

# The EMC effect within the light-front Hamiltonian dynamics for few-nucleon bound systems

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# Based on

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F.F, E.Pace, M.Rinaldi, G.Salmè, S.Scopetta and M.Viviani,  
“The EMC effect for few-nucleon bound systems in Light-Front Hamiltonian Dynamics”,  
in preparation [arXiv:2308.15925 \[nucl-th\]](https://arxiv.org/abs/2308.15925)

E.Pace, M.Rinaldi, G.Salmè and S.Scopetta  
“The European Muon Collaboration effect in Light-Front Hamiltonian Dynamics”,  
**Phys. Lett. B 839(2023) 127810**

R.Alessandro, A.Del Dotto, E.Pace, G.Perna, G.Salmè and S.Scopetta  
“Light-Front Transverse Momentum Distributions for  $J = 1/2$  Hadronic Systems in Valence Approximation”,  
**Phys.Rev.C 104(2021) 6,065204**

A. Del Dotto, E.Pace, G. Salmè and S.Scopetta  
“Light-Front spin-dependent Spectral Function and Nucleon Momentum Distributions for a Three-Body system”,  
**Phys. Rev. C 95,014001 (2017)**

E.Pace, M.Rinaldi, G.Salmè and S.Scopetta  
“EMC effect, few-nucleon system and Poincaré covariance”,  
**Phys. Scr. 95, 064008 (2020)**

MARATHON Coll.

“Measurement of the Nucleon  $F_2^n/F_2^p$  Structure Function Ratio by the Jefferson Lab MARATHON Tritium/Helium-3 Deep Inelastic Scattering Experiment”,  
**Phys. Rev. Lett 128 (2022) 13,132003**

# Outline

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• Overview

• The EMC effect

• The Light-Front Poincaré covariant approach

• Nuclear structure functions with Relativistic Hamiltonian Dynamics

• Numerical results

• Conclusions

# Overview

- Goal: extend the approach applied for  ${}^3\text{He}$  to **any nuclei**  $A$  and calculate the **EMC ratio** for  ${}^4\text{He}$  and  ${}^3\text{H}$  [E. Pace, M. Rinaldi, G. Salmè, S. Scopetta, *Phys. Lett. B* 839 (2023) 137810]
- Since  ${}^4\text{He}$  is a **strongly bound system** this could provide a challenging test to our approach
- Compare EMC effect for  ${}^3\text{He}$ ,  ${}^4\text{He}$  and  ${}^3\text{H}$  obtained by different **modern NN and NNN interactions** (Argonne V18+UIX, NVIa+3N, NVIb+3N)
- Compare the EMC effect for  ${}^4\text{He}$  obtained by different choice of  $F_2^p$  and  $F_2^n$  **parametrization**

[R. B. Wiringa, V. G. J. Stoks, R. Schiavilla, *Phys. Rev. C* 51 (1995) 38–51]

[R. B. Wiringa et al., *Phys. Rev. Lett.* 74 (1995) 4396–4399]

[M. Viviani et al., *Phys. Rev. C* 107 (1) (2023) 014314]

[M. Piarulli et al., *Phys. Rev. Lett.* 120 (5) (2018) 052503]

[M. Piarulli, S. Pastore, R. B. Wiringa, S. Brusilow, R. Lim, *Phys. Rev. C* 107 (1) (2023) 014314]

# The EMC effect

Almost 40 years ago, the European Muon Collaboration (EMC) measured (in DIS processes)

$$R(x) = F_2^{56\text{Fe}}(x) / F_2^{2\text{H}}(x)$$

Expected result:  $R(x) = 1$  up to corrections of the Fermi motion

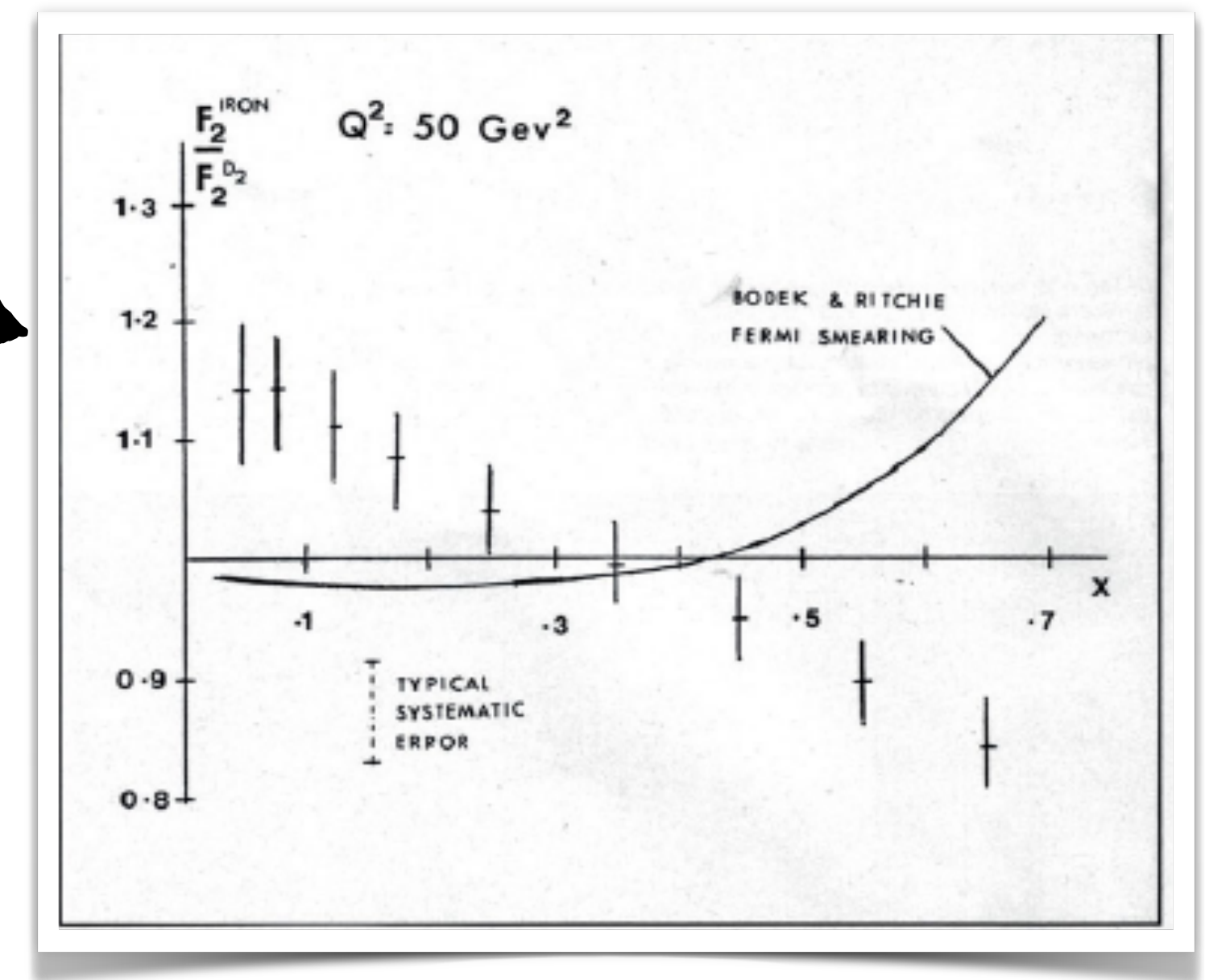
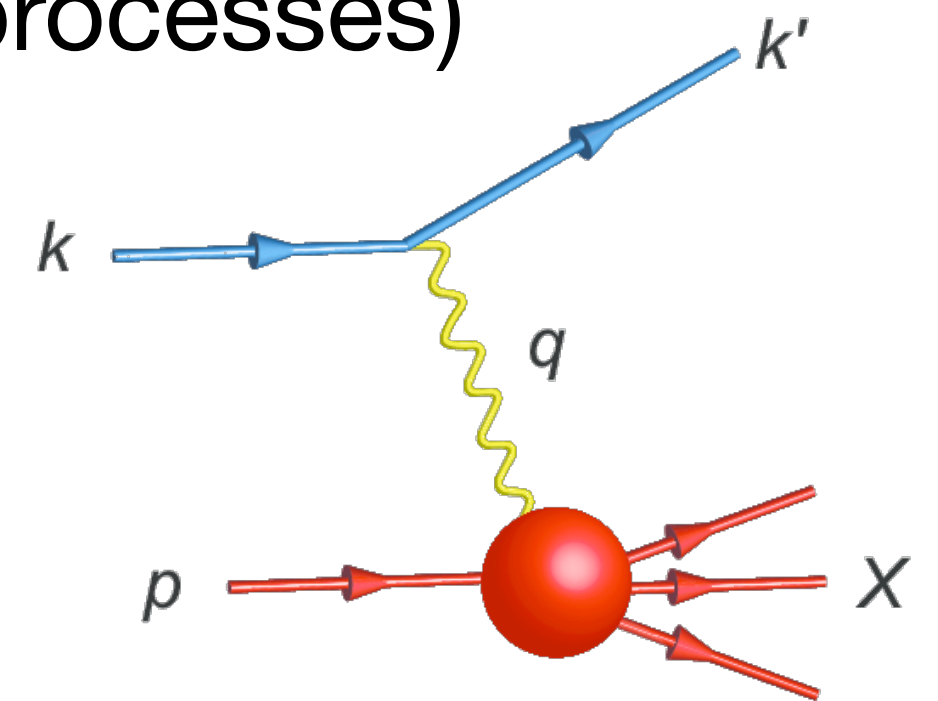
Result:

