references

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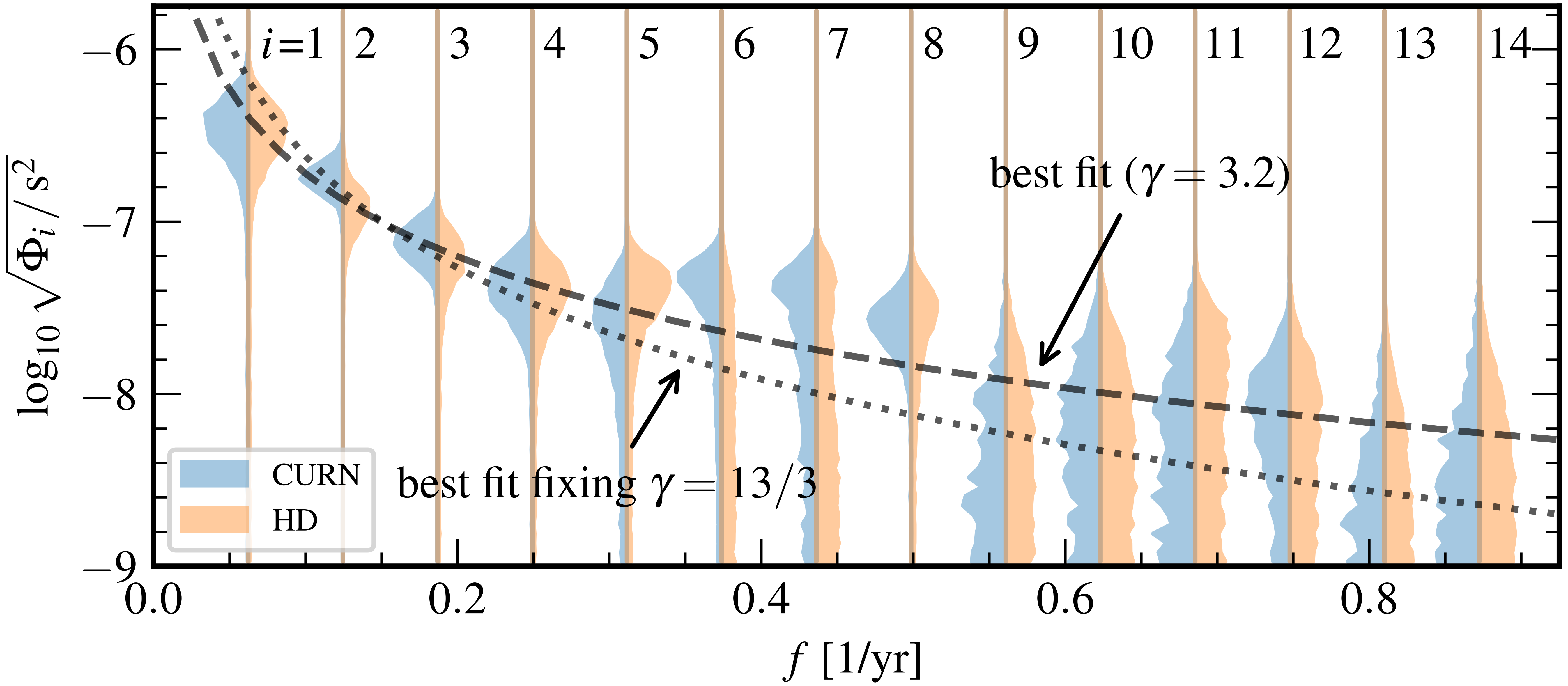
