Gravitational Waves and Fundamental Physics

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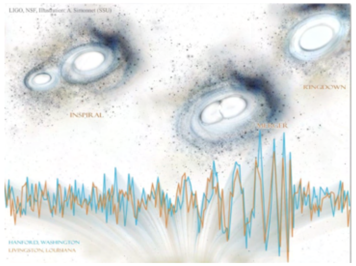
Second MaNiTou Summer School on Gravitational Waves:
A new window to the Universe



3–8 Jul 2023 Nice/Valrose

Outline

- Beyond the Standard Model particle physics
  - Strong First Order Phase Transitions
  - Topological Defects: Cosmic Strings

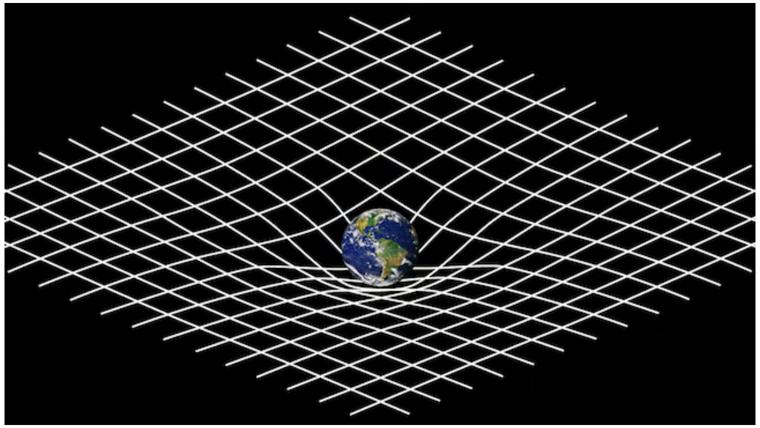
- Dark Matter
  - Axions
  - Primordial Black Holes
  - DM microphysics (CDM versus WDM/IDM/FDM)

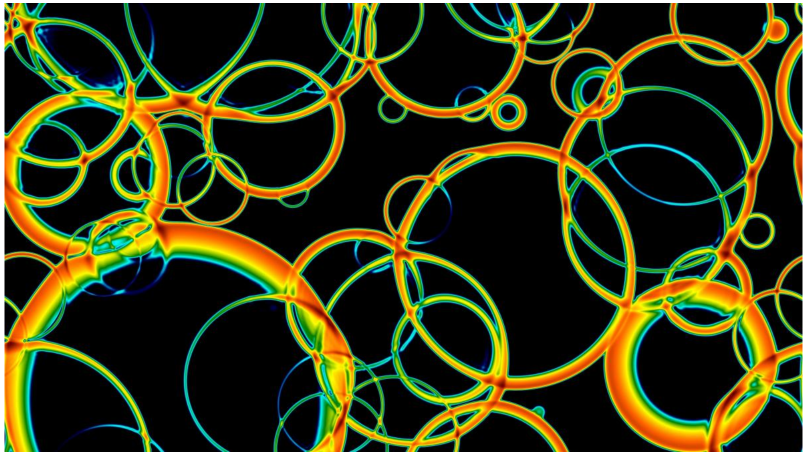
- Cosmological Inflation

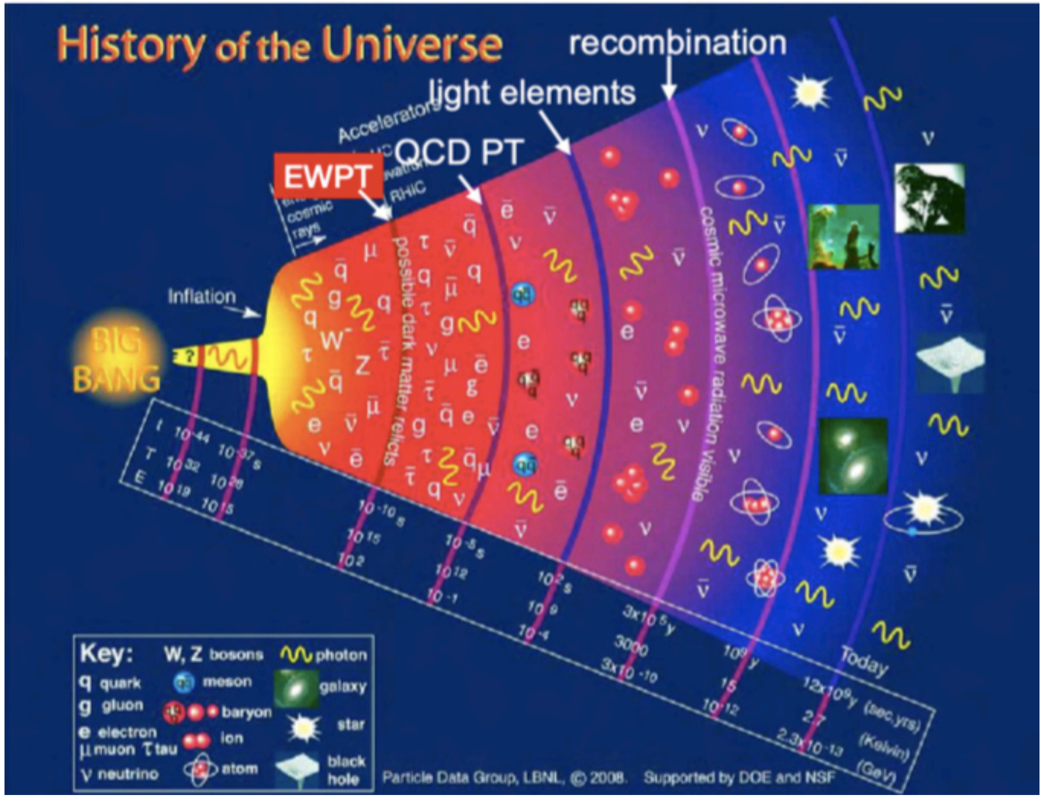
- Classical or Quantum theories of gravity: propagation of GWs
  - signals with EM counterpart
  - signals without EM counterpart

Gravitational Waves and Strong First Order Phase Transitions

→ Constraints on Beyond the Standard Model particle physics at energy scales above the ones reached by LHC







During most of its history the expansion of the Universe was adiabatic, with all processes near equilibrium.
Almost all interesting physics that leave traces in today's Universe happened when **the evolution was non-adiabatic**.
If PTs are sufficiently abrupt/out of equilibrium, they can leave traces: GWs, magnetic fields, baryogenesis, PBH.

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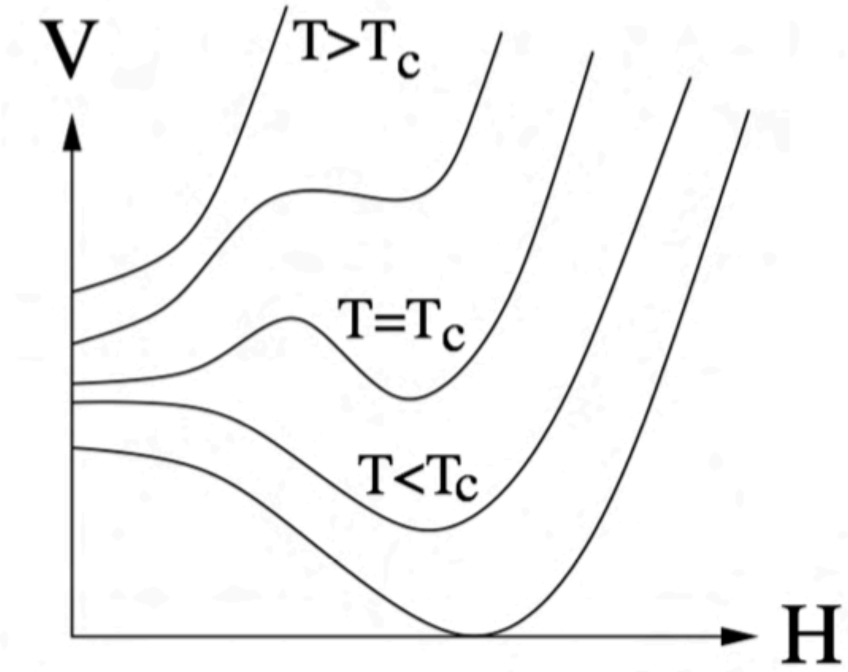
1st order PT

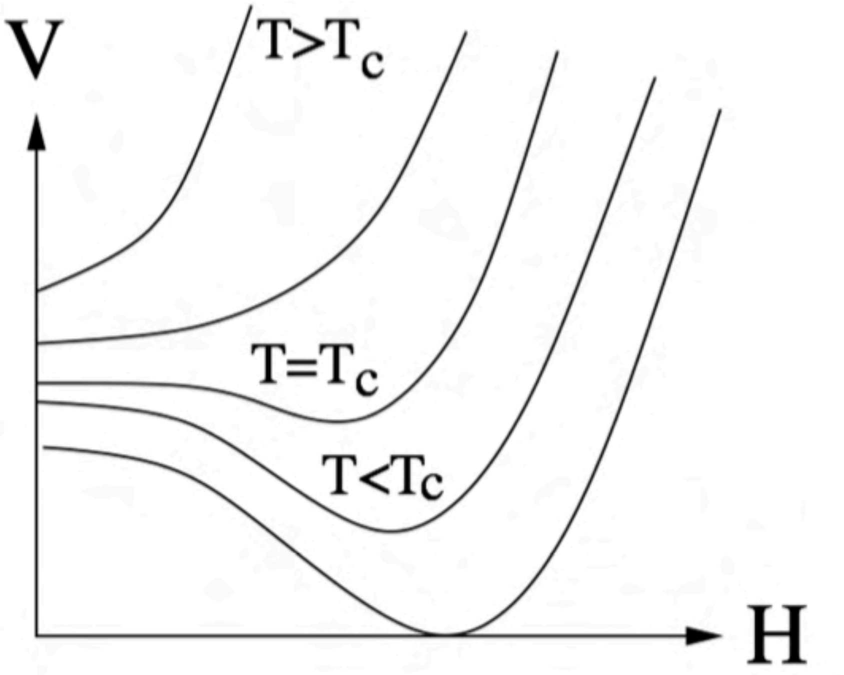
Example: freezing of water into ice



Discontinuity in the first derivative of the free energy with respect to some thermodynamic variable (order parameter)

2nd order PT

Example: Ising model



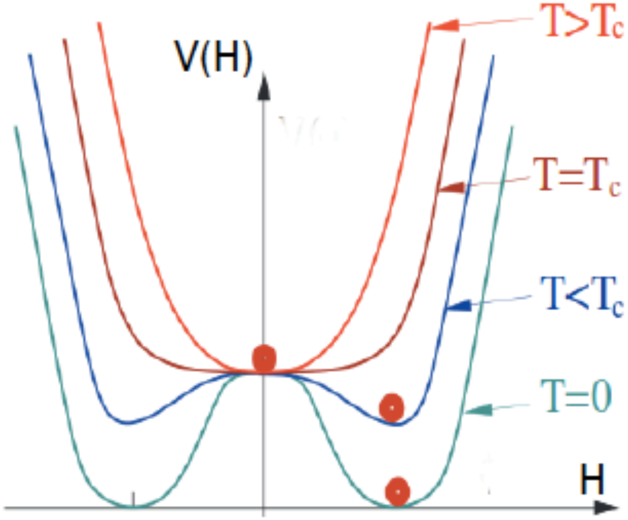
2nd order PT proceeds adiabatically

$$V(H) \;=\; \lambda(H^2-v^2)^2 \;=\; \lambda H^4 - 2\lambda v^2 H^2 + \lambda v^4$$

minima of the potential $H = \pm v$

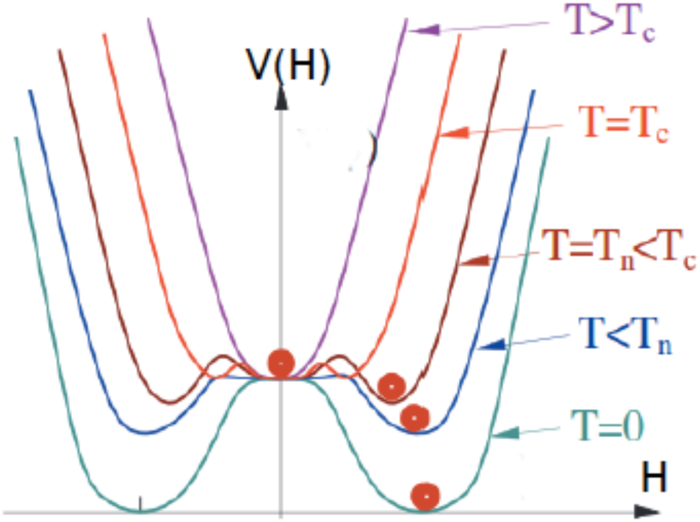
$$V(T,H) \;=\; \lambda(H^2-v^2)^2 \boxed{+\,b\,T^2H^2}$$

2^{nd} order PT

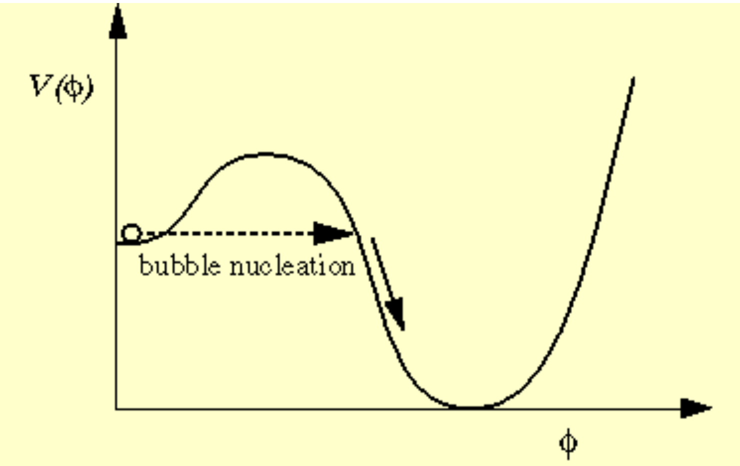


$$V(T,H) \;=\; \lambda(H^2-v^2)^2 + b\,T^2H^2 \boxed{+\,a\,T H^3}$$

1^{st} order PT



The universe might have undergone a series of phase transitions



FOPT: the matter fields get trapped in a "false vacuum" state from which they can only escape by nucleating bubbles of the new phase, i.e the "true vacuum" state

