Gravitational Waves and Cosmology

2nd Manitou School July 7th 2023

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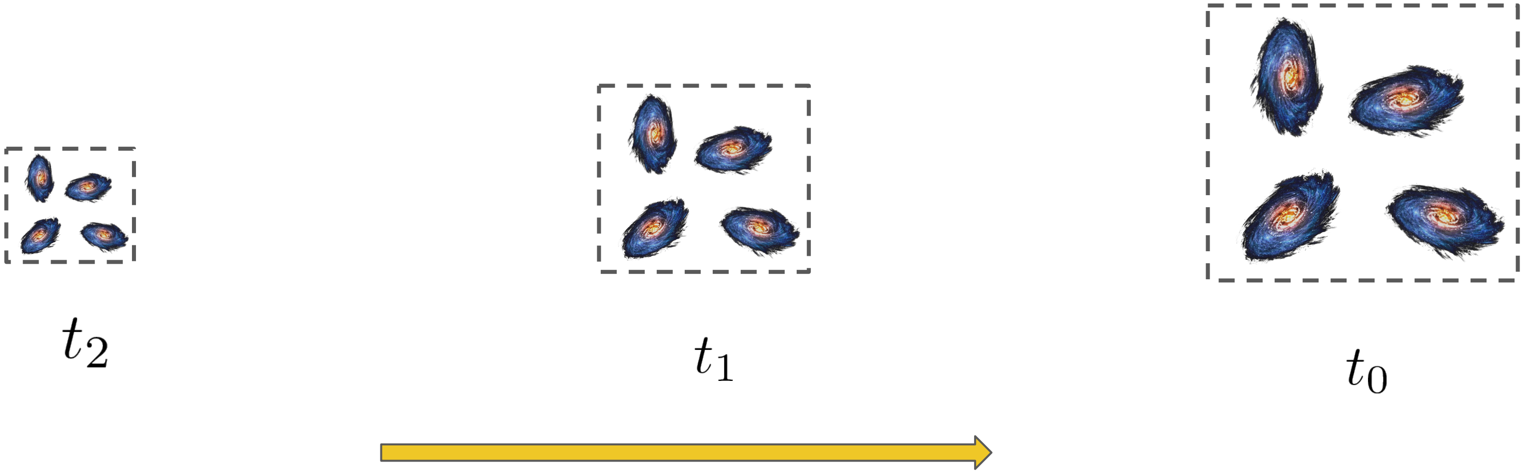
Material for cosmology with GWs

- S. Vitale, “Inferring the Properties of a Population of Compact Binaries in Presence of Selection Effects”, Handbook of GW astronomy (2021).
- S. Mastrogiovanni, “Cosmology with Gravitational Waves: A review”, Annalen der Physik (2022)
- J. Gair, “The Hitchhiker's guide to the galaxy catalog approach for gravitational wave cosmology”, arXiv:2212.08694 (Accepted AJ)
- S. Mastrogiovanni, “ICAROGW: A python package for inference of astrophysical population properties of noisy, heterogeneous and incomplete observations”, arXiv:2305.17973
- S. Mastrogiovanni, “A novel approach to infer population and cosmological properties with gravitational waves standard sirens and galaxy surveys”, arXiv:2305.10488
- S. Mastrogiovanni, “Detection and estimation of the cosmic dipole with the Einstein Telescope and Cosmic Explorer”, MNRAS 914 2 (2022)

Introduction to cosmology

The universe expansion can be using a **scale factor.** Galaxies are placed on an expanding carpet. If we measure the distance between galaxies as a function of time, we find that it increases



t_2 t_1 t_0

Expansion

Introduction to cosmology

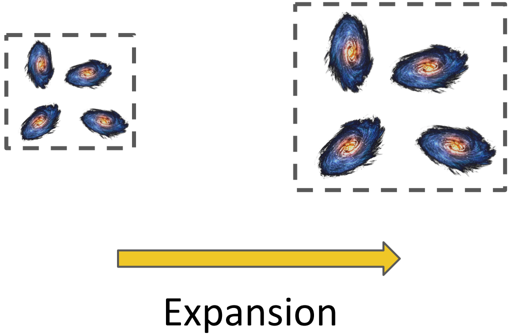
For an expanding universe, we need to define a **physical distance** and and **comoving distance.**

Physical distance **Scale factor**

$$d_p = a(t)d_c$$

Comoving distance

$$\dot{d}_p = \dot{a}(t)d_c + a(t)\dot{d}_c$$



Hubble's parameter

$$\dot{d}_p = \frac{\dot{a}(t)}{a(t)}a(t)d_c = H(t)d_p$$

Hubble's Law

In an expanding Universe, objects naturally run-away from each other with a velocity proportional to their distance.

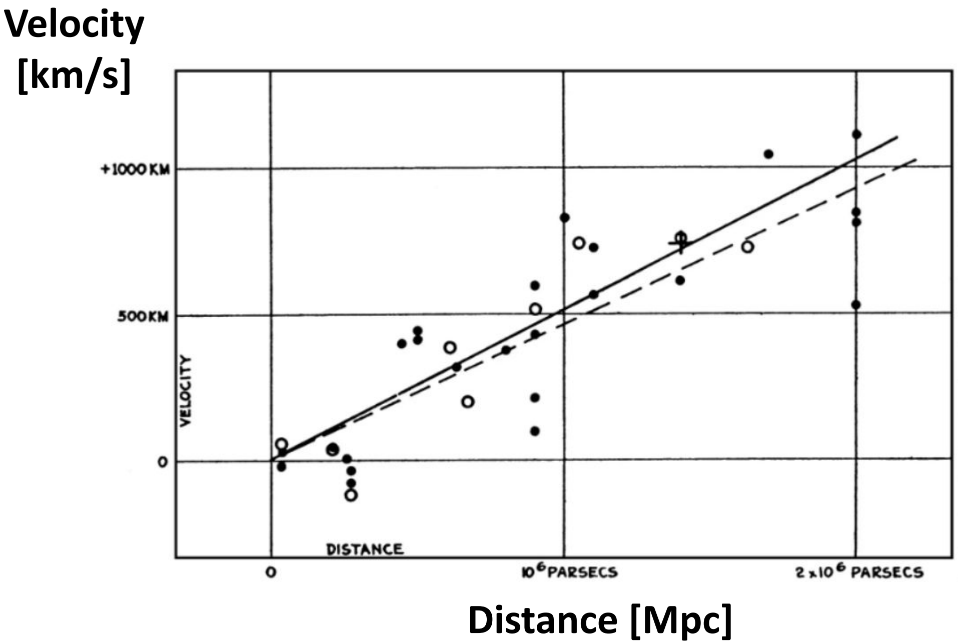
Introduction to cosmology

First observations by Hubble have shown that further galaxies display an higher recessional velocity.

At low distances, the Hubble parameter is constant = > **Hubble constant**

At high distances, the Hubble parameter is not constant anymore (depends on other cosmological parameters)

$$\left[\frac{\dot{a}}{a}\right]^2 = H_0^2[\Omega_m(1+z)^3+\Omega_\Lambda]$$



From Friedman's Equations

Introduction to cosmology

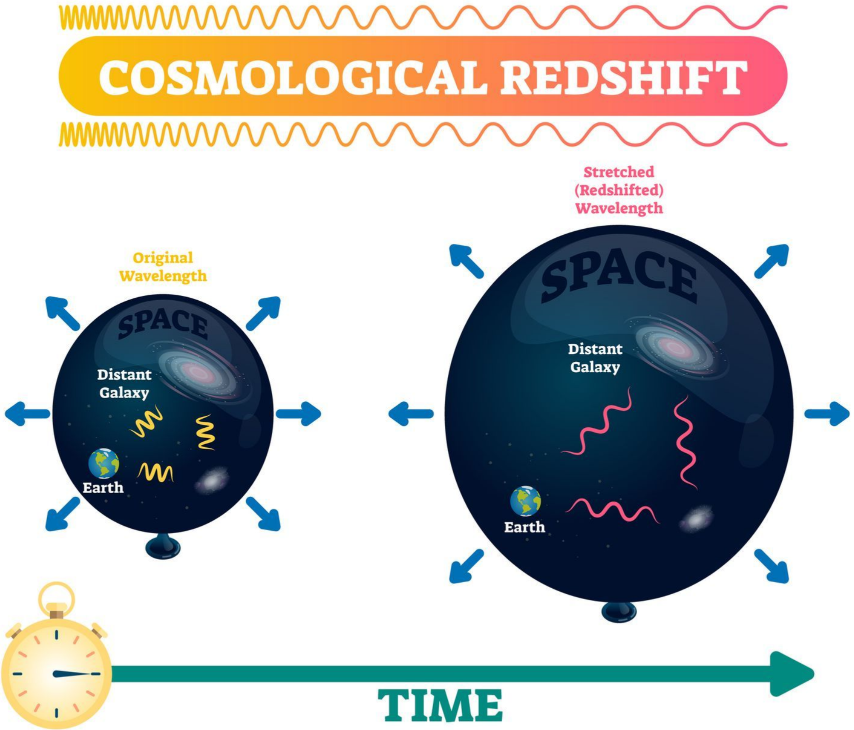
Waves travelling in an expanding universe are redshifted. This can be demonstrated from the definition of comoving distance.

$$\int_{t_s+dt_s}^{t_d+dt_d} \frac{dt'}{a(t')} = \int_{t_s}^{t_d} \frac{dt'}{a(t')}$$

Comoving distance pulse 1 **Comoving distance pulse 2**

$$\frac{dt_d}{a(t_d)} = \frac{dt_s}{a(t_s)}$$

$$dt_d = dt_s \frac{a(t_d)}{a(t_s)} = dt_s(1+z)$$



Introduction to cosmology

In cosmology exists several definitions of distances. The last definition we will see in the luminosity distance.