Aubert et al. Phys.Lett. B123 (1983) 275

Naive parton model interpretation:

“Valence quarks, in the bound nucleon, are in average slower than in the free nucleon”

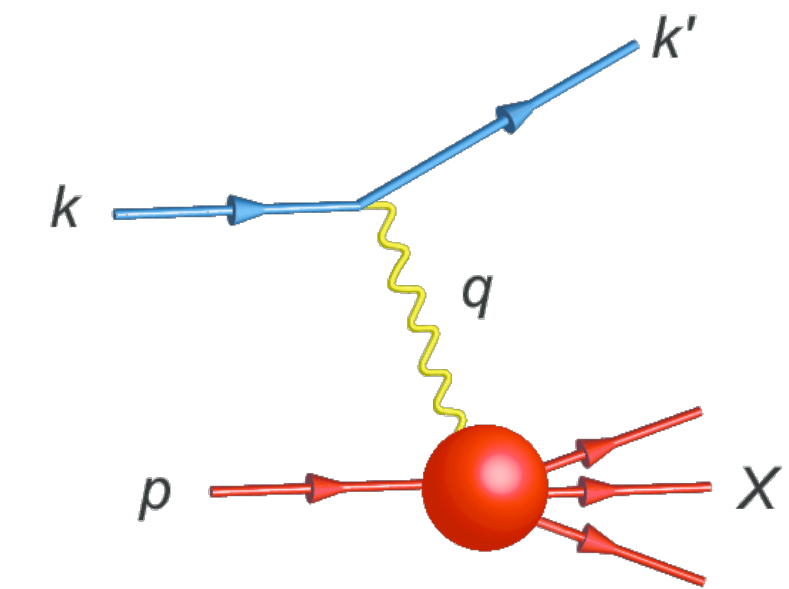
Is the bound proton bigger than the free one??



# The EMC effect

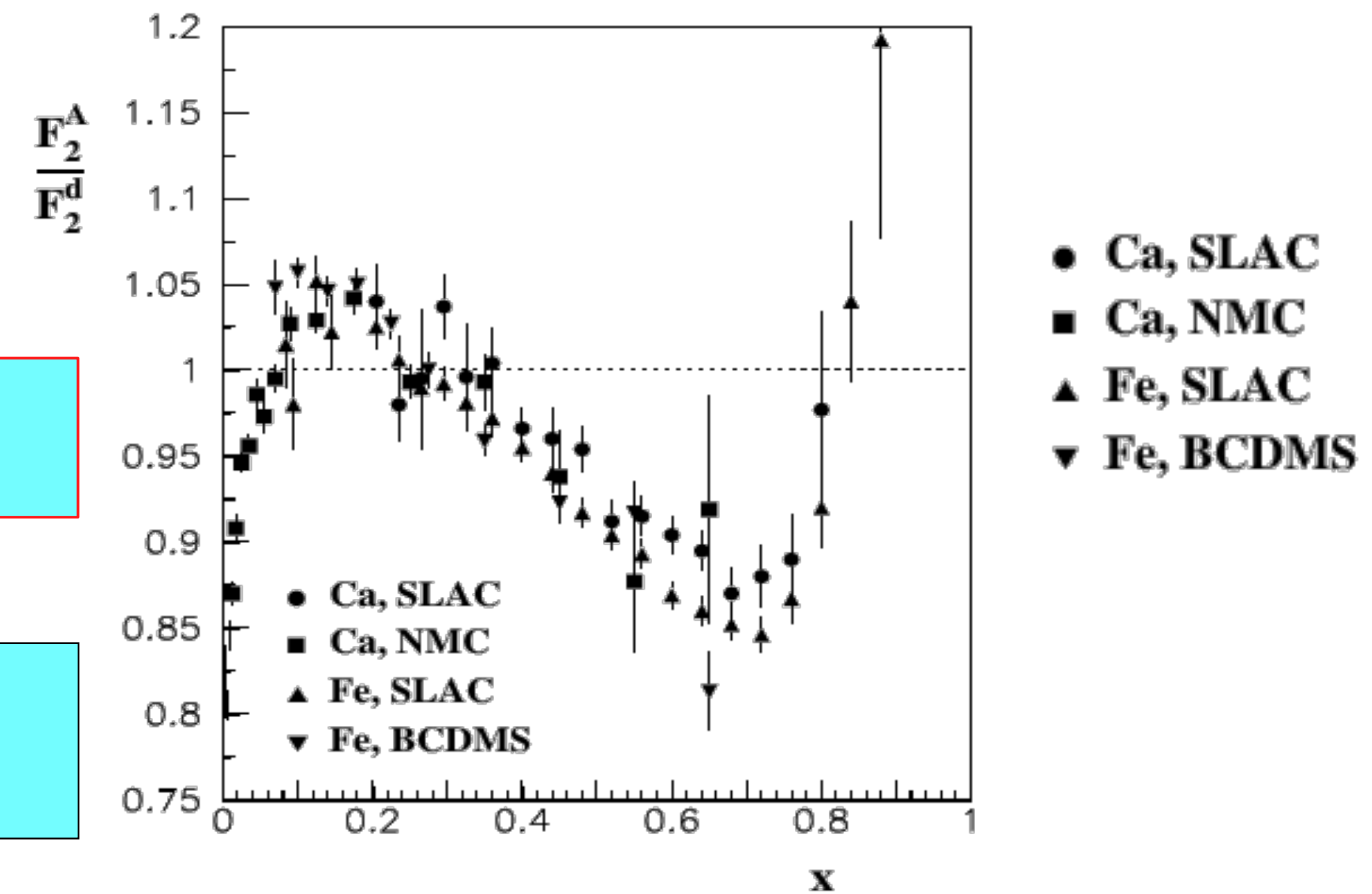
We remind that for DIS off nuclei:  $0 \leq x = \frac{Q^2}{2M\nu} \leq \frac{M_A}{M} \simeq A$

- ◆  $x \leq 0.3$  “**Shadowing region**”: coherence effects, the photon interacts with partons belonging to different nucleons
- ◆  $0.2 \leq x \leq 0.8$  “**EMC (binding) region**”: mainly valence quarks involved
- ◆  $0.8 \leq x \leq 1$  “**Fermi motion region**”



**main features:** universal behavior independent on  $Q^2$ ; weakly dependent on  $A$ ; scales with the density  $\rho \rightarrow$  global property? Or due to SRC  $\rightarrow$  local property?

**Explanation (exotic) advocated:** confinement radius bigger for bound nucleons, quarks in bags with 6, 9, ...,  $3A$  quark, pion cloud effects... Alone or mixed with conventional ones...

