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Holographic Cosmological Reheating

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Bibliography

Ongoing work with:

C. Ecker (Frankfurt), T. Ishii (Tokyo), C. Rosen (Crete), W. Van der Schee (CERN and Utrecht) to appear

C. Ecker (Frankfurt) and W. Van der Schee (CERN and Utrecht)

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Introduction

• Cosmological Inflation is considered as an important part of early universe history.

• Inflation has to end after at least 55 e-foldings.

• By that time, in almost all scenarios, the universe is cold and empty.

• The potential energy has to come back to the dynamics and eventually thermalize, earlier than $T \sim 10 - 40$ MeV, before nucleosynthesis.

• The standard framework used so far (with some variations) involves three stages all happening at weak (quantum) coupling:
(a)Preheating (particle production via Floquet resonances)
(b)(Classical) non-linear evolution
(c) (Very slow) thermalization.

Dolgov+Kirilova, Traschen+Brandenberger, Kofman+Linde+Starobinsky

Preheating and Thermalization at strong coupling

• In the past 20+ years a new class of QFTs was studied: holographic QFTs.

• Such theories are dual to weakly-curved (semiclassical) generalized gravity theories.

• Several types of calculation are possible for such theories using gravitational tools:

♠ Dynamical data like Minkowski signature correlation functions and associated hydrodynamic data like viscosity coefficients and even non-hydrodynamic data like thermal poles and residues.

♠ Far from equilibrium dynamics (quenches).

♠ Thermalization and hydrodynamization as well as the determination of hydrodynamic attractors.

Thermalization

• The process of thermalization in QFT is poorly understood even today.

• It has been brought forward recently with the heavy-ion collisions at RHIC and CERN.

• The data indicate rapid thermalization of the initial energy density and the formation of a quark gluon plasma.

• The thermalization time is an order of magnitude smaller than what was expected at RHIC and is even smaller at LHC.

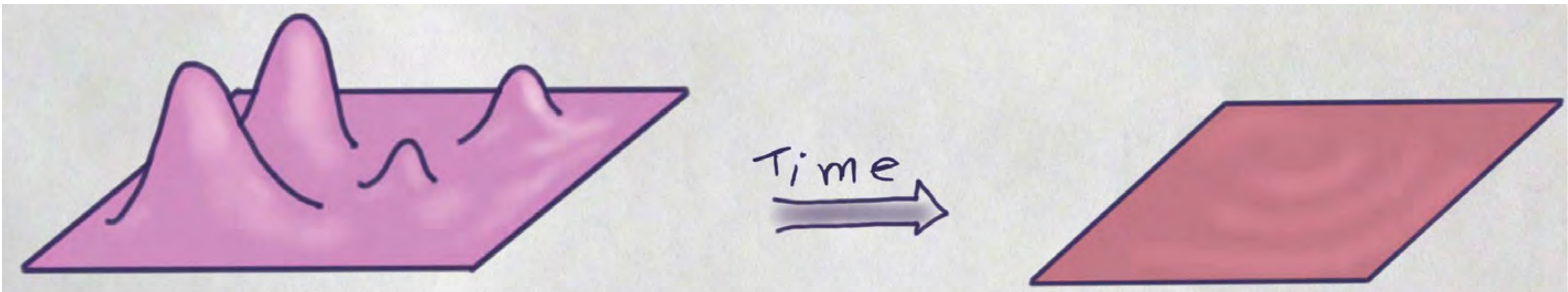
• The theory (QCD) is in a strongly coupled regime for most of the energy range of the experiments.

• Holography has been instrumental to understand this rapid thermalization and hydrodynamization process.

The setup for thermalization

• We consider a QFT in its vacuum state and then perturb it by a time dependent coupling constant (this is a simplification)

$$L_{QFT} + f_0(t) \int d^4x \ O(x) \qquad \to \qquad \nabla^t \langle T_{tt} \rangle = \dot{f}_0 \ \langle O \rangle$$



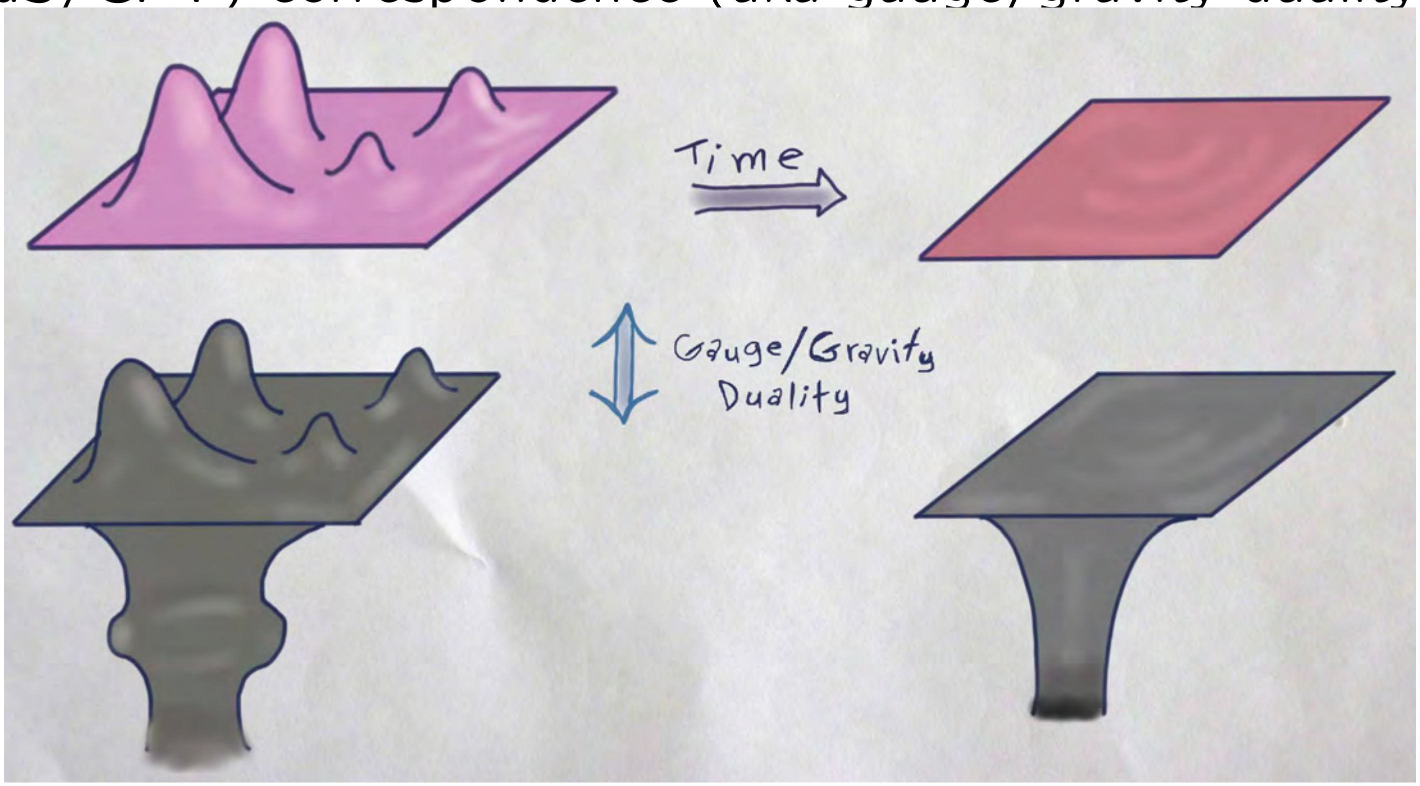
• The approach to equilibration is controlled by the expectation values $\langle T_{tt} \rangle(t)$, $\langle O \rangle(t)$.

• We expect that, if the system thermalizes, then

$$\langle O \rangle(t \to \infty) \quad \to \quad Tr[\rho_{\text{thermal}} \ O]$$

Thermalization at strong coupling

• To calculate the observables at strong coupling we will assume the holographic (AdS/CFT) correspondence (aka gauge/gravity duality).



• Thermalization corresponds to black hole formation in the bulk spacetime.

The cosmological setup

- To build a cosmological model we need (generically) three ingredients:

♠ 4d-gravity

♠ An inflaton theory

♠ A strongly coupled (holographic) “matter theory” (QFT)

♠ A coupling between the inflaton and the QFT

The total action is

$$S = S_{grav} + S_{infl} + S_{holo} + S_{int}$$

with

$$S_{grav} = -\int d^4x\sqrt{g}\left[M_P^2 R + \alpha R^2 + \beta\left(R_{\mu\nu}R^{\mu\nu} - \frac{R^2}{4}\right) + ...\right]$$

$$S_{infl} = -\int d^4x\sqrt{g}\left[\frac{1}{2}(\partial\phi)^2 + V(\phi)\right]$$

$$S_{int} = \int d^4x U(\phi) O(x)$$

- We must renormalize, and therefore the couplings are the renormalized couplings.

- Renormalization affects up to the R^2 terms in Gravity.

- It also renormalizes the inflaton theory.

- We choose a typical (renormalized) inflaton potential

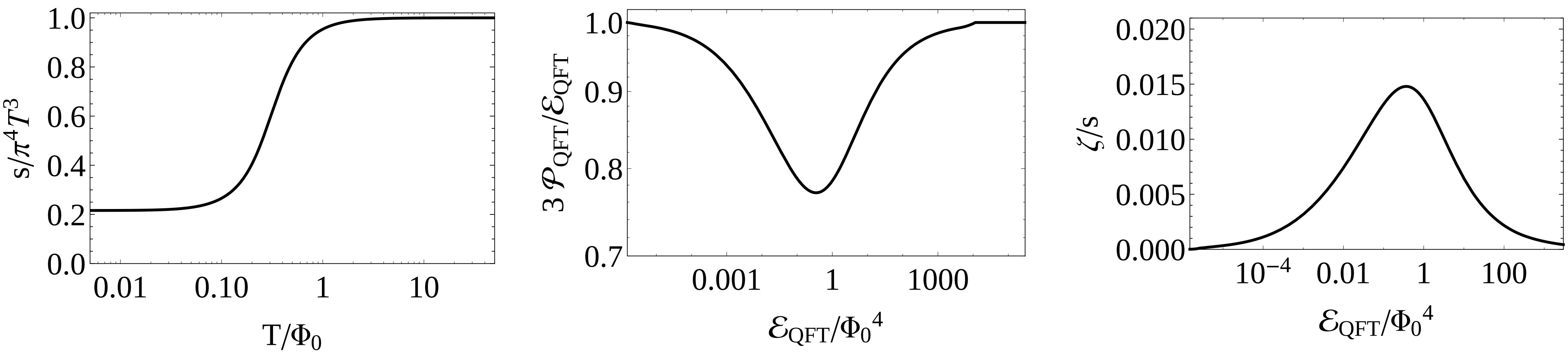
- We choose a typical non-conformal holographic theory. Conformal invariance is broken by S_{int}.

- The "CFT" case is similar, as the inflaton coupling breaks typically conformal symmetry.

The holographic matter theory

- We choose a bottom-up non-conformal theory with a single mass scale, driven by a relevant operator of dimension $\Delta = 3$.

- There is no phase transition in flat space (thermal ensemble) but there is a fast crossover (like QCD)



The Cosmological Evolution Equations

- Homogeneous and isotropic ansatz for the 4d metric

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$$

- 4d Einstein equations

$$H^2 = \frac{\mathcal{E}}{3} \quad , \quad \frac{\ddot{a}}{a} = -\frac{1}{2}\left(\mathcal{P} + H^2\right)$$

with

$$\mathcal{E} = \mathcal{E}_{QFT} + \mathcal{E}_{infl} + U(\phi)\mathcal{O}_{QFT} \quad , \quad \mathcal{E}_{infl} = V_{infl}(\phi) + \frac{\dot{\phi}^2}{2}$$

$$\mathcal{P} = \mathcal{P}_{QFT} + \mathcal{P}_{infl} - U(\phi)\mathcal{O}_{QFT} \quad , \quad \mathcal{E}_{infl} = -V_{infl}(\phi) + \frac{\dot{\phi}^2}{2}$$

$$\ddot{\phi} + 3H\dot{\phi} + V'_{infl}(\phi) = U'(\phi)\mathcal{O}_{QFT} \quad , \quad \mathcal{O}_{QFT} \equiv \langle O \rangle(t)$$

• The unknown parts from the QFT, $\mathcal{E}_{QFT}, \mathcal{P}_{QFT}, \mathcal{O}_{QFT}$ are constrained by energy conservation and the conformal anomaly

$$\dot{\mathcal{E}}_{QFT} - 3H(\mathcal{E}_{QFT} + \mathcal{P}_{QFT}) = U'(\phi)\dot{\phi}\mathcal{O}_{QFT}$$

$$\mathcal{E}_{QFT} - 3\mathcal{P}_{QFT} = -U(\phi)\mathcal{O}_{QFT} + \mathcal{A}(H, \phi)$$

