Subleading asymptotic structure of general relativity

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Asymptotically-flat spacetimes

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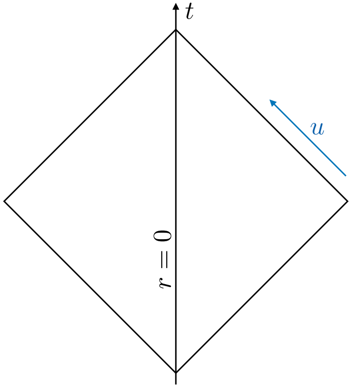

- Consider Minkowski in null coordinates

$$\mathrm{d}s^2 = -\mathrm{d}u^2 - 2\mathrm{d}u\,\mathrm{d}r + r^2 q_{ab}\mathrm{d}x^a\mathrm{d}x^b$$

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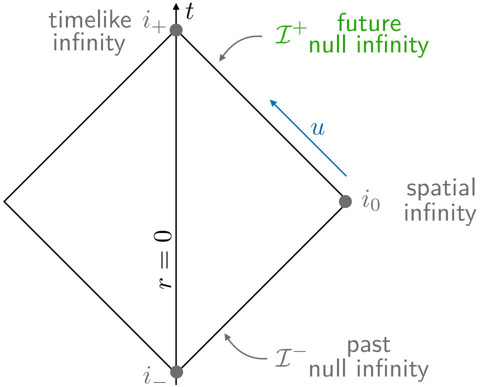

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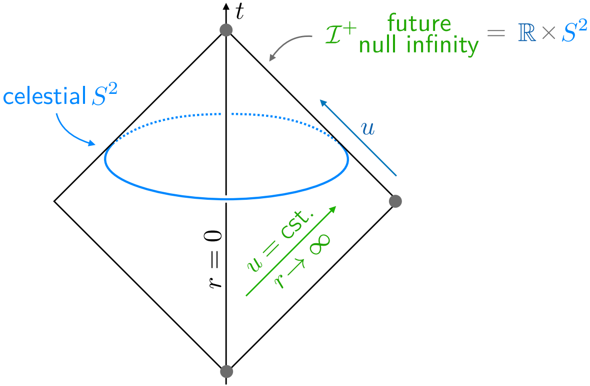

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- Future null infinity $\mathcal{I}^+$ is the ideal region where to read off gravitational radiation [Ashtekar, Bondi, Geroch, Hansen, Metzner, Newman, Penrose, Sachs, Trautman, van der Burg]

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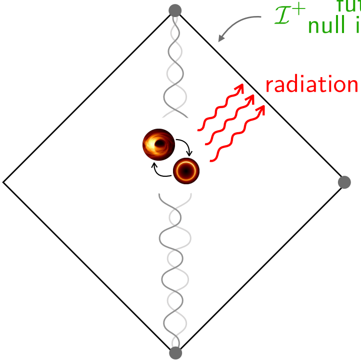

- Consider Minkowski + radiation

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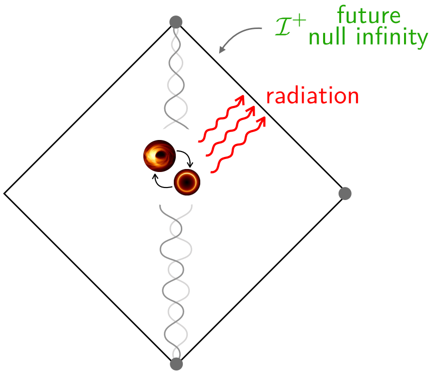

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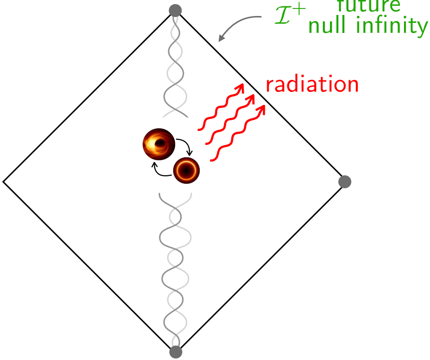
- This is described by the notion of radiative asymptotically-flat spacetimes

Asymptotically-flat spacetimes



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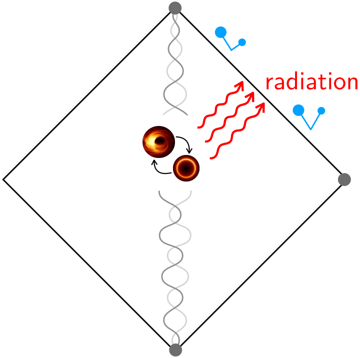
Asymptotically-flat spacetimes



