Black Hole Information Puzzle: Insights from a Quantum Gravity Toy Model

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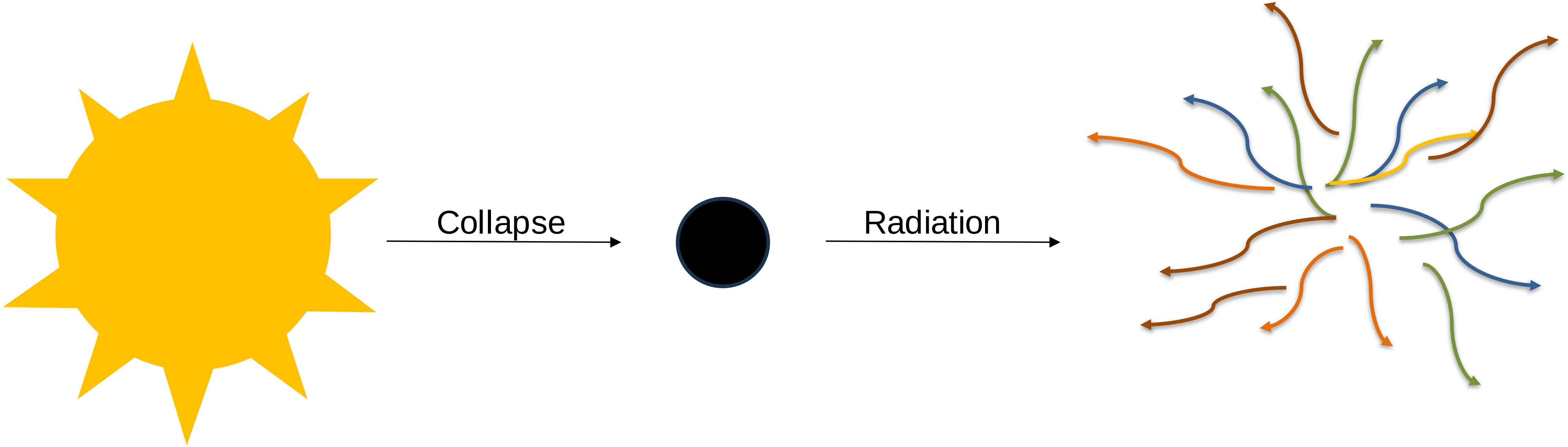
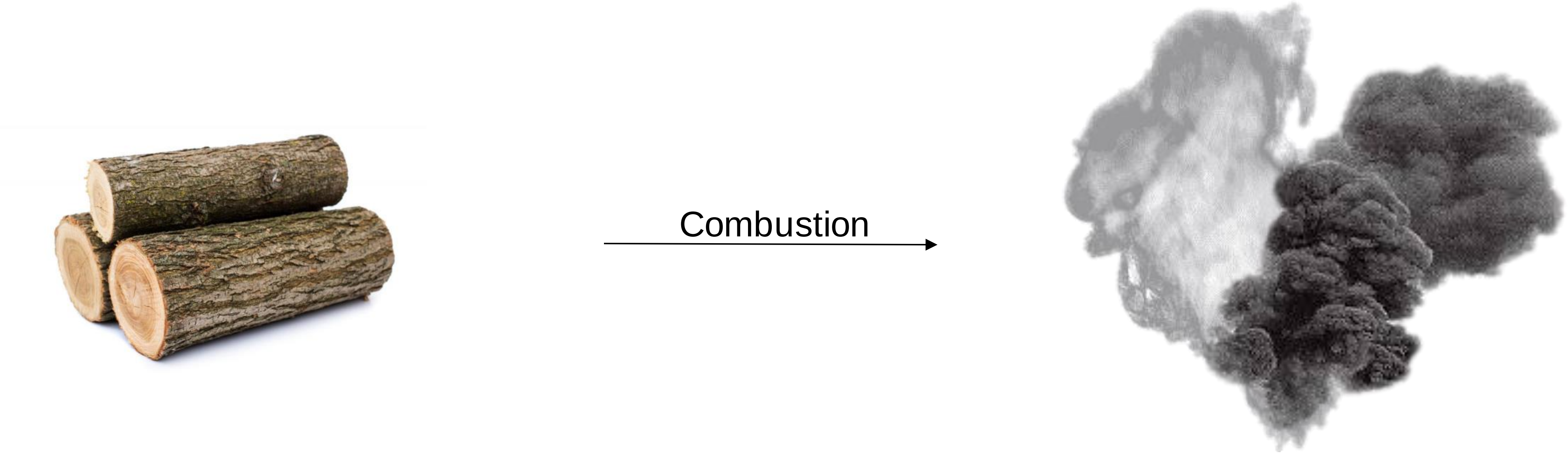
The paradox



→ Any stars of same M Q and J will lead to the same thermal radiation.

→ A pure state evolves into a mixed state.

An analogy



Combustion

→ The process seems irreversible, informations seem to be lost.

→ The information is in fact hidden into correlations between the different microscopic d.o.f of the smoke.