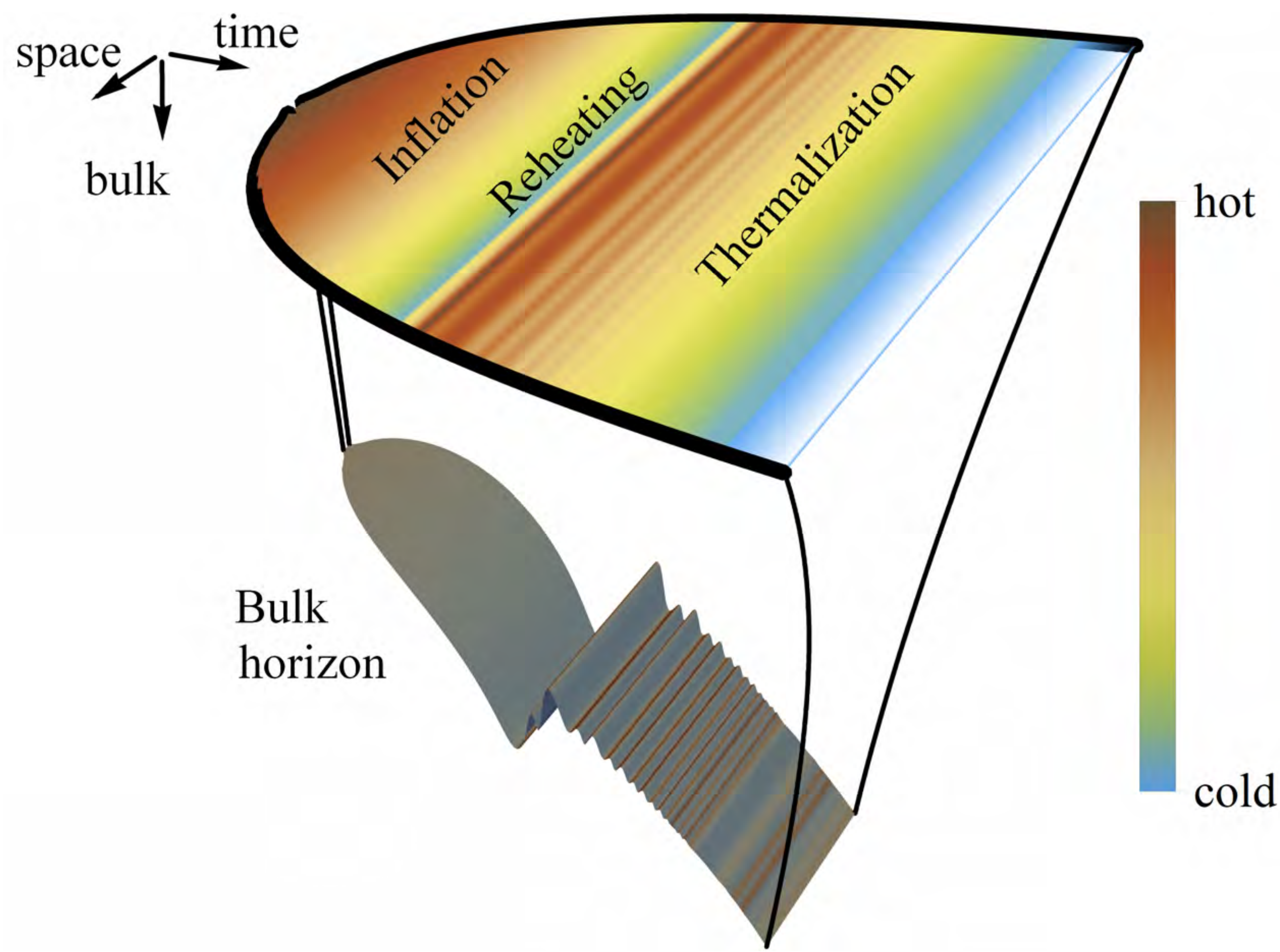
• Therefore, if we know $\mathcal{A}(H, \phi)$ and $\mathcal{O}_{QFT}(H, \phi)$ the system of cosmological equations is closed and can be solved relatively easily.

• To do this we must solve the dynamics of the QFT in the presence of two (arbitrary, time-dependent) sources: $a(t), U(\phi)$

• In the case, where $\Delta < 4$, $\mathcal{A}$ can be calculated and the only non-trivial function to be determined is $\mathcal{O}_{QFT}$

• The procedure is to solve both the 4d, as well as the 5d coupled equations via the Chessler-Yaffe algorithm.

Cartoons

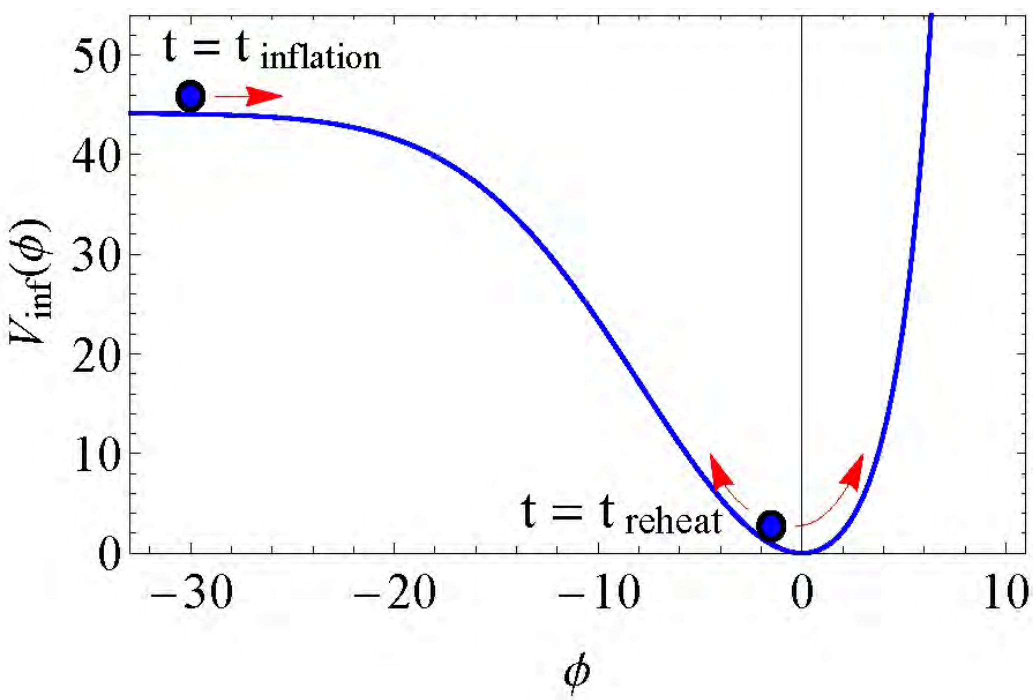


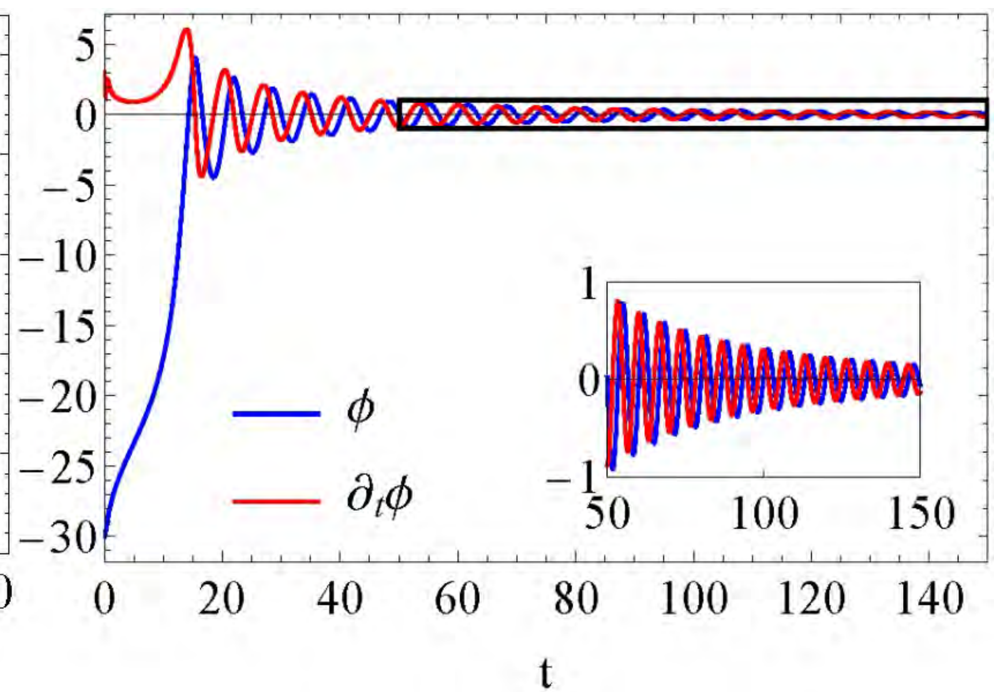
space
time
bulk
Inflation
Reheating
Thermalization
hot
cold
Bulk
horizon

The Inflaton

• Early phase $(t \leq 3)$ dominated by QFT energy. After the universe embarks in an exponential expansion.

• At $t \simeq 14$ the inflaton reaches the bottom of the potential and starts oscillating.



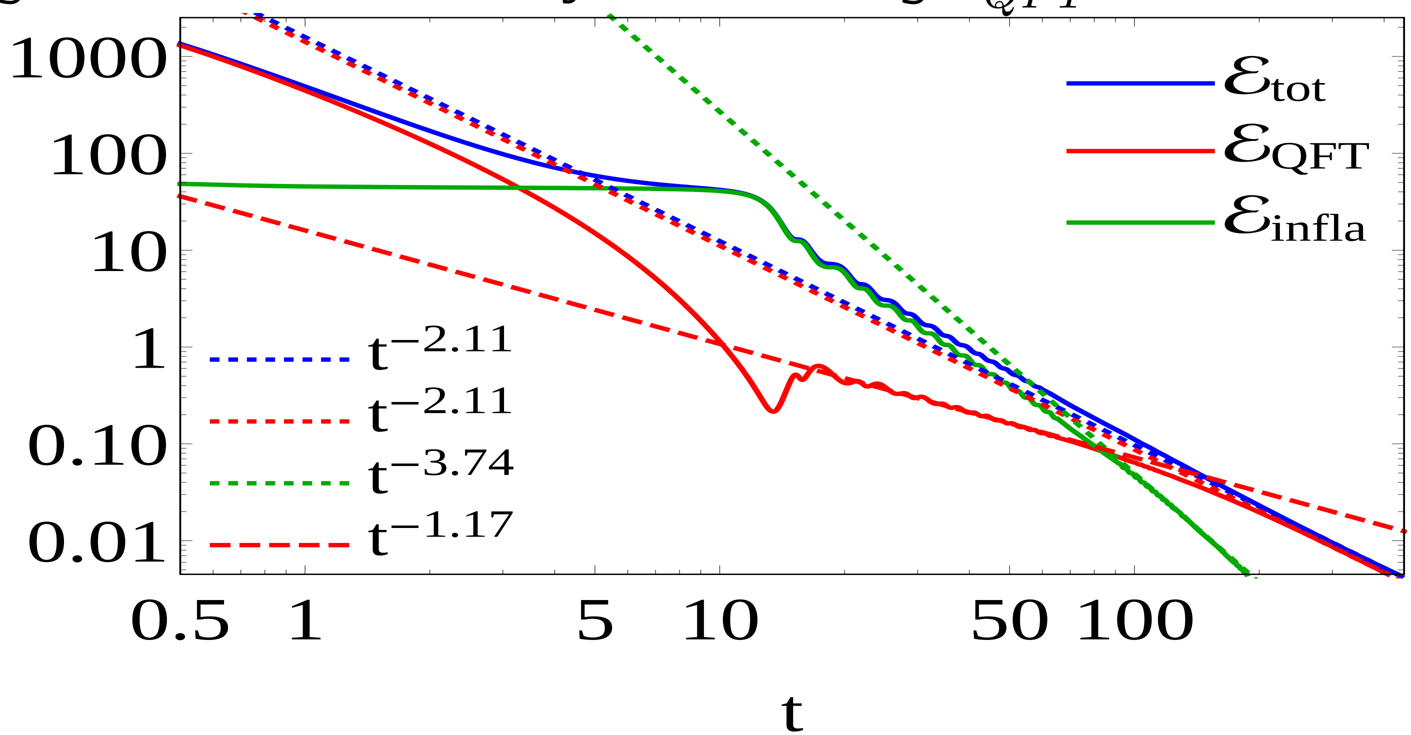


The Energy Density

• QFT energy is dominant until $t \simeq 3$, then the inflaton dominates, until it reaches the bottom of the potential ($t \simeq 14$).

• Inflaton oscillations reheat the QFT from $\mathcal{E}_{QFT} \simeq 0.21$ at $t \simeq 13.5$ to a subsequent maximum of $\mathcal{E}_{QFT} \simeq 0.64$ at $t \simeq 17.3$.