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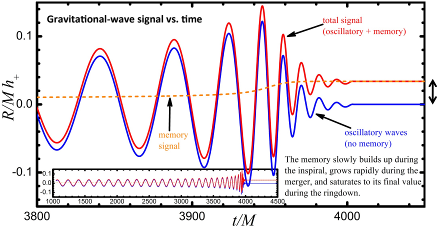
  - memory effects
    [Marc Favata]



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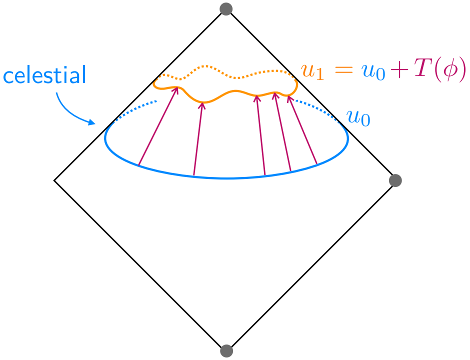

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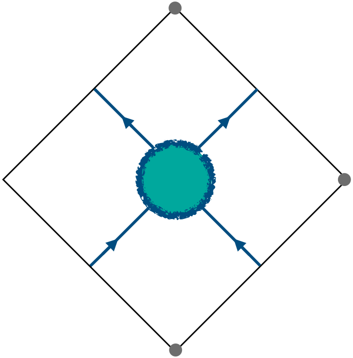

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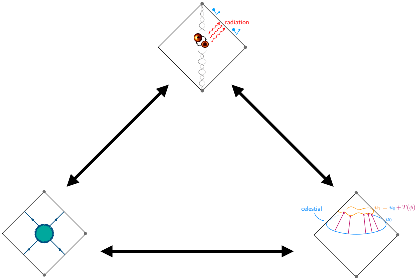

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  } IR physics
- All these aspects are connected through the so-called infrared triangle [Strominger et al.]

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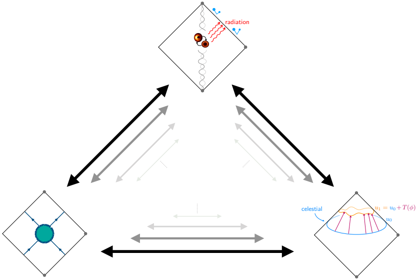

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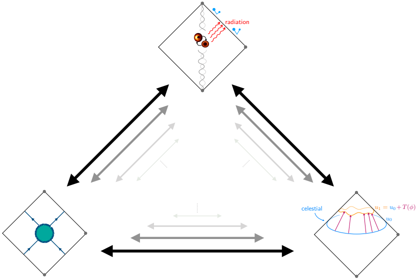

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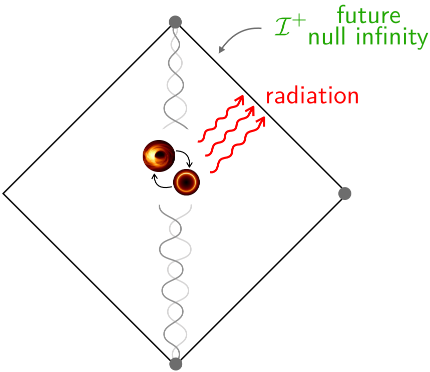
  } IR physics
- All these aspects are connected through the so-called infrared triangle [Strominger et al.]
- There is now strong (yet subtle) evidence that many such subleading triangles exist in gravity
- This underlies the hope for building flat space holography

Asymptotically-flat spacetimes



- Near $\mathcal{I}^+$ it is very convenient to use the Bondi gauge

$$\mathrm{d}s^2 = \left(-1 + \frac{M(u,x^a)}{r} + \dots\right)\mathrm{d}u^2 - (2+\dots)\mathrm{d}u\,\mathrm{d}r + \left(\frac{P_a(u,x^a)}{r} + \dots\right)\mathrm{d}u\,\mathrm{d}x^a + g_{ab}\mathrm{d}x^a\mathrm{d}x^b$$

