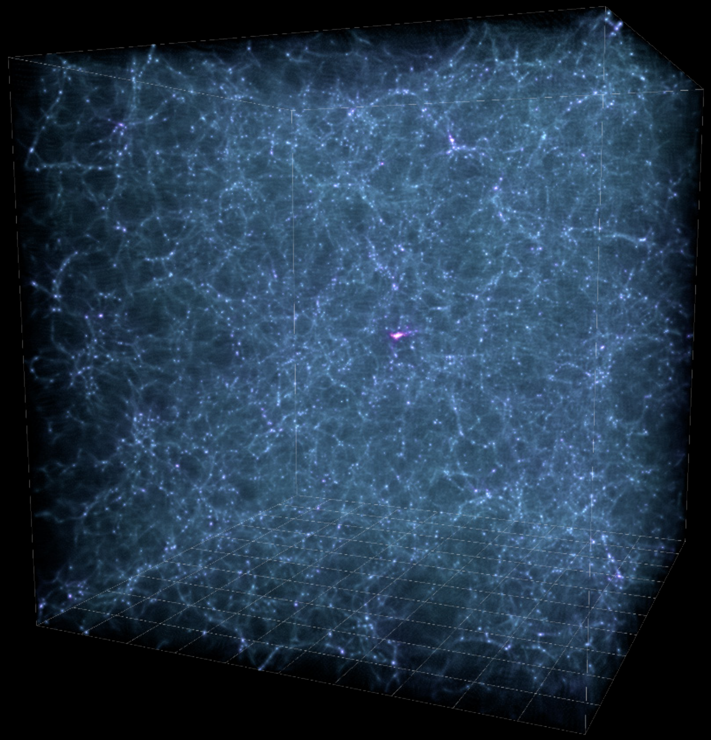
What's in a redshift?

LSST-FRANCE Meeting – June 9th, 2023 – LPSC, Grenoble

Vincent Reverdy



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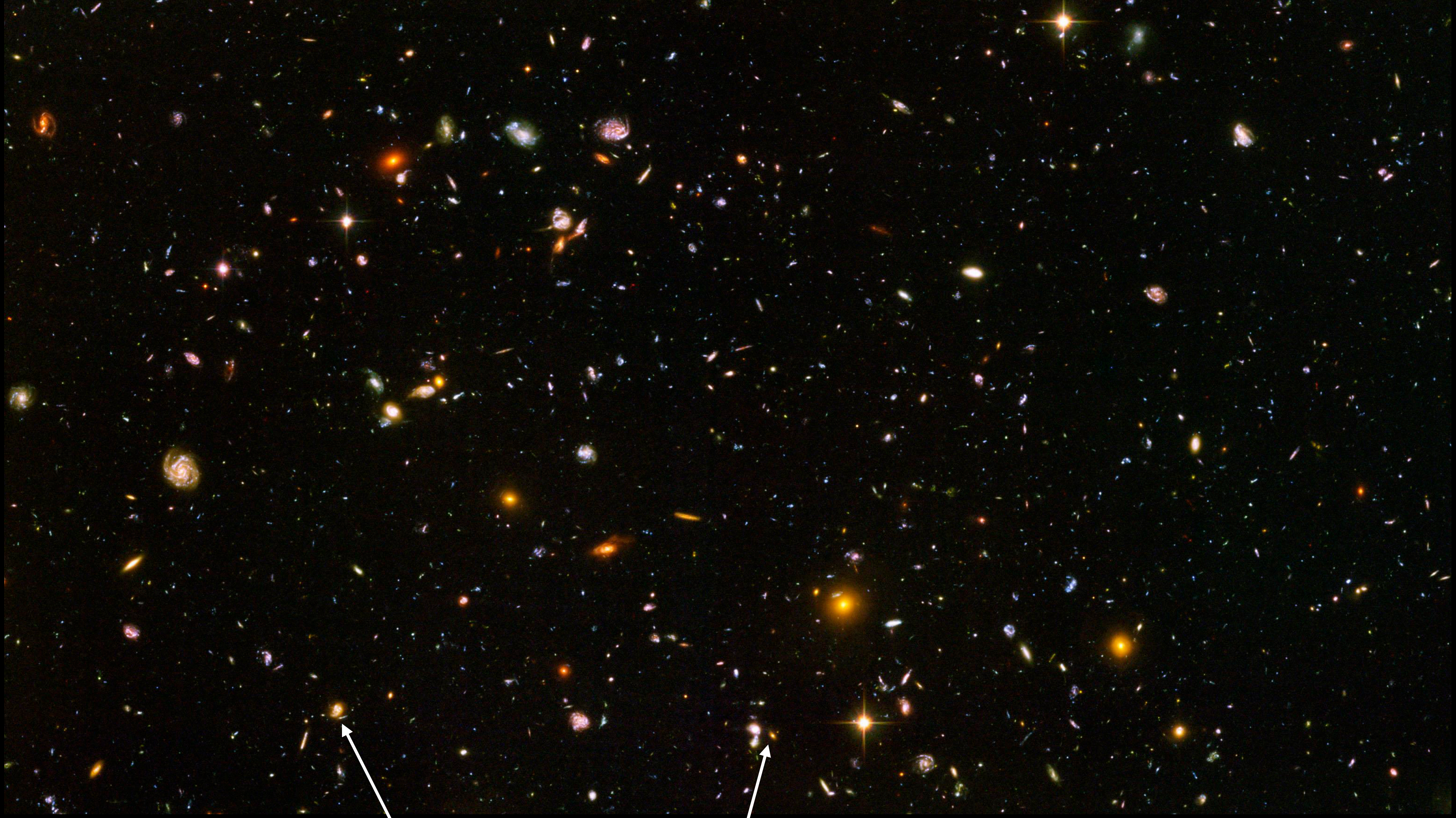
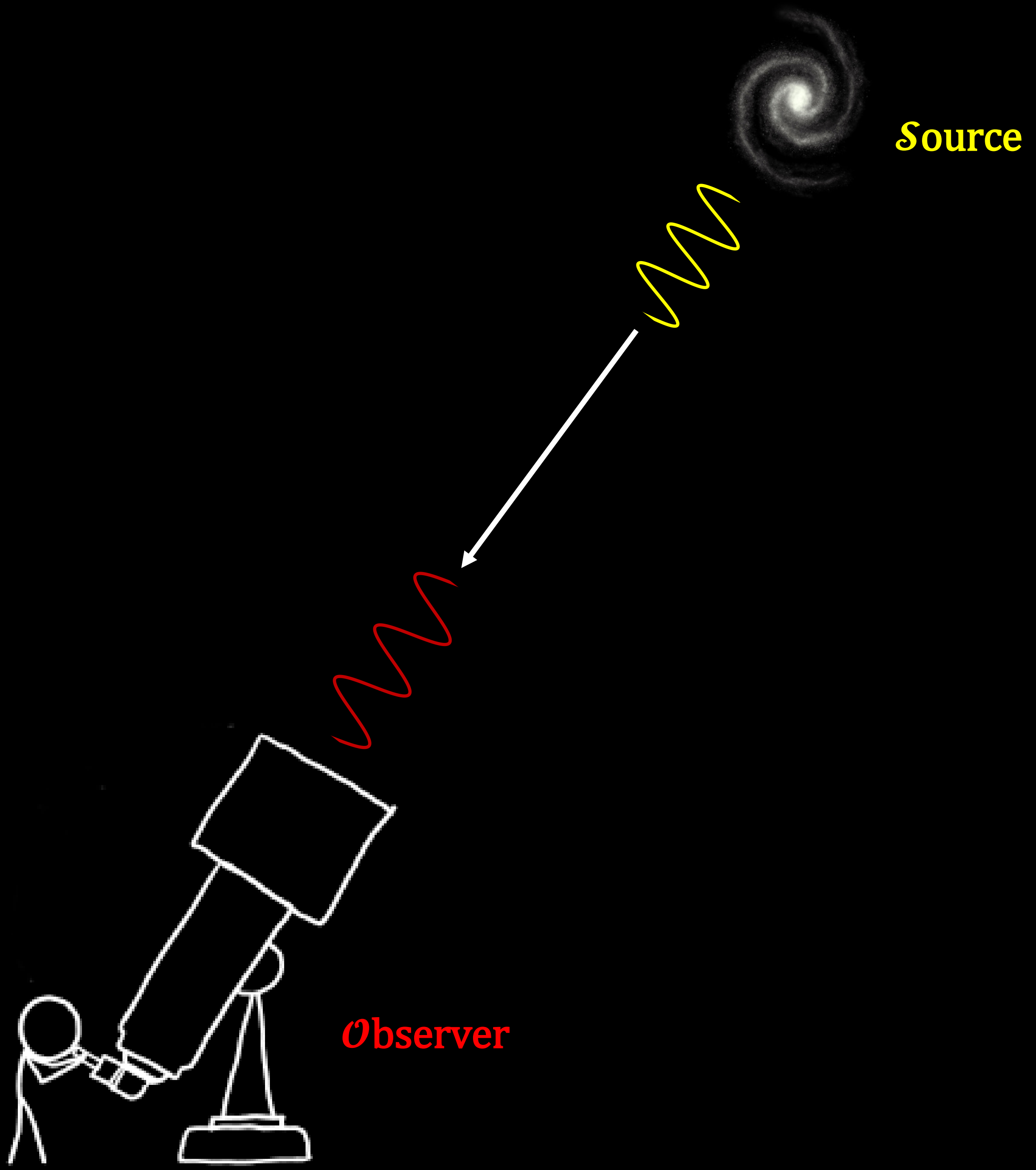
How do we know what we know about the Universe?



