Error correction in mono-qubit quantum computers

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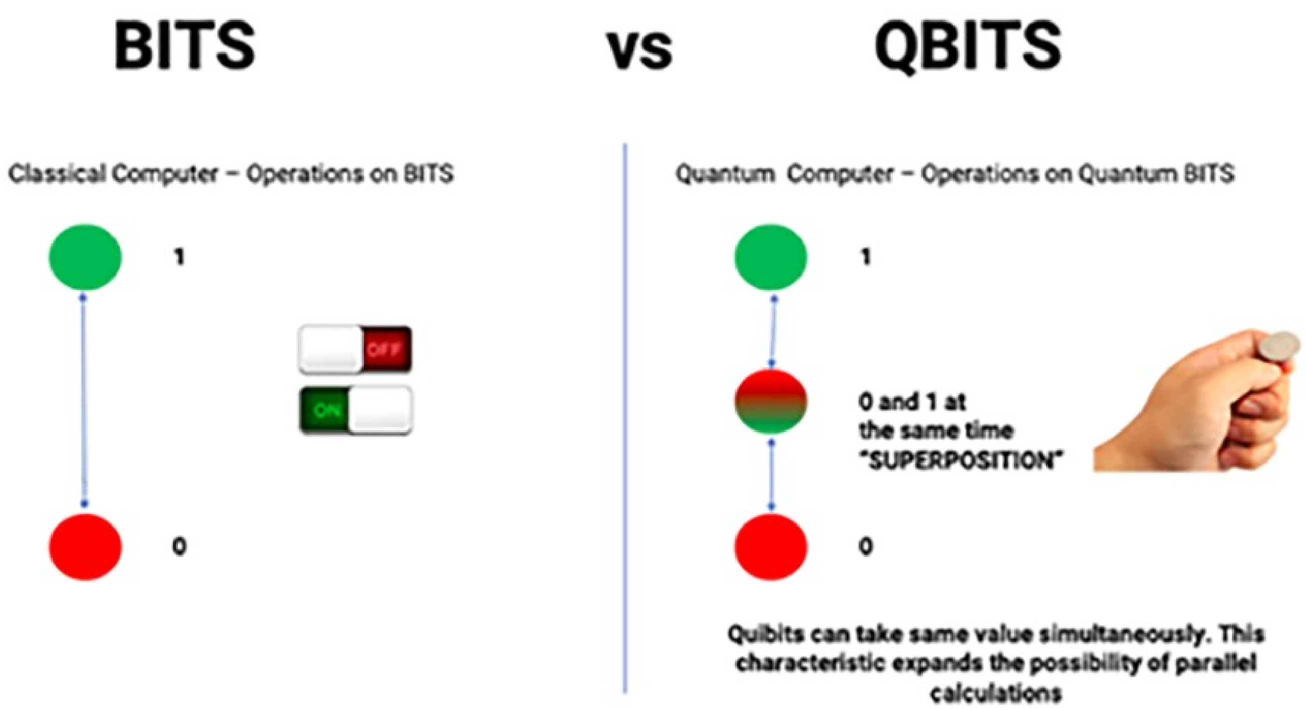
Qubits

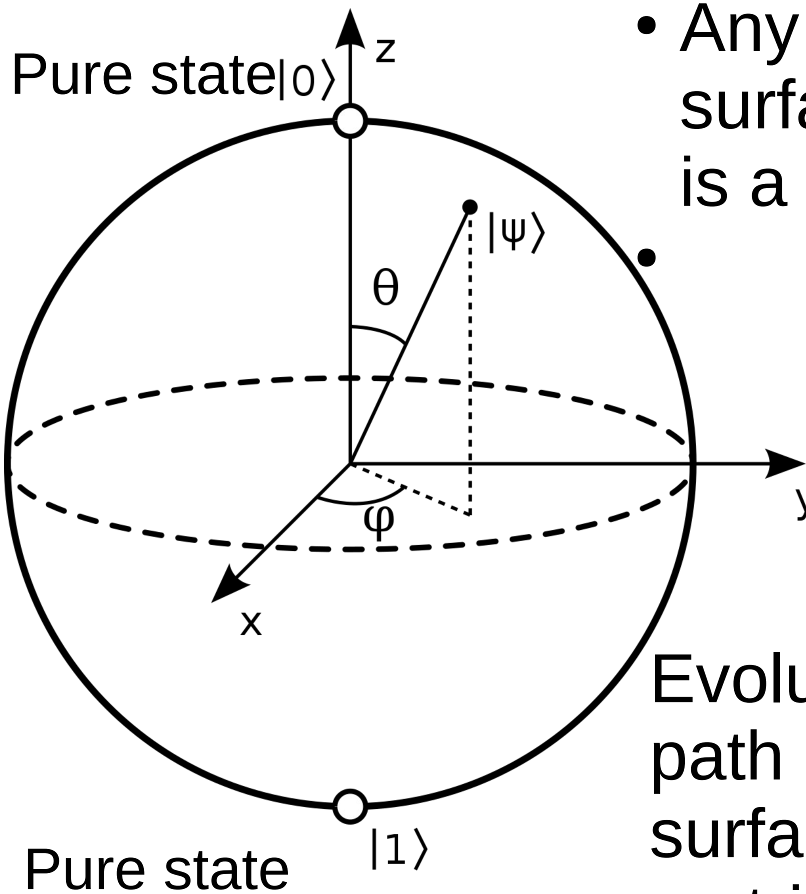


$q=a|0\rangle + b|1\rangle$
with a and b complex

Normalization
$|a|^2+|b|^2=1$

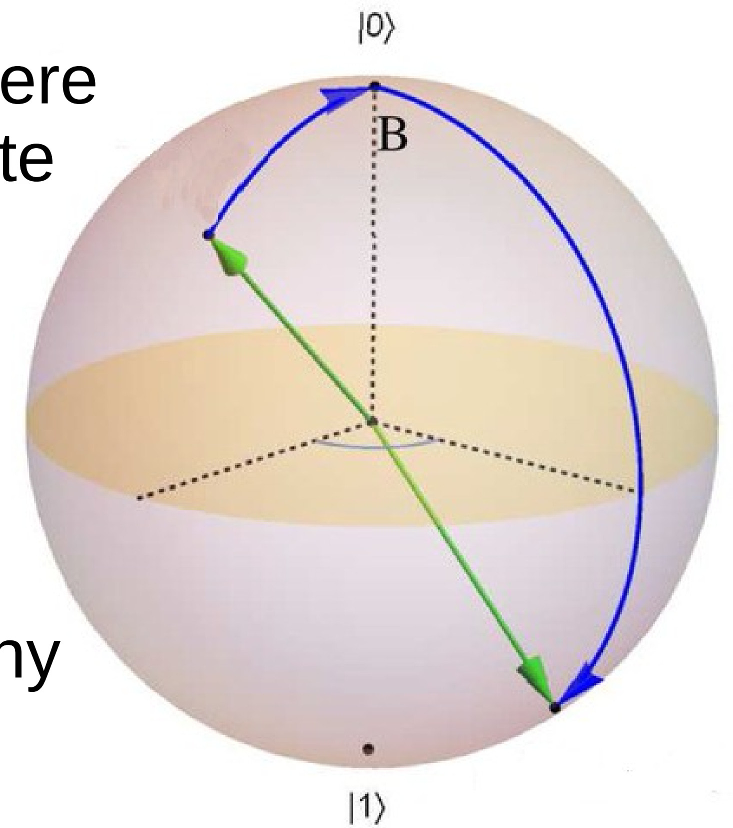
A quantum computer is a collection of qubits that
- can be measured
- can evolve with dedicated operators