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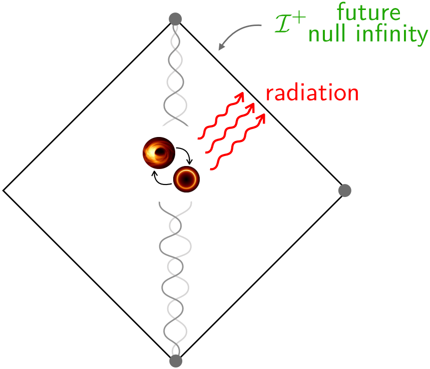

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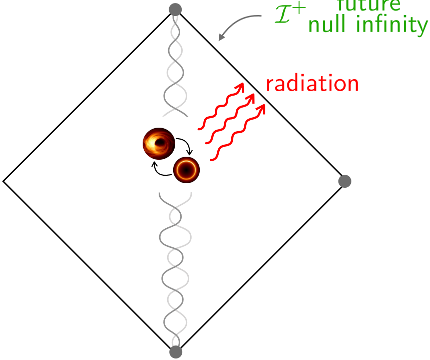

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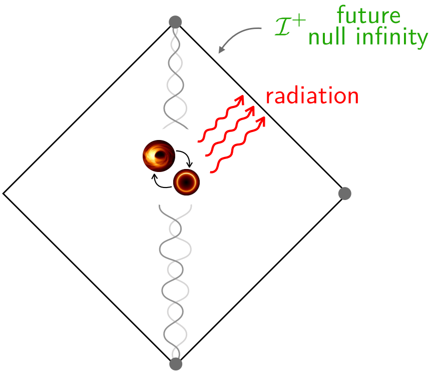

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- 2 types of data: C_{ab} free on $\mathcal{I}^+$ and ∞-amount of data $(M, P_a, E^1_{ab}, \dots)$ satisfying EOMs

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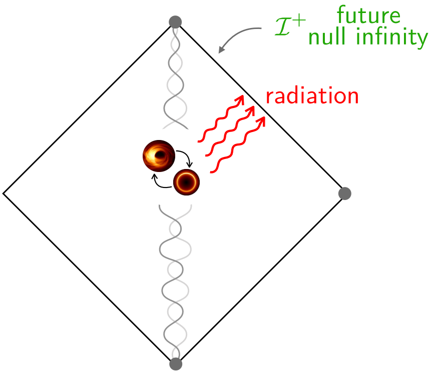

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$$ds^2 = \left(-1 + \frac{M(u,x^a)}{r} + \dots\right) du^2 - (2 + \dots) du\, dr + \left(\frac{P_a(u,x^a)}{r} + \dots\right) du\, dx^a + g_{ab} dx^a dx^b$$

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- 2 types of data: C_{ab} free on $\mathcal{I}^+$ and ∞-amount of data $(M, P_a, E^1_{ab}, \dots)$ satisfying EOMs
- These flux balance laws are e.g. the famous Bondi mass loss $\dot{M} = -\dot{C}_{ab}\dot{C}^{ab} + D_a D_b \dot{C}^{ab}$

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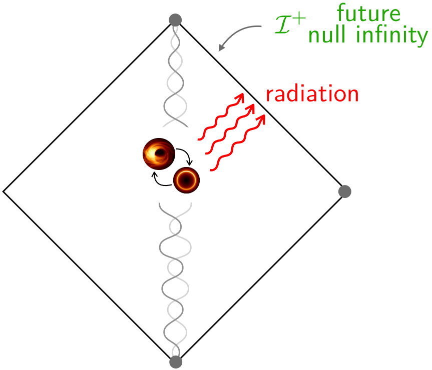

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- On $\mathcal{I}^+$ we have the Ashtekar–Streubel symplectic structure $\Omega = \delta\dot{C}_{ab} \wedge \delta C^{ab}$

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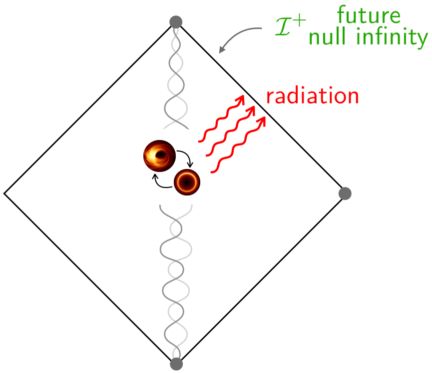

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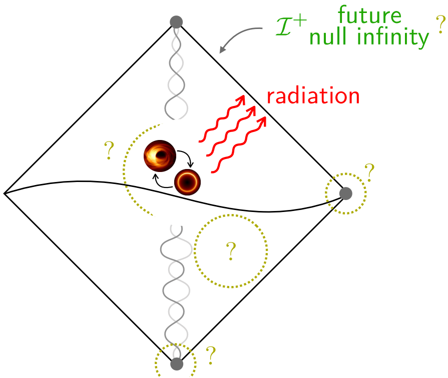

