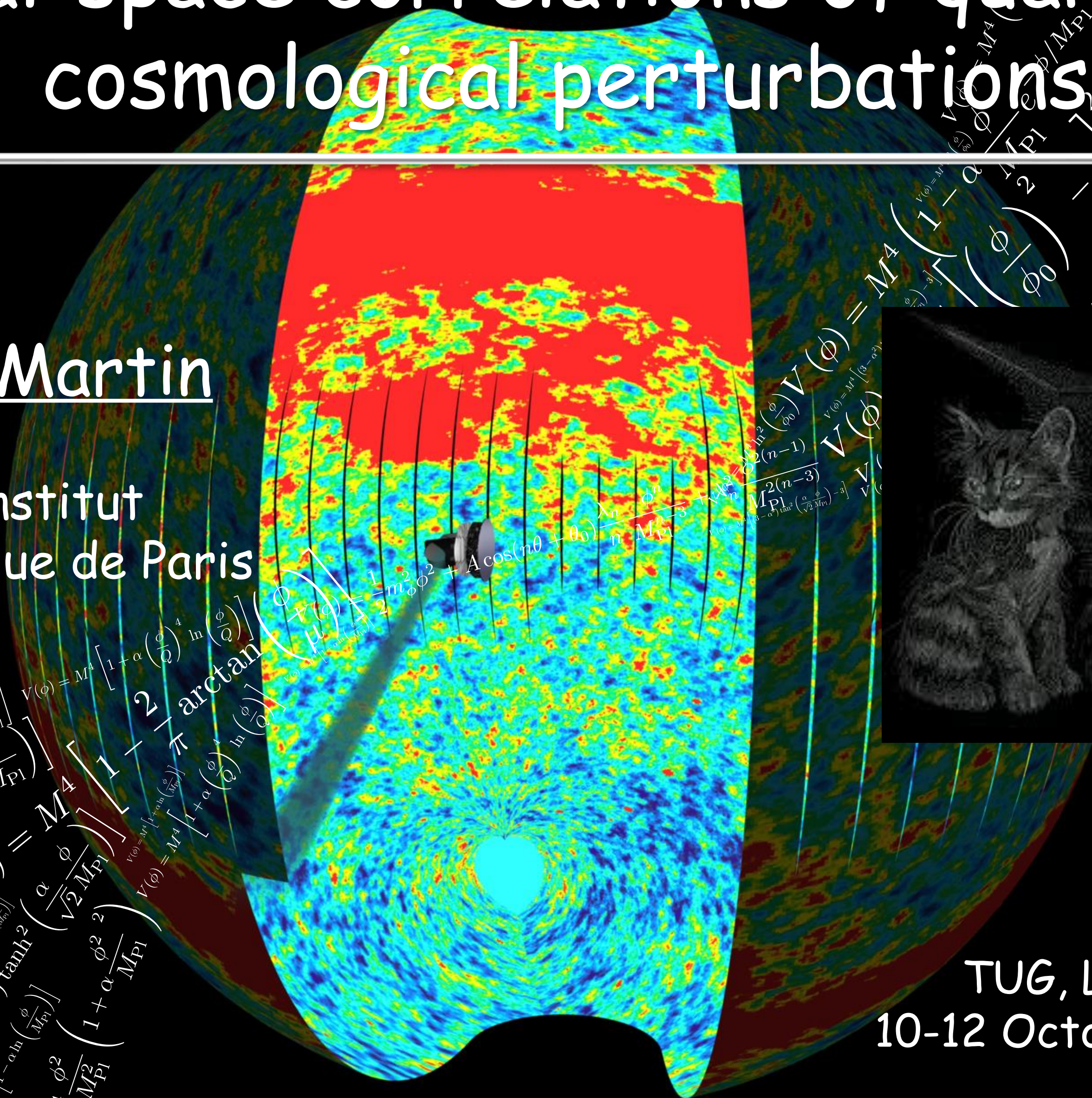
Real-space correlations of quantum cosmological perturbations



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TUG, LPENS
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Based on J. Martin & V. Vennin, "Real-space entanglement in CMB", arXiv:2106.15100
J. Martin & V. Vennin, "Real-space entanglement of quantum field", arXiv:2106.14575

Outline

- Introduction
- Cosmological perturbations of Quantum-Mechanical origin in very brief
- Quantum discord and correlations in real space
- Discussion & Conclusions

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- Additional motivations

  1- would confirm a fundamental insight about our Universe

  2- would confirm that Gravity must be quantized

  3- would indicate that QM operates on cosmological scales

  etc …

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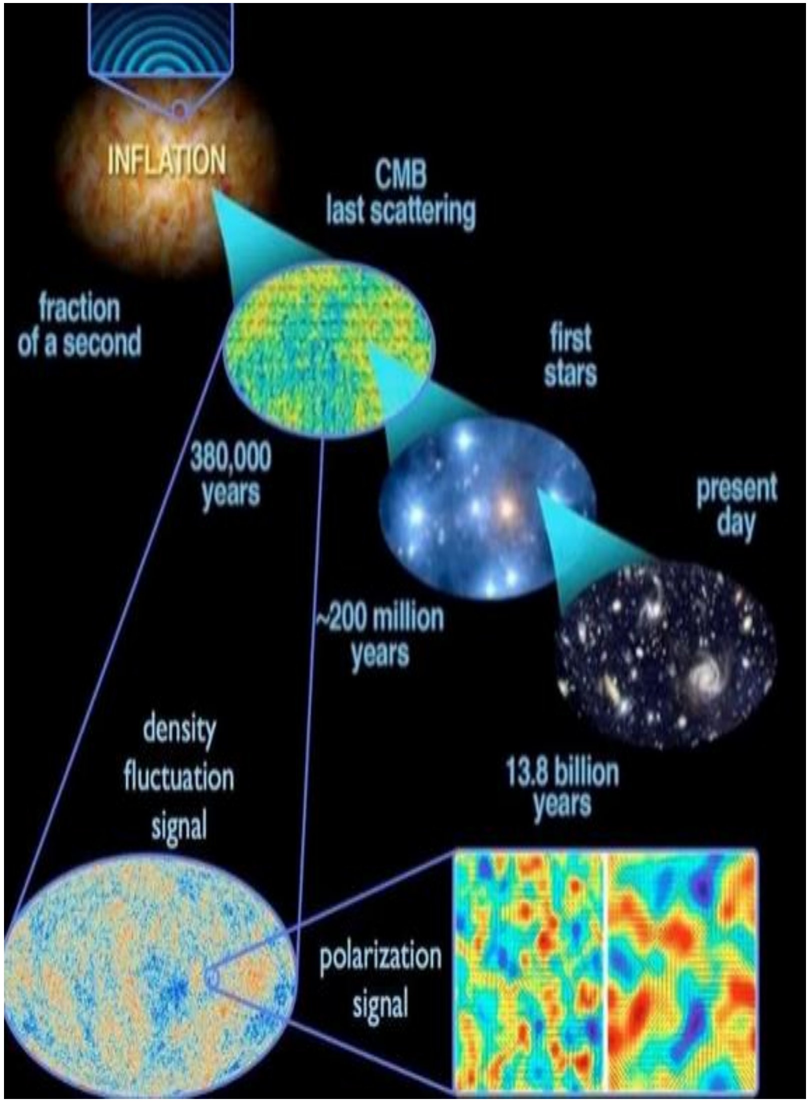