$(\alpha_1, \delta_1, z_1)$ $(\alpha_2, \delta_2, z_2)$

Right ascension (RA) Declination (Dec) Redshift



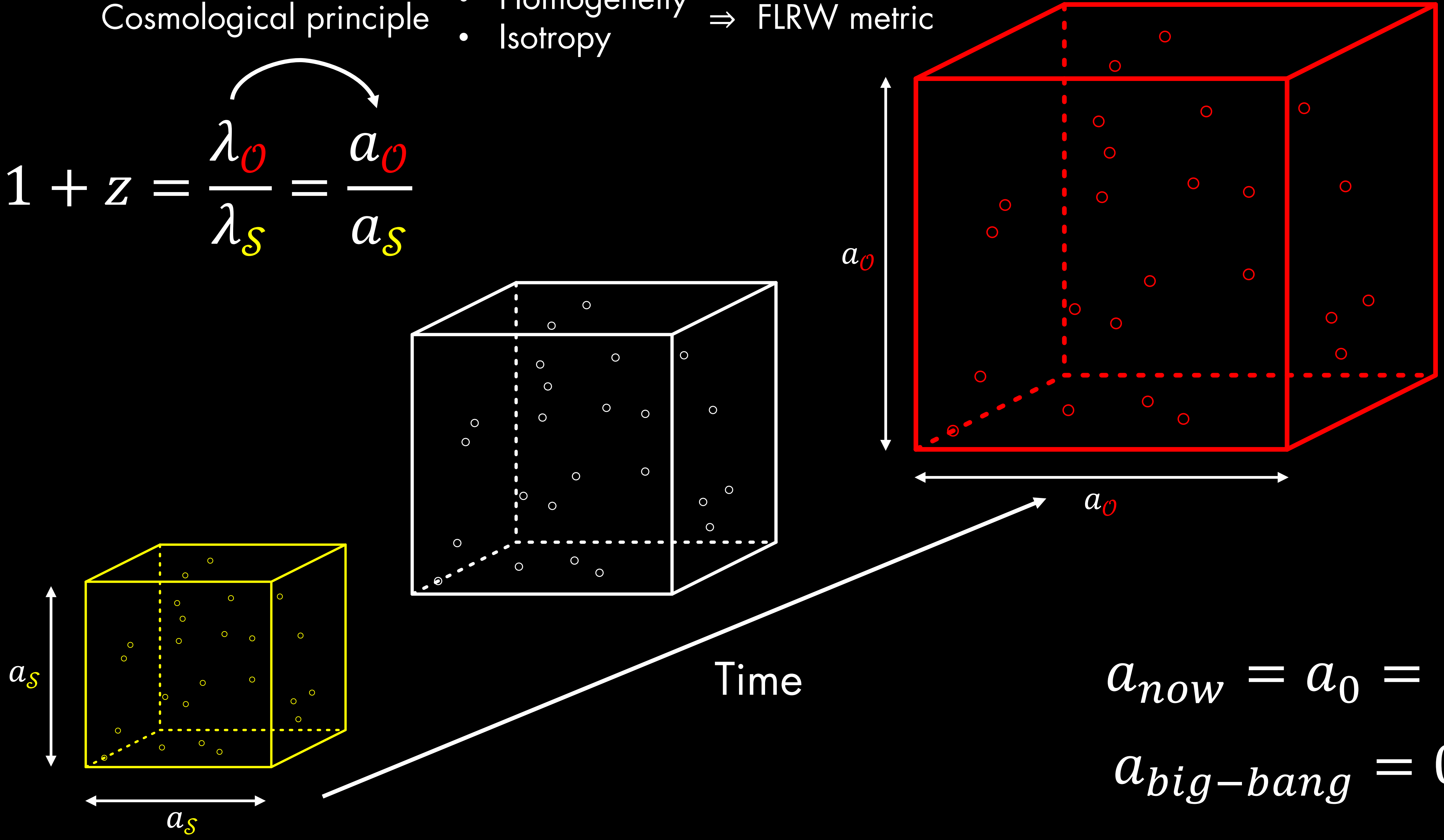
$$z = \frac{\lambda_O - \lambda_S}{\lambda_S}$$

$$1 + z = \frac{\lambda_O}{\lambda_S}$$

Cosmological principle
- Homogeneity
- Isotropy

$\Rightarrow$ FLRW metric

$$1 + z = \frac{\lambda_O}{\lambda_S} = \frac{a_O}{a_S}$$

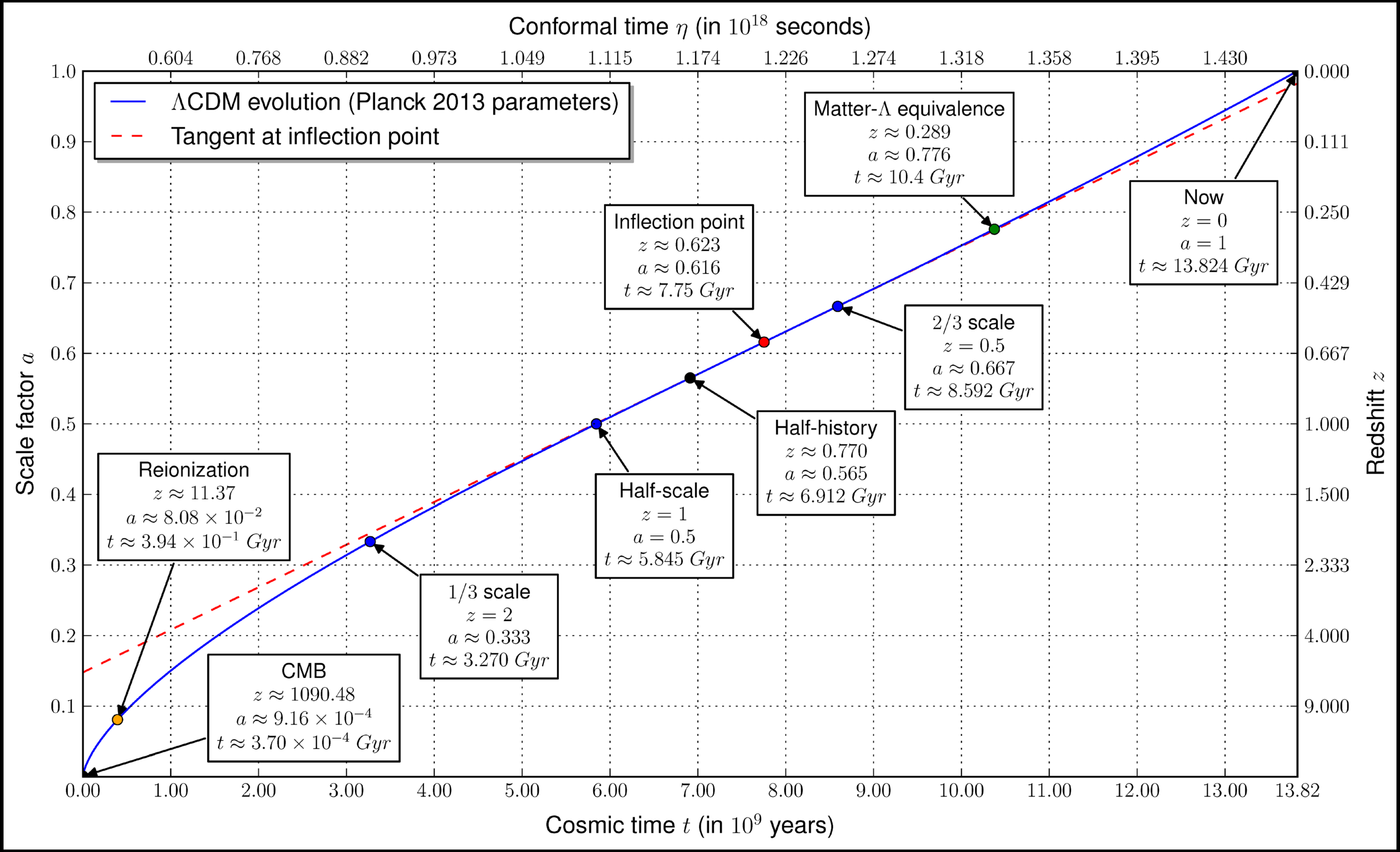


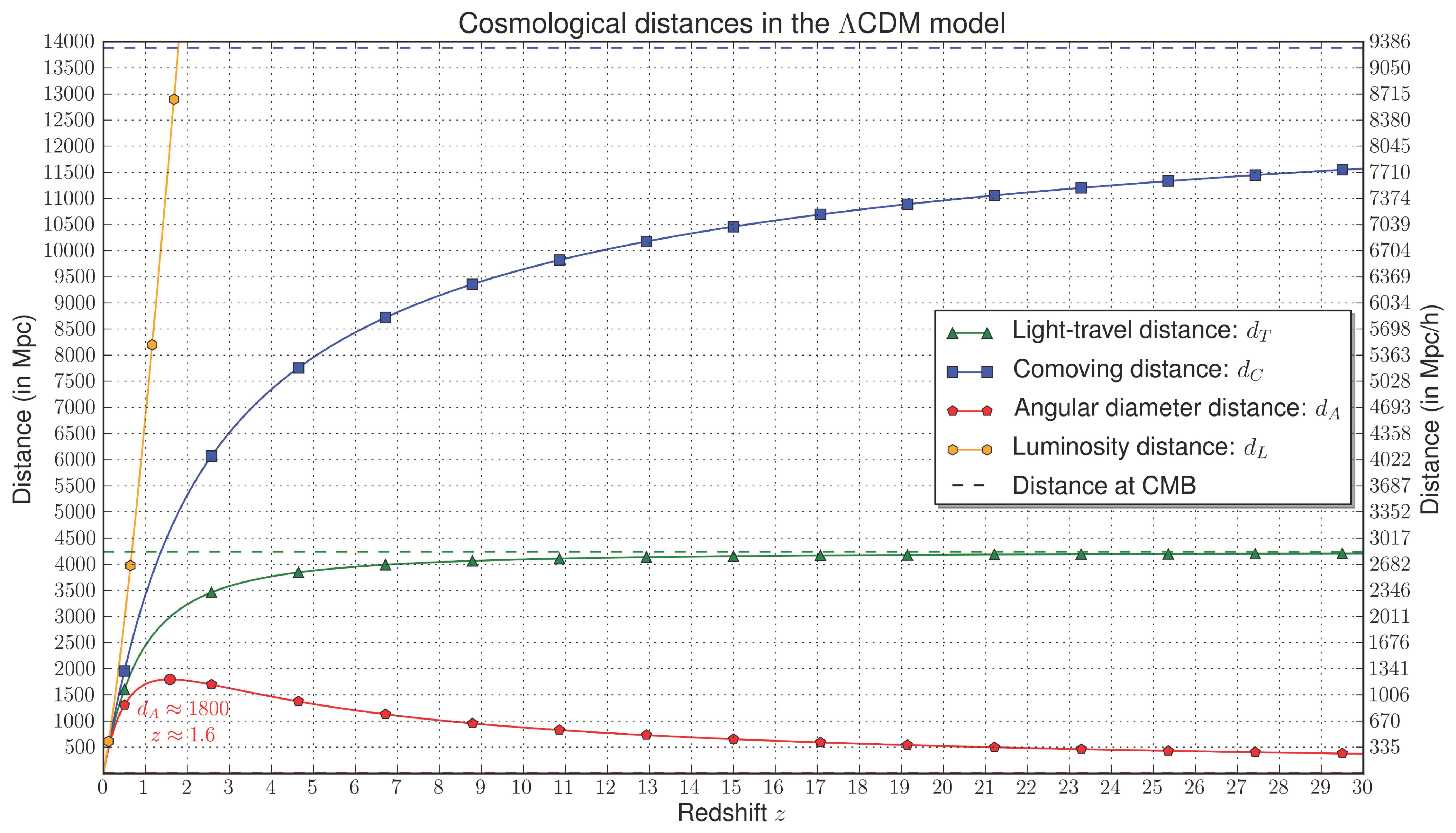
$$a_{now} = a_0 = 1$$

$$a_{big-bang} = 0$$

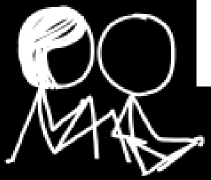
$$z \Longrightarrow a \Longrightarrow d_X \Longrightarrow (\alpha, \delta, d_X)$$

Cosmological parameters Distance (of some kind X)