Luminosity at observer

Luminosity at source

$$\mathcal{F} = \frac{L_o}{4\pi d_c^2} = \frac{L_s}{4\pi d_c^2 (1+z)^2}$$

Flux

$$L_o = \frac{dE_o}{dt_o} = \frac{1}{1+z}\frac{d[h\nu_o]}{dt_s} = \frac{1}{(1+z)^2}\frac{dE_s}{dt_s}$$

If we know the intrinsic luminosity of a source we can measure its **luminosity distance**

$$d_l \equiv d_c(1+z)$$

$$d_c = \frac{c}{H_0}\int_0^z \frac{dz'}{E(z)}$$

Introduction to gravitational waves cosmology

Gravitational Waves at cosmological distances have special properties

$$\frac{df_\mathrm{s}}{dt_s} = \frac{96\pi^{8/3}}{5}(G\mu)^{5/3} f_\mathrm{s}^{11/3}$$

GW frequency evolution at source

$$\frac{dt_\mathrm{s}}{dt_\mathrm{o}} = \frac{f_\mathrm{o}}{f_\mathrm{s}} = (1+z)^{-1}$$

Cosmological redshift

GW frequency evolution at source

$$\frac{df_\mathrm{o}}{dt_o} = \frac{96\pi^{8/3}}{5}(G\mu_0)^{5/3} f_\mathrm{o}^{11/3}$$

$$\mu_o = \mu_s(1+z)$$

Detector’s chirp mass

The frequency evolution of a GW is the same at the source anad at the detector, we just need to define a detector frame mass.

Introduction to gravitational waves cosmology

What about the amplitude of a GW?

$$|h| \propto \frac{\mu_{\rm s}^{5/3}}{d} f_{\rm s}^{2/3} \sin(\phi_{\rm s})$$

Amplitude at the source

We need to replace all the source quantities with detector quantities and we will note that

$$|h| \propto \frac{\mu_{\rm o}^{5/3}}{d(1+z)} f_{\rm o}^{2/3} \sin(\phi_{\rm o}) = \frac{\mu_{\rm o}^{5/3}}{d_L} f_{\rm o}^{2/3} \sin(\phi_{\rm o})$$

From the GW it is possible to directly measure the luminosity distance of the source

Introduction to gravitational waves cosmology

Let’s and summarize what we have learnt

- From the GW sourced by a CBC at cosmological distance, we can measure the detector masses and the luminosity distance.
- GWs are the only source for which we can measure directly a distance.
- If provided with a redshift (recessional velocity), GWs can be used to probe cosmology.

How do we get a redshift?

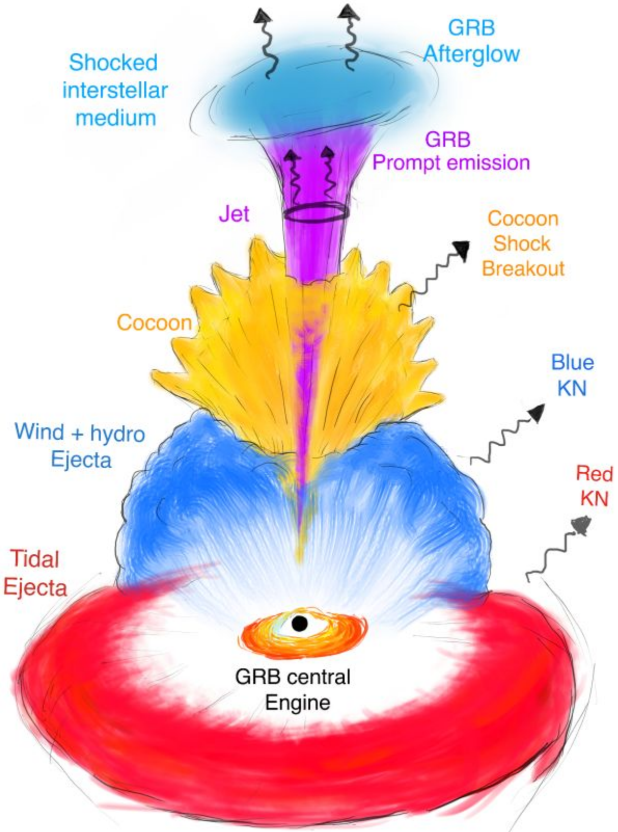
Let’s take a step back for a second

Introduction to gravitational waves cosmology

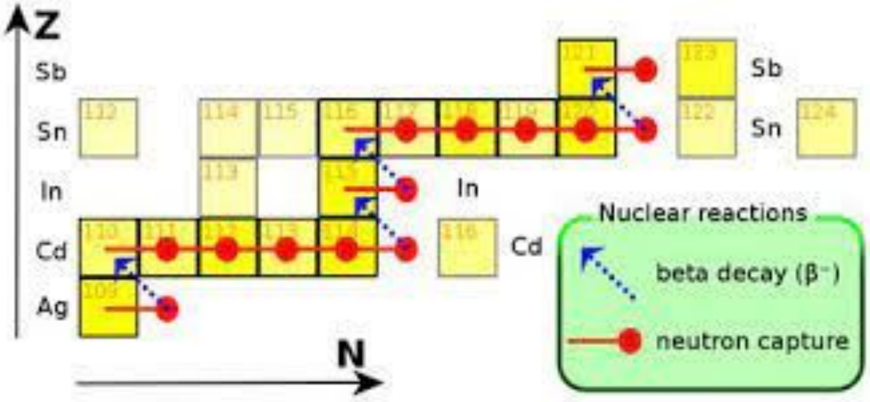
In recent years, we used several methods to assign a redshift to GW sources

- **Bright sirens:** An associated Electromagnetic (EM) counterpart (GRB, Kilonova etc...) can provide the identification of the host galaxy.
- **Dark sirens:** Galaxy surveys can be used to identify possible hosts in the GW localization volume.
- **Spectral sirens:** Knowledge of the source-frame mass distribution can be used to assign a redshift to GW sources.

Introduction to bright sirens

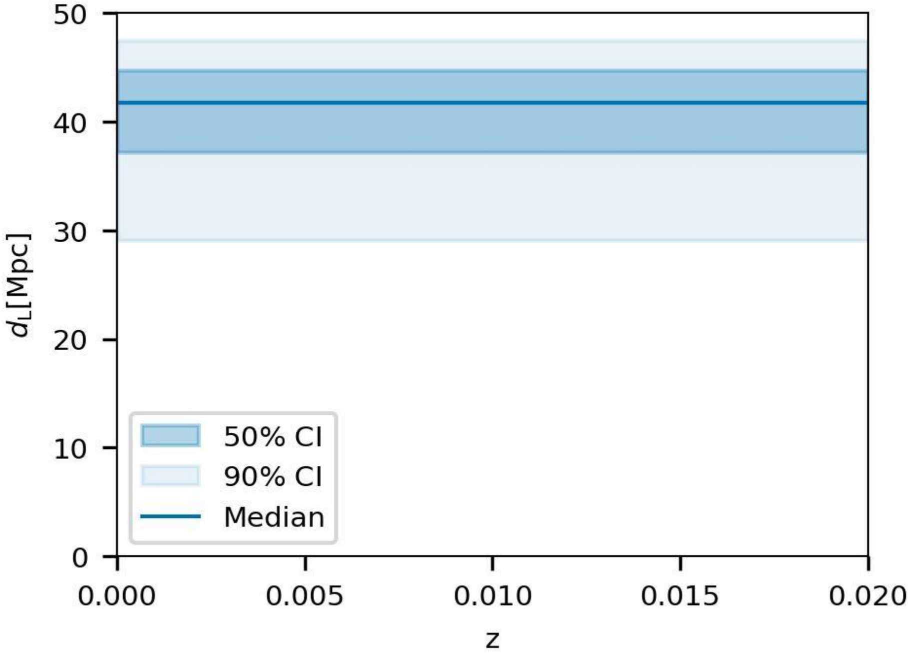


Ascenzi+, Journal of Plasma Physics, 87, 1 2021



Introduction to bright sirens

- **GW170817:** A binary neutron star merger detected by LIGO and Virgo. From the GW, source distance ~ 40 Mpc *[LVK+, ApJL, 848 (2017)]*.