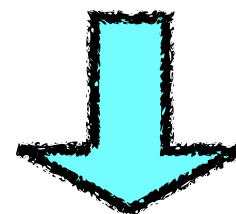


# The EMC effect: what do we know

**Situation: *basically not understood*.** Very unsatisfactory. We need to know the reaction mechanism of hard processes off nuclei and the degrees of freedom which are involved:

**Status of “conventional” calculations for light nuclei:**

**NR Calculations:** qualitative agreement but no fulfillment of both particle and MSR... Not under control



**Our approach is aimed to include only nucleonic dof through conventional nuclear physics in a Poincaré-covariant approach. The only way to fulfill sum rules while using realistic NR nuclear potentials is to embed relativistic effects.**

A completely NR calculation of the effect due to **nucleonic dof** (conventional nuclear physics) could **overestimate** the effect due to **exotic explanations** (involving genuine QCD effects). We want to remark that because of the magnitude of the EMC effect also small **corrections due to relativistic effects** could be significant if we want to study the contribution of partonics dof

# Need for a relativistic treatment

## Why do we need a relativistic treatment ?

**General answer:** to develop an advanced scheme, appropriate for the kinematics of JLAB12 and of EIC

- The **Standard Model of Few-Nucleon Systems**, with nucleon and meson degrees of freedom within a non relativistic (**NR**) framework, has achieved **high sophistication** [e.g. the NR  $^3\text{He}$  and  $^3\text{H}$  Spectral Functions in **Kievsky, Pace, Salmè, Viviani PRC 56, 64 (1997)**].
- Covariance wrt the **Poincaré Group,  $G_P$** , needed for nucleons at large 4-momenta and pointing to high precision measurements.
- **At least**, one should carefully treat the **boosts** of the nuclear states,  $|\Psi_i\rangle$  and  $|\Psi_f\rangle$ !

Our definitely preferred framework for embedding the successful **NR phenomenology**:

**Light-front Relativistic Hamiltonian Dynamics (RHD, fixed dof) + Bakamjian-Thomas (BT)** construction of the Poincaré generators for an interacting theory.



# The relativistic Hamiltonian dynamics framework

In **RHD+BT**, one can address both Poincaré **covariance** and **locality**, general principles to be implemented in presence of interaction:

**Poincaré covariance** → The 10 generators,  $P^\mu \rightarrow 4D$  displacements and  $M^{\nu\mu} \rightarrow$  Lorentz transformation, have to fulfill:

$$[P^\mu, P^\nu] = 0; [M^{\mu\nu}, P^\rho] = -i(g^{\mu\rho}P^\nu - g^{\nu\rho}P^\mu)$$

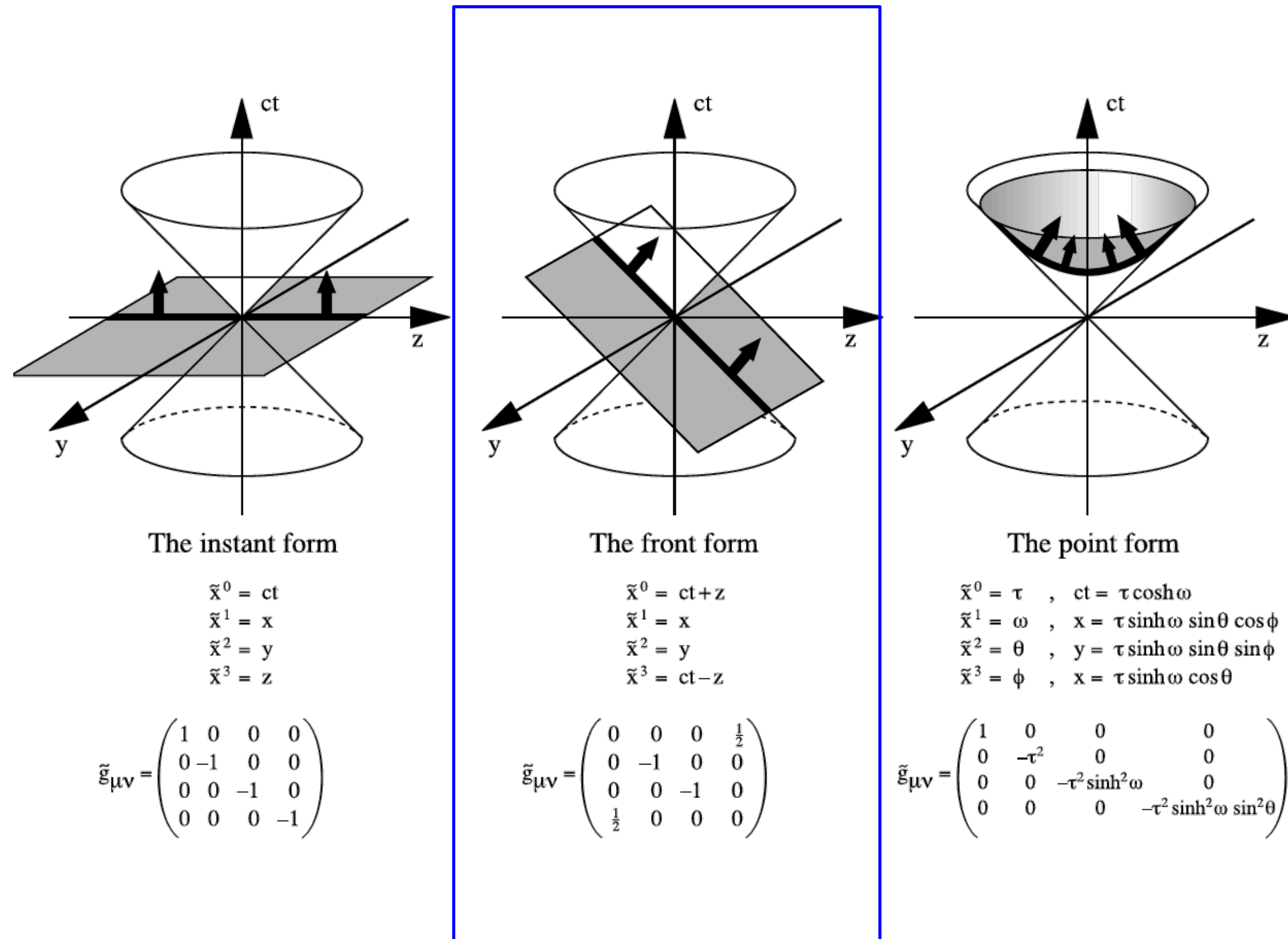
$$[M^{\mu\nu}, M^{\rho\sigma}] = -i(g^{\mu\rho}M^{\nu\sigma} + g^{\nu\sigma}M^{\mu\rho} - g^{\mu\sigma}M^{\nu\rho} - g^{\nu\rho}M^{\mu\sigma})$$

**Macroscopic locality** (= **cluster separability** (relevant in nuclear physics)): i.e. observables associated to different space-time regions must commute in the limit of **large space like separation** (i.e. causally disconnected). In this way, when a system is separated into disjoint subsystems by a sufficiently large space like separation, then the **subsystems behave as independent systems**.

Keister, Polyzou, Adv. Null. Phys. 20,225 (1991)

This requires a careful choice of the intrinsic relativistic coordinates

# Forms of relativistic Dynamics



**We choose the Front Form!**

Fig. from Brodsky, Pauli, Pinsky Phys.Rept. 301 (1998) 299-486

# Advantages of the Light-Front framework

The Light-Front framework has several advantages:

- **7 Kinematical generators:** i) **3 LF boosts** (in instant form they are dynamical!) ;  
ii)  $\tilde{P} = (P^+ = P^0 + P^3, \mathbf{P}_\perp)$  ; iii) Rotation around the z-axis
- The **LF boosts** have a subgroup structure: trivial Separation of intrinsic and global motion, as in the NR case
- $P^+ = 0 \rightarrow$  **meaningful Fock expansion**, once massless constituents are absent
- No square root in the **dynamical operator**  $P^-$ , propagating the state in the LF-time
- The **infinite-momentum frame (IMF)** description of DIS is easily included

**Drawback:** the transfers LF-rotations are dynamical!

**But** within the **Bakamjian-Thomas (BT)** construction of the generators in an interacting theory, one can construct an intrinsic angular momentum fully kinematical

# The Bakamjian-Thomas construction

Bakamjian and Thomas (PR 92 (1953) 1300) proposed an **explicit construction** of 10 Poincaré generators in presence of **interactions**. The key ingredient is the **mass operator**:

- i) Only the **mass operator**  $M$  contains the interaction since  $P^- = P_0^- + \frac{1}{P^+}[M^2 - M_0^2]$
- ii) It generates the dependence of the **3 dynamical generators** ( $P^-$  and LF transverse rotations) upon the interaction
- iii) The eigenvalue equation  $M^2 |\psi\rangle = s |\psi\rangle$  is formally equivalent to the **Schrödinger equation**

The **mass operator** is given by the sum of  $M_0$  with an interaction  $V$ , or  $M_0 + U$ . The interaction,  $U$  or  $V$ , must commute with all the kinematical generators and with the non-interacting angular momentum, as in the **NR** case.