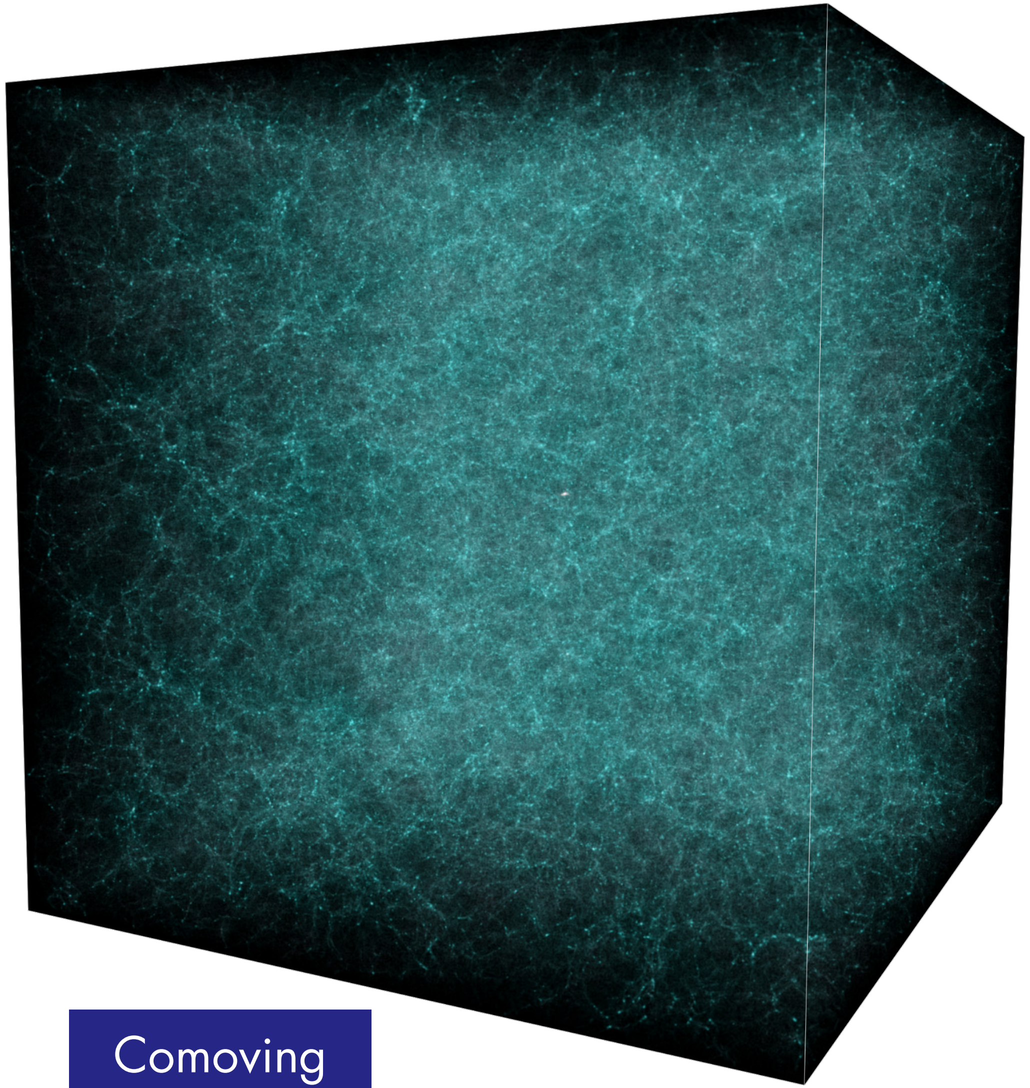




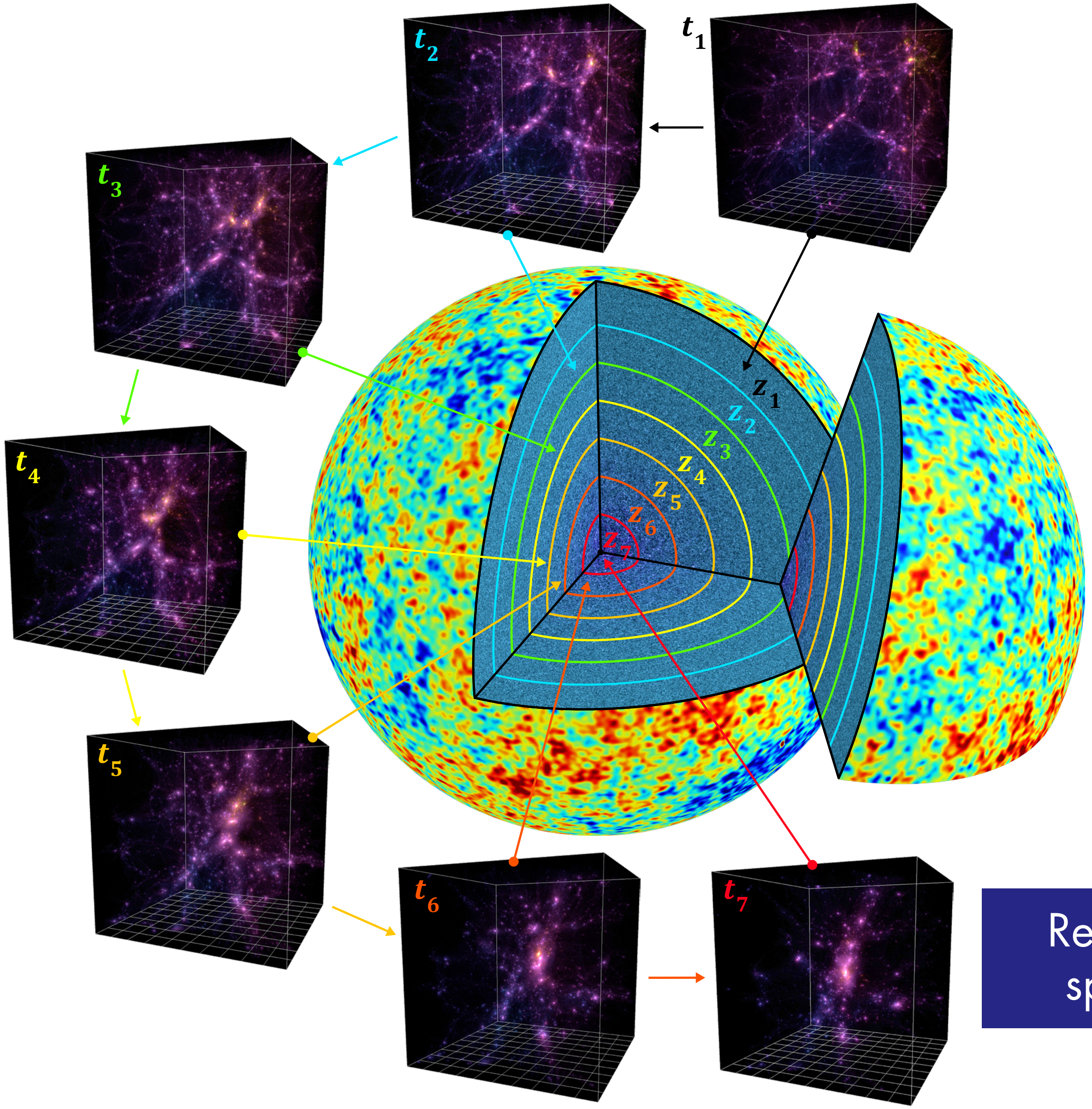
Cosmological distances in the ΛCDM model
Distance (in Mpc)
Distance (in Mpc/h)
Redshift z
14000
13500
13000
12500
12000
11500
11000
10500
10000
9500
9000
8500
8000
7500
7000
6500
6000
5500
5000
4500
4000
3500
3000
2500
2000
1500
1000
500
9386
9050
8715
8380
8045
7710
7374
7039
6704
6369
6034
5698
5363
5028
4693
4358
4022
3687
3352
3017
2682
2346
2011
1676
1341
1006
670
335
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30
Light-travel distance: d_T
Comoving distance: d_C
Angular diameter distance: d_A
Luminosity distance: d_L
Distance at CMB
$d_A \approx 1800$
$z \approx 1.6$



Coordinate spaces (from simulations)