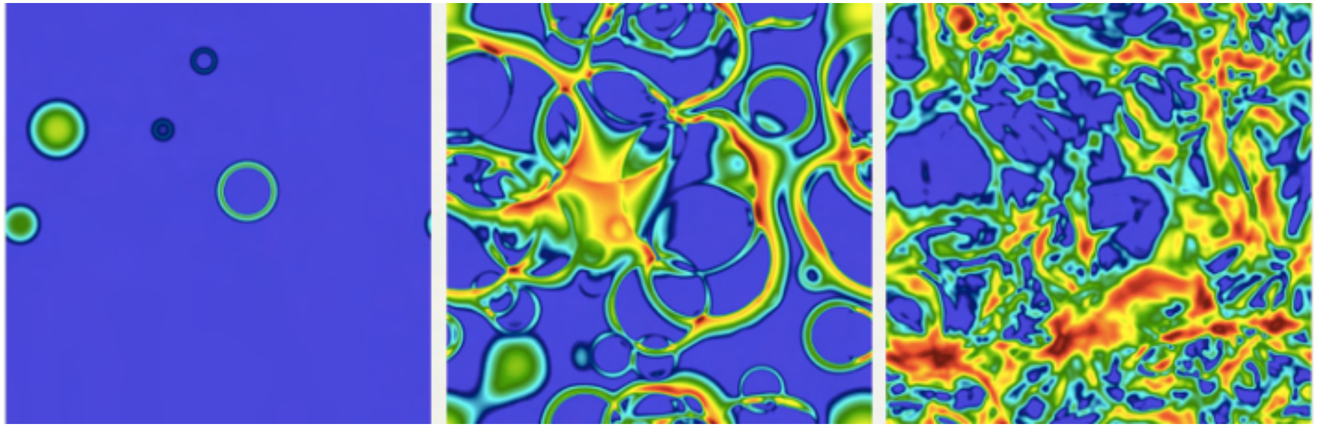
In the case of a FOPT, once the temperature drops below a critical value, the Universe transitions from a meta-stable phase to a stable one, through a sequence of **bubble nucleation**, **growth**, and **merger**

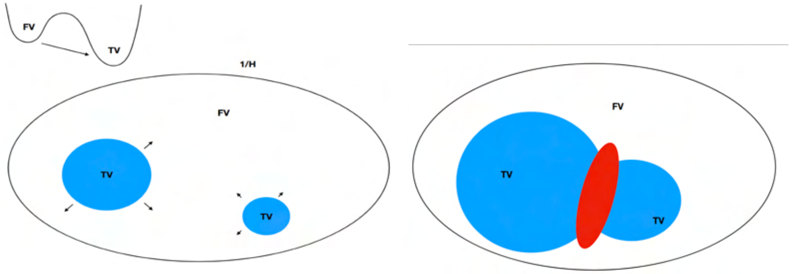
Many compelling extensions of the Standard Model predict strong FOPTs (e.g., GUTs, SUSY, extra dimensions, composite Higgs models, models with extended Higgs sector)

The nature of cosmological PTs depends strongly on the particle physics model at high energy scales

First-order thermal phase transitions:

- bubbles nucleate and grow
- reaction front form
- bubbles + fronts collide
- sound waves in the plasma
- turbulence





Sources of GWs:

- **Bubble collisions**
- **Sound waves** (coupling between scalar field and thermal bath)
- **Magnetohydrodynamic turbulence**

Consider a PT at T=T*. which generates (relativistic) anisotropic stresses which source GWs

- Peak frequency (inversion of the correlation length) is larger than the Hubble rate
- GW energy density

$$\Omega_{gw}(\eta_0) \sim \epsilon\, \Omega_{\rm rad}(\eta_0) \left(\frac{\Omega_X(\eta_*)}{\Omega_{\rm rad}(\eta_*)}\right)^2 \qquad \text{where}$$

$$\epsilon = \begin{cases} (\mathcal{H}_*\Delta\eta_*)^2 & \text{if} \quad \mathcal{H}_*\Delta\eta_* < 1 \\ 1 & \text{if} \quad \mathcal{H}_*\Delta\eta_* \geq 1\,. \end{cases}$$

$\Omega_X = \rho_X/\rho_c$.

energy density of the source

duration of PT

- On large scales, $k \ll k_*$, the spectrum is blue $\frac{d\Omega_{gw}(k)}{d\log(k)} \propto k^3$, $\Omega_{gw} = \int \frac{dk}{k}\frac{d\Omega_{gw}(k)}{d\log(k)}$.
- On smaller scales, $k \gg k_*$, the spectrum decays. The decay law depends on the details of the source

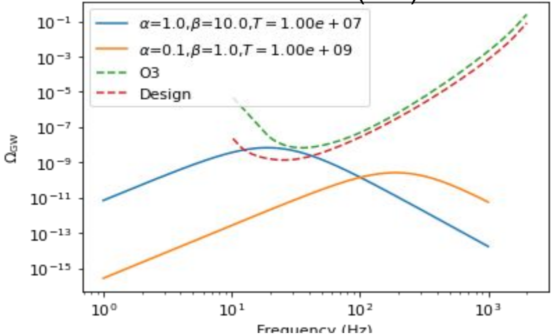
SGWB: broken power law with peak frequency mainly determined by temperature of FOPT

LISA sensitive to first-order phase transitions of 1 GeV- 1000 TeV scale physics
Example: electroweak phase transition at T=100 GeV occurred at about 10^{-10}sec, f ~ (10^{-5} - 10^{-3})Hz
SKA + LISA + ET sensitive to first-order phase transitions of 1 MeV – 1000 PeV scale physics

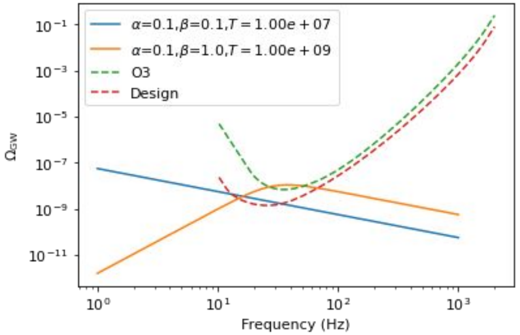
1 TeV=1000 GeV
1 GeV=1000 MeV
1 GeV = 1.0E-6 PeV

If $T_{\rm pt} \sim (10^7 - 10^9)$ GeV (not accessible by LHC –TeV scale) : **SGWB is within aLIGO/aVIRGO**

Sound waves (SW)



Bubble collisions (BC)



α : strength of FOPT
β : inverse duration of FOPT

Confinement transition at a temperature around 100 MeV, about 10^{--5} sec after the big bang, with f~ (10^{-7} - 10^{-5})Hz → timing of radio pulsars

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1 TeV=1000 GeV
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1 GeV = 1.0E-6 **PeV**

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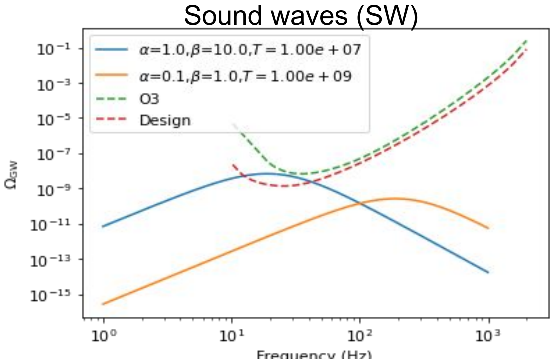
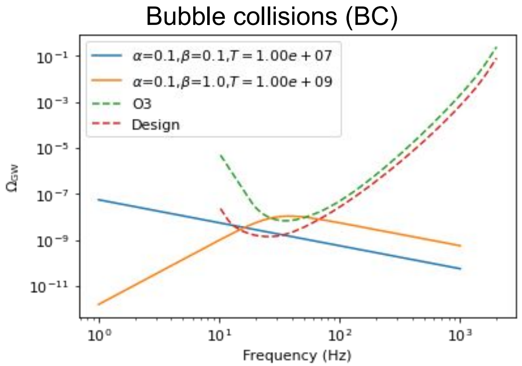

Sound waves (SW)


Bubble collisions (BC)

α : strength of FOPT
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- The SM of particle physics predicts that no strong phase transitions ever occurred
- If we detect GWs from strong phase transitions, we have discovered BSM physics

Broken power-law model

$$\Omega_{\rm bpl}(f) = \Omega_* \left(\frac{f}{f_*}\right)^{n_1} \left[1 + \left(\frac{f}{f_*}\right)^{\Delta}\right]^{(n_2-n_1)/\Delta} .$$

$n_1 = 3$ from causality
$n_2 = -4$ (**sound waves**)
$n_2 = -1$ (bubble collisions)
$\Delta = 2$ (**sound waves**)
$\Delta = 4$ (bubble collisions)

Phenomenological model

$$\boxed{\begin{aligned}\Omega_{\rm sw}(f)h^2 &= 2.65 \times 10^{-6} \left(\frac{H_{\rm pt}}{\beta}\right) \left(\frac{\kappa_{\rm sw}\alpha}{1+\alpha}\right)^2 \left(\frac{100}{g_*}\right)^{1/3} \\ &\times v_{\rm w} \left(\frac{f}{f_{\rm sw}}\right)^3 \left(\frac{7}{4+3(f/f_{\rm sw})^2}\right)^{7/2} \Upsilon(\tau_{\rm sw}) ,\end{aligned}}$$

$$\Omega_{\rm coll}(f)h^2 = 1.67 \times 10^{-5} \Delta \left(\frac{H_{\rm pt}}{\beta}\right)^2 \left(\frac{\kappa_\phi \alpha}{1+\alpha}\right)^2 \left(\frac{100}{g_*}\right)^{1/3} S_{\rm env}(f).$$

Low-frequency: $\Omega_{\rm GW} \propto f^3$ and high frequency $\Omega_{\rm GW} \propto f^{-1}$

$$\Omega_{\rm cbc} = \Omega_{\rm ref}(f/f_{\rm ref})^{2/3}$$

Perform a Bayesian search and model selection

Search for a broken power law in the presence of a CBC background

Broken power-law model

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Low-frequency: $\Omega_{\rm GW} \propto f^3$ and high frequency $\Omega_{\rm GW} \propto f^{-1}$

$$\Omega_{\rm cbc} = \Omega_{\rm ref}(f/f_{\rm ref})^{2/3}$$

O1+O2+O3: $\Omega_{\rm CBC} < 6.1 \times 10^{-9}$

$\Omega_{\rm BPL} < 4.4 \times 10^{-9}.$

Romero, Martinovic, Callister, Guo, Martinez,
***Sakellariadou**, Yang, Zhao, PRL 126 (2021) 15, 151301*

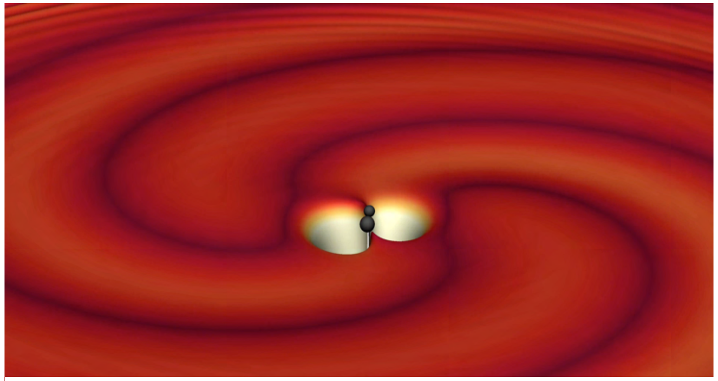
*Badger, **Sakellariadou**, et al, PRD (2022)*

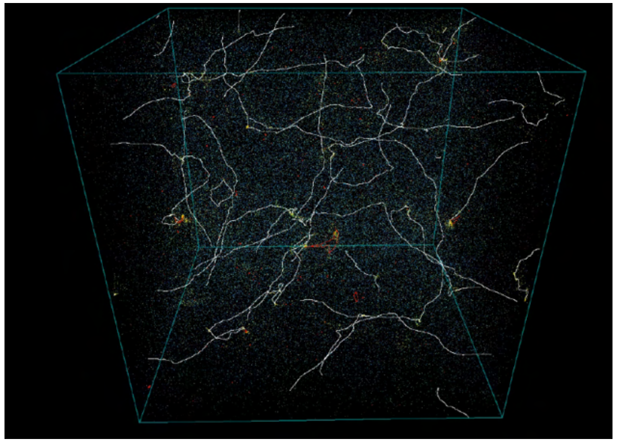
$\Omega_{\rm pt} < 5.8 \times 10^{-9}$ for sound waves $\Omega_{\rm pt} < 5.0 \times 10^{-9}$ for bubble collisions

For PT above 10^8 GeV

Gravitational Waves and Topological Defects: Cosmic Strings

→ Constraints on Beyond the Standard Model particle physics at energy scales above the ones reached by LHC





1dim topological defects formed in the early universe as a result of a PT followed by SSB, characterised by a vacuum manifold with non-contractible closed curves

$G \to \cdots \to G_{\rm SM}$ $\pi_1(\mathcal{M}) \neq 0$

Kibble (1976)

symmetry breaking $G \to H$

vacuum manifold $\mathcal{M} = G/H$

kth homotopy group $\pi_k(\mathcal{M})$ *classifies distinct mappings from k-dim sphere* S^k *into manifold* $\mathcal{M}$

The spacetime dimension d of the defects is given in terms of the order of the nontrivial homotopy group

$$d = 4 - 1 - k$$

1dim topological defects formed in the early universe as a result of a PT followed by SSB, characterised by a vacuum manifold with non-contractible closed curves

$G \to \cdots \to G_{\rm SM}$ $\pi_1(\mathcal{M}) \neq 0$

Kibble (1976)

Generically formed in the context of GUTs

*Jeannerot, Rocher, **Sakellariadou**, PRD68 (2003) 103514*

The Goldstone model

ϕ a complex scalar field with classical Lagrangian density:

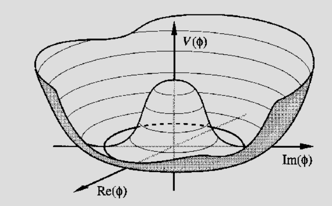
$$\mathcal{L} = (\partial_\mu \bar{\phi})(\partial^\mu \phi) - V(\phi)$$

and potential: $$V(\phi) = \frac{1}{4}\lambda[\bar{\phi}\phi - \eta^2]^2$$

dimensions of mass :
Vacuum Expectation value (VEV)

positive constants

Mexican hat



The Goldstone model is invariant under the U(1) group of global phase transformations $$\phi(x) \longrightarrow e^{i\alpha}\phi(x)$$

constant (independent of spacetime)

The minima of the potential lie on a circle with fixed radius $|\phi| = \eta$

arbitrary phase

The ground state is characterised by: $\langle 0|\phi|0\rangle = \eta e^{i\theta} \neq 0$

1dim topological defects formed in the early universe as a result of a PT followed by SSB, characterised by a vacuum manifold with non-contractible closed curves

$G \to \cdots \to G_{\rm SM}$ $\qquad$ $\pi_1(\mathcal{M}) \neq 0$

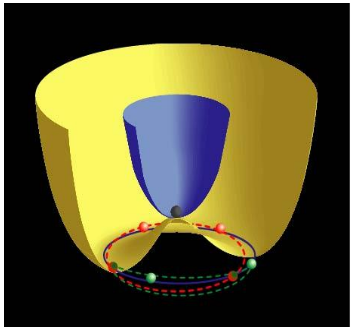
Kibble (1976)

