GRAVITATIONAL LENSING BY MATTER CURRENTS

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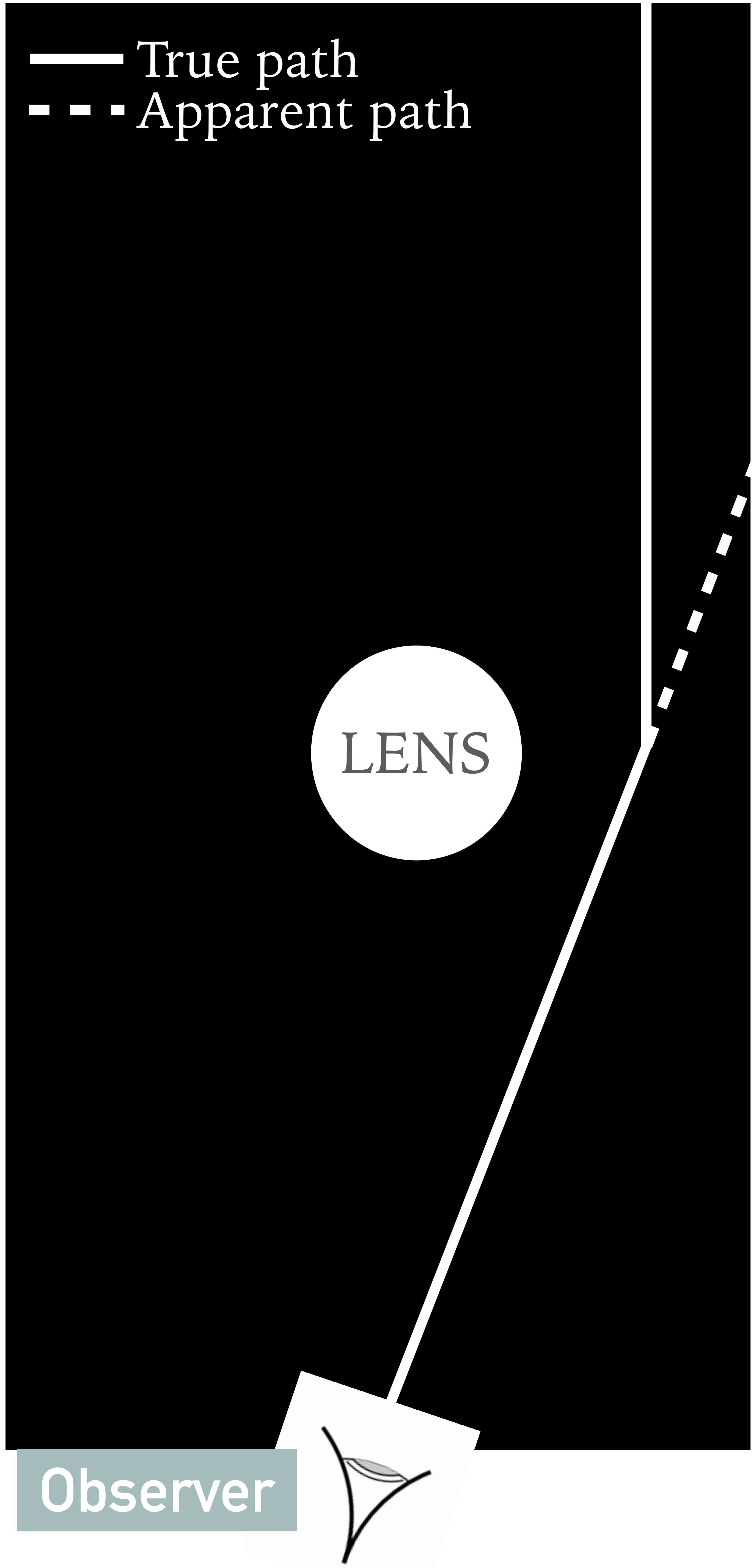
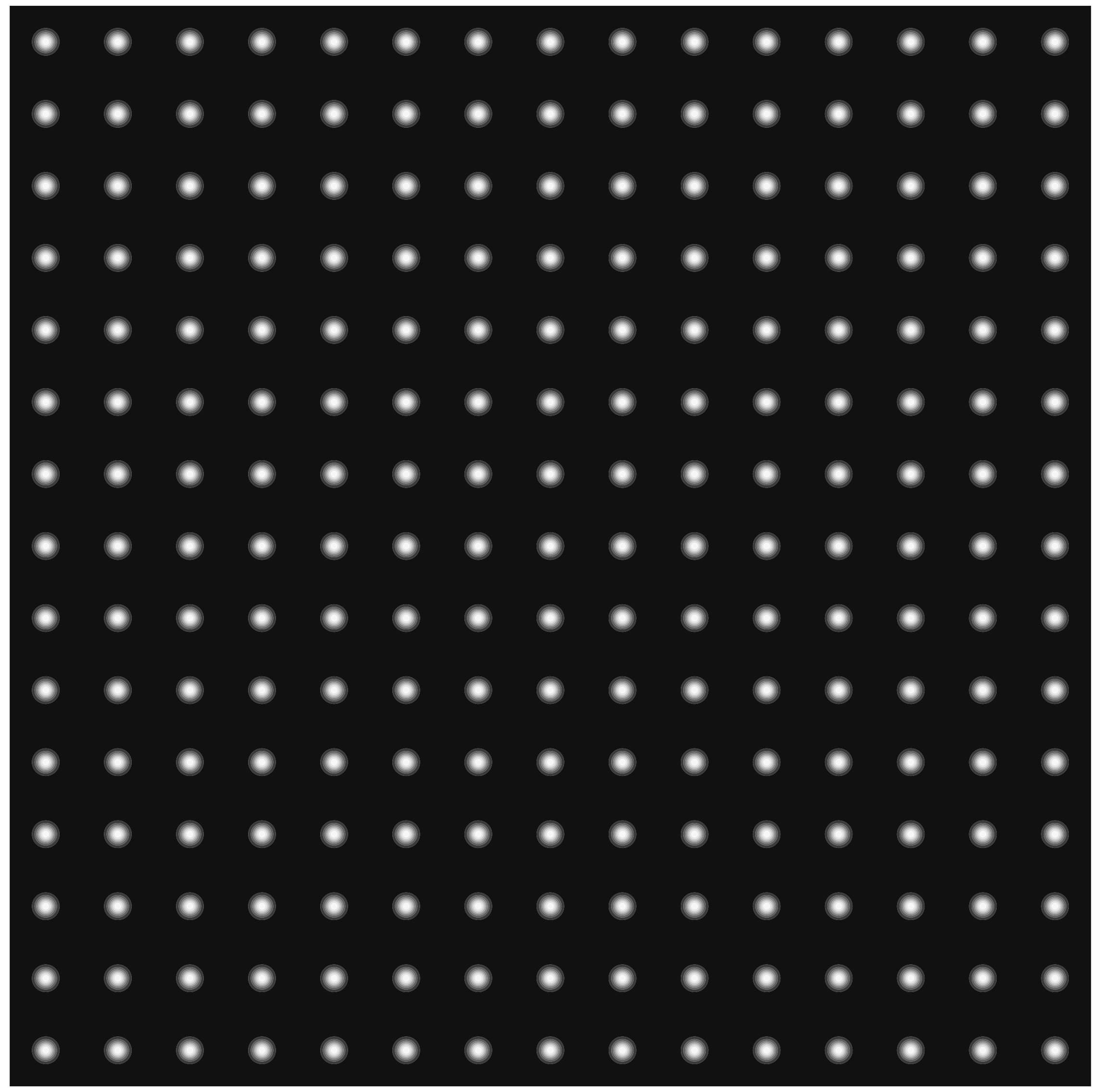
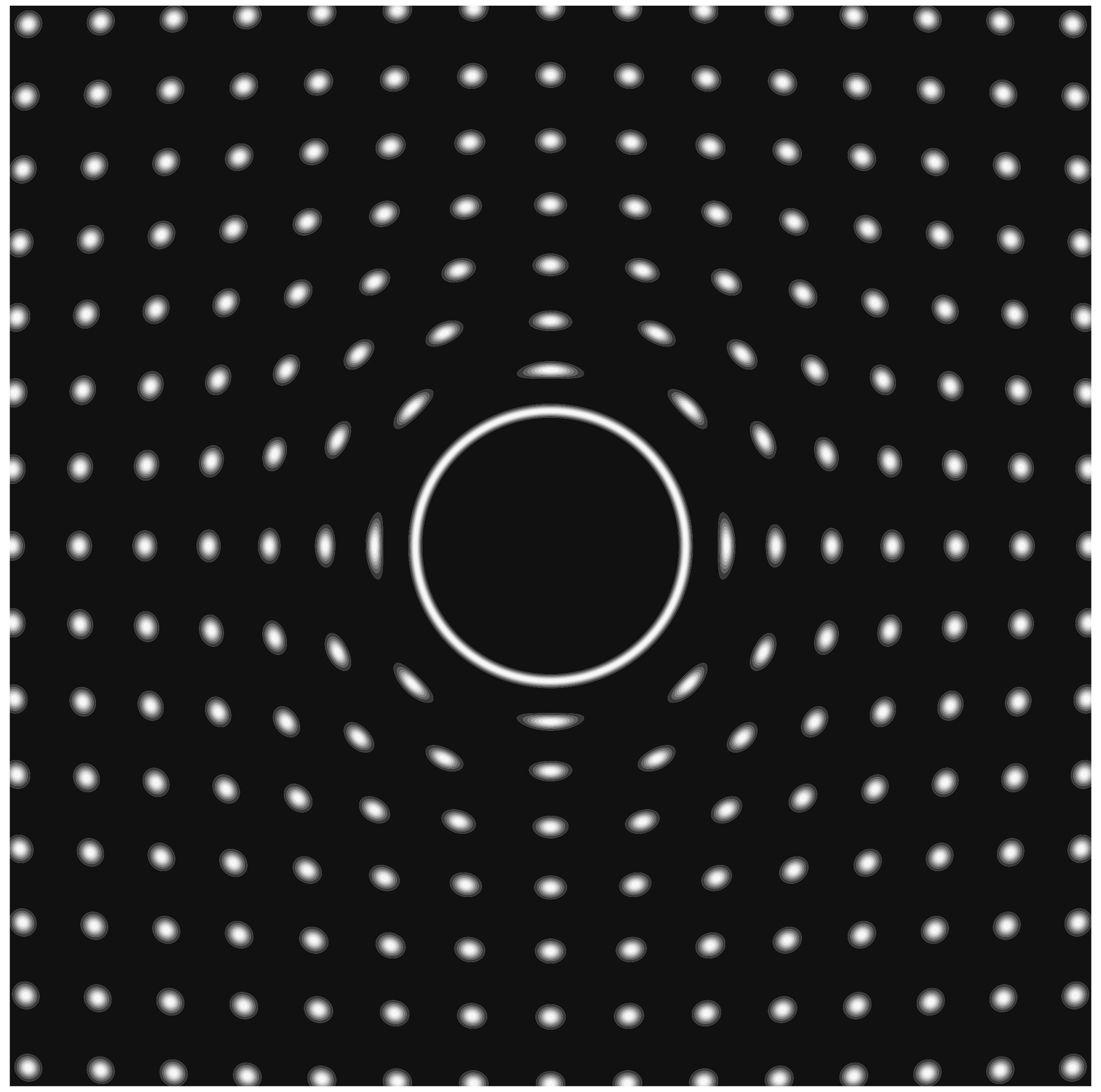