- Scalar perturbations are characterized by one quantity (a combination of metric and inflaton perturbations): curvature perturbations

$$\zeta(\eta, \boldsymbol{x}) = \frac{v(\eta, \boldsymbol{x})}{z(\eta)}$$

Mukhanov-Sasaki variable

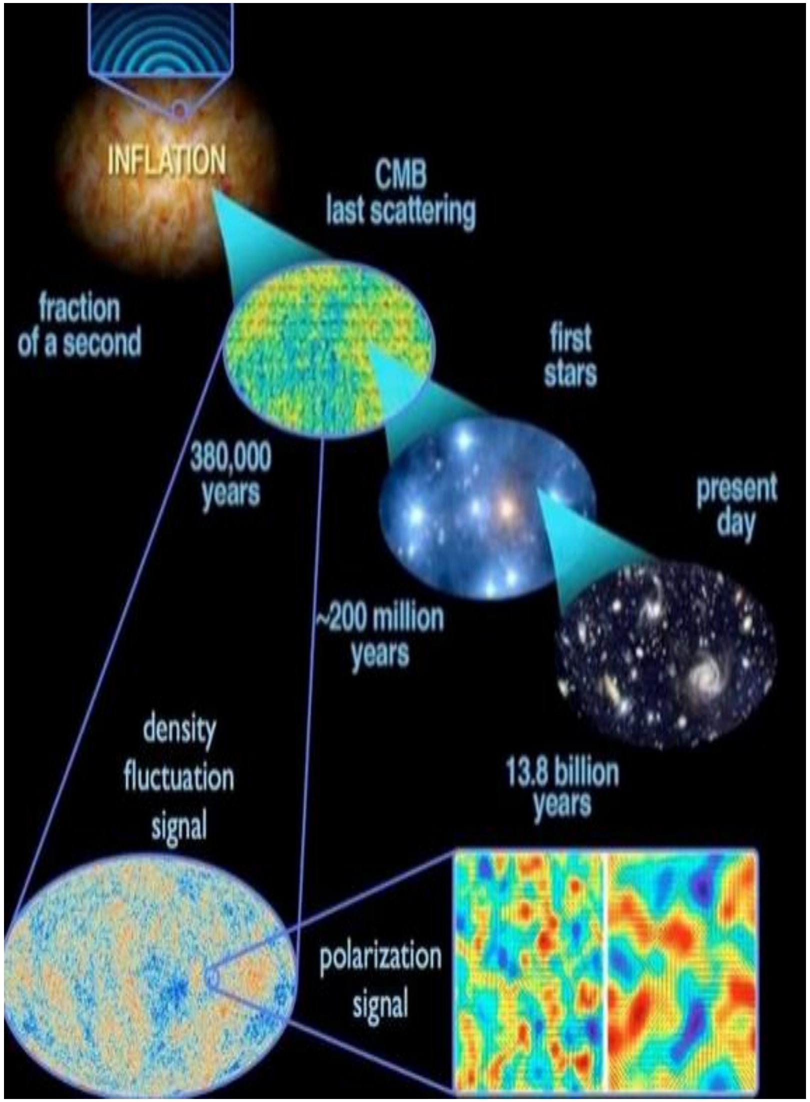
with

$$z(\eta) = a(\eta)\sqrt{2\epsilon_1}M_{\rm Pl}\,, \quad \epsilon_1 = -\frac{\dot{H}}{H^2}$$

- In Fourier space, this is a collection of oscillators, each mode k being described by a "position" and a momentum

$$(q_{\boldsymbol{k}}, \pi_{\boldsymbol{k}})$$

Quantum perturbations

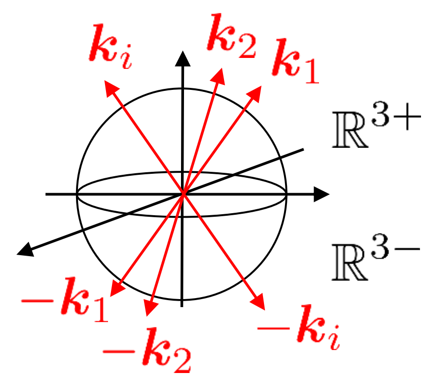


- Hamiltonian of the system

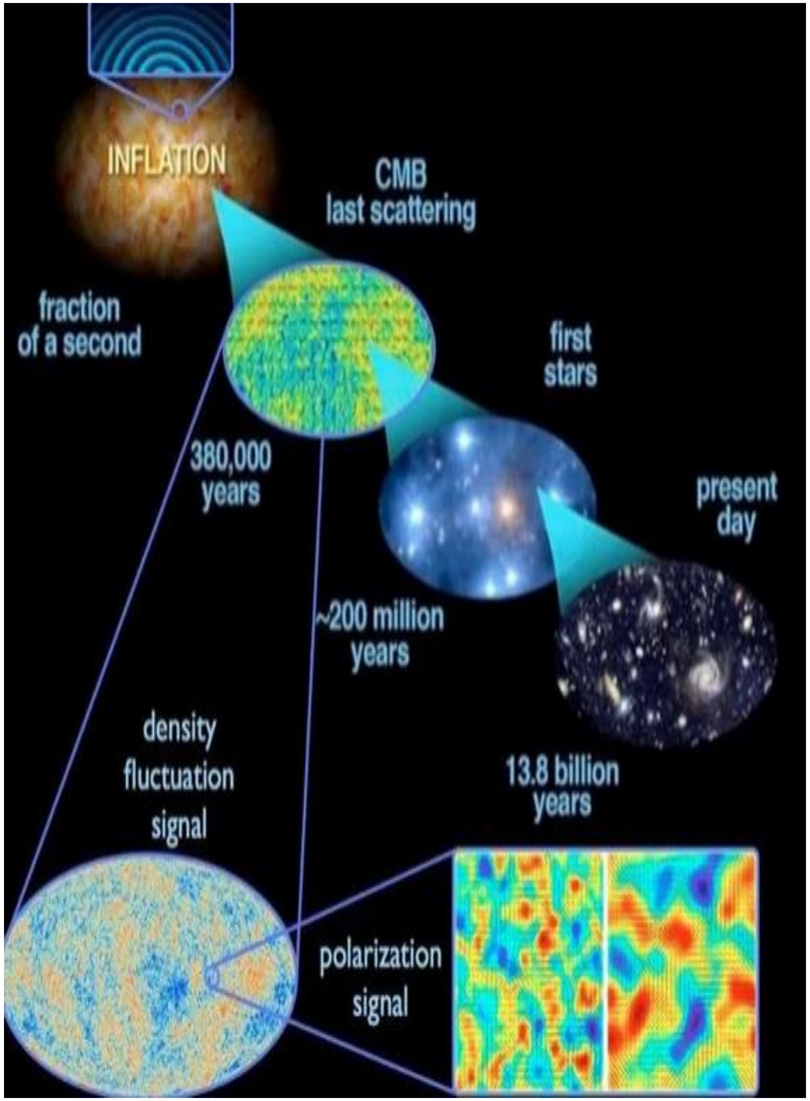
$$S = \frac{M_{\rm Pl}^2}{2}\int {\rm d}^4x\,\sqrt{-g}\,R - \int {\rm d}^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V(\phi)\right]$$