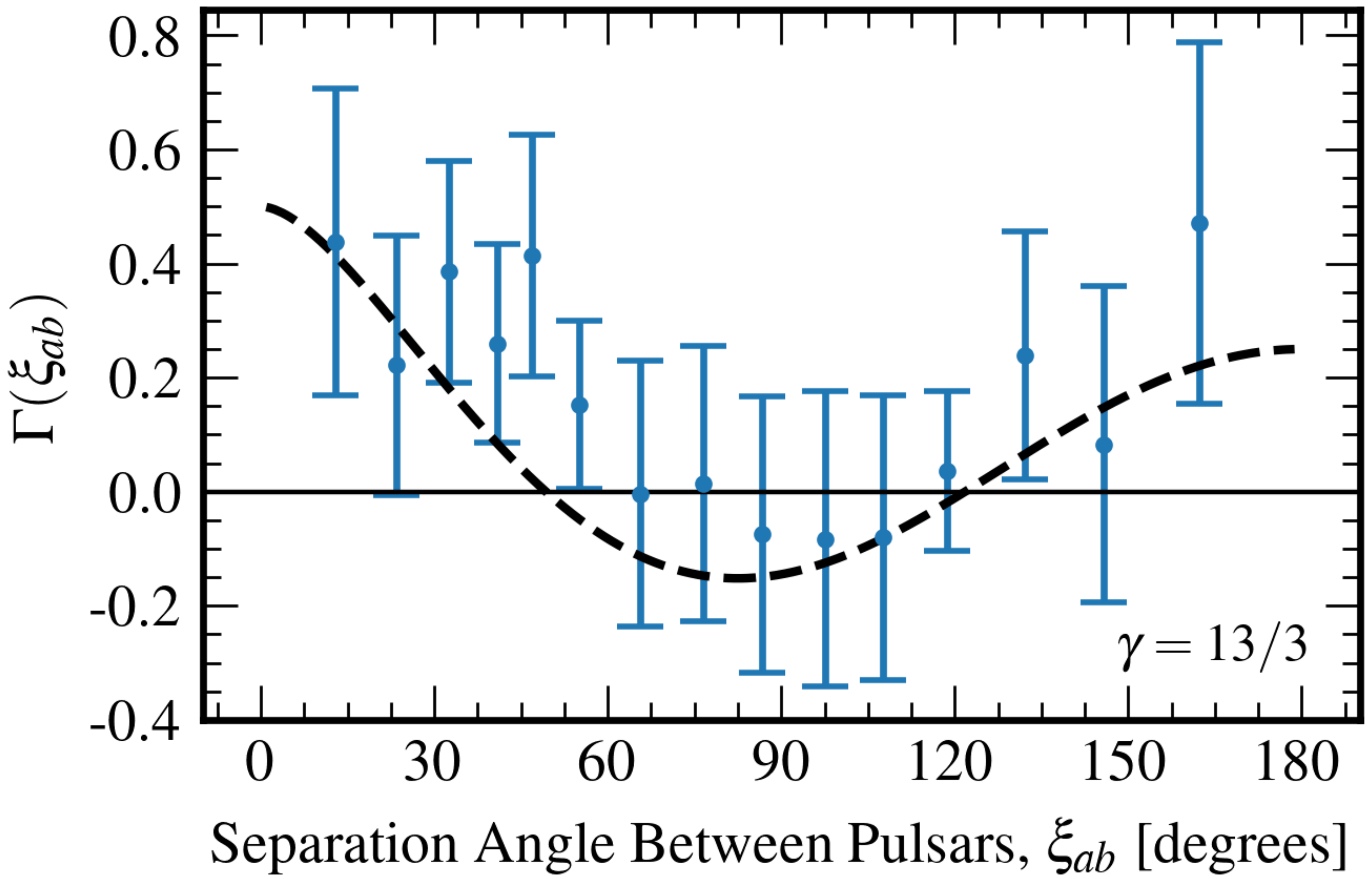
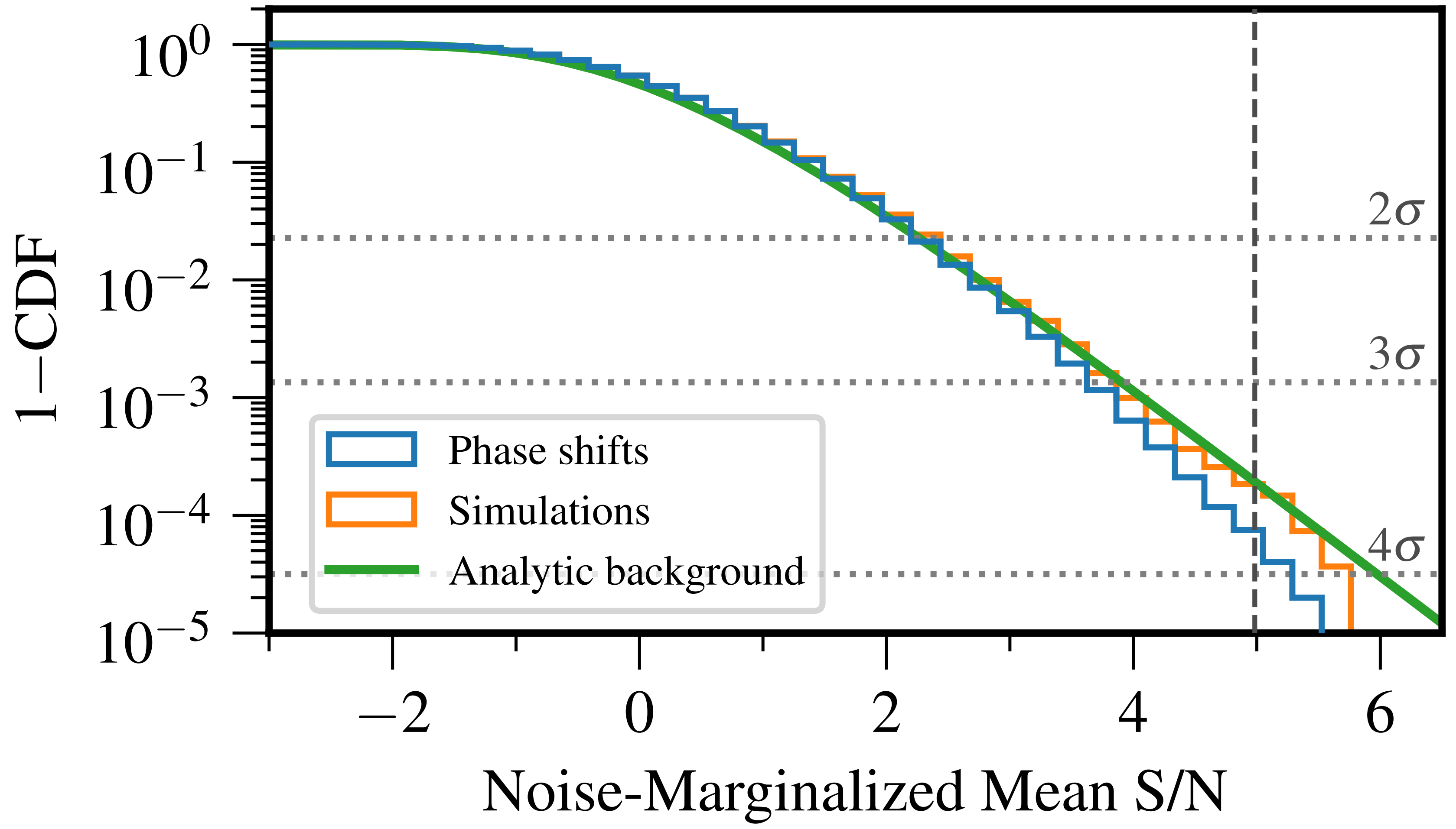
NANOGrav 15yr observations - GW background signal

