Black holes with electroweak hair

Mikhail S. Volkov

Institut Denis Poisson, University of Tours, FRANCE

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Brief history of hairy black holes

- No-hair conjecture /Ruffini and Wheeler, 1969/: black holes formed by gravitational collapse are characterized by their mass, angular momentum, and electric charge = the only parameters that can survive the collapse ⇒ all black holes are described by the Kerr-Newman metrics.

- No-hair theorems /Bekenstein, 1972,.../ confirm the conjecture for a number of special cases. No new black holes holes for gravitating massive scalar, spinor, of vector fields, also for a scalar field with a positive potential, etc.

- First explicit counter-example /M.S.V.+ Gal'tsov, 1989/: static black holes with Yang-Mills hair. Triggered an avalanche of discoveries of other hairy black holes.

Black holes with Yang-Millas hair

Non-Abelian Einstein-Yang–Mills black holes

M. S. Volkov and D. V. Gal'tsov
M. V. Lomonosov Moscow State University

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Pis'ma Zh. Eksp. Teor. Fiz. **50**, No. 7, 312–315 (10 October 1989)

Solutions of the self-consistent system of Einstein-Yang–Mills equations with the SU(2) group are derived to describe black holes with a non-Abelian structure of gauge fields in the external region.

In the case of the electrovacuum, the most general family of solutions describing spherically symmetric black holes is the two-parameter Reissner–Nordström family, which is characterized by a mass M and an electric charge Q. It was recently shown for the Einstein-Yang–Mills systems of equations with the SU(2) group that a corresponding assertion holds when the hold has a nonvanishing color-magnetic charge. In this case the structure of the Yang–Mills hair is effectively Abelian.[1] In the present letter we numerically construct a family of definitely non-Abelian solutions for Einstein-Yang–Mills black holes in the case of zero magnetic charge. These solutions are characterized by metrics which asymptotically approach the Schwarzschild metric far from the horizon but are otherwise distinct from metrics of the Reissner–Nordström family. In addition to the complete Schwarzschild metric, the family of solutions is parametrized by a discrete value of n; the number of nodes of the gauge function. For a

Discovery of hairy black holes

- before 2000: Einstein-Yang-Mills black holes and their generalizations – higher gauge groups, additional fields (Higgs, dilaton), non-spherical solutions, stationary generalizations, Skyrme black holes, Gauss-Bonnet, . . .
  /M.S.V.+Gal'tsov, Phys.Rep. 319 (1999) 1/

- after 2000: black holes with scalar hair – engineered potential, spinning clouds of massive complex scalar /Herdeiro-Radu/, Horndeski black holes, metric-affine theories, higher dimensions, stringy corrections, hairy black holes with massive gravitons /Gervalle+M.S.V., 2020/ , etc, . . .
  /M.S.V., 1601.0823/

- Which of these solutions are physical ?

Present status of hairy black holes

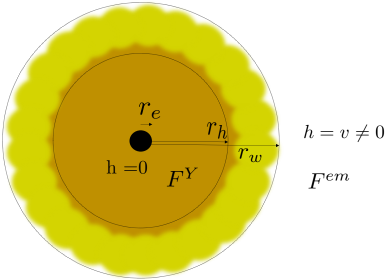
- All known solutions have been obtained within simplified theoretical models. They are nice theoretically but their physical relevance is not obvious.

- To be physically relevant, the solution should be obtained within the context of the physical theory = Einstein's gravity +Standard Model of fundamental interactions (QCD+electroweak).

- Classical configurations in the QCD sector are destroyed by large quantum corrections $\Rightarrow$ no use to study the. There remains the gravitating electroweak theory = Einstein-Weinberg-Salam. This describes the Kerr-Newman black holes. Does it describe some other black holes ?

- Only unphysical limits of the electroweak theory have been analyzed in the black hole context, since in the full theory the spherical symmetry is lost.

Magnetic electroweak black hole /Maldacena 2020/



The U(1) hypermagnetic field near the horizon + electroweak "corona" made of Z,W,Higgs fields + radial magnetic field in the far field. No symmetry.

Magnetic monopoles in Weinberg-Salam

The electroweak sector is more complex than the gravity sector.
What are magnetic electroweak monopoles in flat space ?

