Radiative Corrections to the Gribov-Zwanziger Effective Model for QCD

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UERJ
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Work in collaboration with

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- Marcela Pelaez (UdeLaR – Uruguay)
- Antonio Pereira (UFF – Brazil)

Outline of the talk

- What is Gribov's problem?
- What can we do about it?
- What do we learn from it?

But first,

a little background…

A little background…

- Feynman's sum over stories:

$$\langle \phi(x_1)\phi(x_2)\rangle = \int [D\phi]\, \phi(x_1)\phi(x_2)\exp(-S(\phi))$$

[R. P. Feynman (1918-1988)]

- Free, massless, scalar theory (Euclidean):

$$S = \int d^d x \frac{1}{2}(\partial_\mu \phi)^2 = \int d^d x \frac{1}{2}\phi(-\partial^2)\phi$$

- Free scalar theory:

$$\langle \phi(x_1)\phi(x_2)\rangle_{free} = G(x_1 - x_2)$$

$G(x_1 - x_2)$: Green's function (i.e., "inverse") of $(-\partial^2)$

A little background…

- Classical Electrodynamics:

$$S_{QED} = \int d^4x \left[\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu) + \bar{\psi}(i\gamma_\mu\partial_\mu - m)\psi + J_\mu A_\mu\right]$$

$$= \int d^4x \left[\frac{1}{2}A_\mu(\delta_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu)A_\nu + \bar{\psi}(i\gamma_\mu\partial_\mu - m)\psi + J_\mu A_\mu\right]$$

A little background...

Photon's "kinetic energy" $= \frac{1}{4} F_{\mu\nu} F_{\mu\nu}$

Electron's "kinetic energy"

- Classical Electrodynamics:

$$S_{QED} = \int d^4x \left[\frac{1}{4} (\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu) + \bar{\psi}(i\gamma_\mu \partial_\mu - m)\psi + J_\mu A_\mu \right]$$

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Electron-photon Interaction term

A little background…

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Electron-photon Interaction term

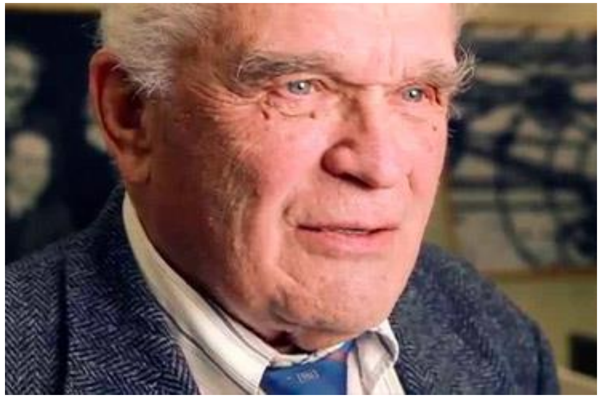
- The operator $(\delta_{\mu\nu}\partial^2 - \partial_\mu \partial_\nu)$ has no inverse (it's a projector)!
- Photon propagator = ?
- Direct quantization is not so simple!

A little background…

- QED: not really a big deal (turnarounds are available)
- QCD, Electroweak (non-abelian): big deal
- A solution to quantization: Faddeev-Popov (1967)

$$Z[J] = \int [DA][D\bar{\psi}][D\psi] \exp(-S_{FP})$$

$$S_{FP} = S_{Classical} + S_{ghosts} + S_{gauge-fixing}$$



[L. D. Faddeev (1934)]

[V. N. Popov (1937-1994)]

A little background…

- Euclidean SU(N) Yang-Mills (i.e., QCD without quarks, if N=3):

$$S_{Classical} = \int d^d x \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A_\mu A_\nu$$

$$D^{ab}_\mu = \delta^{ab} \partial_\mu - g f^{abc} A^c_\mu$$

$$S_{gauge-fixing} = \frac{1}{2\alpha} \left(\partial_\mu A^a_\mu\right)^2$$

(Landau gauge)

$$Z[J] = \int [DA] \det(-\partial D) \exp\left(-S_{gf} + \int J^a_\mu A^a_\mu\right)$$

$$S_{gf} = S_{Classical} + S_{gauge-fixing}$$

A little background…

- Faddeev-Popov operator: $M^{ab} := -\partial_\mu D_\mu^{ab} = -\delta^{ab}\partial^2 + g f^{abc} A_\mu^c \partial_\mu$
- Determinant in terms of Grassmann "scalar" fields c and $\bar{c}$ ("FP ghosts"):

$$\det(M) = \int [D\bar{c}][Dc] \exp\left(\int d^d x\, \bar{c}^a\, M^{ab}\, c^b \right)$$

$$S_{ghosts} = -\int d^d x\;\, \bar{c}^a\, M^{ab}\, c^b$$

$$Z[J] = \int [DA] \exp\left(-S_{FP} + \int J_\mu^a A_\mu^a\right)$$

$$S_{FP} = S_{Classical} + S_{gauge-fixing} + S_{ghosts}$$

Great for perturbative calculations!

What is Gribov’s problem?

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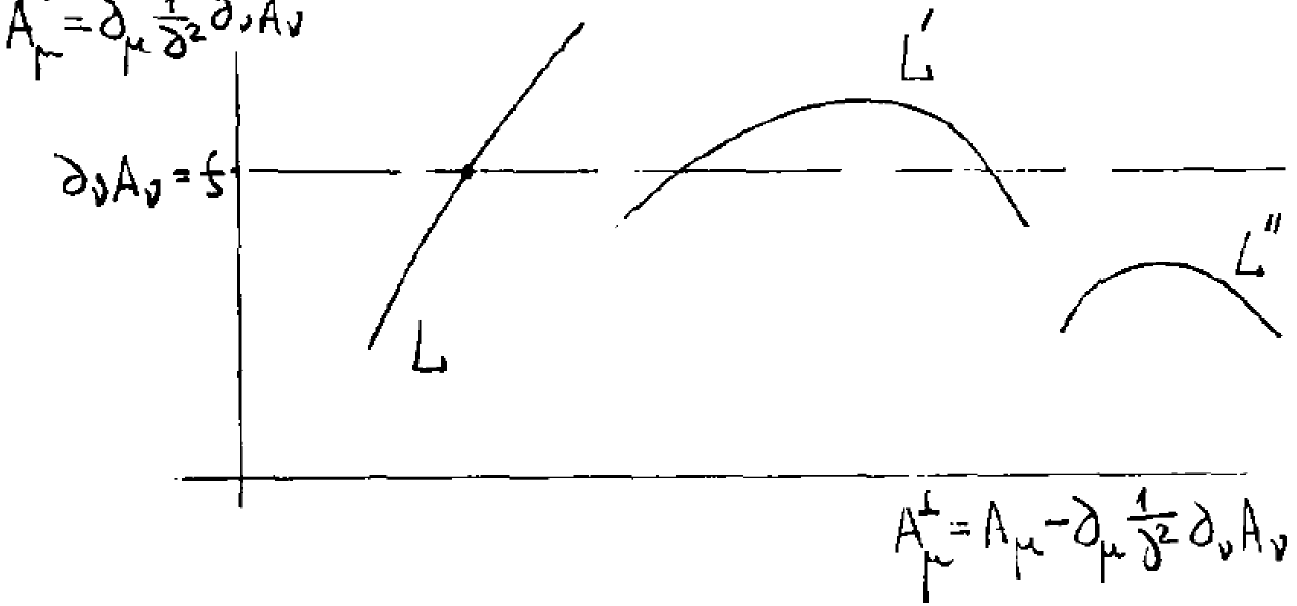
- Gauge fixing (from Gribov’s 1978 paper):



[V. N. Gribov (1930-1997)]

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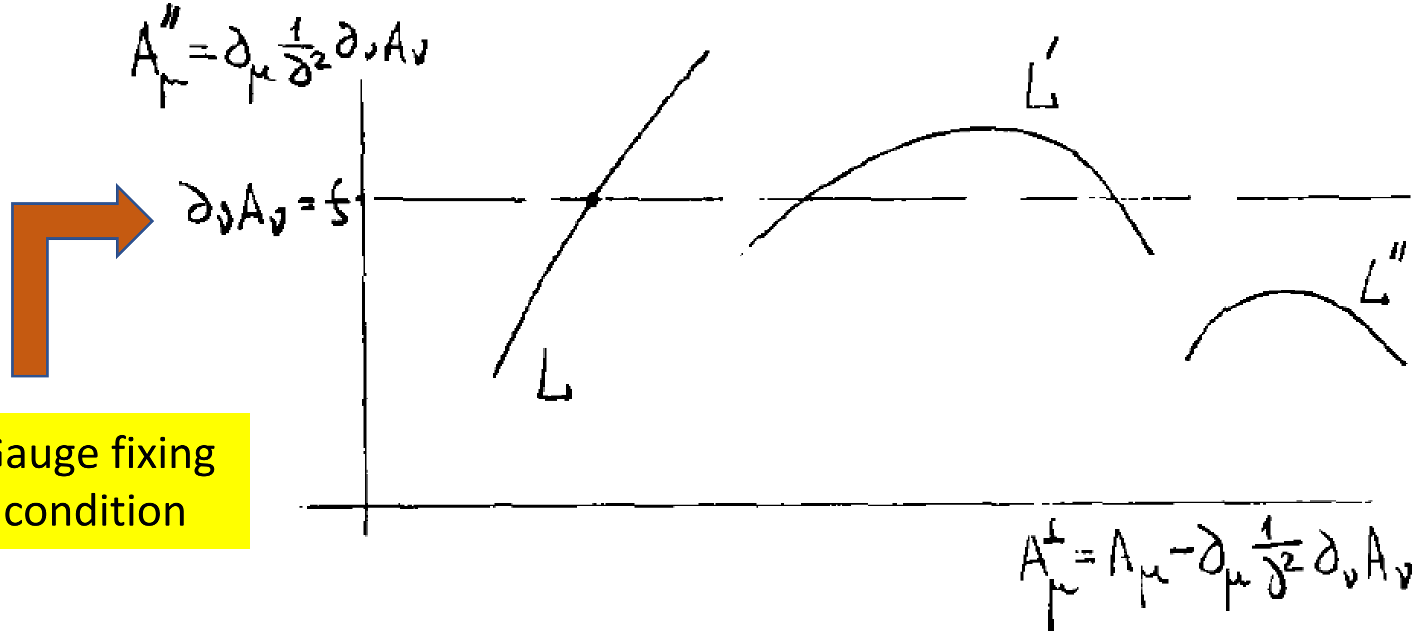

Gauge fixing condition

[V. N. Gribov (1930-1997)]

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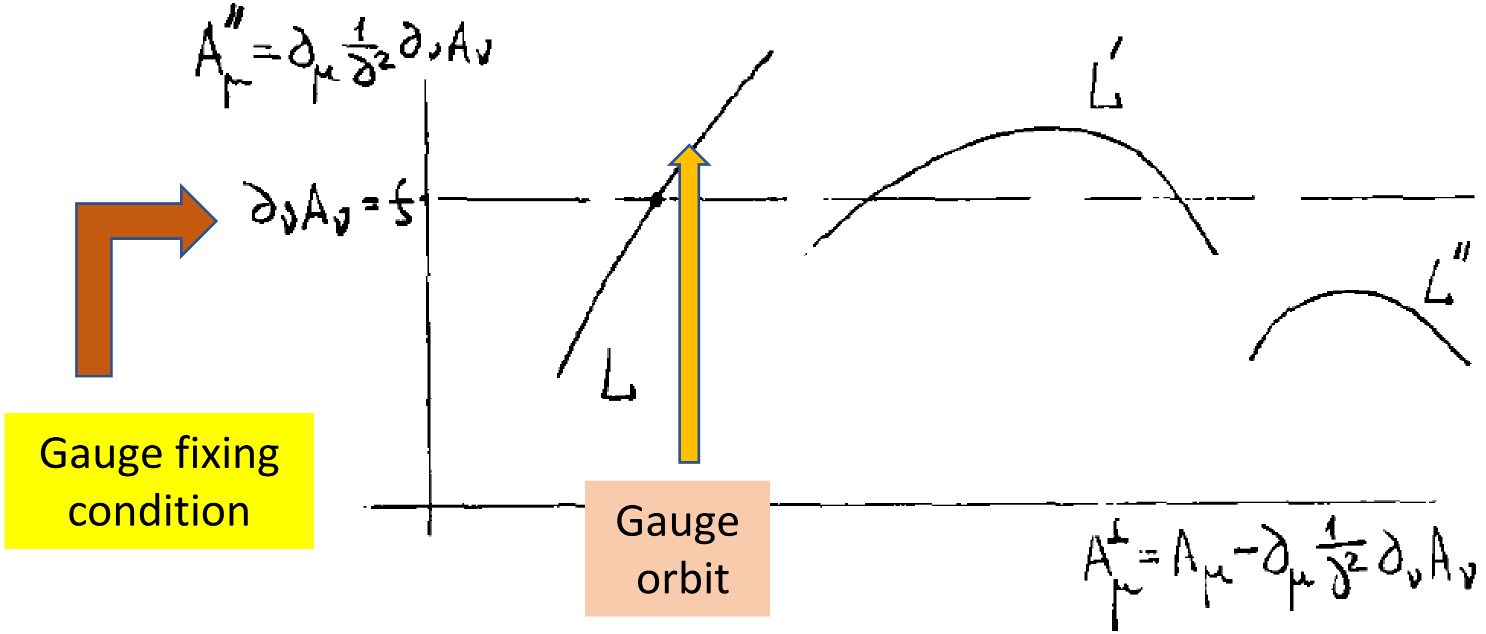