GRAVITATIONAL LENSING

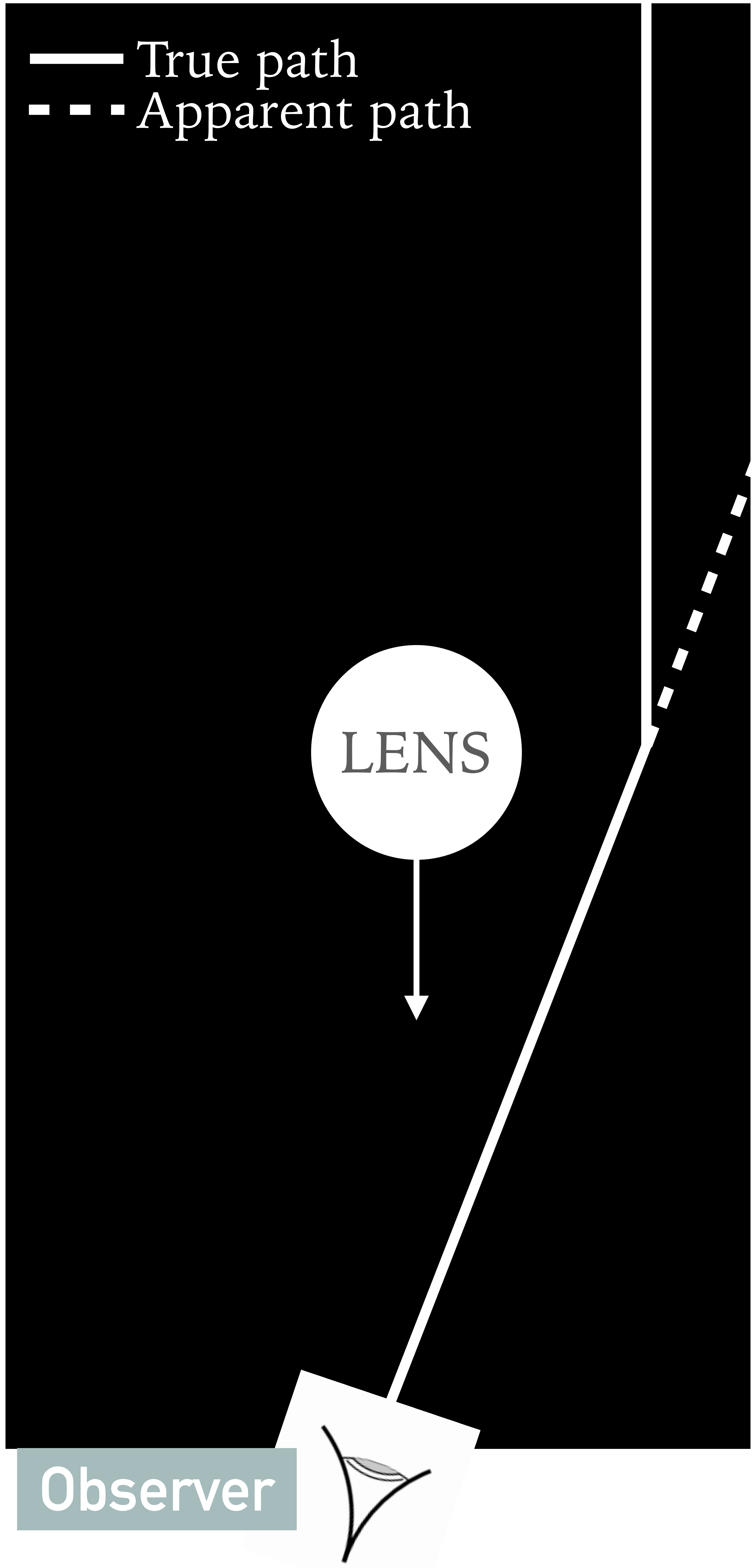
- The path of light is perturbed by gradients in the gravitational potential ϕ
- Distorts the images of distant galaxies



Unlensed field



Lensed field



RADIALLY MOVING LENSES

- The **radial** movement of the mass induces a gravitational force
  - Gravitomagnetic field produced by moving matter
- **Modulates** the magnitude of lensing effects proportional to the velocity

Deflection angle: $$\alpha_\phi \rightarrow \alpha_\phi(1 - v_{||}/c)$$

Lensing convergence: $$\kappa_\phi \rightarrow \kappa_\phi(1 - v_{||}/c)$$

- It's a small effect $v_{||}/c \approx 10^{-3}$

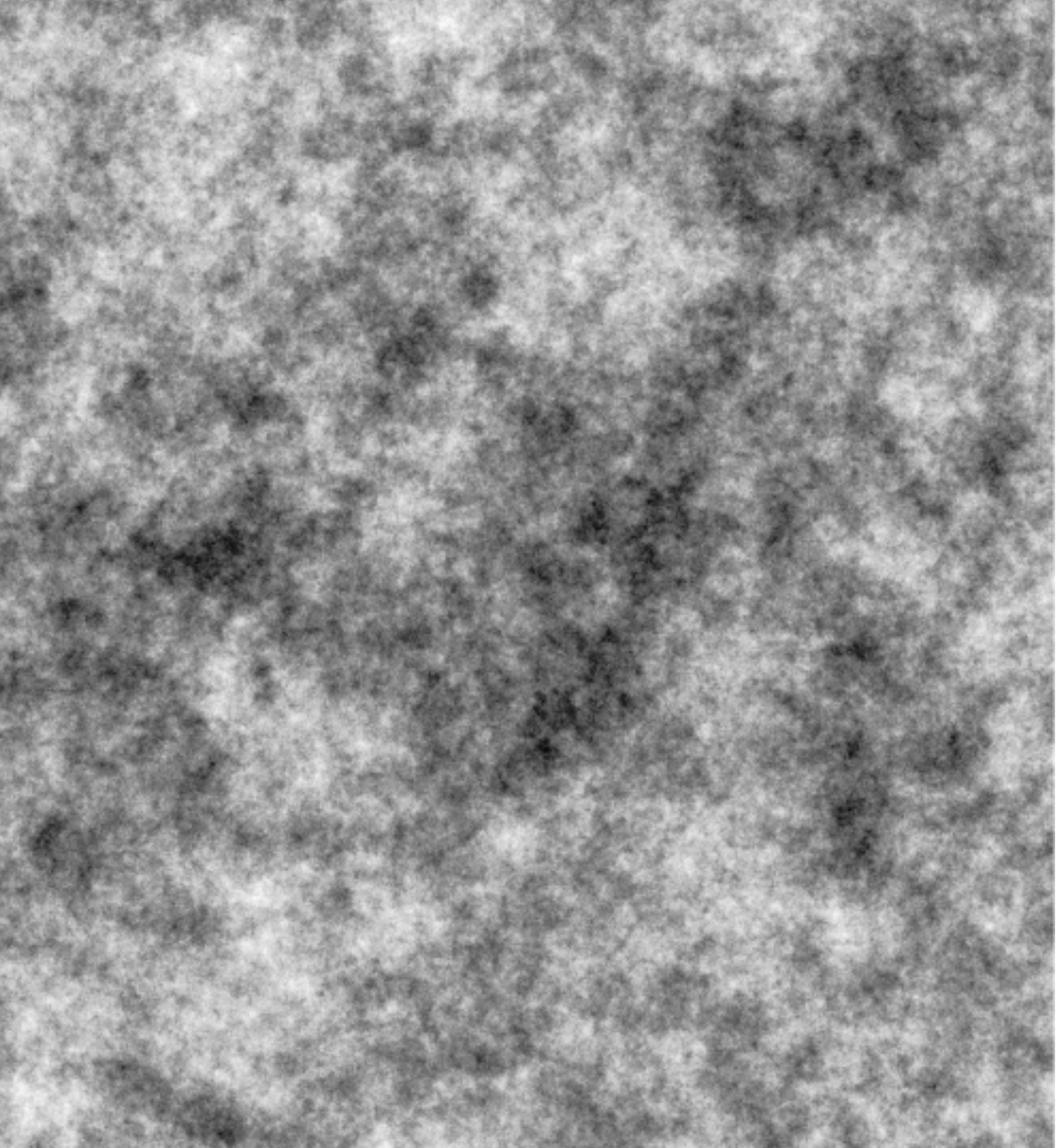
OBSERVATIONAL STATUS

- Measured with the motion of Jupiter (Fomalont and Kopeikin 2003) + Gravity Probe B
- No cosmological scale measurement yet
    - Would provide fairly direct measurement of the motion of dark matter
    - Test of Lorentz invariance

$$ds^2 = a^2(\tau)\Big[-(1+2\psi)d\tau^2 + (1-2\phi)\gamma_{ij}dx^i dx^j\Big]$$

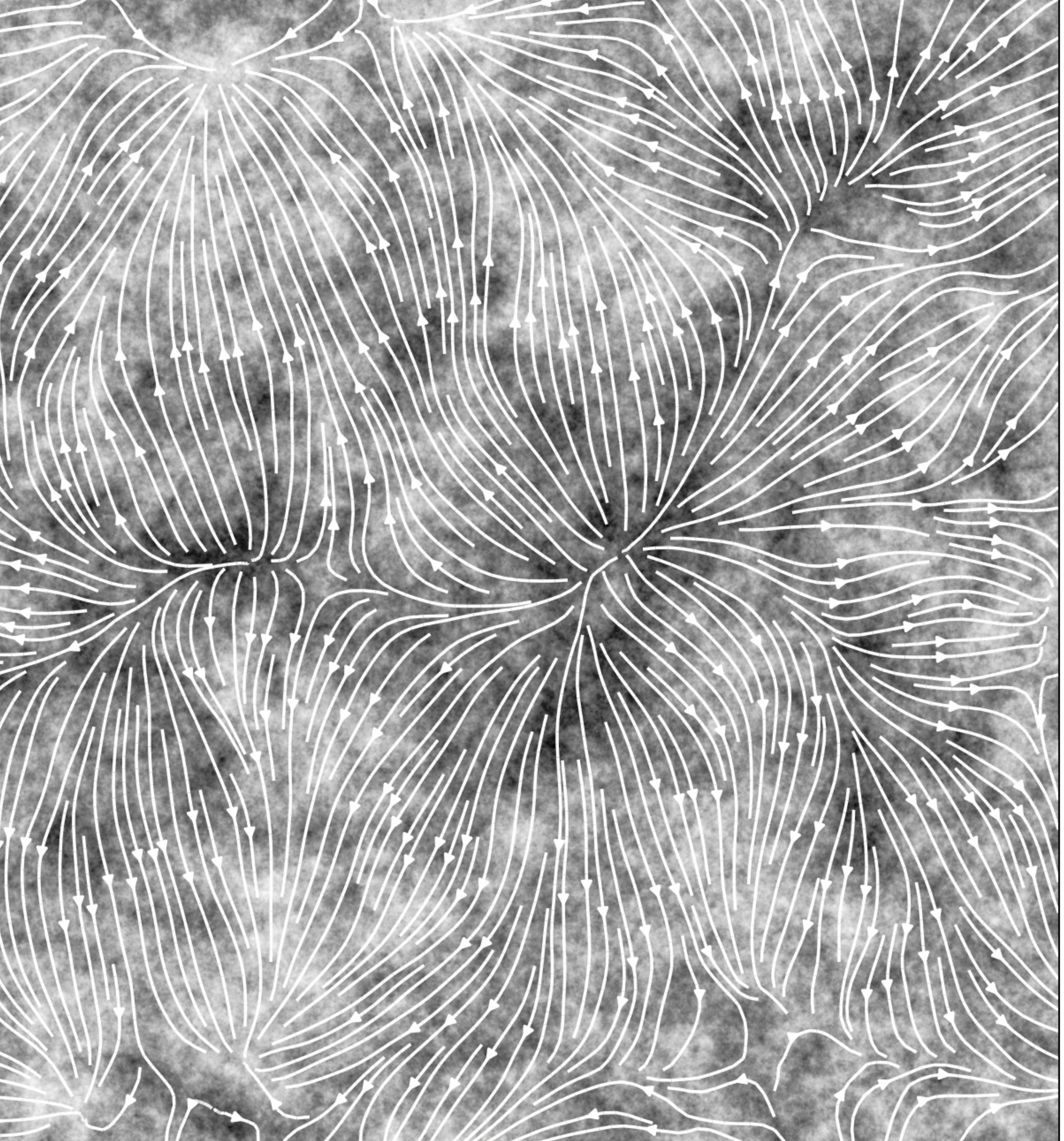
$$\Big\downarrow$$

$$ds^2 = a^2(\tau)\Big[-(1+2\psi)d\tau^2 + 2\mathscr{V}_i d\tau dx^i + (1-2\phi)\gamma_{ij}dx^i dx^j\Big]$$



COSMIC MOMENTUM FIELD

- Scalar/density perturbations source vector perturbations
  - Massive objects cause other objects to move towards them
- Continuity equation, $\mathbf{v}(k) = iaHf\,\frac{\delta(k)}{k^2}\mathbf{k}$