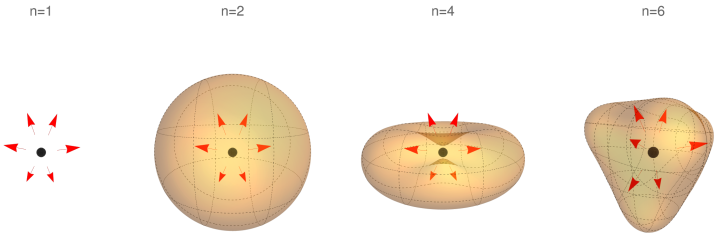
- t'Hooft-Polyakov monopole (topologically stable, finite energy) is not a solution of the Standard Model.
- Electroweak solution of Nambu describing a monopole attached to a vortex cannot be used within the context.
- Electroweak theory in the broken phase reduces to the Maxwell electrodynamics $\Rightarrow$ Dirac monopole is a solution:

$$\vec{\mathcal{B}} = \frac{P\vec{r}}{r^3} \qquad \text{with} \qquad P = \frac{n}{2e} \qquad \text{(Dirac quantization)}$$

where e = electron charge, $n \in \mathbb{Z}$ = "magnetic charge".

New results: Dirac monopole becomes unstable because the electroweak symmetry is restored at small r where $\mathcal{B}$ is large. All monopoles with even $|n| \geq 2$ are unstable with respect to condensing to less-energetic non-Abelian states:

Schematic view of electroweak multi-monopoles



Dirac monopoles with even $|n| \geq 2$ condense to non-Abelian states whose magnetic charge $P = n/(2e)$ splits into a pointlike $P_{\mathrm{U}(1)} = ng/(2g')$ at the origin and $P_{\mathrm{SU}(2)} = ng'/(2g)$ smoothly distributed in space. Only $|n| = 2$ is spherically symmetric (=Cho-Maison monopole, stability is confirmed), the $|n| > 2$ are new. Their energy is still infinite due to the central singularity, but the latter can be shielded by a black hole event horizon, the energy becomes finite $\Rightarrow$ a non-linear superposition of the electroweak multi-monopole with a black hole.

Einstein-Weinberg-Salam theory

$$\mathcal{L} = \frac{1}{2\kappa}\, R + \mathcal{L}_{\rm WS},$$

$$\mathcal{L}_{\rm WS} = -\frac{1}{4g^2}\, {\rm W}^a_{\mu\nu}{\rm W}^{a\mu\nu} - \frac{1}{4g'^2}\, B_{\mu\nu}B^{\mu\nu} - (D_\mu\Phi)^\dagger D^\mu\Phi - \frac{\beta}{8}\left(\Phi^\dagger\Phi - 1\right)^2$$

where Higgs Φ is a complex doublet, $\Phi^{\rm tr} = (\phi_1, \phi_2)$,

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu\,, \qquad {\rm W}^a_{\mu\nu} = \partial_\mu {\rm W}^a_\nu - \partial_\nu {\rm W}^a_\mu + \epsilon_{abc}{\rm W}^b_\mu {\rm W}^c_\nu\,,$$

$$D_\mu\Phi = \left(\partial_\mu - \frac{i}{2}\, B_\mu - \frac{i}{2}\,\tau^a {\rm W}^a_\mu\right)\Phi$$

Length scale $= 1.5\times 10^{-16}$ cm, mass scale $= 128.6$ GeV, couplings

$$g^2 = 0.77, \quad g'^2 = 0.23, \quad \beta = 1.88, \quad \kappa = \frac{8\pi \mathbf{G}\boldsymbol{\Phi}_0^2}{\boldsymbol{c}^4} = 5.42\times 10^{-33}.$$

Fundamental excitations are the graviton, photon, Z, W, Higgs bosons with masses $m_{\rm z} = 1/\sqrt{2}$, $m_{\rm w} = g m_{\rm z}$, $m_{\rm h} = \sqrt{\beta}\, m_{\rm z}$. Electron charge $e = gg'$.