Generically formed in the context of GUTs

Jeannerot, Rocher, ***Sakellariadou****, PRD68 (2003) 103514*

The Abelian-Higgs model

Simplest gauge theory with spontaneously broken symmetry



Lagrangian density:

$$\mathcal{L} = \bar{D}_\mu \phi \mathcal{D}^\mu \phi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - V(\phi)$$

covariant derivative *field strength tensor*

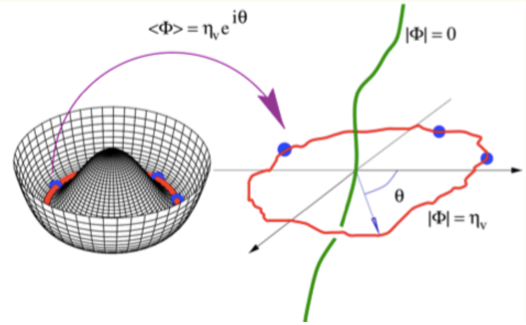
φ complex scalar field with Mexican hat potential

$$\mathcal{D}_\mu = \partial_\mu - ieA_\mu$$

The Abelian-Higgs model is invariant under the group U(1) of local gauge transformations

$$\phi(x) \to e^{i\alpha(x)}\phi(x) \quad ; \quad A_\mu(x) \to A_\mu(x) + \frac{1}{e}\partial_\mu \alpha(x)$$

real single-valued function



The minima of the Mexican hat potential lie on a circle of fixed radius $|\phi| = \eta$, so the symmetry is spontaneously broken and the complex scalar field ϕ acquires a nonzero vacuum expectation value

1dim topological defects formed in the early universe as a result of a PT followed by SSB, characterised by a vacuum manifold with non-contractible closed curves

$G \to \cdots \to G_{\mathrm{SM}}$ $\pi_1(\mathcal{M}) \neq 0$ *Kibble (1976)*

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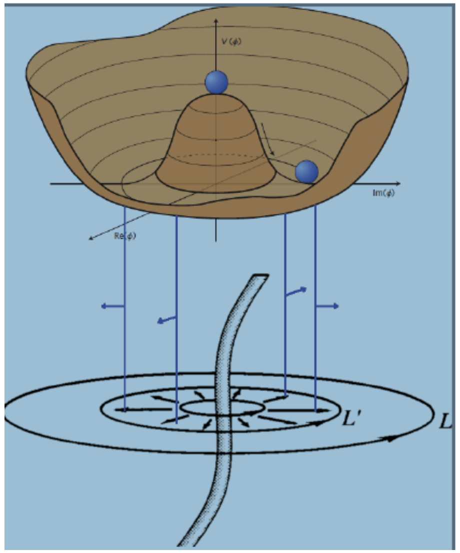


Going around any closed path L in physical space, the phase θ of the Higgs field ϕ develops a nontrivial winding, Δθ = 2π

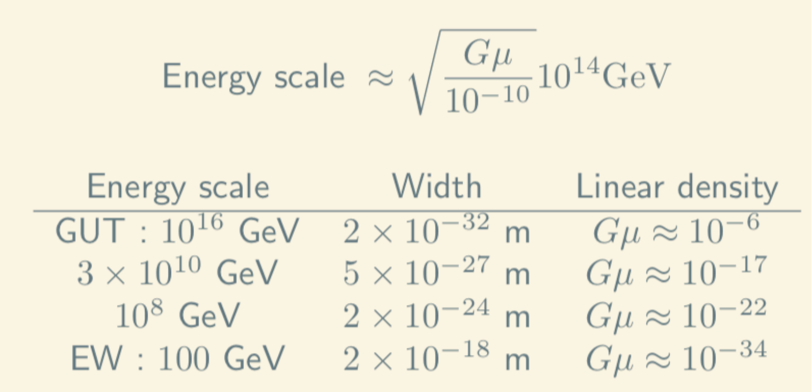
The closed path can be shrunk continuously to a point, only if the field ϕ is lifted to the top of its potential where ϕ=0

Within a closed path for which the total change of the Higgs field ϕ is 2π, a string is trapped

A string must be either closed (loop) or infinitely long; otherwise, you could deform the closed path L and avoid crossing a string

One-dimensional limit (Nambu-Goto action)

$$\mathcal{S} = -\mu \int \sqrt{-\det(\gamma)} \mathrm{d}^2\zeta$$



$$\text{Energy scale} \approx \sqrt{\frac{G\mu}{10^{-10}}} 10^{14} \mathrm{GeV}$$

Energy scale	Width	Linear density
GUT : 10^{16} GeV	2×10^{-32} m	$G\mu \approx 10^{-6}$
3×10^{10} GeV	5×10^{-27} m	$G\mu \approx 10^{-17}$
10^{8} GeV	2×10^{-24} m	$G\mu \approx 10^{-22}$
EW : 100 GeV	2×10^{-18} m	$G\mu \approx 10^{-34}$

String linear mass density for a local (gauge) cosmic string:

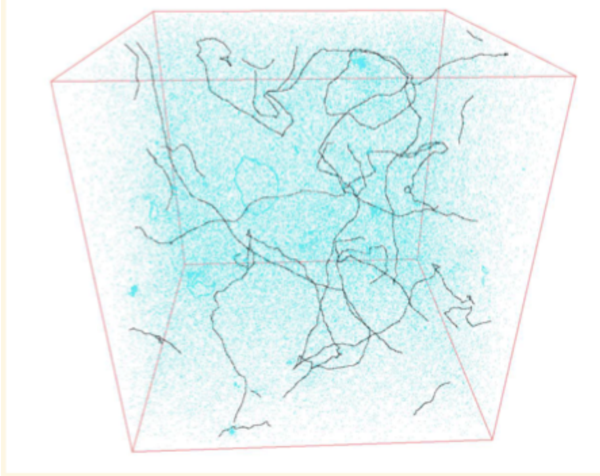
$$\boxed{\mu \sim \eta^2}$$

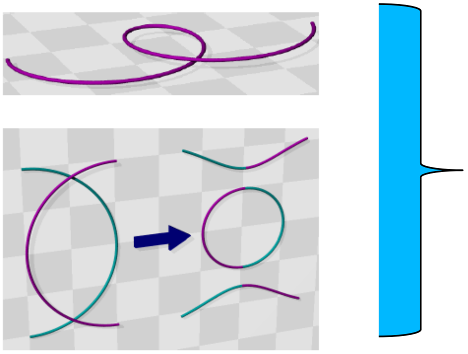
$$G\mu \sim T_{\mathrm{SSB}}^2$$

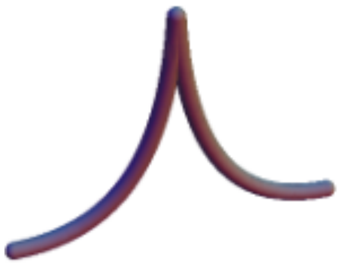
At formation: roughly 80% is in infinite strings and the rest is in loops with a scale-invariant distribution

Scaling of the infinite strings:

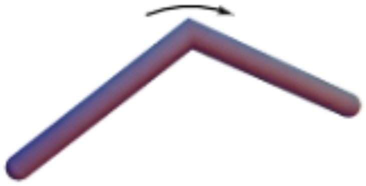
- Attractor solution independent of initial conditions
- All length-scales are proportional to cosmic time



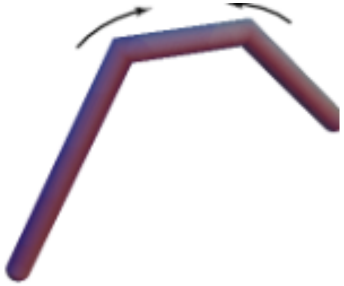




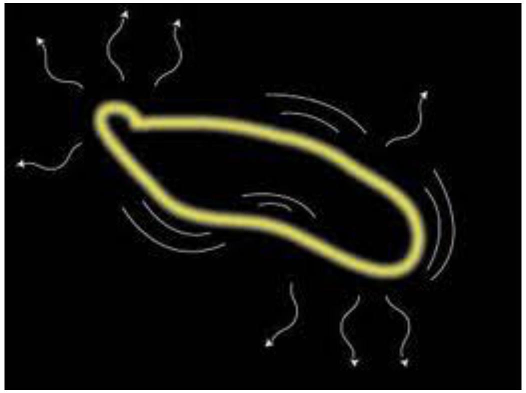
cusp



kink



kink-kink collision



$$\omega_n = 2\pi n/T$$

$$T = l/2$$

$$n = 1, 2, \ldots$$

$$\dot{E} \sim G\left(\frac{d^3D}{dt^3}\right)^2 \sim GM^2L^4\omega^6$$

$D \sim ML^2$ *quadrupole moment*

$M \sim \mu L$ *loop's mass*

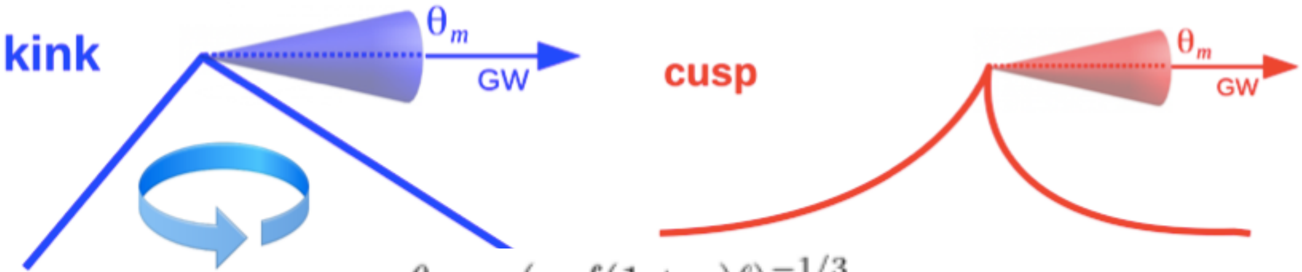
$\omega \sim L^{-1}$ *characteristic frequency*

⇨ $$\dot{E} = \Gamma G\mu^2$$

coefficient independent of loop size; it depends on loop shape and its trajectory

⇨ *lifetime of a loop* $\tau \sim \frac{M}{\dot{E}} \sim \frac{L}{\Gamma G\mu}$

Oscillating loops of cosmic strings generate a SGWB that is strongly non-Gaussian, and includes occasional sharp bursts due to cusps and kinks



$$\theta_m = (g_2 f(1+z)\ell)^{-1/3}$$

For a given loop distribution, the GW burst rate is: $\frac{\mathrm{d}^2 R}{\mathrm{d}z\mathrm{d}h}(h, z, f)$

Incoherent superposition of weaker GW bursts from CS produced over the history of the Universe would create a SGWB

At the frequency of ground-based detectors, the SGWB signal is produced by loops formed during the radiation era

SGWB from cosmic strings:
- All energy radiated by loops is converted to GWs
- Effective average power P_m emitted in mode m

$$\rho_{\mathrm{GW}}(f) = G\mu^2 \sum_{m=1}^{\infty} \mathrm{P}_m C_m(f)$$

$$\Omega_{\mathrm{GW}} = \frac{8\pi G}{3\mathrm{H}_0^2} f \rho_{\mathrm{GW}}$$

$$C_m(f) = \frac{2m}{f^2} \int_0^{z_*} \frac{\mathrm{d}z}{\mathrm{H}(z)(1+z)^6} \frac{\mathrm{d}n}{\mathrm{d}\ell}\left(\frac{2m}{(1+z)f}, t(z)\right)$$

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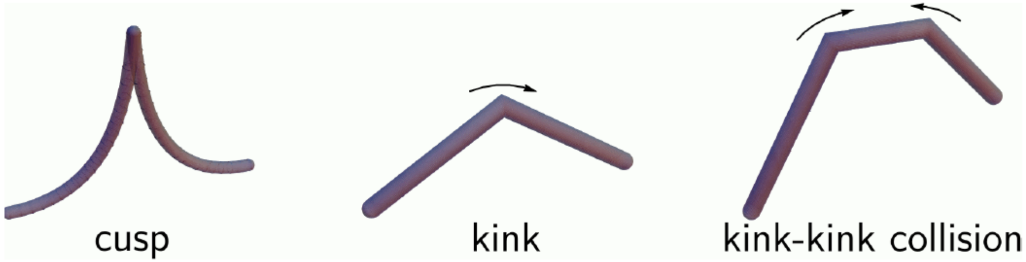
High-frequency regime fl>>1



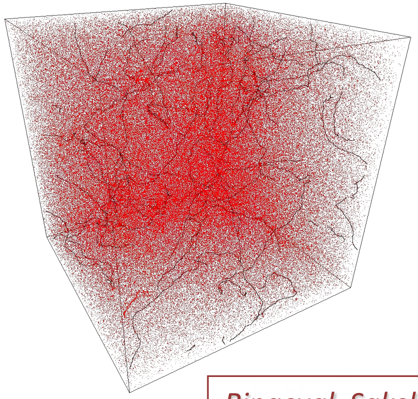
$\tilde{h} \sim f^{-4/3}$ beamed

$\tilde{h} \sim f^{-5/3}$ beamed

$\tilde{h} \sim f^{-2}$ isotropic

Damour, Vilenkin (2001)

$$\bar{\Omega}_{\rm gw} = \frac{2\boxed{(G\mu)^2}}{3\pi^2 H_0^2 \nu_0} \int_0^{t_*} \frac{{\rm d}t}{t^4} a^5 \int_0^{\gamma_*} \frac{{\rm d}\gamma}{\gamma} \boxed{\bar{\mathcal{F}}}\, \Theta\left(\gamma - \frac{2a}{\nu_0 t}\right) \left[N_{\rm k}^2 + 4A\boxed{N_{\rm k}} \left(\frac{\nu_0 \gamma t}{a}\right)^{1/3} + A^2 \boxed{N_{\rm c}} \left(\frac{\nu_0 \gamma t}{a}\right)^{2/3} \right]$$



Ringeval, Sakellariadou, Bouchet (2007)

$$G\mu = \frac{\text{mass}}{\text{length}} \sim \left(\frac{\text{new physics scale}}{\text{Planck scale}}\right)^2 \ll 1$$

$$G\mu \sim T_{\rm SSB}^2$$

Model A: Blanco-Pillado, Olum, Shlaer (2014)
*Model B: Lorenz, Ringeval, **Sakellariadou** (2010)*
*Model C: Auclair, Ringeval, **Sakellariadou**, Steer (2019)*

$$\Omega_{\rm GW}(f) = \frac{f}{\rho_c}\frac{{\rm d}\rho_{\rm GW}}{{\rm d}f}$$

$$\Omega_{\rm GW}(f) = \Omega_{\rm ref}\left(\frac{f}{f_{\rm ref}}\right)^{\alpha}$$

We perform a Bayesian analysis taking into account the precise shape of the background, instead of a power-law and use it to derive upper limits on the CS parameters