• Reheating continues: relatively slow scaling $\mathcal{E}_{QFT} \sim t^{-1.17}$ of the QFT



Temperature

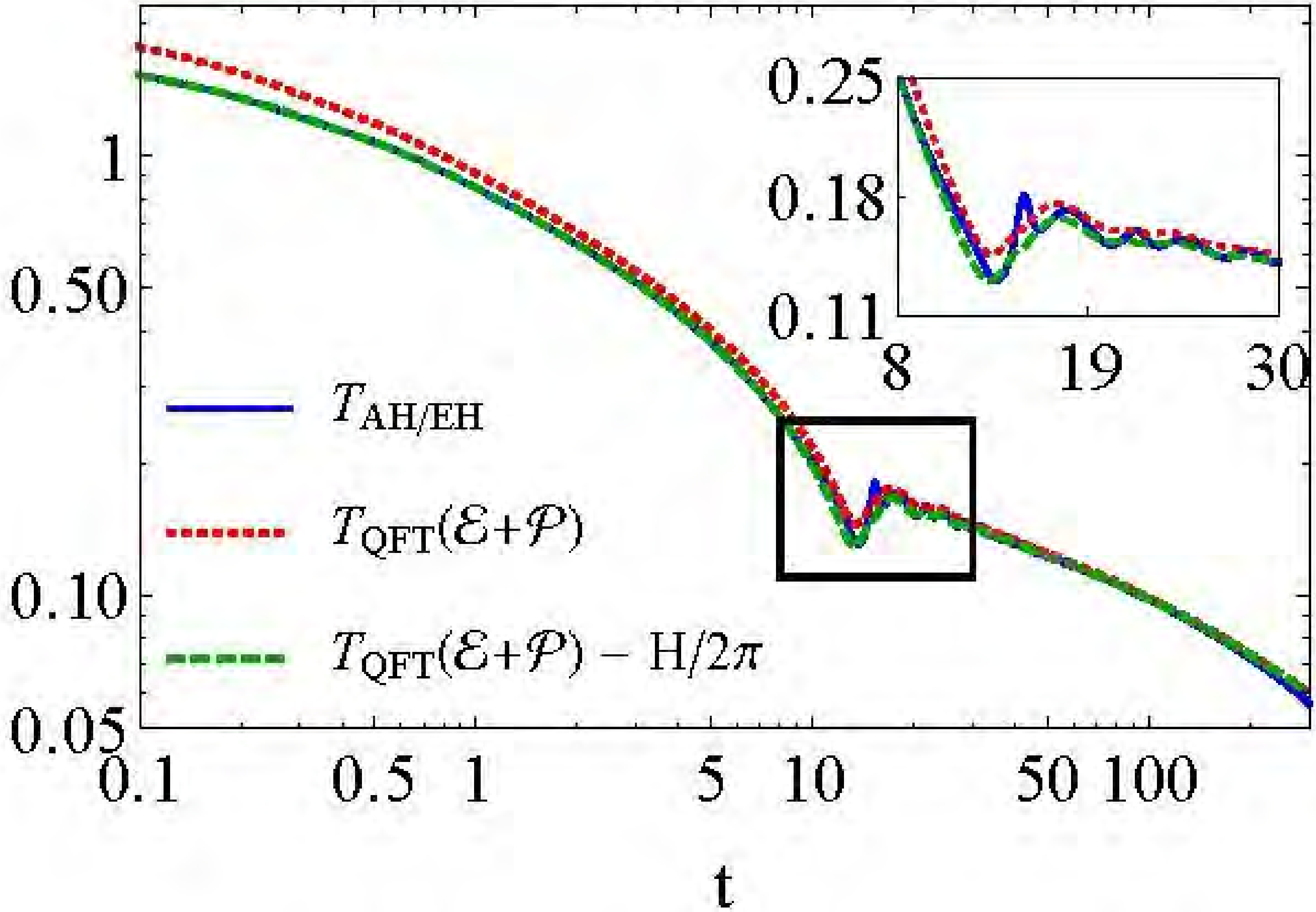
• During the evolution, the bulk solution is a time-dependent solution with an apparent horizon that is different from the (final) event horizon.

• The QFT temperature can be computed from the surface gravity of the bulk apparent horizon.

$$T_{AH} = \frac{\kappa}{2\pi}$$

• Except for a short non-equilibrium period (that can be identified as "reheating") the apparent horizon temperature and the event horizon temperature are numerically indistiguishable.

• The hydrodynamic approximation combined with the equilibrium EoS works well after subtracting the "background" temperature, $T_{dS} = \frac{H}{2\pi}$.



0.25
0.18
0.11
8
19
30
1
0.50
0.10
0.05
0.1
0.5
1
5
10
50
100
t
$T_{\mathrm{AH/EH}}$
$T_{\mathrm{QFT}}(\mathcal{E}+\mathcal{P})$
$T_{\mathrm{QFT}}(\mathcal{E}+\mathcal{P}) - \mathrm{H}/2\pi$

Conclusions

- We have studied cosmological reheating via a strongly coupled (holographic) theory.

- Such theories thermalize/hydrodynamize very fast.

- Moreover, they provide many more options of both couplings and dynamics compared to weakly-coupled theories.

- We have provided a "proof of principle" with a system that had manageable numerics (warm inflation).

- We have found a fast transfer of energy as well as an almost immediate thermalization.

- We have also found that, apart from two short periods, the evolution is well described by viscous (homogeneous) hydrodynamics.

- This opens the way for a systematic study of models and protocols for reheating.

Open Ends

• There are many variations that will provide different scenarios for reheating.

♠ One can use variations on initial conditions.

♠ Variations of the type of strongly-coupled theory (existence of thermal phase transitions, confinement, massive IR etc) as well as of the inflaton portal (different scaling dimensions).

♠ Such variations may provides either fast quenches to the QFT (that can be computed analytically in UV Perturbation theory), or adiabatic quenches that can also be computed analytically.

♠ Tools can be developed to use viscous hydro for the evolution and transition protocols for the short non-hydro periods.

♠ In this connection, the holographic universal hydro attractors may be of use.

♠ On can contemplate using as inflaton a scalar of the QFT, that is associated with a phase transition.

• The role of such a QFT can be played by QCD (this is marginally acceptable) or a higher energy theory like "technicolor".

THANK YOU!

Preheating and Thermalization at strong coupling

• Strongly coupled QFTs are difficult to handle and solve.

• In the past 20+ years a new class of QFTs was studied: holographic QFTs.

• Such theories are dual to weakly-curved (semiclassical) generalized gravity theories.

• They typically have large N_c and strong coupling.

• Several types of calculation are possible for such theories using gravitational tools:

♠ Ground-state and RG-flow calculations.

♠ Dynamical data like Minkowski signature correlation functions and associated hydrodynamic data like viscosity coefficients and even non-hydrodynamic data like thermal poles and residues.

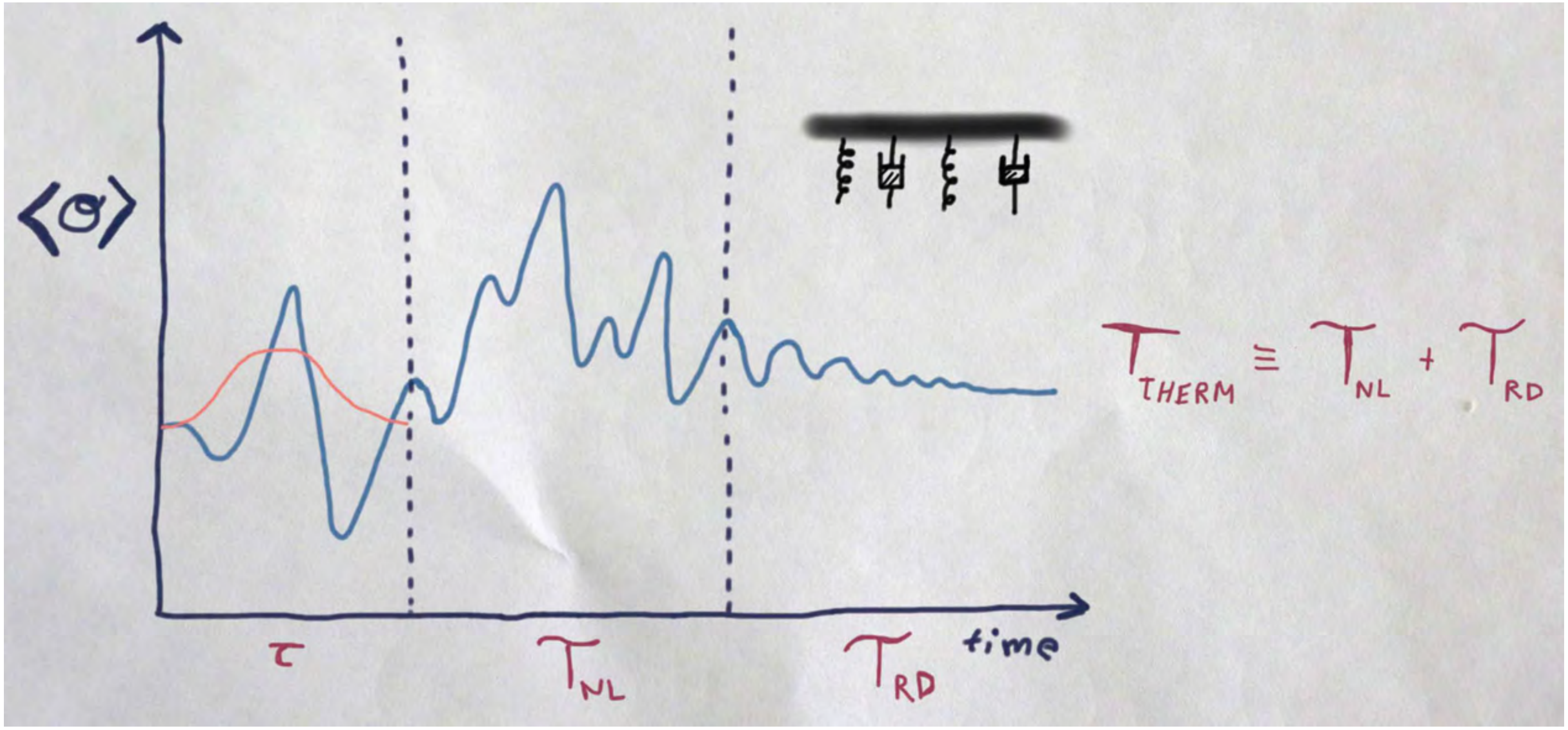
♠ Far from equilibrium dynamics (quenches).

♠ Thermalization and hydrodynamization as well as the determination of hydrodynamic attractors.

♠ Calculations of entanglement via the Ryu-Takayanagi formula and its connection to geometric bridges (wormholes).

♠ The characterization of chaos and the calculation of chaos observables.

Gravitational expectations



- There are three possible characteristic times involved:

$\tau \rightarrow$ duration of quench,

$T_{NL} \rightarrow$ non-linear gravitational evolution,

$T_{RD} \rightarrow$ ring-down of final black hole.

Thermalization calculations

• There have been studies of this setup in holographic CFTs (AdS space). There is no consensus yet but in most cases there is thermalization.

Chessler+Yaffe, Heller+Janik+Witaszczyk
, Bizon+Rostorowski, Buchel+Liebling+Lehner

• There are similarities between a conformal (scale-invariant) gauge theory and a confining gauge theory (like QCD) that has a non-trivial scale, Λ_{QCD} but there are also important differences.

• Confinement is tracked by the Wilson loop that has area behavior in the confining phase.

Quench dynamics

• Consider a quench profile in Improved Holographic QCD (a holographic model for YM):

$$f_0(v) = \tilde{f}_0 - \delta f_0 \, e^{-\frac{v^2}{2\tau^2}}$$

• For numerical simplicity we start with the theory in a thermal state that corresponds to low temperature = the small black hole branch.

