IJCLab Irène Joliot-Curie
Laboratoire de Physique des 2 Infinis

cnrs

université PARIS-SACLAY

Université de Paris

# Black holes in scalar-tensor theories


Eugeny Babichev

*Laboratoire de Physique des 2 Infinis Irène Joliot-Curie*



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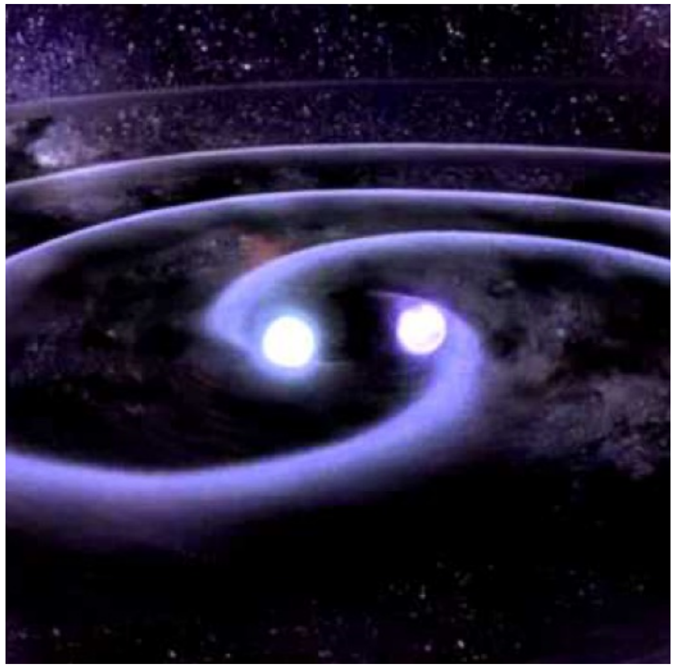
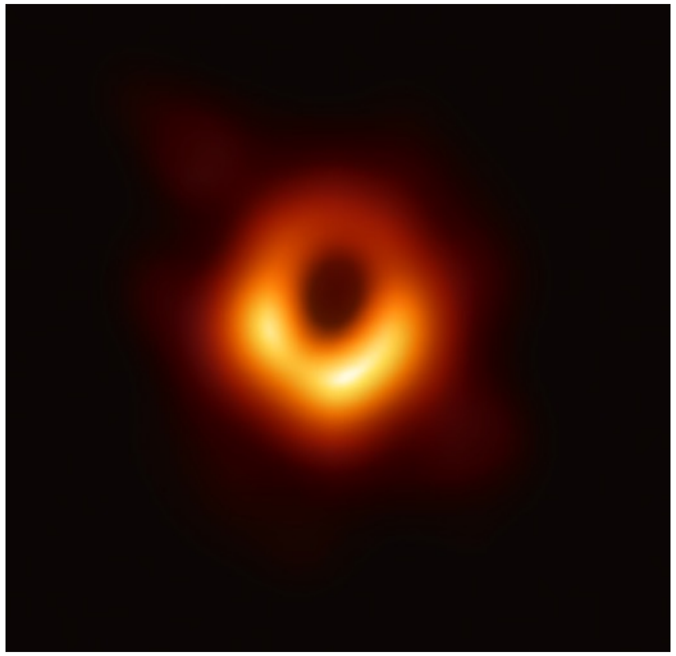
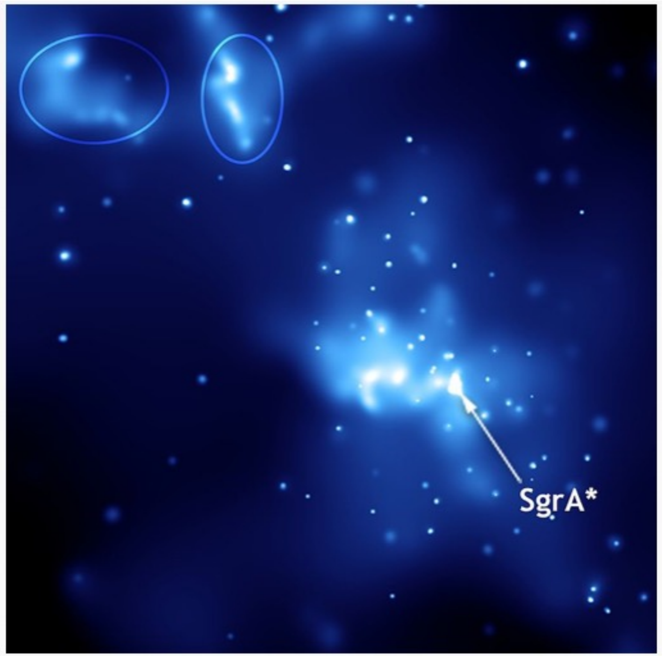
# Motivation