Log-likelihood function assuming Gaussian distributed noise

$$\ln \mathcal{L}(\hat{C}_a^{IJ}|G\mu, N_k) = -\frac{1}{2}\sum_{IJ,a}\frac{(\hat{C}_a^{IJ} - \boxed{\Omega_{\rm GW}^{(M)}(f_a; G\mu, N_k))}^2}{\sigma_{IJ}^2(f_a)}$$

$$\hat{C}_a^{IJ} \equiv \hat{C}^{IJ}(f_a)$$

frequency bins ranging from 20 to 86 Hz

Posterior on parameter Gμ using Bayes' theorem

$$p(G\mu|N_{\rm k}) \propto \mathcal{L}(\hat{C}_a^{IJ}|G\mu, N_{\rm k})p(G\mu|I_{G\mu})p(N_{\rm k}|I_{N_{\rm k}}).$$

$p(G\mu|I_{G\mu})$: Log-uniform prior $10^{-18} \leq G\mu \leq 10^{-6}$

$p(N_{\rm k}|I_{N_{\rm k}})$: δ-function for each value of $N_{\rm k}$

Constraint on Gμ at 95% CL: $$\frac{1}{N}\int_{p\geqslant p_0} p(G\mu|N_{\rm k}){\rm d}\ln G\mu = 0.95$$ $$p(G\mu|N_{\rm k}) = p_0$$

$$\Omega_{\mathrm{GW}}(f) = \frac{f}{\rho_c}\frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}f}$$

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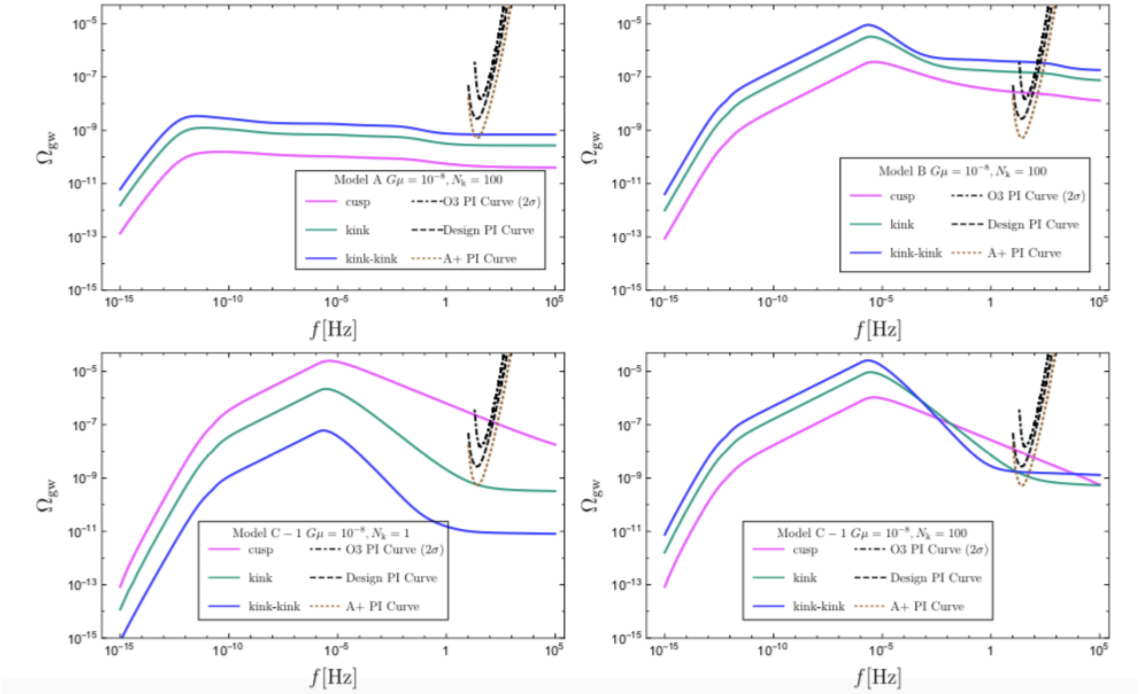

Excluded regions:

Model A: $G\mu \gtrsim (9.6 \times 10^{-9} - 10^{-6})$

strongest limit from PTA $G\mu \gtrsim 10^{-10}$

Model B: $G\mu \gtrsim (4.0 - 6.3) \times 10^{-15}$
strongest limit from LVK stochastic

Model C1: $G\mu \gtrsim (2.1 - 4.5) \times 10^{-15}$
strongest limit from LVK stochastic

Model C2: $G\mu \gtrsim (4.2 - 7.0) \times 10^{-15}$
strongest limit from LVK stochastic

LVK Collaboration, PRL 126 (2021) 24, 241102

$$\Omega_{\mathrm{GW}}(f) = \frac{f}{\rho_c}\frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}f}$$

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We perform a Bayesian analysis taking into account the precise shape of the background, instead of a power-law and use it to derive upper limits on the CS parameters

Note:

These limits are conservative since we have not taken into account the GWs emitted from infinite (super-horizon) cosmic strings

Camargo Neves da Cunha, Ringeval, Sakellariadou (in progress)

The GW spectra generated by ***long cosmic strings*** *are of* ***small amplitude*** *and only the* ***oscillatory plateau*** *is of relevance for today measurements with PTA and laser interferometer*

The reported 2σ upper limit Ω(f=50Hz) < 5.8 x 10^{-9} can be converted for long strings into the upper bound Gμ < 2.5 x 10^{-5}

Camargo Neves da Cunha, Ringeval, Bouchet (2022)

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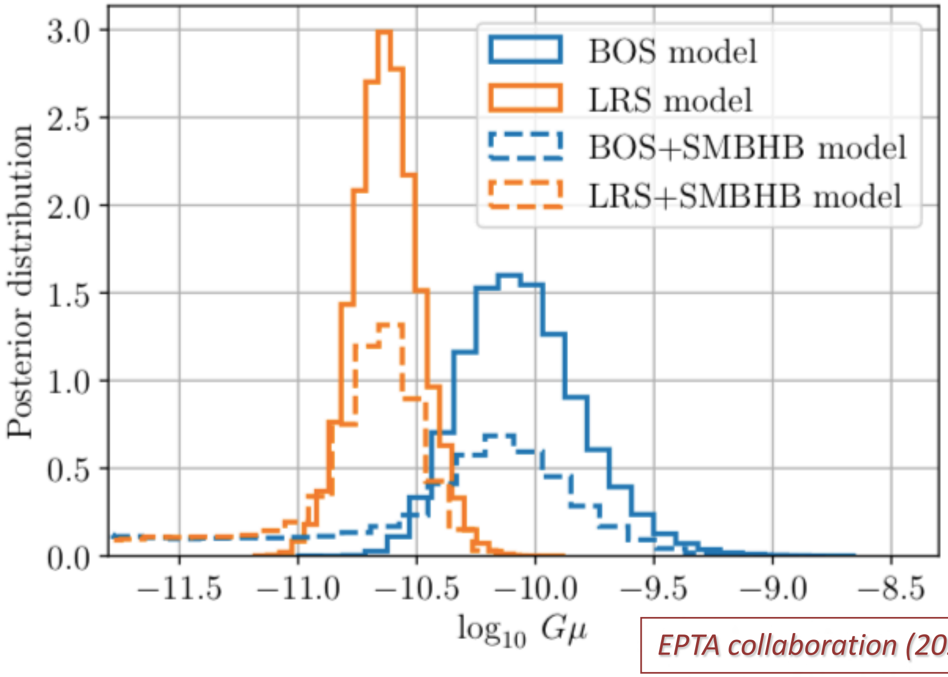
LVK Collaboration, PRL 126 (2021) 24, 241102

At low PTA frequencies, the SGWB signal is dominated by larger loops, namely those formed at recent times, in transition from the radiation to matter era and also in the matter era



EPTA collaboration (2023)

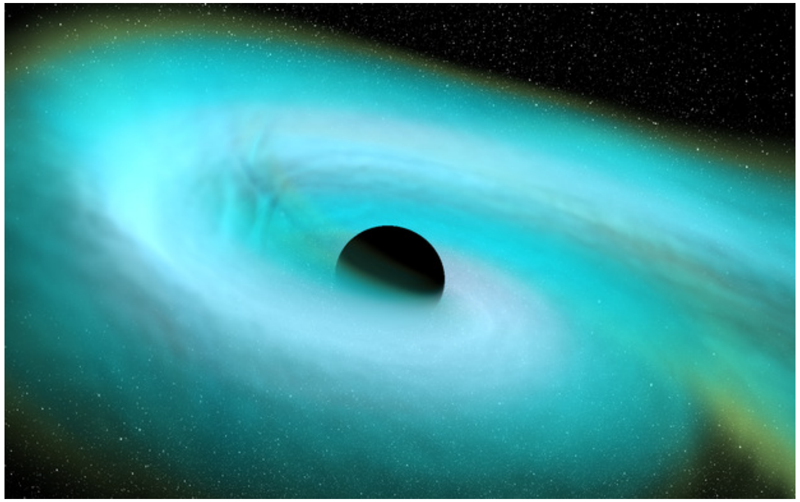
$$N_c = 2 \text{ and } N_k = 0$$

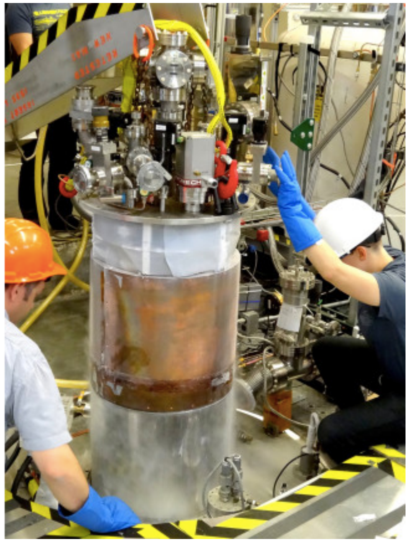
Upper bound $\log_{10} G\mu < -9.77$ (resp. -10.44) for BOS (LRS)

The data can be equally explained by a population of kinky loops with $N_k \gtrsim 120$.
We cannot extract any upper bound on the number of kinks, since this quantity is degenerate with Gμ

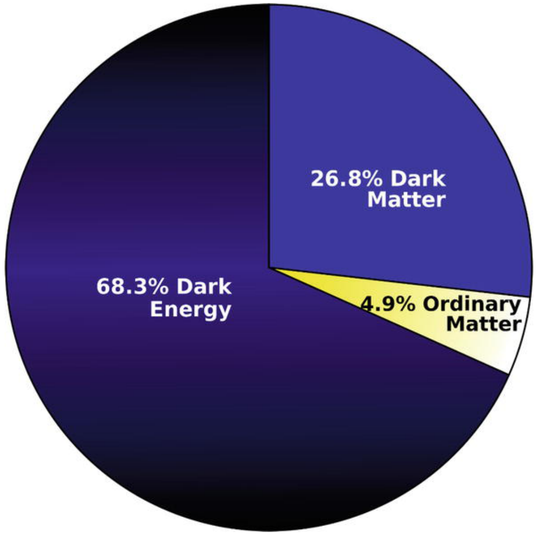
Model A: Blanco-Pillado, Olum, Shlaer (2014)
*Model B: Lorenz, Ringeval, **Sakellariadou** (2010)*

Gravitational Waves and Dark Matter





The problem of dark Matter



Evidence for non-baryonic dark matter

- flat rotation curves – Bullet cluster
- cosmic microwave background radiation

- primordial black holes, axion-like particles, wimpzillas, gravitinos, neutralino, sterile neutrino
- failure of GR (MOND, TeVeS, D-particles)

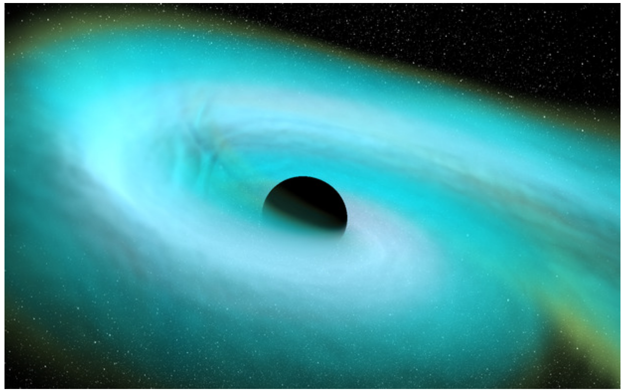
Even if some reasonable candidates exist, we still have not been able to identify dark matter, 90 years after it has been first postulated by Fritz Zwicky

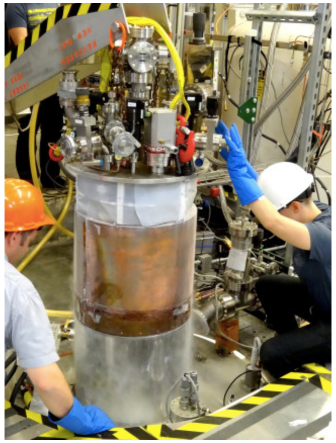
Gravitational Waves and Dark Matter

- Axions

Hypothetical scalar particles that generally appear in many fundamental theories

Example: QCD axion, a pseudoscalar field proposed to solve the strong CP problem





GWs: constraints on light axions

Axions coupled to nuclear matter in a similar way like QCD axion, but with relatively low masses

In vacuum, the axion field is expected to stay at the minimum of its potential $\alpha = 0$.