Bloch Sphere representation

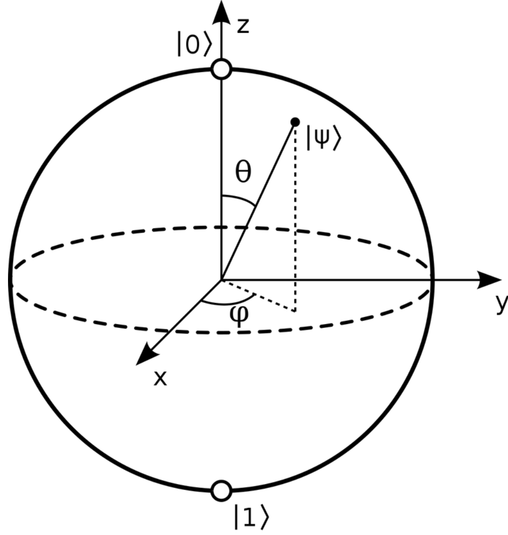


- Any point at the surface of the sphere is a valid qubit state

Evolution can be any path on the sphere surface (unitary matrix)
Combination of axis rotations



Qubit Measurement



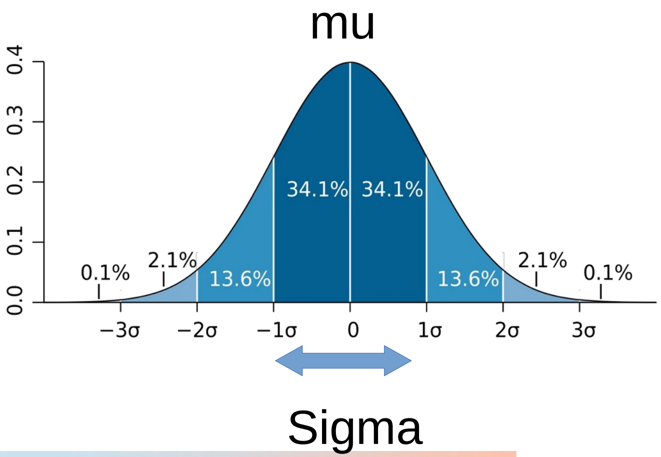
- The internal state cannot be measured directly → obtain only random pure states
- Born rule: 0 with $|a|^2$ probability
  1 with $|b|^2$ probability
- Projection on any axis (usually Z)
- Destructive operation: the qubit value is fixed to the measured value (wave function collapse)
- Quantum computation has to be repetead

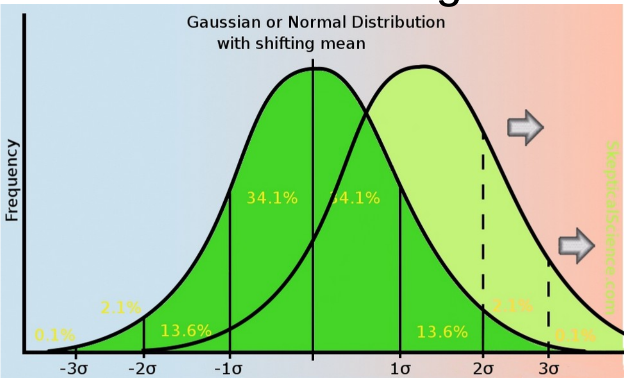
Interest for HEP

- « Nature is quantum! So if we want to simulate it, we need a quantum computer » R. Feynman 1981
- Allow to simulate any quantum phenomenon with linear computational complexity
- Evolution of the quantum computer mimics the target system evolution (Hilbert space)
- Problem : Noise reduces drastically the performance of the simulations

Noise in a mono-qubit

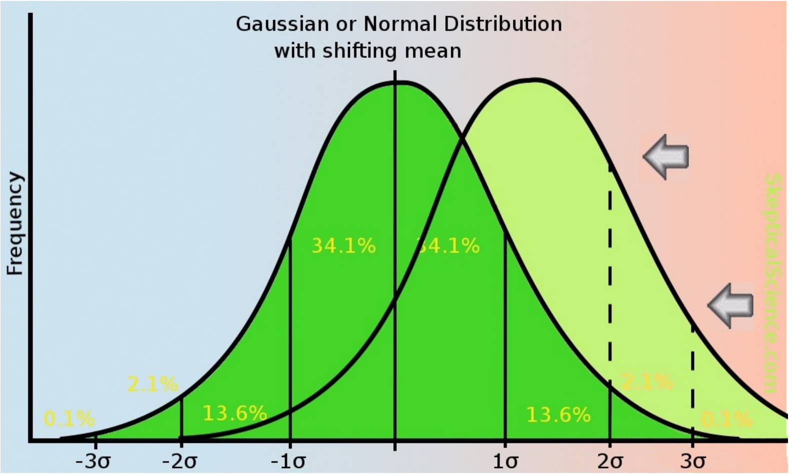
- NISQ era (Noisy Intermediate Scale Quantum)
- Two kind of errors
  - Statistical error : variability of measurement (sigma), includes measurement and decoherence
  - Systematic error : difference on the distribution (mu and sigma)





Our work

- Considering the QC as a black-box
- Only mono-qubit (to get a start)
- Characterization of the stat. noise → precise evaluation of the measurement needs
- Correction of the systematic errors to improve quality of computations