Introduction to bright sirens

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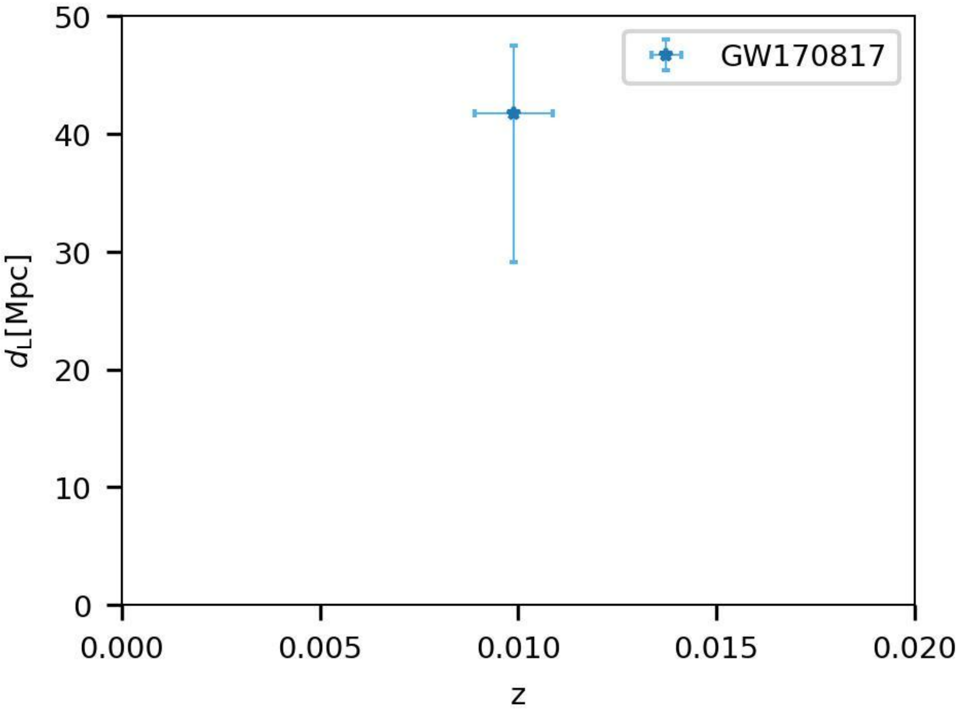

Introduction to bright sirens

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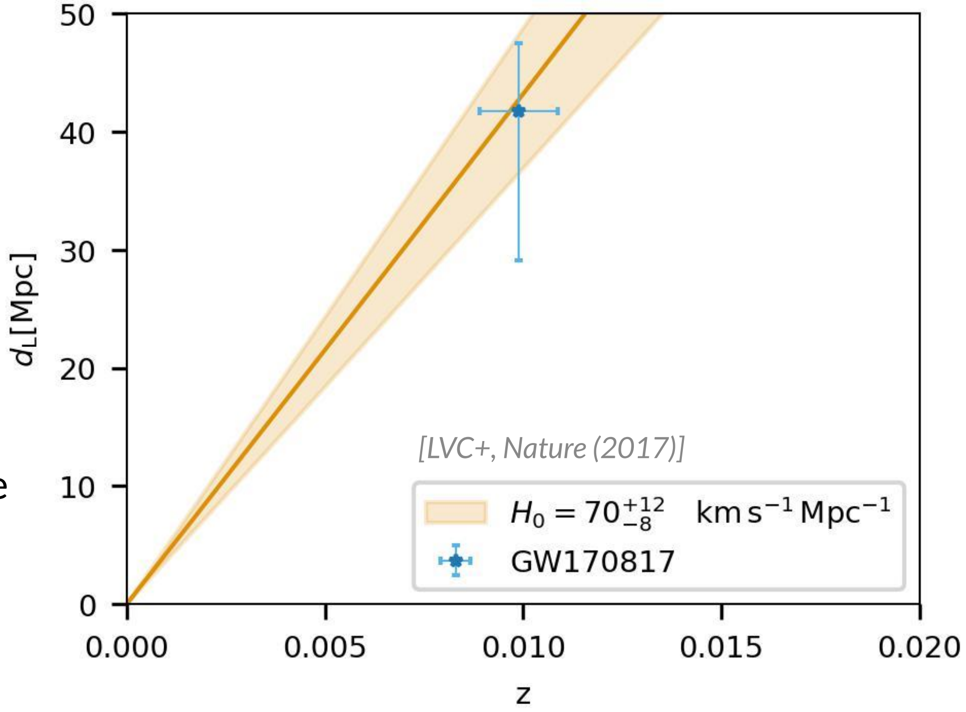
- This type of events will difficult to detect, we might expect to have 0-10 others in the next two observing runs *[SM+ A&A (2017), Patricelli+, MNRAS (2022)]*



[LVC+, Nature (2017)]

Introduction to bright sirens

In the case of an EM counterpart, we need to modify the hierarchical likelihood to take into account the fact that we also measure redshift and sky position from the EM counterpart

Overall likelihood

Rate of CBC with EM counterparts

$$\mathcal{L}(\{x\}|\Lambda) \propto \prod_{i}^{N_{\rm obs}} \frac{\int \mathcal{L}_{\rm EM+GW}(x_i|z,\Omega,\vec{m}) \frac{dN_{\rm CBC}(\Lambda)}{dz d\Omega d\vec{m} dt_s} \frac{1}{1+z} d\vec{m} dz d\Omega}{\int p_{\rm det}^{EM+GW}(\theta,z,\Lambda) \frac{dN_{\rm CBC}(\Lambda)}{dz d\Omega d\vec{m} dt_s} \frac{1}{1+z} d\vec{m} dz d\Omega}.$$

Detection probability

We make two assumptions:

- The detection probability is dominated by the GW.
- The EM and GW likelihoods can be separated in two independent terms.

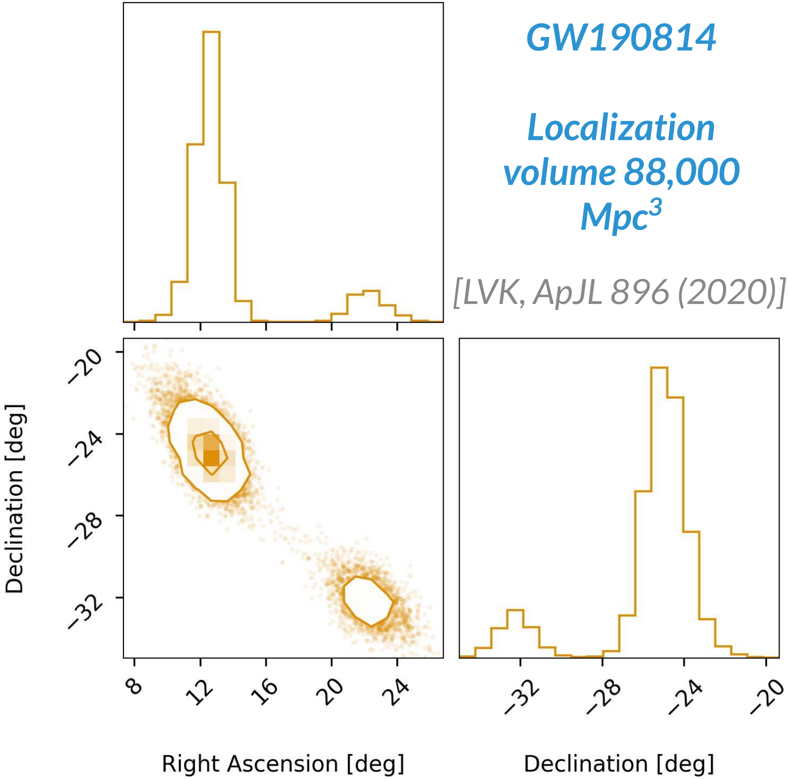
Introduction to dark sirens

- In the case that the GW is not observed with EM counterpart, we can use galaxy catalogs to identify possible galaxy hosts *[Schutz, Nature 1986]*.

- Galaxy surveys will provide possible redshifts.

- GW will provide luminosity distance.

- Best localized events provide better constraints for cosmology.



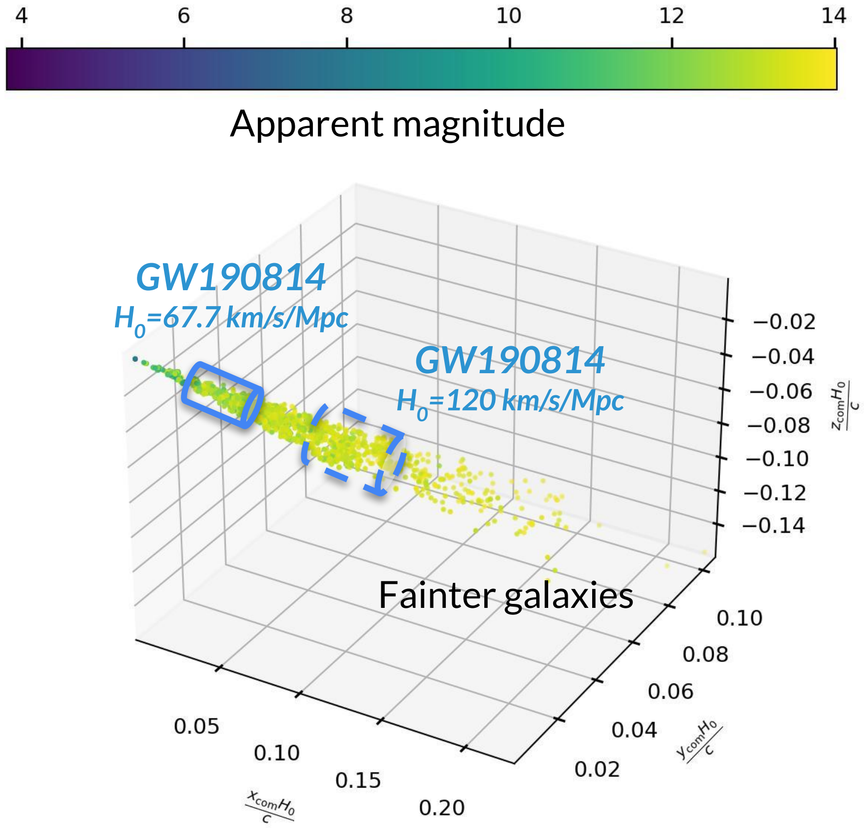
GW190814

Localization volume 88,000 Mpc^3

[LVK, ApJL 896 (2020)]