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- Some features are specific to the case $\Lambda = 0$ and still poorly understood
  - the role of log terms and whether $\mathcal{I}^+$ is smooth [Bieri, Blanchet, Christodoulou, Chrusciel, Damour, Friedrich, MG, Kehrberger, Klainerman, Kroon, Laddha, MacCallum, Singleton, Winicour, Zwikel]

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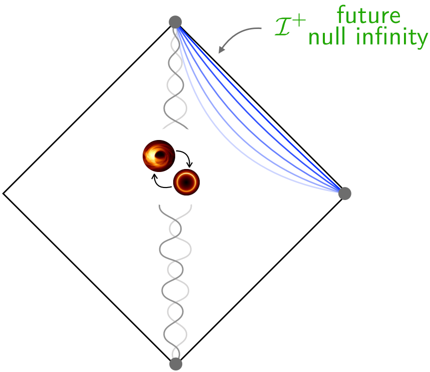

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- Some features are specific to the case $\Lambda = 0$ and still poorly understood
  - the role of log terms and whether $\mathcal{I}^+$ is smooth
  - the structure of the infinite tower of evolution equations for the data on $\mathcal{I}^+_0$

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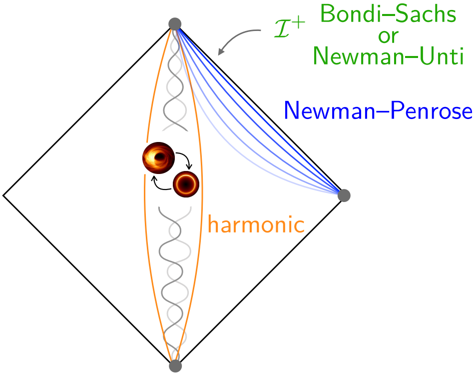

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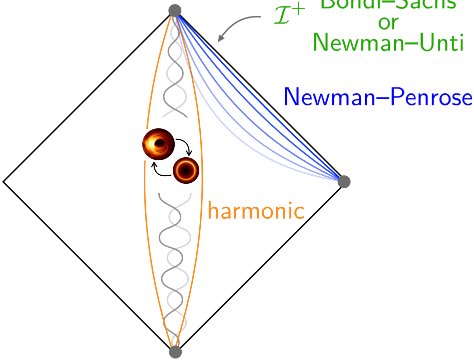

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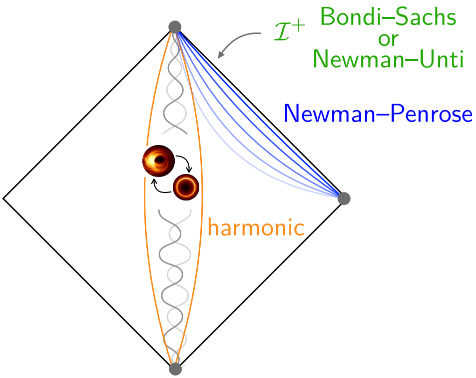

- Depending on the question and the region, different gauges and formalisms are useful
- The translation between gauges and formalisms is very subtle
- The NP formalism allows to neatly repackage the Bondi asymptotic Einstein equations [Barnich, Mao, Ruzziconi] [Freidel, Pranzetti, Raclariu] [MG]

Newman–Penrose formalism

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Weyl scalars

Newman–Penrose formalism

Weyl scalars

- In terms of helicity-weighted scalars and assuming peeling at $\mathcal{I}^+$, the free and initial data is

$$\Psi_0 = \frac{Q_2}{r^5} + \mathcal{O}(r^{-6})$$

$$\Psi_1 = \frac{Q_1}{r^4} + \mathcal{O}(r^{-5})$$

$$\Psi_2 = \frac{Q_0}{r^3} + \mathcal{O}(r^{-4})$$

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- In terms of the previous Bondi data we have

$$Q_{-2}(\ddot{C}_{ab}) \qquad Q_{-1}(\dot{C}_{ab}) \qquad Q_0(M, C_{ab}) \qquad Q_1(P_a, C_{ab}) \qquad Q_2(E^1_{ab}, C_{ab})$$