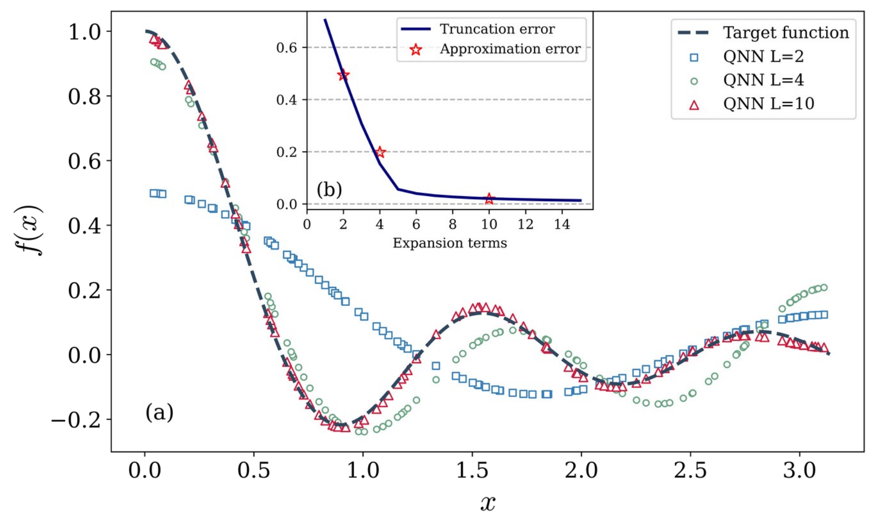


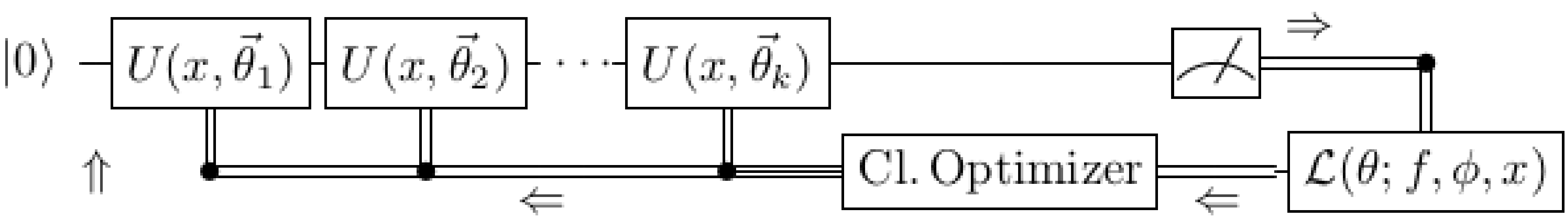
Quantum Machine learning approach

- ML circuit learns the target circuit but also corrects the systematic noise
- Let Γ be a mono-qubit re-uploading circuit
- T is the target circuit
- Train Γ on the result of T on a noisy QC
- Result
  - Correcting operator (Γ)
  - Stat. error distribution from Γ sampling

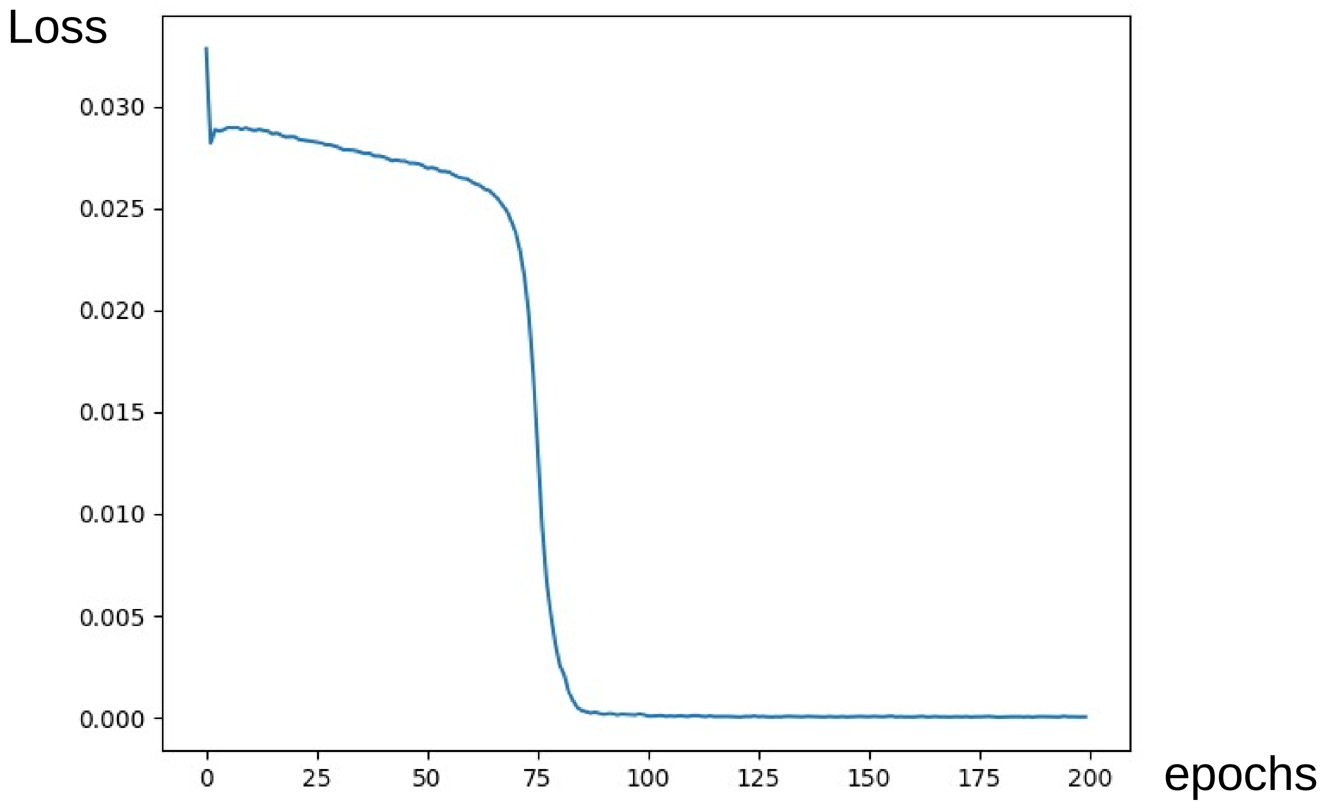
Re-uploading circuit

- Mono-qubit re-uploading circuit (Yu & al 2022)
- Re-upload the data multiple times → non linearity
- Universal approximator
- Fits well in practice
- Quality depends on the circuit depth