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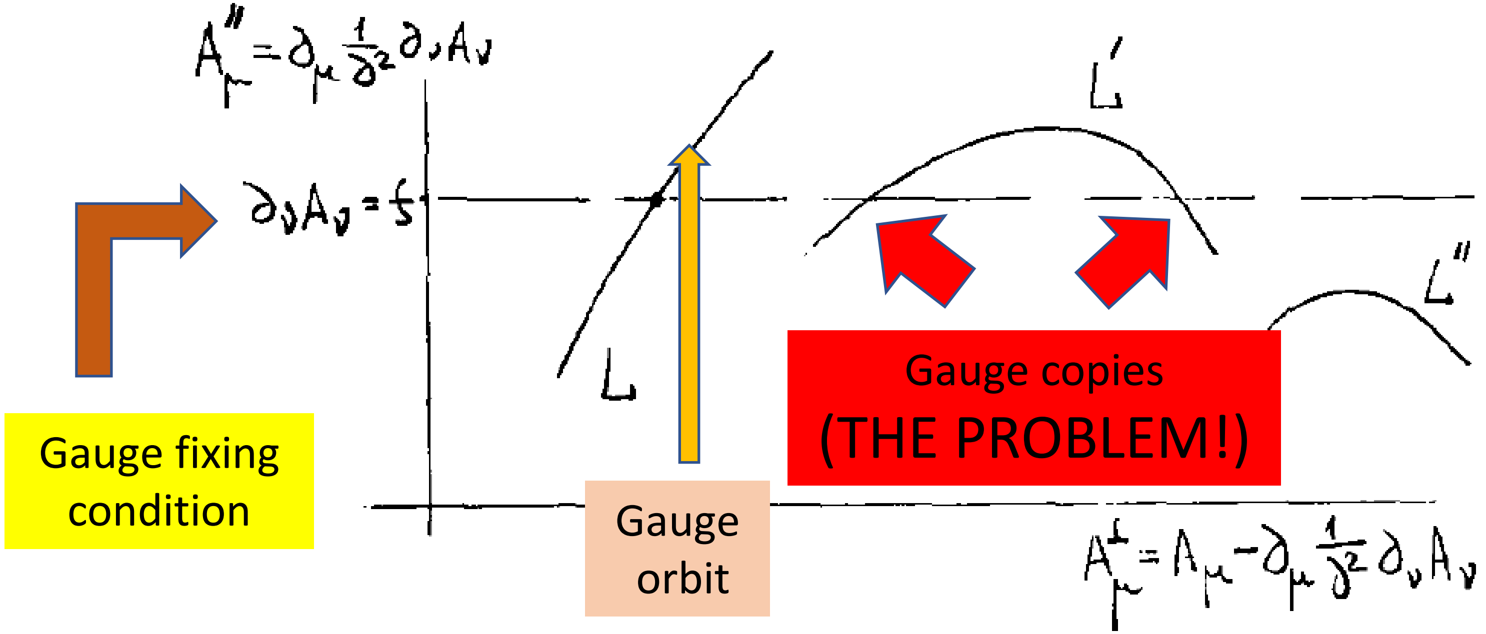

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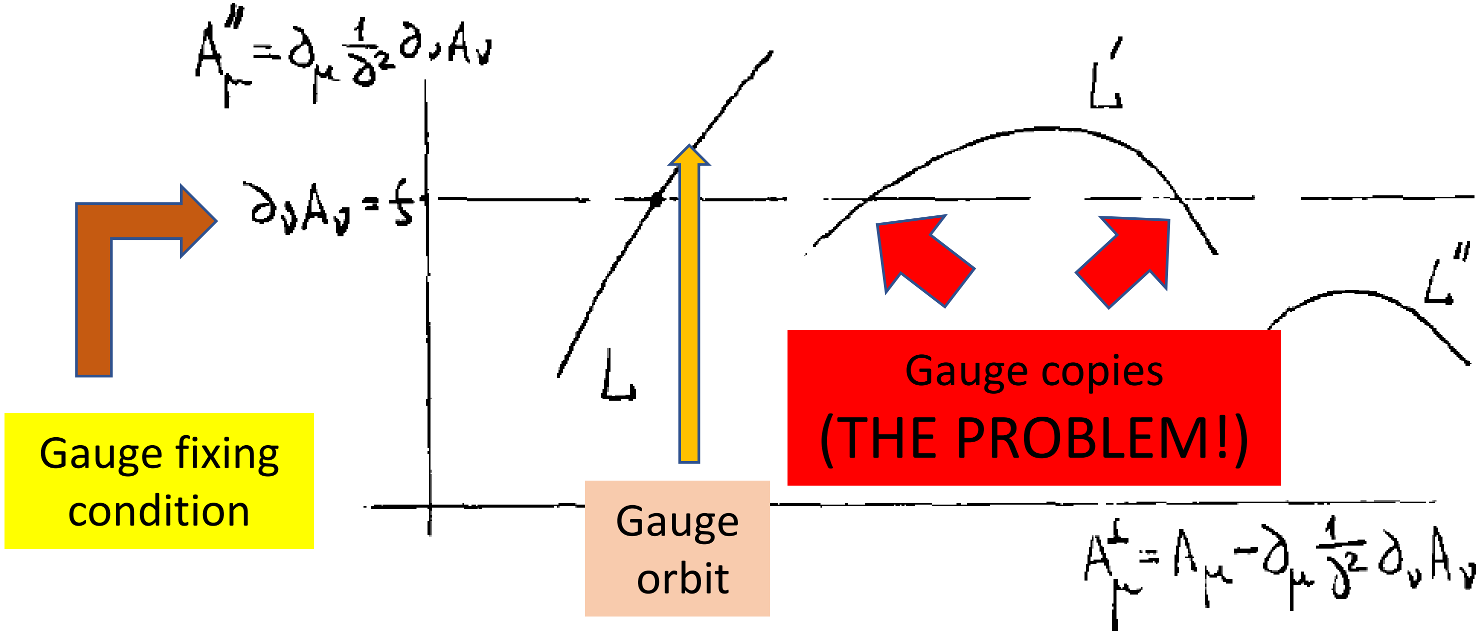
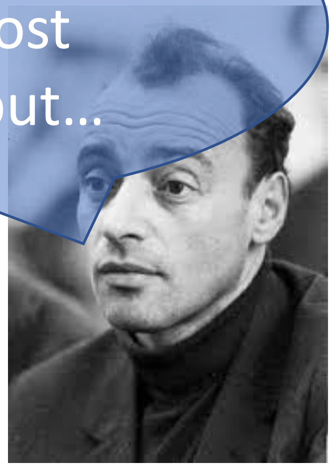


This problem seems to be almost hopeless, but…



[V. N. Gribov (1930-1997)]

… but we shall demonstrate below that there exists a possibility of a sufficiently universal solution leading to physically interesting results. [Gribov, 1978]

What is Gribov's problem?

- If a field A_μ^a has a gauge copy, then the equation

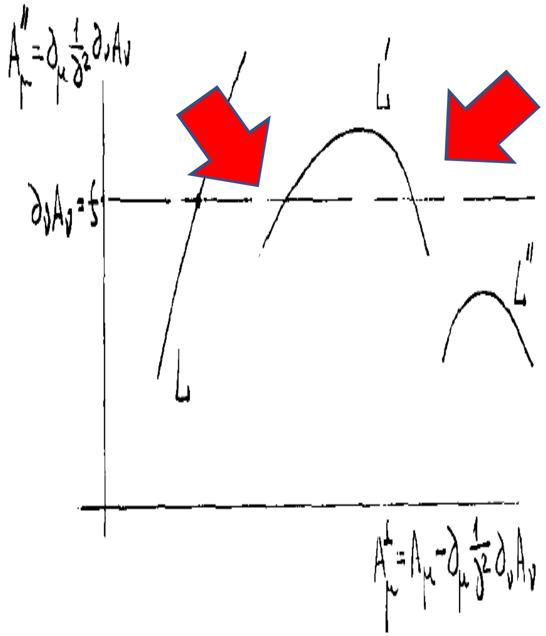
$$M^{ab}\eta^b(x) = 0$$

i.e.,

$$\left[-\delta^{ab}\partial^2 + gf^{abc}A_\mu^c(x)\right]\eta^b(x) = 0$$

has a nontrivial solution $\eta^a(x)$ (i.e., the FP operator has a "zero mode"). (Boundary conditions apply at infinity.)

Gauge copies
(THE PROBLEM!)



- $A = 0$: Laplace equation → no solution → no copies
- $A \neq 0$: More complicated → copies arise (large A / g/ IR)!

What is Gribov's problem?

- Covariant non-abelian Yang-Mills theories do have gauge copies.

[V. N. Gribov (1930-1997)]

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- Covariant non-abelian Yang-Mills theories do have gauge copies.
- FP operator <u>does have</u> zero eigenvalues ("zero modes").

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What is Gribov's problem?

- Covariant non-abelian Yang-Mills theories do have gauge copies.
- FP operator <u>does have</u> zero eigenvalues ("zero modes").
- Determinant: $\det M = \prod_i \lambda_i$ (product of eigenvalues).

[V. N. Gribov (1930-1997)]

What is Gribov's problem?

- Covariant non-abelian Yang-Mills theories do have
- FP operator <u>does have</u> zero eigenvalues ("zero modes").
- Determinant: $\det M = \prod_i \lambda_i$ (product of eigenvalues).
- At least one $\lambda_i = 0$ → zero FP determinant

$$Z[J] = \int [DA] \det\left(-\partial_\mu D_\mu^{ab}\right) \exp\left(-S_{gf} + \int J_\mu^a A_\mu^a\right)$$

[V. N. Gribov (1930-1997)]

<u>More details</u>: Sorella/Sobreiro [arXiv:hep-th/0504095] (Swieca School 2005)

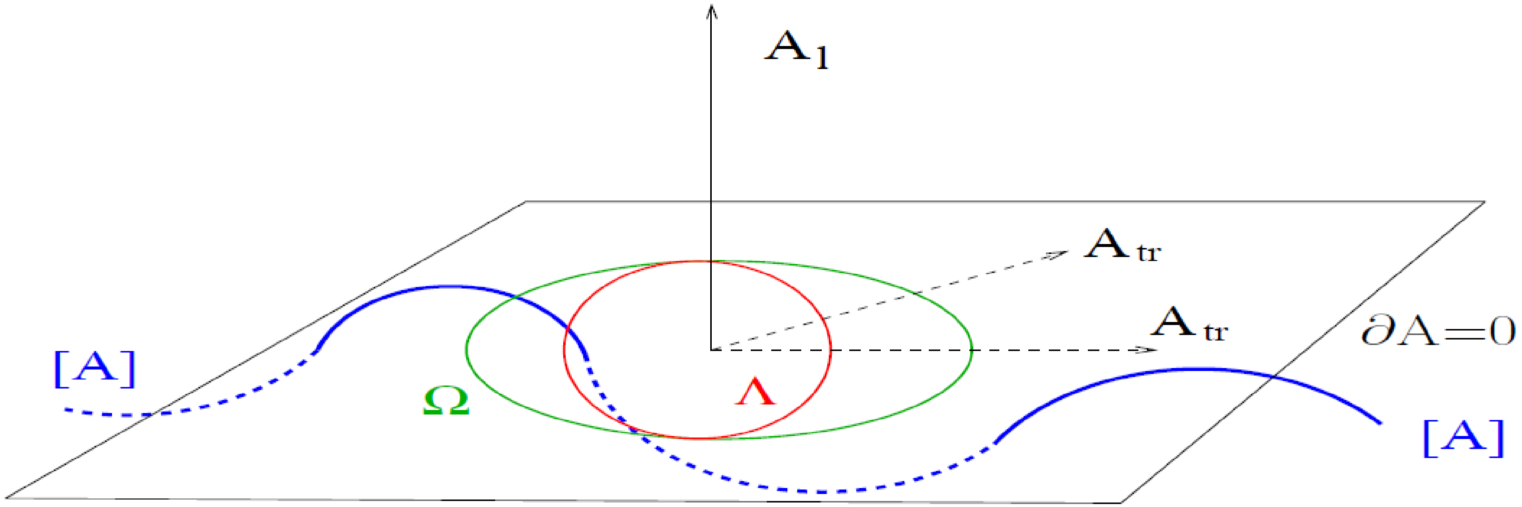
What can we do about Gribov’s problem?

Gribov-Zwanziger effective action (GZ)

Let's get rid of FP zero modes.