## Observation of black holes and neutron stars: a breakthrough







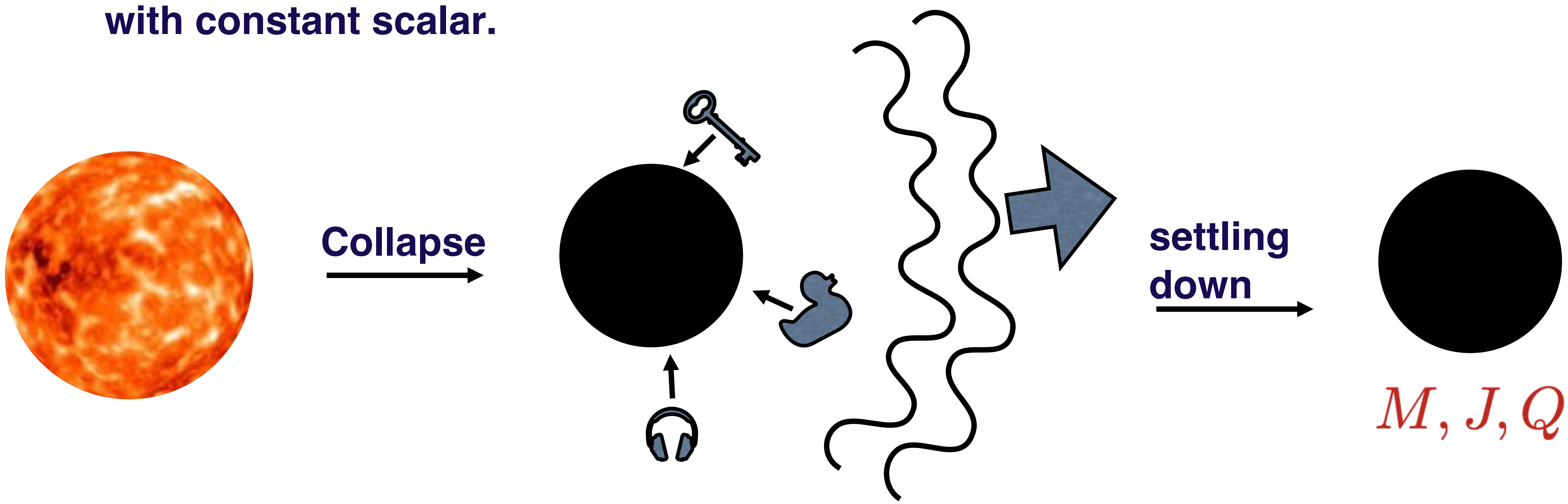

| GW signals from binaries at their ringdown phase (LIGO/Virgo) | Image of M87 black hole with its light ring (from array of radio telescopes, EHT) | Observation of star trajectories orbiting SgrA central black hole (GRAVITY) |
| --- | --- | --- |

- Alternatives to GR black holes and stars as precise rulers of departure from GR?
- Other compact objects like wormholes?

# No hairs in GR

- Gravitational collapse ->
- Black holes eat or expel surrounding matter
- Their stationary phase is characterised by a limited number of charges
- No details about collapse
- Black holes are bald

- **No hair theorems/arguments dictate that adding degrees of freedom lead to trivial (General Relativity) or singular solutions.**
- **E.g. in the standard scalar-tensor theories BH solutions are GR black holes with constant scalar.**

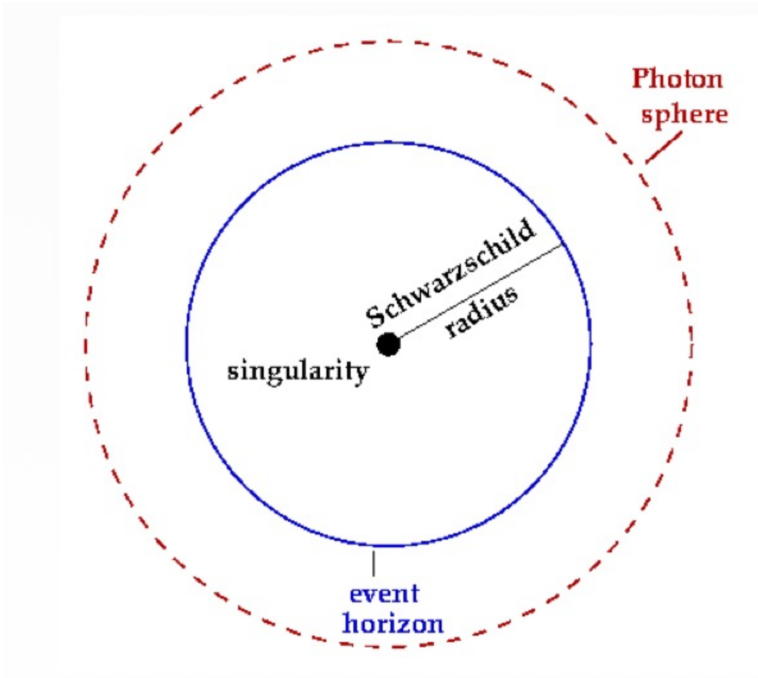


# Schwarzschild solution

- Schwarzschild solution (static and spherically symmetric):

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2, \quad f(r) = 1 - \frac{2M}{r}$$

- The zero of $f(r)$ is the horizon of the black hole ($r_g = 2M$).
- An event horizon is a surface of no return. Nothing can escape the event horizon.
- An interior of the event horizon hides the curvature singularity at $r = 0$.



# Kerr solution

Kerr'63

- Rotating vacuum black holes in General Relativity are described by the Kerr metric.