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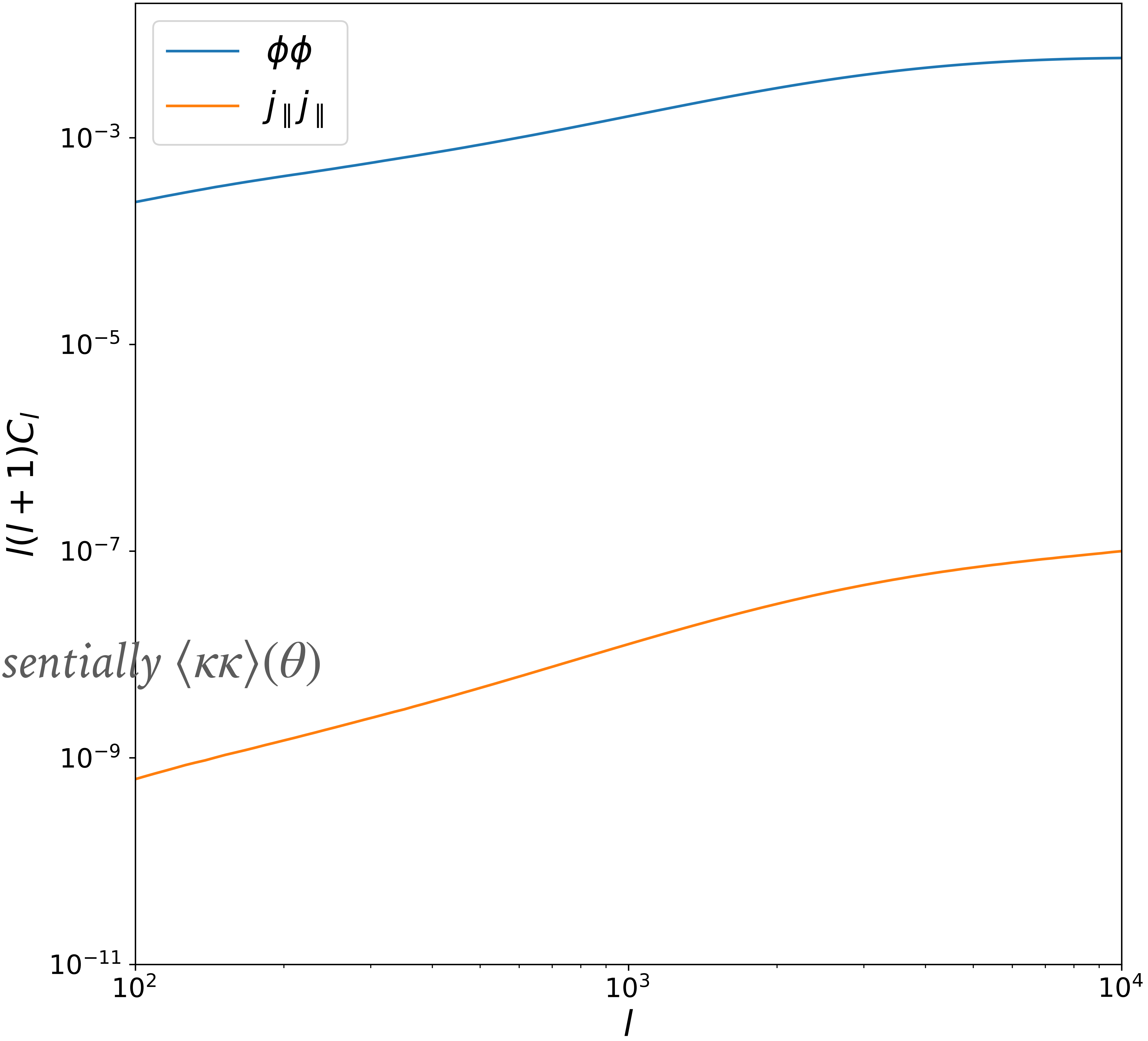
LENSING POWER SPECTRUM

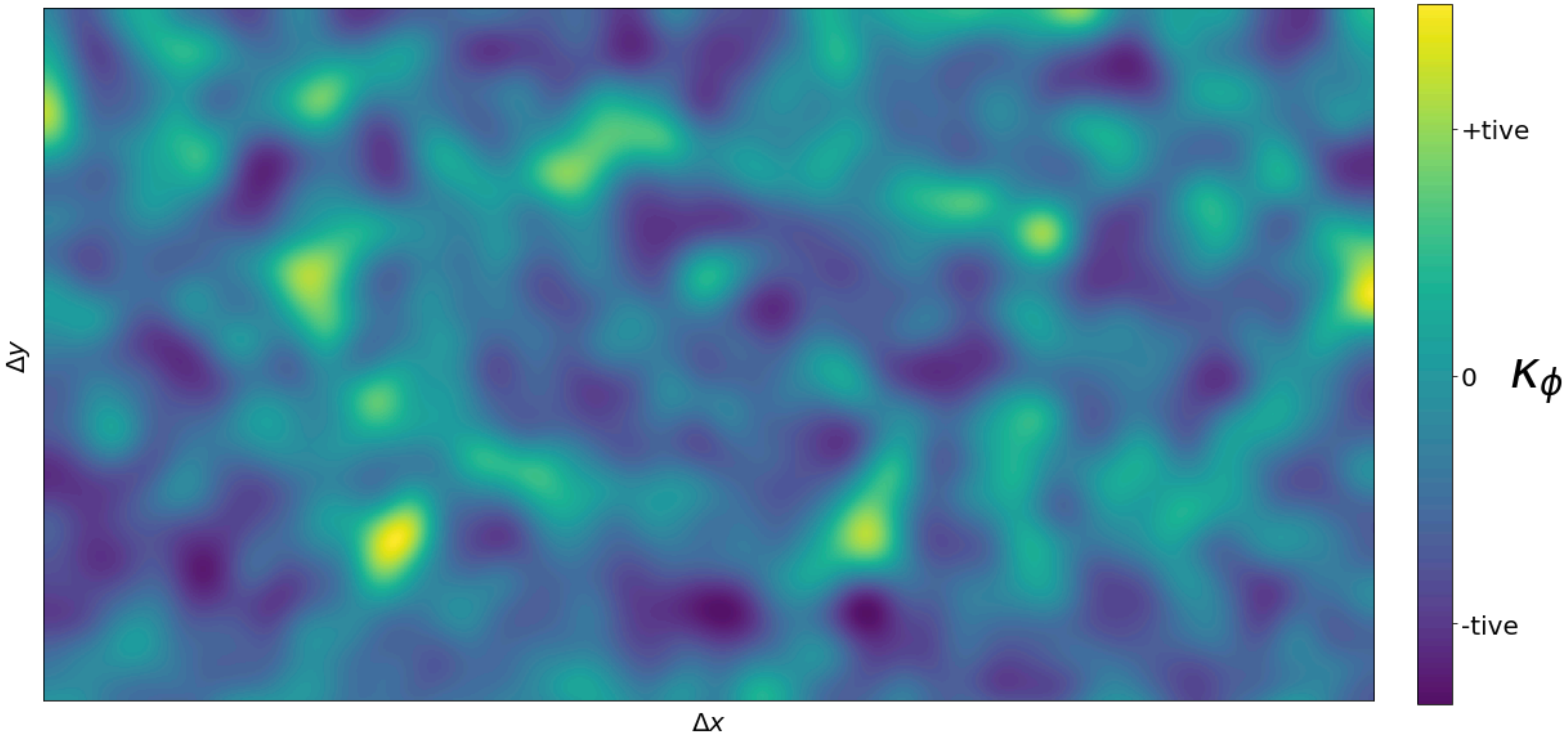


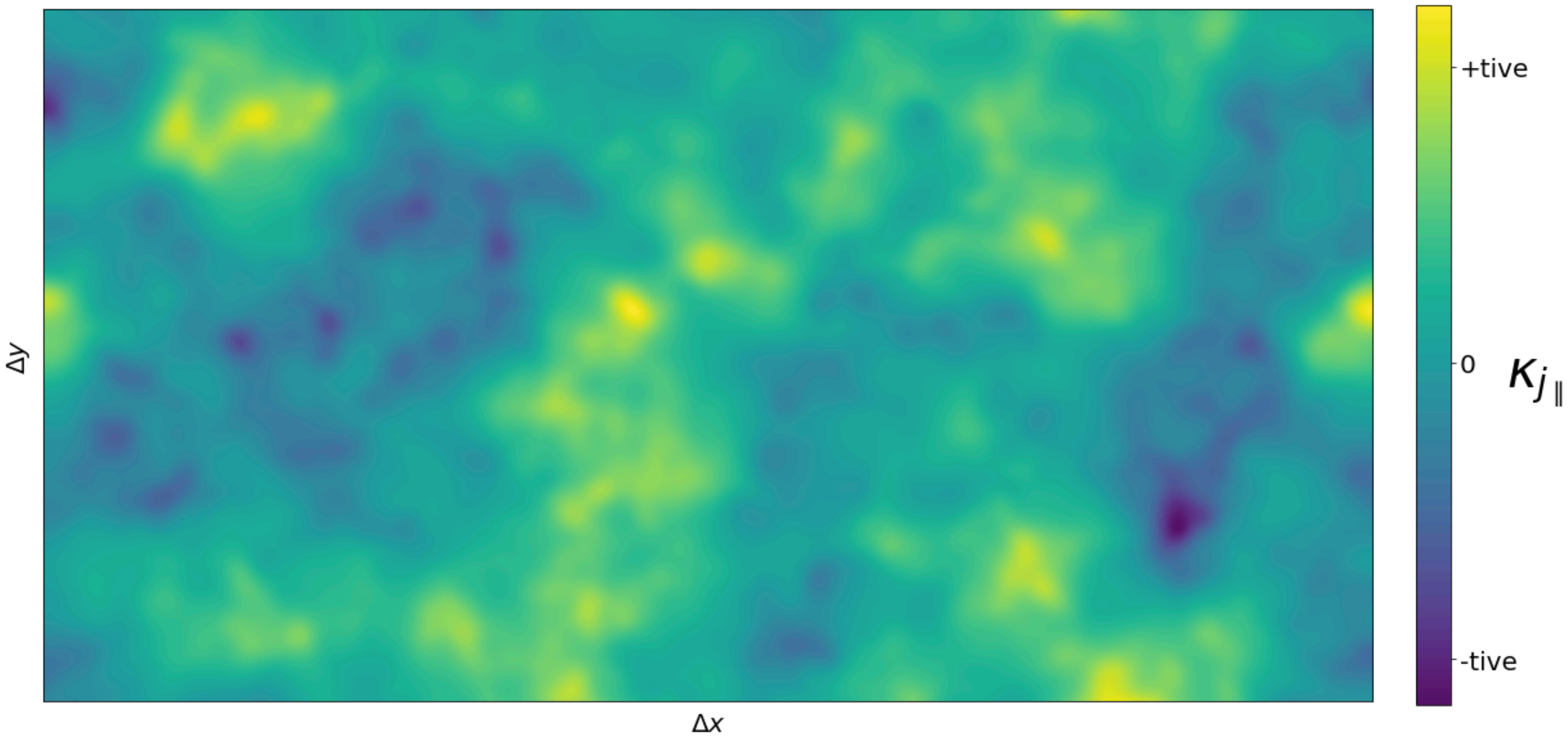
Essentially $\langle\kappa\kappa\rangle(\theta)$

- $\phi\phi$ is the normal term from density homogeneities
- $j_{\|}j_{\|}$ is the induced lensing convergence correlation from the momentum field
- The moving lens term $j_{\|}$ is tiny

$$\kappa_{\Phi}(\theta) = \frac{3}{2}\left(\frac{H_0}{c}\right)^2 \Omega_{m,0}\int_0^{\chi_s} d\chi \frac{\chi(\chi_s-\chi)}{a\chi_s}\delta$$

$$\kappa_{j_{\|}}(\theta) = \frac{3}{2}\left(\frac{H_0}{c}\right)^2 \Omega_{m,0}\int_0^{\chi_s} d\chi \frac{\chi(\chi_s-\chi)}{a\chi_s}j_{\|}$$

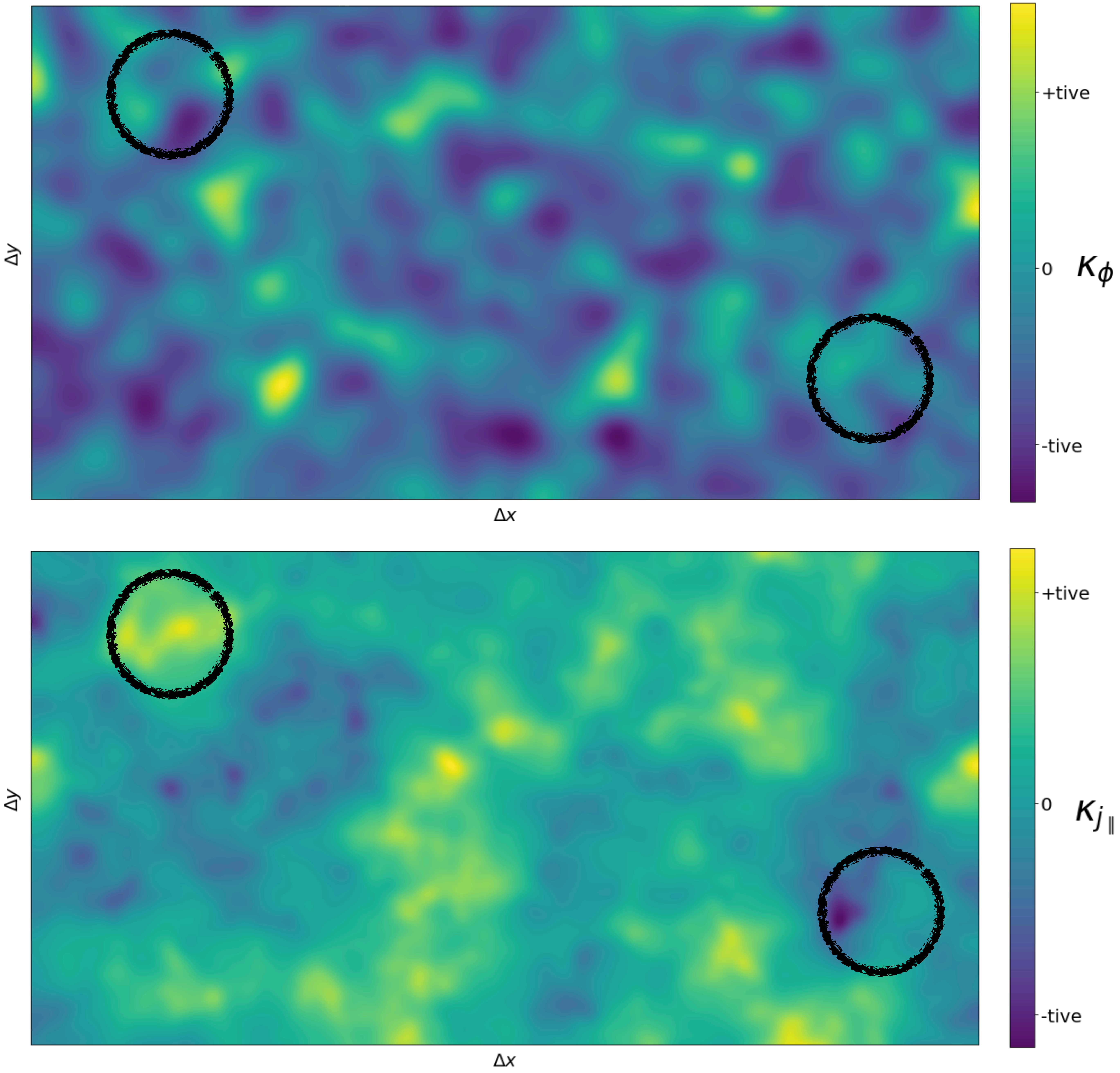




CROSS-CORRELATIONS

- We can cross correlate it with the line-of-sight projected momentum field
- We need to estimate/ find a tracer of the projected momentum density field
- These fields and following work done using the Quijote simulations