Equations to solve

Electroweak:

$$\nabla^\mu B_{\mu\nu} = g'^2 \, \frac{i}{2} \left(\Phi^\dagger D_\nu \Phi - (D_\nu \Phi)^\dagger \Phi \right),$$
$$\mathcal{D}^\mu \mathrm{W}^a_{\mu\nu} = g^2 \, \frac{i}{2} \left(\Phi^\dagger \tau^a D_\nu \Phi - (D_\nu \Phi)^\dagger \tau^a \Phi \right),$$
$$D_\mu D^\mu \Phi - \frac{\beta}{4} (\Phi^\dagger \Phi - 1) \Phi = 0,$$

Einstein:

$$\begin{aligned} G_{\mu\nu} &= \kappa T_{\mu\nu} \qquad \text{where } \kappa \sim 10^{-33}, \\ T_{\mu\nu} &= \frac{1}{g^2} \mathrm{W}^a{}_{\mu\sigma} \mathrm{W}^a{}_\nu{}^\sigma + \frac{1}{g'^2} B_{\mu\sigma} B_\nu{}^\sigma + 2 D_{(\mu} \Phi^\dagger D_{\nu)} \Phi + \mathrm{g}_{\mu\nu} \mathcal{L}_{\mathrm{WS}} \, . \end{aligned}$$

30 coupled equations. A simple solution:

Magnetically charged Reissner-Nordstrom

The U(1) Dirac monopole

$$B = W^3 = \frac{n}{2}\cos\vartheta\, d\varphi, \quad W^1 = W^2 = 0, \quad \Phi = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

with the RN metric

$$ds^2 = -N(r)\, dt^2 + \frac{dr^2}{N(r)} + r^2\left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right),$$

$$N(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}, \qquad Q^2 = \frac{\kappa n^2}{8e^2}, \qquad n \in \mathbb{Z},$$

the event horizon is at $r_H = M + \sqrt{M^2 - Q^2}$.

<u>This solution is stable at large r_H but becomes unstable at small r_H</u>

Stability of Reissner-Nordstrom

The RN background shows instabilities only in a sector with the orbital angular momentum $\boxed{j = |n|/2 - 1 = 0, 1, 2, \ldots \quad (\text{even}|n|)}$ where one obtains the one-channel problem (other 29 channels decouple):

$$\left(-\frac{d^2}{dr_\star^2} + N(r)\left[\frac{g^2}{2} - \frac{|n|}{2r^2}\right]\right)\psi(r) = \omega^2\psi(r) \quad (\star)$$

with $dr_\star = dr/N(r)$. In flat space $N(r) = 1$ and Eq.($\star$) admits infinitely many bound states with $\omega^2 < 0 \Rightarrow$ Dirac monopoles with even $|n|$ are unstable. The $n = 2$ monopole is unstable in the $j = 0$ sector (not splitting !).

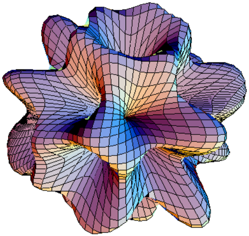
In curved space $N(r) \leq 1$ and Eq.($\star$) admits only a finite number of bound states. No bound states if r_H is large. There is a critical value r_H^0 for which the first bound state appears as a static zero mode with $\omega = 0$. This zero mode solution $\psi_0(r)$ approximates the W-condensate = black hole hair.

Perturbative black hole hair

Values $r_H^0(n)$ for which the zero mode appears

n	2	4	6	10	20	40	100	200
r_H^0	0.89	1.47	1.93	2.69	4.12	6.19	10.33	15.03

The mode is maximal at the horizon and proportional to $Y_{jm}(\vartheta, \varphi)$ with $j = |n/2| - 1$, describes W-currents tangential to the horizon which produce Z-fluxes orthogonal to the horizon= Z-strings of finite length =corona. Schematically,



Non-linear black hole hair with axial symmetry

Hairy solutions cannot be spherically symmetric for $|n| > 2$ but can be axially symmetric, with the metric and the electroweak fields

$$ds^2 = -e^{2U} N(r)\, dt^2 + e^{2k-2U}\left(\frac{dr^2}{N(r)} + r^2 d\vartheta^2 + e^{2w} r^2 \sin^2\vartheta d\varphi^2\right),$$