- In Boyer-Lindquist coordinates:

$$\mathrm{d}s^2 = -\left(1 - \frac{2Mr}{\rho^2}\right)\mathrm{d}t^2 - \frac{4aMr\sin^2\theta}{\rho^2}\mathrm{d}t\mathrm{d}\varphi + \frac{\sin^2\theta}{\rho^2}\left[(r^2+a^2)^2 - a^2\Delta\sin^2\theta\right]\mathrm{d}\varphi^2 + \frac{\rho^2}{\Delta}\mathrm{d}r^2 + \rho^2\mathrm{d}\theta^2$$

where $M$ is the mass, $a$ is the angular momentum and

$$\rho^2 = r^2 + a^2\cos^2\theta$$

$$\Delta = r^2 + a^2 - 2Mr$$

- A ring singularity at $\rho = \sqrt{r^2 + a^2\cos^2\theta} = 0$, i.e.

$$r = 0 \qquad \text{and} \qquad \theta = \frac{\pi}{2}$$

# Properties of the Kerr metric

- The metric is stationary and axi-symmetric, which corresponds to 2 Killing directions

$$\xi_{(t)} = \partial_t \qquad \text{and} \qquad \xi_{(\varphi)} = \partial_\varphi$$

- The spacetime is circular, i.e. symmetric under the reflection $(t, \varphi) \to (-t, -\varphi)$, because the Killing fields verify the condition

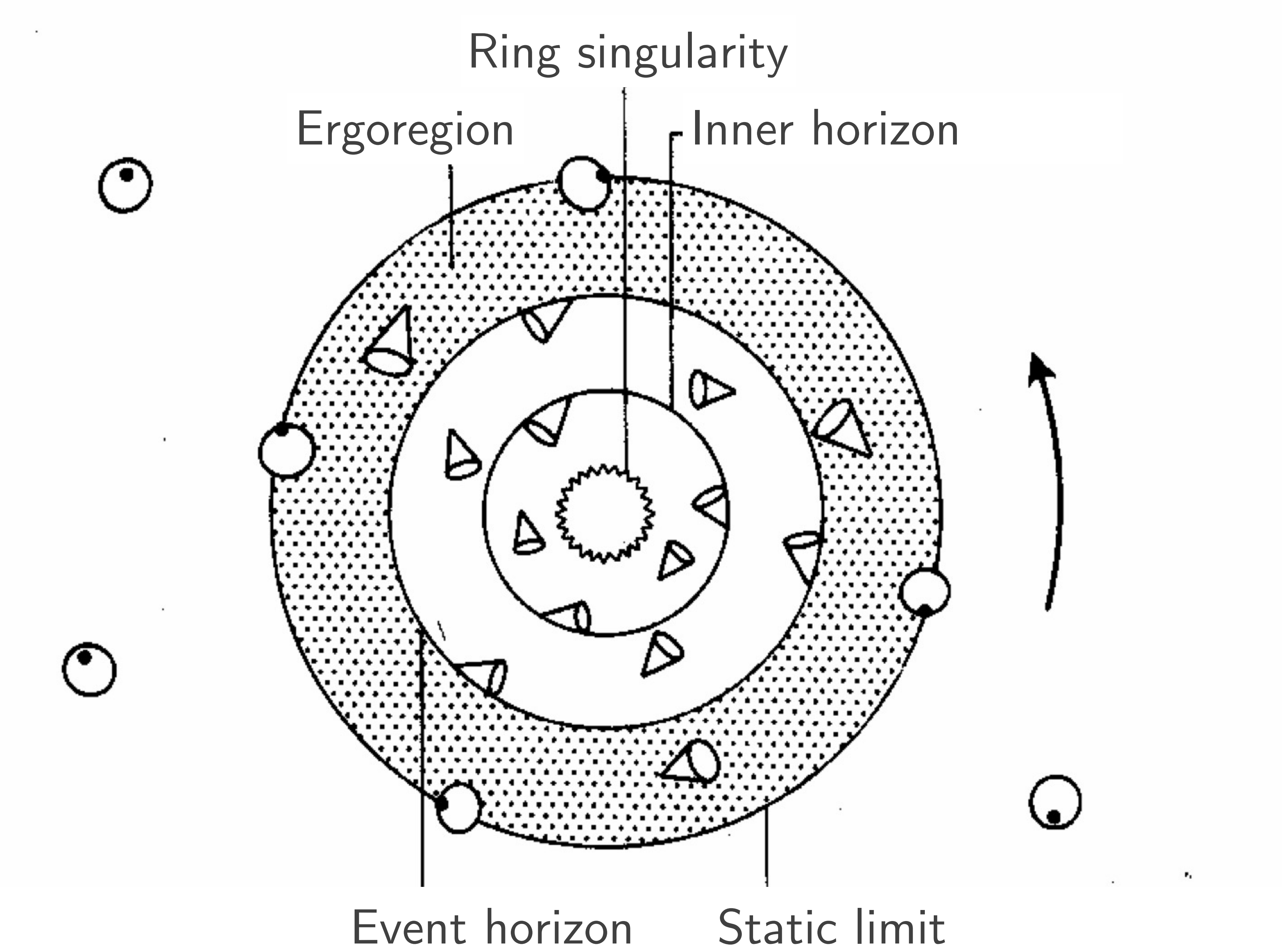
$$\boxed{\xi_{(t)} \wedge \xi_{(\varphi)} \wedge \mathrm{d}\xi_{(t)} = \xi_{(t)} \wedge \xi_{(\varphi)} \wedge \mathrm{d}\xi_{(\varphi)} = 0 \,.}$$

- The Kerr spacetime also admits a nontrivial Killing 2-tensor $K$ verifying the equation

$$\nabla_{(\mu} K_{\nu\sigma)} = 0 \,.$$

- This defines a third nontrivial constant of motion along geodesics (Carter's constant). The geodesic equations thus reduce to a first order system.