$$\kappa = \kappa_\phi + \kappa_{j_\parallel}$$



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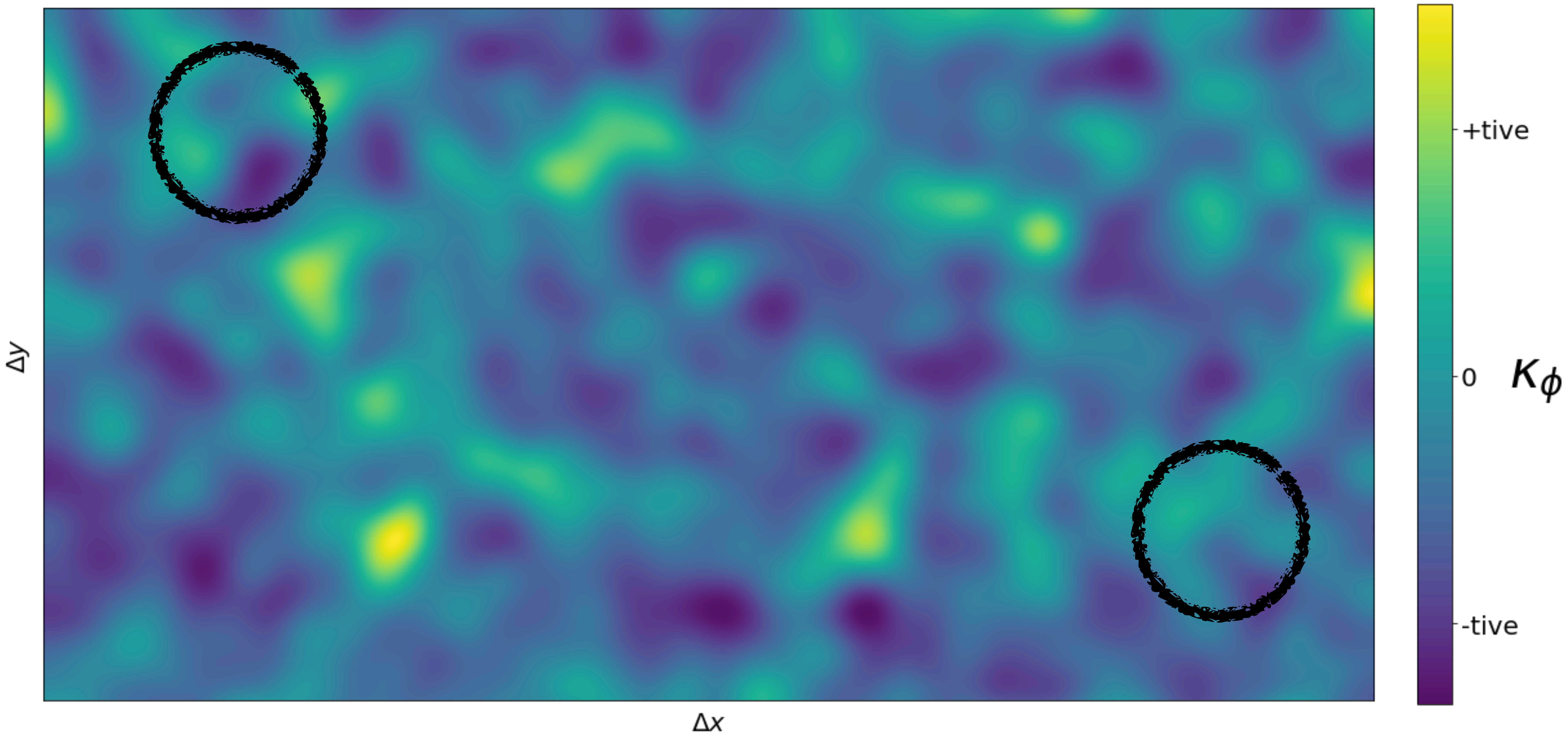
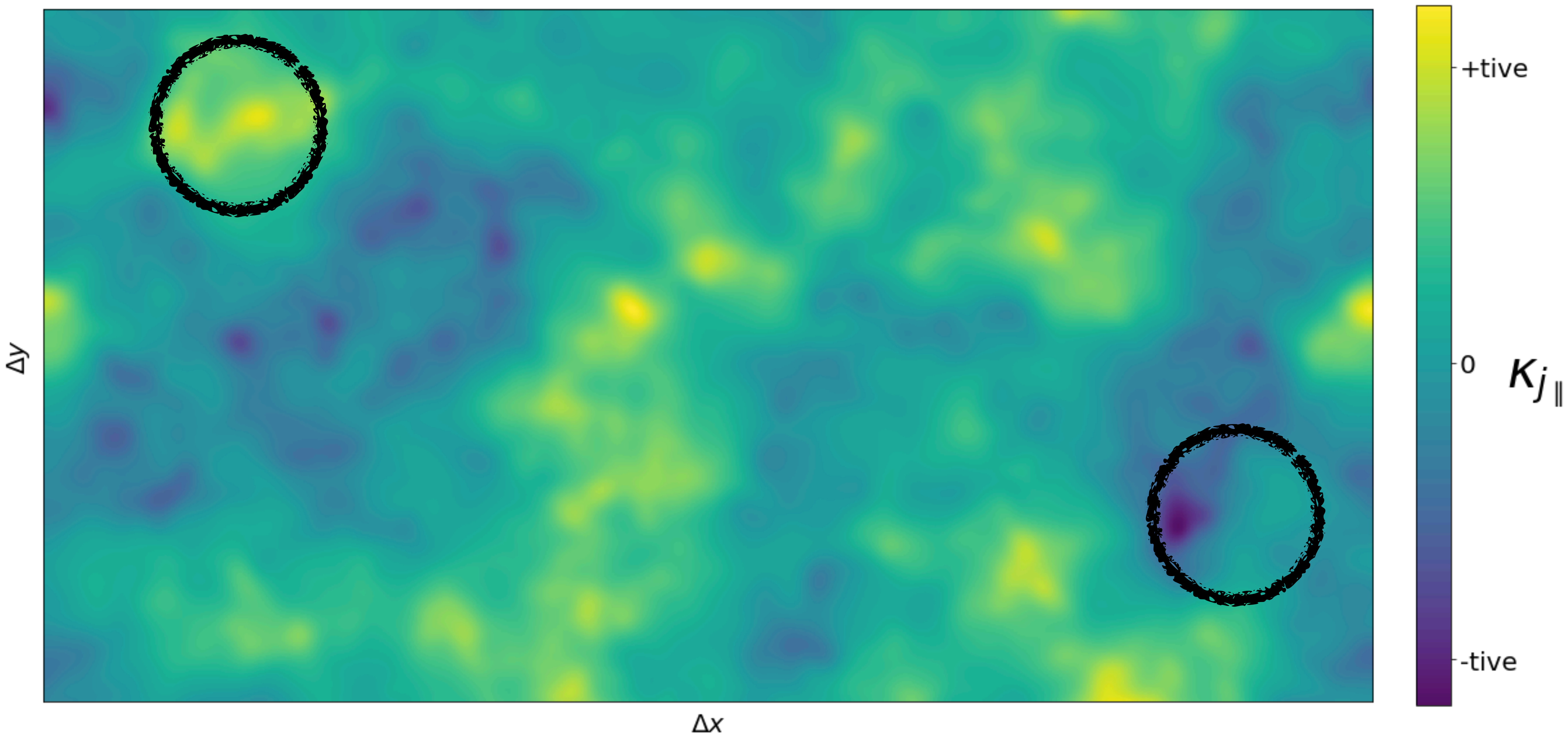
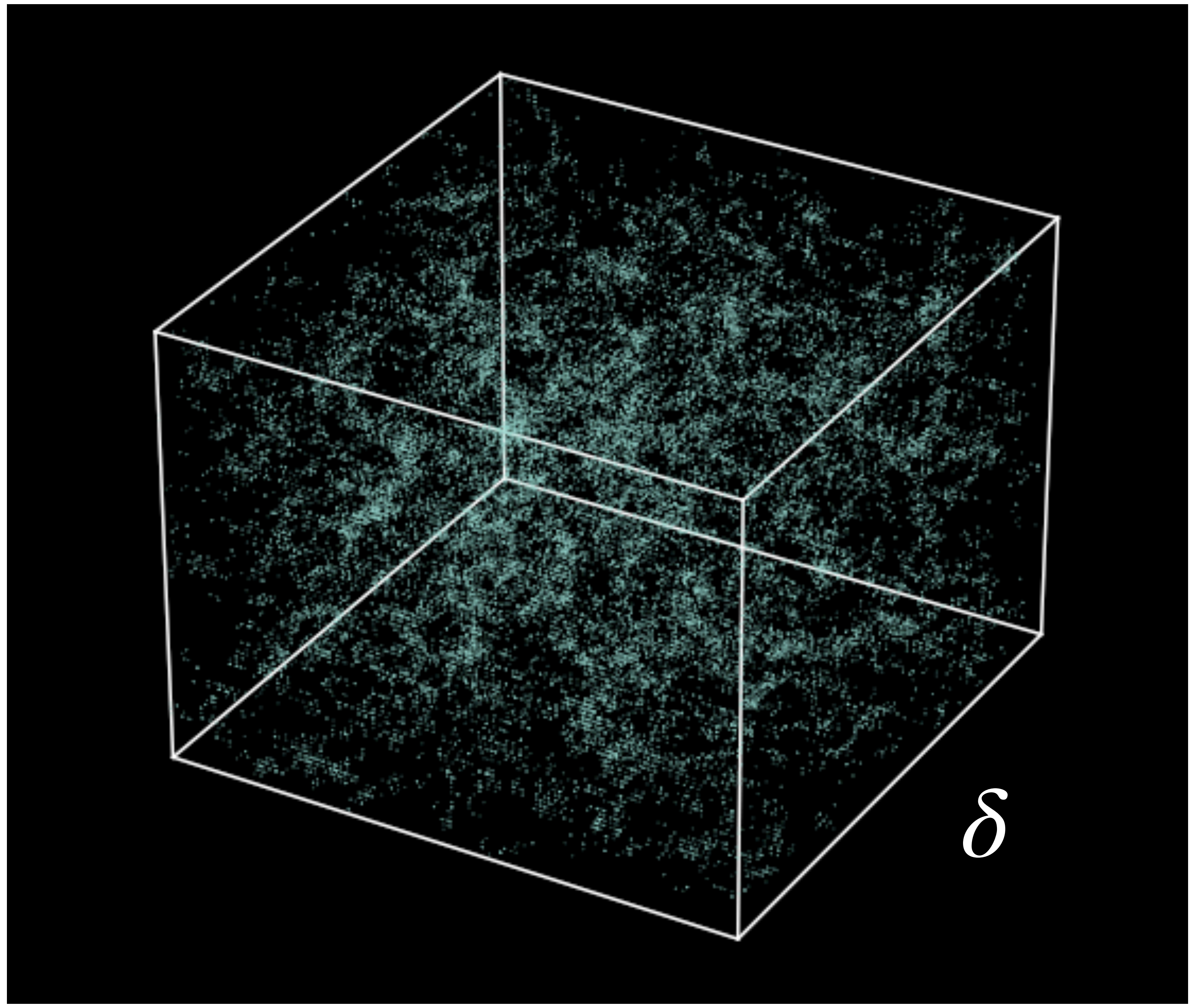