- Gribov's solution: avoid zero-modes of the FP-operator!
- Constrain path integral to Gribov's first region Ω (Landau gauge):

$$\Omega = \{A_\mu^a \; such\ that\colon \partial_\mu A_\mu^a = 0 \; and \; M^{ab} > 0 \; (i.e., \lambda_i > 0)\}$$



[R. Alkofer: BJP 37(1B), 144 (2007)]

Gribov-Zwanziger effective action (GZ)

- Constrain path integral to Gribov's first region Ω (Landau gauge):

$$\Omega = \{A^a_\mu \ such \ that\colon \partial_\mu A^a_\mu = 0 \ and \ M^{ab} > 0 \ (i.e., \lambda_i > 0)\}$$

$$Z[J] = \int_{\Omega} [DA][D\bar{c}][Dc] \exp\left(-S_{FP} + \int J^a_\mu A^a_\mu\right)$$

Restriction to the (1st) Gribov Region ($M > 0$)

Gribov-Zwanziger effective action (GZ)

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Restriction to the (1st) Gribov Region $(M > 0)$

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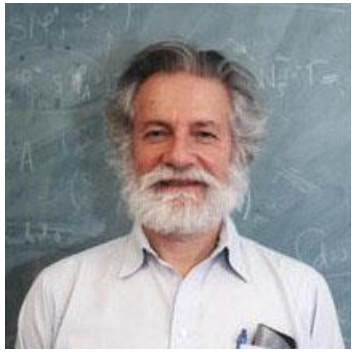
$$Z[J] = \int [DA][D\bar{c}][Dc] \exp\left(-S_{FP} + \int J_\mu^a A_\mu^a\right) \theta\big(1 - \sigma(A)\big)$$

Gribov-Zwanziger effective action (GZ)

Daniel Zwanziger (1935-)



- Gribov restriction (constraint) → modification of the action (à la Lagrange)

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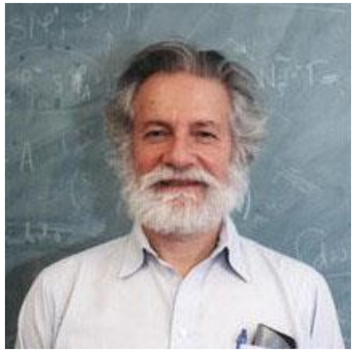


- Gribov restriction (constraint) → modification of the action (à la Lagrange)

Horizon function

$$Z[J] = \int [DA][D\bar{c}][Dc] \exp\left(-S_{FP} + \int J_\mu^a A_\mu^a\right) \exp(-\gamma^4 H(A))$$

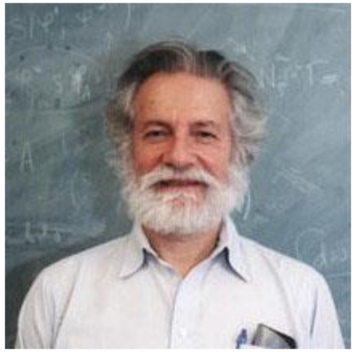
Gribov parameter
(a "Lagrange multiplier")
(not a free parameter)

$$\frac{\partial \log Z[0]}{\partial \gamma} = 0$$

(Gribov gap equation)

Gribov-Zwanziger effective action (GZ)

Daniel Zwanziger (1935-)



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Gribov parameter
(a “Lagrange multiplier”)
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$$H(A) = g^2\gamma^4 \int d^d x\, f^{bal} A_\mu^a(x) [M^{-1}]^{lm}(x)\, f^{bkm} A_\mu^k(x)$$

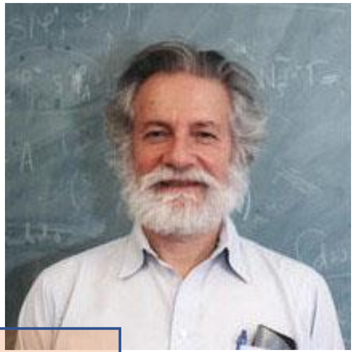
Inverse of the Faddeev-Popov operator

$$\frac{\partial \log Z[0]}{\partial \gamma} = 0$$

[D. Zwanziger, Nucl. Phys. B321, 591 (1989)]

Gribov-Zwanziger effective action (GZ)

Daniel Zwanziger (1935-)



$M^{ab} := -\delta^{ab}\partial^2 + g f^{abc} A_\mu^c \partial_\mu$

- The horizon function is nonlocal (due to M^{-1})!
- "Localization" → Introduce auxiliary fields $(\varphi, \bar{\varphi})$ (Bose) and $(\omega, \bar{\omega})$ (Fermi).

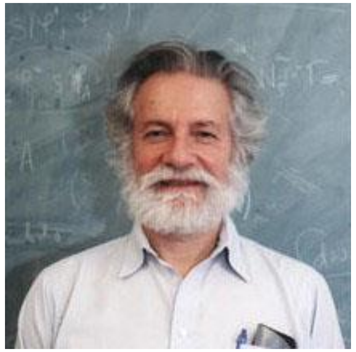
$$Z[J] = \int [D(all\ fields)] \exp\left(-S_{GZ} + \int J_\mu^a A_\mu^a\right)$$

$$S_{GZ} = S_{FP} + \int d^d x \left\{\bar{\varphi}_\mu^{ac} M^{ab} \varphi_\mu^{bc} + \bar{\omega}_\mu^{ac} M^{ab} \omega_\mu^{bc} + i g \gamma^2 f^{abc} A_\mu^a \left(\varphi_\mu^{bc} + \bar{\varphi}_\mu^{bc}\right)\right\}$$

Gribov-Zwanziger (GZ) Action

Gribov-Zwanziger effective action (GZ)

Daniel Zwanziger (1935-)



$$S_{GZ} = S_{FP} + \int d^d x \, \{\bar{\varphi}_\mu^{ac} \, M^{ab} \varphi_\mu^{bc} + \bar{\omega}_\mu^{ac} \, M^{ab} \omega_\mu^{bc} + i g \gamma^2 f^{abc} A_\mu^a (\varphi_\mu^{bc} + \bar{\varphi}_\mu^{bc})\}$$

Tree-level gluon propagator (Landau gauge):

$$D_{\mu\nu}^{ab}(p) = \frac{p^2}{p^4 + 2g^2 N \gamma^4} \delta^{ab} P_{\mu\nu}(p)$$

$$P_{\mu\nu}(p) = \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}$$

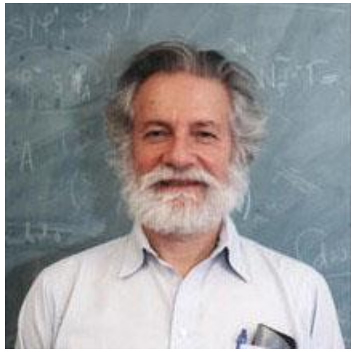
(transverse projector)

"Scaling solution" (GZ)

γ: Gribov parameter

Gribov-Zwanziger effective action (GZ)

Daniel Zwanziger (1935-)



$$S_{GZ} = S_{FP} + \int d^d x \, \{\bar{\varphi}_\mu^{ac} \, M^{ab} \varphi_\mu^{bc} + \bar{\omega}_\mu^{ac} \, M^{ab} \omega_\mu^{bc} + ig\gamma^2 f^{abc} A_\mu^a (\varphi_\mu^{bc} + \bar{\varphi}_\mu^{bc})\}$$

Properties of the GZ action:

- Local
- Renormalizable
- Breaks BRST symmetry (*)
- Complex/not hermitian
- Leads to non-unitary time evolution

Refined Gribov-Zwanziger theory (RGZ)

- Presence of nonzero mass dimension $\mathrm{m_d} = 2$ condensates:

$$\langle A^a_\mu A^a_\mu \rangle \neq 0$$

Dudal *et al.*

[Phys. Rev. D**72** (2005) 014016]

$$\langle \bar{\varphi}^{ab}_\mu \varphi^{ab}_\mu + \bar{\omega}^{ab}_\mu \omega^{ab}_\mu \rangle \neq 0$$

Dudal *et al.*

[Phys. Rev. D **77** (2008) 071501]

- Parameters m and M: "Lagrange multipliers" → add terms to S_{GZ}:

$$S_{RGZ} = S_{GZ} + \int d^d x \left[\frac{m^2}{2} A^a_\mu A^a_\mu + M^2 \left(\bar{\varphi}^{ab}_\mu \varphi^{ab}_\mu + \bar{\omega}^{ab}_\mu \omega^{ab}_\mu \right) \right]$$

Refined Gribov-Zwanziger effective action (RGZ)

$$S_{RGZ} = S_{GZ} + \int d^d x \left[\frac{m^2}{2} A_\mu^a A_\mu^a + M^2 \left(\bar{\varphi}_\mu^{ab} \varphi_\mu^{ab} + \bar{\omega}_\mu^{ab} \omega_\mu^{ab}\right)\right]$$

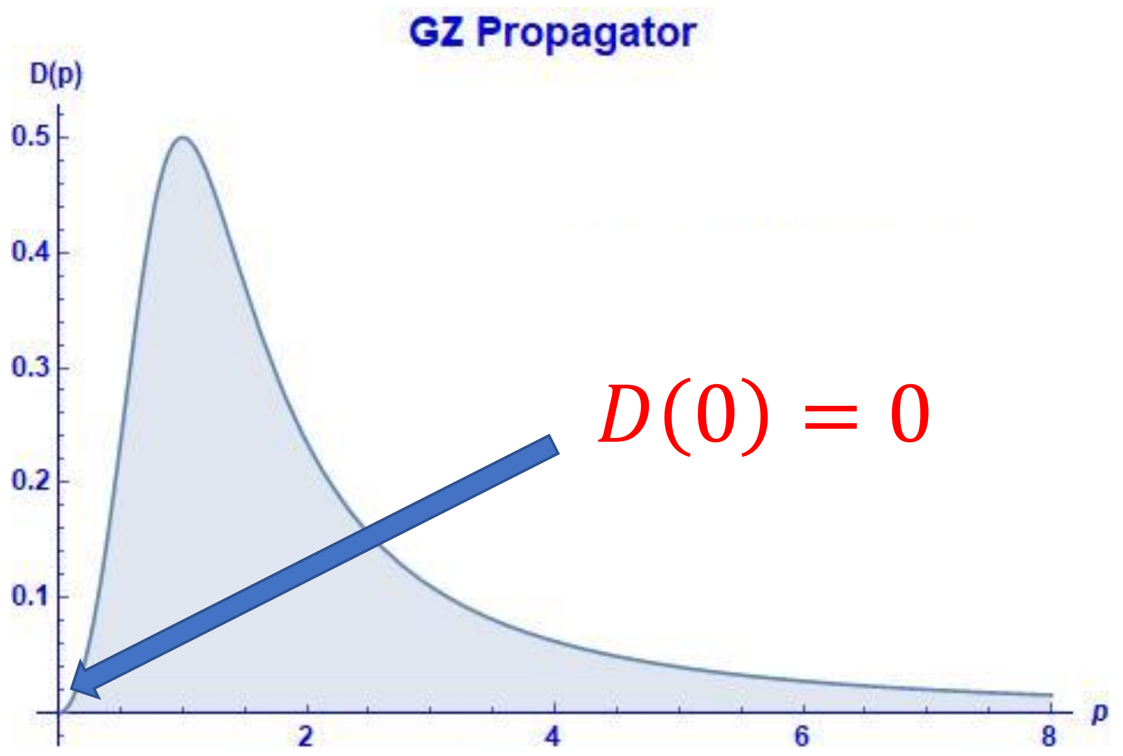
Tree-level gluon propagator (Landau gauge):

$$D_{\mu\nu}^{ab}(p) = \frac{p^2 + M^2}{(p^2 + m^2)(p^2 + M^2) + 2g^2 N \gamma^4} \delta^{ab} P_{\mu\nu}(p)$$

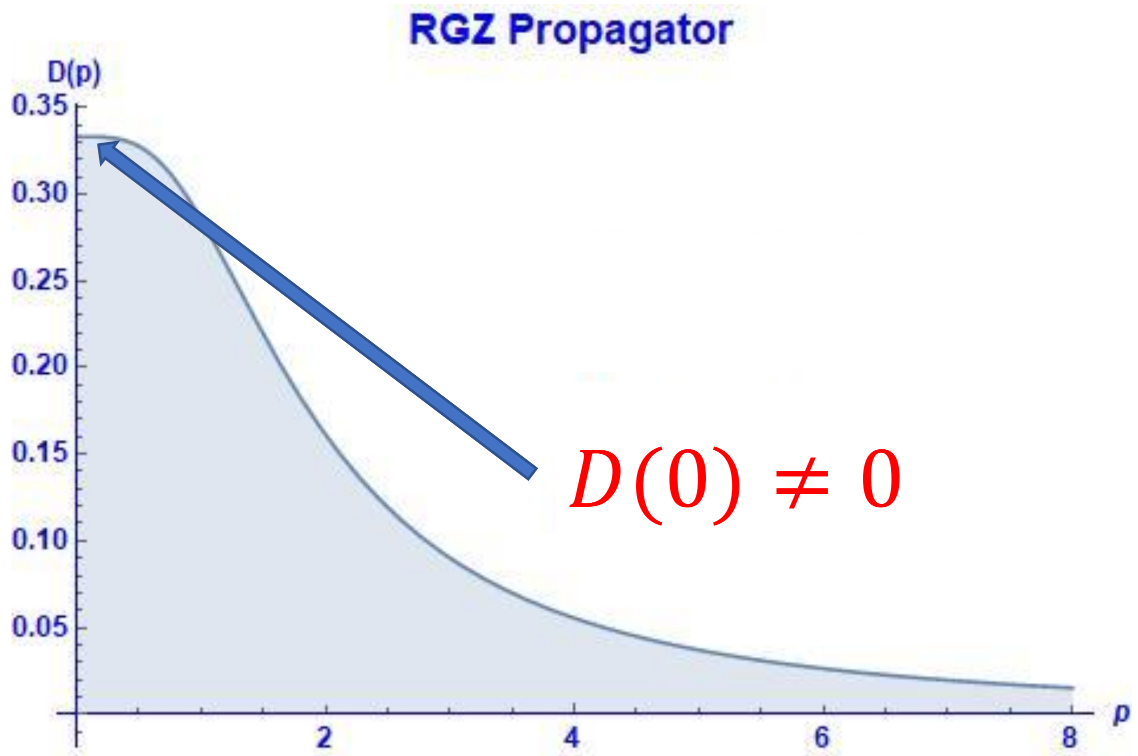
“Decoupling solution” (RGZ)

Dudal *et al.*
[Phys. Rev. D **77** (2008) 071501]

GZ *versus* RGZ gluon (tree-level) propagators



"Scaling solution" (GZ)



"Decoupling solution" (RGZ)

NB: qualitative behavior (don't take values seriously!)

What do we learn from Gribov’s problem?

How to interpret RGZ's gluon propagator?

$$D_{YM}(p) = \frac{1}{p^2}$$

(tree, pert YM)

$$D_{RGZ}(p) = \frac{p^2 + M^2}{(p^2 + m^2)(p^2 + M^2) + 2g^2 N \gamma^4}$$

(tree, RGZ)

$$D_{RGZ}(p) \simeq \frac{1}{p^2} \quad \text{(for } p \gg \text{all scales)}$$

- "Quasi-particle" dressed by interaction with a nonperturbative background.
- Reduces to ordinary Yang-Mills for large momentum.