# Important surfaces in the Kerr metric




from d'Inverno's book

# How to construct exact black hole solutions in modified gravity?

**Shift-symmetry**

**Conformal symmetry**

**Stability of DHOST theory under conf/disformal transformation**

# Horndeski theory

Horndeski'1974

$$S = \int d^4x\, F\left[g, \partial g, \partial^2 g, \partial^3 g, ...\varphi, \partial\varphi, \partial^2\varphi, \partial^3\varphi, ...\right] \quad \rightarrow \quad E[g, \partial g, \partial^2 g, \varphi, \partial\varphi, \partial^2\varphi] = 0$$

$$G_2(X,\phi),\ G_3(X,\phi),\ G_4(X,\phi),\ G_5(X,\phi)$$

$$X = \partial_\mu\phi\partial^\mu\phi$$

$$\mathcal{L}_2 = G_2\left(X,\phi\right)$$
$$\mathcal{L}_3 = G_3\left(X,\phi\right)\Box\phi$$
$$\mathcal{L}_4 = G_4(X,\phi)\,R + G_{4,X}(X,\phi)\left[(\Box\phi)^2 - (\nabla\nabla\phi)^2\right]$$
$$\mathcal{L}_5 = G_{5,X}\left(X,\phi\right)\left[(\Box\phi)^3 - 3\Box\phi\left(\nabla\nabla\phi\right)^2 + 2\left(\nabla\nabla\phi\right)^3\right] - 6G_5\left(X,\phi\right)\,G_{\mu\nu}\nabla^\mu\nabla^\nu\phi$$

Deffayet+'09'11

Kobayashi+'11

# Degenerate higher order Scalar-Tensor theories (DHOST)

Langlois&Noui, Crisostomi+'16

$$S = M_P^2 \int \mathrm{d}^4x \sqrt{-g} \left( f(\phi, X) R + K(\phi, X) - G_3(\phi, X) \Box\phi + \sum_{i=1}^{5} A_i(\phi, X) \mathcal{L}_i \right) + S_\mathrm{m}\left[g_{\mu\nu}, \psi_\mathrm{m}\right]$$

$$\mathcal{L}_1 = \phi_{\mu\nu}\phi^{\mu\nu}, \quad \mathcal{L}_2 = (\Box\phi)^2, \quad \mathcal{L}_3 = \phi_{\mu\nu}\phi^\mu\phi^\nu\Box\phi,$$
$$\mathcal{L}_4 = \phi_\mu\phi^\nu\phi^{\mu\alpha}\phi_{\nu\alpha}, \quad \mathcal{L}_5 = (\phi_{\mu\nu}\phi^\mu\phi^\nu)^2$$
$$X = \phi^\mu\phi_\mu$$

- One subclass of DHOST (subclass Ia) is phenomenologically interesting [Langlois, Noui; Crisostomi+'16]:

$$A_2 = A_2(A_1, A_3)$$
$$A_4 = A_4(A_1, A_3)$$
$$A_5 = A_5(A_1, A_3)$$

# From DHOST to DHOST

Under a disformal transformation

$$g_{\mu\nu} \to C(\phi, X) g_{\mu\nu} + D(\phi, X)\partial_\mu\phi\partial_\nu\phi$$

DHOST $\Rightarrow$ DHOST

[Achour+,Crisostomi+'16]

More precisely,

$$S_{DHOST}[\tilde{g}_{\mu\nu}, \phi] = S_{DHOST}[g_{\mu\nu} + D(X)\partial_\mu\phi\partial_\nu\phi, \phi] = \bar{S}_{DHOST}[g_{\mu\nu}, \phi]$$

**Theory 1**

$$\bar{S}_{DHOST}[g_{\mu\nu}, \phi] + S_{\text{matter}}[g_{\mu\nu}, \Psi_{\text{matter}}]$$

