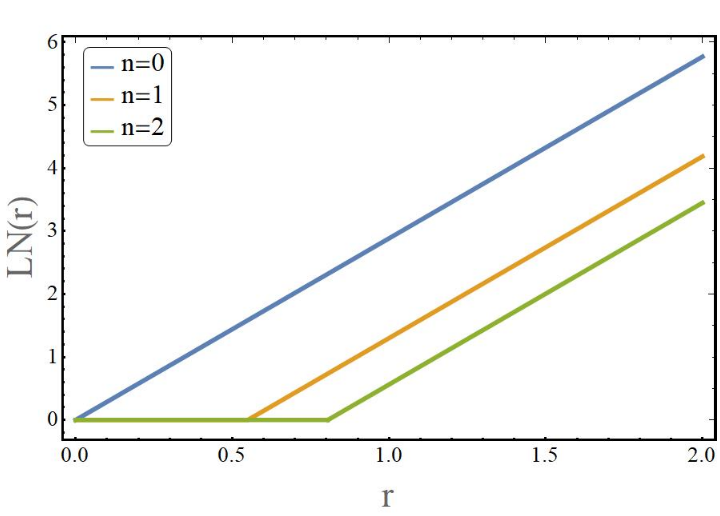

Message: Entanglement increases with r and decreases with n.

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- For **pure** states ($E_{AB} = 0$): $I = 2E_A = 2E_B$

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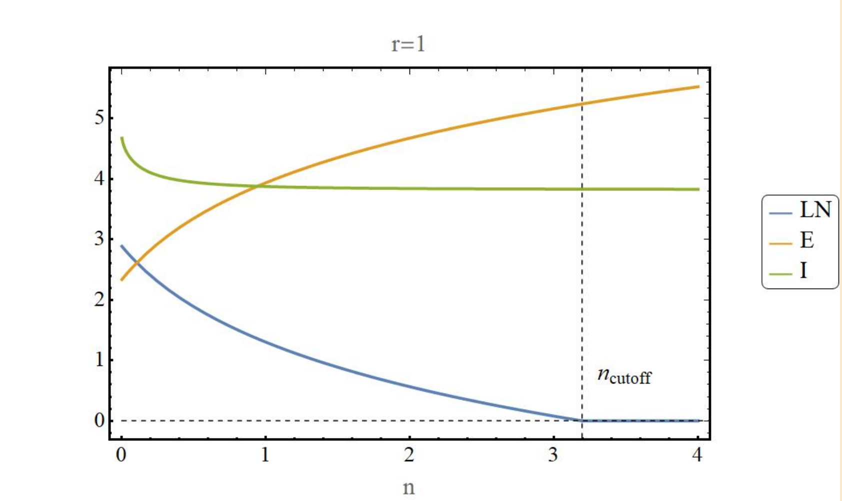

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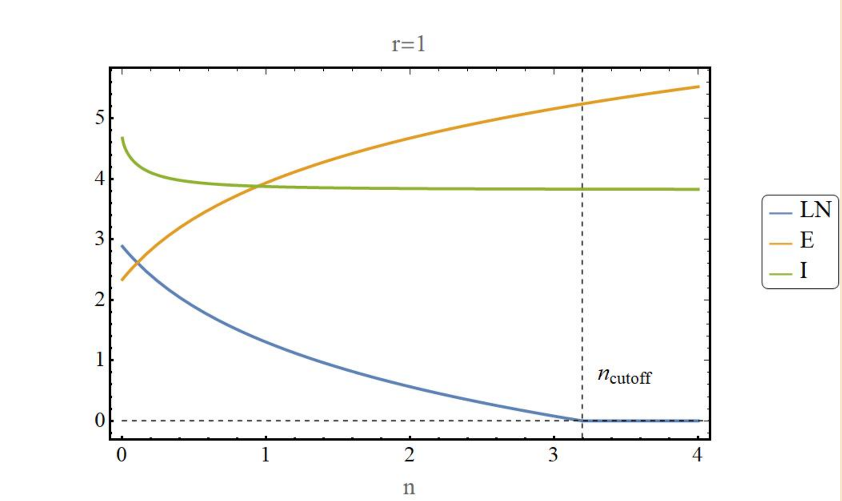


Message: The quantum state contains correlations even when entanglement disappears.