(R)GZ and confinement

- RGZ gluon propagator

$$D_{RGZ}(p) = \frac{p^2 + M^2}{(p^2 + m^2)(p^2 + M^2) + 2g^2 N \gamma^4}$$

- Källen-Lehmann representation

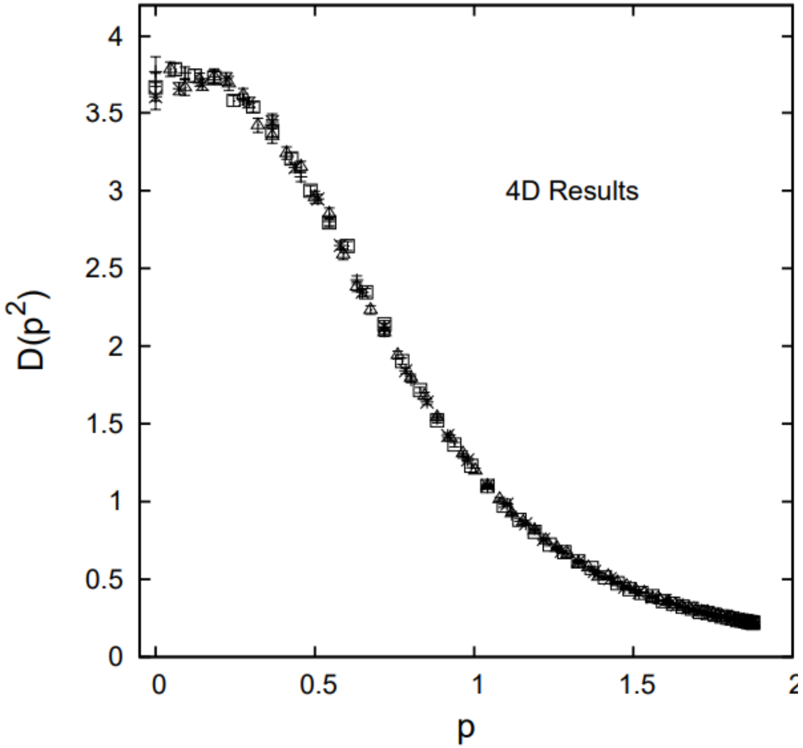
$$D(p) = \int_0^\infty dt \, \frac{\rho(t)}{p^2 + t}$$

$$\rho(t) = \sum_\lambda \delta\left(t - m_\lambda^2\right) |\langle \Omega | \phi(0) | \lambda_0 \rangle|^2 \geq 0$$

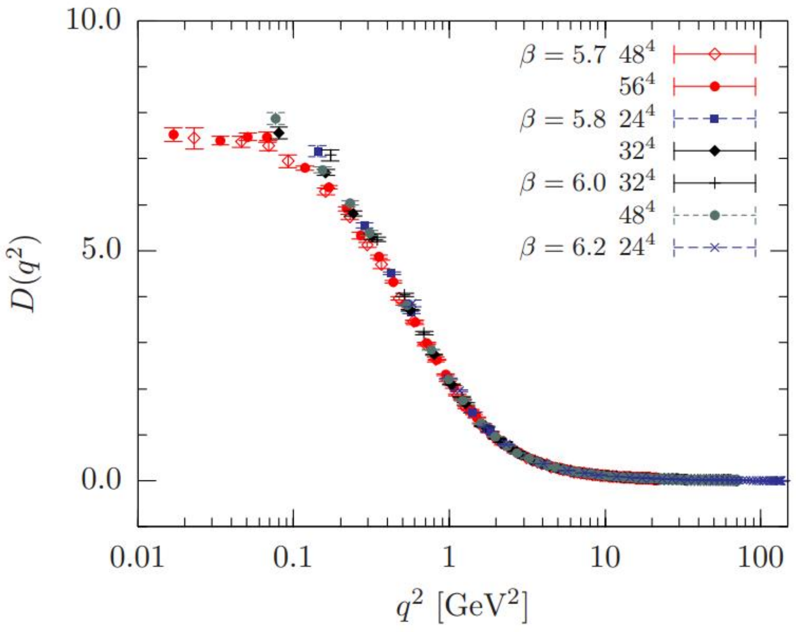
- However, RGZ-like ("lattice-like") gluon propagators: $\rho(t)$ is negative, for some values of t.
- "Positivity violation" i.e., $\rho(t) < 0$ → gluons not possible as asymptotic states → confinement (?)
- Complex masses (poles of the propagator) → confinement (?)

Tree-level RGZ and lattice Yang-Mills

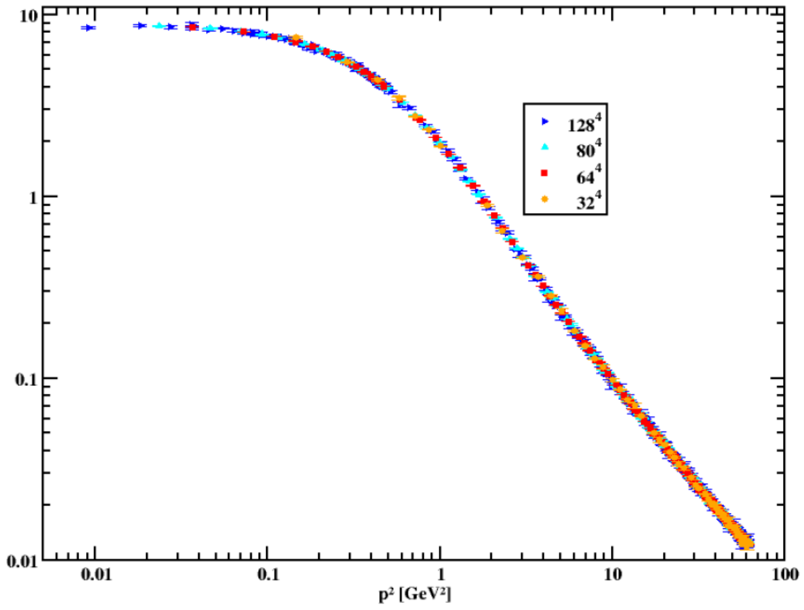
- Gluon propagator form factor from (large) lattices: RGZ-like -> $D(0) \neq 0$



A. Cucchieri and T. Mendes
[arXiv:1001.2584]



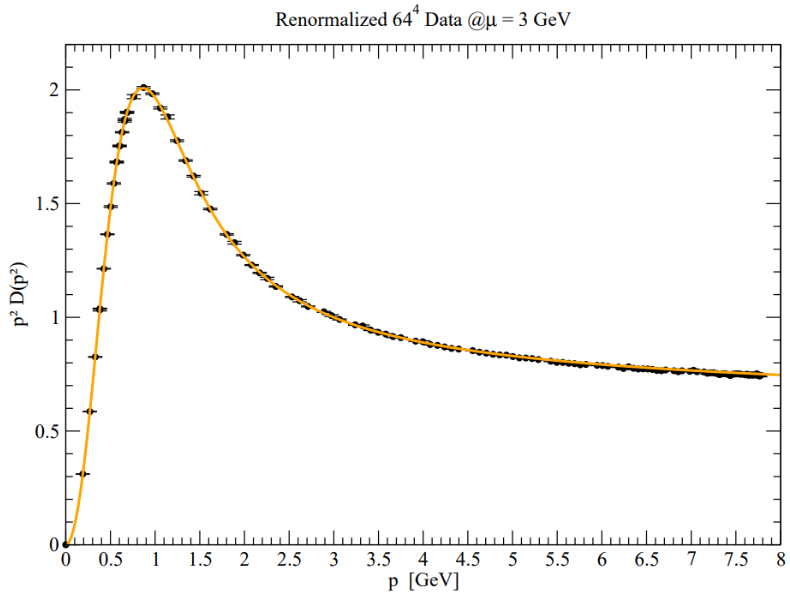
[E.-M. Ilgenfritz *et al.*,
BJP 37 (1B), 193 (2007)]



[A. F. Falcão, O. Oliveira, and P. J. Silva,
PRD102, 114518 (2020)]

Tree-level RGZ and lattice Yang-Mills

- Gluon propagator from (large) lattices: RGZ-like -> $D(0) \neq 0$
- Fitting function: $$D(p^2) = Z \frac{p^2 + M_1^2}{p^4 + M_2^2\, p^2 + M_3^4} \left[\omega \ln \left(\frac{p^2 + m_g^2(p^2)}{\Lambda_{QCD}^2} \right) + 1 \right]^{\gamma_{gl}}$$



$$m_g^2(p^2) = \lambda_0^2 + \frac{m_0^4}{p^2 + \lambda^2}$$

$$\omega = 11\,\mathrm{N} \frac{\alpha(\mu)}{12\pi}$$

$$\gamma_{gl} = -\frac{13}{22}$$

$$\frac{\chi^2}{dof} \simeq 1.1$$

[D. Dudal, O. Oliveira, and P. J. Silva, Ann. Phys. 397, 351 (2018)]

Tree-level RGZ and lattice Yang-Mills

- Gluon propagator from (large) lattices: RGZ-like -> $D(0) \neq 0$

- Fitting function:

$$D(p^2) = Z \underbrace{\frac{p^2+M_1^2}{p^4+M_2^2\,p^2+M_3^4}}_{\text{Tree-level RGZ (dominant @ low p)}} \underbrace{\left[\omega \ln\left(\frac{p^2+m_g^2(p^2)}{\Lambda_{QCD}^2}\right)+1\right]^{\gamma_{gl}}}_{\text{Perturbative QCD-like (dominant @ very high p)}}$$

Tree-level RGZ
(dominant @ low p)

Perturbative QCD-like
(dominant @ very high p)

Good
Agreement with
Lattice QCD

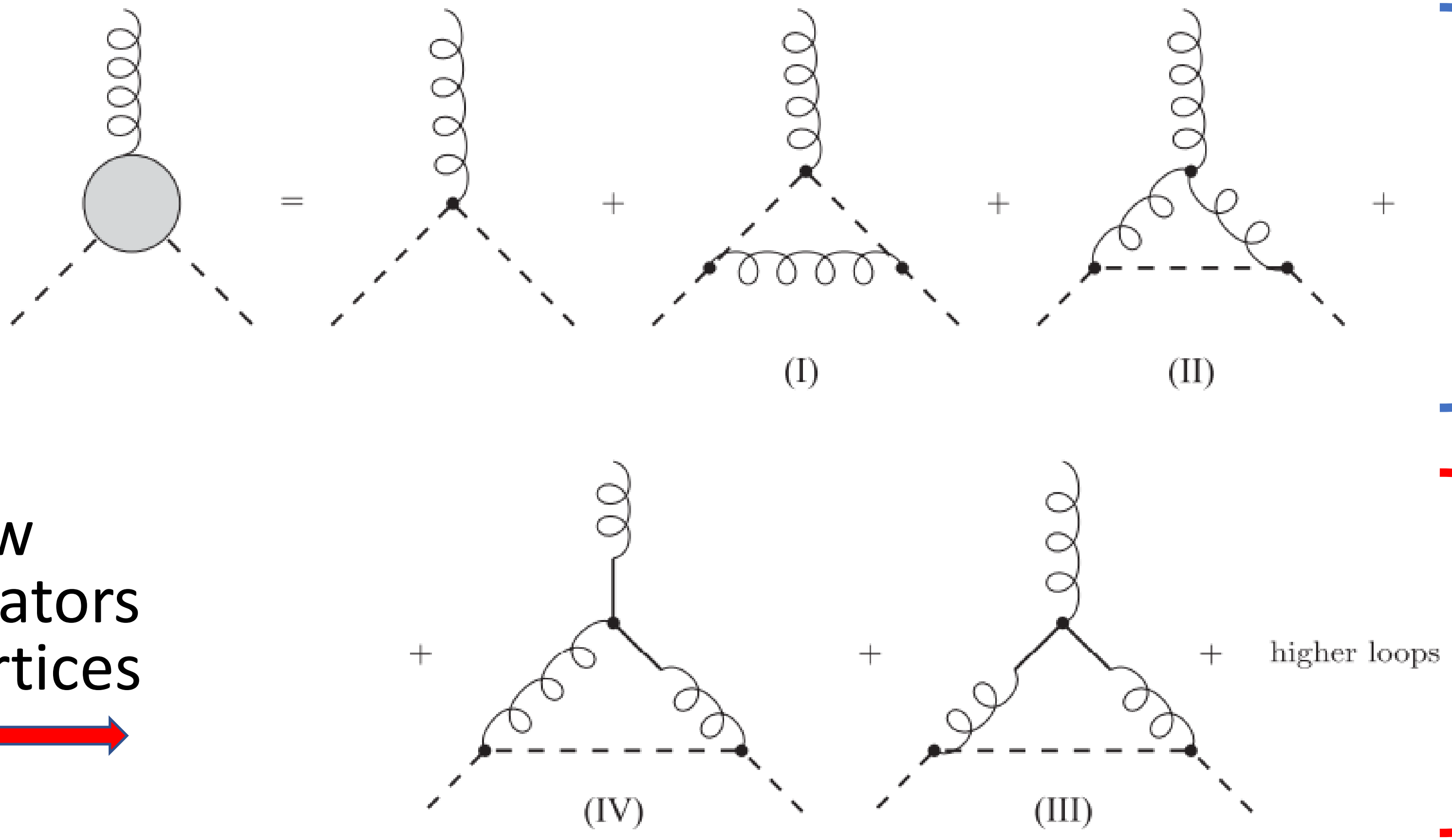
[D. Dudal, O. Oliveira, and P. J. Silva,
Ann. Phys. 397, 351 (2018)]