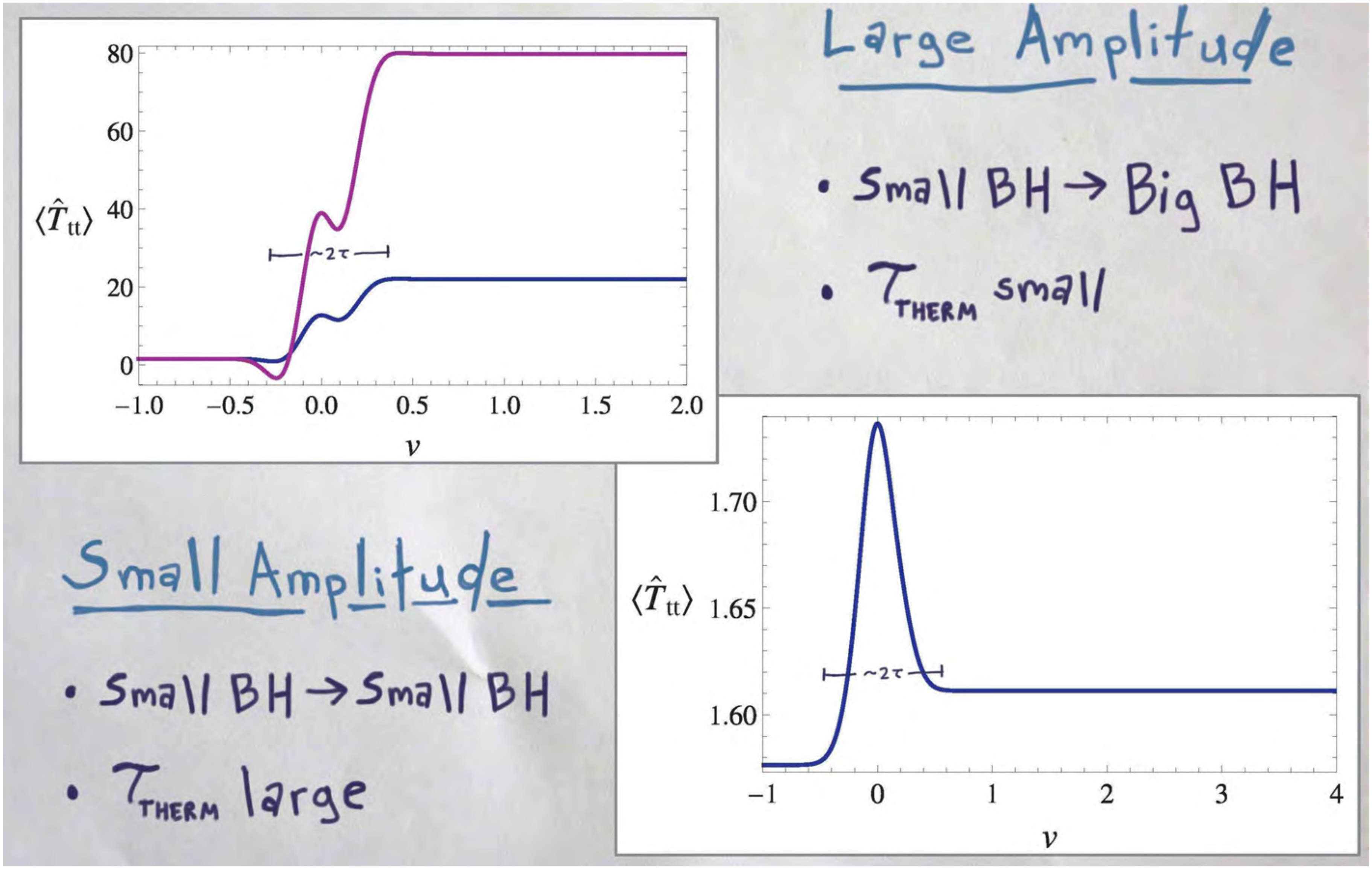
• The "smallest" the initial black hole, the closest we are to the initial ground state of the theory.

• The characteristic time associated with the intermediate non-linear regime is negligible compared to τ and T_{RD}. Why?

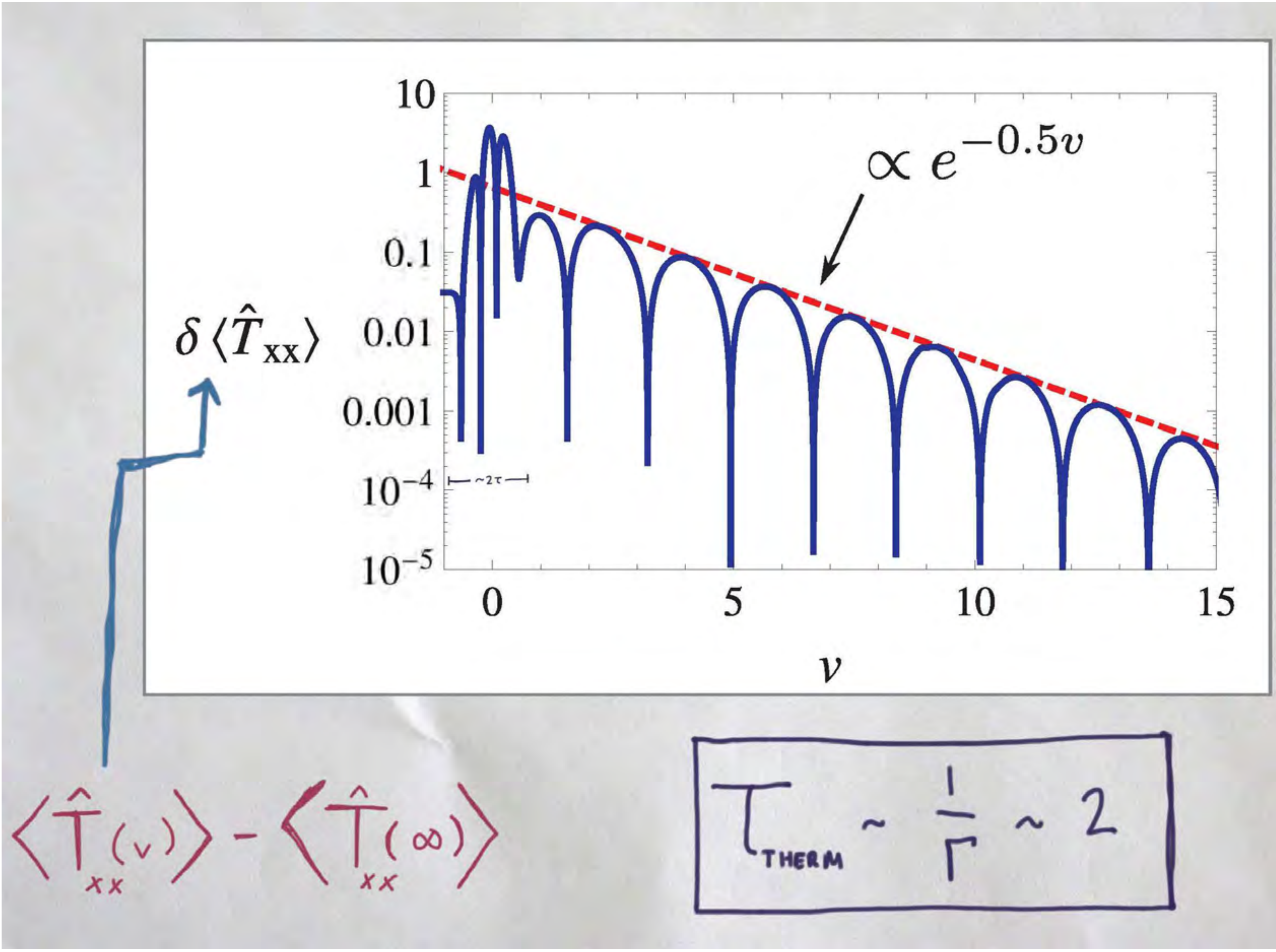
• Therefore

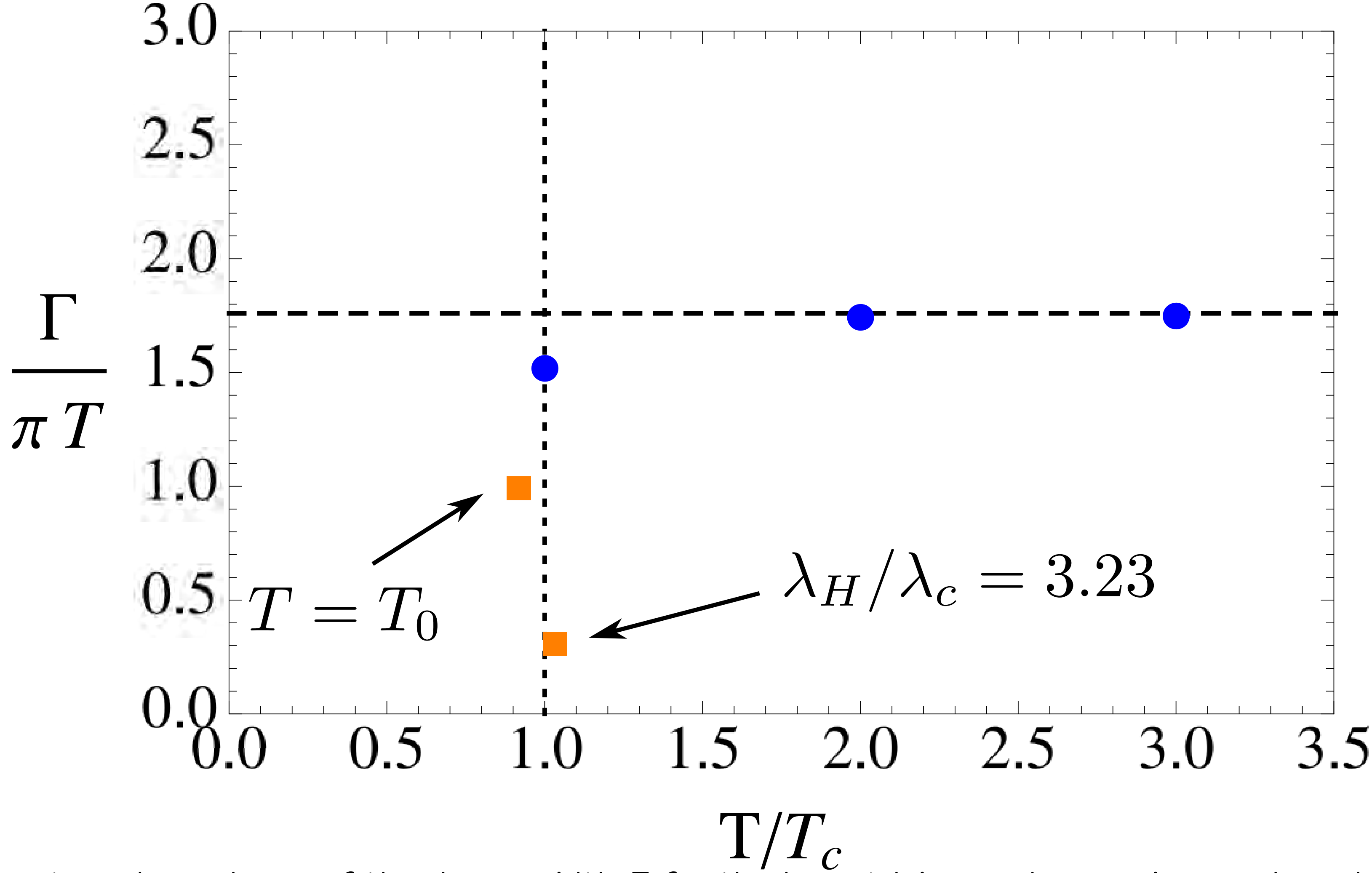
$$T_{\text{thermalization}} \simeq \frac{1}{\Gamma_{RD}}$$

• For adiabatic perturbations, $\tau \gg \Lambda^{-1}$ the system does NOT oscillate but goes continuously to the final-state black hole.


Large Amplitude
• Small BH → Big BH
• τ_THERM small
Small Amplitude
• Small BH → Small BH
• τ_THERM large
⟨T̂_tt⟩
v
~2τ
80
60
40
20
0
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
1.70
1.65
1.60
−1
0
1
2
3
4