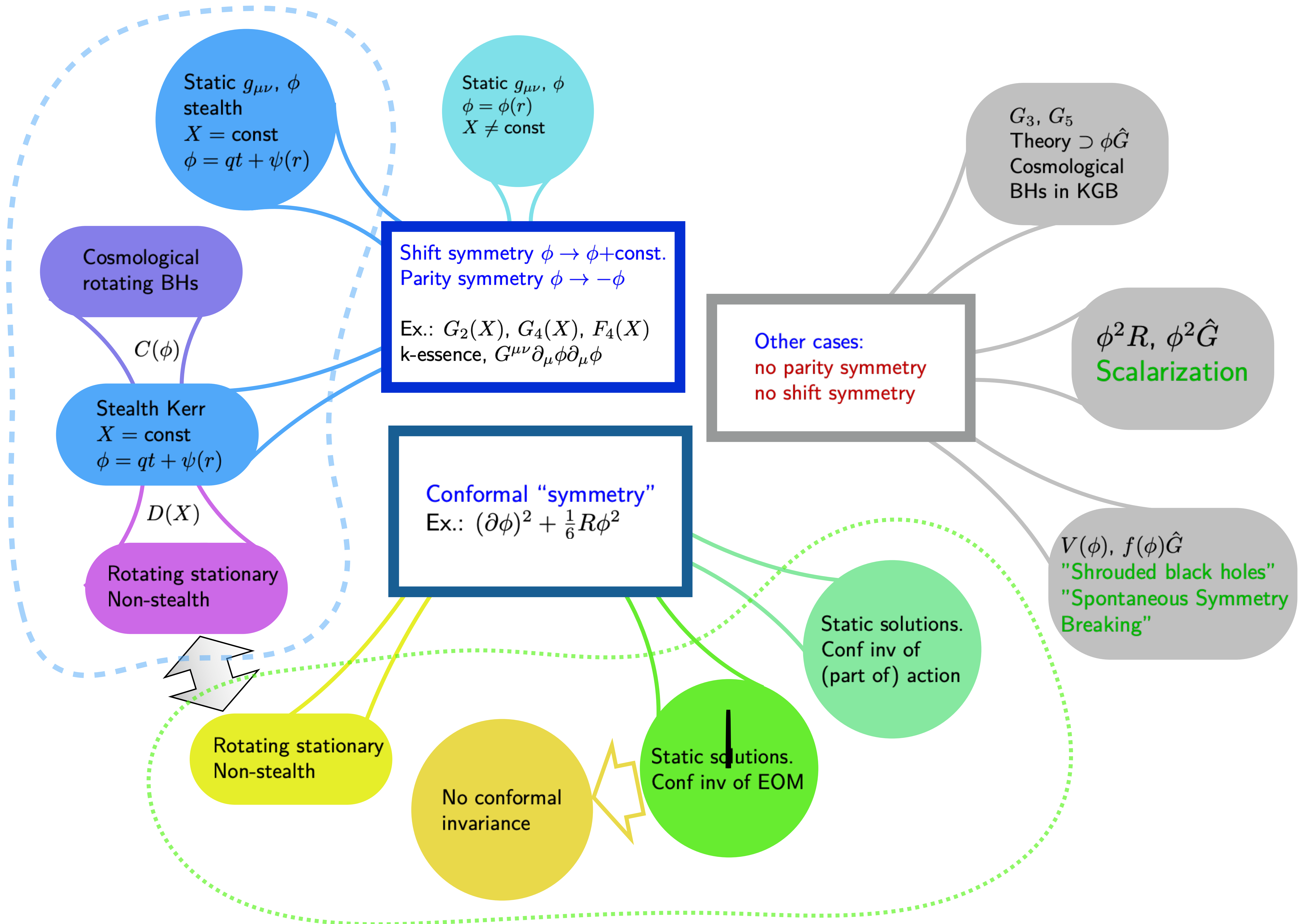
**Theory 2**

$$S_{DHOST}[\tilde{g}_{\mu\nu}, \phi] + S_{\text{matter}}[\tilde{g}_{\mu\nu}, \Psi_{\text{matter}}]$$

**Coupling to matter is different: we get a different theory**

see also a talk by Hugo Roussille
for Non-linear gravitational waves in DHOST

# Hairy solutions in ST theories



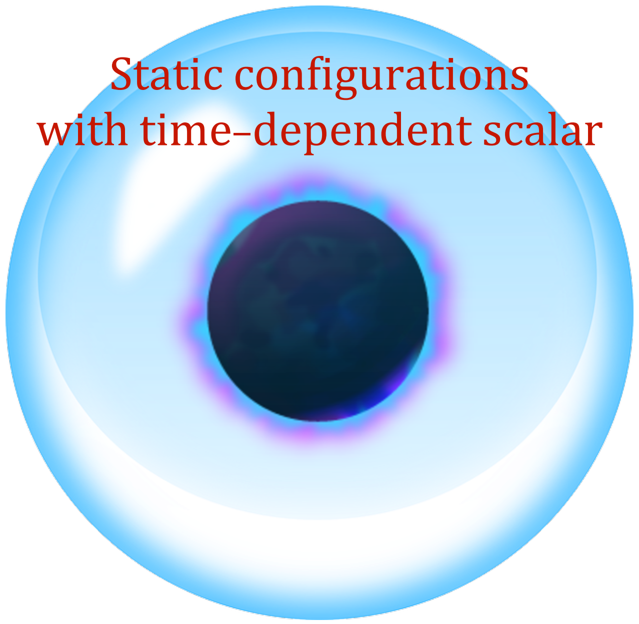
# Shift symmetry

# Shift symmetry and time dependence

$$S = \int d^4 \mathcal{L}\left(g_{\mu\nu}, \partial g_{\mu\nu}, \ldots, \cancel{\phi}, \partial\phi, \partial^2\phi, \ldots\right) \Rightarrow$$

The ansatz $\phi = qt + \ldots$ goes through EOMs, leaving no $t$-dependence (only $q$).