Perturbative RGZ: some results

1) ghost-gluon vertex

Perturbative RGZ (1): ghost-gluon vertex

- Let us take the RGZ action seriously and calculate quantum corrections!
- Example: ghost-gluon vertex



Usual YM diagrams

New propagators and vertices

GZ/RGZ diagrams

[B.W.M. *et al.*, PRD **97**, 034020 (2018)]

Perturbative RGZ (1): ghost-gluon vertex

- Relevant propagators for the ghost-gluon vertex @ 1-loop:

$$\left\langle A_\mu^a(p) A_\nu^b(-p)\right\rangle = \frac{p^2+M^2}{(p^2+m^2)(p^2+M^2)+2g^2N\,\gamma^4}\,\delta^{ab} P_{\mu\nu}(p) \equiv D_{AA}(p^2)\,\delta^{ab} P_{\mu\nu}(p)$$

$$\langle \bar{c}^a(p) c(-p)\rangle = \frac{1}{p^2}\delta^{ab}$$

$$\left\langle A_\mu^a(p) \varphi_\nu^{bc}(-p)\right\rangle = g\gamma^2 f^{abc}\,\frac{D_{AA}(p^2)}{p^2+M^2} P_{\mu\nu}(p) = -\left\langle A_\mu^a(p) \bar{\varphi}_\nu^{bc}(-p)\right\rangle$$

[B.W.M. *et al.,* PRD **97**, 034020 (2018)]

Perturbative RGZ (1): ghost-gluon vertex

- Relevant propagators for the ghost-gluon vertex @ 1-loop:

$$\left\langle A^a_\mu(p) A^b_\nu(-p) \right\rangle = \frac{p^2+M^2}{(p^2+m^2)(p^2+M^2)+2g^2N\gamma^4}\,\delta^{ab} P_{\mu\nu}(p) \equiv D_{AA}(p^2)\,\delta^{ab} P_{\mu\nu}(p)$$

$$\langle \bar{c}^a(p) c(-p) \rangle = \frac{1}{p^2}\delta^{ab}$$

$$\left\langle A^a_\mu(p) \varphi^{bc}_\nu(-p) \right\rangle = g\gamma^2 f^{abc} \frac{D_{AA}(p^2)}{p^2+M^2} P_{\mu\nu}(p) = -\left\langle A^a_\mu(p) \bar{\varphi}^{bc}_\nu(-p) \right\rangle$$

(new GZ/RGZ propagator)

[B.W.M. *et al.*, PRD **97**, 034020 (2018)]

Perturbative RGZ (1): ghost-gluon vertex

- Relevant vertices for the ghost-gluon vertex @ 1-loop:

$$^{\text{tree}}[\Gamma_{AAA}(k,p,q)]^{abc}_{\mu\nu\rho} = -\frac{\delta^3 S_{\text{tree}}}{\delta A^a_\mu(k)\delta A^b_\nu(p)\delta A^c_\rho(q)}\bigg|_{\Phi=0} = igf^{abc}[(k_\nu - q_\nu)\delta_{\rho\mu} + (p_\rho - k_\rho)\delta_{\mu\nu} + (q_\mu - p_\mu)\delta_{\nu\rho}]$$

$$^{\text{tree}}[\Gamma_{A\bar{c}c}(k,p,q)]^{abc}_{\mu} = -\frac{\delta^3 S_{\text{tree}}}{\delta A^a_\mu(k)\delta \bar{c}^b(p)\delta c^c(q)}\bigg|_{\Phi=0} = -igf^{abc}p_\mu$$

$$^{\text{tree}}[\Gamma_{A\bar{\varphi}\varphi}(k,p,q)]^{abcde}_{\mu\nu\rho} = -\frac{\delta^3 S_{\text{tree}}}{\delta A^a_\rho(k)\delta \bar{\varphi}^{bc}_\mu(p)\delta \varphi^{de}_\nu(q)}\bigg|_{\Phi=0} = -igf^{abd}\delta^{ce}\delta_{\nu\rho}p_\mu.$$

[B.W.M. *et al.*, PRD **97**, 034020 (2018)]

Perturbative RGZ (1): ghost-gluon vertex

- Relevant vertices for the ghost-gluon vertex @ 1-loop:

$$ {}^{\text{tree}}[\Gamma_{AAA}(k,p,q)]^{abc}_{\mu\nu\rho} = -\frac{\delta^3 S_{\text{tree}}}{\delta A^a_\mu(k)\delta A^b_\nu(p)\delta A^c_\rho(q)}\bigg|_{\Phi=0} = igf^{abc}[(k_\nu - q_\nu)\delta_{\rho\mu} + (p_\rho - k_\rho)\delta_{\mu\nu} + (q_\mu - p_\mu)\delta_{\nu\rho}] $$

$$ {}^{\text{tree}}[\Gamma_{A\bar{c}c}(k,p,q)]^{abc}_{\mu} = -\frac{\delta^3 S_{\text{tree}}}{\delta A^a_\mu(k)\delta \bar{c}^b(p)\delta c^c(q)}\bigg|_{\Phi=0} = -igf^{abc}p_\mu $$

$$ {}^{\text{tree}}[\Gamma_{A\bar{\varphi}\varphi}(k,p,q)]^{abcde}_{\mu\nu\rho} = -\frac{\delta^3 S_{\text{tree}}}{\delta A^a_\rho(k)\delta \bar{\varphi}^{bc}_\mu(p)\delta \varphi^{de}_\nu(q)}\bigg|_{\Phi=0} = -igf^{abd}\delta^{ce}\delta_{\nu\rho}p_\mu. $$

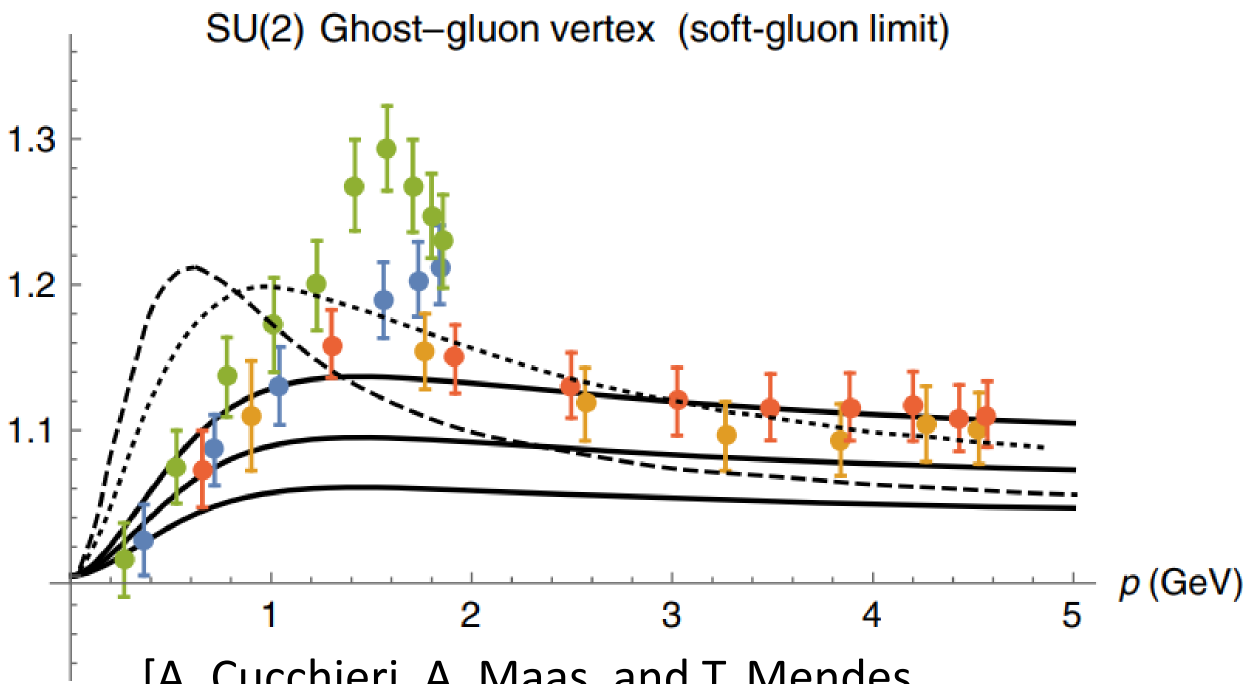
(new GZ/RGZ vertex)

[B.W.M. *et al.*, PRD **97**, 034020 (2018)]

Perturbative RGZ (1): ghost-gluon vertex

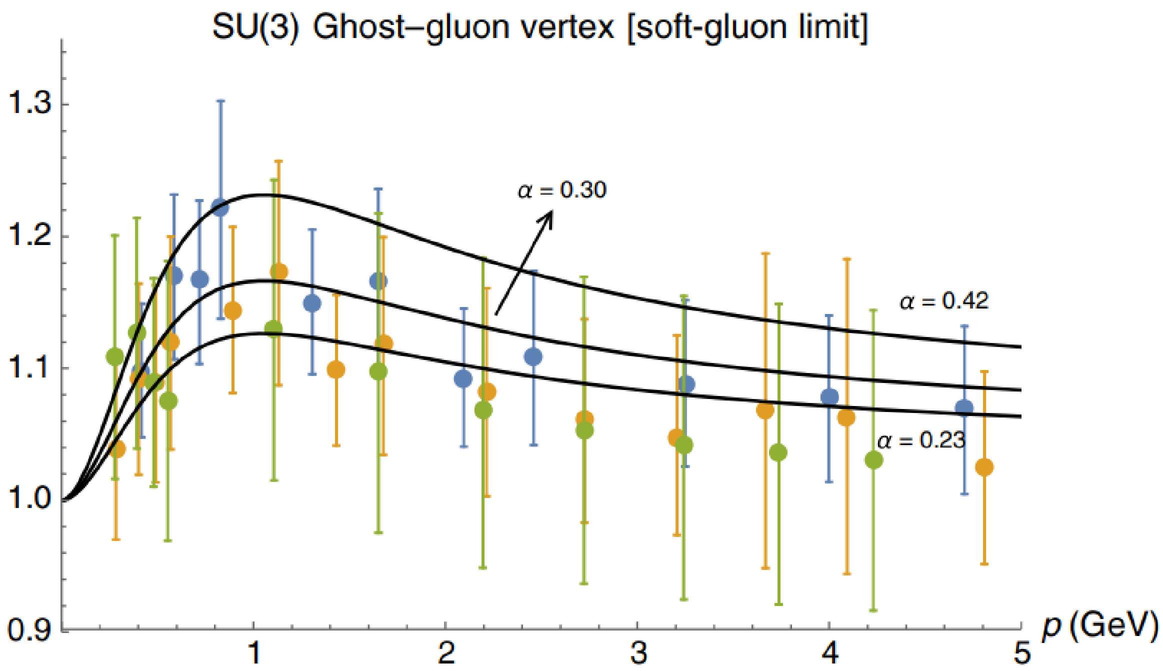
[B.W.M. *et al.*, PRD **97**, 034020 (2018)]

- Soft gluon limit: $k_{gluon} \rightarrow 0$



[A. Cucchieri, A. Maas, and T. Mendes, Phys. Rev. D **77**, 094510 (2008).]

[M. Pelaez, M. Tissier, and N. Wschebor, Phys. Rev. D **88**, 125003 (2013)]

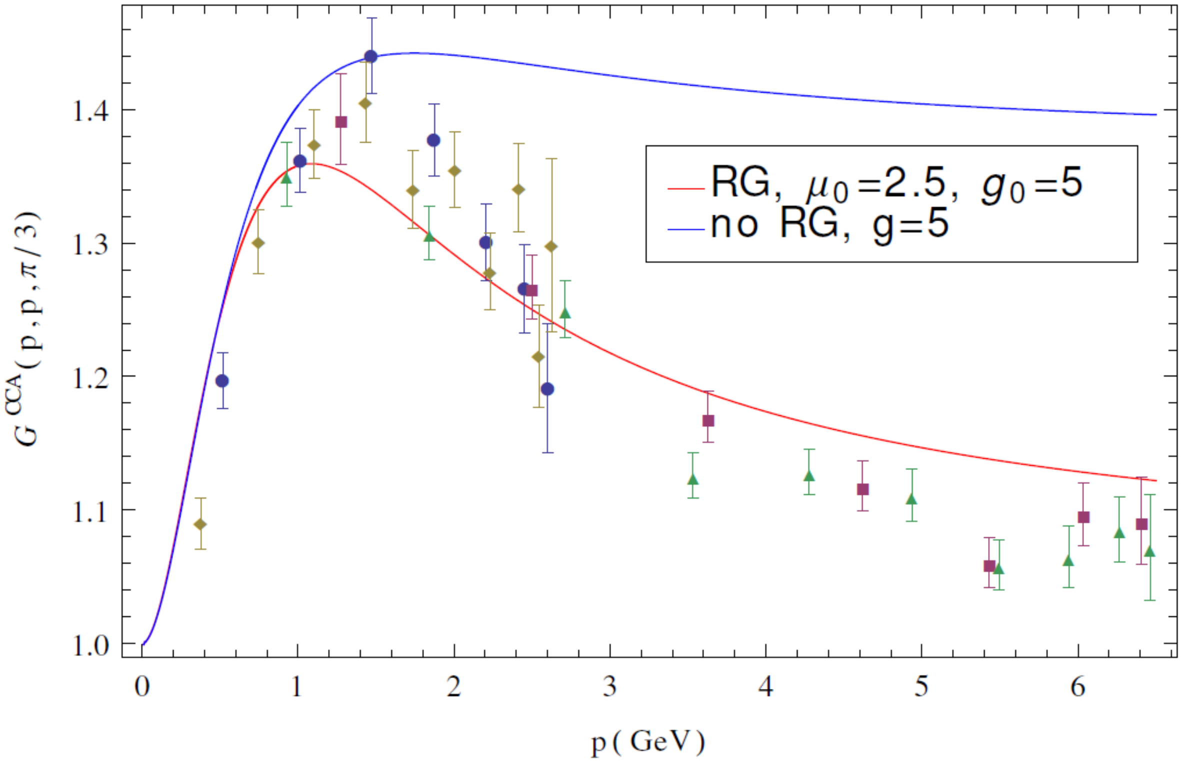


[E. M. Ilgenfritz *et al.*, Braz. J. Phys. **37**, 193 (2007).]