Comoving space

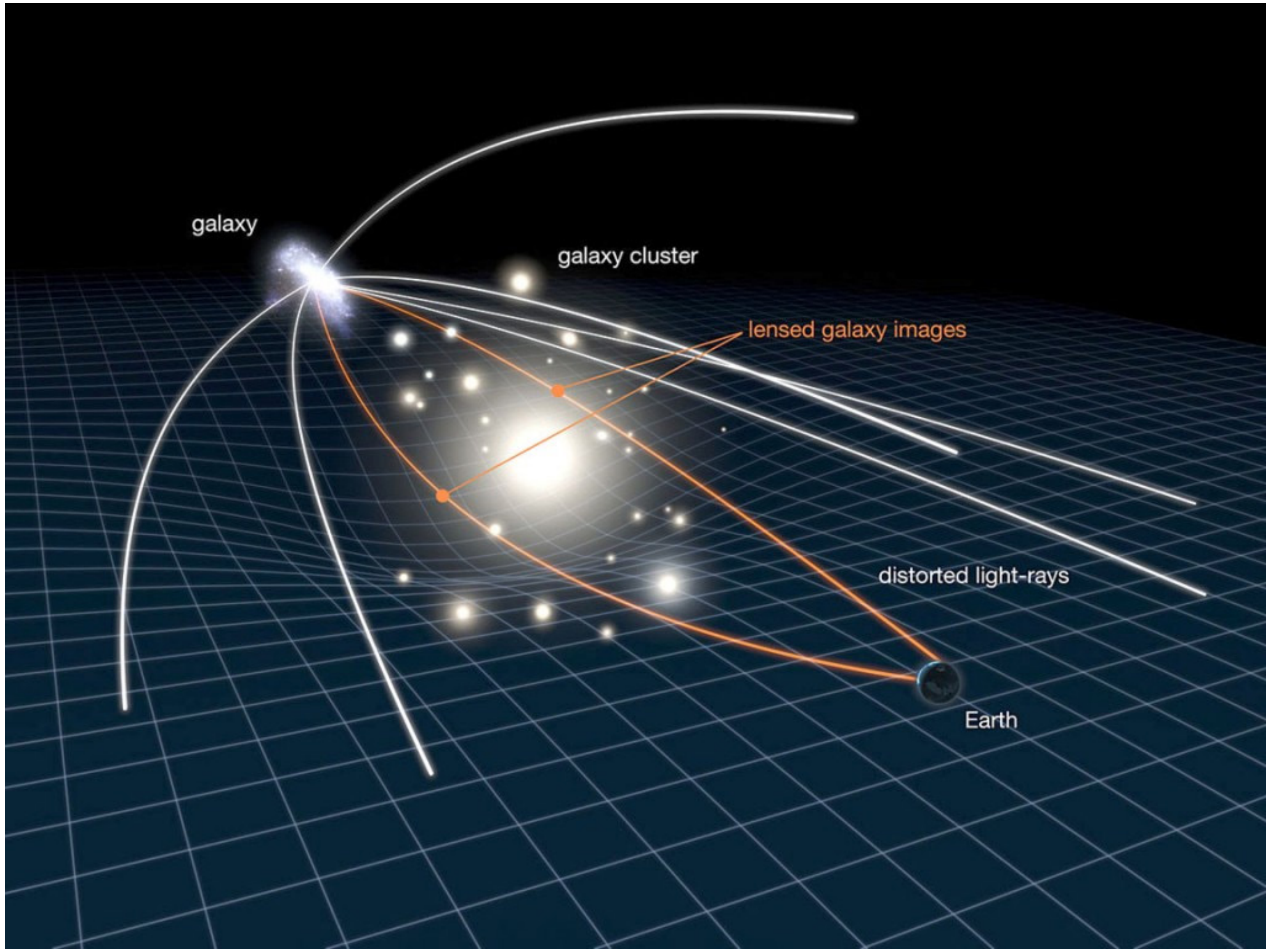


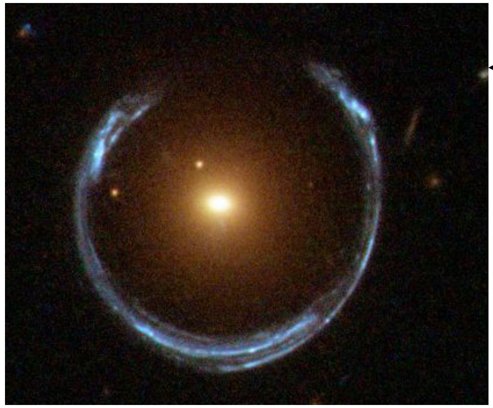
Redshift space

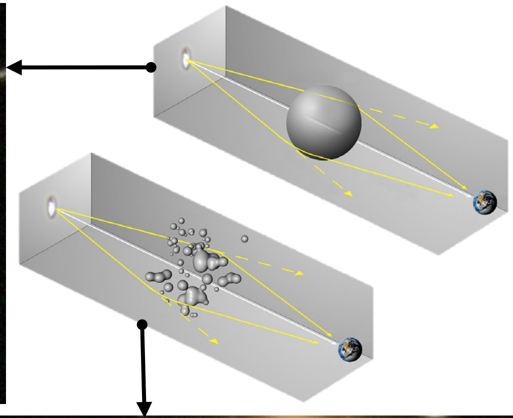
Photons ⟹ Coordinates ⟹ Cosmology

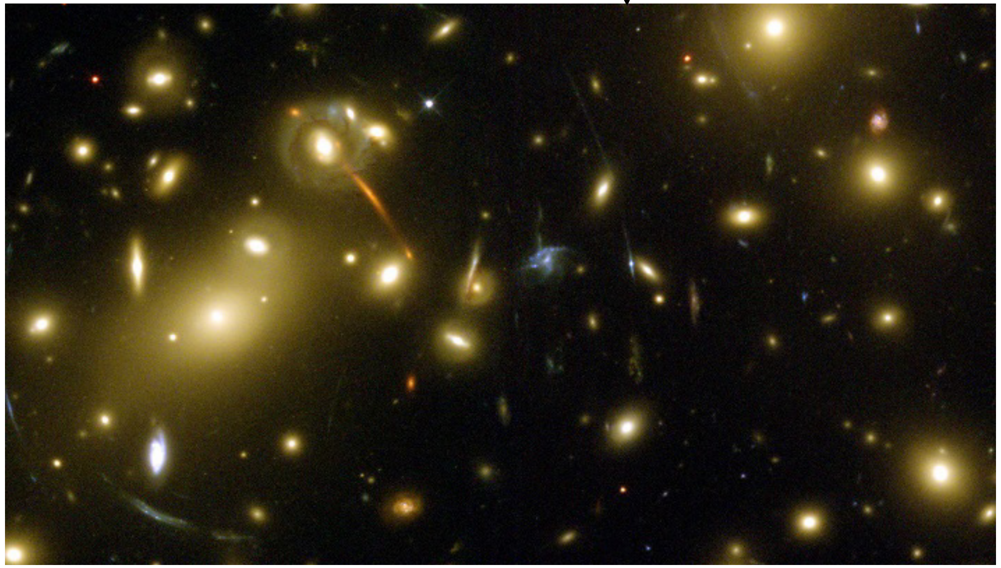
\+ Fancy effects

Effect 1: strong lensing



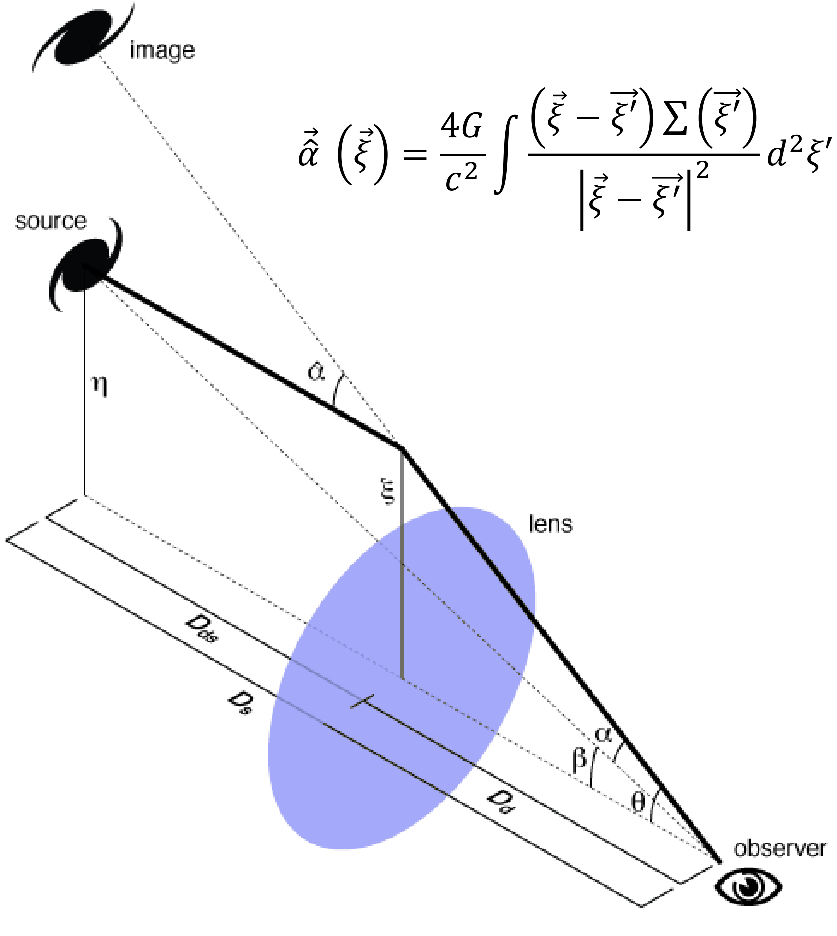




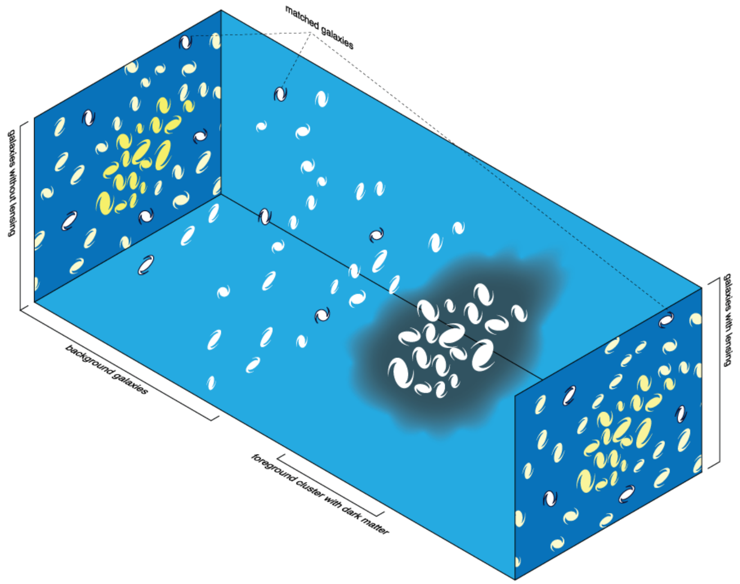


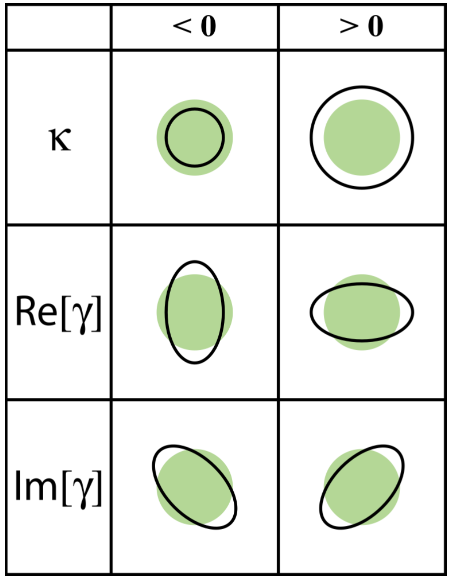
Effect 2: weak lensing

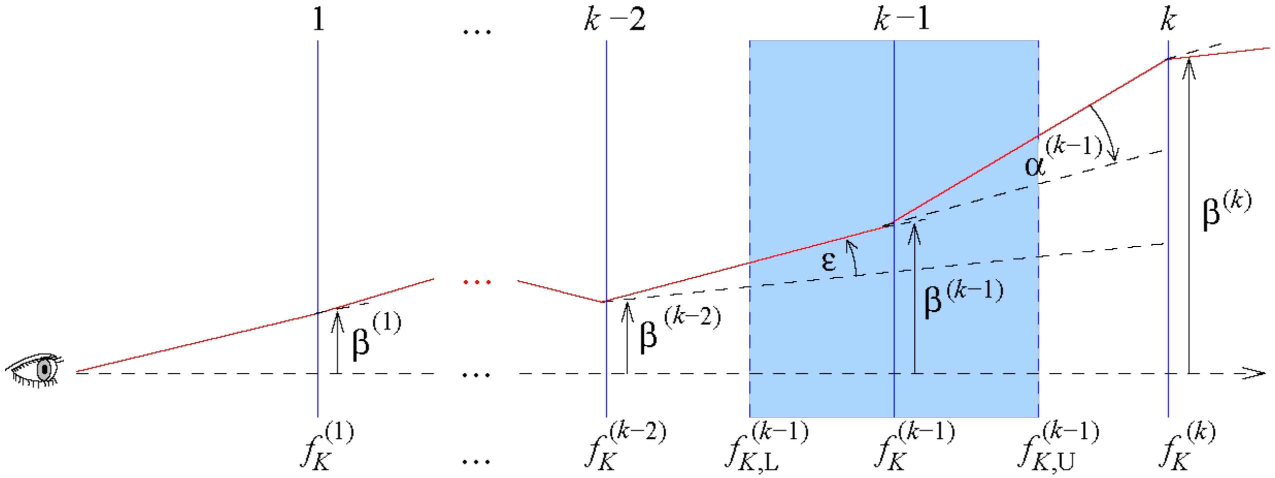
$$\vec{\alpha}\left(\vec{\xi}\right) = \frac{4G}{c^2}\int \frac{\left(\vec{\xi}-\vec{\xi}'\right)\Sigma\left(\vec{\xi}'\right)}{\left|\vec{\xi}-\vec{\xi}'\right|^2}\, d^2\xi'$$