Introduction to dark sirens

- A cosmological model has statistical support when the GW localization matched an *overdensity* of galaxies.
- Galaxy catalogs are not complete at higher redshifts, we need to apply corrections in order to now bias our analyses *[R. Gray+, PRD (2019)]*



Introduction to dark sirens

The CBC likelihood is often parametrized in terms of redshift and **source-frame time**

CBC per galaxy per year

$$\frac{dN_{\rm CBC}(\Lambda)}{dz d\vec{m} d\vec{\chi} d\Omega dt_s} = R^*_{\rm gal,0}\psi(z;\Lambda)p_{\rm pop}(\vec{m},\vec{\chi}|\Lambda)\times$$

Term similar to the vanilla rate

$$\left[\frac{dV_c}{dzd\Omega}\phi_*(H_0)\Gamma_{\rm inc}(\alpha+\epsilon+1, x_{\rm max}(M_{\rm thr}), x_{\rm min})+\right.$$

Integral of Schecter function

Number density of galaxies per steradian (completeness correction)

$$\left.\frac{1}{\Delta\Omega}\sum_{j=1}^{N_{\rm gal}(\Omega)} f_L(M(m_j,z);\Lambda)p(z|z^j_{\rm obs},\sigma^j_{\rm z,obs})\right]$$

Number density of galaxies per steradian (catalog)

Luminosity weight

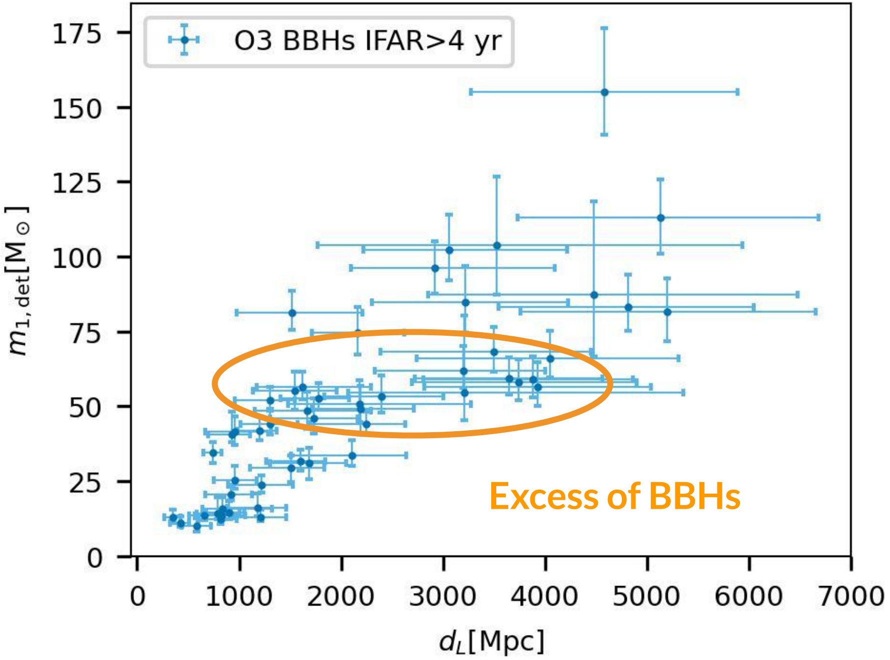
Galaxy localization in redshift

Sky pixel area

Introduction to spectral sirens

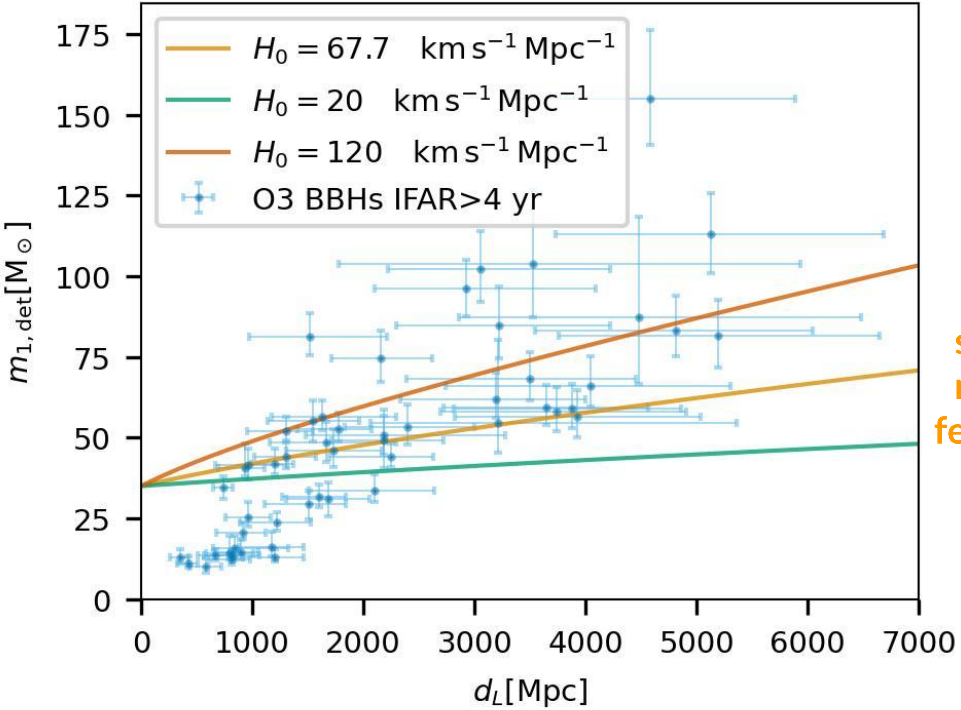
- Many GW are detected with large sky localizations and are very far (galaxy catalogs highly incomplete).
- If BBHs are *preferentially* produced at a given mass, we can exploit the mass-redshift relation to assign a redshift to the GW source *[SM+, PRD 104 (2021)]*.

$$m_{1,\mathrm{det}} = m_{1,\mathrm{s}}(1+z)$$



Introduction to spectral sirens

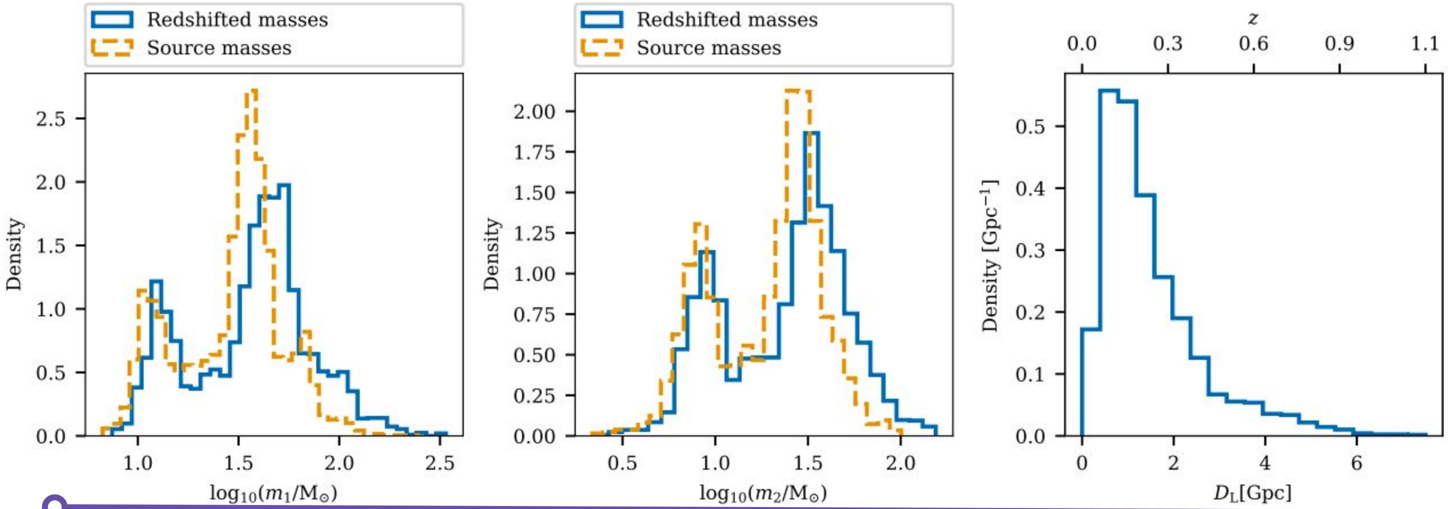
- If we assume an overdensity of BBHs produced at 35 solar masses, some “extreme” cosmologies can not fit the overdensity of BBHs.