Inside a dense object (e.g., NS) the axion potential receives finite density corrections

$$V(a) = -m_\pi^2 f_\pi^2 \left[\left(\epsilon - \frac{\sigma_N n_N}{m_\pi^2 f_\pi^2}\right) \left|\cos\left(\frac{a}{2f_a}\right)\right| \boxed{+\mathcal{O}\left(\left(\frac{\sigma_N n_N}{m_\pi^2 f_\pi^2}\right)^2\right)\Bigg]}$$

Hook, Huang (2017)

GWs: constraints on light axions

Axions coupled to nuclear matter in a similar way like QCD axion, but with relatively low masses

If radius of dense object greater than a critical value, a phase transition occurs, shifting VEV of axion field from 0 to a non-zero value $\pm\pi f_a$ **inside the dense object**

$$R_{\text{crit}} \equiv \frac{2f_a}{\sqrt{\sigma_N n_N - \epsilon m_\pi^2 f_\pi^2}},$$

Hook, Huang (2017)

GWs: constraints on light axions

Axions coupled to nuclear matter in a similar way like QCD, but with relatively low masses

If radius of dense object greater than a critical value, a phase transition occurs, shifting VEV of axion field from 0 to a non-zero value $\pm\pi f_a$ inside the dense object

If radius of NS is about 10 km, this PT happens inside the NS for axions with $f_a \lesssim 10^{18}\mathrm{GeV}$

⇒ The NS develops an axion profile, interpolating from $\pm\pi f_a$ a near the NS surface to 0 at spatial infinity

Hook, Huang (2017)

GWs: constraints on light axions

Axions coupled to nuclear matter in a similar way like QCD, but with relatively low masses

If radius of dense object greater than a critical value, a phase transition occurs, shifting VEV of axion field from 0 to a non-zero value $\pm\pi f_a$ **inside the dense object**

The axion field mediates additional force between two NSs, which can be either **attractive** or **repulsive** depending on whether the axion field has the same or opposite sign on the surface of the two NSs

$$\mathbf{F}_a = -\frac{Q_1 Q_2}{4\pi r^2}(1 + m_a r)\exp[-m_a r]\,\hat{\mathbf{r}}$$

$$Q_{1,2} = \pm 4\pi^2 f_a R_{1,2}$$

If such NSs form binaries, the axion field might also radiate axion waves during binary coalescence

$$P_a = \frac{(Q_1 M_2 - Q_2 M_1)^2}{12\pi\,(M_1 + M_2)^2} r^2 \Omega^4 \left(1 - \frac{m_a^2}{\Omega^2}\right)^{3/2}$$

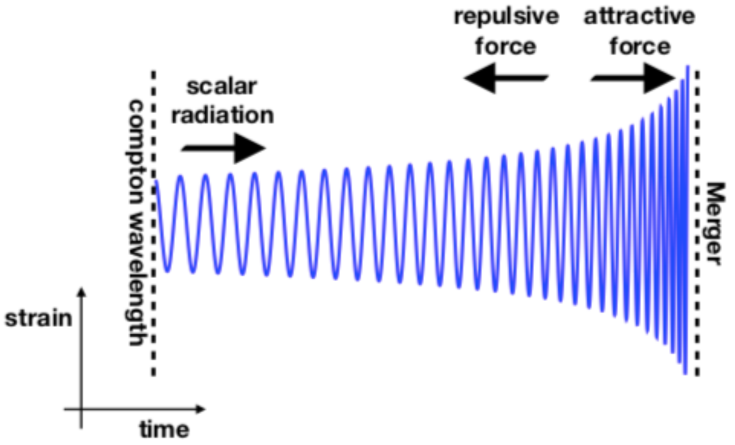
Axion radiation is turned on, only when the orbital frequency Ω > axion mass

Hook, Huang (2017)

GWs: constraints on light axions

Axions coupled to nuclear matter in a similar way like QCD, but with relatively low masses

Schematic plot of strain versus time for a GW waveform emitted during a binary merger in the presence of an axion



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Hook, Huang (2017)

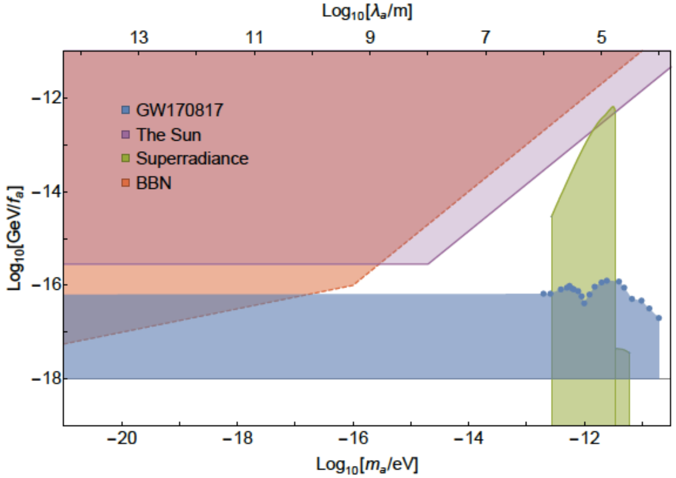
GWs: constraints on light axions

Using an EFT approach, we have calculated the first post-Newtonian corrections to the orbital dynamics, radiated power, and gravitational waveform for binary NS mergers in the presence of an axion

Huang, Johnson, Sagunski, ***Sakellariadou****, Zhang***,** Phys. Rev. D 99, 063013 (2019)

$$h(f) \simeq H(f)\exp\left[i\Psi(f)\right]$$

Constraints on the axion parameter space



$$\lambda_a \equiv 1/m_a$$

$$\Psi = \Psi_{\rm GR} + \Psi_a + \mathcal{O}(Q_{1,2}^4) + \mathcal{O}(Q_{1,2}^2 v^2)$$

The leading order phase correction by the axion field

First constraints on nuclear coupling of axionlike particles from the BNS GW event GW170817

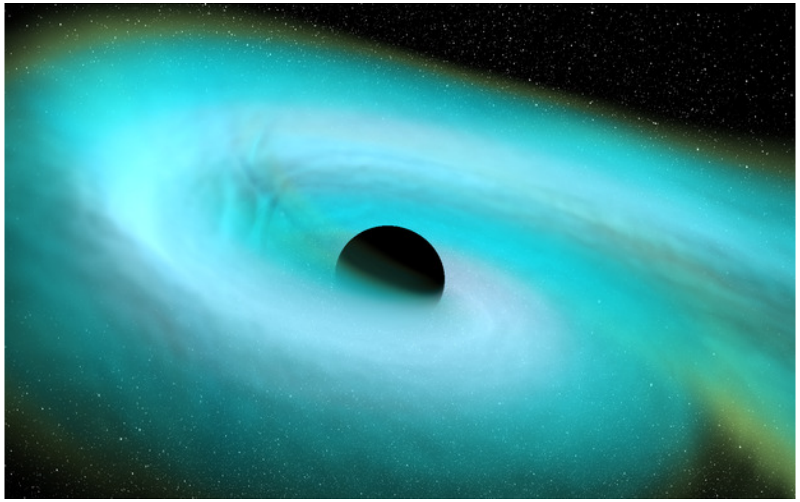
Constraint on f_a will improve by factor $\sqrt{N}$ if the SNR is improved by N

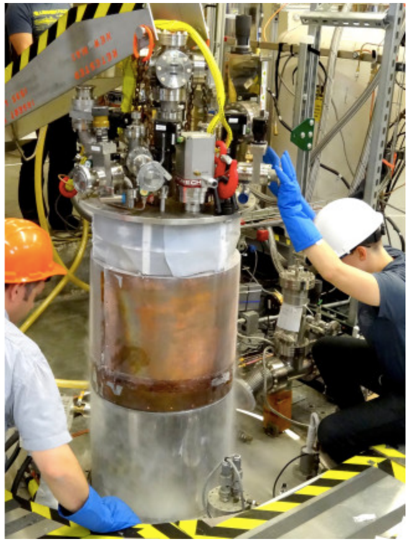
Constraints on axions with masses below $10^{-11}\mathrm{eV}$ by excluding the ones with decay constants ranging from $1.6\times10^{16}\mathrm{GeV}$ to $10^{18}\mathrm{GeV}$ at 3σ confidence level

Zhang, Lyu, Huang, Johnson, Sagunski, ***Sakellariadou****, Yang***,** PRL 127 (2021) 161101

Gravitational Waves and Dark Matter

- Primordial Black Holes





Primordial Black Holes

PBHs would initially have around the cosmological horizon mass

$$M \sim 10^{15} \left(\frac{t}{10^{-23}\,\mathrm{s}} \right) \mathrm{g}$$

PBHs evaporate on a timescale

$$\tau(M) \approx 10^{64} \left(\frac{M}{M_\odot} \right)^3 \mathrm{yr}$$

$f(M)$: the fraction of the dark matter in PBHs of mass M

$$f \equiv \frac{\Omega_{\mathrm{PBH}}}{\Omega_{\mathrm{CDM}}}$$

Constraints suggest that there are only a few ranges where f can be significant:

- asteroidal to sublunar range $10^{17} - 10^{23}\,\mathrm{g}$
- intermediate range $10 - 10^{2}\, M_\odot$
- extremely large range $M > 10^{11}\, M_\odot$ (irrelevant to the dark matter in galaxies)

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lot of attention due to LIGO/Virgo detections of merging binary black holes with mass in the range $10 - 50\, M_\odot$

Primordial Black Holes

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PBH formation

- collapse of primordial density fluctuations *Musco & Miller (2013) ; Harada, Yoo & Kohri (2013)*
- collapse of inflationary fluctuations *Carr & Lidsey (1993); Dolgov & Silk (1993) (2013) ; Ivanov, Naselsky & Novikov (1994) ; Garcia-Bellido, Linde & Wands (1996); Randall, Soljacic & Guth (1996)*
- collapse at the QCD phase transition *Crawford & Schramm (1982) ; Jedamzik (1997) ; Byrnes, Hindmarsh, Young & Hawkins (2018) ; Dvali, Kuehnel & Zantedeschi (2021)*
- collapse of cosmic string loops *Hawking (1989) ; Polnarev & Zembowicz (1991); Garriga & **Sakellariadou** (1993) ; Caldwell & Casper (1996); MacGibbon, Brandenberger & Wichoski (1998) ; Jenkins & **Sakellariadou** (2020)*
- collapse through collisions of bubbles of broken symmetry at a SSB epoch *Khlopov, Konoplich, Rubin & Sakharov (1998, 1999, 2000)*
- collapse of a scalar field *Cotner & Kusenko (2017) ; Cotner, Kusenko & Takhistov (2018) ; Cotner, Kusenko, Sasaki & Takhistov (2019) ; Flores & Kusenko (2021)*
- collapse of domain walls *Garriga, Vilenkin & Zhang (2016) ; Deng, Garriga & Vilenkin (2017) ; Deng & Vilenkin (2017) ; Liu, Guo & Cai (2020)*

SGWB produced by non-linear cosmological perturbations

$$ds^2 = a^2(\tau)[-d\tau^2 + g_{ij}(\mathbf{x},\tau)dx^i dx^j]$$

$$g_{ij} = \delta_{ij} + g_{ij}^{(1)} + \frac{1}{2}g_{ij}^{(2)}$$

linear level

nonlinearity ⇒ *mixing of different modes*
(scalar, vector, tensor)

⇓

second-order tensor modes are dynamically generated by the gravitational instability of scalar fluctuations

(the form of these modes is gauge-dependent)

Matarrese, Mollerach (1997)

scalar induced GWB induced from inflationary scalar perturbations at 2nd order in perturbation theory

- PBH formation through large curvature perturbations during inflation

⇨ **Strong SGWB generated at 2nd order in perturbation theory from scalar perturbations**

$$\mathcal{P}_h \sim \int \mathrm{d}k \int \mathrm{d}k' \left(\int \mathrm{d}t\, f(k,k',t) \right)^2 \mathcal{P}_\zeta(k) \mathcal{P}_\zeta(k')$$

power spectrum of induced GWs

(peaked around same wavenumber
as the curvature power spectrum)

some oscillatory function

power spectrum of primordial curvature perturbations

$$\Omega_{\mathrm{GW}}(\eta,k) = \frac{\rho_{\mathrm{GW}}(\eta,k)}{\rho_{\mathrm{tot}}(\eta)} = \frac{1}{24}\left(\frac{k}{a(\eta)H(\eta)}\right)^2 \overline{\mathcal{P}_h(\eta,k)},$$

Kohri, Teradah (2018)

scalar induced GWB induced from inflationary scalar perturbations at 2nd order in perturbation theory

log-normal shape for the peak in curvature power spectrum

Integrated power of peak

position of peak

$$\mathcal{P}_\zeta(k) = \frac{A}{\sqrt{2\pi}\Delta}\exp\left[-\frac{\ln^2(k/k_*)}{2\Delta^2}\right]$$

width of peak

O1+O2+O3: upper limits on the amplitude of power spectrum and on the fraction of the DM in terms of ultralight PBHs

PBHs produced by critical collapse when large enough curvature perturbations of scale k re-enter the horizon, so the PBH mass is set by the horizon mass at the horizon re-entry time of scale k

For LIGO/Virgo sensitivity:

$$M_{\mathrm{PBH}} \lesssim 10^{16}\,\mathrm{g}.$$

Romero-Rodriguez, Martinez, Pujolas, Sakellariadou, Vaskonen, PRL 128 (2022) 5, 051301

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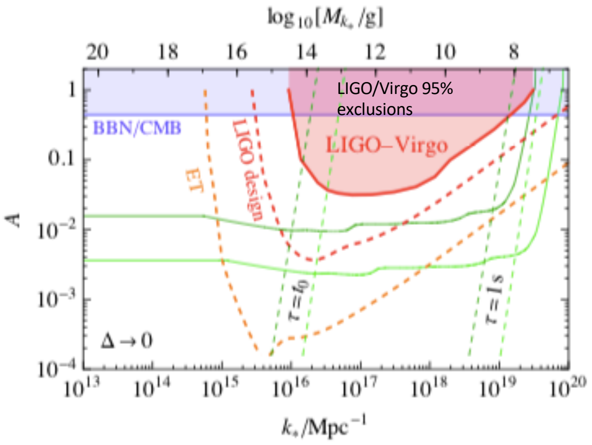

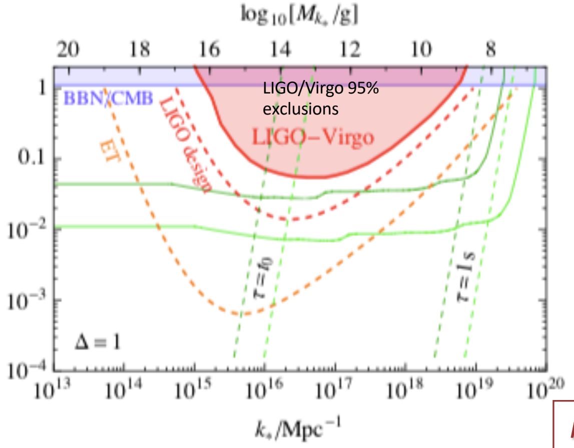

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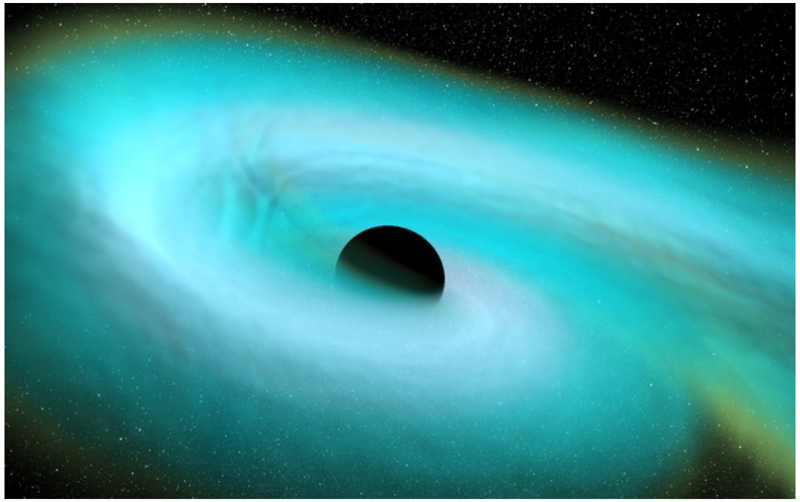
Romero-Rodriguez, Martinez, Pujolas, Sakellariadou, Vaskonen, PRL 128 (2022) 5, 051301

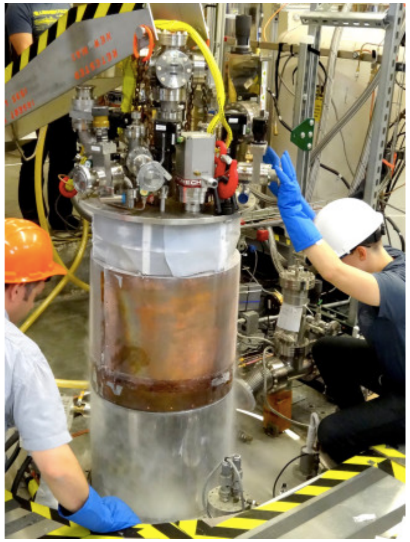
No evidence for such a SGWB

95% CL upper limits on integrated power of the curvature power spectrum peak down to 0.02 at $10^{17}\ \mathrm{Mpc}^{-1}$

Gravitational Waves and Dark Matter

- DM microphysics





GW event rates as a new probe for DM microphysics

Present observations on super-galactic scales are compatible with the hypothesis that the **dark matter is cold**

CDM model: particles also do not have significant non-gravitational interactions

→ particle-like DM typically of mass $\gtrsim$ keV
or wave-like DM of mass $\gtrsim 10^{-22}\ \mathrm{eV}$

However, the key to determining the fundamental nature of DM lies in the sub-galactic scales at large redshifts:
the onset of non-linear structure formation can be very sensitive to the microphysics of the dark matter

Three classes of DM scenarios that predict small-scale signatures that differ from predictions of standard CDM:

- **WDM** (warm DM: negligible interactions but small DM particle mass in the low keV range)
- **IDM** (interacting DM: no strong assumption about particle mass but endows DM particle with non-negligible interactions)
- **FDM** (fuzzy DM: condensate of ultra-light DM particles of mass $\sim 10^{-22}\text{–}10^{-21}\ \mathrm{eV}$ whose collective behaviour is wave-like)

GW event rates as a new probe for DM microphysics

Can DM particles collide with other particles (e.g., neutrinos) or they pass unaffected?