# The BT construction for a nuclear system

For a generic nucleus  $A$ , the mass operator is

$$M[1,2,3,\dots,A] = M_0[1,2,3,\dots,A] + V(\mathbf{k}^2; \mathbf{k} \cdot \mathbf{k}_j; \mathbf{k}_j \cdot \mathbf{k}_i)$$

2 and 3 body forces operator

Where the free mass operator of the system is:

$$M_0[1,2,3,\dots,A] = \sum_i^A \sqrt{m^2 + \mathbf{k}_i^2}$$

Momenta in the intrinsic reference frame  $\sum_{i=1}^A \mathbf{k}_i = 0$

The **commutation rules** impose to  $V$  **invariance for translations and rotations** as well as **independence on the total momentum**, as it occurs for  $V^{NR}$

One can assume  $M[1,2,\dots,A] \sim M^{NR}$

Therefore what has been learned till now about **the nuclear interaction**, within a **non-relativistic framework**, can be re-used in a **Poincaré covariant framework**.

# Reference frames

For a correct description of the structure functions, so that the **Macro-locality** is implemented, it is crucial to distinguish between different frames, moving with respect to each other:

- The **Lab frame**, where  $\tilde{P} = (M, \mathbf{0}_\perp)$
- The intrinsic LF frame of the whole system,  $[1, 2, \dots, A]$ , where  $\tilde{P} = (M_0[1, 2, \dots, A], \mathbf{0}_\perp)$  with  $k_i^+ = \xi_i M_0[1, 2, \dots, A]$  and  $M_0[1, 2, \dots, A] = \sum_{i=1}^A \sqrt{m^2 + \mathbf{k}_i^2}$
- The intrinsic LF frame of the cluster  $[1; 2, 3, \dots, (A - 1)]$  where  $\tilde{P} = (\mathcal{M}_0[1; 2, 3, \dots, A - 1], \mathbf{0}_\perp)$  with  $k^+ = \xi \mathcal{M}_0[1; 2, 3, \dots, A - 1]$  and  $\mathcal{M}_0[1; 2, 3, \dots, A - 1] = \sqrt{m^2 + \kappa^2} + \sqrt{M_s^2 + \kappa^2}$

While  $\mathbf{p}_\perp^{LAB} = \mathbf{k}_{1\perp} = \kappa_\perp$

$M_s = (A - 1)\sqrt{m^2 + m\epsilon}$  is the mass of the spectator system

Since we use an **impulse approximation** assumption (i.e. the scattering involves only a nucleon described by a plane wave), we have to define the **LF spectral function**: the probability to find a particle with a given  $\tilde{\kappa} = (\kappa^+, \kappa_\perp)$  when the rest of the system has energy  $\epsilon$  with a polarization  $S$

The spectral function is written in terms of the overlap  ${}_{LF} \langle tT; \alpha, \epsilon; JJ_z; \tau\sigma', \tilde{\kappa} | \Psi_{\mathcal{M}}; ST_z \rangle_{LF}$

The tensor product of the plane wave of the interacting particle and the state of the spectator system

In the intrinsic reference frame of the cluster  $[1; 2, 3, \dots, A - 1]$

The wave function of the nucleus A (i.e. the eigenstate of  $M[1, 2, \dots, A] \sim M^{NR}$ )

In the intrinsic frame of the system  $[1, 2, \dots, A]$

We can express the **LF overlap** in terms of the **IF overlap** using **Melosh rotations** and then we can approximate the IF overlap into a NR overlap thanks to the **BT construction**:

$$|\{\alpha\}; \phi \rangle_{LF} \rightarrow |\{\alpha\}; \phi \rangle_{IF} \sim |\{\alpha\}; \phi \rangle_{NR}$$

The **LF spectral function** contains the determinant of the Jacobian of the transformation between the intrinsic frames  $[1; 2, 3, \dots, A - 1]$  and  $[1, 2, \dots, A]$ , connected each other by a **LF boost**

# Hadronic tensor

In our approach the **hadronic tensor** is found to be (E.Pace, M.Rinaldi, G.Salmè and S. Scopetta, Phys. Scri. 2020)

$$W_A^{\mu\nu}(P_A, T_{Az}) = \sum_N \sum_\sigma \not\int d\epsilon \int \frac{d\kappa_\perp d\kappa^+}{(2\pi)^3 2\kappa^+} \frac{1}{\xi} \mathcal{P}^N(\tilde{\kappa}, \epsilon) W_{N,\sigma}^{\mu\nu}(p, q)$$

hadronic tensor of the bound nucleon

LF spectral function

In the Bjorken limit the **nuclear structure function** can be obtained from the **hadronic tensor**

$$F_2^A(x) = -\frac{1}{2} x g_{\mu\nu} W^{\mu\nu} = \sum_N \sum_\sigma \int d\epsilon \int \frac{d\kappa_\perp d\kappa^+}{(2\pi)^3 2\kappa^+} P^N(\tilde{\kappa}, \epsilon) F_2^N(z)$$

Free nucleon structure function

Where  $x = \frac{Q^2}{2P_A \cdot q}$  and  $\xi = \frac{\kappa^+}{\mathcal{M}_0[1; 2, 3, \dots, A-1]} \neq x$  with  $z = \frac{Q^2}{2p \cdot q} = \frac{p \cdot x}{P_A^+ \xi}$



# LC momentum distribution

$$F_2^A(x) = -\frac{1}{2}xg_{\mu\nu}W^{\mu\nu} = \sum_N \sum_{\sigma} \int d\epsilon \int \frac{d\mathbf{k}_{\perp}}{(2\pi)^3} \frac{d\kappa^+}{2\kappa^+} P^N(\tilde{\mathbf{k}}, \epsilon) F_2^N(z)$$

In the Bjorken limit  $\int d\epsilon \int d\kappa^+ = \int d\kappa^+ \int d\epsilon$  so we can use the **LC momentum distribution** instead of the **LF spectral function**

A. Del Dotto, E. Pace, G. Salmè, S. Scopetta, Physical Review C 95, 014001 (2017)

$$F_2^A(x) = \sum_N \int_{\xi_{min}}^1 d\xi F_2^N\left(\frac{m x}{\xi M_A}\right) f_1^N(\xi)$$

Light-cone momentum distribution

Free nucleon structure function

Determinant of the Jacobian matrix. LF boost: effect of a Poincaré covariance approach

Squared nuclear wave function. Thanks to the BT construction, one is allowed to use the NR one

$$\text{With: } f_1^N(\xi) = \int d\mathbf{k}_{\perp} n^N(\xi, \mathbf{k}_{\perp})$$

LF momentum distribution:

$$n^N(\xi, \mathbf{k}_{\perp}) = \frac{1}{2\pi} \int \prod_{i=2}^{A-1} [d\mathbf{k}_i] \left| \frac{\partial k_z}{\partial \xi} \right| \mathcal{N}^N(\mathbf{k}, \mathbf{k}_2, \dots, \mathbf{k}_{A-1})$$

# Convolution formula for the nuclear structure function

To calculate the EMC ratio  $R_{EMC}^A(x) = \frac{F_2^A(x)}{F_2^d(x)}$  for any nucleus A, we need a **NR realistic wave function** and a **parametrization for the free-nucleon structure functions**

$$F_2^A(x) = \sum_N \int_{\xi_{min}}^1 d\xi \left[ F_2^N\left(\frac{m x}{\xi M_A}\right) f_1^N(\xi) \right]$$

Calculated through 3 different potentials: Av18+UIX and 2 versions of the Norfolk  $\chi EFT$  interactions NVIa+3N and NVIb+3N

We need both  $F_2^n$  and  $F_2^p$

One can choose a parametrization for  $F_2^p$  and a parametrization for the ratio  $\frac{F_2^n}{F_2^p}$  because  $F_2^n$  could be only extracted by nuclear DIS data. We used a parametrization extracted by MARATHON data.