35 solar mass feature

Latest LIGO, Virgo KAGRA GW results

- **During O1 (~4 months):**
  - 3 confident BBHs
- **During O2 (~8 months):**
  - 7 confident BBHs
  - 1 confident BNS+EM counterpart
- **During O3 (~12 months):**
  - 1 consistent with BNS masses (GW190425)
  - 4 events compatible with NSBH masses
  - 2 events compatible with BNS masses
  - ~80 confident BBHs.
  - Tentative EM counterpart from GW190521



See also Tab 1 from 2111.03604

Introduction to spectral sirens

The CBC likelihood is often parametrized in terms of redshift and **source-frame time**

$$\frac{dN_{\rm CBC}(\Lambda)}{d\vec{m}d\vec{\chi}d\Omega dz dt_s} = R_0$$

Rate of CBC [#mergers Gpc^-3 yr^-1]

$$\psi(z;\Lambda)$$

Rate evolution function, e.g. (1+z)^gamma. Two models available

$$p_{\rm pop}(\vec{m},\vec{\chi}|\Lambda)$$

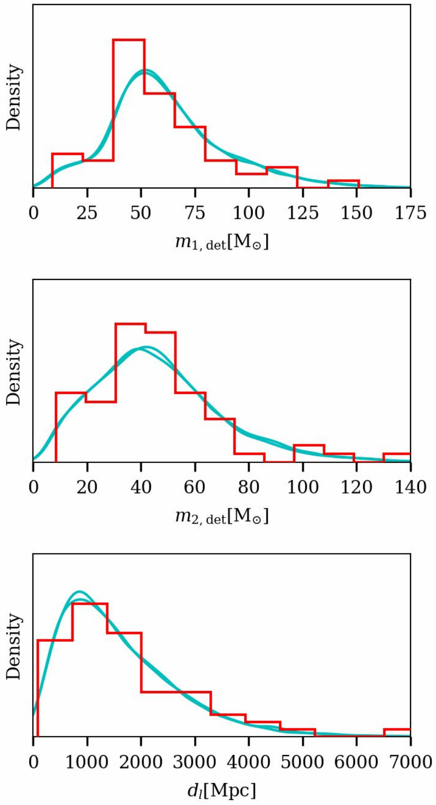
Probabilities for source-frame masses and spins, 8 models for masses, 2 for spins

$$\frac{dV_c}{dz d\Omega}$$

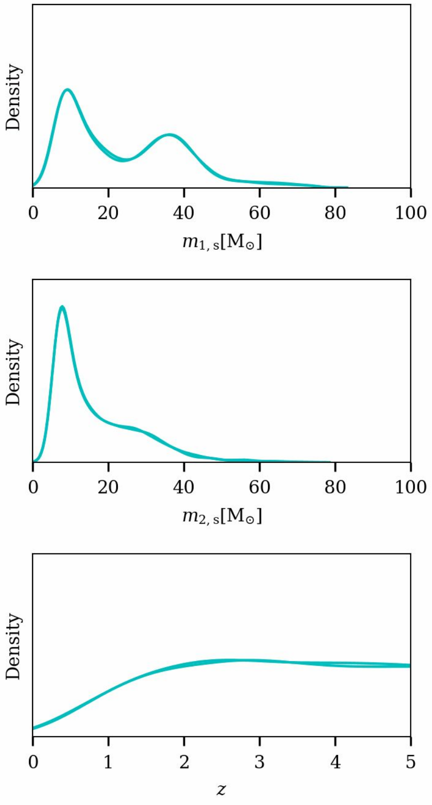
Comoving volume (depends on cosmology)

Introduction to spectral sirens

Detector Frame



Source Frame

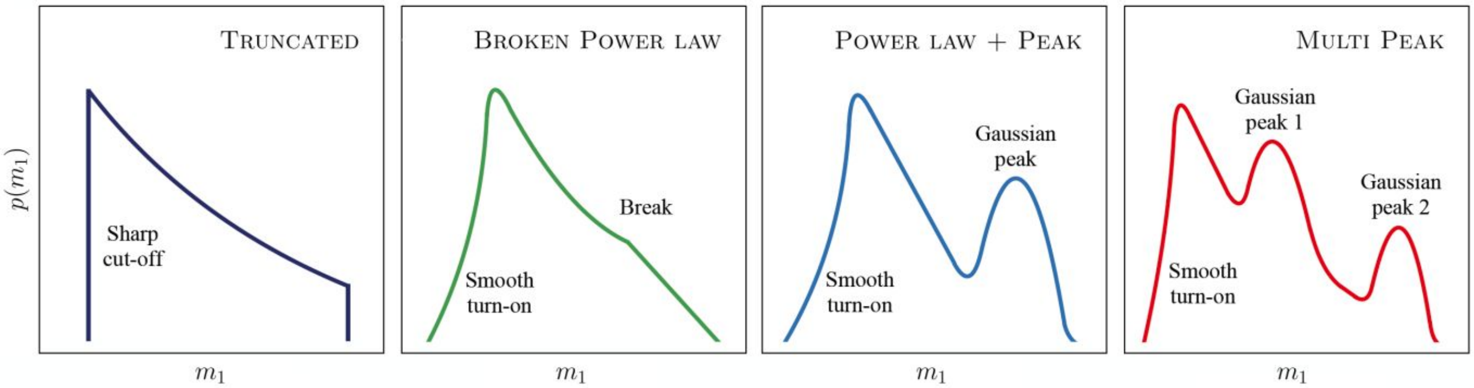


Cosmology

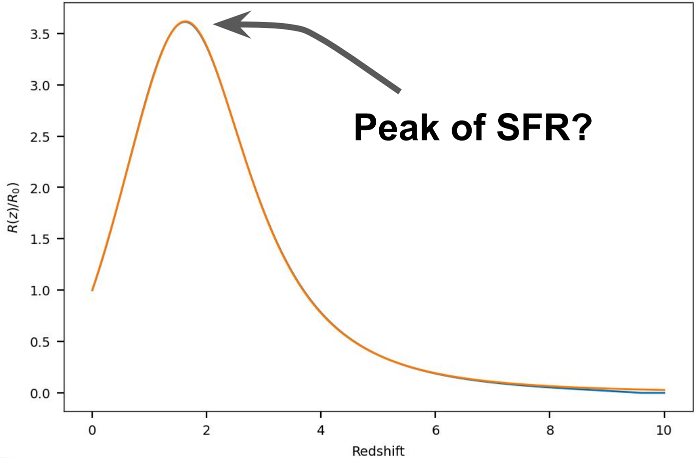
Selection biases

Simulation

Latest LIGO, Virgo KAGRA GW results



- Several mass models include various scales to describe stellar processes *[LVK+ 2021 ApJL 913 L7]*.
- As a consequence, the masses of GW observations are redshifted by the expansion of the Universe *[Ye+ PRD 104 2021]*.

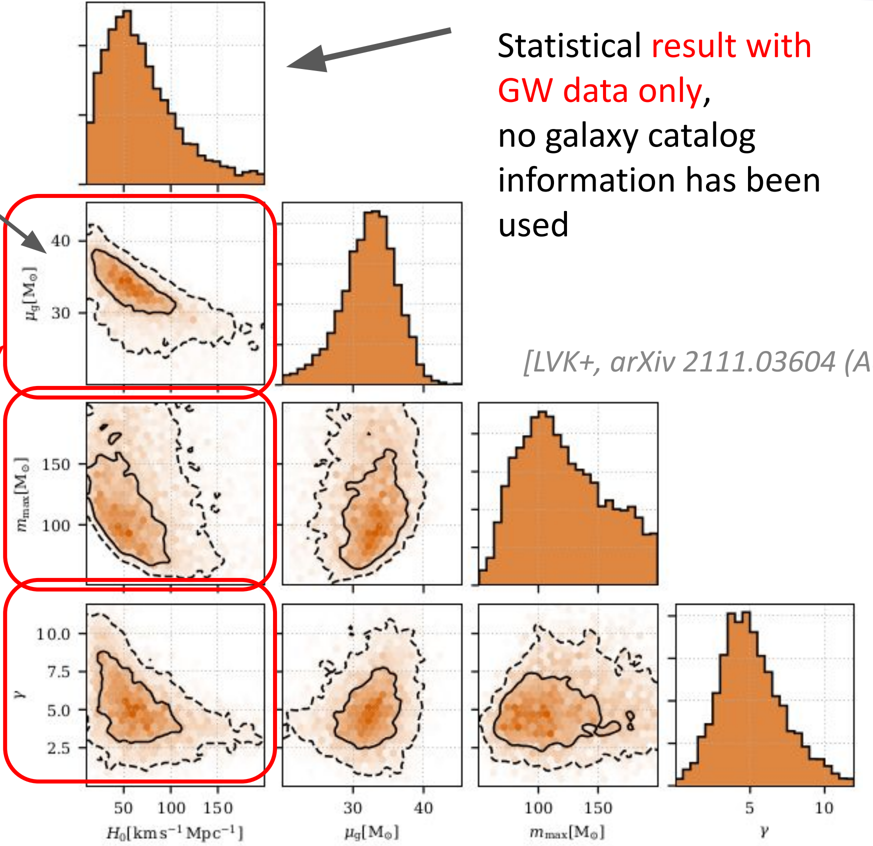


Peak of SFR?

Latest LIGO, Virgo KAGRA GW results

The excess of BBHs around 35 solar masses sets a scale for the redshift and provides constraints on H0.