Look at how galaxies form in dense clouds of DM haloes

If DM scatters off of particles (e.g., neutrinos) --> DM is washed out --> fewer galaxies

GWs : indirect measure of the abundance of missing galaxies (very small and very distant)

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*Mosbech, Jenkins, Bose, Boehm, **Sakellariadou**, Wong, PRD (2023)*

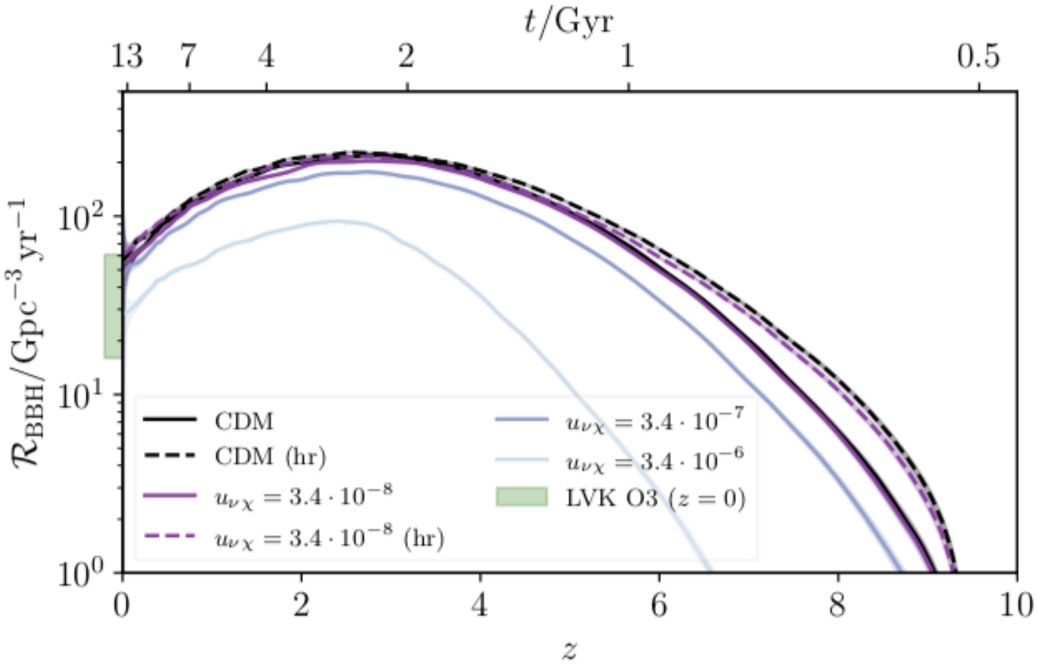
GW event rates as a new probe for DM microphysics

The BBH merger rate is highly sensitive to the suppression of small-scale structure induced by DM microphysics

Example: **DM neutrino interacting model**

$$u_{\nu\chi} \equiv \frac{\sigma_0}{\sigma_{\rm Th}}\left(\frac{m_\chi}{100\,{\rm GeV}/{\rm c}^2}\right)^{-1}$$

BBH merger rate density over cosmic time, as predicted by our pipeline



Suppression of small-scale structure—such as that caused by interacting, warm, or fuzzy dark matter—leads to a significant **reduction in the rate of binary black hole mergers at redshifts** z>5

*Mosbech, Jenkins, Bose, Boehm, **Sakellariadou**, Wong, PRD (2023)*

GW event rates as a new probe for DM microphysics

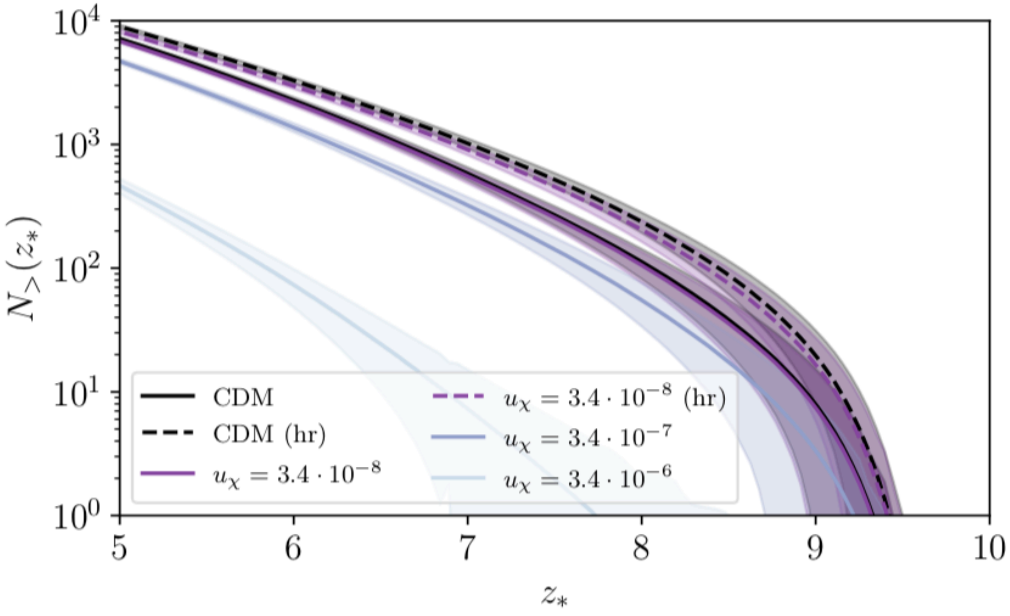
The BBH merger rate is highly sensitive to the suppression of small-scale structure induced by DM microphysics

These differences in the high-z BBH merger rate will be detectable with future gravitational-wave observatories

One year of observations with a 3g network (1 ET + 2 CE)

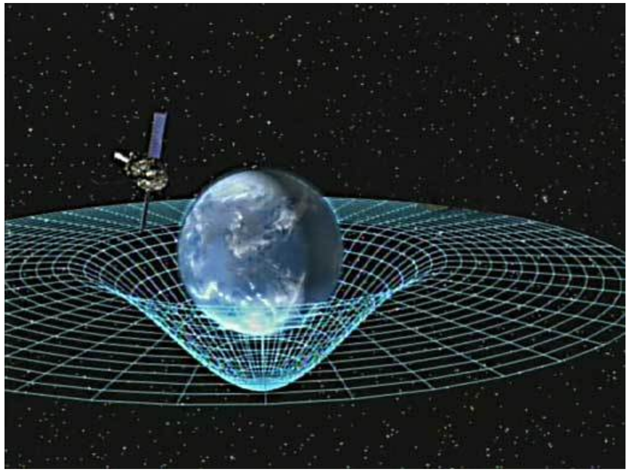
We can clearly distinguish between the different N>(z*) predictions, allowing us to confirm or rule out a small-scale suppression of the scale caused by DM-neutrino interactions down to the level of $u_{\nu\chi} \sim 10^{-7}$.

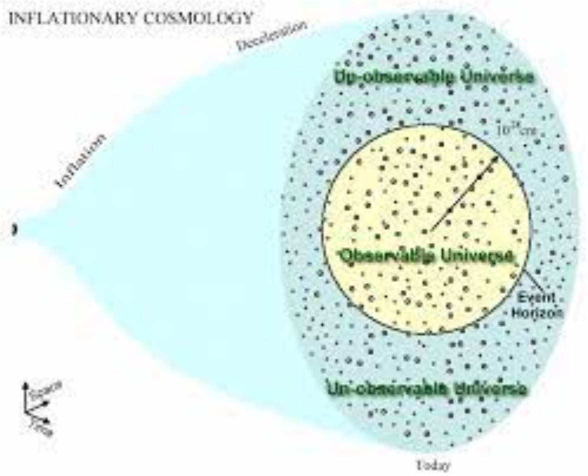
Expected number of BBH mergers observed



Mosbech, Jenkins, Bose, Boehm, Sakellariadou, Wong, PRD (2023)

Cosmological Inflation





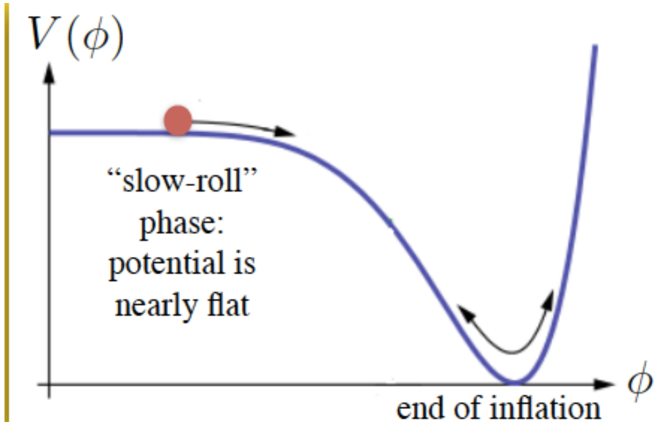
Stage of accelerated expansion of the Universe when gravity acts as a repulsive force

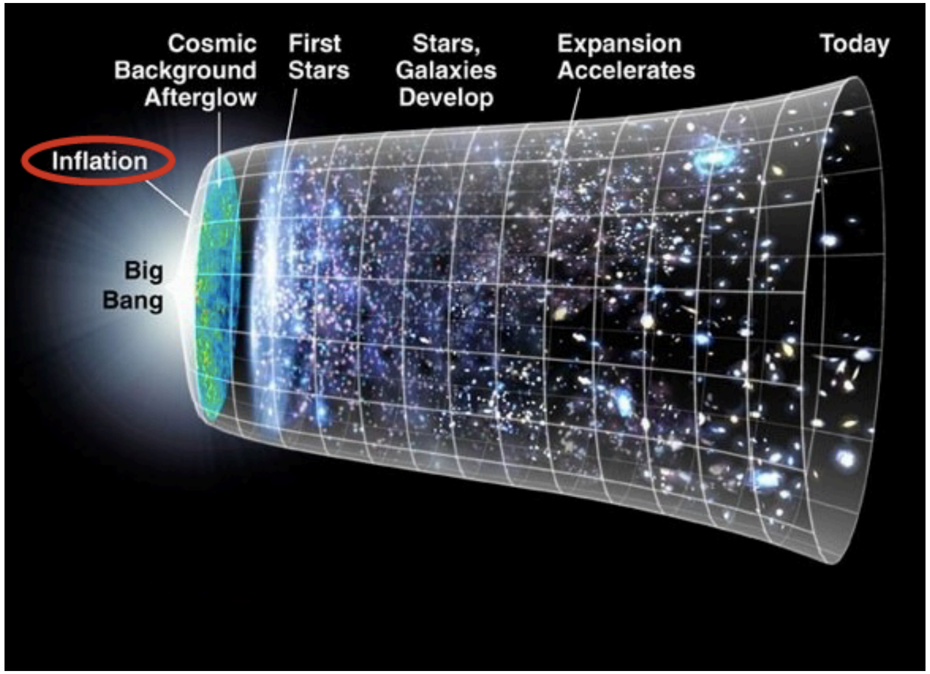
$$\text{inflation} \quad \Leftrightarrow \quad \ddot{a} > 0$$

$$\text{inflation} \quad \Leftrightarrow \quad \frac{d}{dt}\left(\frac{1}{aH}\right) < 0$$