Thin lens approximation

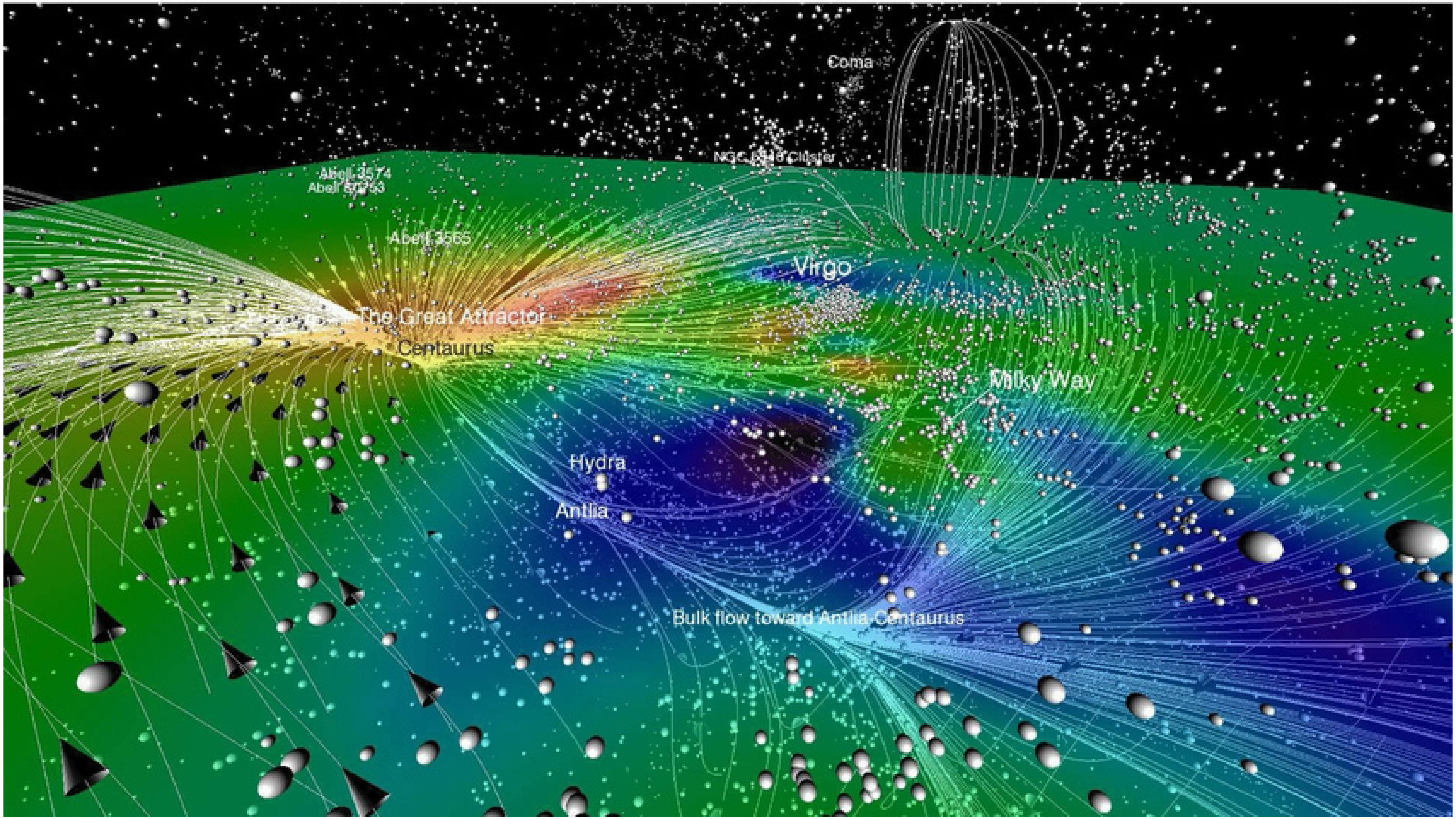




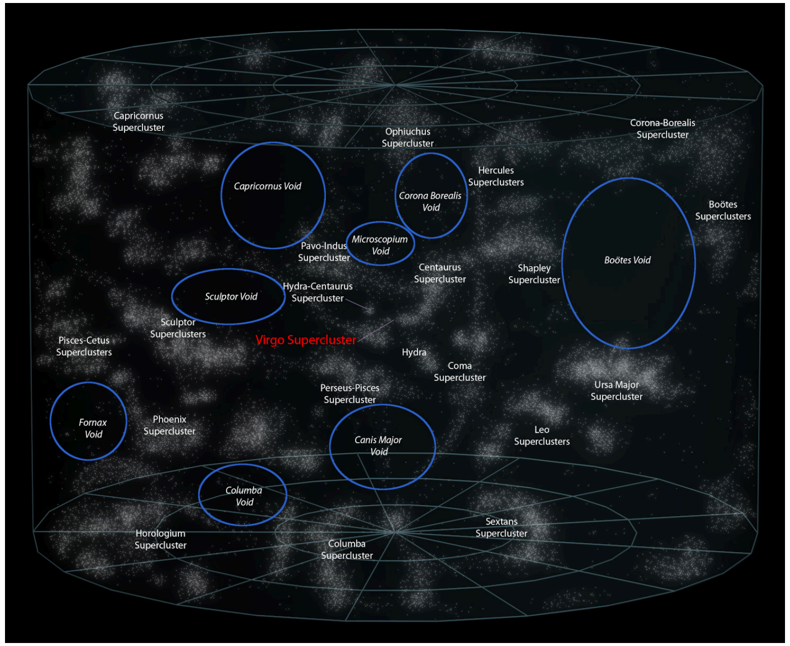


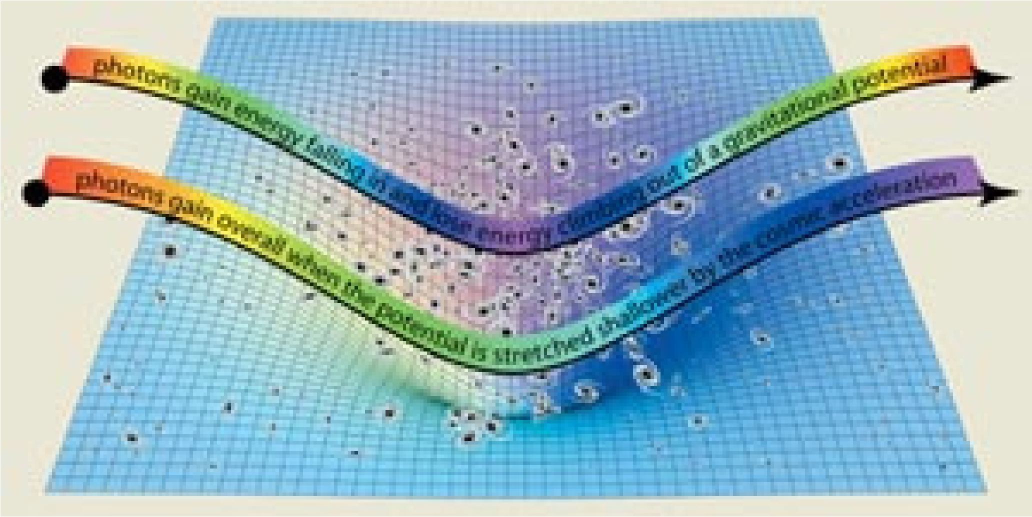
Multi-lens plane approximation

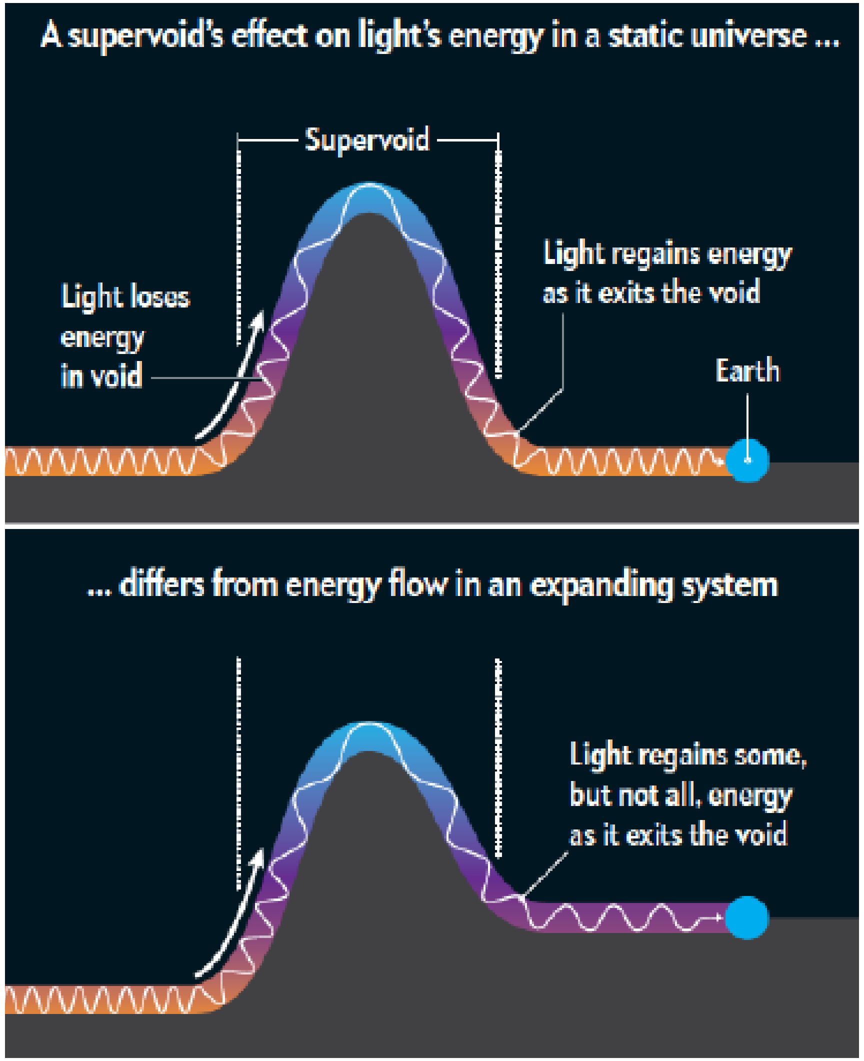
Effect 3: Peculiar velocities



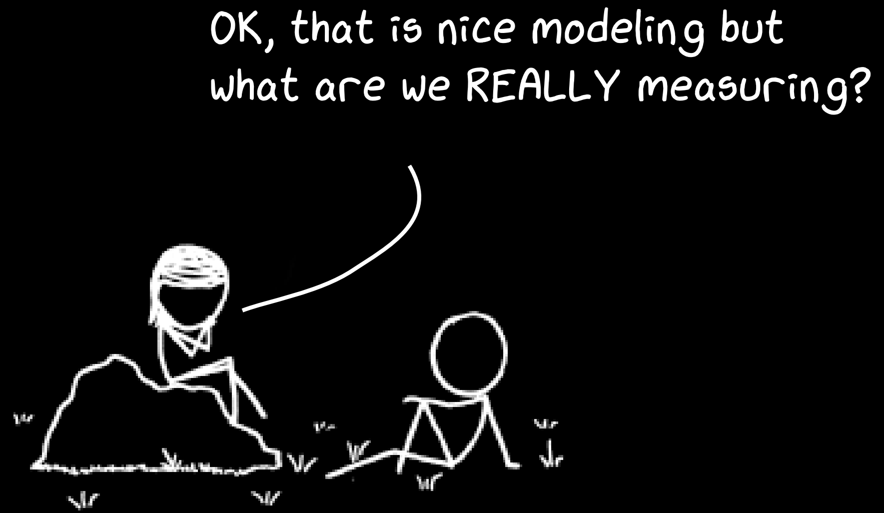
Effect 4: Late-time Integrated Sachs-Wolfe effect (ISW)

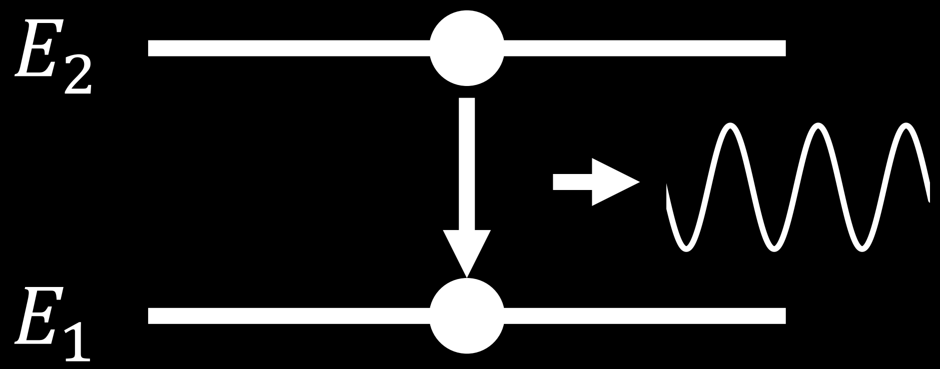




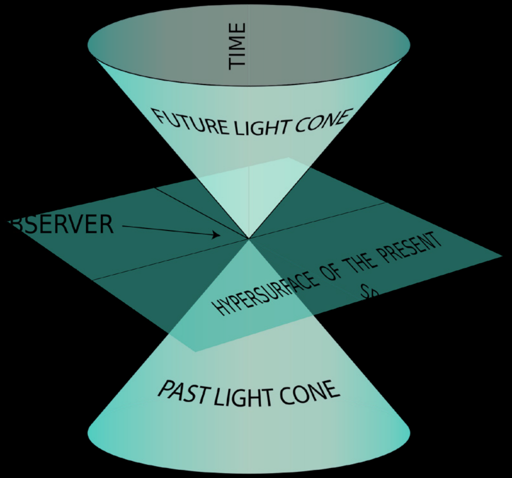


And more... (like Shapiro time delay)


OK, that is nice modeling but
what are we REALLY measuring?