Population parameters that correlate with H0



Statistical result with GW data only, no galaxy catalog information has been used

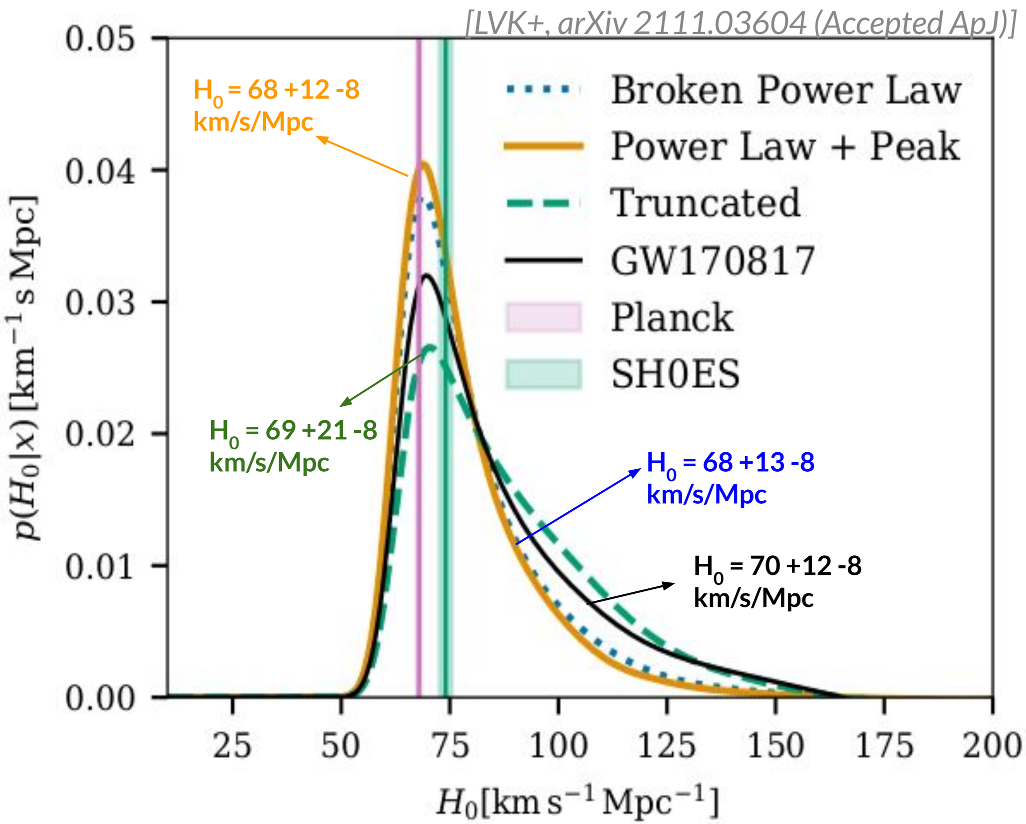
[LVK+, arXiv 2111.03604 (Accepted ApJ)]

Posteriors of H_0, mu of the Gaussian peak, maximum mass of the BH and the merger rate evolution.

Latest LIGO, Virgo KAGRA GW results

The only EM information is the counterpart of GW170817

H_0 posterior of the 3 mass models combined with GW170817 posterior



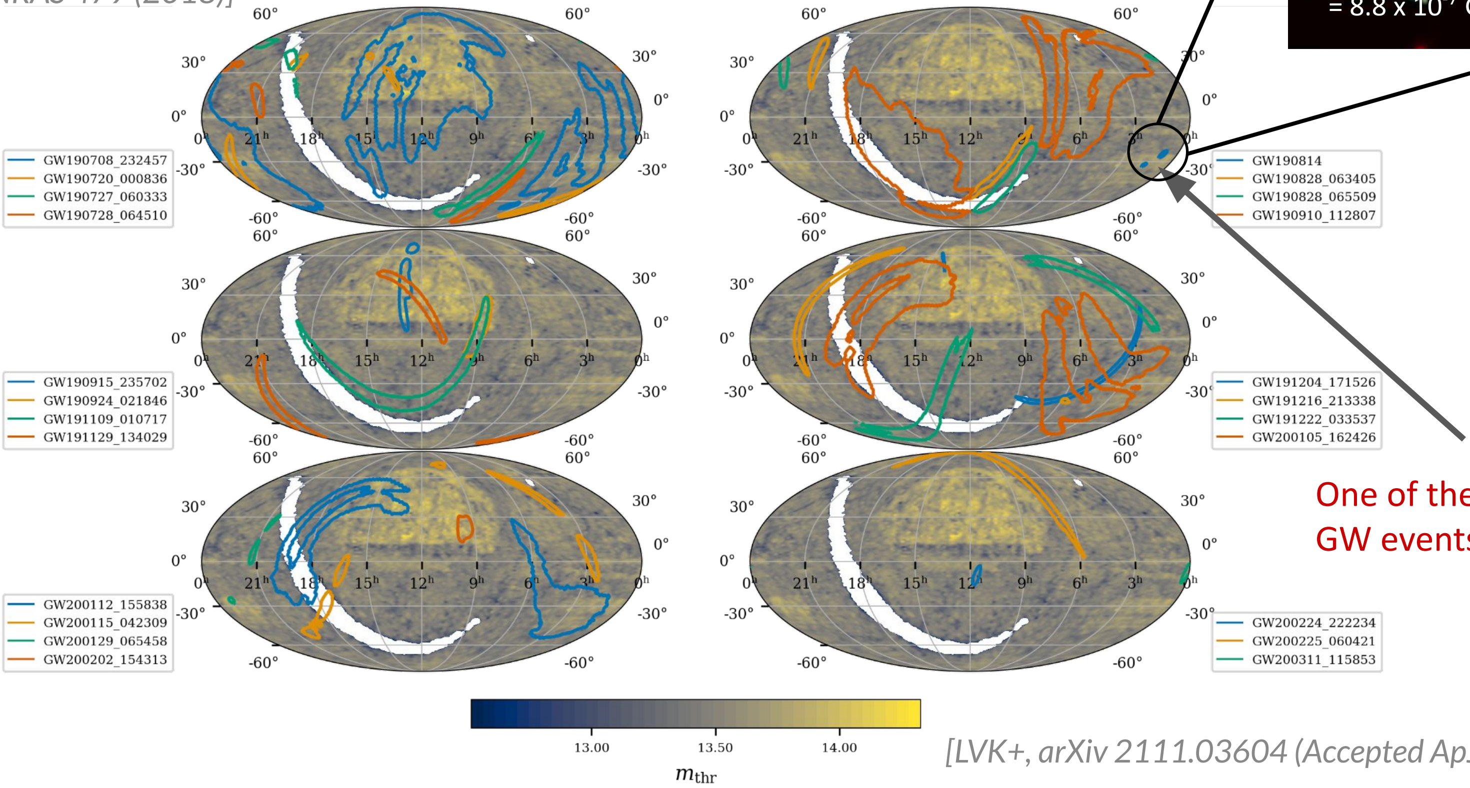
Latest LIGO, Virgo KAGRA GW results

GW190814

Sky area ~ 20 sq. deg

Localisation volume = 8.8 x 10^{-7} Gpc^3

[Dalya+, MNRAS 479 (2018)]

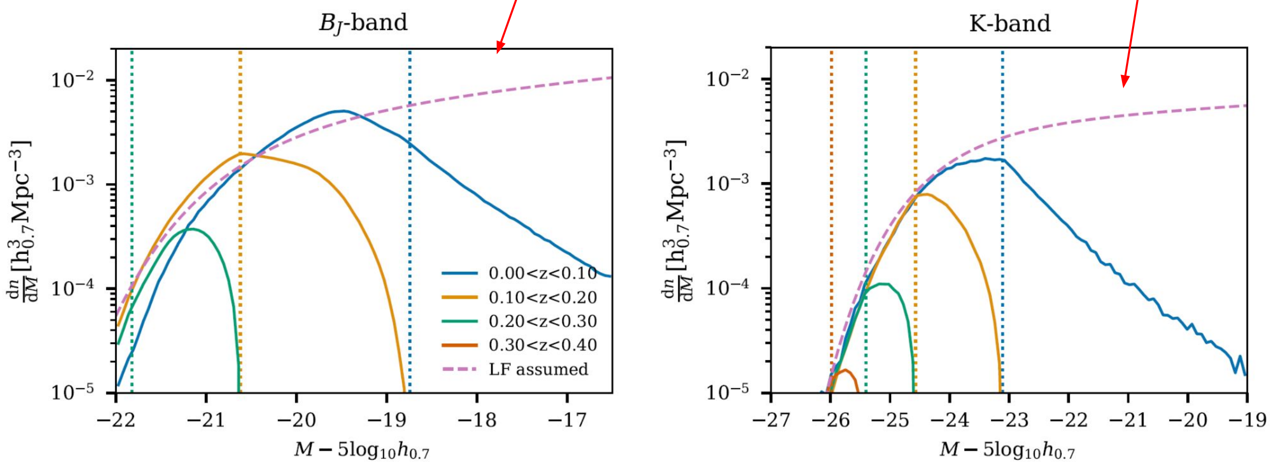


One of the highly localised GW events

[LVK+, arXiv 2111.03604 (Accepted ApJ)]

Latest LIGO, Virgo KAGRA GW results

The galaxy catalog is incomplete, namely it does not contain all the galaxies



Schechter luminosity function

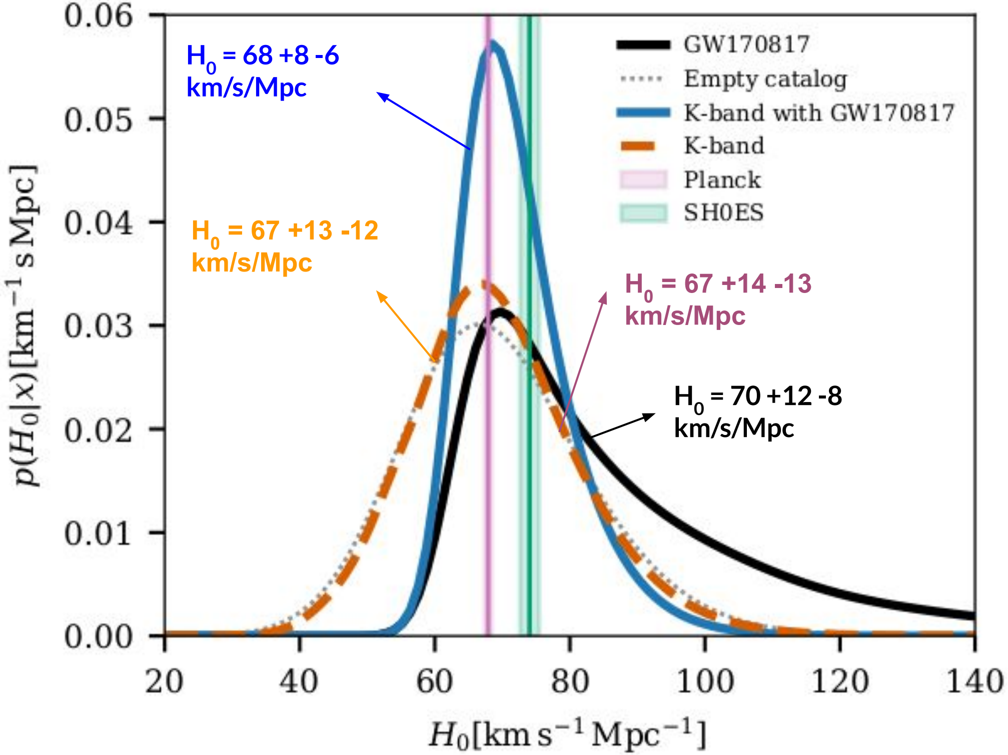
Schechter luminosity function

Latest LIGO, Virgo KAGRA GW results

Main result of the paper showing various H0 posteriors.