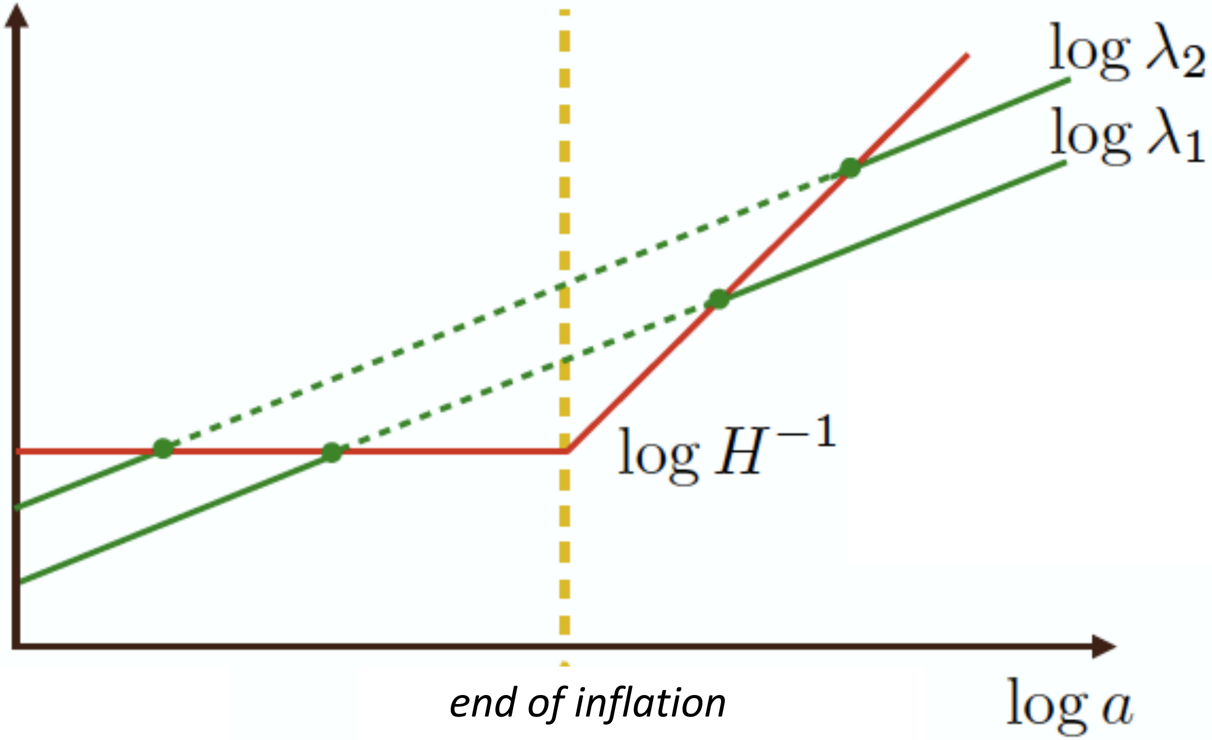
comoving Hubble length shrinks

$$\text{inflation} \quad \Leftrightarrow \quad \rho + 3p < 0$$





Perturbations cross the horizon twice



$$ds^2 = -dt^2 + a^2(t)\,(\delta_{ij} + \gamma_{ij})\,dx^i dx^j$$

$$\ddot{\gamma}_{ij} + 3H\dot{\gamma}_{ij} + k^2\gamma_{ij} = 16\pi G\,\Pi_{ij}^{TT}$$

Homogeneous solution: GWs from vacuum fluctuations

Inhomogeneous solution: GWs from sources

Primordial GW: indirect detection

In the presence of GW, photon propagation occurs along **perturbed** geodesics

→ temperature anisotropies

Thomson scattering of radiation with quadrupole anisotropy by free electrons

→ B modes

Inflationary GW from vacuum fluctuations (single field slow roll)

Red tilt : $n_T \simeq -2\epsilon = -r/8$

Nearly gaussian: $f_{\rm NL} \ll 1$

Non-chiral

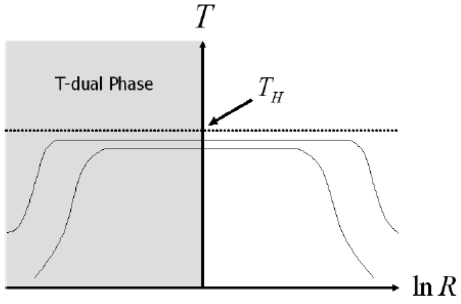
Energy scale of inflation: $V_{\rm inf}^{1/4} \simeq 10^{16}{\rm GeV}(r/0.01)^{1/4}$

GW can give info about inflationary models:

- GW from amplification of vacuum fluctuations
- Generation of GW from additional fields during inflation
- Second order GW from peaks in scalar power spectrum

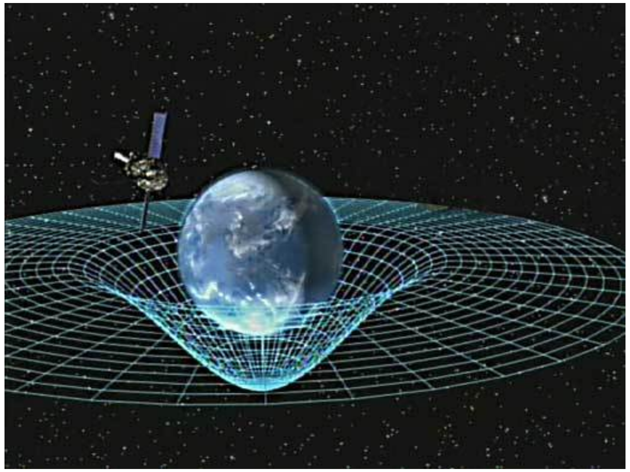
Differentiate between cosmological inflation and alternatives to inflation

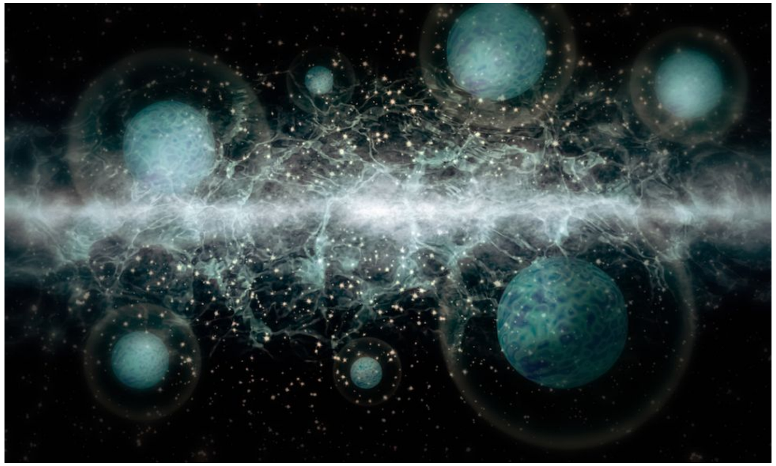
Example: string gas



Blue spectrum of GWs

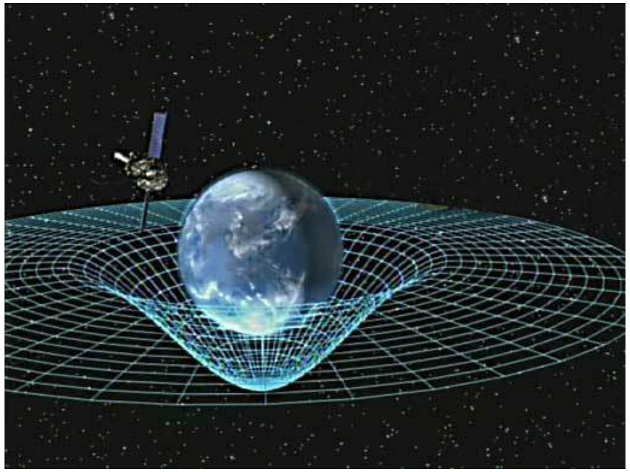
Gravitational Waves and (classical or quantum) theories of gravity

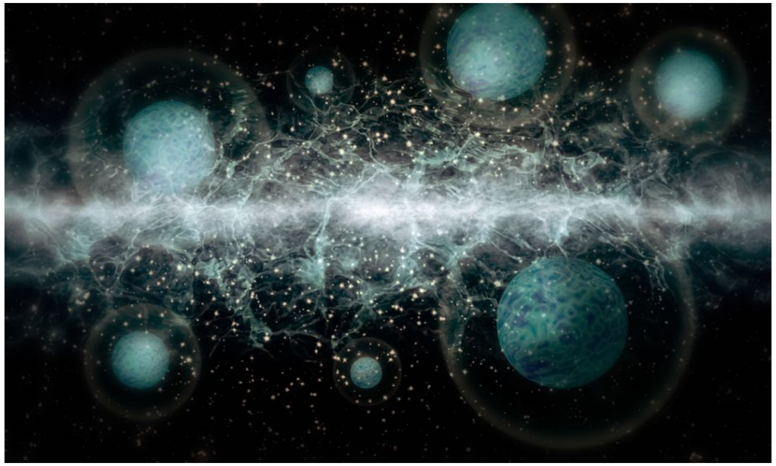




Gravitational Waves and (classical or quantum) theories of gravity

- Signals with electromagnetic counterparts

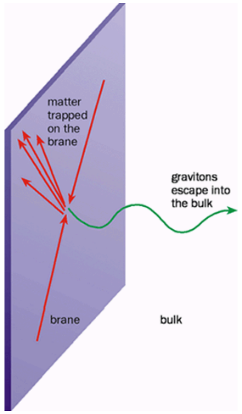




Brane/String theory - Extra dimensions

Constraints on the number of spacetime dimensions from GWs

Damping of the waveform due to gravitational leakage (beyond R_c) into extra dim

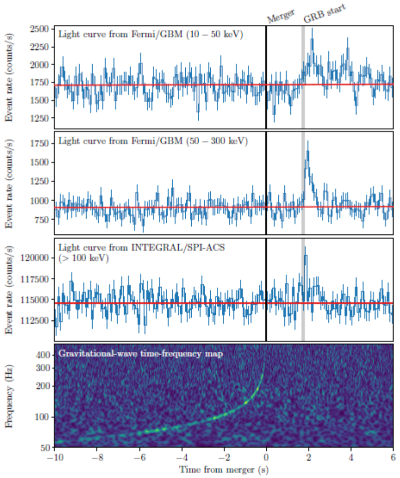


Deviation depends on the number of dimensions D and would result to a systematic **overestimation of the source d_L^{EM} inferred from GW data**

$$h \propto \frac{1}{d_L^{\mathrm{GW}}} = \frac{1}{d_L^{\mathrm{EM}}}\left[1+\left(\frac{d_L^{\mathrm{EM}}}{R_c}\right)^n\right]^{-(D-4)/(2n)}$$

$$d_L^{\mathrm{EM}} \simeq \frac{z(1+z)}{H_0} \overset{z\ll1}{\simeq} \frac{z}{H_0}$$

***Strain** measured in a GW interferometer* ↔ ***Luminosity distance** measured for the optical counterpart of the standard siren*



- Consistency with GR in D=4 dim
- Some models (e.g. the Dvali-Gabadadze-Porrati (DGP) model) are ruled out

LVC PRL (2019)

GRB 170817A and GW170817

BNS merger at 40 Mpc

GW event 1.7 s before γ-ray observation

Testing gravity theories through GW propagation

phenomenological parameters

$$E^2 = p^2c^2 + \boxed{A_\alpha p^\alpha c^\alpha}$$

massive gravity

$$A_0 > 0$$
$$m_g = A_0^{1/2}/c^2$$

doubly special relativity

$$\alpha = 0,0.5,1,1.5,2.5,3,3.5,4$$

Lorentz-violating dispersion relation

multi-fractal ST

Horava-Lifshitz & extra dim

Frequency dependence of speed of GWs

$$\tilde{h}(\nu) = A(\nu)e^{i\Phi(\nu)}$$

Frequency-domain GW phase evolution Φ(ν) described by PN expansion in terms of u/c

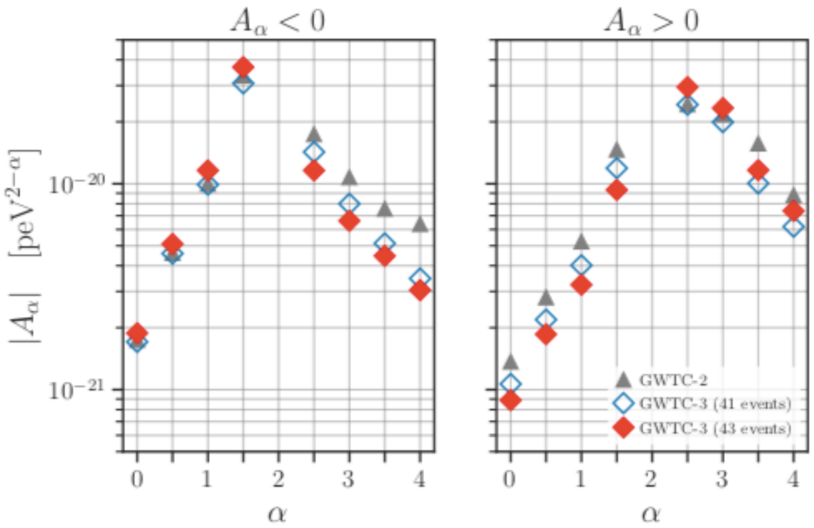
$$\boxed{\tilde{h}(\nu) = A(\nu)e^{i(\Phi(\nu)+\delta\Phi_\alpha(\nu))}}$$

90% credible upper bounds on the absolute value of the modified dispersion relation parameter A_α

$$m_g \leq 1.27 \times 10^{-23}\mathrm{eV}/c^2.$$

~ 2 times more stringent than the most recent Solar System bound

LVKC (2021)



Propagation of GWs in the context of Quantum Gravity

Long-range nonperturbative mechanism found in most QG candidates:
Dimensional flow (change of spacetime dimensionality) - *short distance dimensional reduction in quantum gravity*

ST distorted by QG effects characterised by ST measure ρ (how volume scales) and kinetic term K (modified dispersion relations)

Perturbed action for a small perturbation h over background

$$S = \frac{1}{2\ell_*^{2\Gamma}} \int d\varrho \sqrt{-g^{(0)}} \left[h_{\mu\nu} \mathcal{K} h^{\mu\nu} + O(h_{\mu\nu}^2) + \mathcal{J}^{\mu\nu} h_{\mu\nu} \right]$$

characteristic scale of geometry

scaling parameter

generic source term

$$\boxed{\Gamma(\ell) := \frac{d_{\rm H}(\ell)}{2} - \frac{d_{\rm H}^k(\ell)}{d_{\rm S}(\ell)}}$$

spectral dim: indicator of geometry and topology of spacetime

Hausdorff dim: how volumes scale with their linear size

*Calcagni, Kyroyamagi, Morsat, **Sakellariadou**, Tamanini, Tasinato , PLB 2019; JCAP2019*

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$$\boxed{h \propto \int d\varrho\, \mathcal{J}\, G}$$

The GW amplitude is determined by the convolution of the source with the retarded Green function

$$G(t,r) \sim f_G(t,r)\, r^{-\Gamma}, \text{ where } f_G \text{ is dimensionless.}$$

$$\Downarrow$$

In radial coordinates, and in the local wave zone

$$\boxed{h(t,r) \sim f_h(t,r)\ (\ell_*/r)^{\Gamma}}$$

GW amplitude h is the product of a dimensionless function and a power-law distance behavior

depends on the source J and on the type of correlation function (advanced or retarded)

*Calcagni, Kyroyamagi, Morsat, **Sakellariadou**, Tamanini, Tasinato , PLB 2019; JCAP2019*

Propagation of GWs in the context of Quantum Gravity

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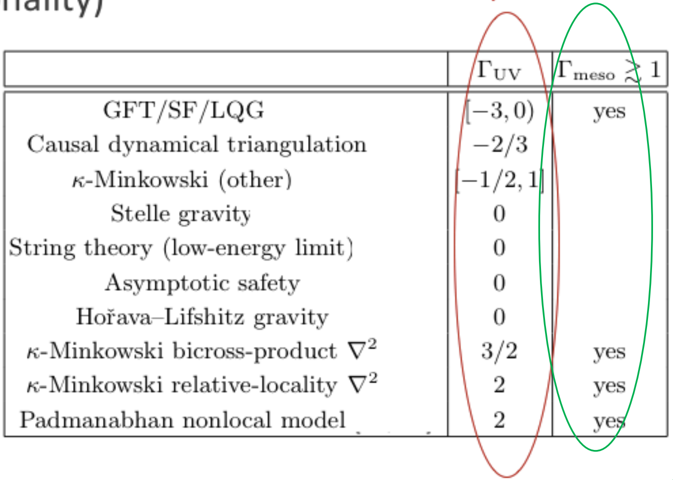
Dimensional flow (change of spacetime dimensionality)