$$\downarrow$$

$$\mathcal{H} = \int_{\mathbb{R}^{3+}} {\rm d}^3\boldsymbol{k}\,\mathcal{H}_{\pm\boldsymbol{k}}$$



$$\mathcal{H}_{\pm\boldsymbol{k}} = \frac{1}{2}\pi_{\boldsymbol{k}}^2 + \frac{1}{2}k^2q_{\boldsymbol{k}}^2 + \frac{1}{2}\pi_{-\boldsymbol{k}}^2 + \frac{1}{2}k^2q_{-\boldsymbol{k}}^2 + \frac{z'}{z}\left(q_{\boldsymbol{k}}\pi_{-\boldsymbol{k}} + q_{-\boldsymbol{k}}\pi_{\boldsymbol{k}}\right)$$



- The (pure) state of the system is a Gaussian two-mode squeezed state

$$\Psi[\zeta(\eta,\boldsymbol{x})] = \prod_{\boldsymbol{k}\in\mathbb{R}^{3+}} \psi\left(\eta, q_{\boldsymbol{k}}, q_{-\boldsymbol{k}}\right)$$

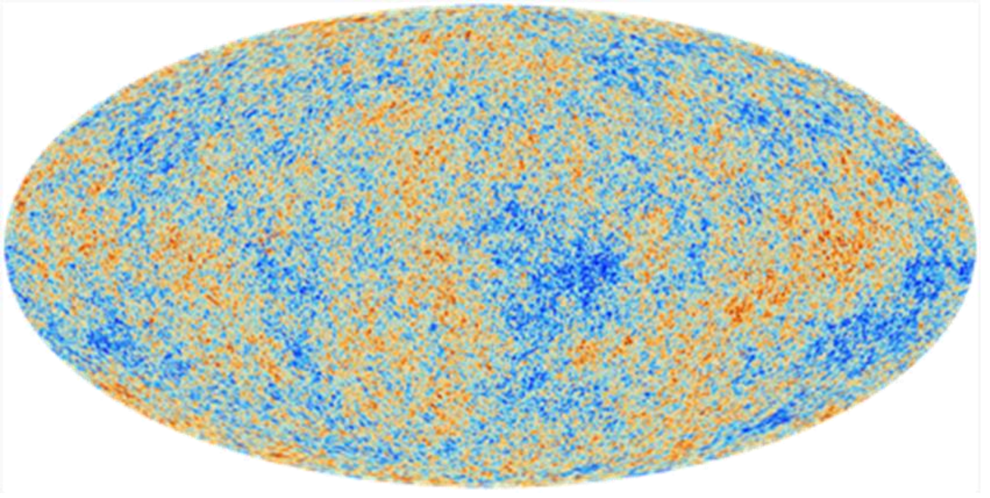
with

$$\psi = \frac{e^{A(r_{\boldsymbol{k}},\varphi_{\boldsymbol{k}})k(q_{\boldsymbol{k}}^2+q_{-\boldsymbol{k}}^2)-B(r_{\boldsymbol{k}},\varphi_{\boldsymbol{k}})kq_{\boldsymbol{k}}q_{-\boldsymbol{k}}}}{\sqrt{\pi}\cosh(r_{\boldsymbol{k}})\sqrt{1-e^{-4i\varphi_{\boldsymbol{k}}}\tanh^2(r_{\boldsymbol{k}})}}$$

It is an entangled state

$$\psi(\eta, q_{\boldsymbol{k}}, q_{-\boldsymbol{k}}) \neq \psi(\eta, q_{\boldsymbol{k}})\psi(\eta, q_{-\boldsymbol{k}})$$

The cosmological two-mode squeezed state is (very!) strongly squeezed



CMB anisotropy is the strongest squeezed state ever produced in Nature

$$r_k = \mathcal{O}\left(10^2\right)$$

$$-10\log_{10}\left(e^{-2r_k}\right)\ \mathrm{dB}$$

- ~ 15 dB in the lab
- > 500 dB inflation

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- <u>Idea</u>: the system is quantum if there are correlations that cannot be explained classically. Exemple: Bell’s inequality. There are correlations that cannot be reproduced with a classical random variable.

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- Quantum discord: measure of quantumness of a system based on the above idea. In this work: application to cosmological inflationary perturbations

- Mutual information I between systems A and B

$$I(A,B) = S(A) + S(B) - S(A,B)$$

$$S(A) = -\sum_i \underbrace{p(a_i)}_{\text{pdf}} \ln[p(a_i)]$$

$$S(A,B) = -\sum_{ij} \underbrace{p(a_i,b_j)}_{\text{joined pdf}} \ln[p(a_i,b_j)]$$

$$p(a_i,b_j) = p(a_i)p(b_j) \longrightarrow I(A,B) = 0$$

- Mutual information J (in a different way with Bayes theorem)

$p(a_i|b_j)$ = probability of a_i given b_j $\longrightarrow$ $-\sum_i p(a_i|b_j)\ln[\underbrace{p(a_i|b_j)}]$ "entropy of A given b_j "

$$S(A|B) = \sum_j p(b_j)\left\{-\sum_i p(a_i|b_j)\ln[p(a_i|b_j)]\right\}$$

$$J(A,B) = S(A) - S(A|B)$$

Bayes theorem: $p(a_i,b_j) = p(b_j|a_i)p(a_i) \longrightarrow S(A|B) = S(A,B) - S(B)$

$$J(A,B) \equiv S(A) - S(A|B) = S(B) + S(A) - S(A,B) = I(A,B)$$

- Mutual information I between systems A and B in Quantum Mechanics

$$S(A,B) = -\mathrm{Tr}\left(\hat{\rho}\log_2\hat{\rho}\right)$$

$$S(A) = -\mathrm{Tr}\left(\hat{\rho}_{\mathrm{A}}\log_2\hat{\rho}_{\mathrm{A}}\right) \text{ with } \hat{\rho}_{\mathrm{A}} = \mathrm{Tr}_B\hat{\rho}$$