Observations?

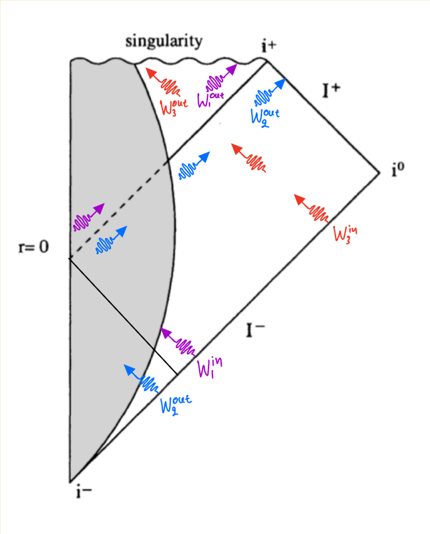
- **Entanglement**: Impossible as it would require extracting information from the interior of the black hole.
- **Hawking radiation**: Potentially...But, for "standard" black holes the resulting signal is extremely weak. Recall

$$T_{\mathrm{H}} = \frac{\hbar c^3}{8\pi G k_{\mathrm{B}} M}.$$

For $M = M_\odot \rightarrow T_{\mathrm{H}} = 61.7\,\mathrm{nK}$. On the other hand, $T_{\mathrm{CMB}} = 2.7\,\mathrm{K}$

Conclusion: Hawking radiation emitted by BHs of a typical mass is extremely weak and, thus, will be buried under other cosmic signals (e.g. CMB).

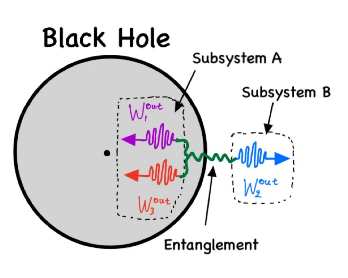
Hawking effect in a nutshell



- The Hawking process is a 3-mode interaction of a field (for concreteness we consider a masssless field).
- We associate $(\hat{a}_i, \hat{a}_i^\dagger)$, $i = 1, 2, 3$ to the three modes.
- At I^-, the field is in the vacuum state $|0\rangle_{\text{in}}$, i.e. $\hat{a}_i^{\text{in}} |0\rangle_{\text{in}} = 0$, $\forall\ i$.No quanta initially: $\langle 0| \hat{n}_i^{\text{in}} |0\rangle_{\text{in}} = \langle 0| (\hat{a}_i^{\text{in}})^\dagger \hat{a}_i^{\text{in}} |0\rangle_{\text{in}} = 0$.
- At I^+, a detector would measure $\langle 0| \hat{n}_2^{\text{out}} |0\rangle_{\text{in}} = \langle 0| (\hat{a}_2^{\text{out}})^\dagger \hat{a}_2^{\text{out}} |0\rangle_{\text{in}} = \Gamma_\omega \left(e^{\frac{\hbar\omega}{k_B T_H}} - 1 \right)^{-1}$.
- Black holes radiate as blackbodies of temperature $T_H = \frac{\hbar c^3}{8\pi G k_B M}$.
- The Hawking mode W_2^{out} is entangled with the interior modes W_1^{out} and W_3^{out}.

Entanglement in the Hawking effect

Let us compute the entanglement between Hawking radiation and the modes falling inside the black hole.



- Entanglement is directly produced in modes W_2^{out} and W_1^{out} by the two-mode squeezer.
- Due to the gravitational barrier (modeled by a beam splitter), some of the Hawking quanta are backscattered and follow into the black hole via the mode W_3^{out}. Hence, this mode will also be entangled with the W_1^{out}.