MARATHON Coll., Phys. Rev. Lett 128 (2022) 13,132003

Since our approach fulfill both **macro-locality** and **Poincaré covariance** the LC momentum distribution must satisfies 2 essential **sum rules**:

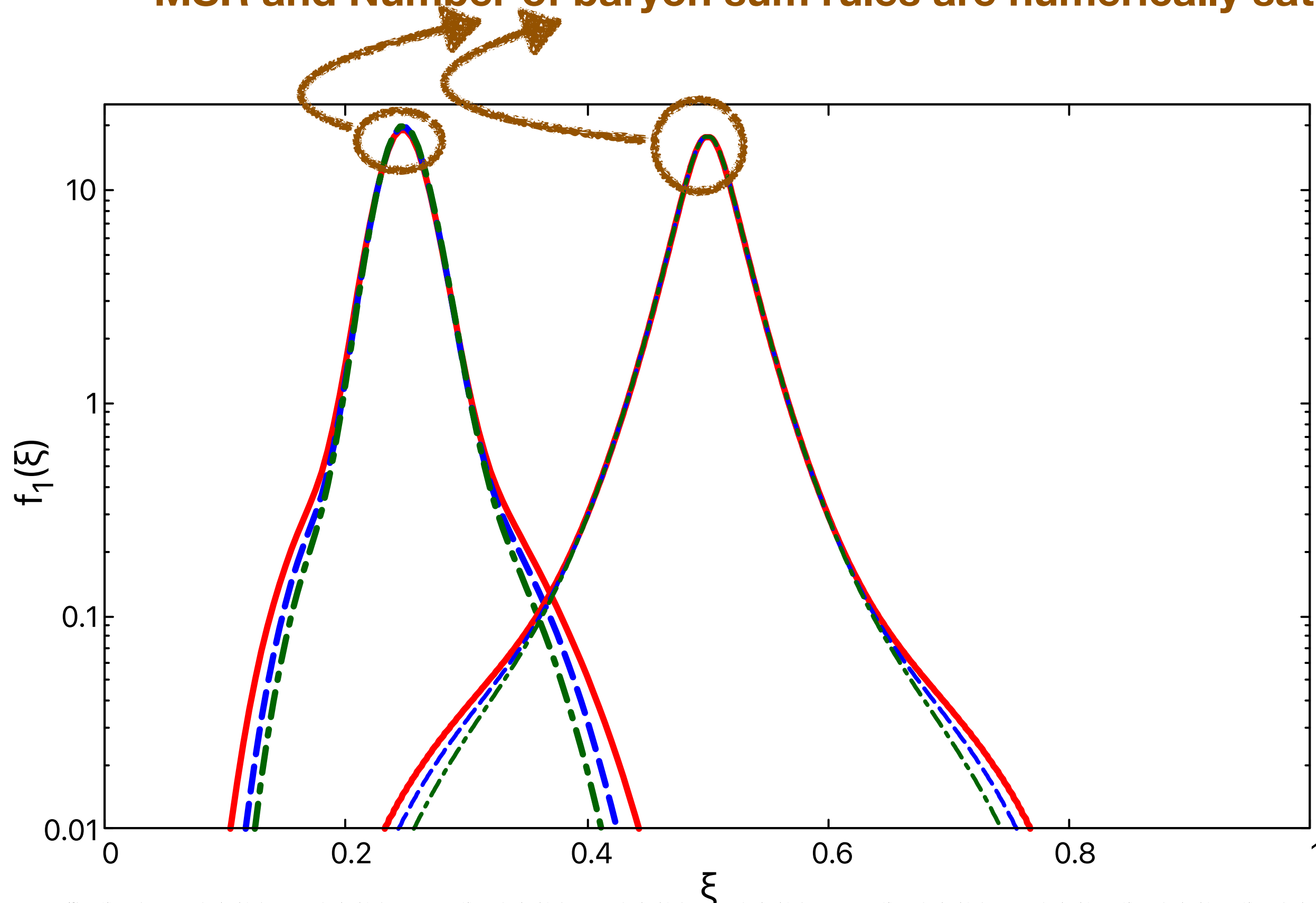
$$A = \int_0^1 d\xi [Z f_1^p(\xi) + (A - Z) f_1^n(\xi)]: \text{Baryon number sum rule;}$$

$$1 = Z \langle \xi \rangle_p + (Z - N) \langle \xi \rangle_n ; \langle \xi \rangle_N = \int_0^1 d\xi \xi f_1^N(\xi): \text{MSR}$$

# LC momentum distribution: numerical results for ${}^4\text{He}$

The distributions are peaked in  $1/A$  with an accuracy of  $1/1000$ :  
**MSR and Number of baryon sum rules are numerically satisfied**

arXiv:2308.15925 [nucl-th]



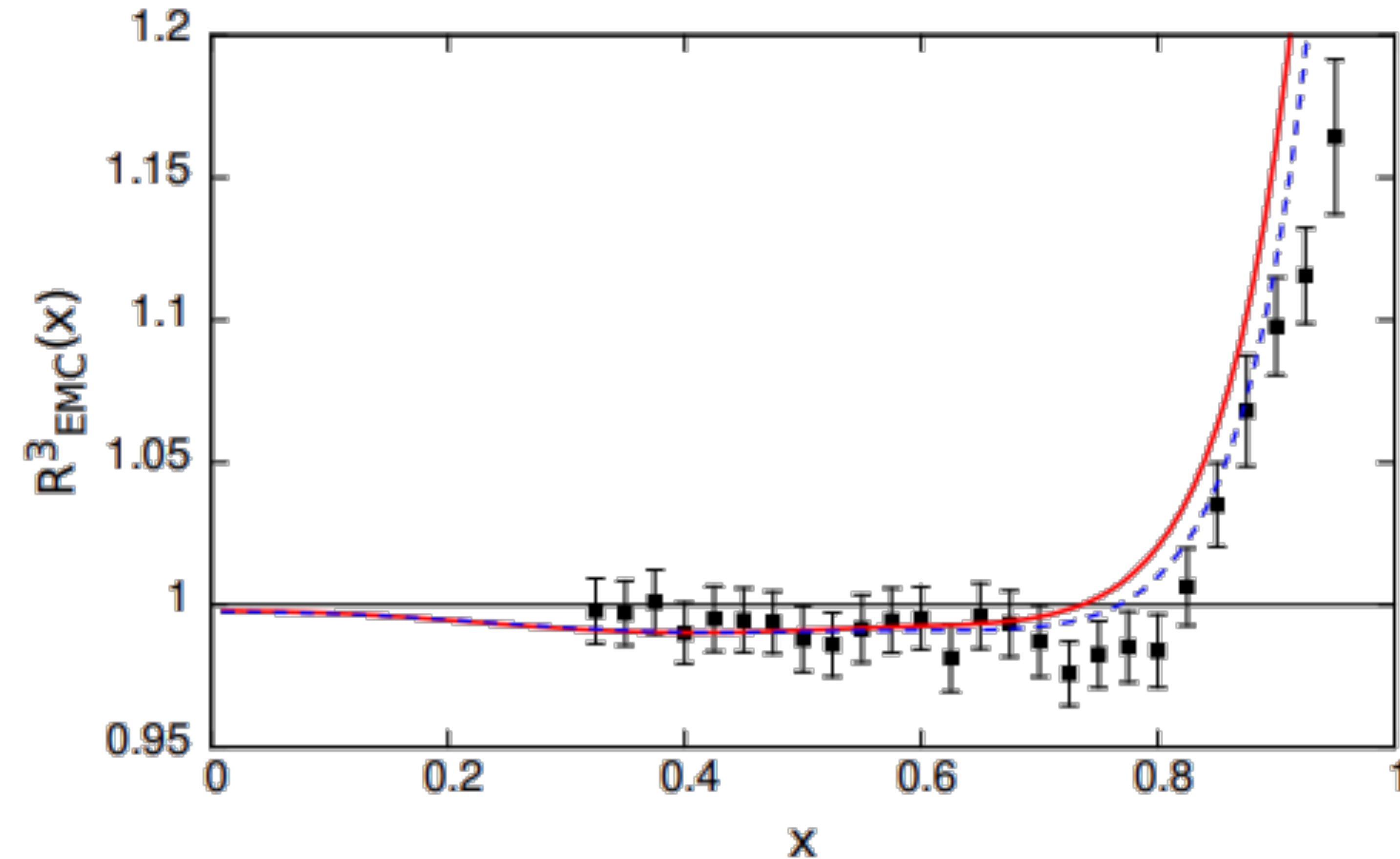
- The tails of the distributions are generated by the **short range correlations (SRC)** induced by the potentials (i.e the **high-momentum content** of the 1-body momentum distribution)
- The tails of the LC momentum distribution calculated by the **Av18/UIX** potential is larger than the ones obtained by the  $\chi$ **EFT** potentials for both  ${}^4\text{He}$  and deuteron
- This difference will partially cancel out on the EMC ratio