We select the K-band for the luminosities of galaxies and the preferred mass model (powerlaw+Gaussian peak)

[LVK+, arXiv 2111.03604 (Accepted ApJ)]

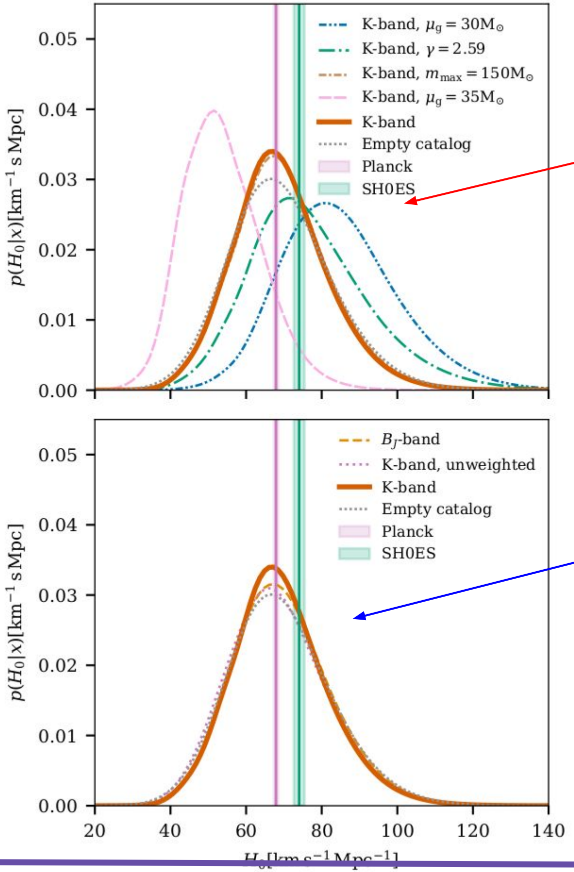


Latest LIGO, Virgo KAGRA GW results

Systematic plot showing the H0 different posteriors when **varying**:

1) **population parameters** (top plot)

2) **galaxy catalog parameters** (bottom plot)



Varying certain population parameters affects the H0 posterior significantly

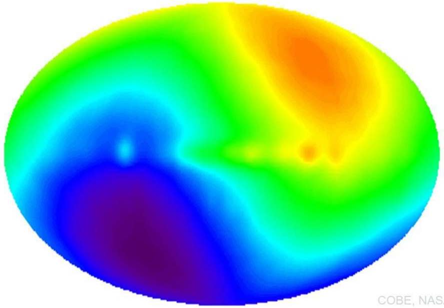
Varying galaxy catalog parameters affects the H0 posterior only marginally

The cosmic kinematic dipole: another tension?

- Our motion with respect to the Hubble flow creates an anisotropy in temperature fluctuation of the Cosmic Microwave Bakground (CMB) *[Planck+ 2020]* and distribution of electromagnetic (EM) sources *[Colin+ 2017, Benglay+ 2018, Secrest+ 2022]*.

- The dipole estimation from the CMB and quasars at low and intermediate redshifts agrees on direction but not on amplitude (5-sigma tension).

- We expect the two dipole estimations to agree, unless a **systematic** is present or the AGN distribution **is not isotropic**. This can be due to a possible redshift evolution of EM sources *[Dalang+ 2022]*.



COBE, NASA

Quasars (AGN)

$$\frac{v_o}{c} = 6.0 \cdot 10^{-3}$$

CMB

$$\frac{v_o}{c} = 1.2 \cdot 10^{-3}$$

The effect of the observer velocity on GW detections

While the GW source is propagating from cosmological distances, the Earth is moving in the Hubble follow. GW sources will be subject to aberration and Doppler shift.

Sky position

$$d\Omega = d\bar{\Omega}\left(1 - 2\mathbf{n}\cdot\frac{\mathbf{v}_o}{c}\right)$$

Luminosity distance

$$\frac{\delta D_L}{\bar{D}_L} = -\mathbf{n}\cdot\frac{\mathbf{v}_o}{c}$$

Redshift

$$\frac{\delta z}{1+\bar{z}} = -\mathbf{n}\cdot\frac{\mathbf{v}_o}{c}$$

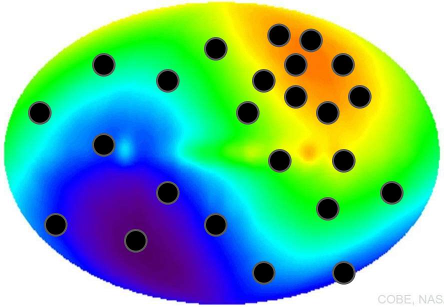
We expect to observe a distribution of sources that is "beamed" in the dipole direction

Discrete version of the estimator

$$\hat{v}_{\mathbf{n}'} = \frac{3}{2N_{\rm tot}}\sum_{i=1}^{N_{\rm sky}} N^i_{\rm det}\cdot(\mathbf{n}_i\cdot\mathbf{n}')$$

Expected value of the estimator

$$\langle\hat{v}_{\mathbf{n}'}\rangle = \frac{\alpha v_o}{2c}\cos(\theta')$$



COBE, NASA

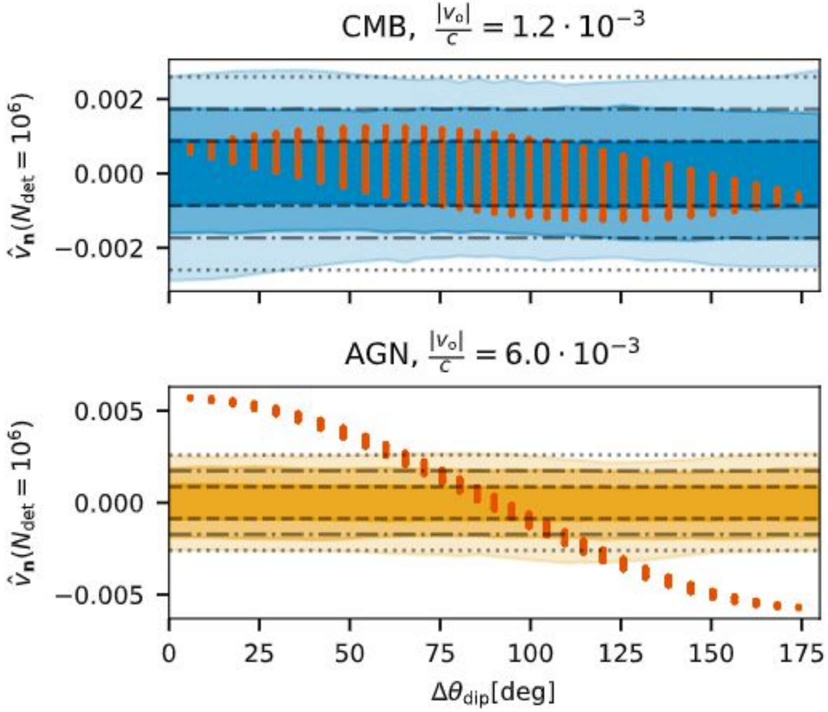
Simone Mastrogiovanni

Simulating compact binaries with XG detectors

- The dipole estimator maximizes when it is computed along the direction of the observer velocity.

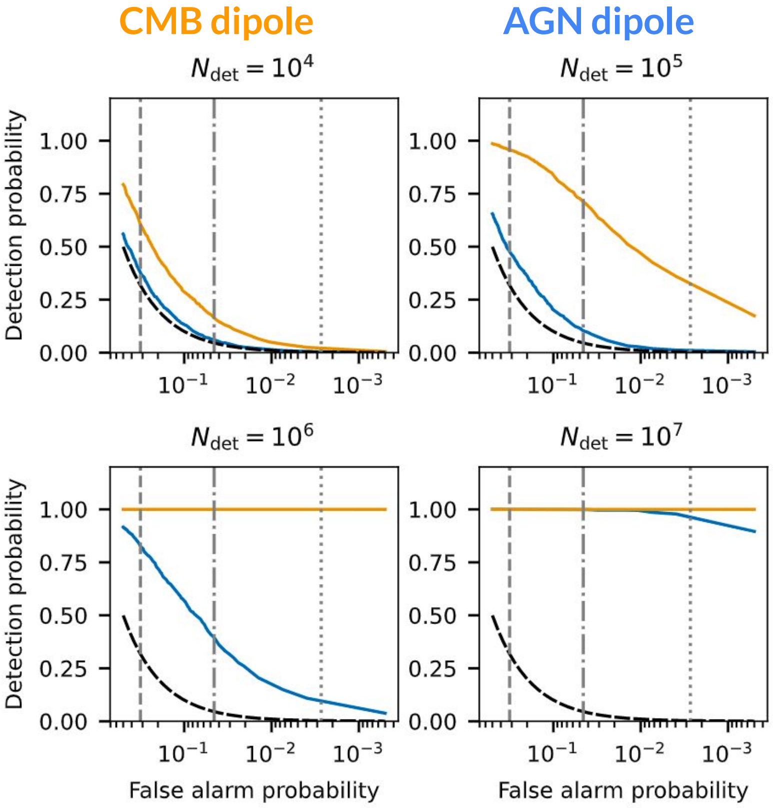
- However, the estimator can fluctuate due to Poissonian statistic. It is important to understand if we are detecting a dipole or a noise fluctuations.