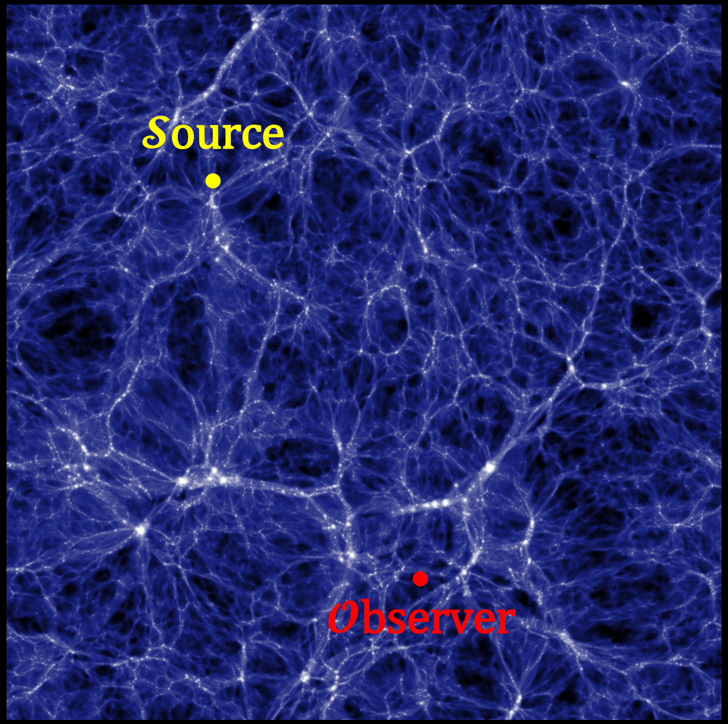


$$E = h\nu$$



At the beginning there was a photon

traveling in a Universe that is neither rigorously homogeneous and isotropic



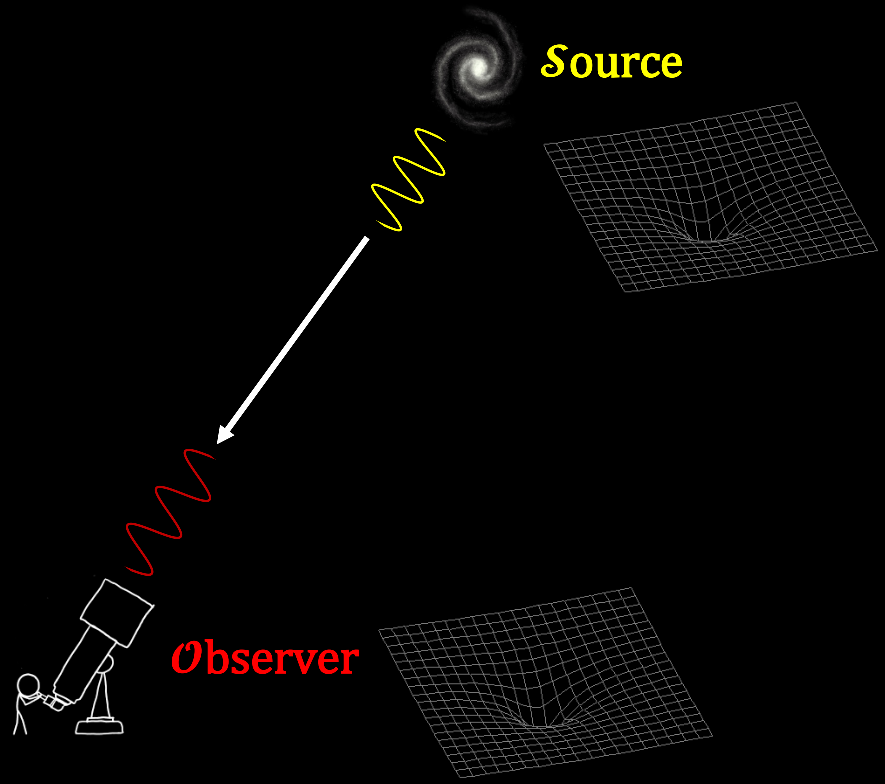
The true expression of redshift

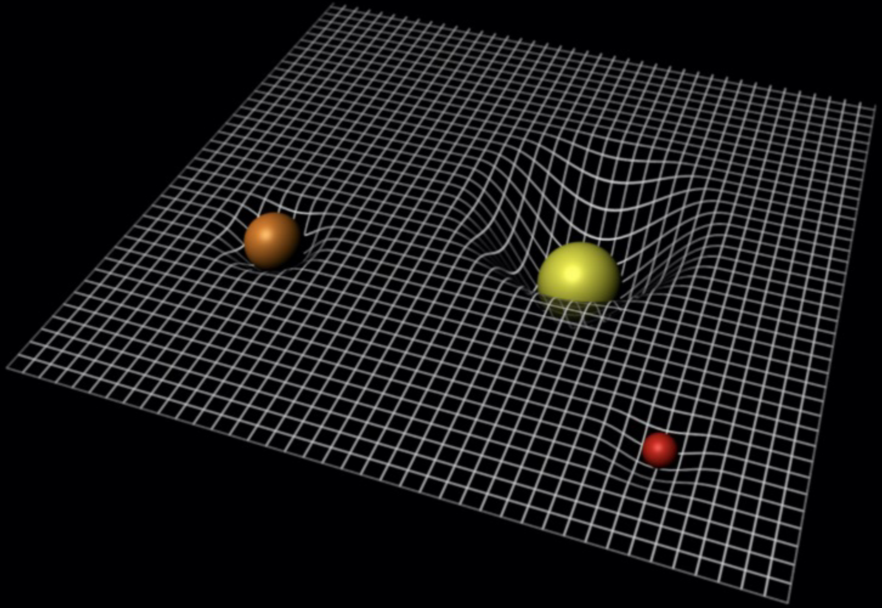
$$1 + z = \frac{h\nu_{\mathcal{S}}}{h\nu_{\mathcal{O}}} = \frac{(k_\alpha u^\alpha)_{\mathcal{S}}}{(k_\alpha u^\alpha)_{\mathcal{O}}} = \frac{\left(g_{\mu\nu} k^\mu u^\nu\right)_{\mathcal{S}}}{\left(g_{\mu\nu} k^\mu u^\nu\right)_{\mathcal{O}}}$$

Metric tensor

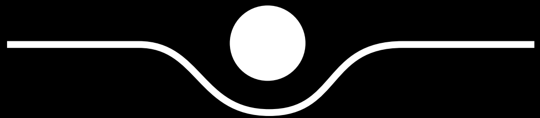
4-velocity of the cosmic fluid

Coordinate derivative $k^\mu = \frac{dx^\mu}{d\lambda}$ regarding the affine parameter λ





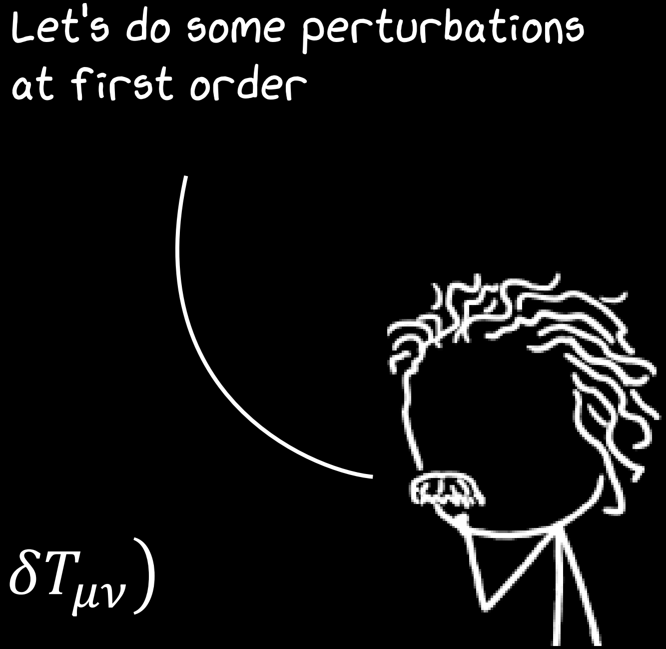
$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$



$$T_{\mu\nu} = \bar{T}_{\mu\nu} + \delta T_{\mu\nu}$$

$$G_{\mu\nu} = \bar{G}_{\mu\nu} + \delta G_{\mu\nu}$$

$$\bar{G}_{\mu\nu} + \delta G_{\mu\nu} = \frac{8\pi G}{c^4}\left(\bar{T}_{\mu\nu} + \delta T_{\mu\nu}\right)$$



Weak-field metric

	Cosmic time	Conformal time
g_{00}	$-c^2\left(1+2\frac{\Psi}{c^2}\right)$	$-a^2c^2\left(1+2\frac{\Psi}{c^2}\right)$
g_{ii}		
g^{00}		
g^{ii}		

Derivatives of the weak-field metric

	Cosmic time	Conformal time
$\frac{\partial g_{00}}{\partial x^0}$	$-2\frac{\partial\Psi}{\partial t}$	$-2aa'c^2-4aa'\Psi-2a^2\frac{\partial\Psi}{\partial\eta}$
$\frac{\partial g_{00}}{\partial x^i}$	$-2\frac{\partial\Psi}{\partial x^i}$	$-2a^2\frac{\partial\Psi}{\partial x^i}$
$\frac{\partial g_{ii}}{\partial x^0}$	$2a\dot{a}-\frac{4a\dot{a}}{c^2}\Phi-\frac{2a^2}{c^2}\frac{\partial\Phi}{\partial t}$	$2aa'-\frac{4aa'}{c^2}\Phi-\frac{2a^2}{c^2}\frac{\partial\Phi}{\partial\eta}$
$\frac{\partial g_{ii}}{\partial x^i}$	$-\frac{2a^2}{c^2}\frac{\partial\Phi}{\partial x^i}$	
$\frac{\partial g_{ii}}{\partial x^j}$	$-\frac{2a^2}{c^2}\frac{\partial\Phi}{\partial x^j}$	