$I(A,B)$ can easily be generalized to QM

- Mutual information J between systems A and B in Quantum Mechanics

Upon measurement of $|b_j\rangle$:

$$\hat{\rho} \rightarrow \frac{1}{\mathrm{Prob}(|b_j\rangle)}\hat{\Pi}_{|b_j\rangle}\hat{\rho}\hat{\Pi}_{|b_j\rangle}$$

$$\hat{\rho}(A|\hat{\Pi}_{|b_j\rangle}) = \mathrm{Tr}_B\left[\frac{1}{\mathrm{Prob}(|b_j\rangle)}\hat{\Pi}_{|b_j\rangle}\hat{\rho}\hat{\Pi}_{|b_j\rangle}\right]$$

$$S(A|\hat{\Pi}_{|b_j\rangle}) = -\mathrm{Tr}\left\{\hat{\rho}(A|\hat{\Pi}_{|b_j\rangle})\ln\left[\hat{\rho}(A|\hat{\Pi}_{|b_j\rangle})\right]\right\}$$

$$S(A|B) = \sum_j \mathrm{Prob}(|b_j\rangle) S(A|\hat{\Pi}_{|b_j\rangle})$$

This allows us to generalize J in QM but, now, crucially, I≠J

$$\boxed{\mathcal{D}(A,B) \equiv I(A,B) - J(A,B)}$$

Quantum discord

- In general, for an arbitrary system and/or quantum state, the calculation of the discord is complicated

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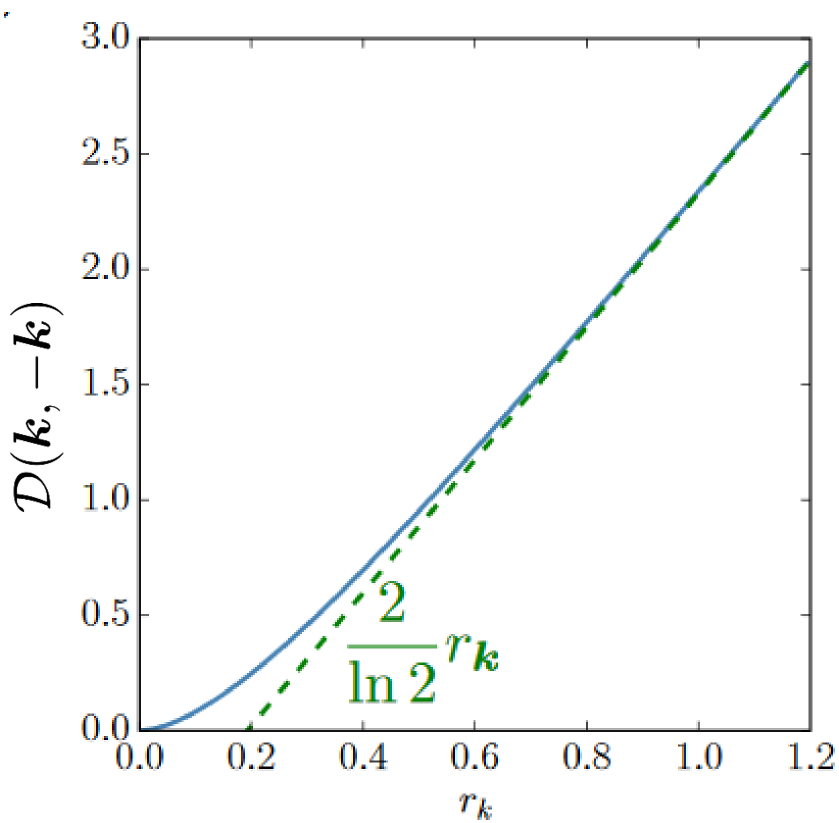
- However, a Gaussian state is completely characterized by its covariance matrix (the matrix of all the two-point correlation functions)

- There exists an algorithm allowing the calculation of the discord from the covariance matrix, see

G. Adesso and A. Datta, "Quantum versus Classical Correlations in Gaussian States", Phys. Rev. Letter 105 (July, 2010) 030501, arXiv:1003.4979

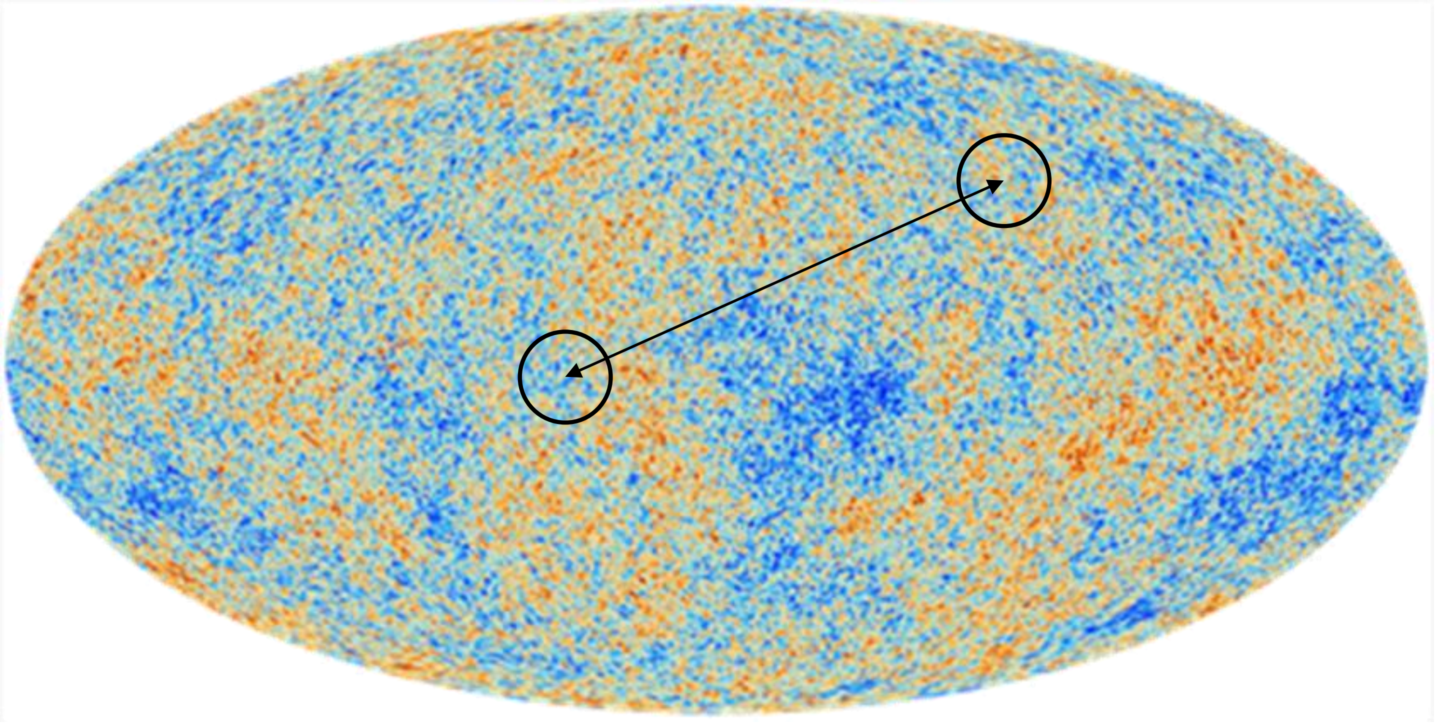
Application: Quantum discord in Fourier space for a two-mode squeezed state



$$\mathcal{D}(\boldsymbol{k},-\boldsymbol{k}) = \cosh^2 r_k \log_2\left(\cosh^2 r_k\right) - \sinh^2 r_k \log_2\left(\sinh^2 r_k\right)$$