**LC momentum distribution for  ${}^4\text{He}$  (peaked in 0.25) and deuteron (peaked in 0.5). Remind that for symmetric nuclei  $f_1^n = f_1^p$**

**Solid lines: Av18/UIX. Dashed lines: NVIb+3N. Dot-dashed lines: NVIa+3N**

# The EMC effect: results for ${}^3\text{He}$

E.Pace, M.Rinaldi, G.Salmè and S.Scopetta Phys. Lett. B 839(2023) 127810



**Result:** small but solid effect, comparable to the experimental one

**Full squares: JLab data from experiment E03103**  
[J. Arrington, et al, Phys. Rev. C 104 (6) (2021) 065203] **as reanalyzed in** [S. A. Kulagin and R. Petti, Phys. Rev. C 82, 054614 (2010)]

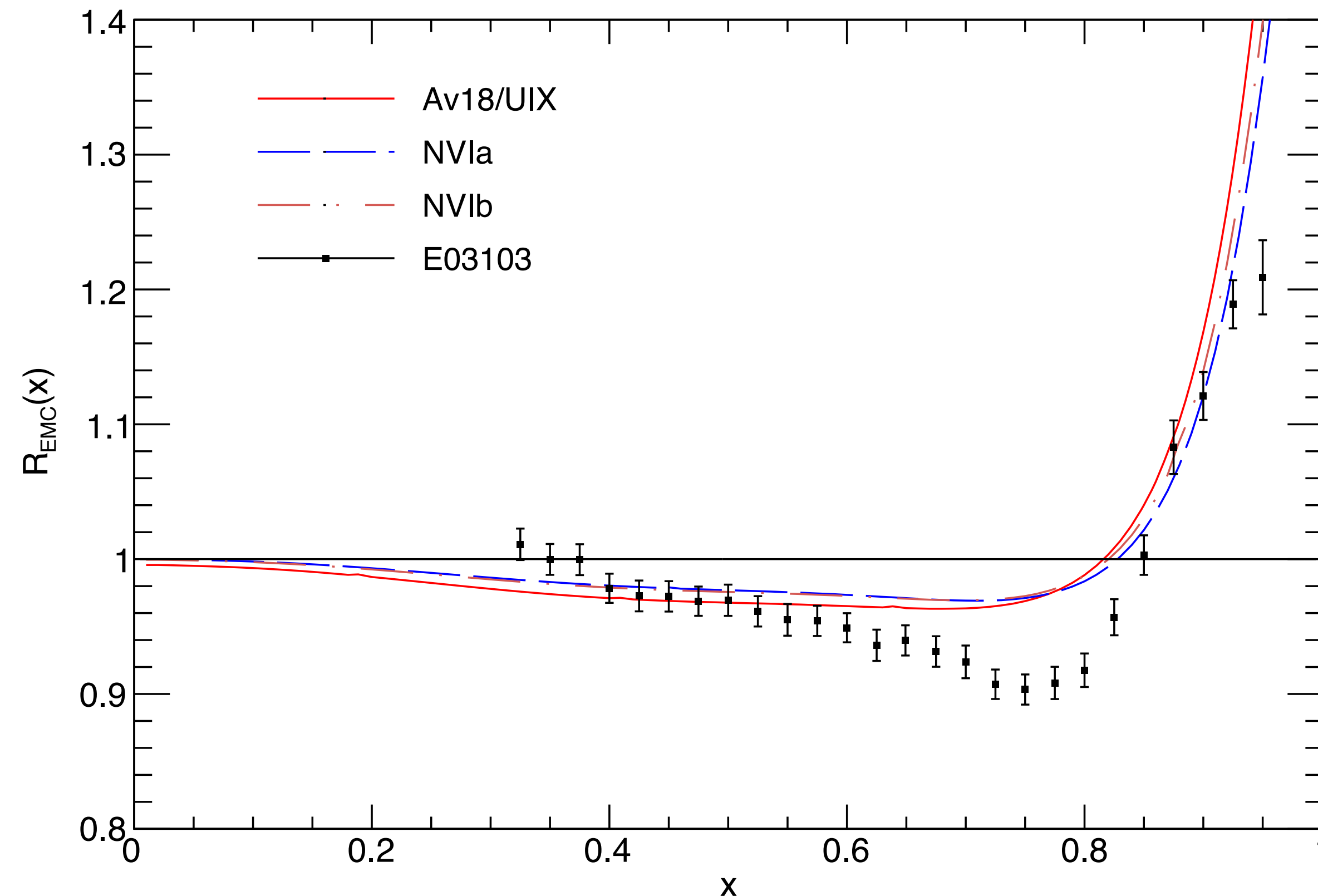
**Solid lines: Av18/UIX. Dashed lines: Av18**

# The EMC effect: numerical results for ${}^4\text{He}$

arXiv:2308.15925 [nucl-th]

Full squares: JLab data from  
experiment E03103

[J. Arrington, et al, Phys. Rev. C 104 (6)  
(2021) 065203]



Analogous results obtained  
also for  ${}^3\text{He}$  and  ${}^4\text{He}$

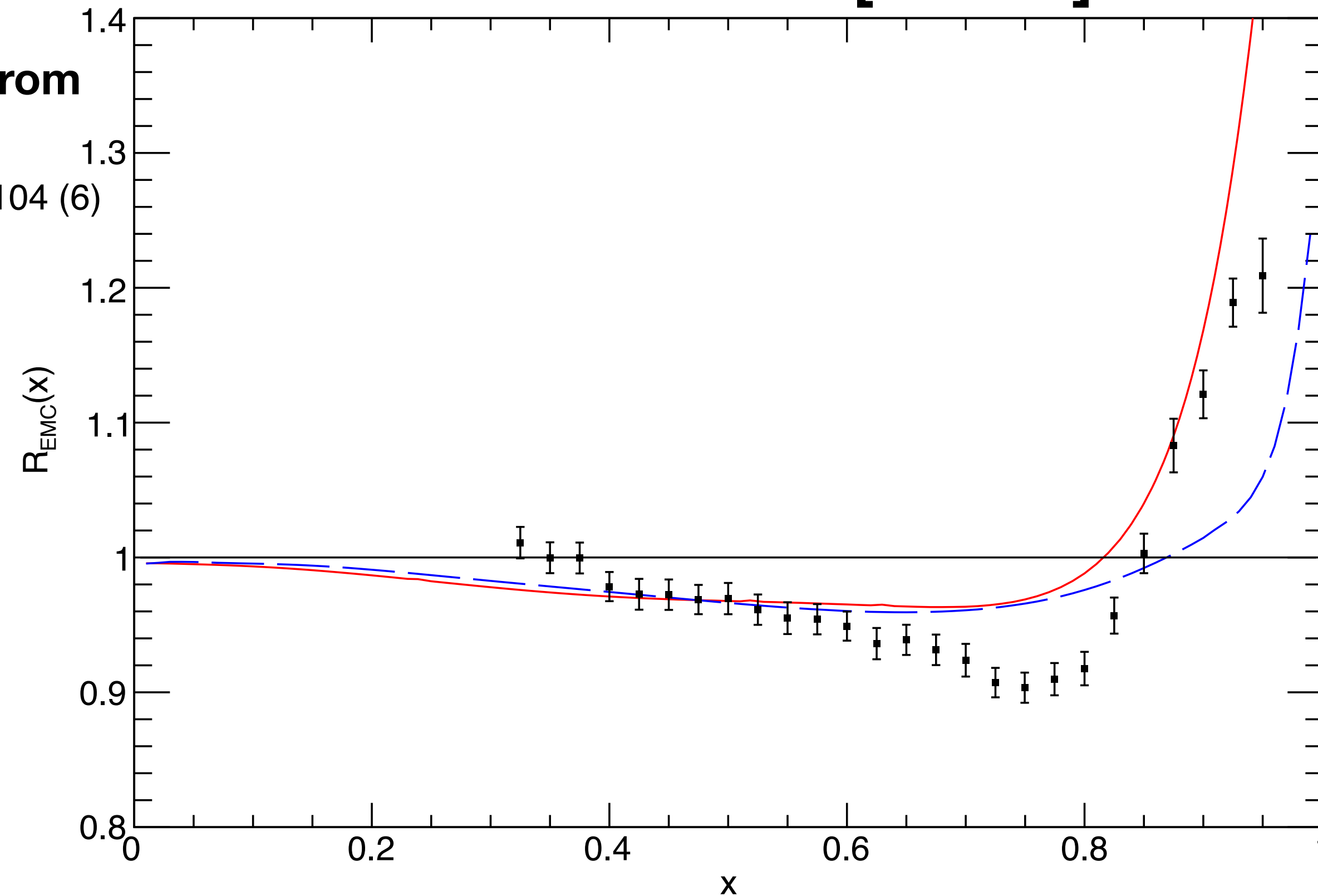
- The differences between the calculations from different potentials are of the **same order for both nuclei**
- They are definitely **smaller than the difference between data and theoretical prediction**