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Newman–Penrose formalism

Weyl scalars

- In terms of helicity-weighted scalars and assuming peeling at $\mathcal{I}^+$, the free and initial data is

$$
\begin{aligned}
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\Psi_1 &= \frac{Q_1}{r^4} - \frac{\bar{\eth} Q_2}{r^5} + \mathcal{O}(r^{-6}) \\
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- In terms of the previous Bondi data we have $Q_{s\geq 2}(E_{ab}^{s-1}, C_{ab})$

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Newman–Penrose formalism

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- $Q_{s\geq 2}$ = Newman–Penrose charges [Newman, Penrose]
  = subleading BMS charges [Godazgar, Godazgar, Long] [MG]
  = canonical multipole moments [Compère, Oliveri, Seraj]

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- This structure can be recovered from symmetry considerations [Freidel, Pranzetti] [MG, Zwikel]
  - the Q_s are primaries under the conformal and Diff(S^2) parts of BMS$_4$
  - the EOMs are primaries under the time evolution (supertranslations)

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- Up to many subtleties, these EOMs are the sub^s-leading soft graviton theorems [Lysov, Pasterski, Strominger] [Campiglia, Laddha] [Freidel, Pranzetti, Raclariu]

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- Introducing $\sigma = C_{ab} m^a m^b$, the linearized asymptotic Einstein equations are

$$\boxed{\dot{Q}_s \approx \eth Q_{s-1}}$$

- In linearized gravity, these EOMs immediately give $\forall s$ ∞-many conserved quantities

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- In the non-linear theory, truncating the EOMs to this form $\forall s$ leads to a $w_{1+\infty}$ loop algebra [Adamo, Ball, Freidel, Guevara, Mason, Narayanan, Pranzetti, Raclariu, Salzer, Sharma, Strominger, . . .]

$$\{Q_{s_1}(\tau_1), Q_{s_2}(\tau_2)\} = Q_{s_1+s_2-1}\left[(s_2+1)\tau_2 D\tau_1 - (s_1+1)\tau_1 D\tau_2\right]$$

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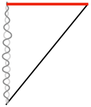
- Simple proof: the $w_{1+\infty}$ transformation laws are consistent with the truncated EOMs [MG]
- A conjecture is that this describes the single helicity / self-dual sector of gravity [wip]

Perspectives

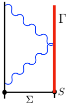
Perspectives

Holography

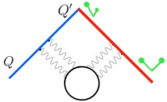
- What are the types of holography?
  - dS: spacelike boundary, potential radiation, link with cosmological memories
  - AdS: timelike boundary, rigid boundary condition, no outgoing radiation, unitary by design
  - flat: null boundary, open system with energy loss via radiation, relation with S-matrix
  - local: finite boundary of causal diamond, no boundary conditions [Freidel, MG, Wieland]



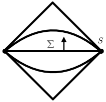
dS



AdS



flat



local

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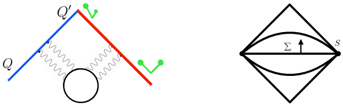

dS AdS flat local

Surprises in asymptotically-flat spacetimes

- Many open questions left: role of log terms, asymptotic symmetry algebra with flux, . . .
- Using the NP formalism, one can rearrange the Bondi data to find a tower of flux balance laws
- This reveals a symmetry connecting sub^s soft theorems, multipoles, and non-local symmetries

Perspectives

Recent activity

- Carrollian physics [Bagchi, Ecker, Grumiller, Hartong, Obers, Pérez, Prohazka, . . .]
- celestial/Carrollian holography [Donnay, Herfray, Petropoulos, Puhm, Raclariu, Strominger, . . .]
- classical and quantum soft theorems [Campiglia, He, Laddha, Lysov, Mitra, Sen, . . .]
- covariant phase space [Barnich, Ciambelli, Freidel, MG, Pranzetti, Speranza, Speziale, . . .]
- dual charges [Godazgar, Godazgar, Long, Oliveri, Pope, . . .]
- extensions to (A)dS [Compère, Fiorucci, Pool, Ruzziconi, Skenderis, Taylor, Zwikel, . . .]
- extensions to FLRW [Bonga, Enriquez-Rojo, Heckelbacher, Oliveri, Prabhu, Schroeder, . . .]
- horizon tomography [Ashtekar, Khera, Kolanowski, Lewandowski, . . .]
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- new memory effects [Flanagan, Grant, Nichols, Oblak, Pasterski, Seraj, . . .]
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Thanks for your attention!