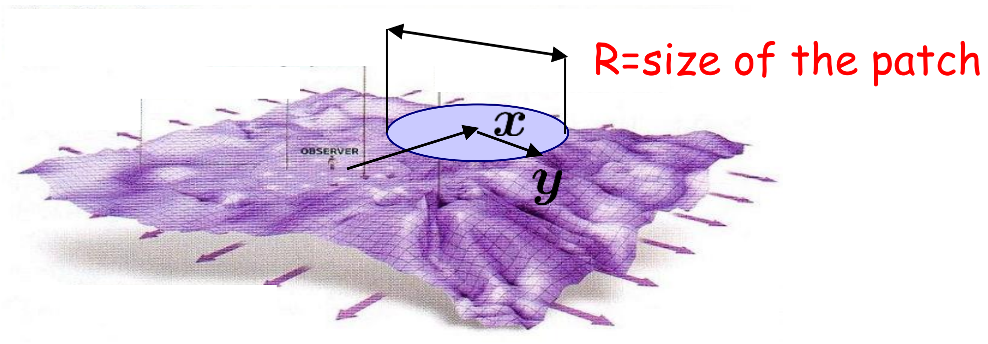
J. Martin & V. Vennin, PRD 93 (2016), 1023505, arXiv:1510.04038

Quantum correlations in real space?



We study the correlations between two patches in real space

Coarse-grained Mukhanov-Sasaki field in real space



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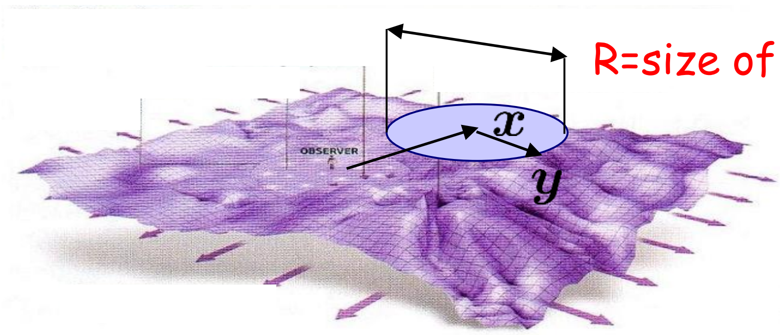

R=size of the patch

$$\hat{v}_{R,i}(\boldsymbol{x}) = \left(\frac{a}{R}\right)^3 \int \mathrm{d}^3\boldsymbol{y}\, \hat{v}_i(\boldsymbol{y}) W\left(\frac{a|\boldsymbol{y}-\boldsymbol{x}|}{R}\right)$$

Window function

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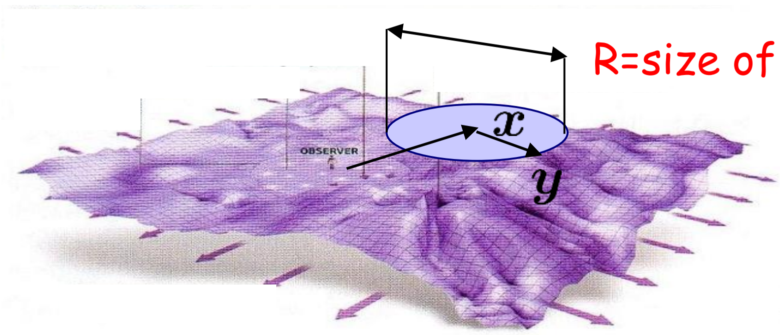

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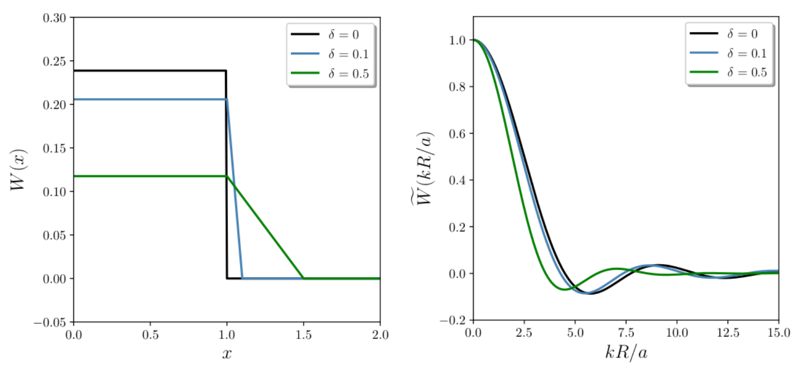
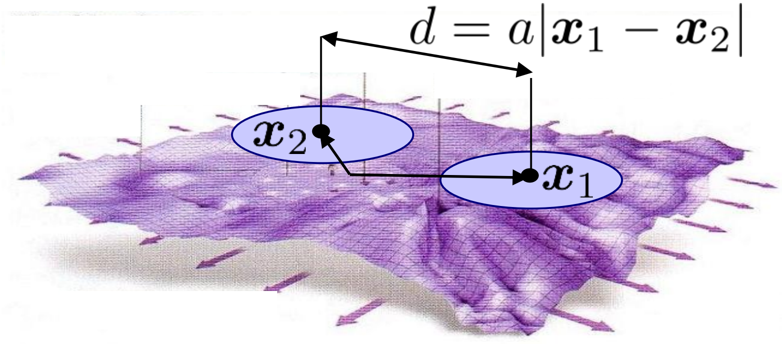


$$W(x) = \frac{3}{4\pi \mathcal{F}(\delta)} \begin{cases} 1, & x \leq 1 \\ -\frac{1}{\delta}(x-1)+1, & 1 < x \leq 1+\delta \\ 0, & x > 1+\delta \end{cases}$$

with $\mathcal{F}(\delta) = \frac{1}{4}(\delta+2)(\delta^2+2\delta+2)$

"Bipartite" coarse-grained real scalar field



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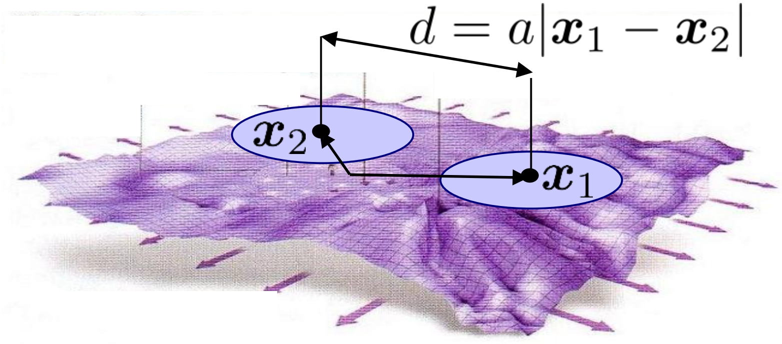