CROSS-CORRELATIONS

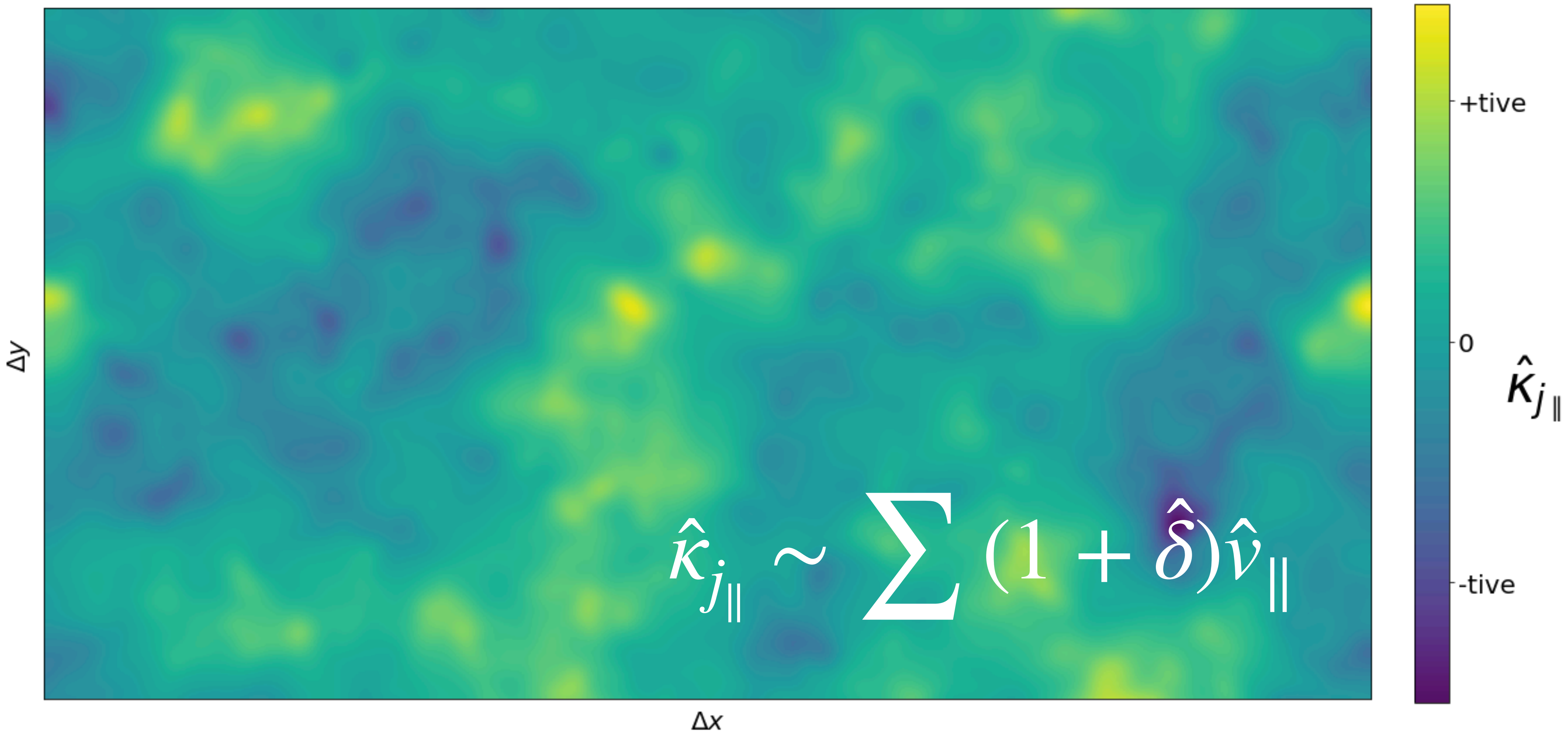
- We need a tracer of the cosmic momentum field
- Two fields
  - $\kappa = \kappa_\phi + \kappa_{j_\parallel}$
  - $\hat{\kappa}_{j_\parallel} \approx \sum \frac{\hat{v}_\parallel}{c}\delta$
  - $\left\langle \hat{\kappa}_{j_\parallel}\kappa \right\rangle = \cancel{\left\langle \kappa_\phi \frac{\hat{v}_\parallel}{c}\hat{\kappa}_\phi \right\rangle} + \left\langle 4\frac{\widehat{v_\parallel^2}}{c^2}\hat{\kappa}_\phi\kappa_\phi \right\rangle$
- Objects equally likely to have positive/negative momentum along LOS

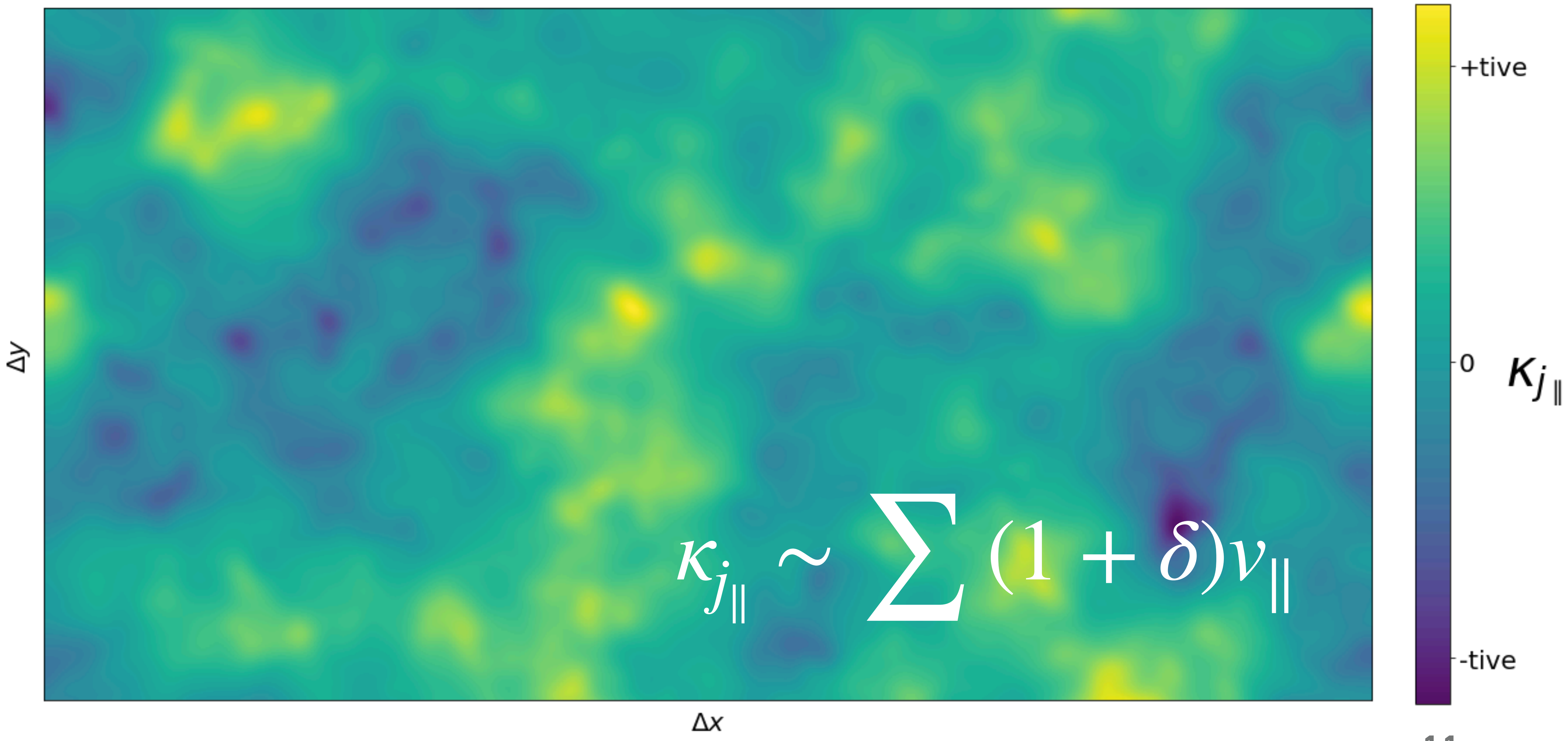


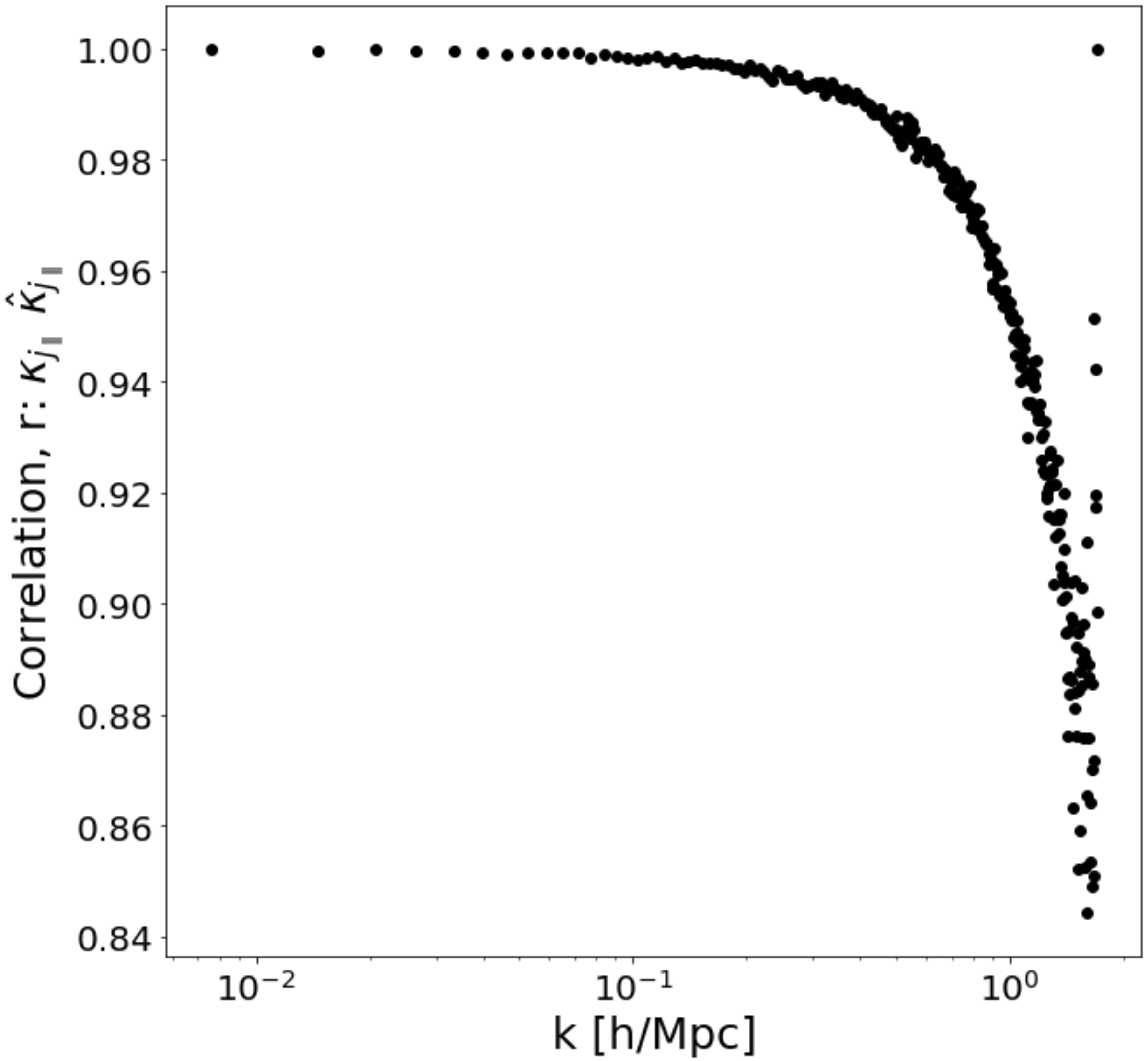
ESTIMATING THE COSMIC MOMENTUM FIELD WITH GALAXIES

- Estimate $\delta(\mathbf{x}) = \delta_g(\mathbf{x})/b_g,\ \ \delta(\mathbf{x}) \rightarrow \delta(\mathbf{k})$
- Construct the momentum field
  - $\hat{\mathbf{q}}(\mathbf{k}) = iaHf\delta(\mathbf{k})\dfrac{\mathbf{k}}{k^2}\left(1 + \hat{\delta}(\mathbf{k})\right)$
- Limited by the shot noise of galaxy surveys at small scales