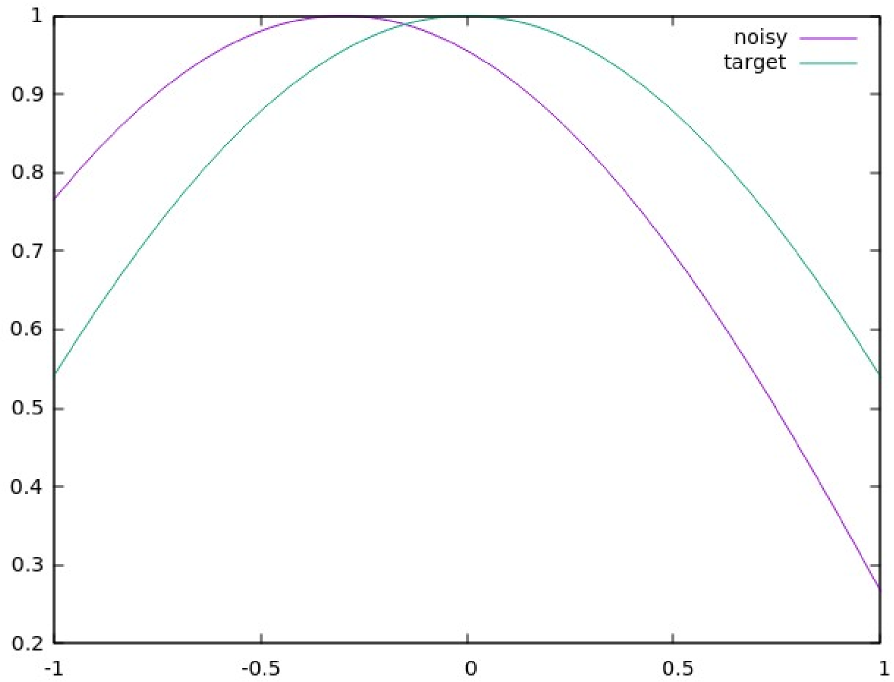
Γ circuit training



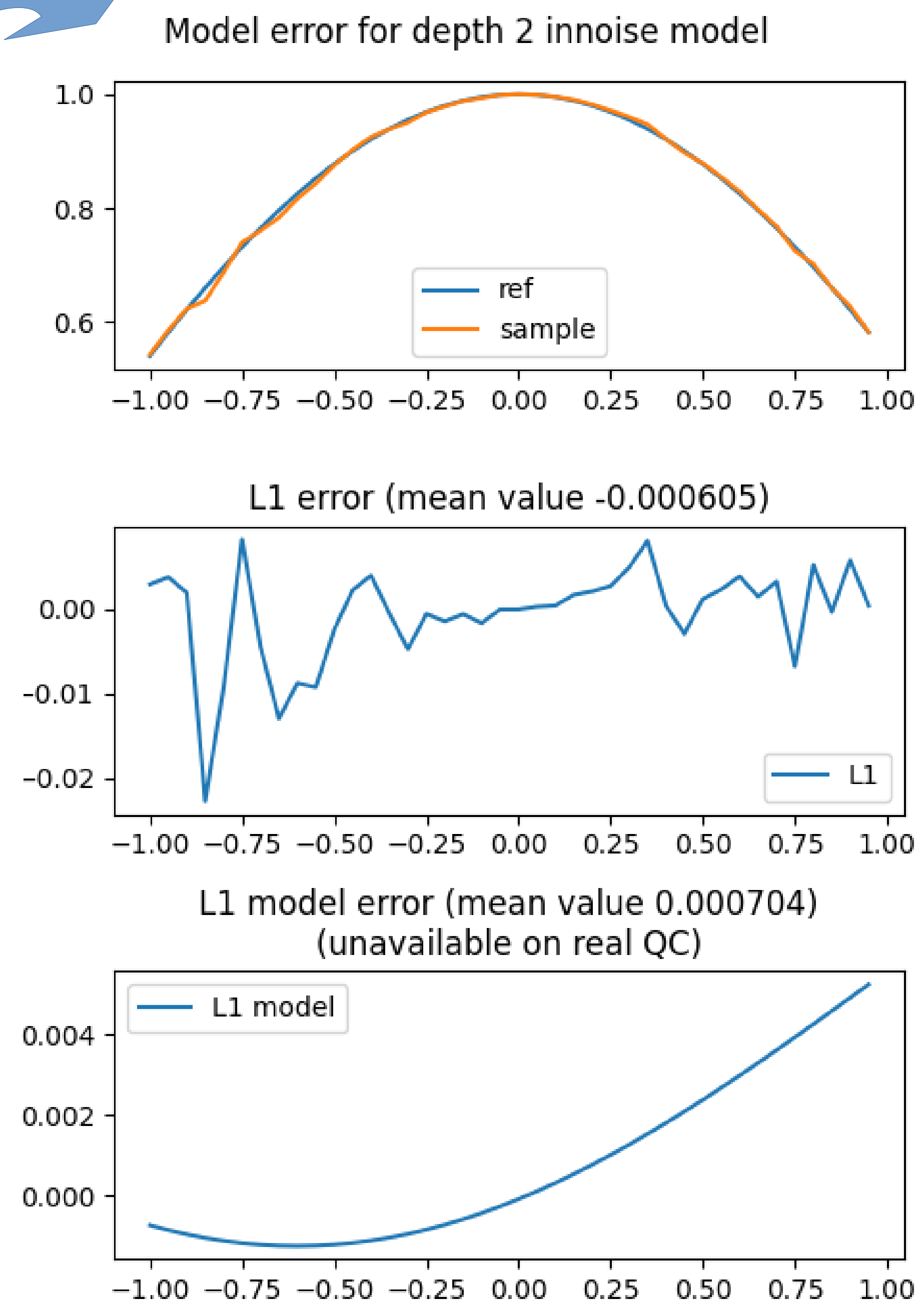
- Noise model : angle shift + Gaussian noise
- ML circuit : re-uploader of depth 2
- Target circuit : $R_X(t)$