System described by the vector

$$\hat{Z}_R(\boldsymbol{x}_1, \boldsymbol{x}_2) = \begin{pmatrix} \hat{v}_R(\boldsymbol{x}_1) \\ \hat{\pi}_R(\boldsymbol{x}_1) \\ \hat{v}_R(\boldsymbol{x}_2) \\ \hat{\pi}_R(\boldsymbol{x}_2) \end{pmatrix}$$

First subsystem: $\hat{v}_R(\boldsymbol{x}_1)$, $\hat{\pi}_R(\boldsymbol{x}_1)$

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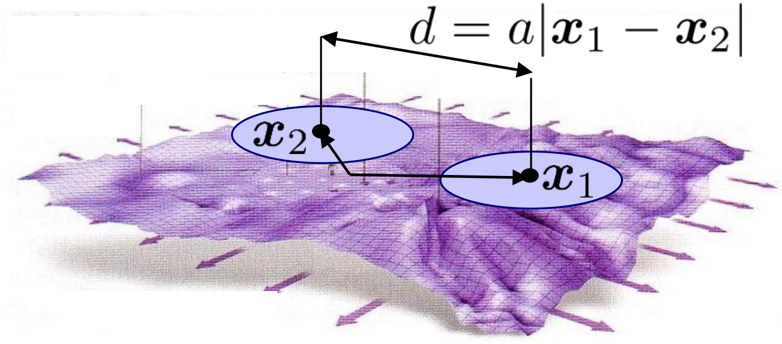

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Covariance matrix:

$$\gamma_{ab} = \left\langle \left\{ \hat{\hat{Z}}_{R,a}(\boldsymbol{x}_1, \boldsymbol{x}_2), \hat{\hat{Z}}_{R,b}(\boldsymbol{x}_1, \boldsymbol{x}_2) \right\} \right\rangle$$

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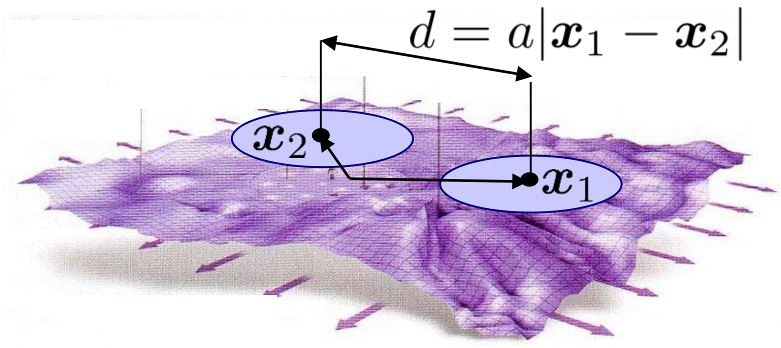

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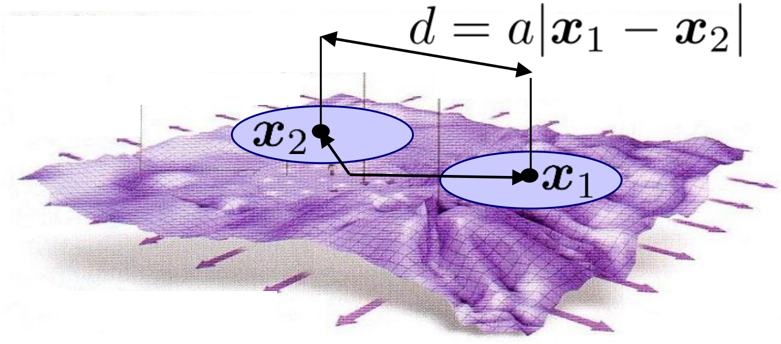
~ Fourier transform of the window function

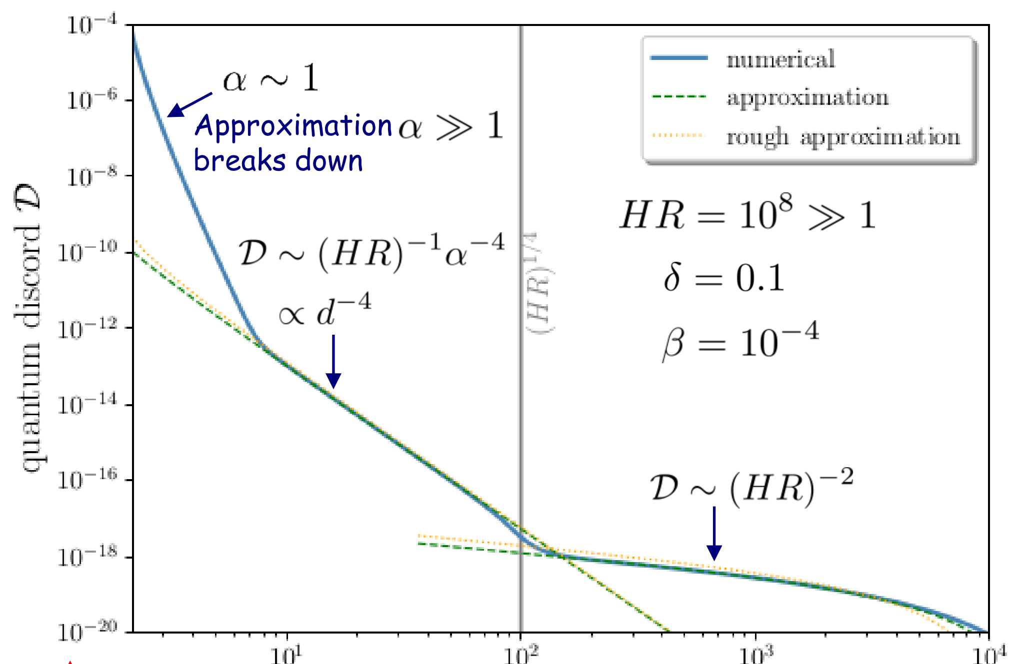
Power spectrum in Fourier space

$$\gamma = \frac{8\pi}{3G(\delta)} \left(\frac{R}{a}\right)^3 \int \frac{\mathrm{d}k}{k} \tilde{W}^2\left(\frac{R}{a}k\right) \begin{pmatrix} \mathcal{P}_{vv}(k) & \mathcal{P}_{v\pi}(k) & \mathcal{P}_{vv}(k)\,\mathrm{sinc}\left(\frac{kd}{a}\right) & \mathcal{P}_{v\pi}(k)\,\mathrm{sinc}\left(\frac{kd}{a}\right) \\ - & \mathcal{P}_{\pi\pi}(k) & \mathcal{P}_{\pi v}(k)\,\mathrm{sinc}\left(\frac{kd}{a}\right) & \mathcal{P}_{\pi\pi}(k)\,\mathrm{sinc}\left(\frac{kd}{a}\right) \\ - & - & \mathcal{P}_{vv}(k) & \mathcal{P}_{v\pi}(k) \\ - & - & - & \mathcal{P}_{\pi\pi}(k) \end{pmatrix}$$