- With 1 million detections, the AGN dipole is definitely detectable but not the CMB one.



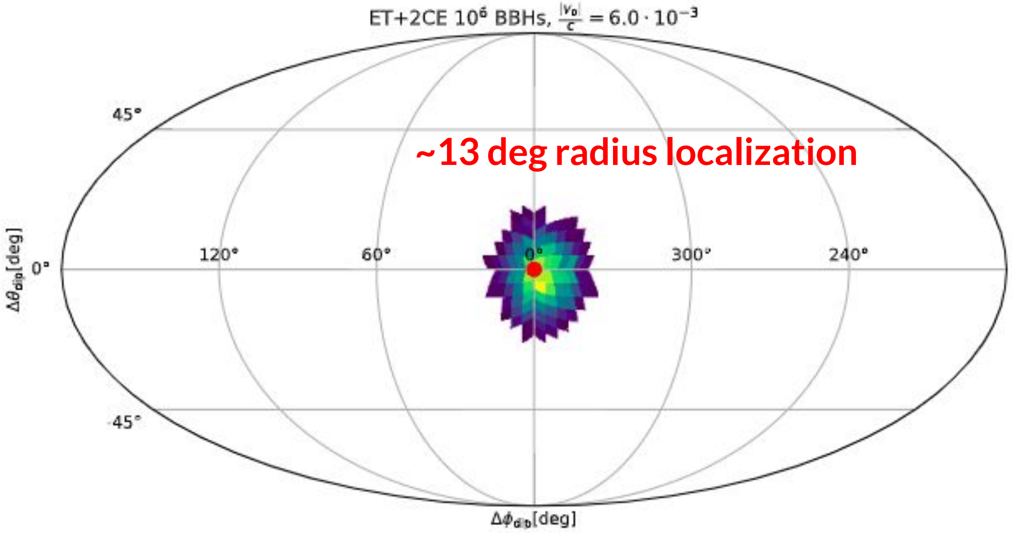
Simulating compact binaries with XG detectors

- Using Monte Carlo simulations we evaluate the detection False Alarm Rate (FAP) vs Detection probability curves for the cosmic dipole.
- Lower FAR, more confident detection but more difficult to detect the dipole.
- With 1 million detections (~3 years of ET+2CE) the **AGN dipole will be detectable (3 sigma)**, while the CMB will be marginally detectable.
- With 10 million (~30 years of ET+2CE) detections both dipoles would be detectable.

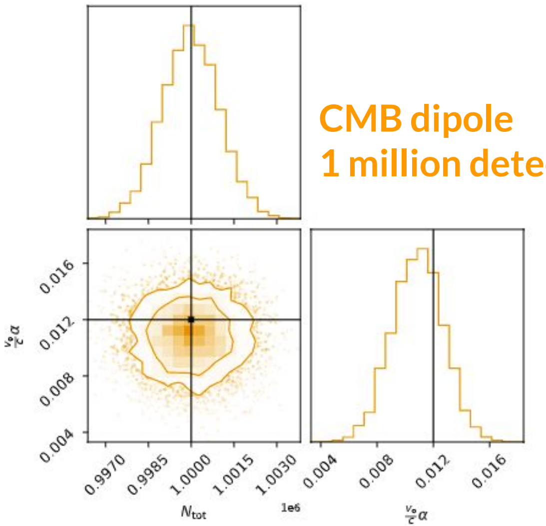


Simulating compact binaries with XG detectors: a bayesian tool

- We can also use Bayesian inference to estimate the direction and magnitude of the cosmic dipole.



~13 deg radius localization



CMB dipole
1 million detections

Simulating compact binaries with XG detectors: a bayesian tool

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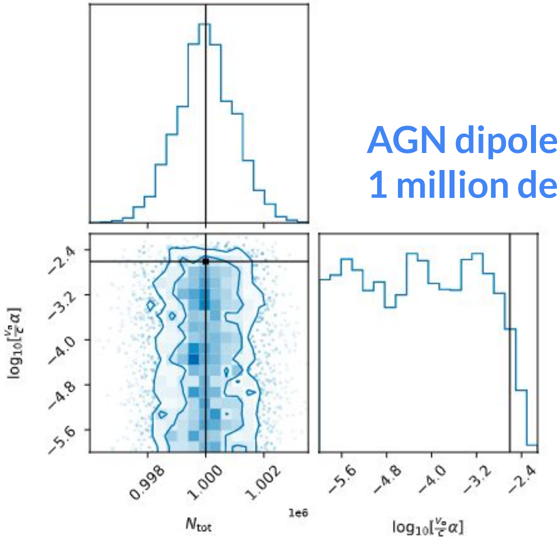
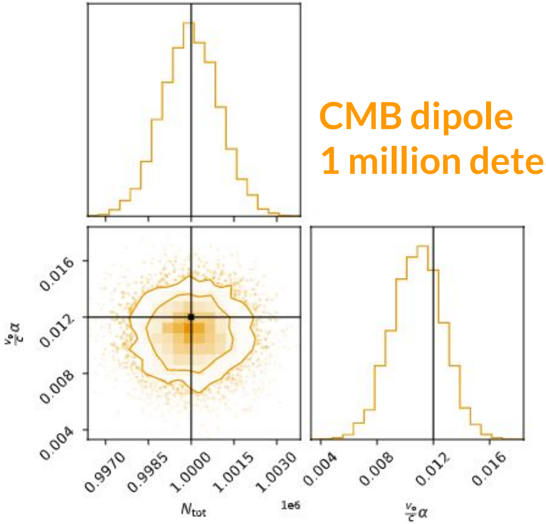
- Bayesian statistic can provide also an upper-limit in case of non-detections (Bayes factors inconclusive).



AGN dipole
1 million detections



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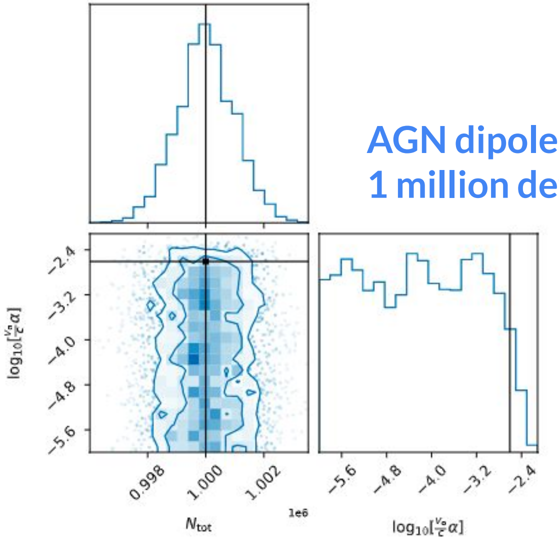

AGN dipole
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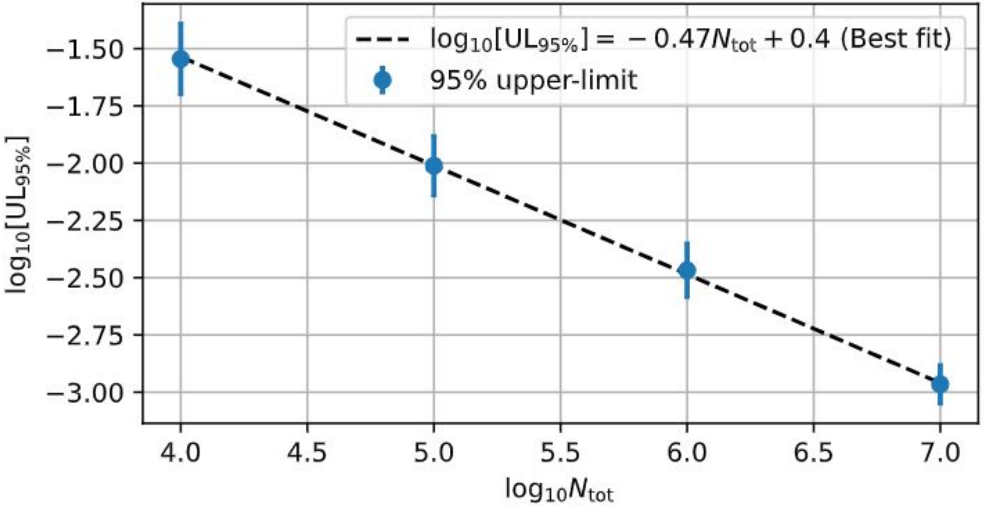

Learn how to estimate H0 with EM counterparts and galaxy catalogs

Link