Motivations

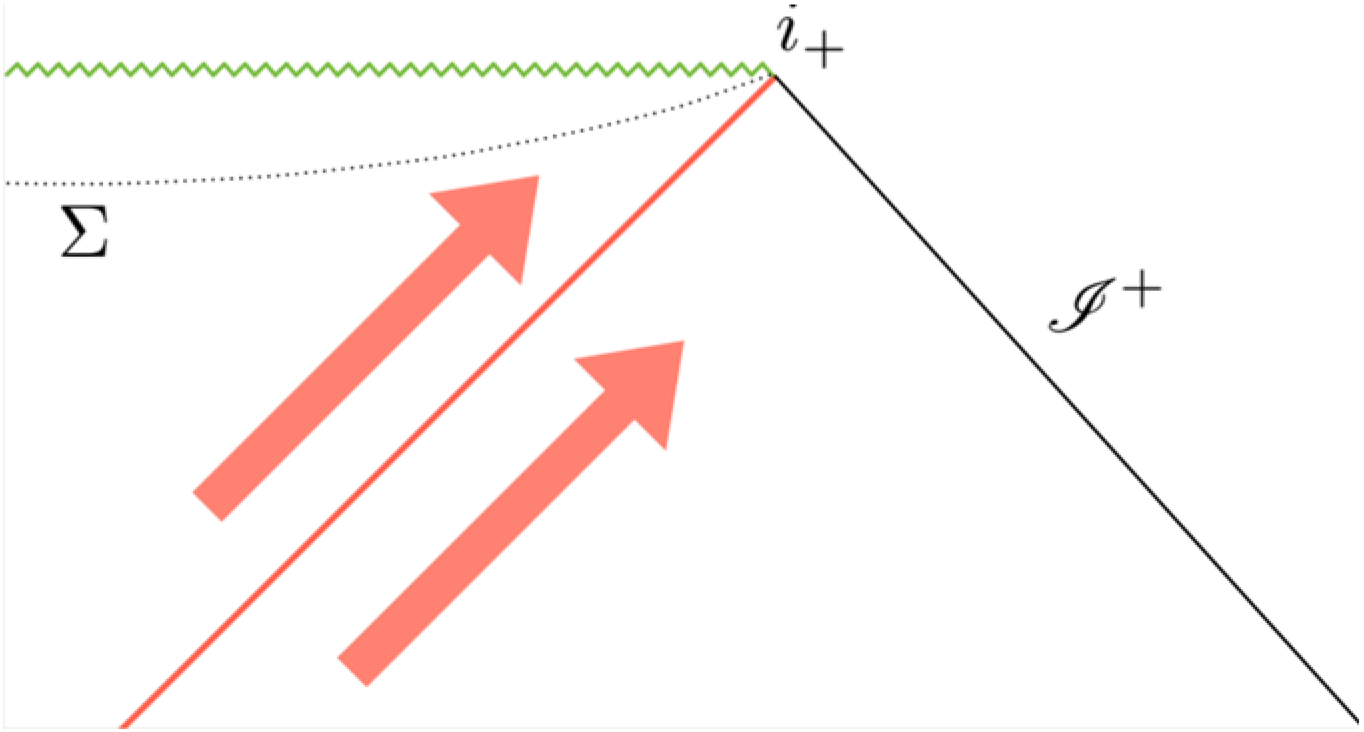
Construct a toy model to study the information loss paradox

-Describe the classical dynamics of a particle falling into a black hole.

-Focus on the near singularity regime (i.e. $r \ll 2M$).

-Quantize the dynamics and implement the discretness of the geometry suggested by LQG.

The system of interest



The ingoing particle falls with a 4 wave vector

$$k^a = -E(\partial_t)^a + p_r(\partial_r)^a$$

The physical moment in the $(\partial_t)^a$ direction is

$$p_t = \frac{k_a(\partial_t)^a}{\sqrt{|\partial_t.\partial_t|}} = -E\sqrt{\frac{r}{2M-r}} \xrightarrow{r\to 0} 0$$

The physical moment of the particle vanishes in the t, θ and ϕ direction

The KG scalar field excitation describing the particle is homogeneous in the t, θ and direction ϕ

The classical model

Inside of black hole + ingoing particle = Homogenous & Anisotropic S.T. + massless ϕ field

↓

$$ds^2 = -f(r)dt^2 + h(r)dr^2 + r^2 d\Omega^2 \quad \text{and} \quad \phi = \phi(r)$$

The action is $$S = \frac{1}{16\pi}\int_R d^4x\sqrt{-g}R + 2\int_{\partial_R} K - \frac{1}{2}\int_R d^4x\sqrt{-g}\partial_a\phi\partial^a\phi$$

The Hamiltonian is : $H(f, \mathrm{p_f}, \mathrm{r}, \mathrm{p_r}, \phi, \mathrm{p}_\phi) \approx 0$

The near singularity regime

We work in the limit $r \ll 2M$ (near singularity regime) and we perform a canonical transformation

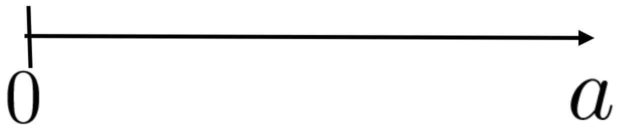
$$a = 4\pi r^2 \qquad p_a = \frac{p_r}{8\pi r} \qquad m = -f p_f \qquad p_m = -\log(-f)$$