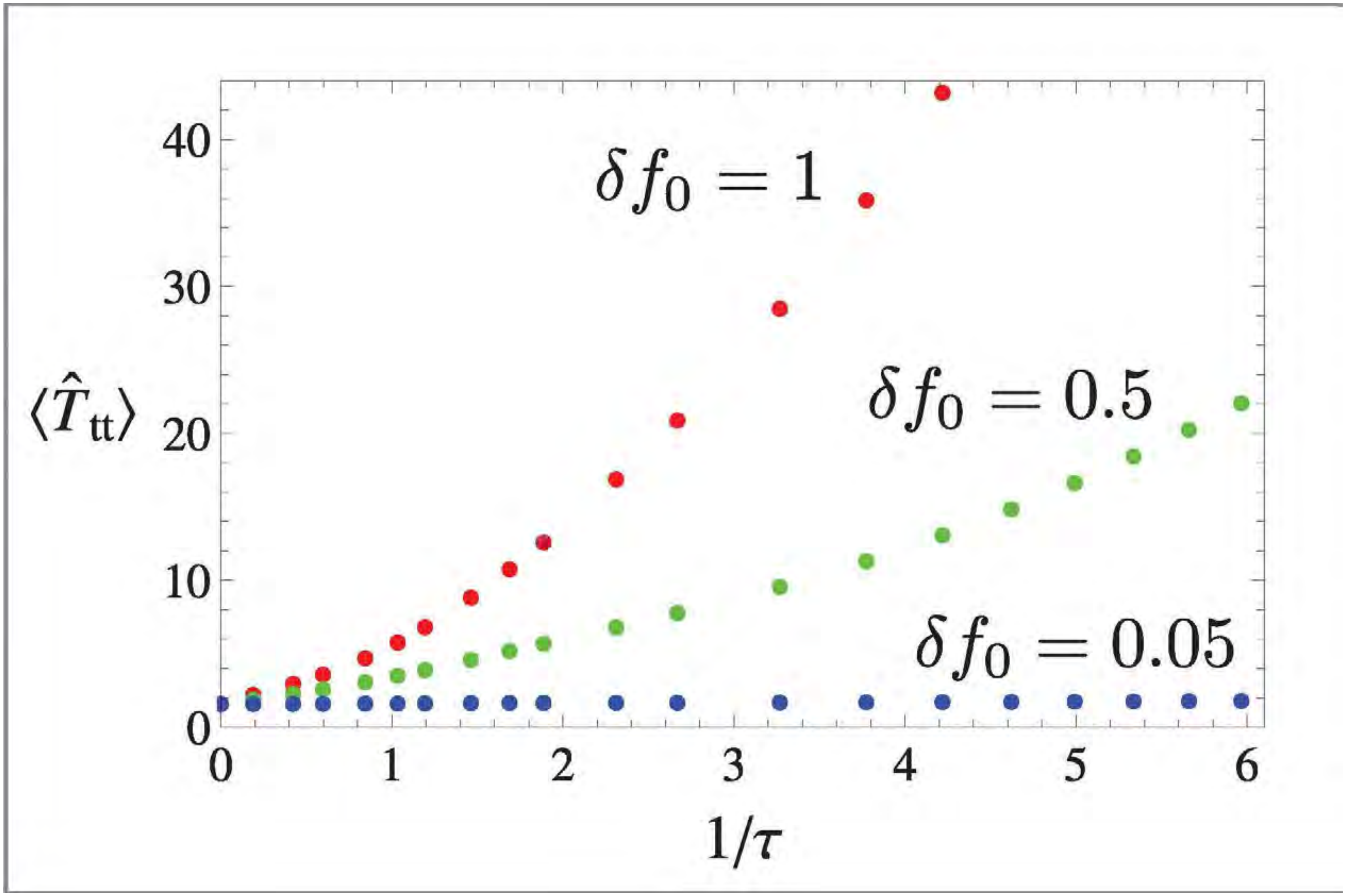
The ring-down phase



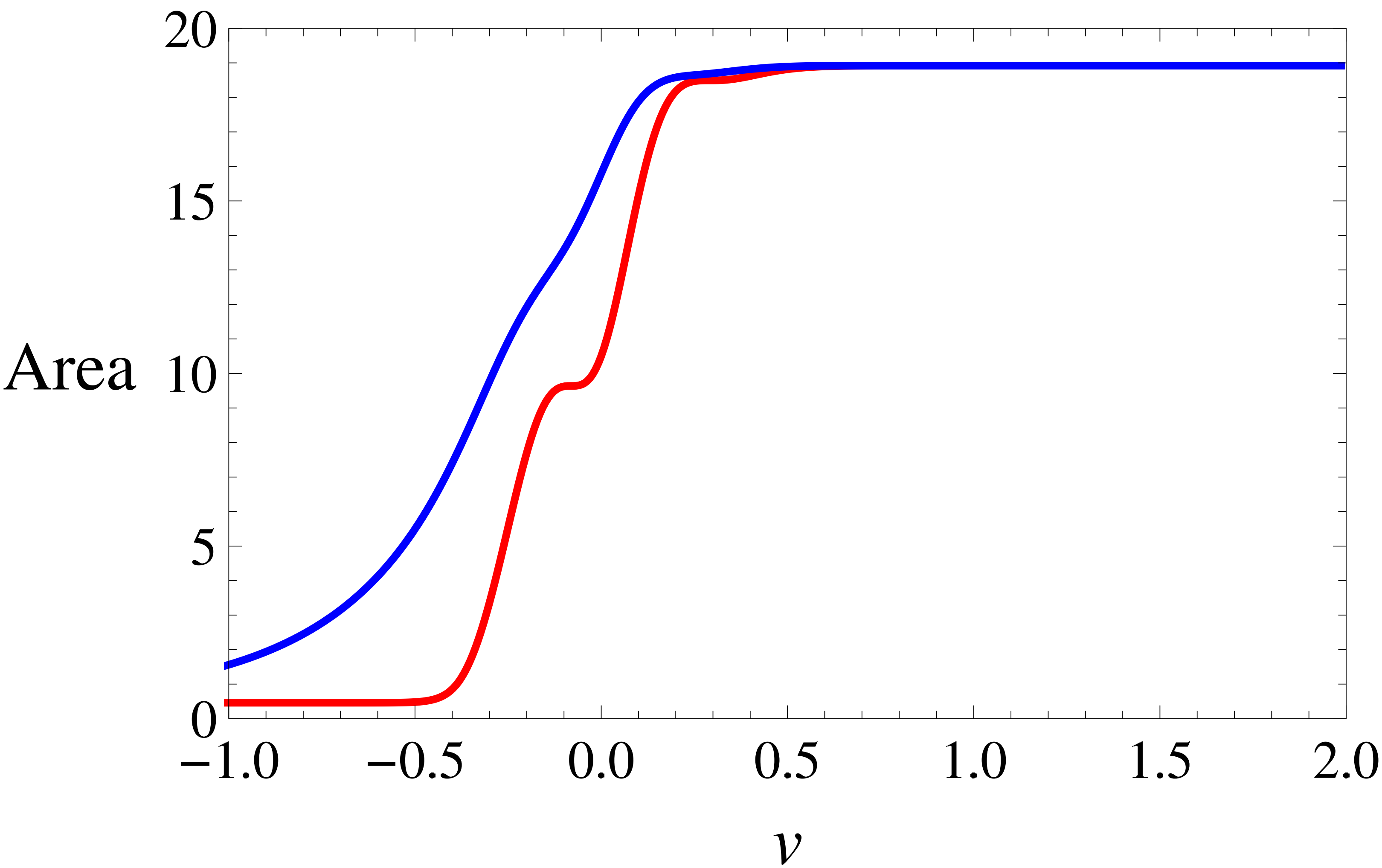


The temperature dependence of the decay width Γ for the lowest lying scalar quasi-normal mode in several states of our theory. The blue circles are large black branes whose temperature is an integer multiple of T_c. The orange squares correspond to the minimum temperature black brane (top) and the smallest black hole we perturb in our study (bottom). The ratio $\Gamma/\pi T$ approaches 1.75953 (the dashed line) at high temperatures, which coincides with the expected value for perturbations of AdS_5 Schwarzschild by a dimension 3 scalar operator

Quench numerical data

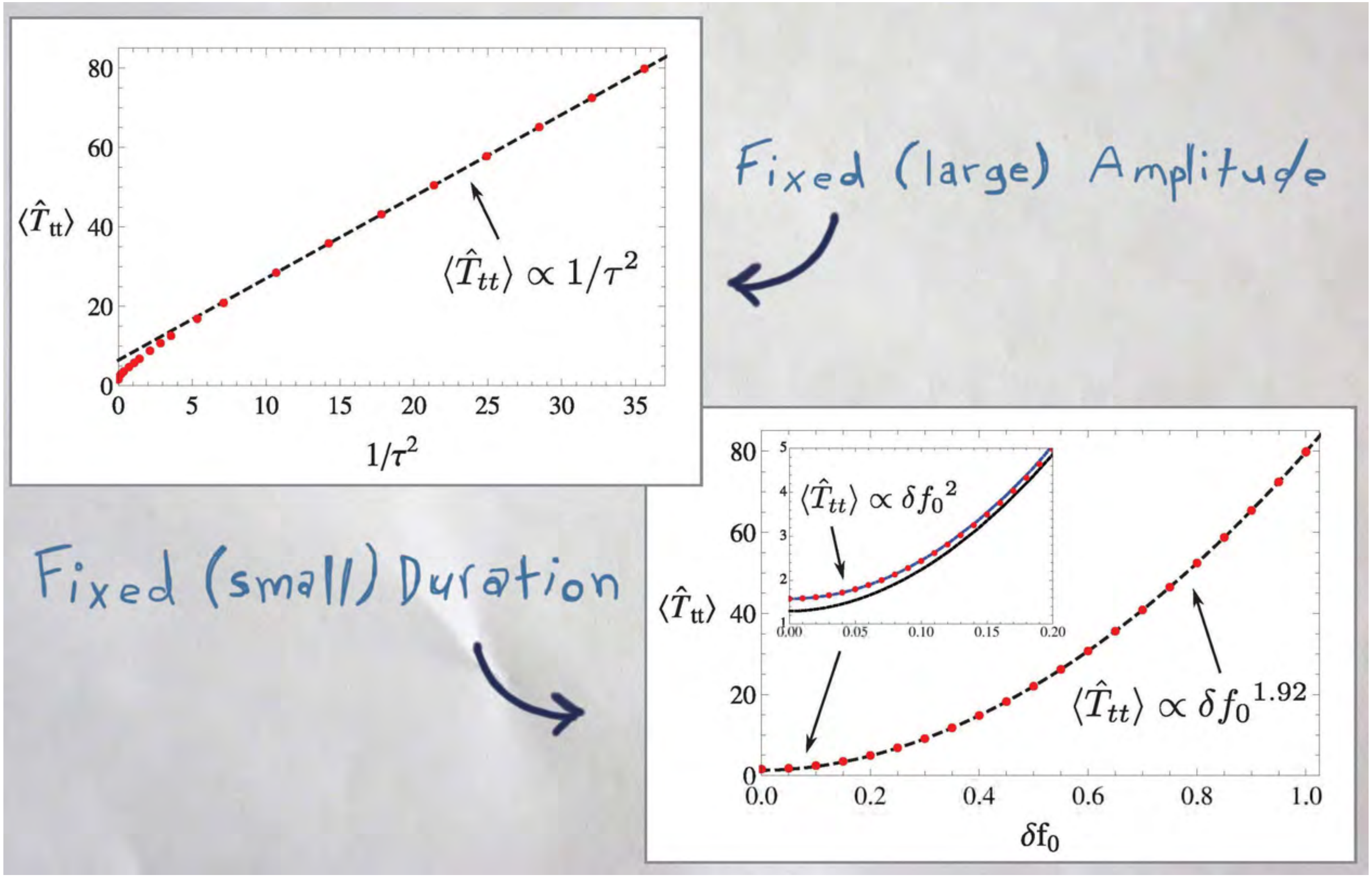


The evolution of bulk horizons during quenches



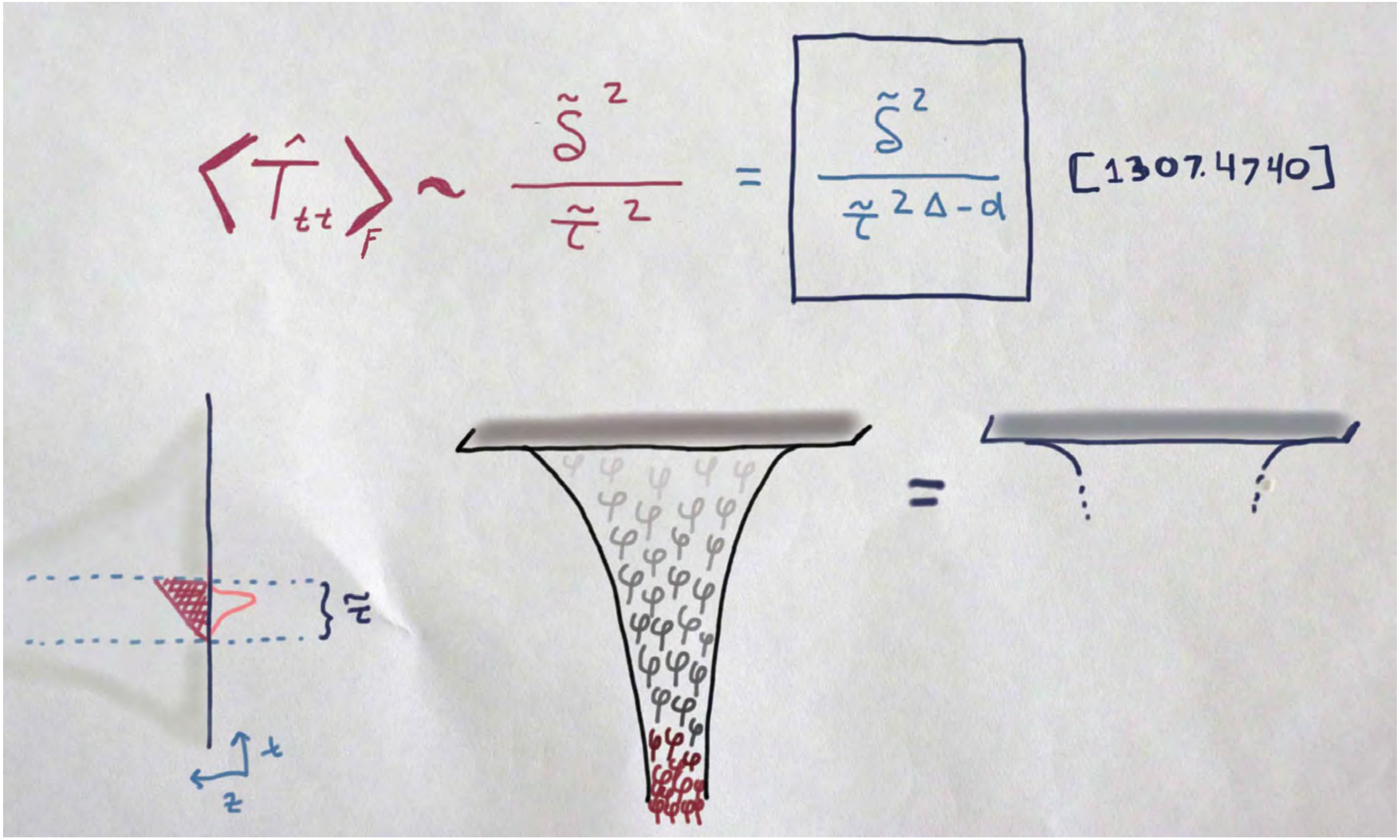
Example of the time evolution of the apparent (red) and event (blue) horizons. The event horizon coincides with the apparent horizon when the bulk solution is static, at $v \to \pm\infty$.