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- We study the energy scale (frequency) where **effects of dispersion** become important and the Hawking particle creation loses its thermal character.

Black holes immersed in a thermal bath

→ The previous calculations were made for black holes in isolation. What about black holes immersed in a thermal bath of photons (such as the CMB)?

→ Does the thermal bath affect particle production and generation of entanglement?

- The initial quantum state of the field is not the vacuum anymore, but rather a mixed state.
- The covariance matrix of each mode is $(2n_{\text{env},i} + 1)\boldsymbol{I}_2$. But, modes W_1^{in} and W_2^{in} have an ultra-high frequency and therefore $n_{\text{env},1} = n_{\text{env},2} \approx 0$. For W_3^{in}, $n_{\text{env},3} \equiv n_{\text{env}} = \left(e^{-\omega/T_{\text{env}}} - 1\right)^{-1}$. The initial state is $\boldsymbol{\mu}^{\text{in}} = (0,0)$, $\boldsymbol{\sigma} = \boldsymbol{I}_4 \oplus (2n_{\text{env}} + 1)\boldsymbol{I}_2$.(I should probably remove this last bullet as it is technical and doesn't offer much in the global discussion.)

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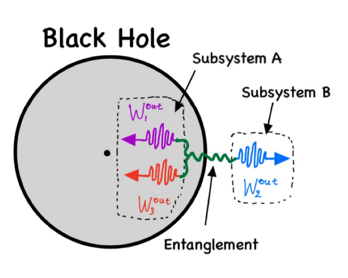
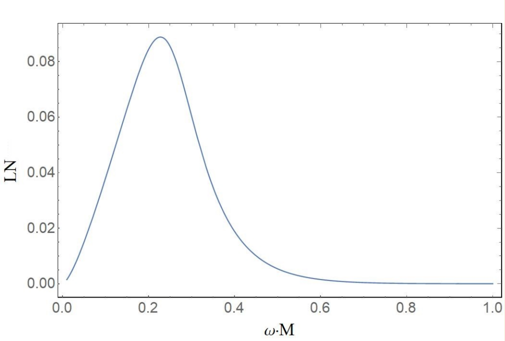
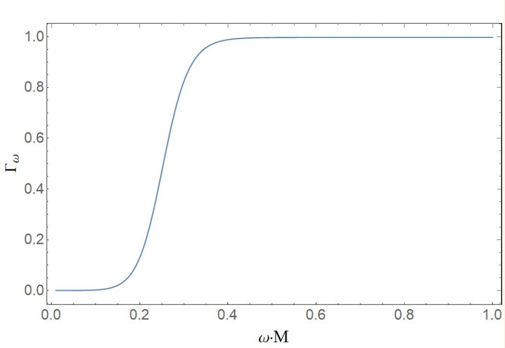
Let us compute the entanglement between Hawking radiation and the modes falling inside the black hole.



- Entanglement is directly produced in modes W_2^{out} and W_1^{out} by the two-mode squeezer.
- Due to the gravitational barrier (modeled by a beam splitter), some of the Hawking quanta are backscattered and follow into the black hole via the mode W_3^{out}. Hence, this mode will also be entangled with the W_1^{out}.

Entanglement in the Hawking effect





- At low ω, $\Gamma_\omega \to 0$: the gravitational barrier becomes fully reflective $\to$ No Hawking quanta escape.
- At high ω, $\Gamma_\omega \to 1$: the gravitational barrier becomes fully transparent $\to$ All Hawking quanta escape.
- The number of Hawking quanta produced at the horizon decreases monotonically with ω (since it follows a Bose-Einstein distribution).
- The competition of the last two functional forms results in a maximum value of LN at $\omega = 0.228\, M^{-1}$.

Black holes immersed in a thermal bath

→ The previous calculations were made for black holes in isolation. What about black holes immersed in a thermal bath of photons (such as the CMB)?

→ The initial quantum state of the field is not the vacuum anymore, but rather a mixed state.

→ Does the thermal bath affect particle production and generation of entanglement?