Perturbative RGZ (1): ghost-gluon vertex

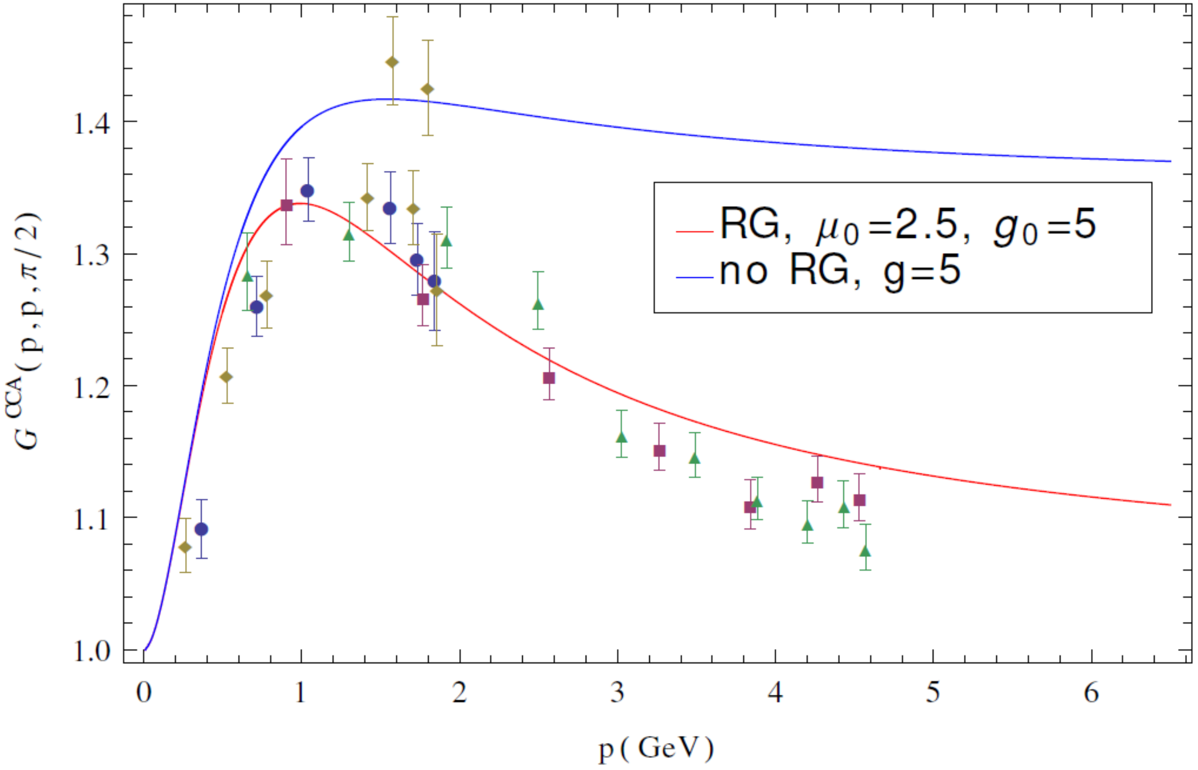
[M. Pelaez *et al.*, To be submitted]

- Symmetric configuration:



Lattice points: [A. Cucchieri and T. Mendes, PoS Confinement 8, 040 (2008); arXiv:0812.3261]

- Orthogonal configuration:



Model IR flow: $g(\mu)^2 = \dfrac{g_0^2}{1 + \frac{11N}{3}\frac{g_0^2}{16\pi^2}\mathrm{Log}\left(\frac{\mu^2+\mu_0^2}{\mu_0^2}\right)}$

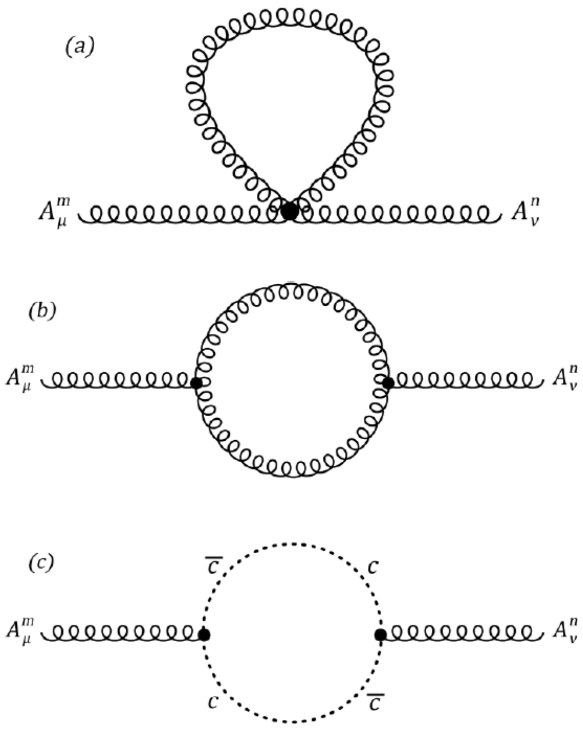
Perturbative RGZ: some results

2) Gluon propagator

- Warning: Preliminary results

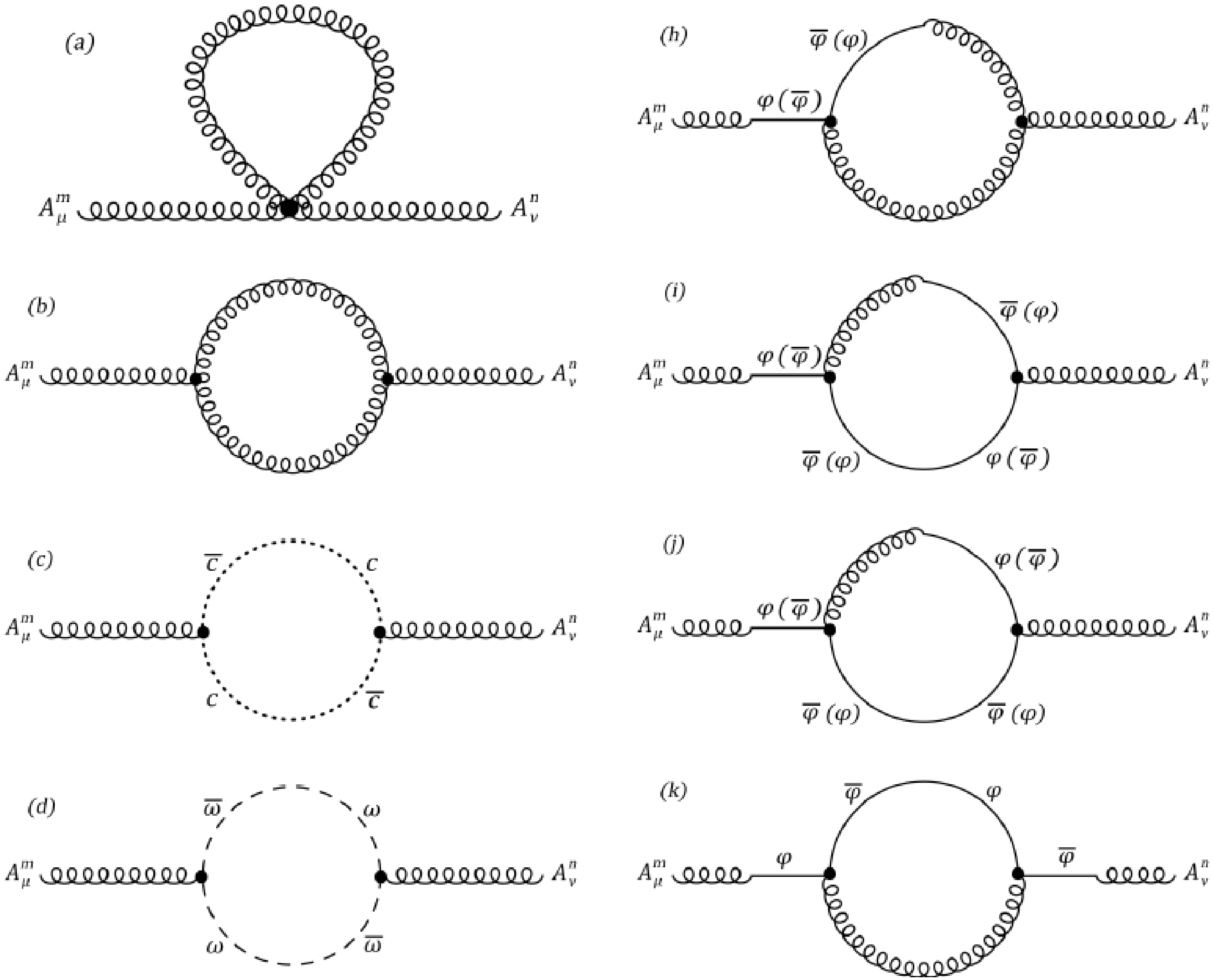
Perturbative RGZ(2): 1-loop gluon propagator

GZ: [J. A. Gracey, JHEP 0605:052 (2006)]



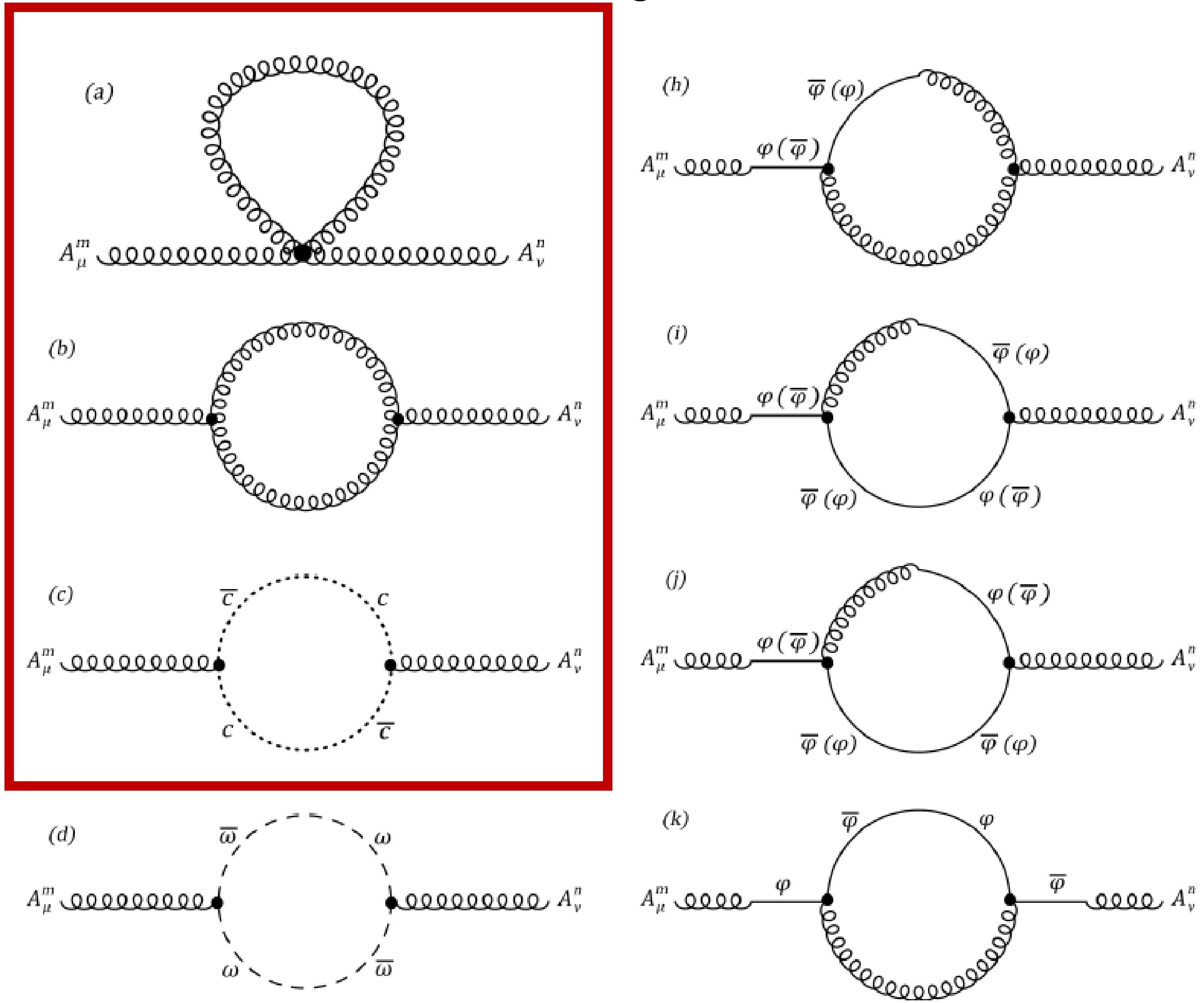
Usual YM diagrams

(but with RGZ-modified gluon propagators)