# The EMC effect: numerical results for ${}^4\text{He}$

arXiv:2308.15925 [nucl-th]

Full squares: JLab data from experiment E03103

[J. Arrington, et al, Phys. Rev. C 104 (6) (2021) 065203]



Both lines calculated with Av18/UIX

Solid line: SMC parametrization of  $F_2^p$

Dashed line: NV1b+3N: CJ15 +TMC

Parametrization of  $F_2^p$

$F_2^n$  extracted from MARATHON data

[B. Adeva, et al., Phys. Lett. B 412 (1997) 414–424.]

[A. Accardi, L. T. Brady, W. Melnitchouk, J. F. Owens, N. Sato, Phys. Rev. D 93 (11) (2016) 114017]

[MARATHON, PRL 128,132003 (2022) ]

[E.Pace, M.Rinaldi, G.Salmè and S.Scopetta Phys. Lett. B 839(2023) 127810]

The dependance on the ratio  $F_2^n/F_2^p$  is **largely under control** as well the dependance on the parametrization of  $F_2^p$  in the properly EMC region

# Conclusions

- The results increase confidence in our **light-front approach**, which includes **only the nucleonic dof**
- The difference between the  $R_{EMC}$  generated by different **realistic nuclear potentials** is relatively **smaller than the EMC effect itself**
- The deviations from experimental data could be ascribed to **genuine QCD effects**
- Our results could provide a **reliable baseline to study exotic phenomena involving partonic dofs**

## In preparation

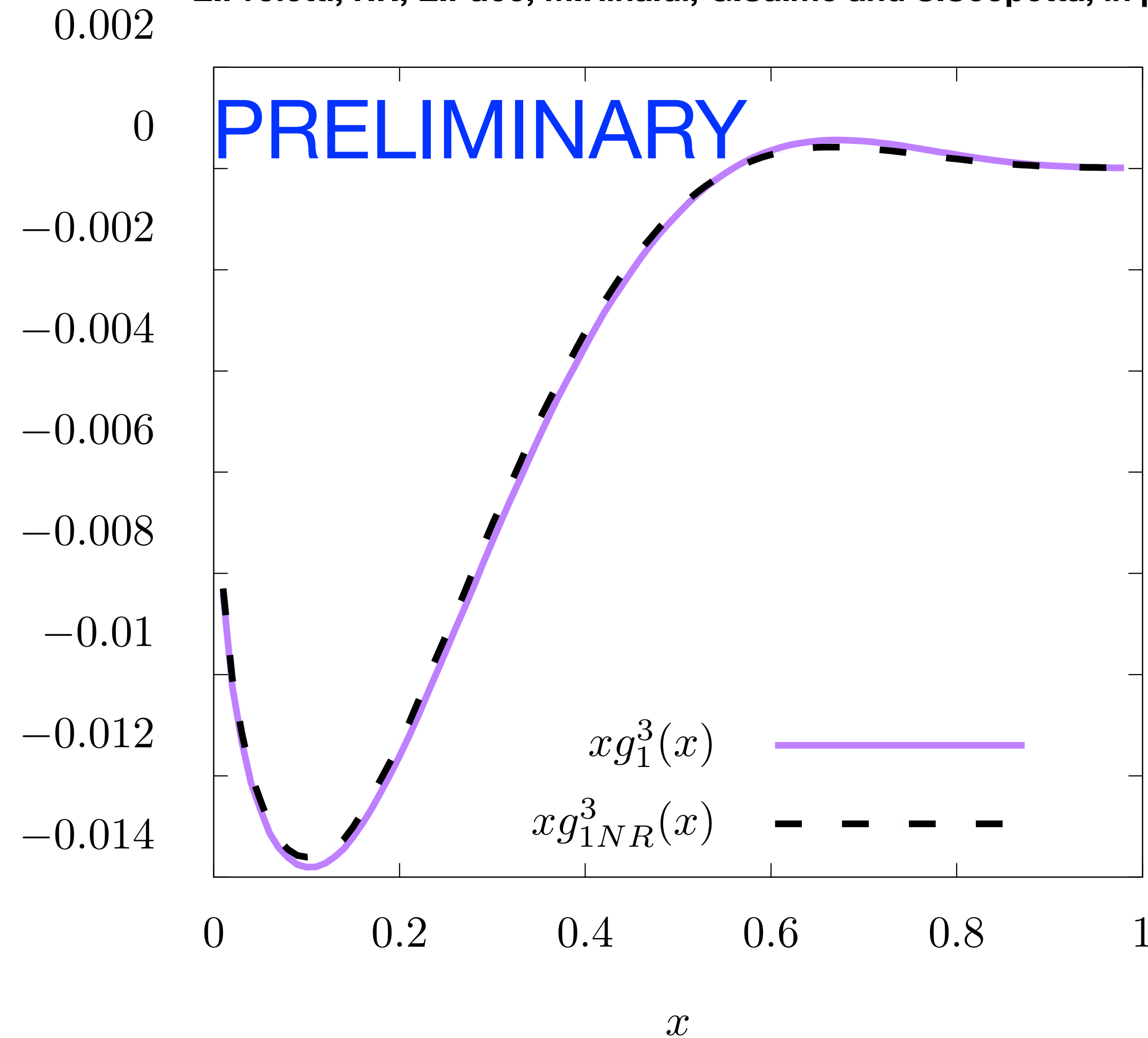
- **Spin-dependent** structure functions for  ${}^3\text{He}$

## To do next:

- Include **off-shell** corrections to our approach
- Repeat the calculation of the EMC-effect for **heavier nuclei**