The Hamiltonian constraint simply becomes : $\boxed{H = p_a + \frac{1}{2a}\left(m + \frac{p_\phi^2}{m}\right) \approx 0}$

and $ds^2 = e^{-\mathrm{p}_m} dt^2 - \frac{\ell_0^2}{(4\ell_p)^4 \pi^2} \frac{e^{-\mathrm{p}_m}}{m^2} da^2 + \frac{a}{4\pi} d\Omega^2$ with $M \propto \frac{1}{\sqrt{a}} m^2 e^{\mathrm{p}_\mathrm{m}}$

The quantization scheme

Schrödinger quantisation : $\left[-i\hbar\frac{\partial}{\partial a}+\frac{1}{2a}\left(m+\frac{\mathrm{p}_\phi^2}{m}\right)\right]\psi(m,\mathrm{p}_\phi,\mathrm{a})=0$



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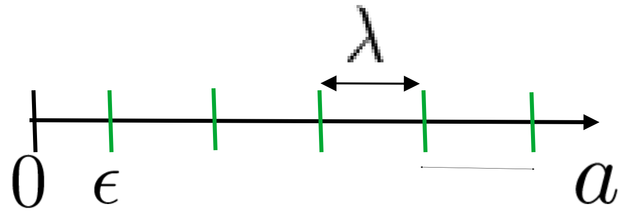
Input : no infinitesimal shift in a allowed

$\Gamma_{\epsilon,\lambda} =$



with $\epsilon \in [0,\lambda)$

Discrete quantisation : $$\psi(m,\mathrm{p}_\phi,\mathrm{a}+\lambda)-\mathrm{e}^{\frac{\mathrm{i}}{2}\int_{\mathrm{a}}^{\mathrm{a}+\lambda}\frac{\mathrm{da}}{\mathrm{a}}\left(\mathrm{m}+\frac{\mathrm{p}_\phi^2}{\mathrm{m}}\right)}\psi(\mathrm{m},\mathrm{p}_\phi,\mathrm{a})=0$$

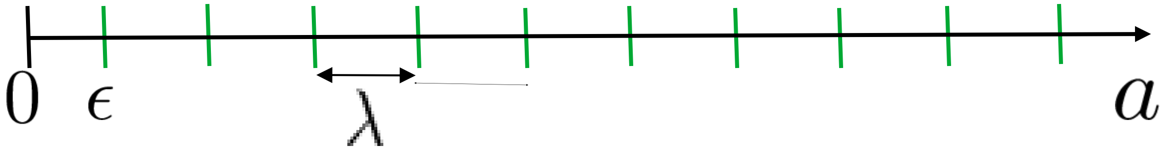
The mass operator

$$M \propto \frac{1}{\sqrt{a}} m^2 e^{\mathrm{p_m}} \xrightarrow{\hbar} \hat{M} \propto \widehat{\frac{1}{\sqrt{a}}}\, \widehat{m}\, \widehat{e^{\mathrm{p_m}}}\, \widehat{m}$$

Eigenvector : $|M, \epsilon\rangle = \sum_{a \in \Gamma_{\epsilon,\lambda}} \int \phi_M(\mathrm{p}_m, a) |\mathrm{p}_m\rangle |a\rangle d\mathrm{p}_m$ such that $\hat{M}|M, \epsilon\rangle = M|M, \epsilon\rangle$

Bessel function

$\Gamma_{\epsilon,\lambda} =$



with $\epsilon \in [0, \lambda)$

$\Rightarrow$ The mass operator is infinitly degenerate due to this ϵ parameter representing a Planckian geometric d.o.f !

Evolution of the correlations

How the correlations of an Hawking pair evolve in this quantum geometry ?

For the matter part : $|\psi_{mat}\rangle = \frac{1}{\sqrt{2}}(|\mathrm{p}_\phi = \mathrm{A}, -\mathrm{A}\rangle + |\mathrm{p}_\phi = \mathrm{B}, -\mathrm{B}\rangle)$ → Maximally entangled pair

Particle inside

Particle outside

For the geometric part : $|\psi_{geo}\rangle = \frac{1}{\sqrt{2}}(|m, \epsilon_1\rangle + |m, \epsilon_2\rangle)$

Total initial state : $|\psi_I\rangle = |\psi_{mat}\rangle \otimes |\psi_{geo}\rangle$

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When $a \searrow$ $I_{\mathrm{out,in}}(|\psi\rangle) \searrow$ and $I_{\mathrm{out},\epsilon}(|\psi\rangle) \nearrow$

Conclusion

-A quantum description of the inside of a BH is proposed and completely solvable.

-For a Schwarzschild solution of mass M, there is an infinite number of quantum states.

-The correlations of the outgoing particle with the ingoing one become correlations with the Planckian geometric d.o.f.

-At the end of the evaporation, it does not only remains the Hawking radiation but also Planckian geometric d.o.f, restoring the purity of the total system.