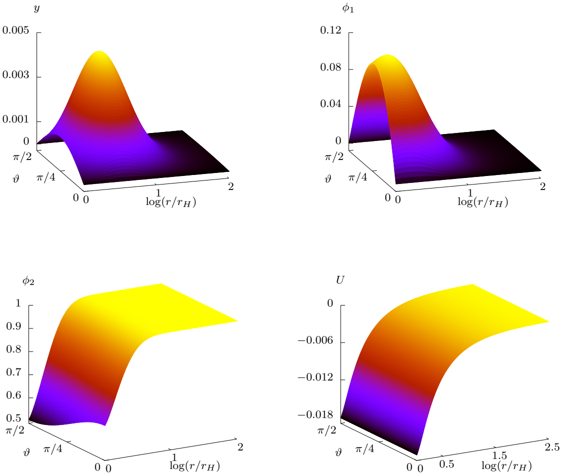
$$W = \mathrm{T}_a \mathrm{W}^a_\mu dx^\mu = \mathrm{T}_2\,(F_1\, dr + F_2\, d\vartheta) + \frac{n}{2}\,(\mathrm{T}_3\, F_3 - \mathrm{T}_1 F_4\,)\, d\varphi$$

$$B_\mu dx^\mu = \frac{n}{2}\, Y d\varphi\,, \qquad \Phi = \begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix} \qquad \mathrm{T}_a = \tau_a/2\,,$$

where $U, k, w, F_1, F_2, F_3, F_4, Y, \phi_1, \phi_2$ are 10 real functions of r, ϑ and $N(r) = 1 - r_H/r$ where r_H is the black hole "size". The Dirac string can be removed if $n/2 \in \mathbb{Z}$.

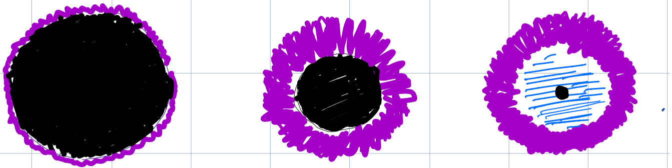
The 10 coupled PDE's are solved with the FreeFem++ numerical solver (Paris VI) based on the finite element method. For $n = \pm 2$ the solution is spherically symmetric. Iteratively increasing n gives solutions with higher magnetic charge.

Numerical solutions



Decreasing the horizon size r_H

$$r_H^{min}(n) \leq r_H \leq r_H^{max}(n)$$

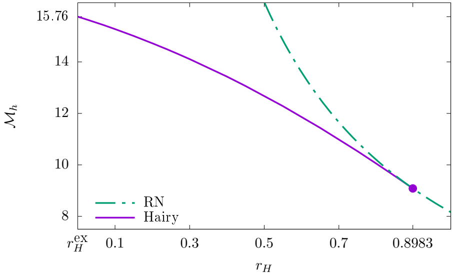


when horizon decreases, more and more energy and charge are stored in the hair.
The size of the hair region does not change
As $r_H \rightarrow r_H^{min}(n)$, a buble of symmetric phase with Reissner-Nordström-de Sitter geometry appears in the center

ADM mass

$$-g_{00} = 1 - \frac{2M}{r} + \ldots,$$
$$M = \frac{\mathrm{k_H A_H}}{4\pi} + \frac{\kappa}{4\pi}\int_{r>r_\mathrm{H}} (2\,T_{\hat{0}\hat{0}} + T)\sqrt{-g}\, d^3x$$

Hairy solutions are less energetic than the RN of the same size.



Energy density

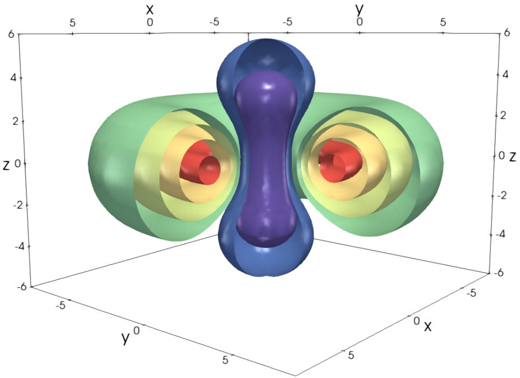


Figure: Surfaces of constant $T_{\hat{0}\hat{0}}$ for $n = 10$.