Scaling



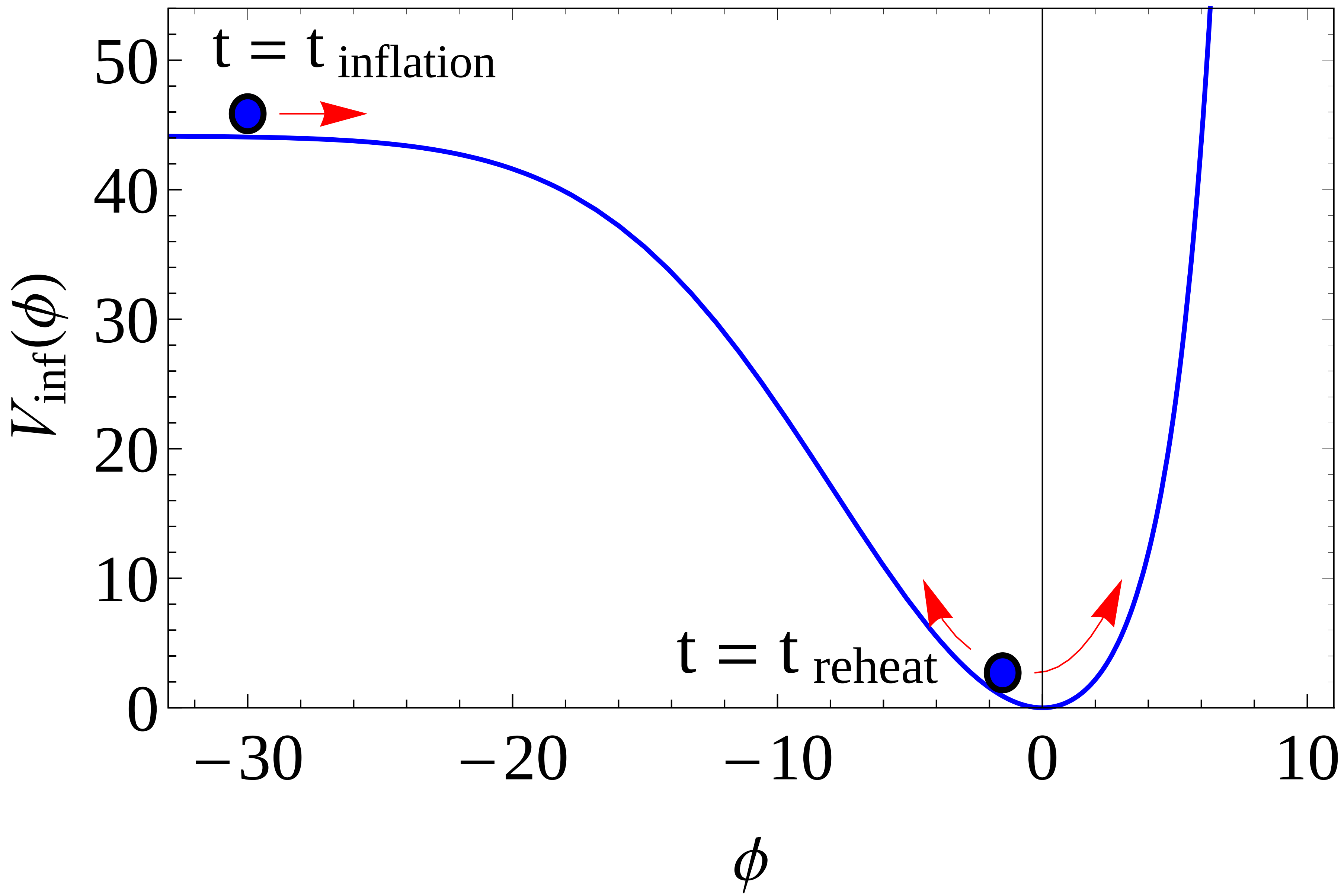
80
60
40
20
0
$\langle \hat{T}_{tt} \rangle$
0 5 10 15 20 25 30 35
$1/\tau^2$
$\langle \hat{T}_{tt} \rangle \propto 1/\tau^2$
Fixed (large) Amplitude
Fixed (small) Duration
80
60
40
20
0
$\langle \hat{T}_{tt} \rangle$
0.0 0.2 0.4 0.6 0.8 1.0
δf_0
$\langle \hat{T}_{tt} \rangle \propto \delta f_0{}^{1.92}$
$\langle \hat{T}_{tt} \rangle \propto \delta f_0{}^2$
0.00 0.05 0.10 0.15 0.20

Fast Quenches



Buchel+Lehner+Myers+Niekerk, Das+Galante+Myers

The inflaton Potential



The holographic theory:details

• We choose a bottom-up non-conformal theory with a single mass scale, driven by arelevant operator of dimension $\Delta = 3$.

• There is no phase transition in flat space (thermal ensemble) but there is a fast crossover (like QCD)

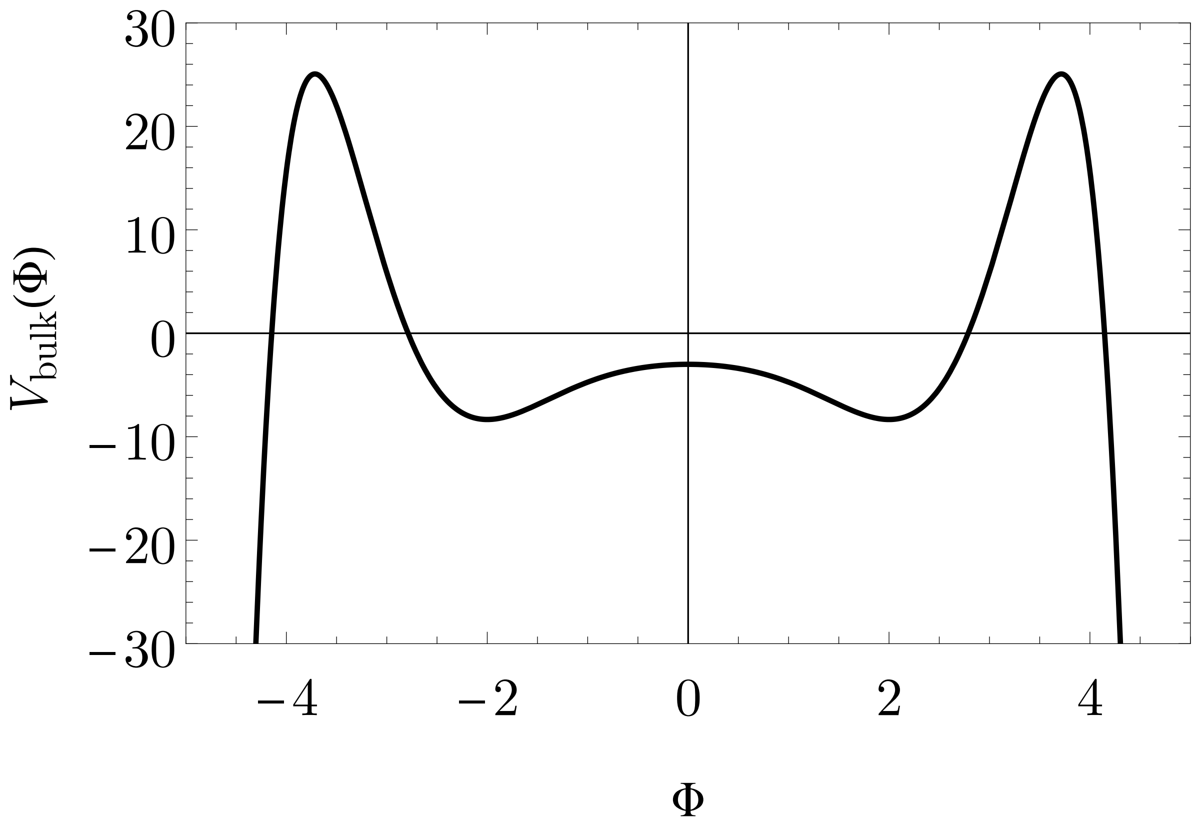
The bulk 5d action is Einstein-Dilaton gravity

$$S_{bulk} = \frac{2}{\kappa_5} \int d^5 \sqrt{g} \left[\frac{R}{4} - \frac{1}{2} (\partial \Phi)^2 - V(\Phi) \right]$$

and the bulk potential is

$$V(\Phi) = \frac{1}{\ell^2} \left[-3 - \frac{3}{2}\Phi^2 - \frac{\Phi^4}{3} + \frac{11}{96}\Phi^6 - \frac{\Phi^8}{192} \right]$$

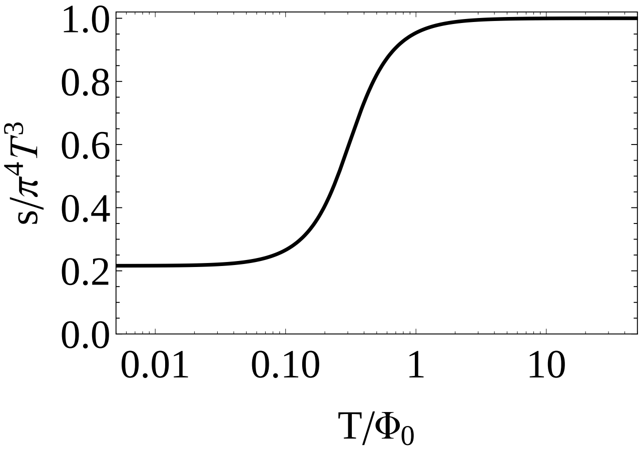
which looks like:

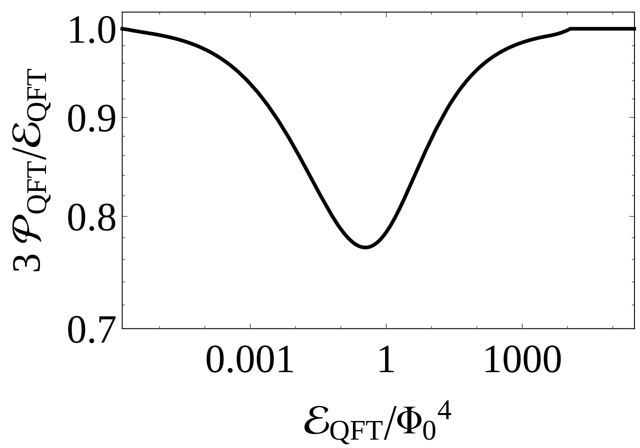


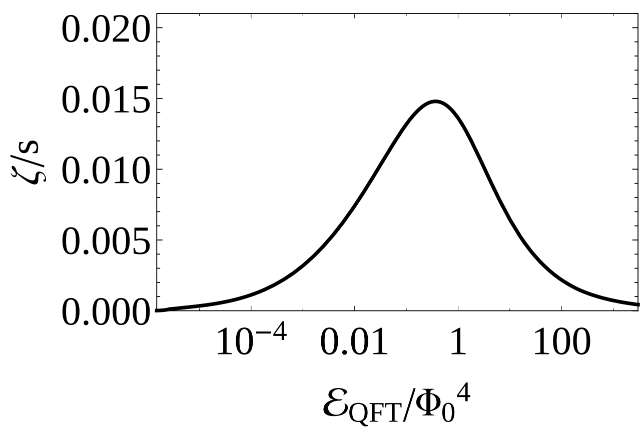
• The holographic theory corresponds to the flow between the maximum and one of the minima.