$$\delta_g = n_g/\bar{n}_g - 1$$

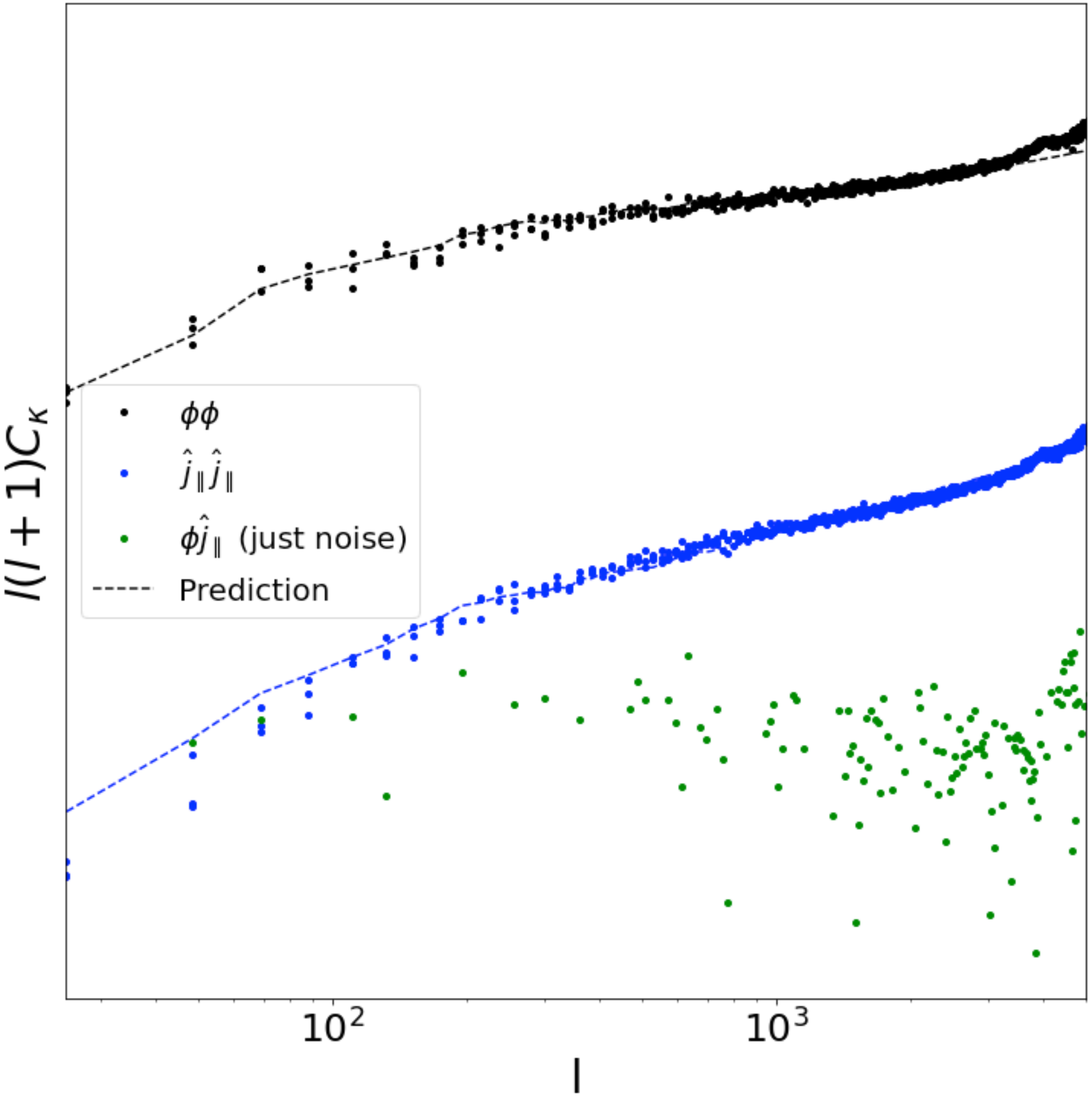






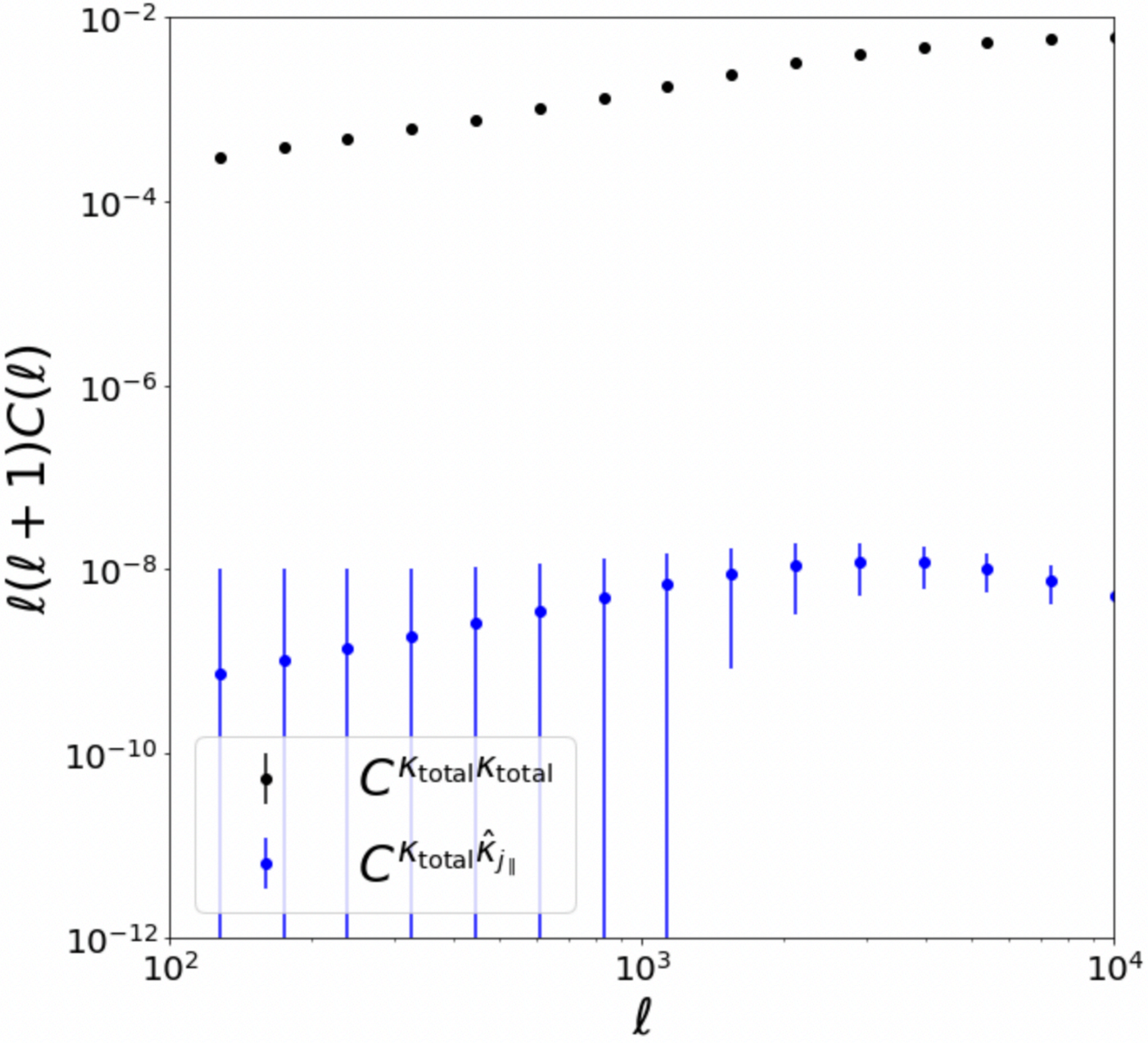
CORRELATION BETWEEN: $\hat{\kappa}_{j_\parallel}$ AND $\kappa_{j_\parallel}$

- Estimate $\delta(\mathbf{x}) = \delta_g(\mathbf{x})/b_g$, $\delta(\mathbf{x}) \rightarrow \delta(\mathbf{k})$
- Construct the momentum field
  - $\hat{\mathbf{q}}(\mathbf{k}) = iaHf\delta(\mathbf{k})\frac{\mathbf{k}}{k^2}(1+\hat{\delta}(\mathbf{k}))$
- $\hat{\kappa}_{j_\parallel} \sim \sum(1+\hat{\delta})\hat{v}_\parallel$
- $\kappa_{j_\parallel} \sim \sum(1+\delta)v_\parallel$



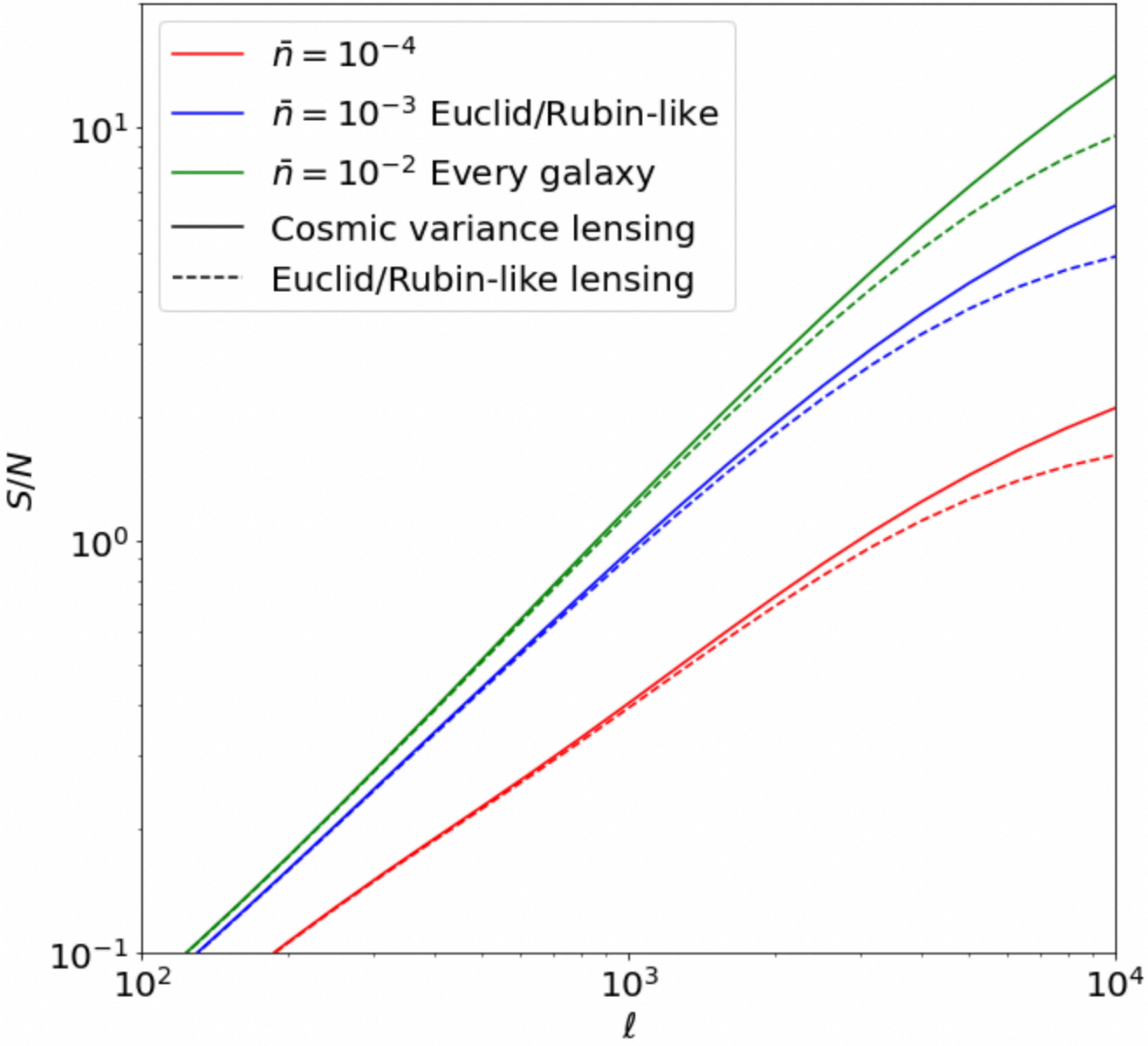
CROSS CORRELATIONS WITH SIMULATIONS

- Estimate $\delta(\mathbf{x}) = \delta_g(\mathbf{x})/b_g, \;\; \delta(\mathbf{x}) \to \delta(\mathbf{k})$
- Construct the momentum field
    - $\hat{\mathbf{q}}(\mathbf{k}) = iaHf\delta(\mathbf{k})\frac{\mathbf{k}}{k^2}(1+\hat{\delta}(\mathbf{k}))$
- $\hat{\kappa}_{j_\parallel} \sim \sum (1+\hat{\delta})\hat{v}_\parallel$
- $\kappa_{j_\parallel} \sim \sum (1+\delta)v_\parallel$
- $\left\langle \hat{\kappa}_{j_\parallel} \kappa_\phi \right\rangle = \cancel{\left\langle \kappa_\phi \frac{\hat{v}_\parallel}{c} \hat{\kappa}_\phi \right\rangle} = 0$



FORECASTS: IS THIS MEASURABLE?