Magnetic charge and electric current

The electromagnetic field tensor and its dual,

$$
\begin{aligned}
F_{\mu\nu} &= \frac{g}{g'} B_{\mu\nu} - \frac{g'}{g} N^a \mathrm{W}^a_{\mu\nu} \quad \text{with} \quad N^a = \frac{\Phi^\dagger \tau^a \Phi}{\Phi^\dagger \Phi}, \\
\tilde{F}^{\mu\nu} &= \frac{1}{2\sqrt{-\mathrm{g}}} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} \,,
\end{aligned}
$$

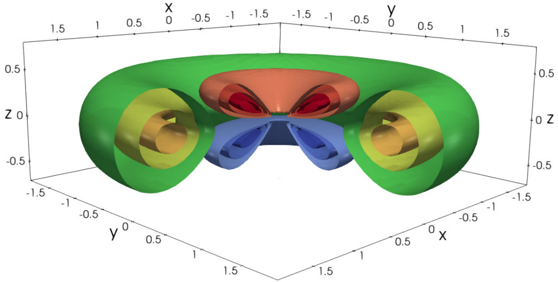
the electric and magnetic 4-currents

$$
J^\mu = \frac{1}{4\pi} \frac{1}{\sqrt{-\mathrm{g}}} \partial_\nu \left(\sqrt{-\mathrm{g}}\, F^{\mu\nu}\right), \qquad \tilde{J}^\mu = \frac{1}{4\pi} \frac{1}{\sqrt{-\mathrm{g}}} \partial_\nu \left(\sqrt{-\mathrm{g}}\, \tilde{F}^{\mu\nu}\right),
$$

magnetic 3-current is $\vec{J}$ and the magnetic charge density $\rho = \tilde{J}^0$.

When the central black hole is small, the hair shows the following structure:

Structure of the hair, $n = 4$.



A small black hole is in the center. Magnetic charge density ρ and positive J_φ and negative J_φ density of the azumuthal electric current. The magnetic charge forms a ring whose magnetic field forces the charged W-bosons to Larmore-orbit, creating two electric currents. These currents create the magnetic field which squeezes the magnetic charge toward equatorial plane.

Extreme limit

- As r_H approaches the lower bound $r_H \to r_H^{\rm min}$, the horizon becomes degenerate and the temperature vanishes.
- The size of the extreme black hole $r_H^{\rm min} \sim \sqrt{\kappa} \sim 10^{-17}$ is parametrically small as compared to the size of the hair region. The massive hair live in the $r \ggg r_H$ region where the geometry is flat, it does not feel the black hole, it feels only the U(1) hypermagnetic field created by the black hole.
- Close to the horizon the hypermagnetic field $B \sim |n|/r^2$ is very strong and drives to zero the SU(2) and Higgs fields, $W^a_\mu \to 0$, $\Phi \to 0$. This creates a bubble of symmetric phase around the horizon where the geometry is the extreme RN-de Sitter for the charge $P_{\rm U(1)} = ng/(2g')$ and cosmological constant $\Lambda = \kappa\beta/8$. The horizon size

$$r_H^{\rm min} = \frac{\sqrt{\kappa}|n|}{2\sqrt{2}g'} \lll r_{\rm hair} \sim \sqrt{|n|}$$

False vacuum bubble

The strong hypermagnetic field drives to zero the Higgs field in the horizon vicinity:

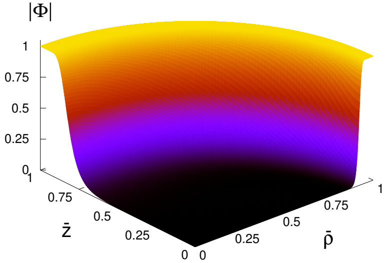


Figure: The norm of the Higgs field $|\Phi|$ for $n = 100$. The black hole is at the origin.