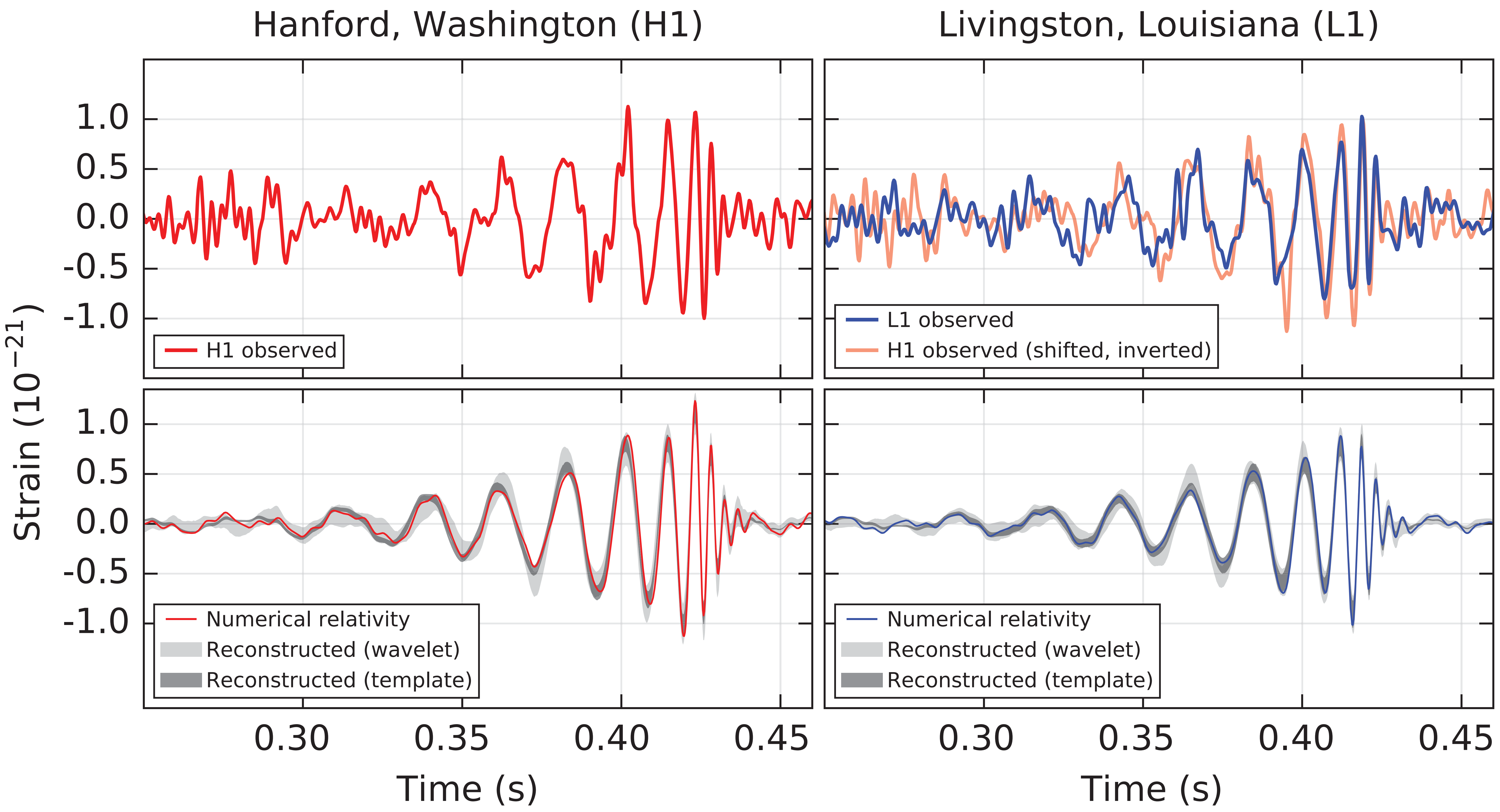


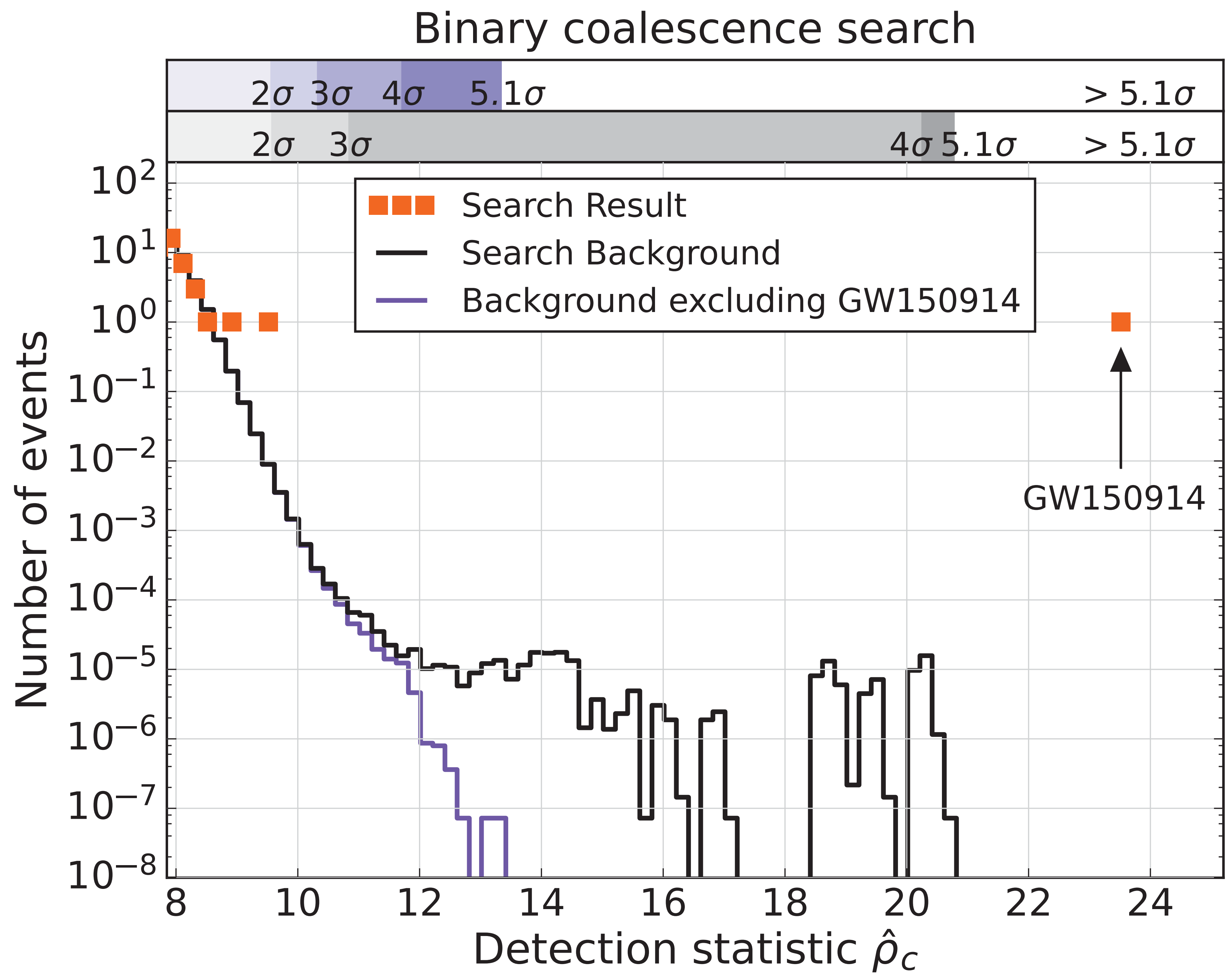




- **observed common spectrum consistent with predicted signal from population of SMBH binaries**
- **observed correlation pattern consistent with "stretching and squeezing" of space predicted by GR**
- **roughly 1/10,000 chance of observing such a large value of the detection statistic**