Affine connections at first order

	Cosmic time	Conformal time
Γ^0_{00}	$\frac{1}{c^2}\frac{\partial\Psi}{\partial t}$	$\frac{a'}{a}+\frac{1}{c^2}\frac{\partial\Psi}{\partial\eta}$
Γ^i_{ii}	$-\frac{1}{c^2}\frac{\partial\Phi}{\partial x^i}$	$-\frac{1}{c^2}\frac{\partial\Phi}{\partial x^i}$
Γ^0_{ii}	$\frac{a\dot{a}}{c^2}-\frac{2a\dot{a}}{c^4}(\Psi+\Phi)-\frac{a^2}{c^4}\frac{\partial\Phi}{\partial t}$	$\frac{a'}{ac^2}-\frac{2a'}{ac^4}(\Psi+\Phi)-\frac{1}{c^4}\frac{\partial\Phi}{\partial\eta}$
Γ^i_0		
$\Gamma^0_{0i}=$		
$\Gamma^i_{i0}=$		

Geodesics equations in cartesian conformal coordinates

$\mathrm{d}s^2$	$a^2\left[-c^2\left(1+2\frac{\Psi}{c^2}\right)\mathrm{d}\eta^2+\left(1-2\frac{\Phi}{c^2}\right)(\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2)\right]$
$\frac{\mathrm{d}^2\eta}{\mathrm{d}\lambda^2}$	$-\left(\frac{a'}{a}+\frac{1}{c^2}\frac{\partial\Psi}{\partial\eta}\right)\left(\frac{\mathrm{d}\eta}{\mathrm{d}\lambda}\right)^2-\frac{2}{c^2}\frac{\mathrm{d}\eta}{\mathrm{d}\lambda}\left[\frac{\partial\Psi}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}\lambda}+\frac{\partial\Psi}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}\lambda}+\frac{\partial\Psi}{\partial z}\frac{\mathrm{d}z}{\mathrm{d}\lambda}\right]$ $-\left[\frac{a'}{ac^2}-\frac{2a'}{ac^4}(\Psi+\Phi)-\frac{1}{c^4}\frac{\partial\Phi}{\partial\eta}\right]\left[\left(\frac{\mathrm{d}x}{\mathrm{d}\lambda}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}\lambda}\right)^2+\left(\frac{\mathrm{d}z}{\mathrm{d}\lambda}\right)^2\right]$
$\frac{\mathrm{d}^2x}{\mathrm{d}\lambda^2}$	$-\frac{\partial\Psi}{\partial x}\left(\frac{\mathrm{d}\eta}{\mathrm{d}\lambda}\right)^2-2\left(\frac{a'}{a}-\frac{1}{c^2}\frac{\partial\Phi}{\partial\eta}\right)\frac{\mathrm{d}\eta}{\mathrm{d}\lambda}\frac{\mathrm{d}x}{\mathrm{d}\lambda}+\frac{2}{c^2}\frac{\mathrm{d}x}{\mathrm{d}\lambda}\left[\frac{\partial\Phi}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}\lambda}+\frac{\partial\Phi}{\partial z}\frac{\mathrm{d}z}{\mathrm{d}\lambda}\right]$ $+\frac{1}{c^2}\frac{\partial\Phi}{\partial x}\left[\left(\frac{\mathrm{d}x}{\mathrm{d}\lambda}\right)^2-\left(\frac{\mathrm{d}y}{\mathrm{d}\lambda}\right)^2-\left(\frac{\mathrm{d}z}{\mathrm{d}\lambda}\right)^2\right]$
$\frac{\mathrm{d}^2y}{\mathrm{d}\lambda^2}$	$-\frac{\partial\Psi}{\partial y}\left(\frac{\mathrm{d}\eta}{\mathrm{d}\lambda}\right)^2-2\left(\frac{a'}{a}-\frac{1}{c^2}\frac{\partial\Phi}{\partial\eta}\right)\frac{\mathrm{d}\eta}{\mathrm{d}\lambda}\frac{\mathrm{d}y}{\mathrm{d}\lambda}+\frac{2}{c^2}\frac{\mathrm{d}y}{\mathrm{d}\lambda}\left[\frac{\partial\Phi}{\partial z}\frac{\mathrm{d}z}{\mathrm{d}\lambda}+\frac{\partial\Phi}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}\lambda}\right]$ $+\frac{1}{c^2}\frac{\partial\Phi}{\partial y}\left[\left(\frac{\mathrm{d}y}{\mathrm{d}\lambda}\right)^2-\left(\frac{\mathrm{d}z}{\mathrm{d}\lambda}\right)^2-\left(\frac{\mathrm{d}x}{\mathrm{d}\lambda}\right)^2\right]$
	$\frac{\partial\Psi}{\partial}\left(\frac{\mathrm{d}\eta}{}\right)^2 \ldots \left(\frac{a'}{}\ \frac{1}{}\frac{\partial\Phi}{}\right)\frac{\mathrm{d}\eta}{}\frac{\mathrm{d}z}{} \ldots \frac{2}{}\frac{\mathrm{d}z}{}\left[\frac{\partial\Phi}{}\frac{\mathrm{d}x}{} \ldots \frac{\partial\Phi}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}\lambda}\right]$

Curvature scalar

R	$\frac{6\dot{a}^2}{a^2c^2}+\frac{6\ddot{a}}{ac^2}-\frac{12}{c^4}\left(\frac{\dot{a}^2}{a^2}+\frac{\ddot{a}}{a}\right)\Psi-\frac{6\dot{a}}{ac^4}\left(\ldots\right.$ $-\frac{6}{c^4}\frac{\partial^2\Phi}{\partial t^2}-\frac{2}{a^2c^2}\nabla^2\Psi+\frac{4}{a^2c^2}\nabla^2\Phi$
$\frac{1}{2}g_{00}R$	$-\frac{3\dot{a}^2}{a^2}-\frac{3\ddot{a}}{a}+\frac{3\dot{a}}{ac^2}\left(\frac{\partial\Psi}{\partial t}+4\frac{\partial\Phi}{\partial t}\right)+\frac{3}{c^2}\frac{\partial^2\Phi}{\partial t^2}+\ldots$
$\frac{1}{2}g_{ii}R$	$\frac{3\dot{a}^2}{c^2}+\frac{3a\ddot{a}}{c^2}-\frac{6}{c^4}\left(\dot{a}^2+a\ddot{a}\right)(\Psi+\Phi)-\frac{3a\dot{a}}{c^4}\ldots$ $-\frac{3a^2}{c^4}\frac{\partial^2\Phi}{\partial t^2}-\frac{1}{c^2}\nabla^2\Psi+\frac{2}{c^2}\nabla^2\Phi$

T_{00}	
T_{ii}	
$T_{0i}=T_{i0}$	
$T_{ij}=T_{ji}$	0

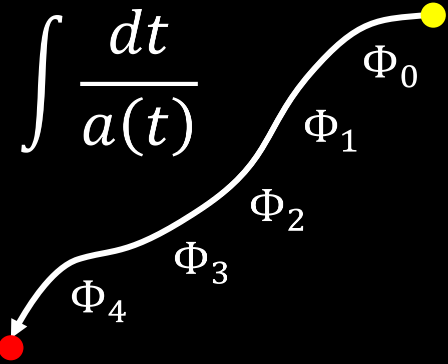
Einstein tensor

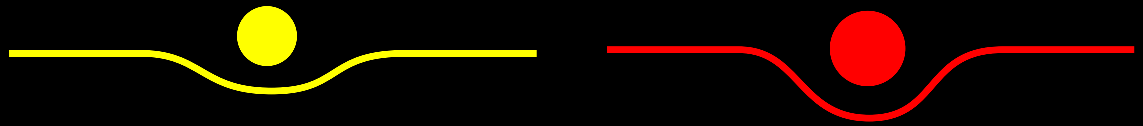
G_{00}	$\frac{3\dot{a}^2}{a^2}-\frac{6\dot{a}}{ac^2}\frac{\partial\Phi}{\partial t}+\frac{2}{a^2}\nabla^2\Phi$
G_{ii}	$-\frac{2a\ddot{a}}{c^2}-\frac{\dot{a}^2}{c^2}+\frac{2}{c^4}\left(\dot{a}^2+2a\ddot{a}\right)(\Psi+\Phi)+\frac{2a\dot{a}}{c^4}\left(\frac{\partial\Psi}{\partial t}+3\frac{\partial\Phi}{\partial t}\right)$ $+\frac{2a^2}{c^4}\frac{\partial^2\Phi}{\partial t^2}+\frac{1}{c^2}\left(\nabla^2\Psi-\frac{\partial^2\Psi}{\partial x^{i^2}}-\nabla^2\Phi+\frac{\partial^2\Phi}{\partial x^{i^2}}\right)$
$G_{0i}=G_{i0}$	$\frac{2\dot{a}}{ac^2}\frac{\partial\Psi}{\partial x^i}+\frac{2}{c^2}\frac{\partial^2\Phi}{\partial x^i\partial t}$
$G_{ij}=G_{ji}$	$-\frac{1}{c^2}\frac{\partial^2\Psi}{\partial x^i\partial x^j}+\frac{1}{c^2}\frac{\partial^2\Phi}{\partial x^i\partial x^j}$

Ricci curvature tensor

R_{00}	$-\frac{3\ddot{a}}{a}+\frac{3\dot{a}}{ac^2}\left(\frac{\partial\Psi}{\partial t}+2\frac{\partial\Phi}{\partial t}\right)+\frac{3}{c^2}\frac{\partial^2\Phi}{\partial t^2}+\frac{1}{a^2}\nabla^2\Psi$
R_{ii}	$\frac{a\ddot{a}}{c^2}+\frac{2\dot{a}^2}{c^2}-\left(\frac{2a\ddot{a}}{c^4}+\frac{4\dot{a}^2}{c^4}\right)(\Psi+\Phi)-\frac{a\dot{a}}{c^4}\left(\frac{\partial\Psi}{\partial t}+6\frac{\partial\Phi}{\partial t}\right)$ $-\frac{a^2}{c^4}\frac{\partial^2\Phi}{\partial t^2}+\frac{1}{c^2}\left(-\frac{\partial^2\Psi}{\partial x^{i^2}}+\frac{\partial^2\Phi}{\partial x^{i^2}}+\nabla^2\Phi\right)$
$R_{0i}=R_{i0}$	$\frac{2\dot{a}}{ac^2}\frac{\partial\Psi}{\partial x^i}+\frac{2}{c^2}\frac{\partial^2\Phi}{\partial x^i\partial t}$
$R_{ij}=R_{ji}$	$-\frac{1}{c^2}\frac{\partial^2\Psi}{\partial x^i\partial x^j}+\frac{1}{c^2}\frac{\partial^2\Phi}{\partial x^i\partial x^j}$

Conformal time $\eta = \int \frac{dt}{a(t)}$





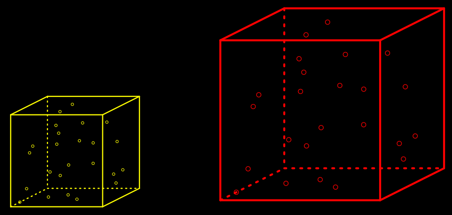
Difference of local potentials

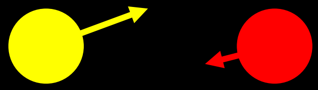
Integral of the potential

$$z = \frac{a_{\mathcal{O}}}{a_{\mathcal{S}}}\left[1 + \left(\frac{\Phi_{\mathcal{O}} - \Phi_{\mathcal{S}}}{c^2}\right) + \left(\frac{k^i_{\mathcal{O}} v^i_{\mathcal{O}} - k^i_{\mathcal{S}} v^i_{\mathcal{S}}}{c}\right) - \frac{2}{c^2}\int_{\lambda_{\mathcal{S}}}^{\lambda_{\mathcal{O}}} \frac{\partial \Phi}{\partial \eta} d\lambda\right] - 1$$