Γ performance

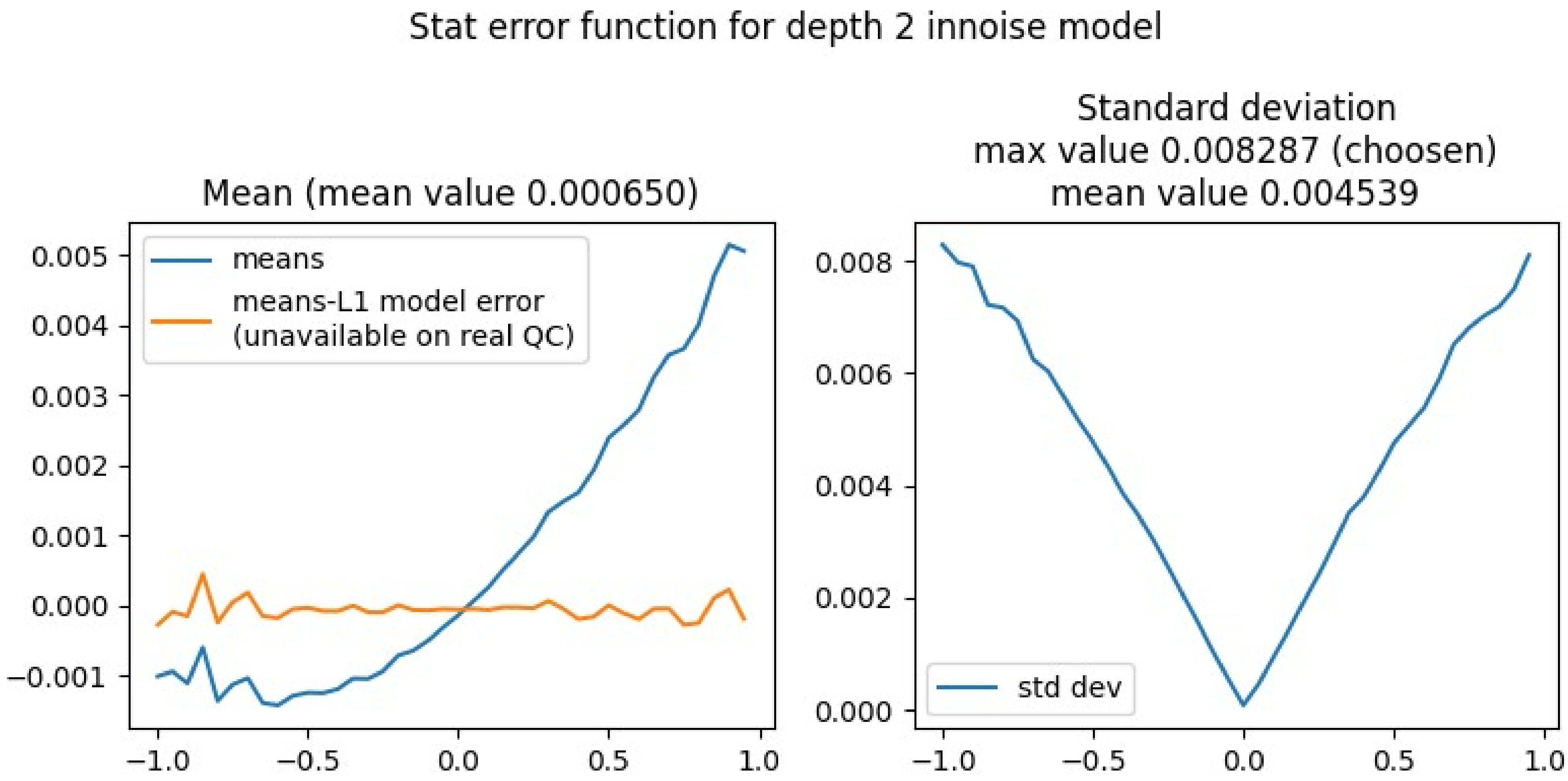
Correction



Observed systematic error



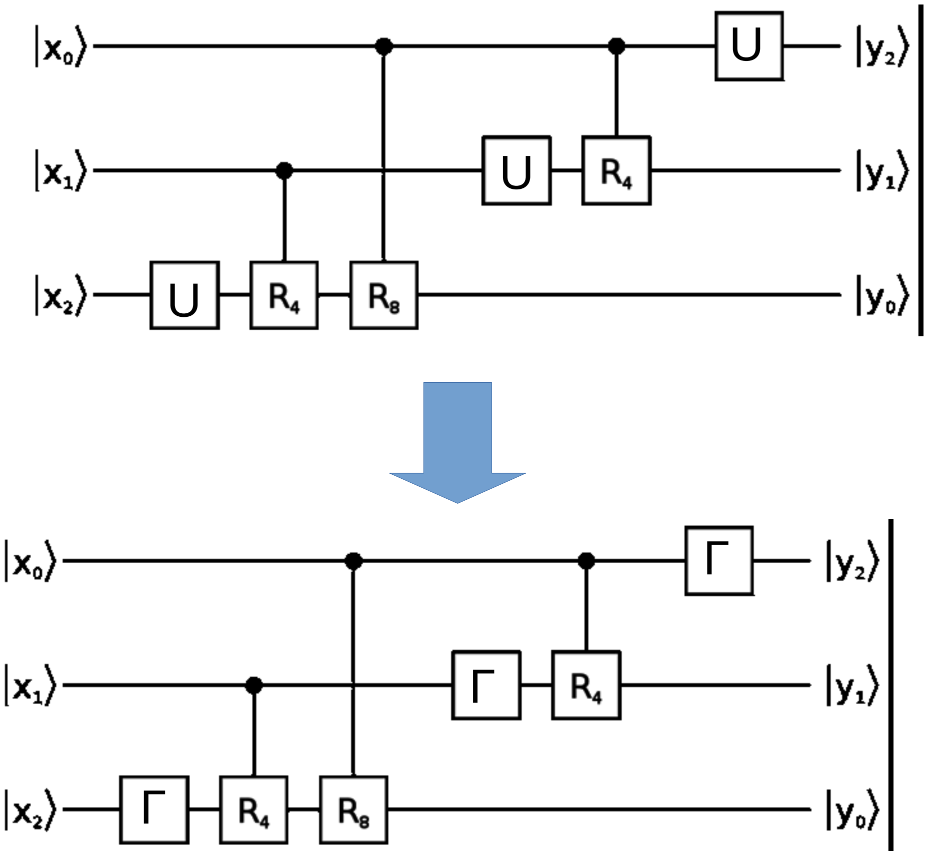
Stat Error Measurement



Γ is a corrected operator

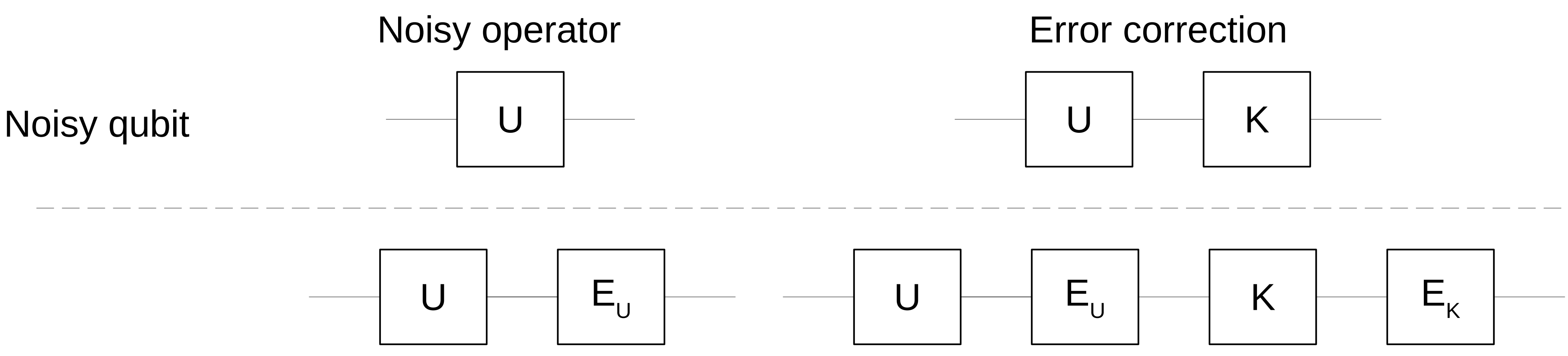
- Γ can be used instead of the initial target circuit
- But long circuit especially for complicated functions
  - increase the stat. error (decoherence)