Solution structure

- Inside the central bubble: a tiny RNdS black hole supporting the hypermagnetic field of charge $P_{\mathrm{U}(1)} = ng/(2g')$ which suppresses the other fields and restores the full electroweak symmetry:

$$B = \frac{n}{2r^2}, \quad W^a_\mu = 0, \quad \Phi = 0, \quad \text{RNdS geometry}$$

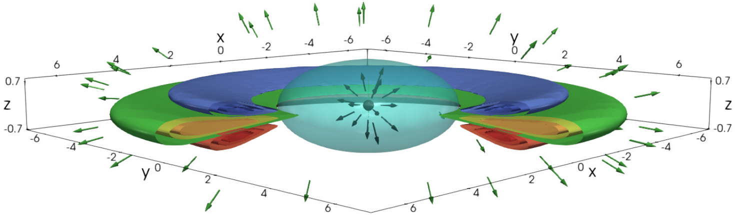
- Bubble wall = hair: a condensate of massive W, Z, Φ carrying the magnetic charge $P_{\mathrm{SU}(2)} = ng'/(2g)$.
- Far field region: All massive fields vanish, radial magnetic field of charge $P_{\mathrm{U}(1)} + P_{\mathrm{SU}(2)}$,

$$\vec{\mathcal{B}} = \frac{n}{2e}\frac{\vec{r}}{r^3}, \qquad Z = W = 0, \quad \Phi = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- Estimates for the hair size, energy, and quadrupole moments:

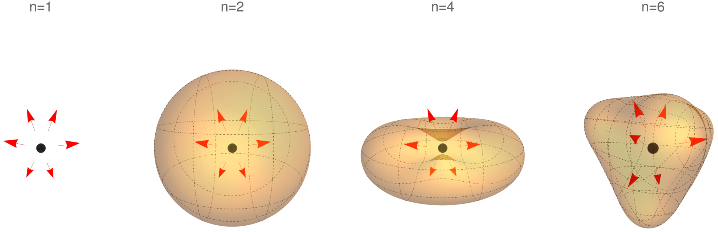
$$r_{\mathrm{hair}} \sim \sqrt{|n|}, \quad M_{\mathrm{hair}} \sim 10|n|^{3/2}, \quad Q_{\mathrm{WS}} \sim n^2, \quad Q_{\mathrm{G}} \sim \kappa Q_{\mathrm{WS}}$$

View of extreme black hole with $n = 40$.



The extreme RNdS black hole at the center is surrounded by the vacuum bubble filled with the U(1) hypermagnetic field of charge $P_{\mathrm{U}(1)}$. Outside the bubble the W-bosons condense to a green ring of magnetic charge $P_{\mathrm{SU}(2)}$ squeezed between two superconducting electric currents =hair. Farther away there remains only the radial magnetic field of total charge $P_{\mathrm{U}(1)} + P_{\mathrm{SU}(2)} = n/(2e)$.

Then $r_H = r_H^{\min} \to 0$ and the black hole configurations approach the flat space multi-monopoles pointwise for $r > r_H$.



Increasing the magnetic charge n

- Unless n is very large, the geometry is very close to RN, the hair contribution to the total mass being negligible.
- Maximal charge n: The minimal value of the event horizon $r_H^{\mathrm{min}} \propto \sqrt{\kappa}\,|n|$ increases with n faster than the maximal value $r_H^{\mathrm{max}} \propto \sqrt{n}$. The two merge for $\boxed{n \sim 1/\kappa \approx 10^{32}}$, then the hair contribution to the total mass is equal to the U(1) contribution. The black holes size and mass are then $r_H \propto \kappa^{-1/2} \approx 1\ cm$ and $\mathcal{M} \propto \mathcal{M}_{\mathrm{Pl}}/\kappa \approx 10^{25} kg$ = planetary size (not small) hairy black holes.
- For a given n there should be also $|n|$ different non-axially symmetric hairy black holes with symmetries of $Y_{jm}(\vartheta, \varphi)$, $j = |n/2| - 1 \Rightarrow$ CORONA. Their number is the same as the black hole entropy $S \sim r_H^2 \sim n$ = /also entropy?/.

Conclusion

Solutions describing black hole with electroweak hair which may perhaps be astrophysically relevant are constructed.