Ratio of scale factors
(Universe's expansion)

Difference of peculiar velocities





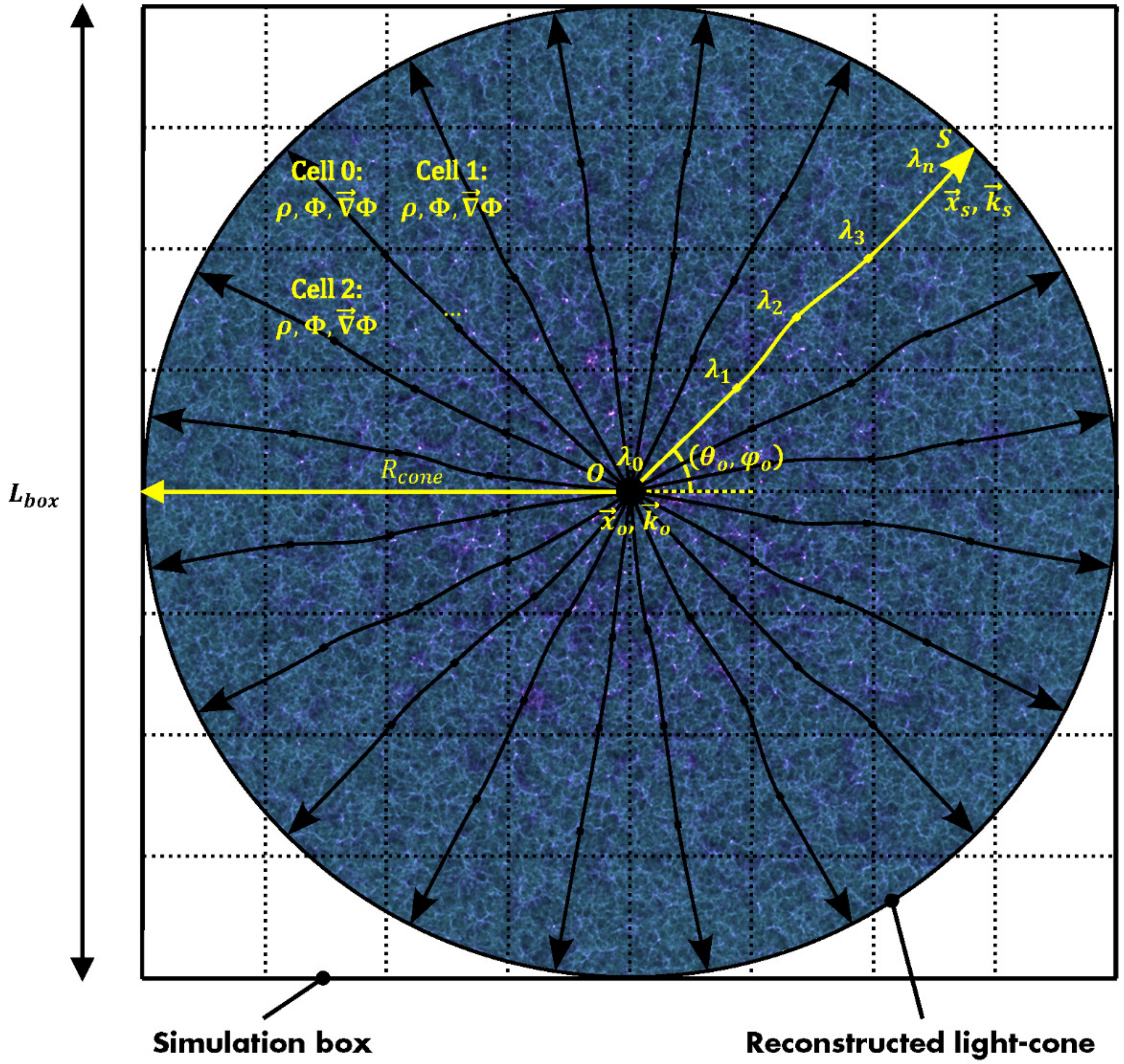
Photons ⇝ Coordinates ⟹ Cosmology

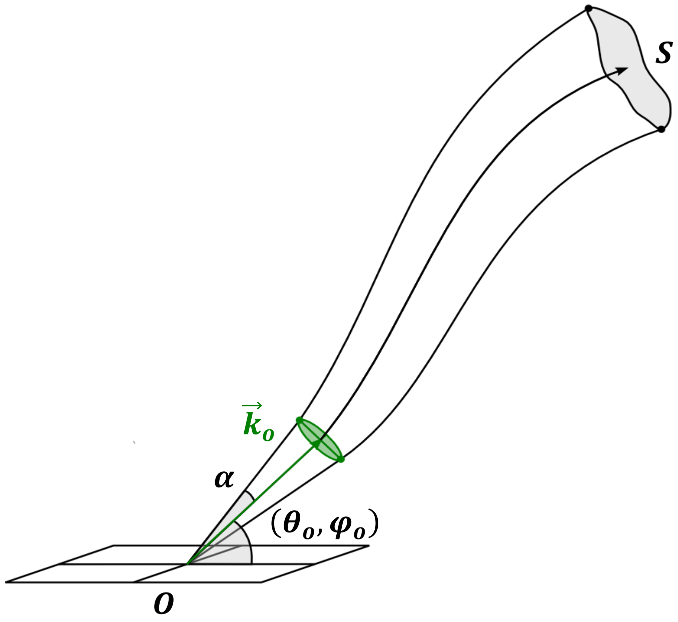
+ ~~Fancy effects~~

Emerging effects

- Strong lensing
- Weak lensing
- Peculiar velocities
- Late-time integrated Sachs-Wolfe effect
- Shapiro time-delay
- and more...

The right way of doing things: getting everything at once!





Geodesics equations at 1st order

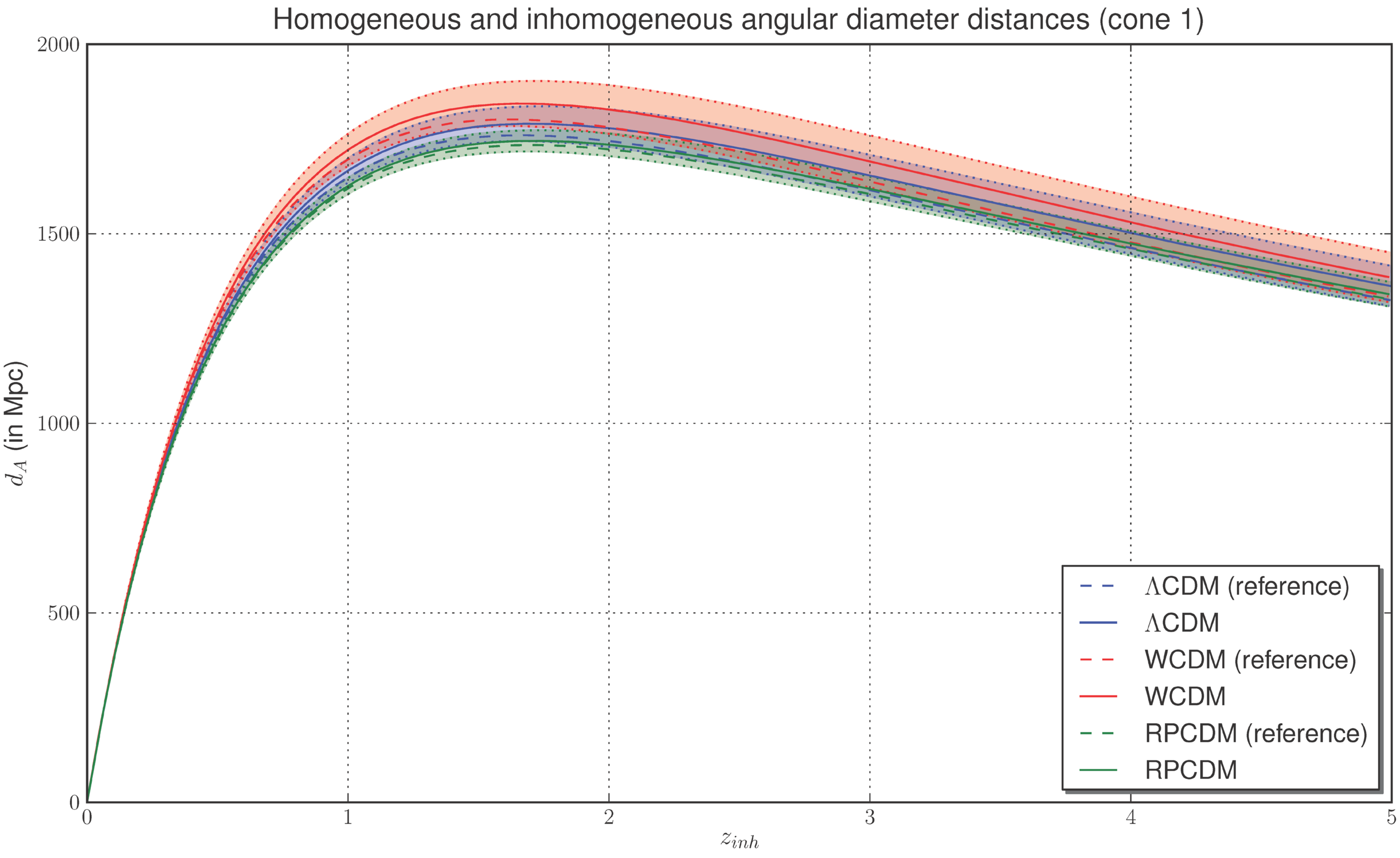
$$\frac{d^2\eta}{d\lambda^2} \approx -\frac{2a'}{a}\frac{d\eta}{d\lambda}\frac{d\eta}{d\lambda} - \frac{2}{c^2}\frac{d\Phi}{d\lambda}\frac{d\eta}{d\lambda} + 2\frac{\partial\Phi}{\partial\eta}\left(\frac{d\eta}{d\lambda}\right)^2$$

$$\frac{d^2x}{d\lambda^2} \approx -\frac{2a'}{a}\frac{d\eta}{d\lambda}\frac{dx}{d\lambda} + \frac{2}{c^2}\frac{d\Phi}{d\lambda}\frac{dx}{d\lambda} - 2\frac{\partial\Phi}{\partial x}\left(\frac{d\eta}{d\lambda}\right)^2$$

$$\frac{d^2y}{d\lambda^2} \approx -\frac{2a'}{a}\frac{d\eta}{d\lambda}\frac{dy}{d\lambda} + \frac{2}{c^2}\frac{d\Phi}{d\lambda}\frac{dy}{d\lambda} - 2\frac{\partial\Phi}{\partial y}\left(\frac{d\eta}{d\lambda}\right)^2$$

$$\frac{d^2z}{d\lambda^2} \approx -\frac{2a'}{a}\frac{d\eta}{d\lambda}\frac{dz}{d\lambda} + \frac{2}{c^2}\frac{d\Phi}{d\lambda}\frac{dz}{d\lambda} - 2\frac{\partial\Phi}{\partial z}\left(\frac{d\eta}{d\lambda}\right)^2$$

Be careful about the interpretation of distances...



$$z = \frac{a_O}{a_S}\left[1 + \left(\frac{\Phi_O - \Phi_S}{c^2}\right) + \left(\frac{k_O^i v_O^i - k_S^i v_S^i}{c}\right) - \frac{2}{c^2}\int_{\lambda_S}^{\lambda_O} \frac{\partial \Phi}{\partial \eta} d\lambda\right] - 1$$