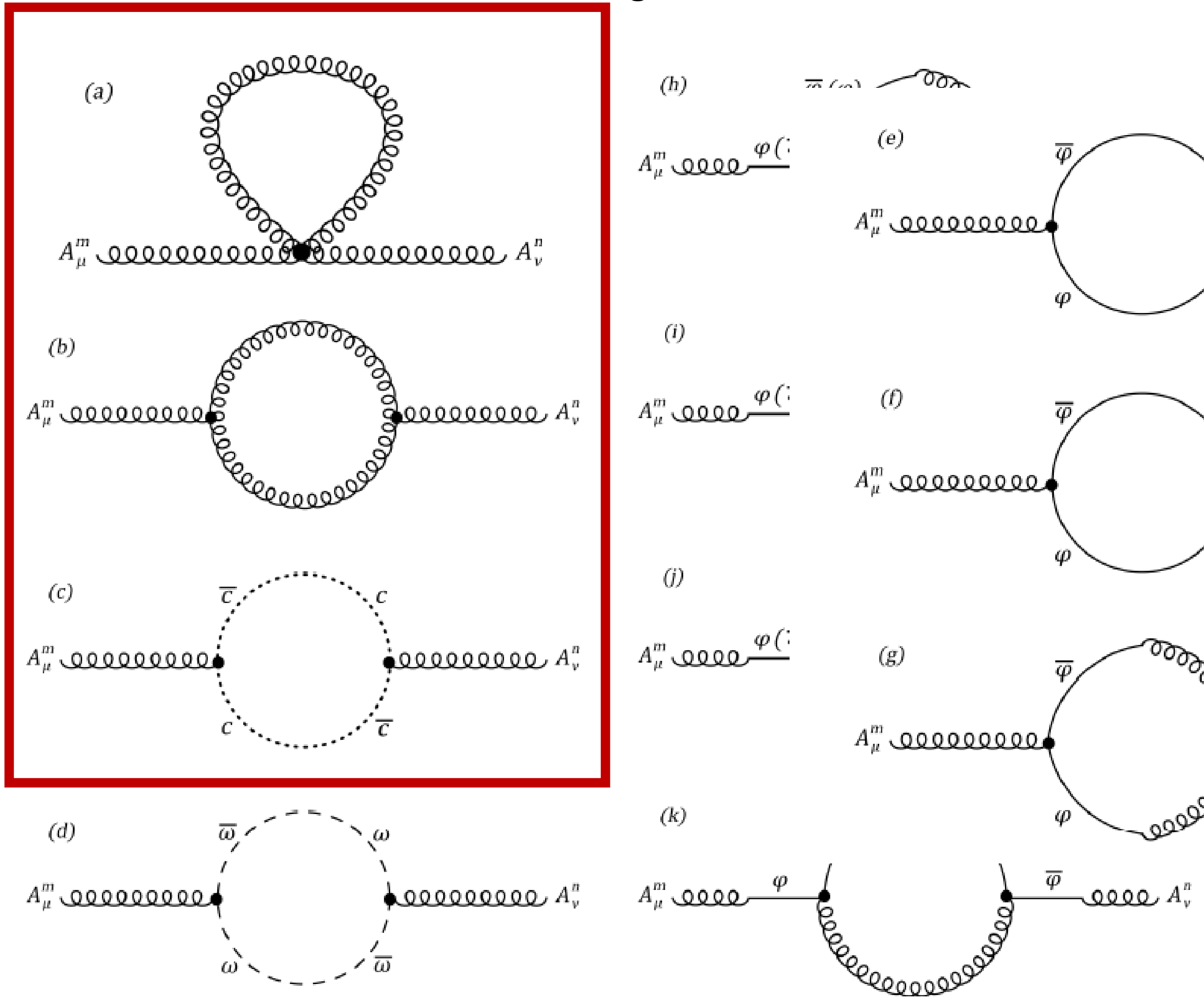
(a)
A_μ^m
A_ν^n
(b)
A_μ^m
A_ν^n
(c)
$\overline{c}$
c
A_μ^m
A_ν^n
c
$\overline{c}$
(d)
$\overline{\omega}$
ω
A_μ^m
A_ν^n
ω
$\overline{\omega}$
(h)
$\overline{\varphi}\ (\varphi)$
$\varphi\ (\overline{\varphi})$
A_μ^m
A_ν^n
(i)
$\overline{\varphi}\ (\varphi)$
$\varphi\ (\overline{\varphi})$
A_μ^m
A_ν^n
$\overline{\varphi}\ (\varphi)$
$\varphi\ (\overline{\varphi})$
(j)
$\varphi\ (\overline{\varphi})$
$\varphi\ (\overline{\varphi})$
A_μ^m
A_ν^n
$\overline{\varphi}\ (\varphi)$
$\overline{\varphi}\ (\varphi)$
(k)
$\overline{\varphi}$
φ
φ
$\overline{\varphi}$
A_μ^m
A_ν^n

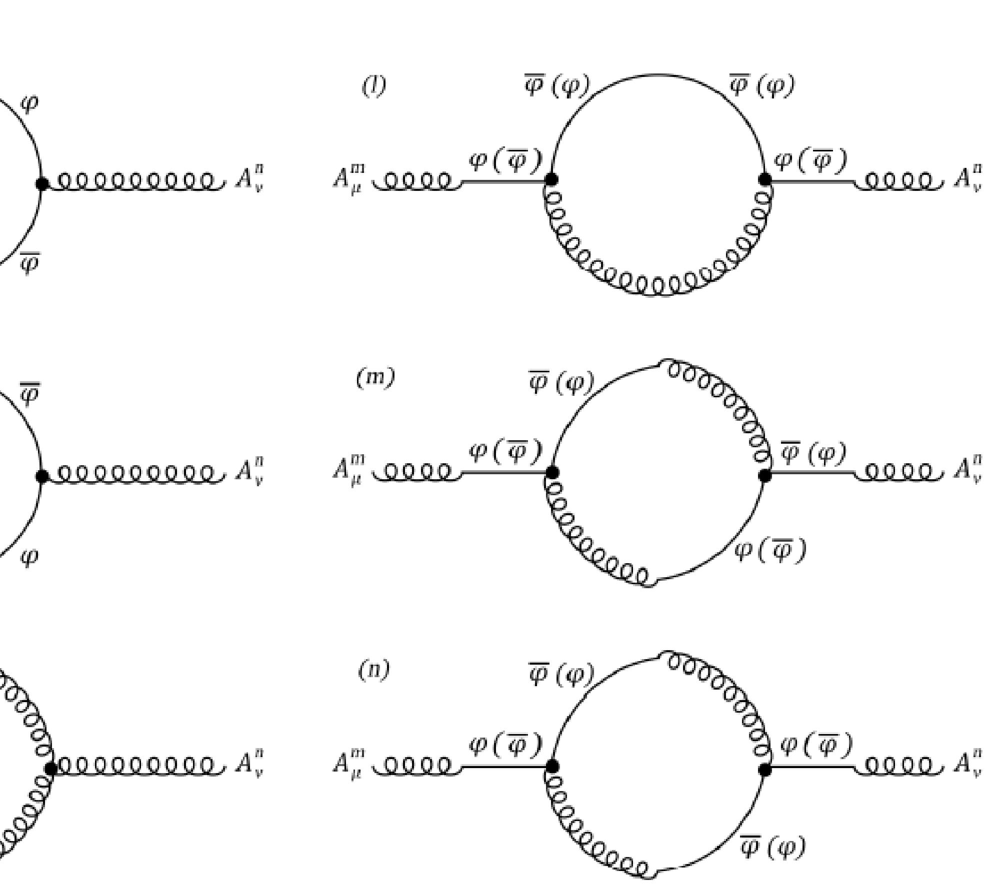
Yang-Mills



(a)
A_μ^m
A_ν^n
(b)
A_μ^m
A_ν^n
(c)
$\overline{c}$
c
c
$\overline{c}$
A_μ^m
A_ν^n
(d)
$\overline{\omega}$
ω
ω
$\overline{\omega}$
A_μ^m
A_ν^n
(h)
$\overline{\varphi}\,(\varphi)$
$\varphi\,(\overline{\varphi})$
A_μ^m
A_ν^n
(i)
$\overline{\varphi}\,(\varphi)$
$\varphi\,(\overline{\varphi})$
$\overline{\varphi}\,(\varphi)$
$\varphi\,(\overline{\varphi})$
A_μ^m
A_ν^n
(j)
$\varphi\,(\overline{\varphi})$
$\varphi\,(\overline{\varphi})$
$\overline{\varphi}\,(\varphi)$
$\overline{\varphi}\,(\varphi)$
A_μ^m
A_ν^n
(k)
$\overline{\varphi}$
φ
φ
$\overline{\varphi}$
A_μ^m
A_ν^n

Yang-Mills





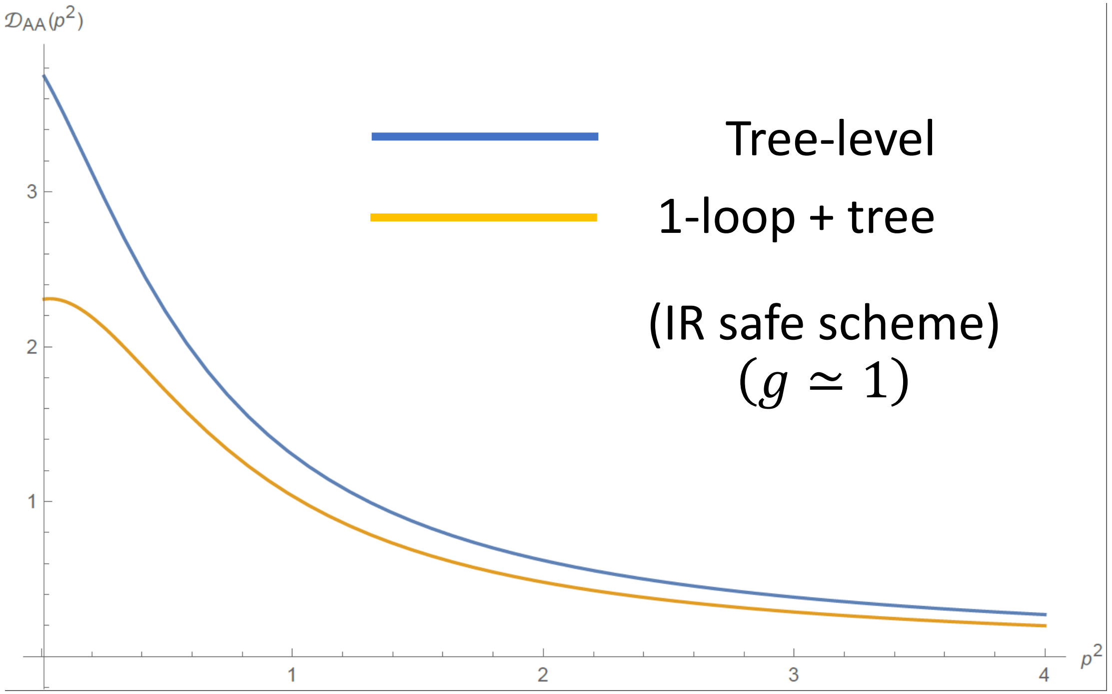
RGZ, 1-loop: 20 diagrams

[A. D. Pereira *et al.*, work in progress]

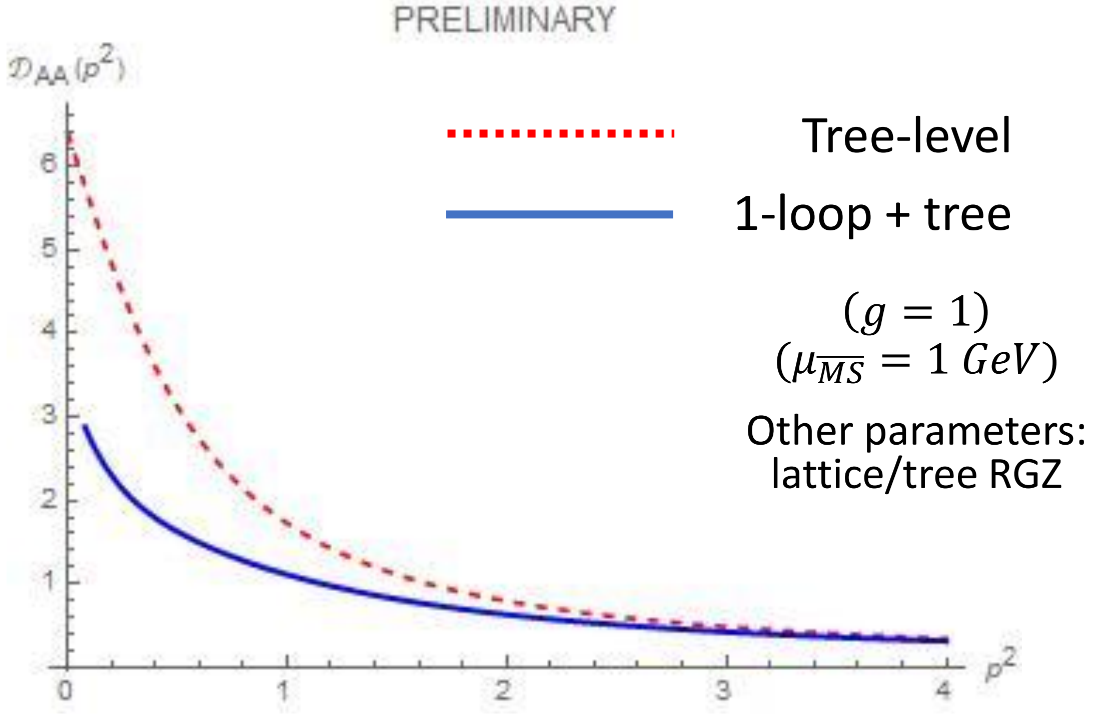
[B. W. M., work in progress]