$T_{\mu\nu}$ is $t$-independent.




Shift symmetry of the theory implies conserved current $\nabla_\mu J^\mu = 0$. Need to impose

$$J^r = 0$$

because $J^r \propto E^t_r$.

**Linear time-dependence $\phi = qt + \psi(r, \theta)$:**

- **Possibility to build non-trivial solutions**
- **Matching to cosmology**
- **Static (stationary) metric**

# Example of exact solution

EB, Charmousis'13

- Subclass of Horndeski theory:

$$S = \int d^4x\sqrt{-g}\,(R - 2\Lambda - \eta X + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi)$$

- Simple (stealth) solution reads

$$f = h = 1 - \frac{2M}{r} + \frac{\eta}{3\beta}r^2, \quad \phi = qt \pm \int dr\frac{q}{h}\sqrt{1-h}.$$

Secondary hair $q^2 = \frac{\zeta\eta+\Lambda\beta}{\beta\eta}$

- $X = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi = -q^2$ is constant for such solutions [Kobayashi&Tanahashi'14]. Leads to nice generalization to include arbitrary $G_2$ and $G_4$.

- Also there are further generalisations to beyond Horndeski, DHOST.

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- Disformal transformation $g_{\mu\nu} \to g_{\mu\nu} + D(X)\phi_\mu\phi_\nu$, e.g. to get the speed of gravity = speed of light [EB, Charmousis, Esposito-Farèse, Lehébel]:

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{\beta}{\zeta + \frac{\beta}{2}\varphi_\lambda^2}\varphi_\mu\varphi_\nu$$

A coordinate change shows that
$\mathcal{D}$(spherical stealth) = spherical stealth

# Rotating solution?

Charmousis+'19

- The idea is to associate the scalar $\phi$ with the geodesics in Kerr space.
- Hamilton-Jacobi equation

$$g^{\mu\nu}_{\mathrm{Kerr}} \partial_\mu S \partial_\nu S = -m^2$$

- If we assume for the scalar $X = g^{\mu\nu}_{\mathrm{Kerr}} \partial_\mu \phi \partial_\nu \phi = -q^2$ (like in spherical symmetry), one can look for the solution $\phi = S$.
- Ensure that there is no backreaction so Kerr solution remains to be valid. Restricts considerably the class of the DHOST theories.
- Choose geodesics such that $\phi$ is regular everywhere (at least outside the horizon). Fix constants of integration of geodesics.

# Stealth Kerr solution in DHOST

Charmousis+'19

- A stealth Kerr solution, where the metric is Kerr and the scalar field such that

$$g = g_{\mathsf{Kerr}}$$
$$X = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi = X_0 = \text{const.}$$
$$\phi = q\left[t + \int \frac{\sqrt{2Mr(a^2+r^2)}}{\Delta}\mathrm{d}r\right]$$

- The metric $g_{\mathsf{Kerr}}$ is regular everywhere apart from the ring singularity and
- The scalar field is regular at $r > 0$.

# Cosmological black holes

EB, Charmousis, Lecoeur'23

- Time-dependent solutions with $\phi = qt + \psi(r,\theta)$ with flat asymptotic: $g_{\mu\nu} \to \eta_{\mu\nu}$ and $\phi = qt$ as $r \to \infty$.
- Perform a conformal transformation of the solution $g_{\mu\nu} \mapsto \widetilde{g}_{\mu\nu} = C\,(\phi)\, g_{\mu\nu}$. $\mathcal{C}$(DHOST) $=$ DHOST.
- $\phi$ plays a role of conformal time of expanding universe: asymptotically $\eta_{\mu\nu} \mapsto \widetilde{g}_{\mu\nu} = C\,(\phi)\, \eta_{\mu\nu}$ with $C(\phi) \equiv a^2_{\mathrm{FLRW}}(\phi)$.
- Choice of $C$ corresponds to a cosmological evolution.
- Regular $\phi$ (at the horizon) leads to regular resulting conformal solution.
- Black hole embedded in FLRW universe.

# Disformed Kerr black hole

Anson, EB, Charmousis, Hassaine'20
[see also Achour+'20]

- Starting from the stealth Kerr solution, we perform the transformation:

$$\tilde{g}_{\mu\nu} = g_{\mu\nu}^{(\mathrm{Kerr})} - \frac{D}{q^2}\,\partial_\mu\phi\,\partial_\nu\phi, \qquad \phi = q\left[t + \int \frac{\sqrt{2Mr(a^2+r^2)}}{\Delta}\mathrm{d}r\right]$$

where $D$ and $q$ are constants.