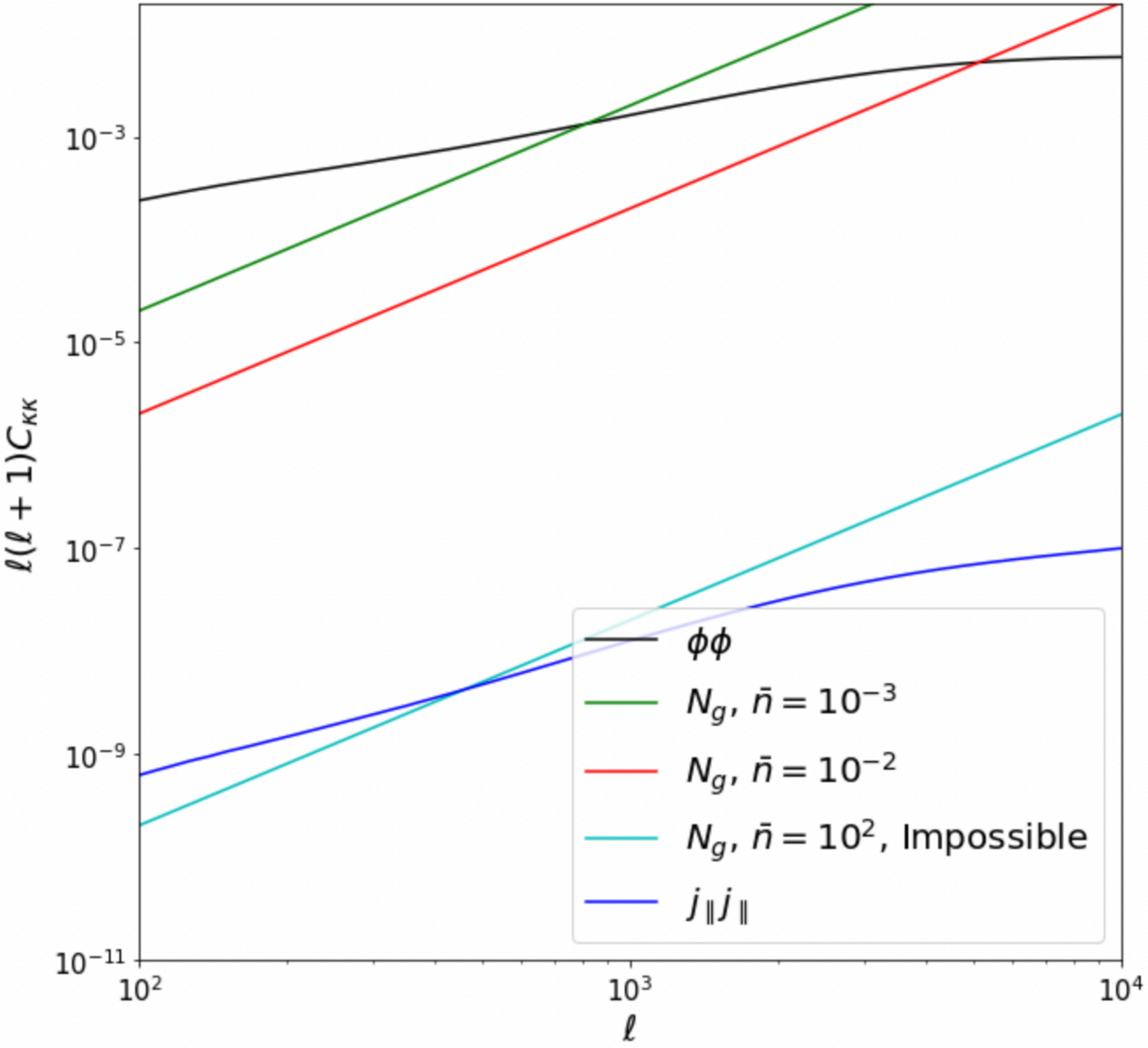
- $\kappa_{\rm total} = \kappa_\phi + \kappa_{j_\parallel}$
- $C^{\kappa\kappa}$ is the convergence auto-correlation, dominated by the density perturbation
- $C^{\hat{\kappa}_{j_\parallel}\kappa}$ is the reconstructed momentum field-convergence correlation function
    - $f_{sky} = 1/4$
    - $\bar{n}_{\rm gal} = 10^{-3}$ [Mpc^{-3}] for momentum reconstruction
    - 30 galaxies per arcminute^2 for the lensing measurement



FORECASTS

- For next generation surveys, Euclid/Rubin we have a SNR ~ 5
  - Can improve this by messing around with cosmological parameters
- The SNR is dominated by the cosmic variance from κ_ϕ
- $\kappa = \kappa_\phi + \kappa_{j_\parallel}$

$$\left(\frac{S}{N}\right)^2 = f_{sky} \sum_l (2l+1) \frac{\left(C_{\kappa_j \kappa}\right)^2}{C_{\kappa_j \kappa_j}\left(C_{\kappa\kappa} + N_l\right)}$$

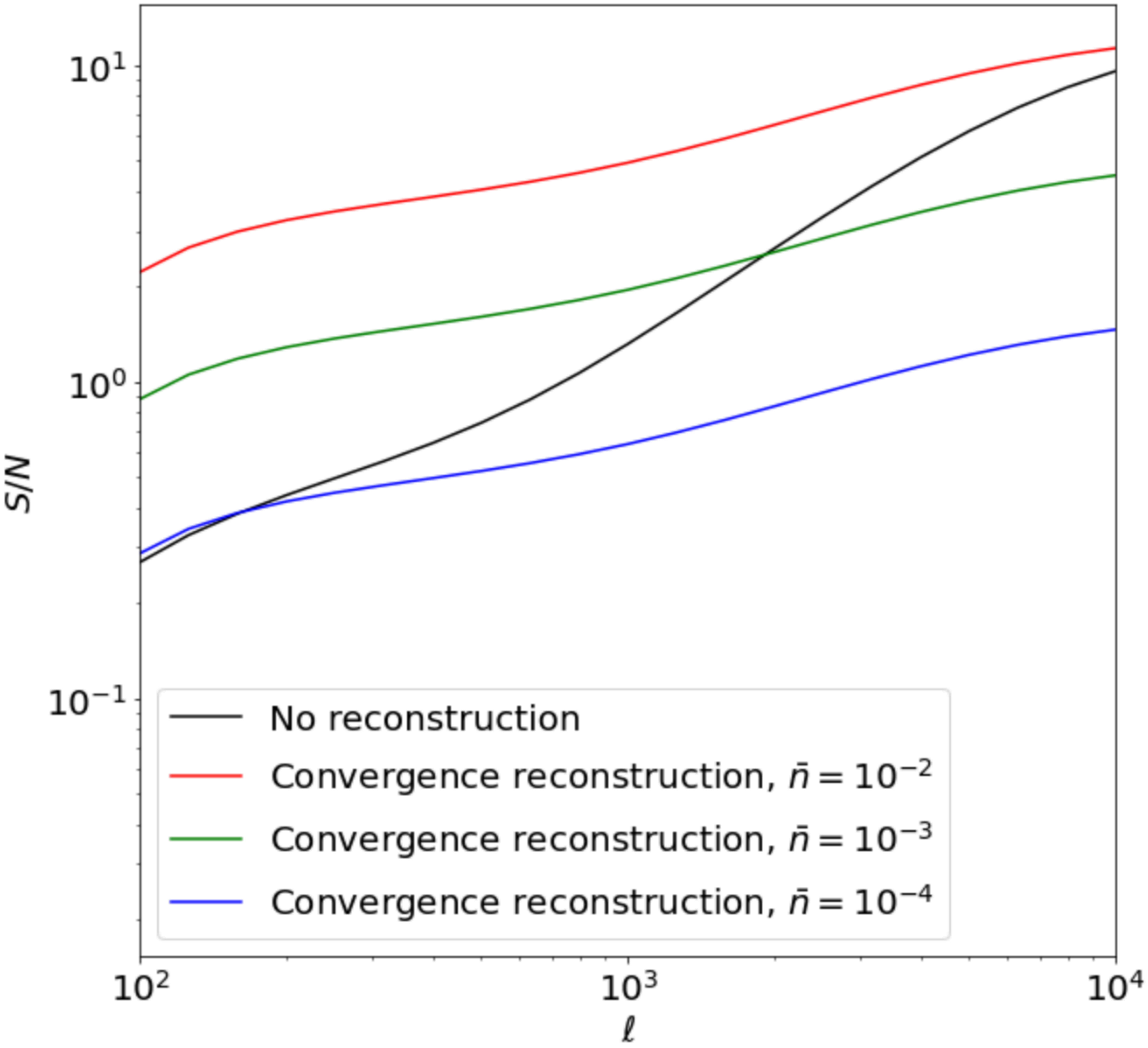


LET'S REMOVE κ_ϕ

- Main source of noise is the primary convergence contribution
- $\kappa_{\text{measured}} \approx \kappa_\phi + \kappa_{j_\parallel}$
- We could also estimate this too…
- $\hat{\kappa}_{j_\parallel} = \kappa_{\text{measured}} - \hat{\kappa}_\Phi$

$$\hat{\kappa}_\Phi(\theta) = \frac{3}{2}\left(\frac{H_0}{c}\right)^2 \Omega_{m,0} \int_0^{\chi_s} d\chi \frac{\chi(\chi_s - \chi)}{a\chi_s} \hat{\delta}_g / b$$

$$\delta_g / b = \delta_m$$



CONVERGENCE RECONSTRUCTION

- Lensing noise $C_{\kappa\kappa}$ → galaxy shot noise N_{shot}

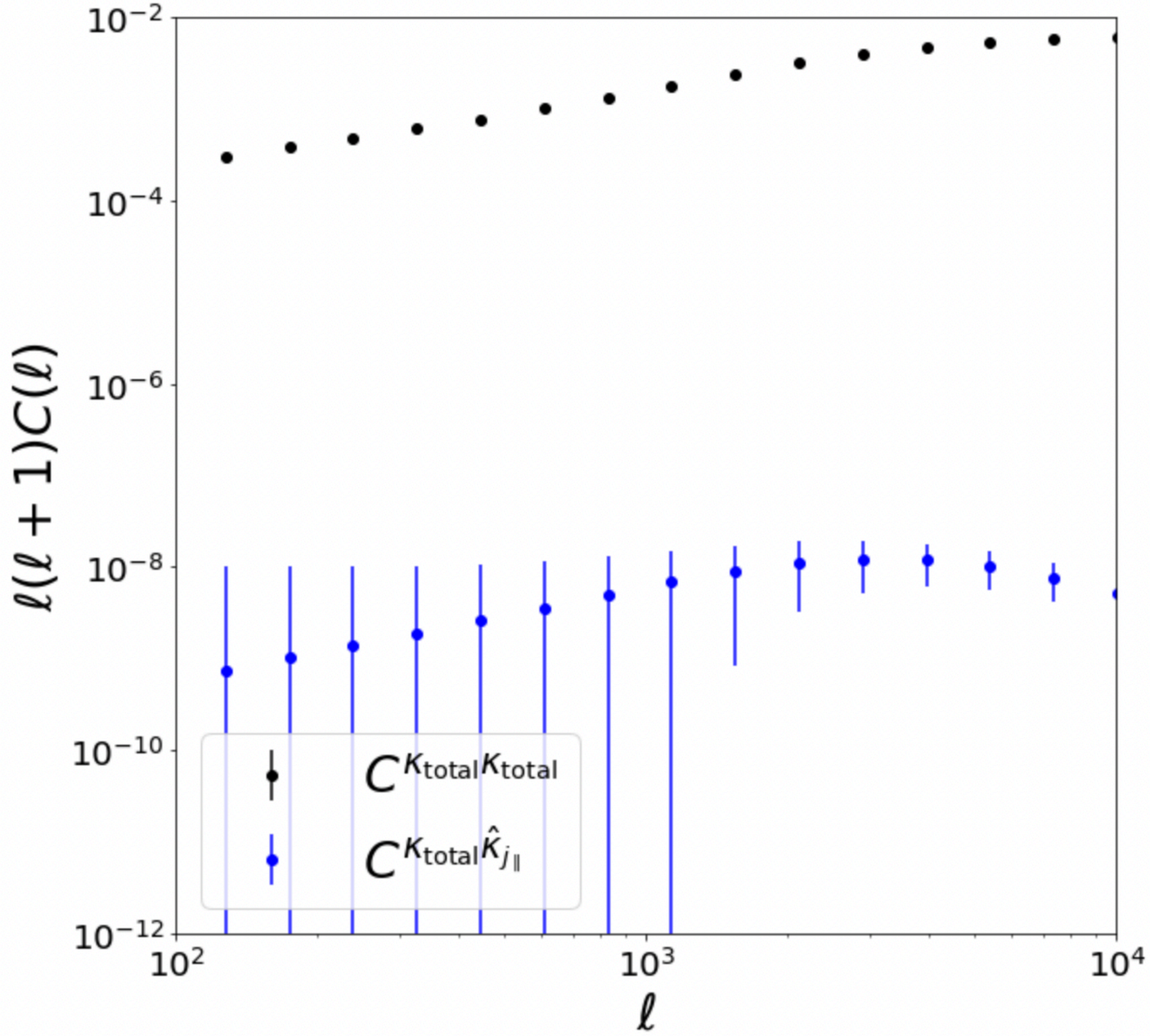
$$\left(\frac{S}{N}\right)^2 = f_{sky} \sum_l (2l+1) \frac{\left(C_{\kappa_j \kappa}\right)^2}{C_{\kappa_j \kappa_j}\left(C_{\kappa\kappa} + N_l\right)}$$