"Bipartite" coarse-grained real scalar field



Since the system is Gaussian and its covariance matrix known, we can use the standard techniques to calculate the discord in de Sitter spacetime



$\alpha = d/R$ $\left(2R(1+\delta) < d < R_{\rm obs}\right.$ or $\left.\alpha \in [2(1+\delta), \beta^{-1}]\right)$

Discord is strongly suppressed in real space

J. Martin & V. Vennin, PRD94, 085012 (2021); JCAP 10, 036 (2021)

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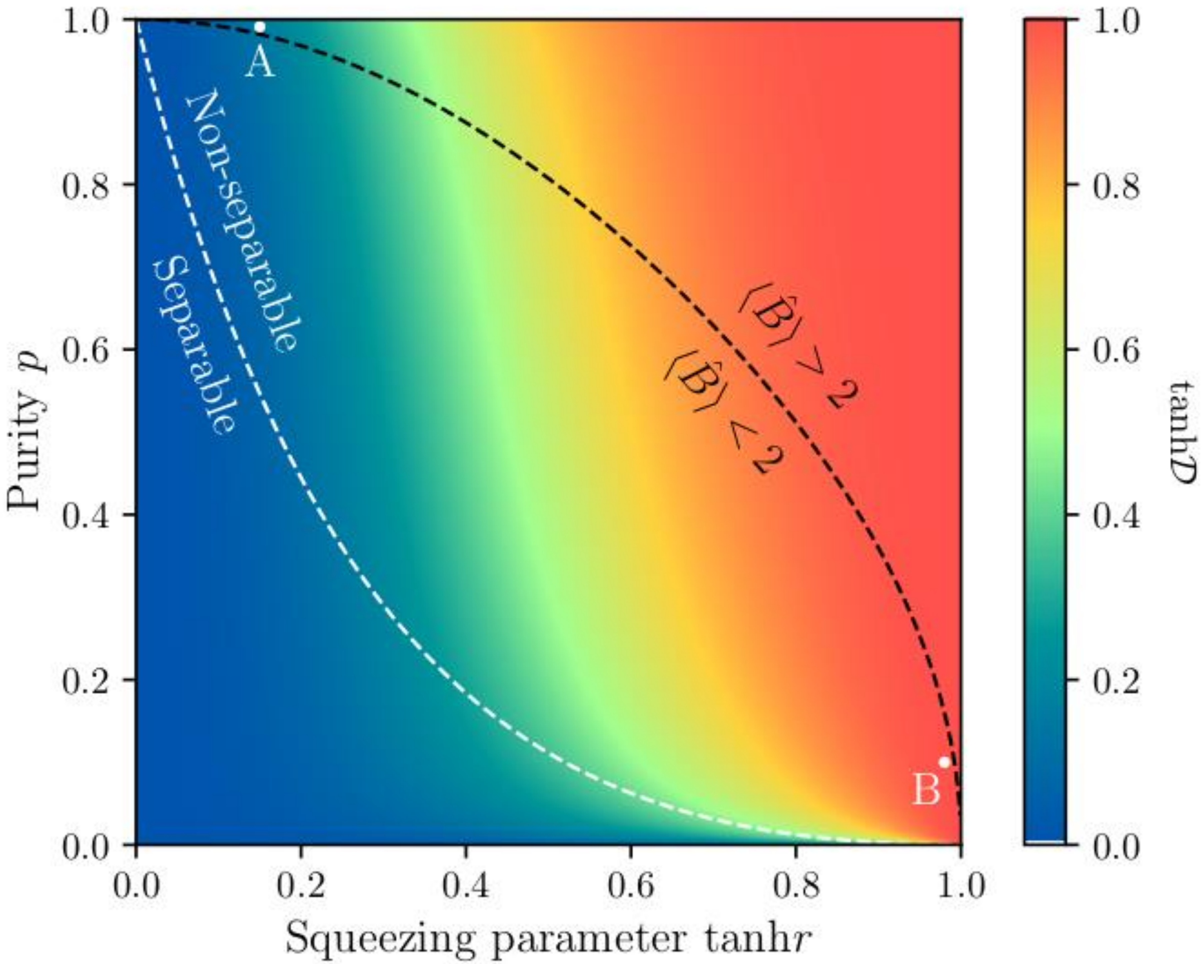
Thank you for your attention

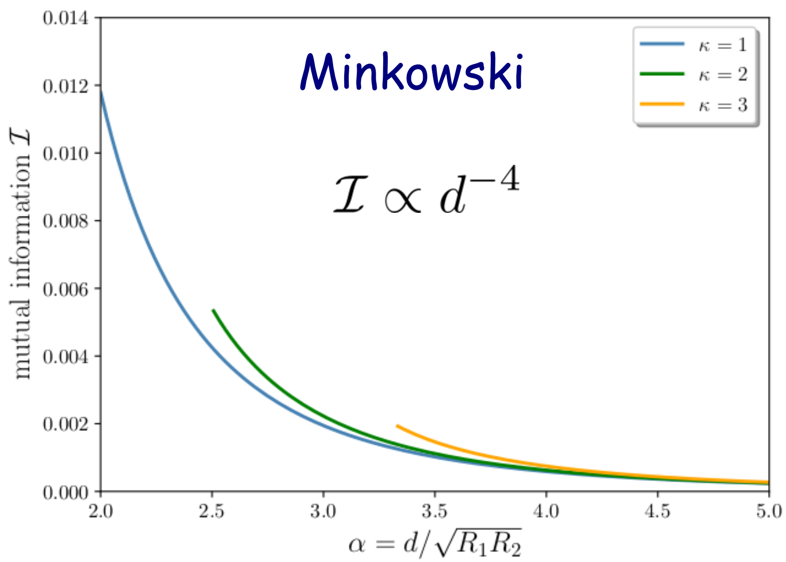
Criterions for quantumness are not all equivalent