Scaling parameter

$$\boxed{\Gamma(\ell) := \frac{d_{\mathrm{H}}(\ell)}{2} - \frac{d_{\mathrm{H}}^{k}(\ell)}{d_{\mathrm{S}}(\ell)}}$$

QG corrections are important



	Γ_{UV}	$\Gamma_{\mathrm{meso}} \gtrsim 1$
GFT/SF/LQG	$[-3, 0)$	yes
Causal dynamical triangulation	$-2/3$	
κ-Minkowski (other)	$[-1/2, 1]$	
Stelle gravity	0	
String theory (low-energy limit)	0	
Asymptotic safety	0	
Hořava–Lifshitz gravity	0	
κ-Minkowski bicross-product ∇^2	$3/2$	yes
κ-Minkowski relative-locality ∇^2	2	yes
Padmanabhan nonlocal model	2	yes

Contributions to GR
small but non-negligible

Scales at which QG corrections are important: UV regime
Intermediate scales where corrections to GR are small but not negligible: mesoscopic regime

*Calcagni, Kyroyamagi, Morsat, **Sakellariadou**, Tamanini, Tasinato , PLB 2019; JCAP2019*

Propagation of GWs in the context of Quantum Gravity

*The **strain** measured in a GW interferometer* ↔ *The **luminosity distance** measured for the optical counterpart of the standard siren*

$$\boxed{h \propto \frac{1}{d_L^{\rm GW}}, \qquad d_L^{\rm GW} = d_L^{\rm EM}\left[1+\varepsilon\left(\frac{d_L^{\rm EM}}{\ell_*}\right)^{\gamma-1}\right], \qquad \gamma \neq 0,} \quad \varepsilon = \pm(\gamma-1)$$

Standard sirens: - NS merger GW170817 (LIGO/Virgo & Fermi)
--simulated z=2 supermassive BH merger within LISA detectability

When $\gamma = \Gamma_{\rm UV}$ we cannot constrain the deep UV limit of QG, since $\ell_* = \mathcal{O}(\ell_{\rm Pl})$.
(*deviations from classical geometry occur at microscopic scales unobservable in astrophysics)*

The only theories that can be constrained in this way are those with $\Gamma_{\rm meso} > 1 > \Gamma_{\rm UV}$

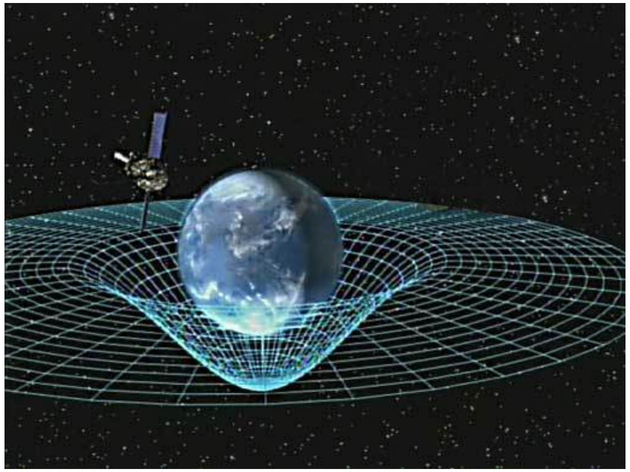
⇒ **Only GFT, SF or LQG *could* generate a signal detectable with standard sirens**

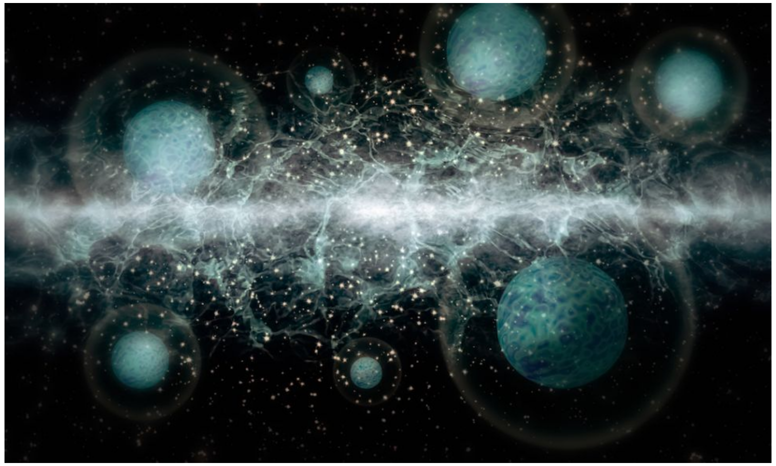
⇒ Look for realistic quantum states of geometry giving rise to such a signal

*Calcagni, Kyroyamagi, Morsat, **Sakellariadou**, Tamanini, Tasinato , PLB 2019; JCAP2019*

Gravitational Waves and (classical or quantum) theories of gravity

- Signals without electromagnetic counterparts





Propagation of GWs in the context of Extended Classical Gravity or Quantum Gravity:

The propagation speed of GWs may vary as a function of the energy scale

Low energies: many theories **spontaneously break Lorentz invariance** through a time-dependent vacuum expectation value (essential for driving cosmic acceleration) of an additional field(s) ➡ tensor speed $c_T < 1$

Examples: *Horndeski theories and their extensions, DHOST (degenerate higher order scalar-tensor theories)*

If the UV completion of an extended gravity theory is required to be Lorentz invariant, then **the graviton speed becomes luminal at high energies**.

A frequency-dependent propagation speed can also arise in any scenario of gravity (typical for many QG theories) where the **spectral dimension of spacetime changes with the probed scale**.
Also, a frequency dependent GW speed arises in **brane-world models** motivated by string theory.

A massive graviton (or the related bigravity) scenario can lead to a frequency-dependent GW velocity, with interesting and testable consequences for GW waveforms.

Propagation of GWs in the context of Extended Classical Gravity or Quantum Gravity:

LVC: BNS GW170817 ⇒ $-3\times10^{-15} \leq c_T - 1 \leq 7\times10^{-16}$ (in $c = 1$ units)

LVKC (2019)

Construct a function for $c_T(f)$ which satisfies the LIGO-Virgo bounds whilst modifying the millihertz regime (LISA)

⇒ sharp transitions for $c_T(f)$ in the frequency band between LISA and LIGO frequencies

Framework:
Dynamics of GW at emission and detection is described by GR (possibly thanks to screening mechanisms)
Deviations from GR can occur during the propagation of GW through cosmological spacetime from source to observation

Method that does not rely on the presence of an electromagnetic counterpart: for long-duration sources, analysis could be applied on-the-fly months or years before merger

*Baker, **Sakellariadou**, et al JCAP 2022*

Propagation of GWs in the context of Extended Classical Gravity or Quantum Gravity:

Assume massless GW propagating freely through cosmological background from source (inspiralling binary) to detection

Quadratic action for the linearised transverse-traceless GW modes

$$\boxed{S_T = \frac{M_{\rm Pl}^2}{8} \int dt\, d^3x\, a^3(t)\, \bar{\alpha} \left[\dot{h}_{ij}^2 - \frac{c_T^2(f)}{a^2(t)} (\vec{\nabla} h_{ij})^2\right]}$$

dimensionless normalisation constant

$$\bar{\alpha} = c_T^{-1}(f_s)$$

the frequency of GW as emitted by an inspiralling binary process

effective metric to describe propagation of GW

$$ds^2 = c_T(f)\, \bar{\alpha} \left[-c_T^2(f)\, dt^2 + a^2(t)\, d\vec{x}^2\right]$$

Ansätze for $c_T(f)$

LIGO bound implies:

$$|\beta_n| \lesssim 10^{-15-n}(f_*/\mathrm{Hz})^n$$

- Polynomial ansatz

motivated from a perturbative expansion in powers of $(f/f_\star)$

$$c_T(f) = 1 + \sum_n \beta_n \left(\frac{f}{f_*}\right)^n$$

positive or negative integer

fixed frequency scale controlling the onset of the deviations

set of parameters controlling deviations from GR

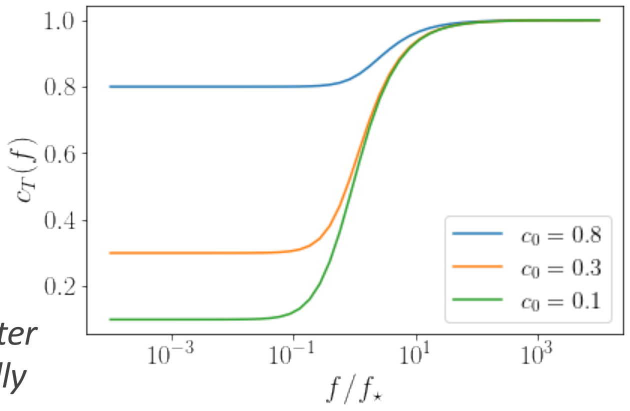
- EFT-inspired ansatz

Smooth transition in c_T *from some lower value to c, taking place inside or close to the LISA band*

$$c_T(f) = \left[1 + \frac{f_\star^2}{f^2} - \frac{f_\star^2}{f^2}\sqrt{1 + 2\left(1 - c_0^2\right)\frac{f^2}{f_\star^2}}\right]^{1/2}$$

low-frequency speed with $0 < c_0 \leq 1$

fiducial frequency, a free parameter around which c_T *changes rapidly*



Mairi Sakellariadou

GW amplitude

The two helicities of GW waveform for the binary compact object inspiral in Fourier space:

$$h_+(f) = A(f)\frac{1+\cos^2\iota}{2}e^{i\Psi(f)}, \qquad h_\times(f) = iA(f)\cos\iota\ e^{i\Psi(f)}$$

redshifted GW amplitude

$$\boxed{A^{\mathrm{GR}}(f_z) = \sqrt{\frac{5\pi}{24}\frac{\mathcal{M}_z^2}{(1+z)r_{\mathrm{com}}}}(\pi\mathcal{M}_z f_z)^{-7/6}}$$

$$\mathcal{M}_z = (1+z)\mathcal{M}_s$$

$$\mathcal{M}_s = M_{\mathrm{tot}}\eta^{3/5}$$

chirp mass of binary at the source

$$\eta = m_1 m_2/M_{\mathrm{tot}}$$

reduced mass parameter

$$f_z = f_s/(1+z)$$

redshift frequency

modified GW amplitude

$$\boxed{A^{\mathrm{MG}}(f_o) = \sqrt{\frac{5\pi}{24}\frac{\mathcal{M}_o^2}{d_L^{\mathrm{GW}}}}(\pi\mathcal{M}_o f_o)^{-\frac{7}{6}}\left[\frac{c_T(f_o)}{c_T(f_s)}\right]^{\frac{3}{2}}}$$

$$d_L^{\mathrm{GW}} = (1+z_e)\ (1-\Delta)^{-\frac{1}{2}}\ r_{\mathrm{com}}^{\mathrm{GW}}$$

$$f_o = f_z\frac{c_T(f_o)}{c_T(f_s)}$$

frequency at detection

$$\mathcal{M}_o = \mathcal{M}_z\frac{c_T(f_s)}{c_T(f_o)}$$

observed chirp mass

GW phase

The phase of GW during inspiral can be computed analytically using **PN expansion**

We focus on GW propagation effects, so **we do not consider modifications to the physics of the merging process at the source position** ⟹ **the rate of change of GW frequency in the source frame should match that of GR**

Consider **non-spinning binary systems on circular orbits**

$$\boxed{\frac{df_o}{dt_o} = (1-\Delta)^2 \left(\frac{1}{1 + \frac{f_o}{1-\Delta} \frac{\partial \Delta}{\partial f_o}} \right) \frac{96}{5\pi\, \mathcal{M}_z^2}\, u^{\frac{11}{3}} \left[1 + \psi_1 u^{\frac{2}{3}} + \psi_{1.5} u + \psi_2 u^{\frac{4}{3}} + \psi_{2.5} u^{\frac{5}{3}} \right]}$$

red-shifted chirp mass

$u = \pi \mathcal{M}_s f_s = \pi \mathcal{M}_z f_z = \pi \mathcal{M}_o f_o$ *frame-independent*

$\psi_k \;\; (k \; = \; 1, 1.5, 2, 2.5)$ *the PN phase parameters*

$$\Delta = \frac{f_s - (1+z)\, f_o(f_s, z)}{f_s}$$

to parametrise deviations from GR

Note: - The 3PN term remains subdominant in all our calculations

Integrate to find time to coalescence and then the GW phase

Mairi Sakellariadou

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Integrate to find time to coalescence and then the GW phase

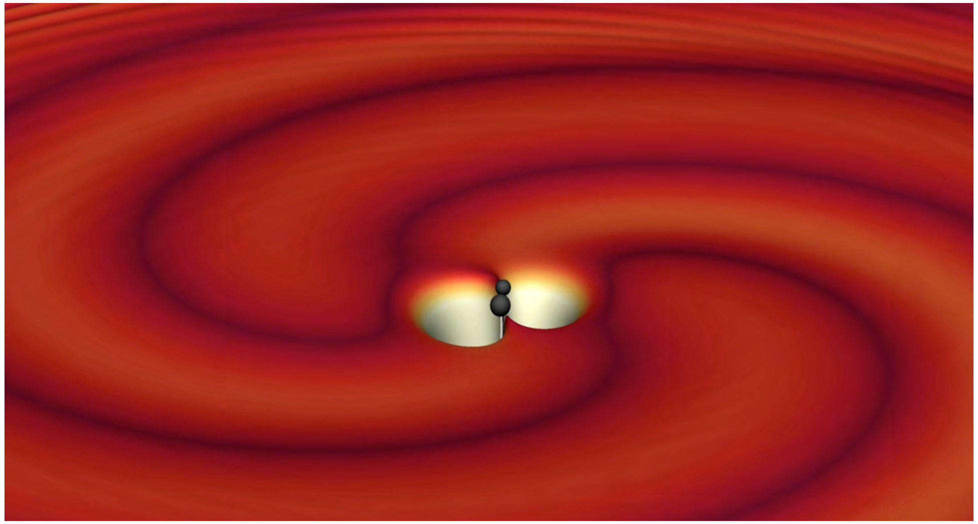
Tests of gravity at low frequency can be carried out with LISA in (almost) any scenario

Mairi Sakellariadou

Conclusions

The implications of gravitational-wave detections can hardly be overestimated

For instance:



- beyond the standard model particle physics
  - topological defects: cosmic strings
  - strong first order phase transitions
- nature of dark matter
  - axions
  - primordial black holes
  - DM microphysics
- classical and quantum theories of gravity
  - theories where GWs are accompanied by EM counterparts
  - theories where GWs are not necessarily accompanied by EM counterparts