# Polarized Structure Functions for ${}^3\text{He}$

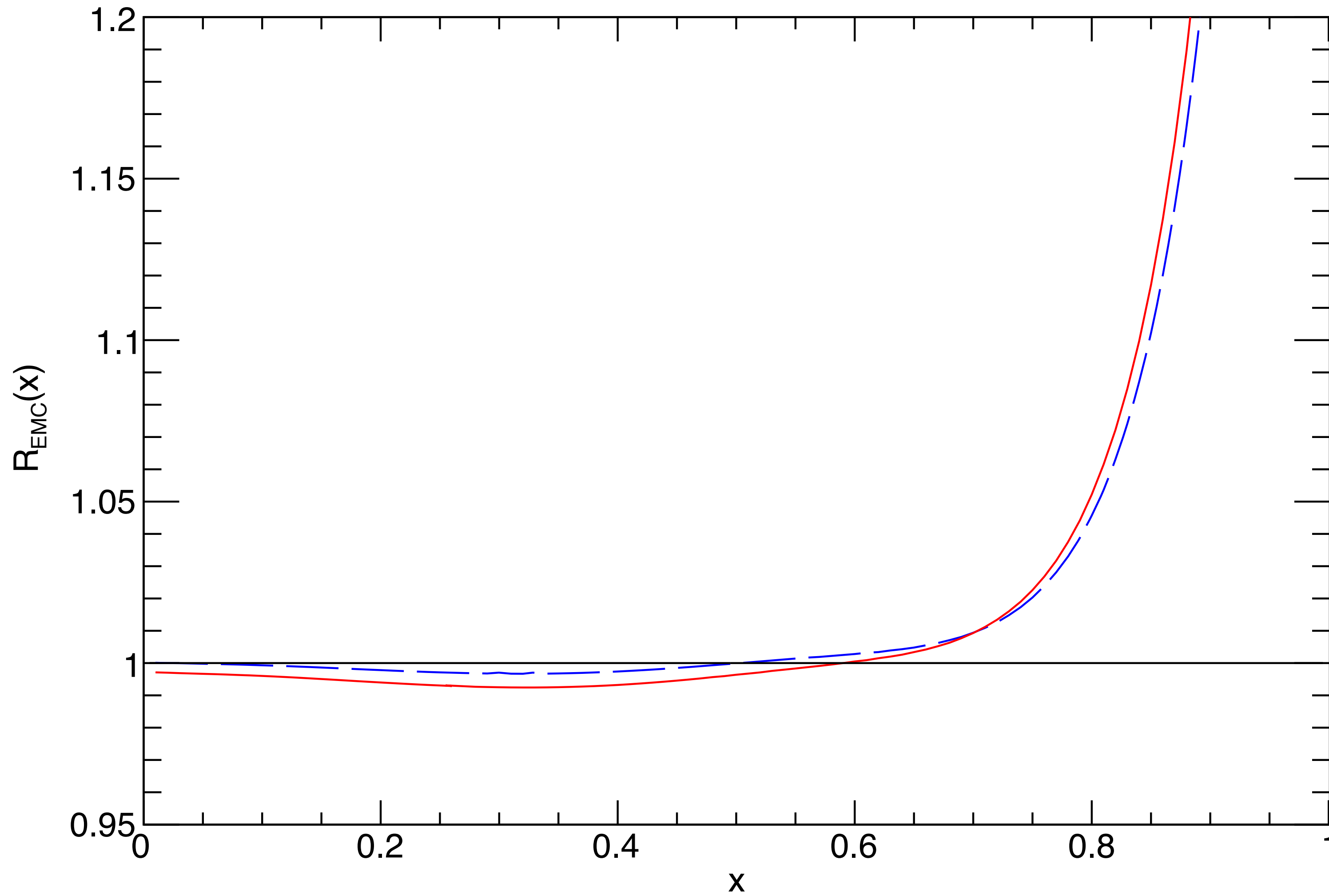
E.Proietti, F.F., E.Pace, M.Rinaldi, G.Salmè and S.Scopetta, in preparation



Comparison between the spin-dependent structure function of  ${}^3\text{He}$   $g_1^3(x)$  obtained by a NR approach (dashed-line) in [**C.Ciofi degli Atti, S.Scopetta, E.Pace, G.Salme, Phys.Rev.C48:968-972(1993)**] and RHD approach (solid line)



# Tritium EMC effect



Results similar to  ${}^3He$  and  ${}^4He$

No experimental data

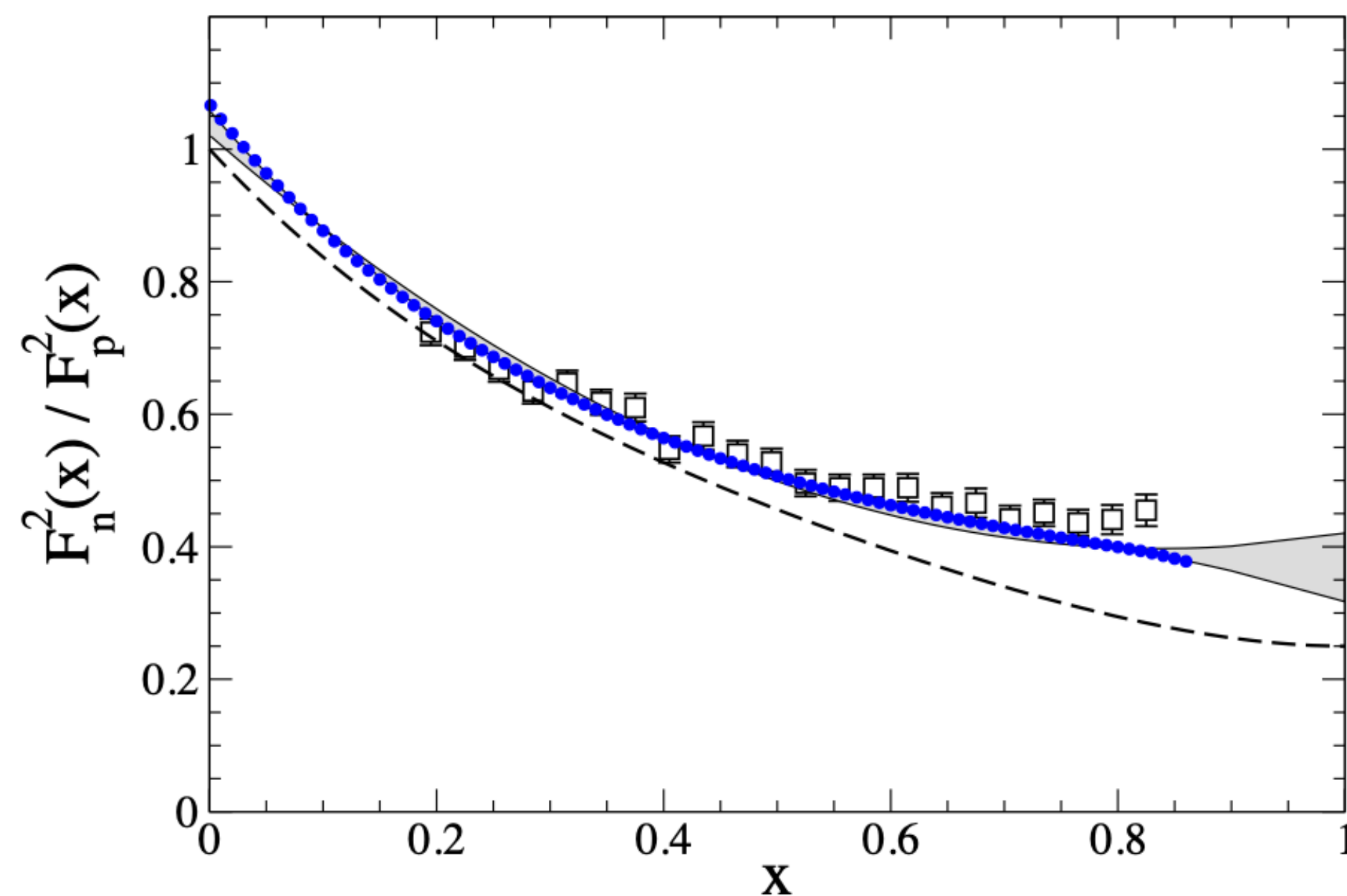
Solid line: Av18/UIX; Dashed-line: NVIb/UIX

# Extraction of $F_2^n/F_2^p$ via MARATHON data

MARATHON coll. : experimental data of the super-ratio  $R^{ht}(x) = F_2^{3He}(x)/F_2^{3H}(x)$

${}^3He$ :  $2p + n$ ;  ${}^3H$ :  $n + 2p$

Is possible to extract the ratio  $F_2^n(x)/F_2^p(x)$  through the super-ratio



**E.Pace, M.Rinaldi, G.Salmè and S.Scopetta Phys. Lett. B 839(2023) 127810**

Dashed line: ratio from SMC collaboration  
Empty squares: MARATHON extraction  
Solid line: cubic and conic extractions from  $F_2^p$  SMC parametrization, fitted to MARATHON data

# NR potentials

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2-body case (equal mass):

$$M^2 |\psi\rangle = s |\psi\rangle$$

$$[4(m^2 + k^2) + U] |\psi\rangle = s |\psi\rangle$$

$$\left[ \frac{k^2}{4m} + \frac{U}{4m} \right] |\psi\rangle = \frac{s - 4m^2}{4m} |\psi\rangle$$

For the states of the continuum spectrum  $\frac{s - 4m^2}{2m} = T_{Lab}$ , i.e. the asymptotic kinetic energy in the Lab frame when the particle 1 is at rest

The phase shifts are given as a function of  $T_{Lab}$

NN interaction extracted from the experimental phase shifts via the Lippmann-Schwinger equation