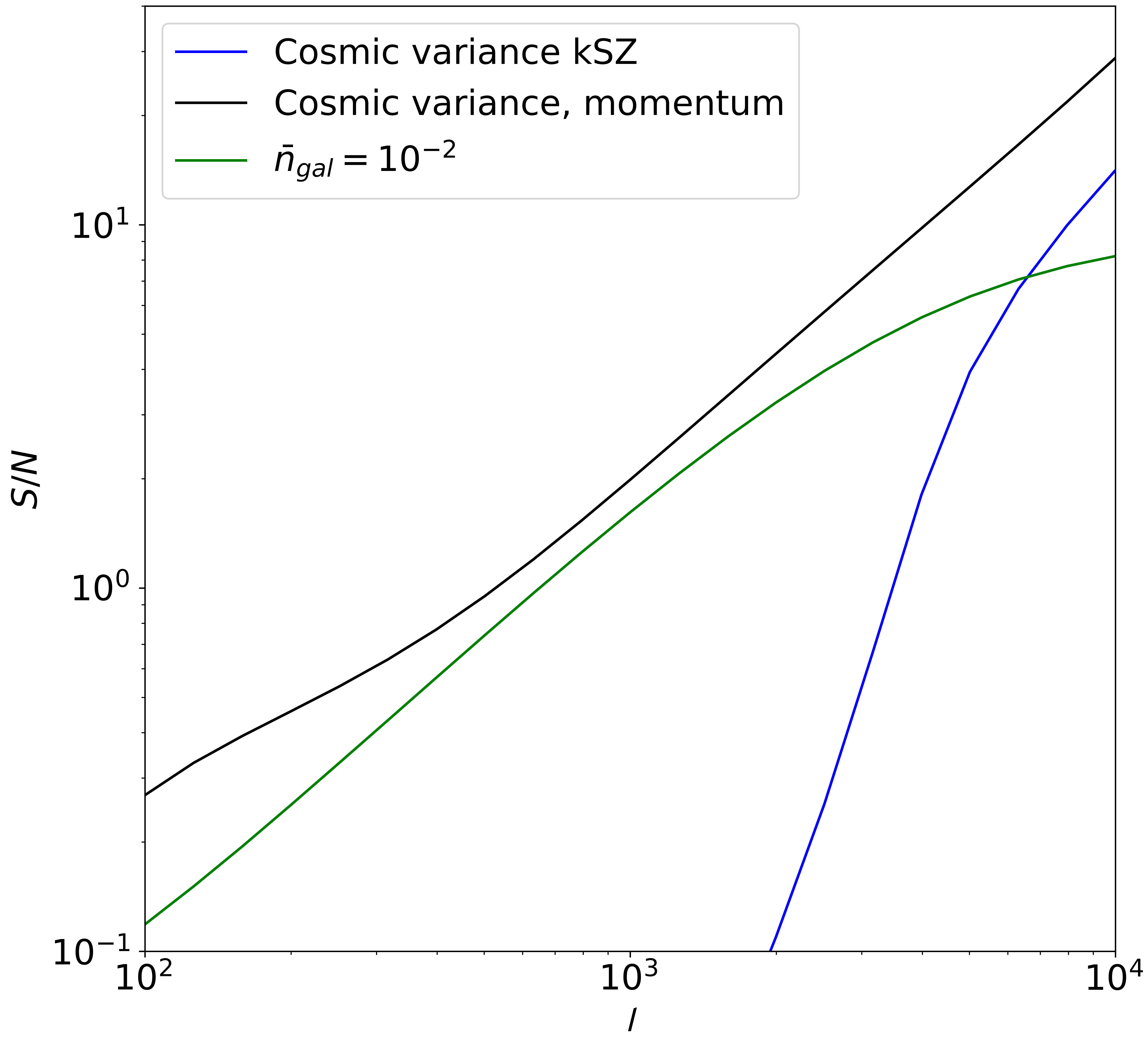
$$\downarrow$$

$$\left(\frac{S}{N}\right)^2 = f_{sky} \sum_l (2l+1) \frac{\left(C_{\kappa_j \kappa}\right)^2}{C_{\kappa_j \kappa_j}\left(N_{\text{shot}} + N_l\right)}$$

CONCLUSIONS

- Matter currents have a small effect on the measured gravitational lensing signal
- A fairly direct measurement of the motion of dark matter
- A test of Lorentz invariance on cosmological scales
- Difficult to measure but hopefully possible with the next generation of cosmological surveys!

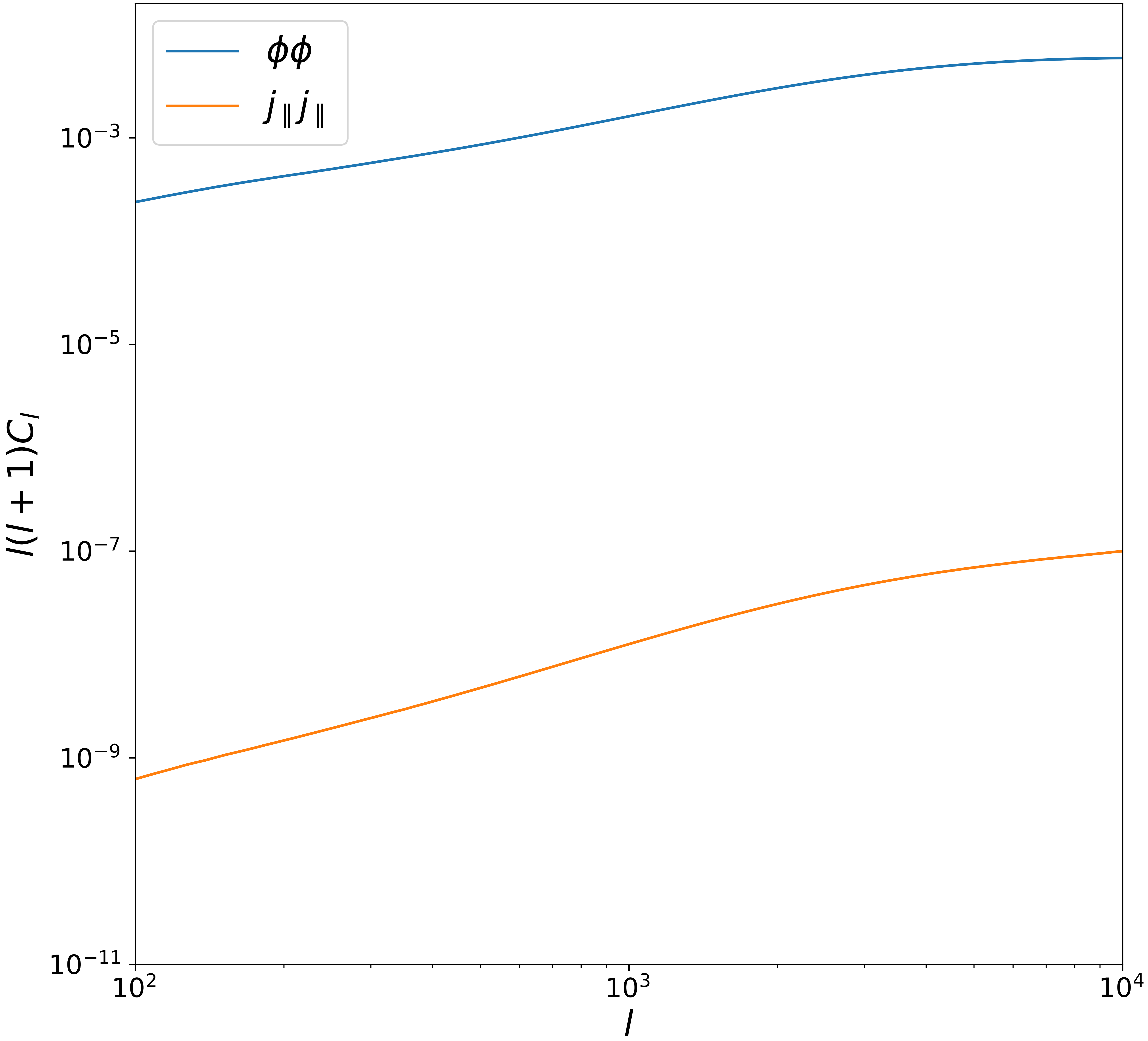




FORECASTS COMPARISON

- ➤ $\bar{n}_{gal} = 10^{-2}$ is likely the cosmic variance limit for a galaxy survey
- ➤ Momentum refers to exact reconstruction of the momentum field
- ➤ Galaxy reconstruction gives access to larger scales
- ➤ kSZ gives access to smaller scales

$$\frac{\Delta T_{kSZ}(\hat{n})}{T_{cmb}} \approx \int_0^{sls} d\chi e^{-\tau(z)} v_{\parallel} \delta_e(\hat{n}, \chi)$$

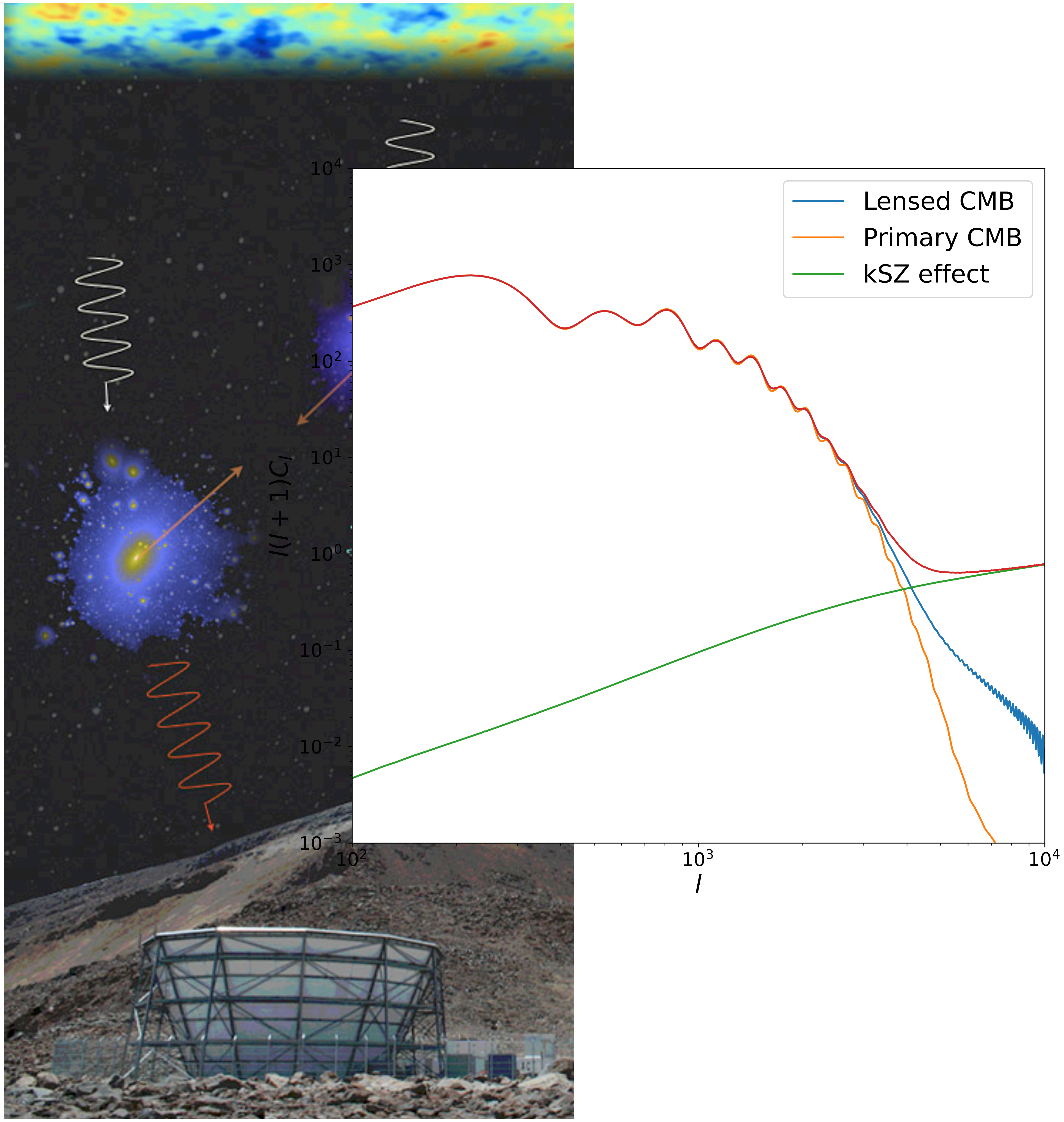


WE NEED TO CROSS CORRELATE!

$$\left(\frac{S}{N}\right)^2 = f_{sky} \sum_l (2l+1) \frac{\left(C_{\kappa_j \kappa_j}\right)^2}{\left(C_{\kappa\kappa} + N_l\right)^2}$$

$$\left(\frac{S}{N}\right)^2 = f_{sky} \sum_l (2l+1) \frac{\left(C_{\kappa_j \kappa}\right)^2}{C_{\kappa_j \kappa_j}\left(C_{\kappa\kappa} + N_l\right)}$$

$C_{\kappa_j \kappa_j}$ is much smaller than $C_{\kappa\kappa}$



KSZ EFFECT

- ➤ Doppler shift of the CMB as seen by electrons with a bulk velocity relative to the CMB
- ➤ Direct access to the projected momentum to very small scales
- ➤ Not the same kernel as lensing

$$\frac{\Delta T_{kSZ}(\hat{n})}{T_{cmb}} \approx \int_0^{sls} d\chi e^{-\tau(z)} v_{\|} \delta_e(\hat{n}, \chi)$$