• The theory is massless in the IR due to the non-triviality of the IR CFT

• The thermodynamics in flat space is shown below







Holographic vev's

$$ds^2 = L^2 \frac{d\rho^2}{4\rho^2} + \bar{g}_{ij}(\rho, x) dx^i dx^j ,$$

$$\begin{aligned}
\bar{g}_{ij}(\rho, x) &= \frac{1}{\rho}\Big[\gamma_{ij}(x) + \rho\, \gamma_{(2)ij}(x) + \rho^2 \gamma_{(4)ij}(x) \\
&+ \rho^2 \log\rho\, h_{(4)ij}(x) + O(\rho^3)\Big] , \\
\Phi(\rho, x) &= \rho^{1/2}\Big[\Phi_{(0)}(x) + \rho\, \Phi_{(2)}(x) + \rho \log\rho\, \psi_{(2)}(x) + O(\rho^2)\Big] .
\end{aligned}$$

$$\begin{aligned}
\langle T^{\rm QFT}_{ij} \rangle &= \frac{2}{\kappa_5}\Big\{ \gamma_{(4)ij} + \frac{1}{8}\left[{\rm Tr}\gamma_{(2)}^2 - ({\rm Tr}\gamma_{(2)})^2\right]\gamma_{ij} \\
&- \frac{1}{2}\gamma_{(2)}^2 + \frac{1}{4}\gamma_{(2)ij}{\rm Tr}\gamma_{(2)} + \frac{1}{2}\partial_i\Phi_{(0)}\partial_j\Phi_{(0)} \\
&+ \left(\Phi_{(0)}\Phi_{(2)} - \frac{1}{2}\Phi_{(0)}\psi_{(2)} - \frac{1}{4}\partial_k\Phi_{(0)}\partial^k\Phi_{(0)}\right)\gamma_{ij} \\
&+ \alpha\left(\mathcal{T}^\gamma_{ij} + \mathcal{T}^\phi_{ij}\right) + \left(\frac{1}{18} + \beta\right)\Phi^4_{(0)}\gamma_{ij}\Big\} .
\end{aligned}$$

$$\langle \mathcal{O} \rangle = \frac{2}{\kappa_5}\left[(1 - 4\alpha)\psi_{(2)} - 2\Phi_{(2)} - 4\beta\Phi^3_{(0)}\right] .$$

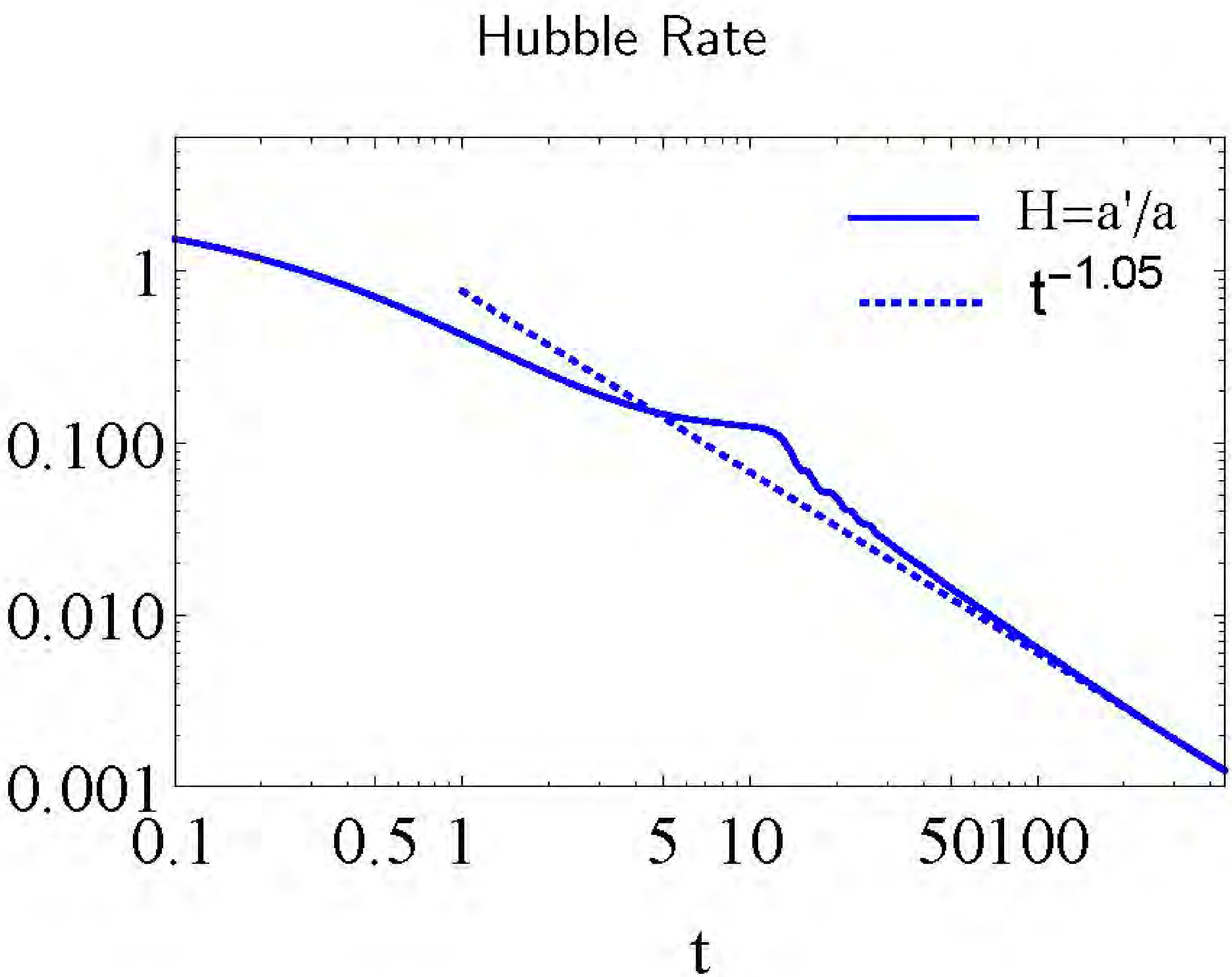
Bulk Equations of Motion

$$\begin{aligned} ds^2_{\rm bulk} &= g_{\mu\nu} dx^\mu dx^\nu = -A(r,t)dt^2 + 2drdt + S(r,t)^2 d\vec{x}^2 , \\ \Phi &= \Phi(r,t) , \end{aligned}$$

$$\begin{aligned} S'' &= -\frac{2}{3} S \left(\Phi'\right)^2 , \\ \dot{S}' &= -\frac{2\dot{S}S'}{S} - \frac{2SV}{3} , \\ \dot{\Phi}' &= \frac{V'}{2} - \frac{3\dot{S}\Phi'}{2S} - \frac{3S'\dot{\Phi}}{2S} , \\ A'' &= \frac{12\dot{S}S'}{S^2} + \frac{4V}{3} - 4\dot{\Phi}\Phi' , \\ \ddot{S} &= \frac{\dot{S}A'}{2} - \frac{2S\dot{\Phi}^2}{3} , \end{aligned}$$

$$f' \equiv \partial_r f , \quad \dot{f} \equiv \partial_t f + \frac{1}{2} A \partial_r f .$$

Hubble Rate



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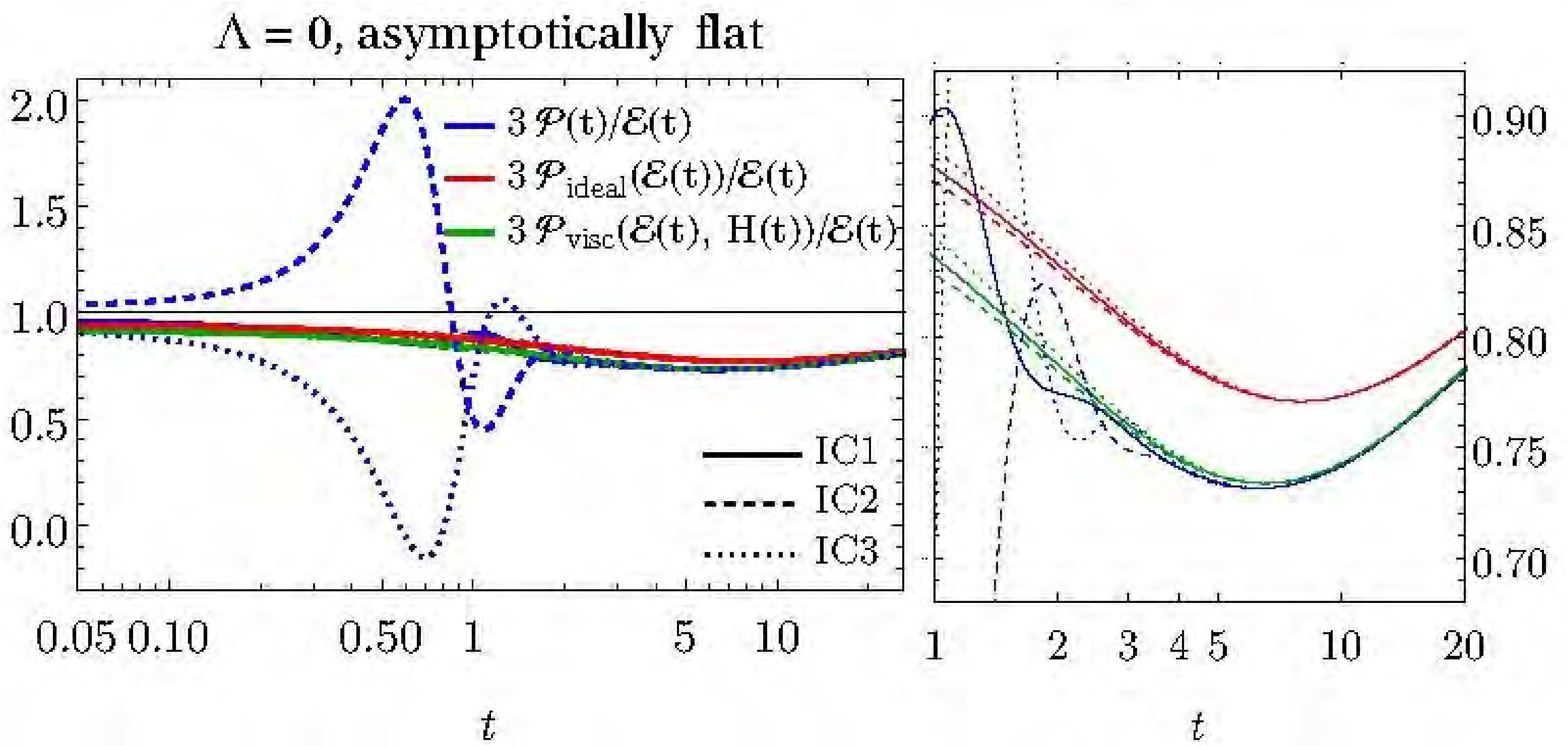
Hydrodynamization with Frozen Inflaton



Ecker+Casalderrey-Solana+Mateos+Van der Schee

The conformal anomaly

• The conformal anomaly in a QFT$_4$ gives the trace of the energy-momentum tesnor

$$T^{\mu}{}_{\mu} = \mathcal{A}$$

• It depends on all external sources: background metric, coupling functions etc.

• The metric dependence is universal

$$\mathcal{A}_g = a(Gauss - Bonnet) + c(Weyl^2) + b\Box R$$

• The coupling dependent part depends crucially on the QFT

$$\mathcal{A}_{extra} = \beta(\Phi)\langle O\rangle + \mathcal{A}_{\Phi}$$

• $\mathcal{A}_{\Phi}$ exists when $\mathcal{O}$ is "anomalous". In our case ($\Delta = 3$)

$$\mathcal{A}_g = \frac{1}{16}(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2) \quad , \quad \beta(\Phi) = -\Phi \quad , \quad \mathcal{A}_{\Phi} = -\frac{1}{2}\left(\partial_{\mu}\Phi\partial^{\mu}\Phi + \frac{1}{6}R\Phi^2\right)$$

The model data

- We use the following values and initial conditions

$$\kappa_5 = \frac{1}{9} \quad , \quad \kappa_4 = \frac{2\pi}{5625} \quad , \quad U(\phi) = \frac{1}{30}\phi$$

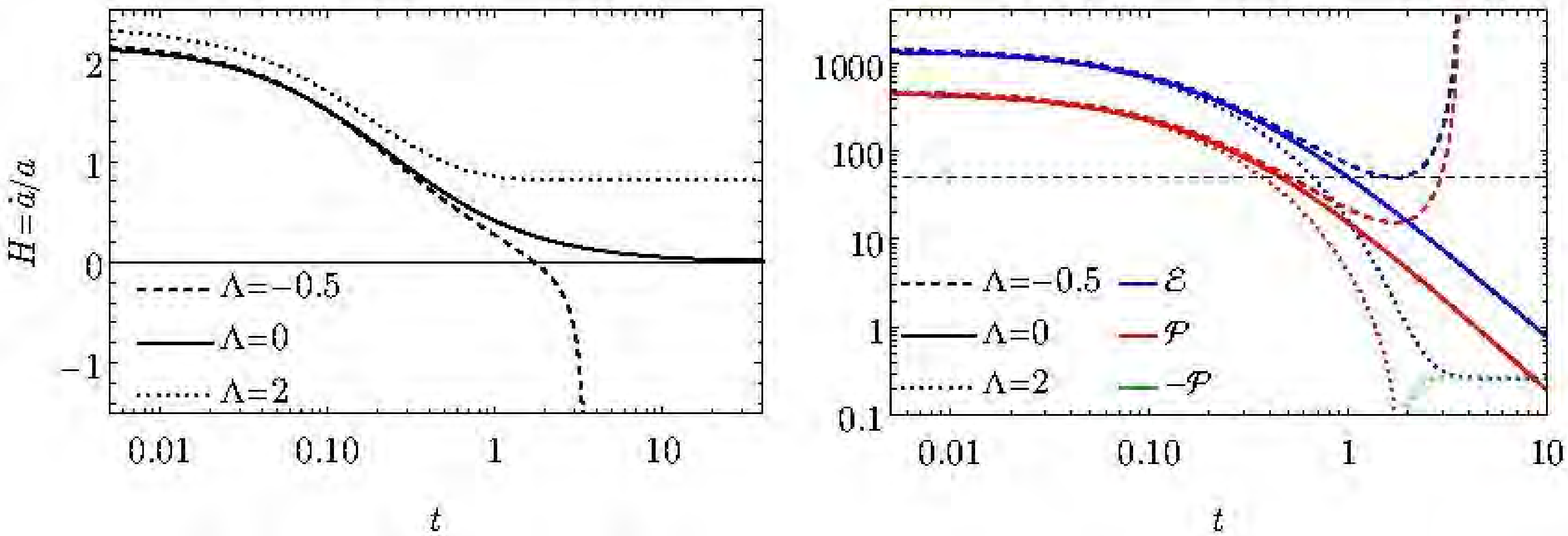
$$\mathcal{E}_{QFT}^{ini} = 13275 \quad , \quad \phi_{ini} = -30 \quad , \quad \dot{\phi}_{ini} = \frac{3}{10}$$

$$\Phi_{ini}(r) = r\left[\Phi_0(t) + r^2\Phi_2 + 2r^2 \log r\Psi_2 + \cdots + \frac{1}{r^3}\left(-6 + \frac{120}{r} - \frac{300}{r^3}\right)\right]$$

$$\Psi_2 = \frac{1}{4}\left(\Box\Phi_0 - \frac{1}{6}R\Phi_0\right)$$

Static Inflaton

- Let's first see what happens when we freeze the inflaton: $\phi(t) = \text{const.}$.
- Pick some value for cosmological constant Λ and initialize QFT.
- Depending on the value of Λ, the universe ends up in a Big Crunch ($\Lambda < 0$), in flat space ($\Lambda = 0$) or in de Sitter ($\Lambda > 0$).
- de Sitter solution has some Casimir energy $\mathcal{E}_{\text{dS}} = -\mathcal{P}_{\text{dS}}$.