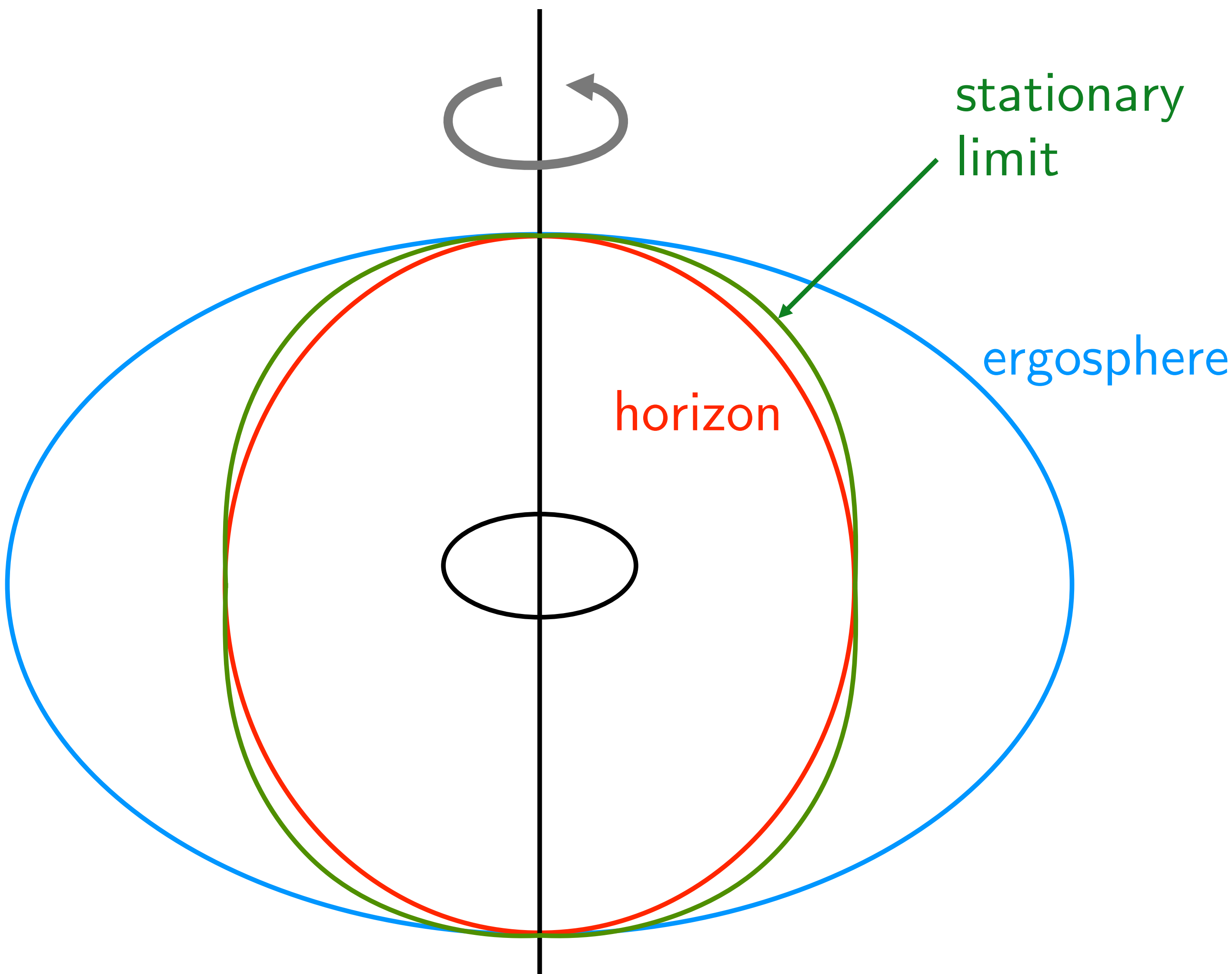
- The line element is now

$$\mathrm{d}\tilde{s}^2 = -\left(1 - \frac{2\tilde{M}r}{\rho^2}\right)\mathrm{d}t^2 - 2D\frac{\sqrt{2\tilde{M}r(a^2+r^2)}}{\Delta}\mathrm{d}t\mathrm{d}r + \frac{\rho^2\Delta - 2\tilde{M}(1+D)rD(a^2+r^2)}{\Delta^2}\mathrm{d}r^2$$
$$- \frac{4\sqrt{1+D}\tilde{M}ar\sin^2\theta}{\rho^2}\mathrm{d}t\mathrm{d}\varphi + \frac{\sin^2\theta}{\rho^2}\left[\left(r^2+a^2\right)^2 - a^2\Delta\sin^2\theta\right]\mathrm{d}\varphi^2 + \rho^2\mathrm{d}\theta^2$$

with $\tilde{M} = M/(1+D)$ and the rescaling $t \to \sqrt{1+D}t$

# Disformed Kerr black hole

- The solution is not Ricci-flat, but the only singularity is at $\rho = 0$, like Kerr.
- The spacetime is globally causal, since there is $\phi(t, r)$ which serves as a global time.
- Non-circular space-time
- There are *three* important surfaces: static limit (egrosphere), stationary limit and the event horizon (in case of Kerr spacetime the two latter coinside).



# Conformal symmetry

# BBMB solution

Bocharova, Bronnikov, Melnikov'70; Bekenstein'74

- Scalar field with non-minimal coupling:

$$S = \int d^4x\sqrt{-g}\left(R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{12}R\phi^2\right)$$

- The BBMB solution is

$$ds^2 = -\left(1-\frac{M}{r}\right)^2 dt^2 + \frac{dr^2}{(1-\frac{M}{r})^2} + r^2 d\Omega^2, \quad \phi = \pm\frac{M}{r-M}$$

- Properties: Metric of the extremal Reissner-Nordstrom; scalar diverges at $r_h = M$; it is unique; hair with the choice $\pm$ due to the discrete symmetry $\phi \to -\phi$.