GW150914 - BBH “chirp” signal





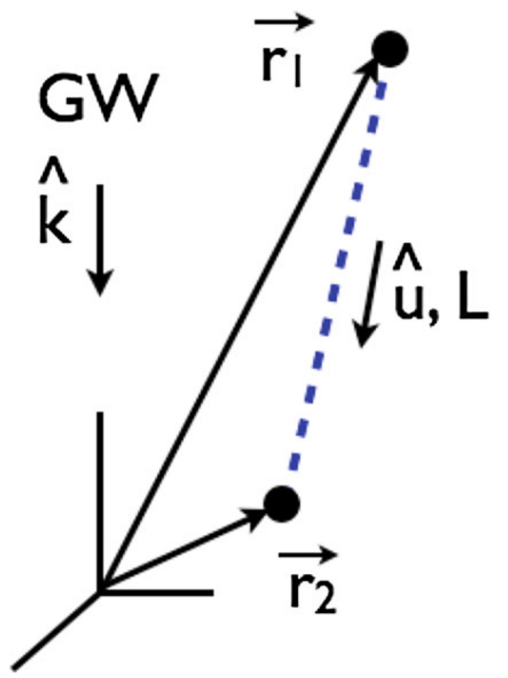
- observed data agrees with predicted GW signal from BBH merger

- observed data are consistent across detectors

- probability of observing such a large value of the detection statistic $< 1/5$ **million**

PTA observation	GW150914, etc
stochastic / persistent signal	deterministic / transient signal
power spectra & cross-correlations	waveforms & coincidence
combined signal from a population of approx monochromatic inspiraling binaries	single binary black hole merger
supermassive black holes (10^9 solar masses)	stellar mass black holes (1 - 100 solar masses)
nanohertz frequencies (10^{-9} - 10^{-7} Hz) [periods: months -> decades]	audio frequencies (10's - 1000 Hz)
galactic-scale detector using msec pulsars, with "arm" lengths ~100 - few x 1000 light-years	laser interferometers with km-scale arms
GW wavelength << arm length	GW wavelength >> arm length
"evidence for …" (3-4 sigma)	"detection of …" (>5 sigma)

other items of interest



- timing residual response:

$$\Delta T(t) = \frac{1}{2c} u^i u^j \int_0^L \mathrm{d}s\, h_{ij}(t(s), \vec{x}(s)) = \int \mathrm{d}f \int \mathrm{d}\hat{k} \sum_{A=+,\times} h_A(f,\hat{k}) R^A(f,\hat{k}) \mathrm{e}^{i2\pi f t_2}$$

$$R^A(f,k) = \frac{1}{2}\left(\frac{u^i u^j e^A_{ij}(\hat{k})}{1-\hat{k}\cdot\hat{u}}\right)\frac{1}{i2\pi f}\left[1-\mathrm{e}^{-i2\pi f \frac{L}{c}(1-\hat{k}\cdot\hat{u})}\right]\mathrm{e}^{-i2\pi f \hat{k}\cdot\vec{r}_2/c} \quad \text{with} \quad F^A(k) = \frac{1}{2}\left(\frac{p^i p^j e^A_{ij}(\hat{k})}{1+\hat{k}\cdot\hat{p}}\right)$$

for the Earth-term-only response where $\hat{p} = -\hat{u}$

- Hellings and Downs correlation as special case of an "overlap function" for cross-correlated power:

$$C_{ab}(f) = \Gamma_{ab}(f)\, S_h(f)$$

$$\Gamma_{ab}(f) = \frac{1}{8\pi}\int \mathrm{d}\hat{k} \sum_{A=+,\times} R^A_a(f,\hat{k}) R^{A\star}_b(f,\hat{k}) \quad \text{with} \quad \Gamma_{ab} = \frac{1}{3}\left[\frac{1}{2} - \frac{1}{4}\left(\frac{1-\cos\gamma_{ab}}{2}\right) + \frac{3}{2}\left(\frac{1-\cos\gamma_{ab}}{2}\right)\ln\left(\frac{1-\cos\gamma_{ab}}{2}\right)\right](1+\delta_{ab})$$

- optimal cross-correlation detection statistic (to determine detection significance):

$$S = \frac{\sum_{ab}\rho_{ab}\Gamma_{ab}/\sigma^2_{ab,0}}{\sqrt{\sum_{ab}\Gamma^2_{ab}/\sigma^2_{ab,0}}} \quad \text{where} \quad \rho_{ab} = \delta t_a^T Q_{ab}\delta t_b \quad \text{with} \quad \langle\rho_{ab}\rangle = A^2_{\mathrm{gw}}\Gamma_{ab} \quad \text{and} \quad \sigma^2_{ab,0} = \langle\rho^2_{ab}\rangle_0$$

- optimal binned estimator of the Hellings and Downs correlation (for testing consistency with GWB model):