→ try to find a compact correction operator



Mono-operator approach

- Characterize numerically the systematic noise
  - Evaluating error of a target circuit U
  - Calculating K_U, the correction operator for U



Charaterize the syst. error

- E generic operator equivalent to

  -Rz(Φ)-Ry(θ)-Rz(λ)- circuit

$$E = \begin{bmatrix} e^{-i\frac{\phi+\lambda}{2}} cos(\frac{\theta}{2}) & -e^{-i\frac{\phi-\lambda}{2}} sin(\frac{\theta}{2}) \\ e^{i\frac{\phi-\lambda}{2}} sin(\frac{\theta}{2}) & e^{i\frac{\phi+\lambda}{2}} cos(\frac{\theta}{2}) \end{bmatrix}$$

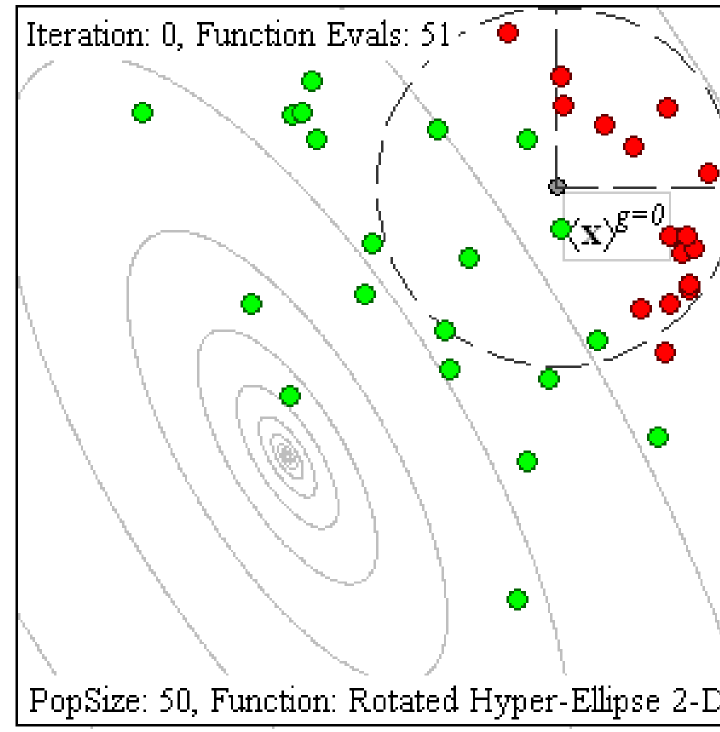
- System of equations to solve
  - $\langle 0|U^{+}E^{+}AEU|0\rangle = \langle 0|U^{+}AU|0\rangle$ (A=X,Y,Z axis)

    equations   measures on noisy qubit

- Problem : this system is really not smooth → need powerfull numerical technique

CMA-ES

- Covariance Matrix Adaptation Evolution Strategy
- Stochastic, derivative-free
- Generational adaptation of a population of points
- Elimination of worst point → covariance matrix estimation
- Very efficient if function is cheap to compute O(dim²)



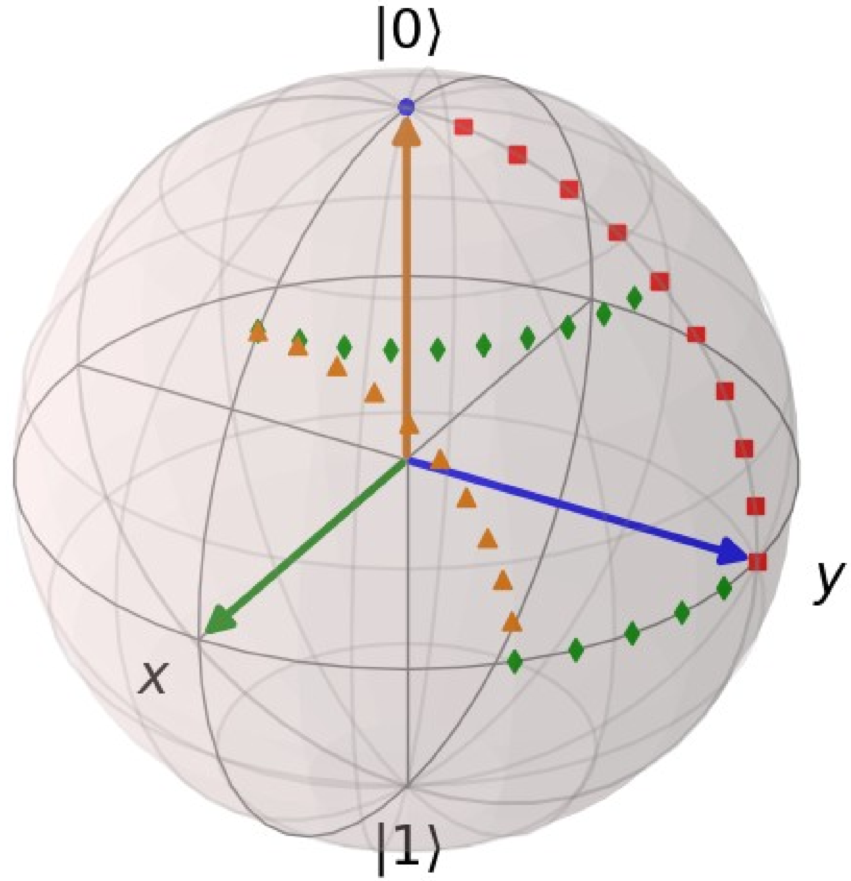
Hansen & Ostermeier, Completely Derandomized Self-Adaptation in Evolution Strategies, 2001