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- Properties: Metric of the extremal Reissner-Nordström; scalar diverges at $r_h = M$; it is unique; hair with the choice $\pm$ due to the discrete symmetry $\phi \to -\phi$.

- The key in finding the solution is in the conformal invariance of the scalar part of the action, $g_{\mu\nu} \to e^{2\sigma}g_{\mu\nu}$ and $\phi \to e^{-\sigma}\phi \Rightarrow S_\phi \to S_\phi +$ b.t.
  As a consequence of the invariance

$$R = 0 \quad \text{(pure geometric constraint)}$$

$$\Box\phi = \frac{1}{6}R\phi \quad \Rightarrow \Box\phi = 0 \quad \text{(first integral)}$$

- This allows to derive the most general asymptotically flat solution [Xanthopoulos & Zannias'91]

# Generalization of the action

Lu-Pang'20, Fernandes'21

- Generalized action:

Martinez, Troncoso, Zanelli'03

$$S = \int d^4x\sqrt{-g}\left\{R - 2\Lambda - 6\beta\left((\partial\phi)^2 + \frac{1}{6}R\phi^2\right) - 2\lambda\phi^4 \right.$$
$$\left. -\alpha\left[\ln(\phi)\mathcal{G} - \frac{G^{\mu\nu}\phi_\mu\phi_\nu}{\phi^2} - \frac{4\Box\phi\,(\partial\phi)^2}{\phi^3} + \frac{2\,(\partial\phi)^4}{\phi^4}\right]\right\}$$

where $\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$ is the Gauss-Bonnet invariant.

- The $\alpha-$ contribution breaks the conformal invariance of the action for the scalar. The scalar field equation remains conformally invariant.

- Look for the solution

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2, \quad \phi = \phi(r).$$

# Geometric constraint from conformal EOM

1

Conformal invariance of the scalar EOM $\Rightarrow$ pure geometric constraint:

$$R - 2\Lambda + \frac{\alpha}{2}\mathcal{G} = 0$$

From which the solution for $f(r)$ immediately follows:

$$f(r) = 1 + \frac{r^2}{2\alpha}\left[1 \pm \sqrt{1 + 4\alpha\left(\frac{2M}{r^3} - \frac{q}{r^4} + \frac{\Lambda}{3}\right)}\right]$$

**Geometric constraint comes from conformal symmetry of the scalar EOM, without conformal invariance of the scalar action.**

Non-Noetherian scalar field
[Ayon-Beato & Hassaine'23]

# Geometric constraint from conformal EOM

2 Scalar field equation is has a "simple" form to integrate (assuming $\alpha \neq 0$):

$$\left(\frac{\phi'}{\phi^2}\right)'\left(f\left[(r\phi)'\right]^2-\phi^2\left(1+\frac{\beta}{2\alpha}r^2\phi^2\right)\right)=0.$$

Two disconnected branches of solutions [Fernandes'21]

**Extensions:** [Babichev, Charmousis, Hassaine Lecoeur'22]

- Slowly rotating solutions
- Radiating solutions (Vaidya-like)
- Wormholes by disformal transformation
- Gravitational monopole-like solution

# Rotating solution

Fernandes'23

- Kerr-Schild ansatz:

$$ds^2 = ds^2_{\text{flat}} + H(\mathbf{x})\,(l_\mu dx^\mu)^2\,,$$

where $H$ is a scalar (to look for) and $l^\mu$ is the tangent vector to a geodesic null congruence.

- The solution contains arbitrary functions $M(\theta)$ and $q(\theta)$ (a sign of strong coupling?)

**Very similar to the disformed Kerr solution:**

- Non-circular
- The horizon is given by a similar equation.

# No symmetry (but simple scalar EOM?)

EB, Charmousis, Hassaine & Lecoeur '23

Give up the requirement of the symmetries?
But construct a theory that yields a similar scalar field equation with factorization.

$$S=\int \mathrm{d}^4x\sqrt{-g}\left\{(1+W(\phi))R-\frac{1}{2}V_k(\phi)(\nabla\phi)^2+Z(\phi)+V(\phi)\mathcal{G}+V_2(\phi)G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi\right.$$
$$\left.+V_3(\phi)(\nabla\phi)^4+V_4(\phi)\Box\phi(\nabla\phi)^2\right\}.$$

- The combination $E_t^t-E_r^r=0$ can be factorized:

$$\left[\frac{\phi''}{(\phi')^2}-1\right]\left[r^2W_\phi+4(1-f)V_\phi+2frV_2\phi'+fr^2V_4(\phi')^2\right]=0,$$

  provided specific relations between the potentials (still leaving 3 arbitrary potentials at this step $Z$, $V$ and $W$).

- Fix the potentials $Z$, $V$ and $W$ so that the remaining 2 equations admit the solution for $f=f(r)$

# Conclusions

- Use symmetries of gravity theories to construct analytic solutions.
- Shift symmetry of a theory leads to a conserved current.
- Conformal symmetry leads to a geometric constraint.
- General disformal transformation as a way to construct new solutions.