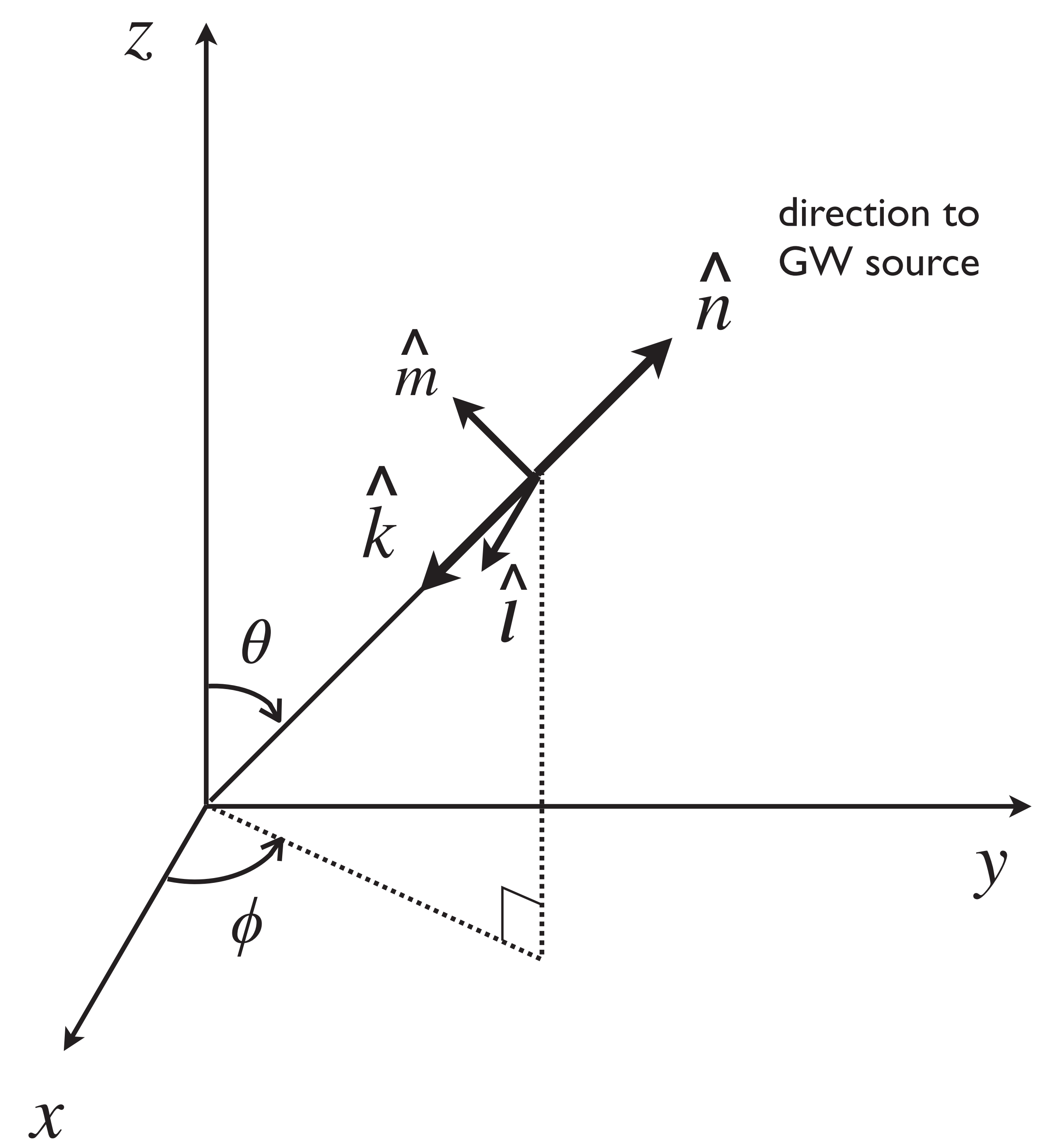
$$\Gamma_{\mathrm{opt}} = \frac{\Gamma_{\mathrm{bin}}}{A^2_{\mathrm{gw}}}\frac{\sum_{ab\in j}\sum_{cd\in j}\rho_{ab}C^{-1}_{ab,cd}\Gamma_{cd}}{\sum_{ab\in j}\sum_{cd\in j}\Gamma_{ab}C^{-1}_{ab,cd}\Gamma_{cd}} \quad \text{where} \quad C_{ab,cd} = \langle\rho_{ab}\rho_{cd}\rangle - \langle\rho_{ab}\rangle\langle\rho_{cd}\rangle \quad \text{with} \quad \langle\Gamma_{\mathrm{opt}}\rangle = \Gamma_{\mathrm{bin}} \quad \text{and} \quad \sigma^2_{\mathrm{opt}} = \frac{\Gamma^2_{\mathrm{bin}}}{A^4_{\mathrm{gw}}}\frac{1}{\sum_{ab\in j}\sum_{cd\in j}\Gamma_{ab}C^{-1}_{ab,cd}\Gamma_{cd}}$$

extra slides

Plane wave expansion, ensemble average

- Plane wave expansion:

$$h_{ij}(t, \vec{x}) = \int_{-\infty}^{\infty} \mathrm{d}f \int \mathrm{d}^2\Omega_{\hat{k}} \sum_{A=+,\times} h_A(f, \hat{k}) e_{ij}^A(\hat{k})\, \mathrm{e}^{i2\pi f(t - \hat{k}\cdot\vec{x}/c)}$$



- Polarization tensors:

$$e_{ij}^{+}(\hat{k}) = \hat{l}_i\hat{l}_j - \hat{m}_i\hat{m}_j\,, \qquad e_{ij}^{\times}(\hat{k}) = \hat{l}_i\hat{m}_j + \hat{m}_i\hat{l}_j$$

$$\hat{k} = -\,\hat{r}\,, \quad \hat{l} = -\,\hat{\phi}\,, \quad \hat{m} = -\,\hat{\theta}$$