Non-unicity

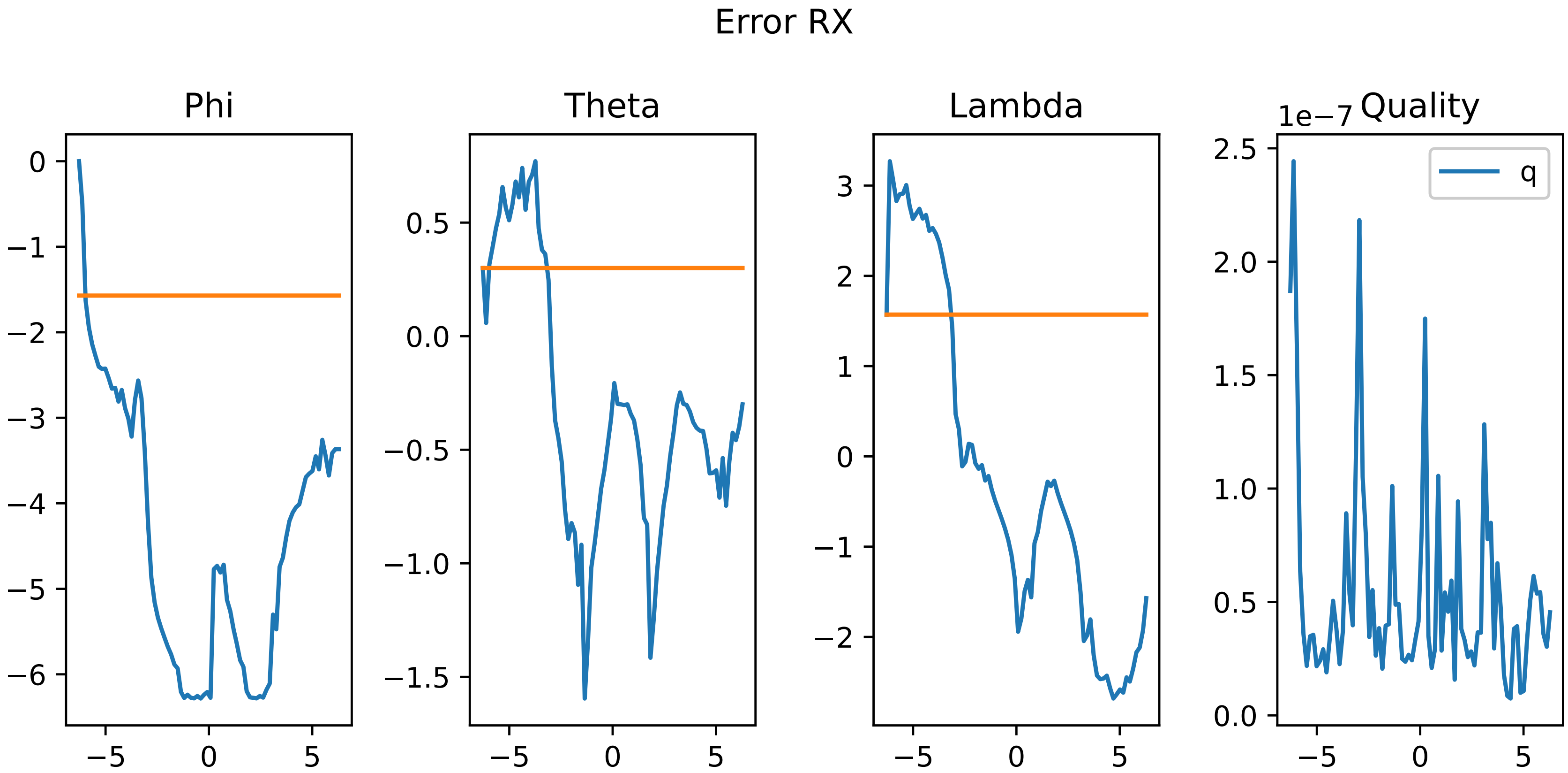
- E is not unique
- An infinity of 3 rotation paths join 2 points on the sphere