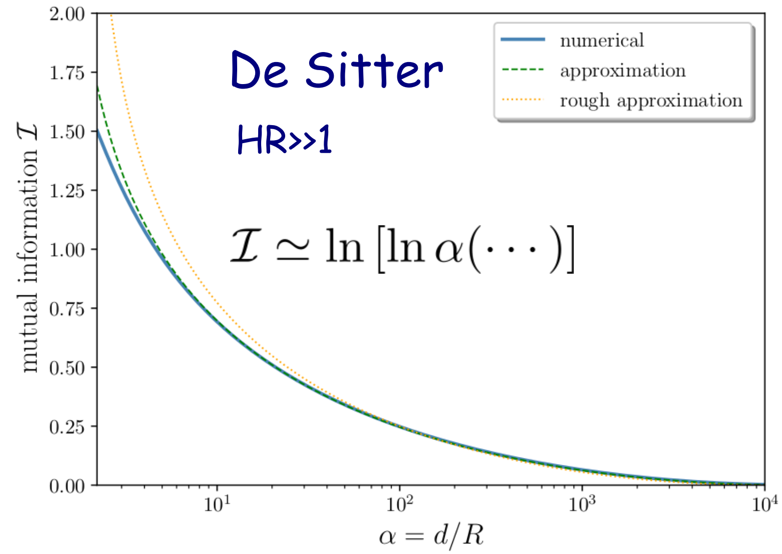
$$\gamma = \frac{\gamma^{\mathrm{TMSV}}}{\sqrt{p}} \longrightarrow \text{purity: } p = \mathrm{Tr}\left(\rho^2\right) = \frac{1}{\sqrt{\det \gamma}}$$

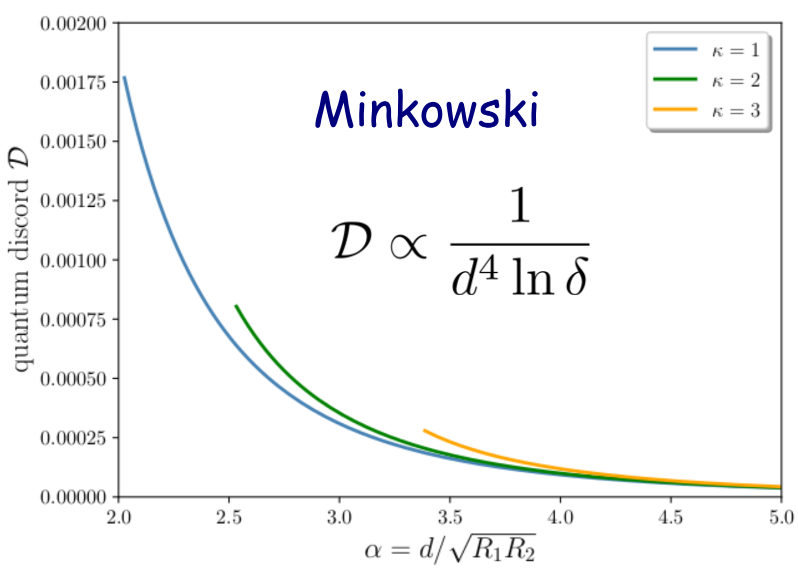


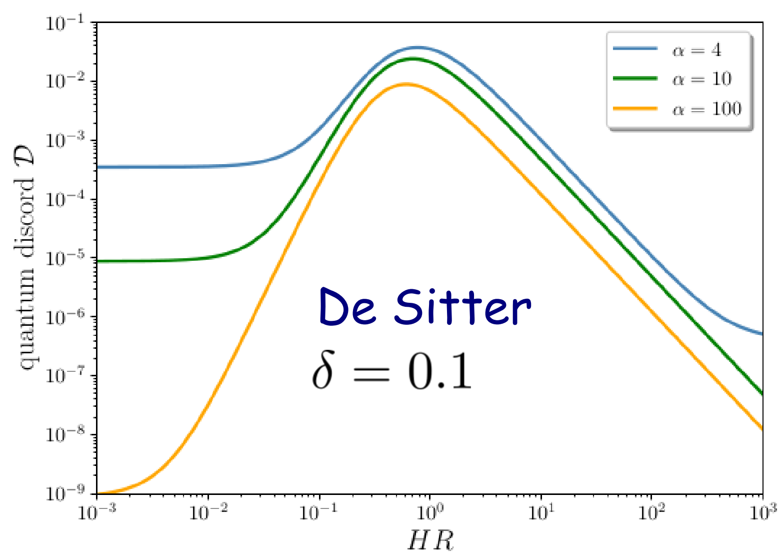
J. Martin, A. Micheli & V. Vennin, EPL 142, 18001 (2023)

Mutual information & Discord

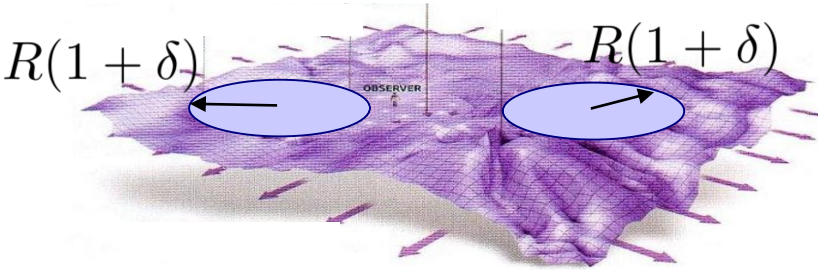








Rescaled coarse-grained real scalar field



$$\hat{z}_{R,i}(\boldsymbol{x}) = \left(\frac{a}{R}\right)^3 \int \mathrm{d}^3\boldsymbol{y}\, \hat{z}_i(\boldsymbol{y}) W\left(\frac{a|\boldsymbol{y}-\boldsymbol{x}|}{R}\right)$$

- $$\left[\hat{\phi}_R(\boldsymbol{x}_1), \hat{\pi}_R(\boldsymbol{x}_2)\right] = i\left(\frac{a}{R}\right)^6 \int \mathrm{d}\boldsymbol{y}\, W\left(\frac{a}{R}|\boldsymbol{y}-\boldsymbol{x}_1|\right) W\left(\frac{a}{R}|\boldsymbol{y}-\boldsymbol{x}_2|\right)$$
  $= 0$ for $d = a|\boldsymbol{x}_1 - \boldsymbol{x}_2| > 2R(1+\delta)$

- $$\left[\hat{\phi}_R(\boldsymbol{x}), \hat{\pi}_R(\boldsymbol{x})\right] = 4i\pi\left(\frac{a}{R}\right)^3 \int \mathrm{d}u\, W^2(u)\, u^2 = \frac{3i}{4\pi}\left(\frac{a}{R}\right)^3 G(\delta)$$

$\tilde{z}_R = \Lambda^{(1)} z_R$ with $\Lambda^{(1)} = \left(\frac{R}{a}\right)^{3/2}\sqrt{\frac{4\pi}{3G(\delta)}}$

$[\tilde{z}_{R,i}(\boldsymbol{x}_a), \tilde{z}_{R,j}(\boldsymbol{x}_b)] = iJ_{ij}^{(1)}\,\delta_{ab}$ with $\delta_{ab} = 0$ for $d = a|\boldsymbol{x}_1 - \boldsymbol{x}_2| > 2R(1+\delta)$