- Ensemble average (for an unpolarized, isotropic, stationary-Gaussian GWB):

$$\langle h_A(f, \hat{k}) \rangle = 0$$

$$\langle h_A(f, \hat{k}) h_{A'}^{\star}(f', \hat{k}') \rangle = \frac{1}{16\pi} S_h(f) \delta(f - f') \delta_{AA'} \delta^2(\hat{k}, \hat{k}')$$

Simple example: Omni-directional detectors

$$R_a^A(f, \hat{k}) = C\, \mathrm{e}^{-i2\pi f \hat{k}\cdot \vec{x}_a / c}$$

$$\Gamma_{ab}(f) = \frac{1}{8\pi} \int \mathrm{d}\hat{k} \sum_A R_a^A(f, \hat{k}) R_b^{A\star}(f, k)$$

$$= \frac{C^2}{4\pi} \int \mathrm{d}\hat{k}\, \mathrm{e}^{-i\frac{2\pi f D_{ab}}{c} \hat{k}\cdot \hat{\Delta} x_{ab}}$$

$$= \frac{C^2}{4\pi} \int_0^{2\pi} \mathrm{d}\phi \int_{-1}^{1} \mathrm{d}(\cos\theta)\, \mathrm{e}^{i\frac{2\pi f D_{ab}}{c}\cos\theta}$$