→ No unicity of the system solution

→ the noise model cannot be reverse fitted

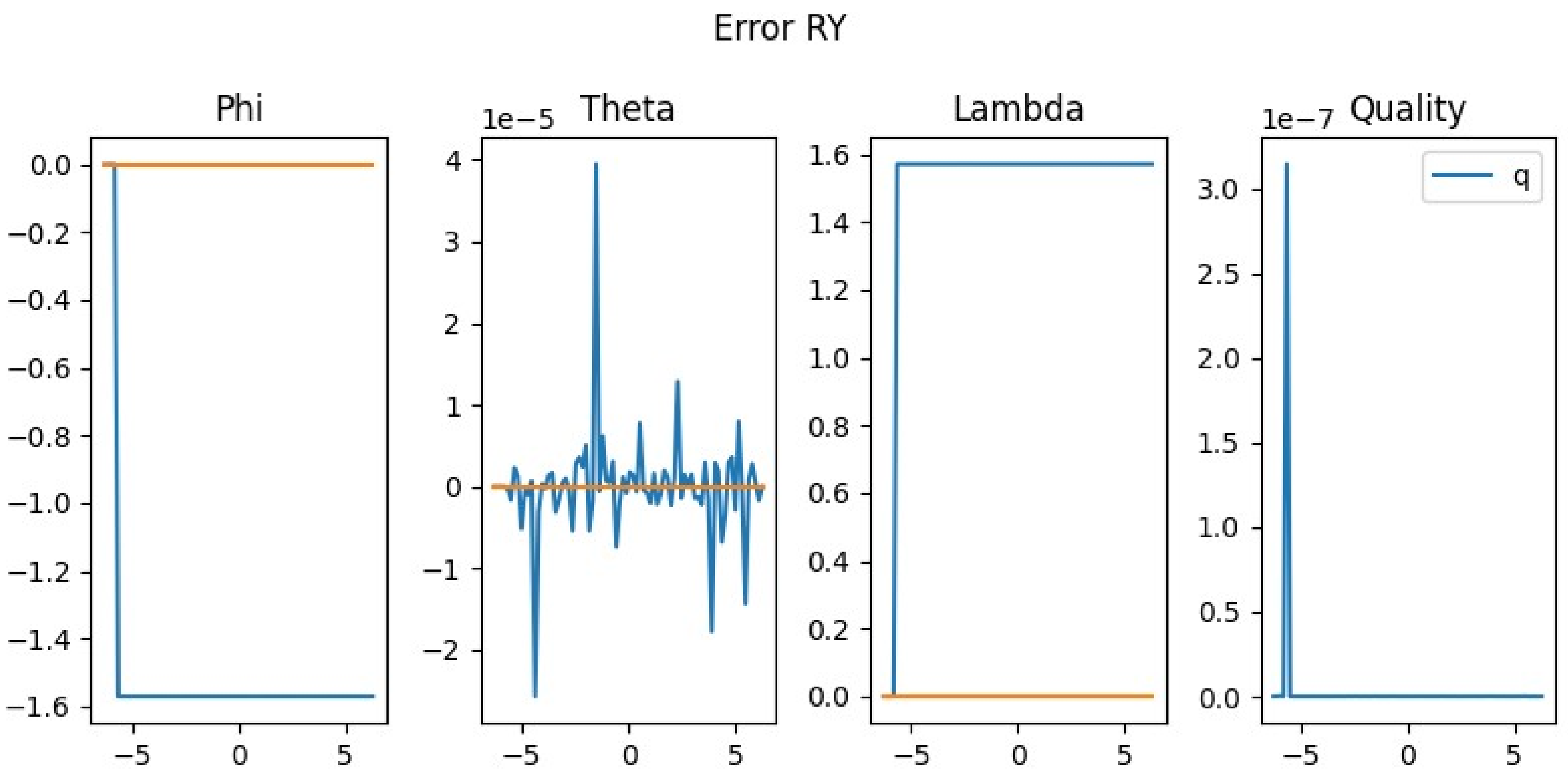


Result RX



- Yellow : theoritical result
- Blue : computed result
- Quality is good

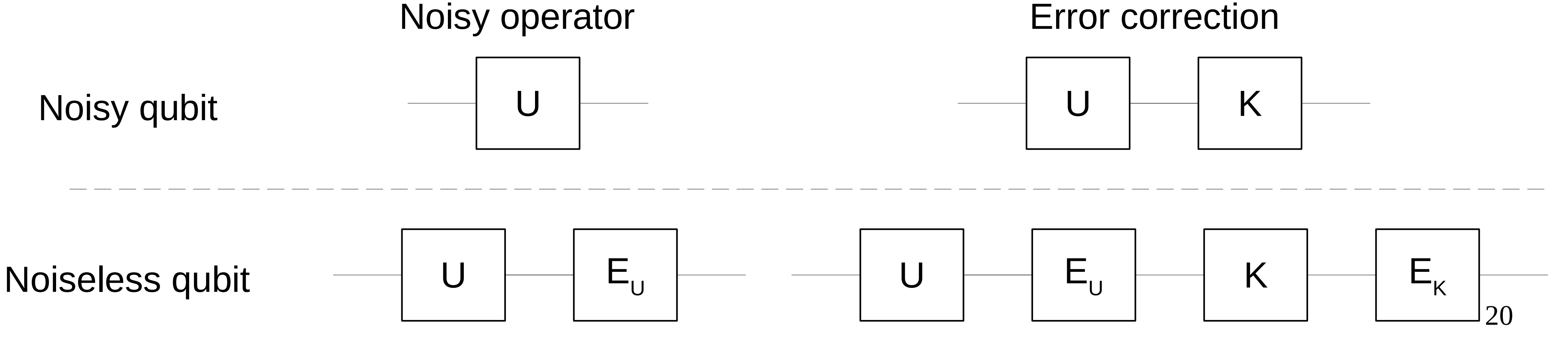
Result RY



The -1.6 RZ shift is compensated by a +1.6 RZ shift

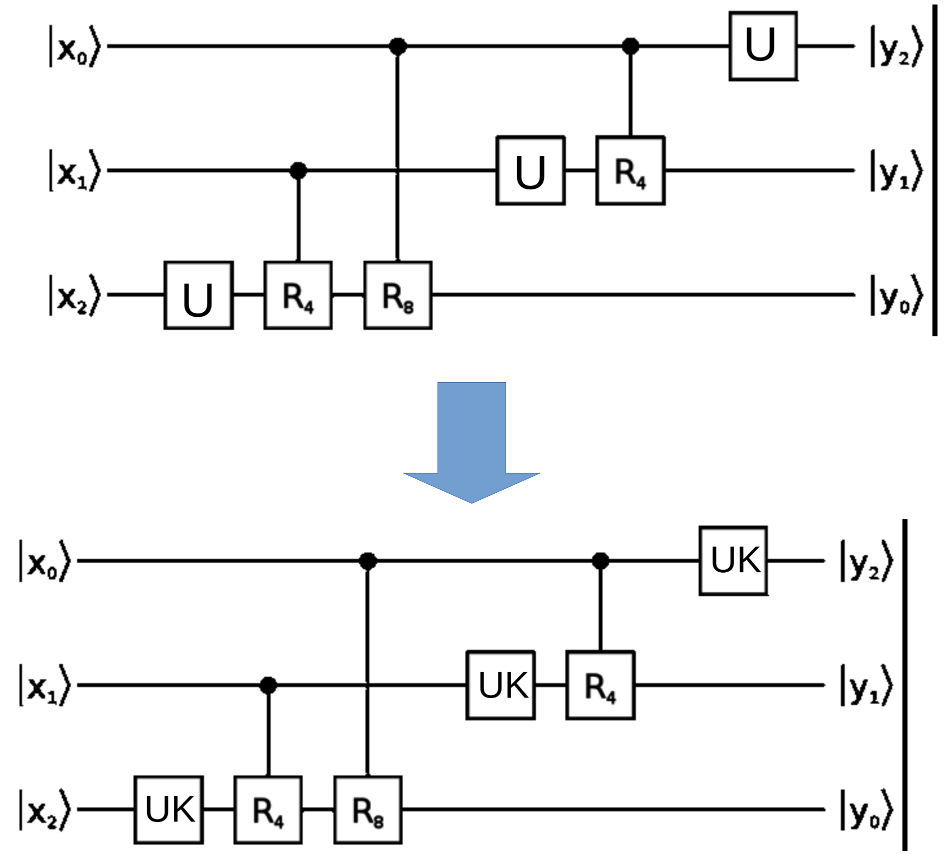
Computing correction

- Correction operator also induces an error E_K
- Matrix equation to solve : $E_K K E_U U = U$
- Unitarity → 2 complex equations → 4 real equations → solving with CMA-ES



Hadamard operator correction

- Evaluating K_H the Hadamard operator correction → coherent with target precision
- Could be extended to any U circuit
- Can be used as a correction in any circuit → improve performance
- minimal impact on circuit length (3 gates overhead per U)



H_KH noisy
X tensor(0.9978, dtype=torch.float64)
Y tensor(-0.0383, dtype=torch.float64)
Z tensor(0.0054, dtype=torch.float64)

H noiseless
X tensor(1.0000, dtype=torch.float64)
Y tensor(0., dtype=torch.float64)
Z tensor(0., dtype=torch.float64)

Perspectives

- Extending correction to arbitrary circuit
- Finalizing and releasing the error and correction python code
- Search for a collaboration with a QC company to test the method on a real machine
- Extending the technique to multi-qubits operators (CNOT)