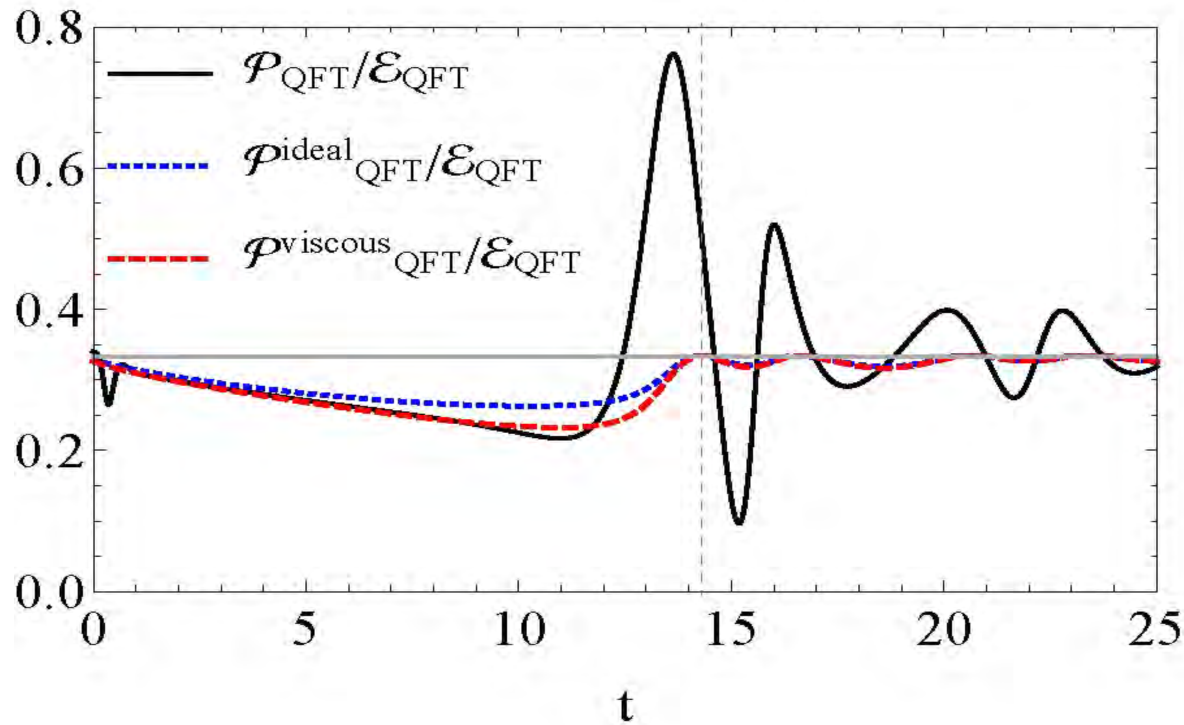
Ecker+Casalderrey-Solana+Mateos+Van der Schee

The QFT Pressure

• After an initial short far-out-of-equilibrium stage, the system is well described by hydrodynamics until the inflaton drives back the QFT out-of-equilibrium.

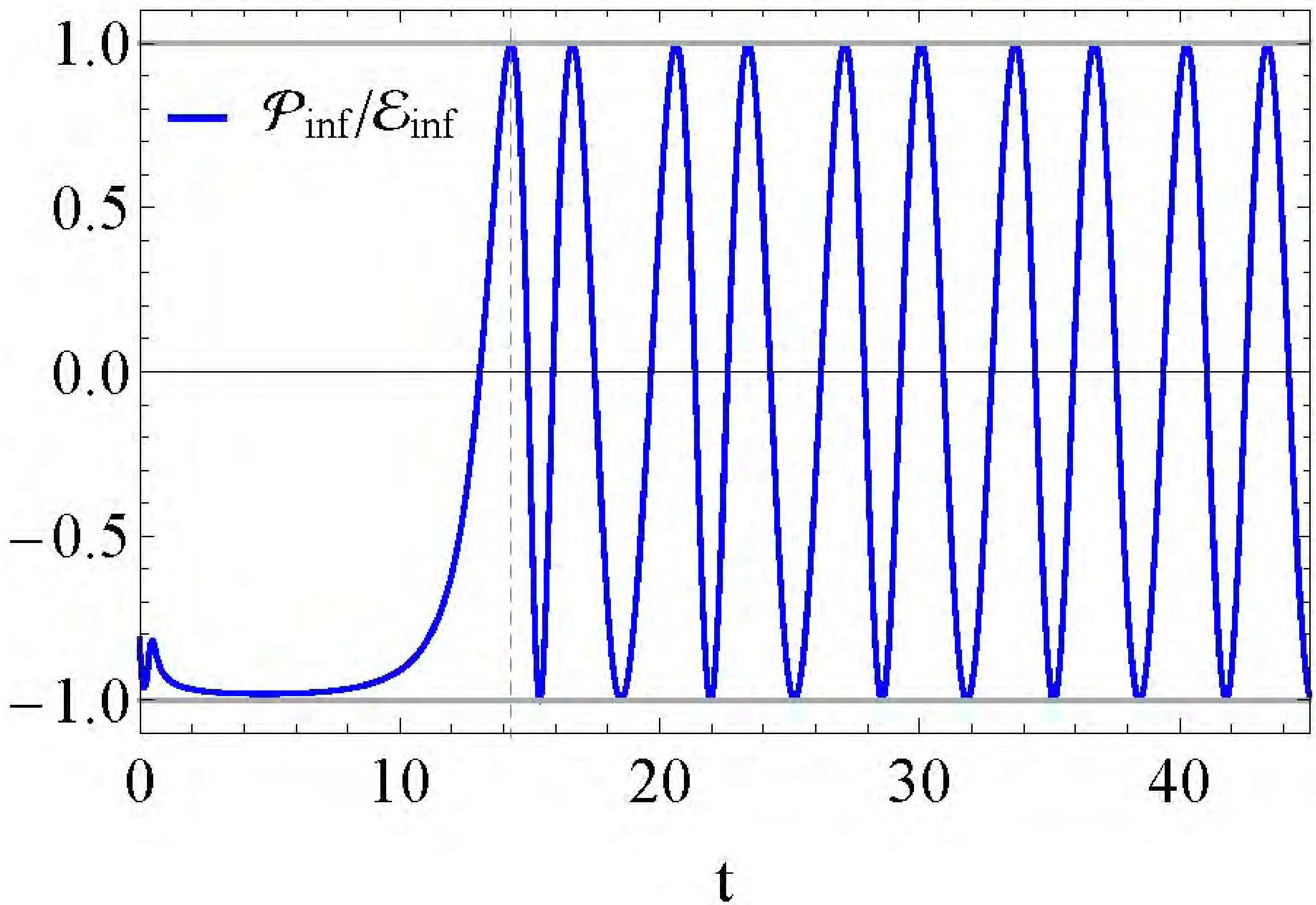
$$\mathcal{P}^{viscous}_{QFT}(t) = \mathcal{P}^{ideal}_{QFT}(t) - 3H\zeta\mathcal{E}_{QFT}(t) + \mathcal{O}(H^2)$$

• The QFT evolves from the UV to the IR fixed point where $\mathcal{P}_{QFT} = \frac{1}{3}\mathcal{E}_{QFT}$.



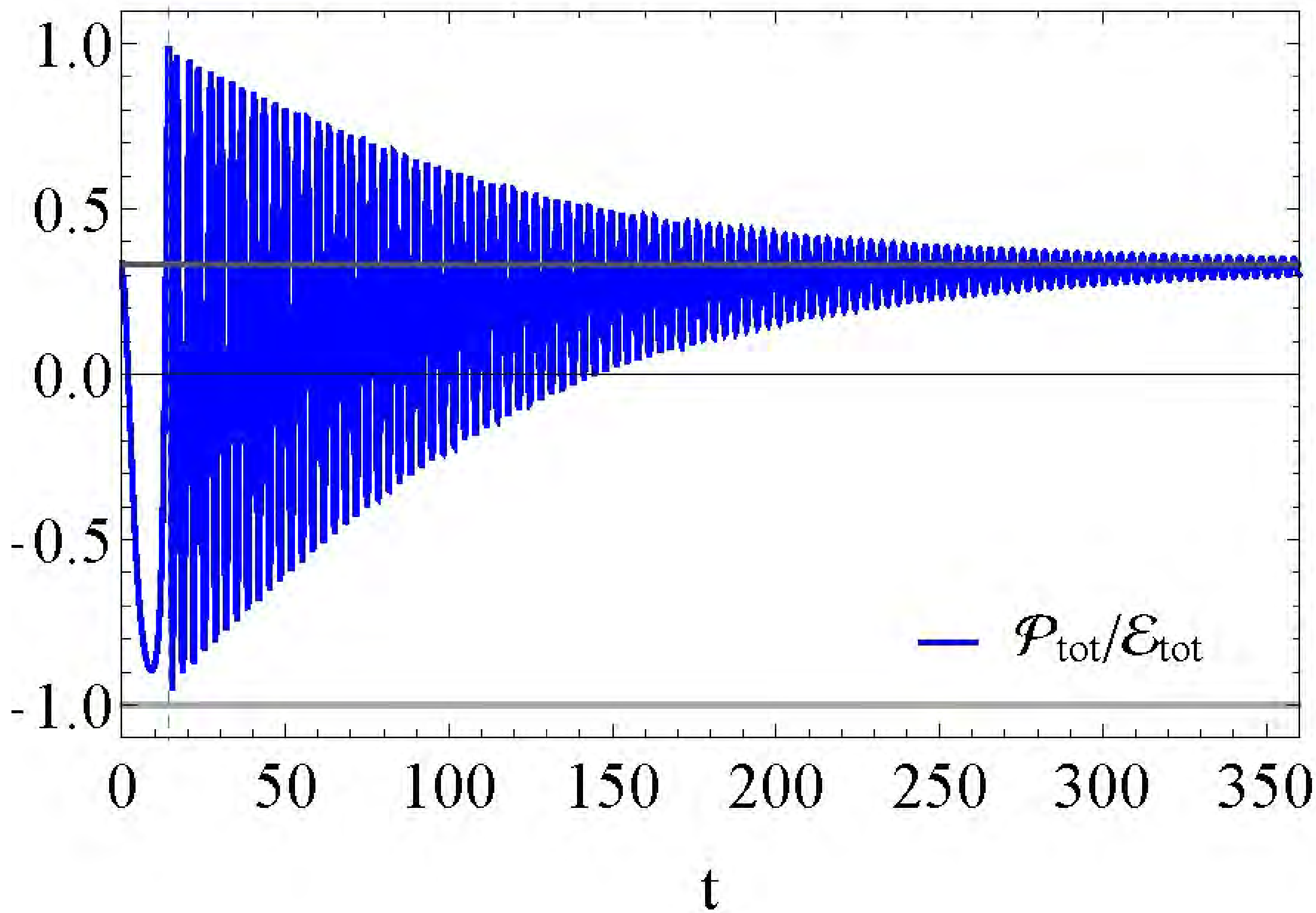
The Inflaton Pressure

• In the initial (inflation) stage $\mathcal{E}_{infl} \simeq -\mathcal{E}_{infl}$



The total pressure

• The total pressure is initially dominated by the QFT, then by the inflaton, and finally by the reheated QFT.



Detailed plan of the presentation

- Title page 0 minutes
- Bibliography 1 minutes
- Introduction 3 minutes
- Preheating and Thermalization at Strong Coupling 4 minutes
- Thermalization 5 minutes
- The setup for thermalization 6 minutes
- Thermalization at strong coupling 6 minutes
- The cosmological setup 8 minutes
- The holographic matter theory 10 minutes

- The Cosmological Evolution Equations 13 minutes
- Cartoons 14 minutes
- The Inflaton 15 minutes
- The Energy Density 16 minutes
- Temperature 18 minutes
- Conclusions 19 minutes
- Open ends 21 minutes