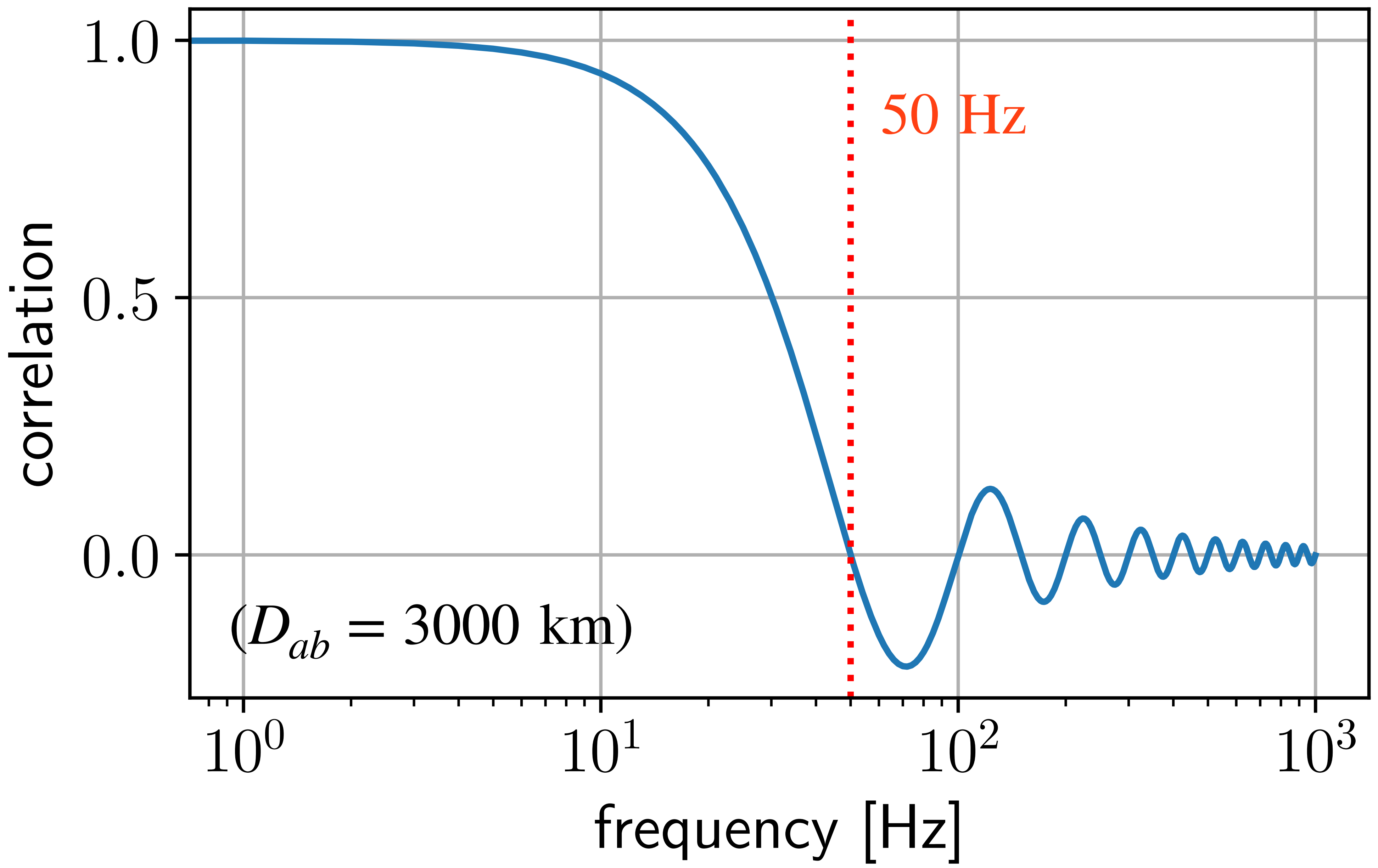
$$\ldots$$

$$= C^2 \operatorname{sinc}\left(\frac{2\pi f D_{ab}}{c}\right)$$

where

$$\vec{x}_a - \vec{x}_b \equiv D_{ab}\, \hat{\Delta} x_{ab}$$

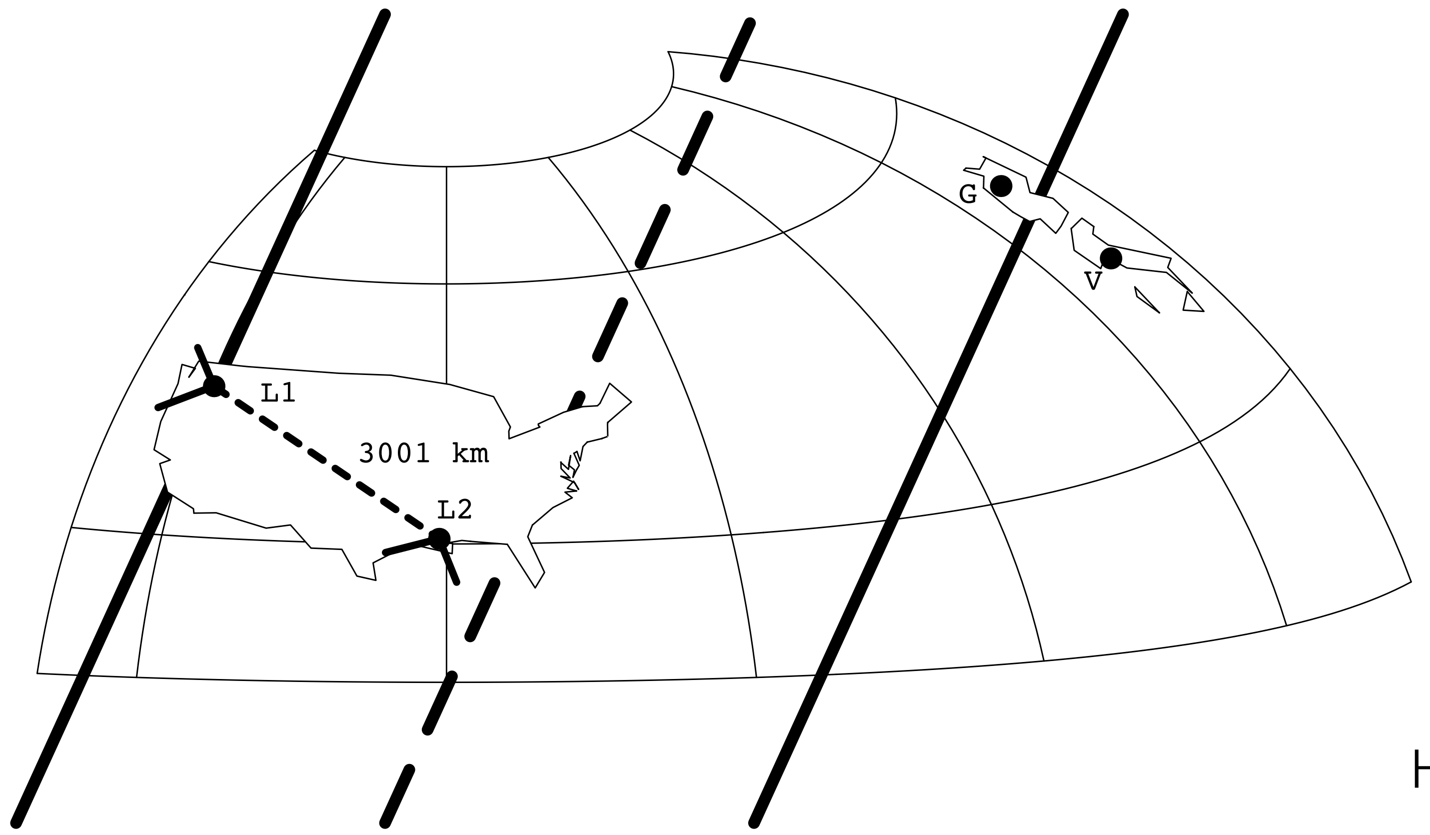
$$\operatorname{sinc} x \equiv \frac{\sin x}{x}$$



Example: Ground-based interferometers

$$R_a^A(f,\hat{k}) \simeq \frac{1}{2}(u_a^i u_a^j - v_a^i v_a^j)e_{ij}^A(\hat{k})\, \mathrm{e}^{-i2\pi f\hat{k}\cdot\vec{x}_a/c}$$

(short-antenna limit)



(B. Allen, Les Houches 1995)