Perturbative RGZ (2): gluon propagator

- Landau gauge 1-loop RGZ gluon propagator: **PRELIMINARY RESULTS**



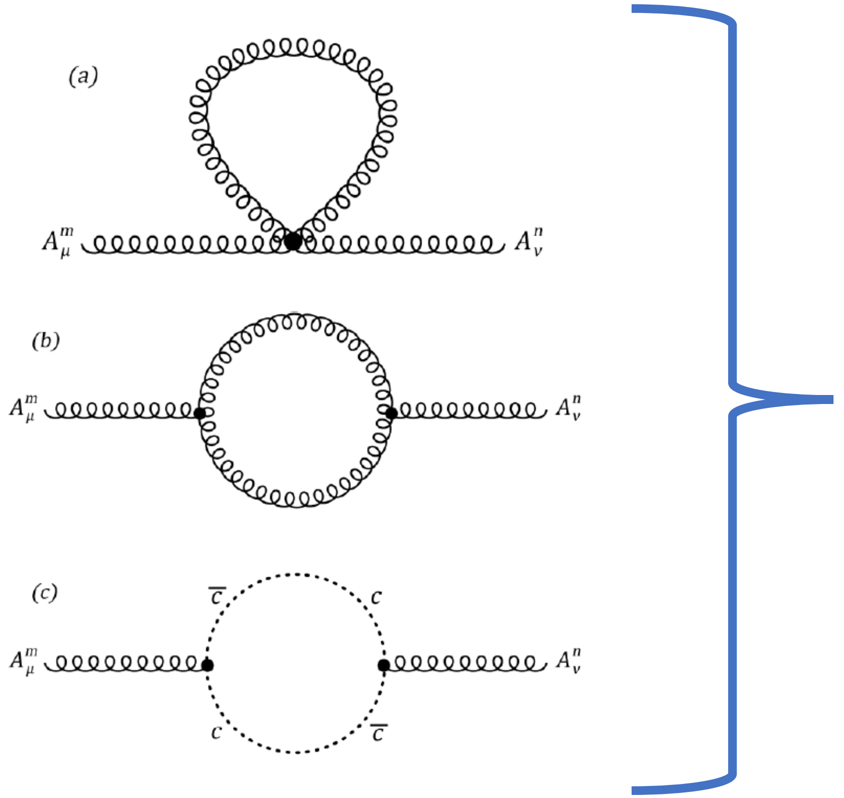
[A. D. Pereira and G. P. Brito, work in progress]



[B.W.M., work in progress]

Perturbative RGZ(2): 1-loop gluon propagator

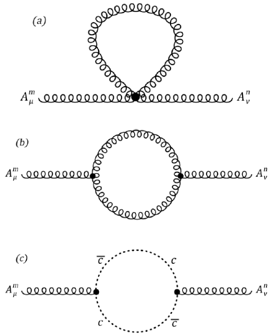
- Another possible approach: use the (nonlocal) horizon function



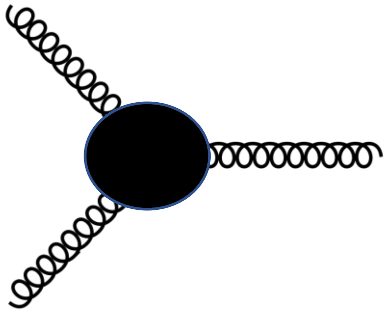
Usual YM diagrams

(but with RGZ-modified gluon propagators and vertices)

Perturbative RGZ(2): 1-loop gluon propagator



- Three-gluon vertex modification (with Gribov's horizon)

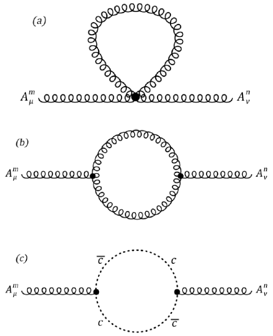


$$= \frac{ig^3 N\gamma^4}{2} f^{abc}\,\delta(p_1+p_2+p_3)\left\{ \frac{[(p_1)_\gamma - (p_2)_\gamma]\,\delta_{\alpha\beta}}{(p_1^2+M^2)(p_2^2+M^2)} + \frac{[(p_2)_\alpha - (p_3)_\alpha]\,\delta_{\beta\delta}}{(p_2^2+M^2)(p_3^2+M^2)} + \frac{[(p_3)_\beta - (p_1)_\beta]\,\delta_{\gamma\alpha}}{(p_3^2+M^2)(p_1^2+M^2)} \right\}$$

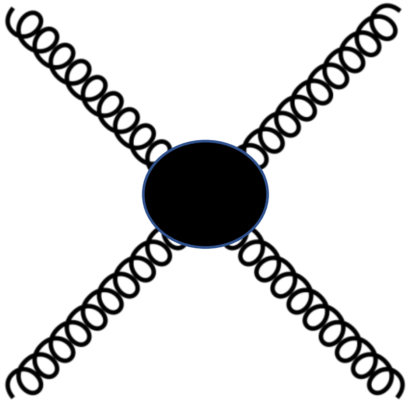
Local RGZ: auxiliary fields + vertices (previous slides)

Nonlocal RGZ: momentum-dependent vertices

Perturbative RGZ(2): 1-loop gluon propagator



- Four-gluon vertex modification (with Gribov's horizon)



$$= -g^4\gamma^4\delta(p_1+p_2+p_3+p_4)\left\{F^{abcd}\left[\left(\frac{(p_4)_\gamma(p_4+p_3)_\beta}{(p_4^2+M^2)[(p_4+p_3)^2+M^2][(p_4+p_3+p_2)^2+M^2]}\right.\right.\right.$$

$$\left.+\frac{(p_1)_\beta(p_1+p_2)_\gamma}{(p_1^2+M^2)[(p_1+p_2)^2+M^2][(p_1+p_2+p_3)^2+M^2]}\right)\delta_{\delta\alpha}$$

$$+\left(\frac{(p_4)_\alpha(p_4+p_1)_\beta}{(p_4^2+M^2)[(p_4+p_1)^2+M^2][(p_4+p_2+p_1)^2+M^2]}\right.$$

$$\left.+\frac{(p_3)_\beta(p_3+p_2)_\alpha}{(p_3^2+M^2)[(p_3+p_2)^2+M^2][(p_3+p_2+p_1)^2+M^2]}\right)\delta_{\delta\gamma}$$

$$+\left(\frac{(p_3)_\delta(p_3+p_4)_\alpha}{(p_3^2+M^2)[(p_3+p_4)^2+M^2][(p_3+p_4+p_1)^2+M^2]}\right.$$

$$\left.+\frac{(p_2)_\alpha(p_2+p_1)_\delta}{(p_2^2+M^2)[(p_2+p_1)^2+M^2][(p_2+p_1+p_4)^2+M^2]}\right)\delta_{\gamma\beta}$$

$$+\left(\frac{(p_1)_\delta(p_1+p_4)_\gamma}{(p_1^2+M^2)[(p_1+p_4)^2+M^2][(p_1+p_4+p_3)^2+M^2]}\right.$$

$$\left.\left.+\frac{(p_2)_\gamma(p_2+p_3)_\delta}{(p_2^2+M^2)[(p_2+p_3)^2+M^2][(p_2+p_3+p_4)^2+M^2]}\right)\delta_{\beta\alpha}\right]$$

$$+F^{acbd}\left[\left(\frac{(p_4)_\beta(p_4+p_2)_\gamma}{(p_4^2+M^2)[(p_4+p_2)^2+M^2][(p_4+p_2+p_3)^2+M^2]}\right.\right.$$

$$\left.+\frac{(p_1)_\gamma(p_1+p_3)_\beta}{(p_1^2+M^2)[(p_1+p_3)^2+M^2][(p_1+p_3+p_2)^2+M^2]}\right)\delta_{\delta\alpha}$$

$$+\left(\frac{(p_4)_\alpha(p_4+p_1)_\gamma}{(p_4^2+M^2)[(p_4+p_1)^2+M^2][(p_4+p_1+p_2)^2+M^2]}\right.$$

$$\left.+\frac{(p_2)_\gamma(p_2+p_3)_\alpha}{(p_2^2+M^2)[(p_2+p_3)^2+M^2][(p_2+p_3+p_1)^2+M^2]}\right)\delta_{\delta\beta}$$

$$+\left(\frac{(p_1)_\delta(p_1+p_4)_\beta}{(p_1^2+M^2)[(p_1+p_4)^2+M^2][(p_1+p_4+p_2)^2+M^2]}\right.$$

$$\left.+\frac{(p_3)_\beta(p_3+p_2)_\delta}{(p_3^2+M^2)[(p_3+p_2)^2+M^2][(p_3+p_2+p_4)^2+M^2]}\right)\delta_{\gamma\alpha}$$

$$+\left(\frac{(p_2)_\delta(p_2+p_4)_\alpha}{(p_2^2+M^2)[(p_2+p_4)^2+M^2][(p_2+p_4+p_1)^2+M^2]}\right.$$

$$\left.\left.+\frac{(p_3)_\alpha(p_3+p_1)_\delta}{(p_3^2+M^2)[(p_3+p_1)^2+M^2][(p_3+p_1+p_4)^2+M^2]}\right)\delta_{\gamma\beta}\right]$$

$$+F^{acdb}\left[\left(\frac{(p_4)_\gamma(p_4+p_3)_\alpha}{(p_4^2+M^2)[(p_4+p_3)^2+M^2][(p_4+p_3+p_1)^2+M^2]}\right.\right.$$

$$\left.+\frac{(p_2)_\alpha(p_2+p_1)_\gamma}{(p_2^2+M^2)[(p_2+p_1)^2+M^2][(p_2+p_1+p_3)^2+M^2]}\right)\delta_{\delta\beta}$$

$$+\left(\frac{(p_4)_\beta(p_4+p_2)_\alpha}{(p_4^2+M^2)[(p_4+p_2)^2+M^2][(p_4+p_2+p_1)^2+M^2]}\right.$$

$$\left.+\frac{(p_3)_\alpha(p_3+p_1)_\beta}{(p_3^2+M^2)[(p_3+p_1)^2+M^2][(p_3+p_1+p_2)^2+M^2]}\right)\delta_{\delta\gamma}$$

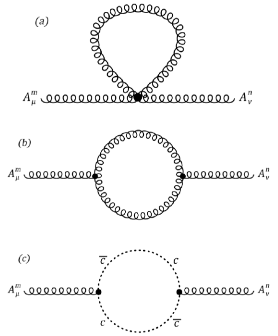
$$+\left(\frac{(p_3)_\delta(p_3+p_4)_\beta}{(p_3^2+M^2)[(p_3+p_4)^2+M^2][(p_3+p_4+p_2)^2+M^2]}\right.$$

$$\left.+\frac{(p_1)_\beta(p_1+p_2)_\delta}{(p_1^2+M^2)[(p_1+p_2)^2+M^2][(p_1+p_2+p_4)^2+M^2]}\right)\delta_{\gamma\alpha}$$

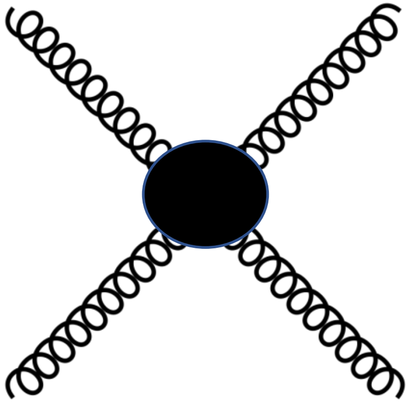
$$+\left(\frac{(p_2)_\delta(p_2+p_4)_\gamma}{(p_2^2+M^2)[(p_2+p_4)^2+M^2][(p_2+p_4+p_3)^2+M^2]}\right.$$

$$\left.\left.\left.+\frac{(p_1)_\gamma(p_1+p_3)_\delta}{(p_1^2+M^2)[(p_1+p_3)^2+M^2][(p_1+p_3+p_4)^2+M^2]}\right)\delta_{\beta\alpha}\right]\right\}.$$

Perturbative RGZ(2): 1-loop gluon propagator



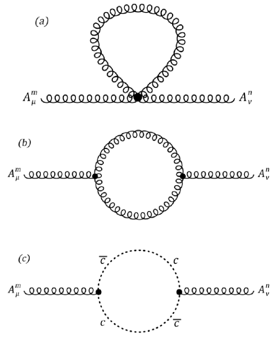
- Four-gluon vertex modification (with Gribov's horizon)



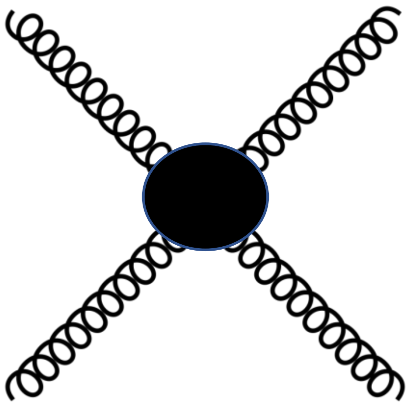
$$
\begin{aligned}
= &-g^4\gamma^4\delta(p_1+p_2+p_3+p_4)\Big\{F^{abcd}\Big[\Big(\frac{(p_4)_\gamma(p_4+p_3)_\beta}{(p_4^2+M^2)[(p_4+p_3)^2+M^2][(p_4+p_3+p_2)^2+M^2]}\\
&+\frac{(p_1)_\beta(p_1+p_2)_\gamma}{(p_1^2+M^2)[(p_1+p_2)^2+M^2][(p_1+p_2+p_3)^2+M^2]}\Big)\delta_{\delta\alpha}\\
&+\Big(\frac{(p_4)_\alpha(p_4+p_1)_\beta}{(p_4^2+M^2)[(p_4+p_1)^2+M^2][(p_4+p_2+p_1)^2+M^2]}\\
&+\frac{(p_3)_\beta(p_3+p_2)_\alpha}{(p_3^2+M^2)[(p_3+p_2)^2+M^2][(p_3+p_2+p_1)^2+M^2]}\Big)\delta_{\gamma\delta}\\
&+\Big(\frac{(p_3)_\delta(p_3+p_4)_\alpha}{(p_3^2+M^2)[(p_3+p_4)^2+M^2][(p_3+p_4+p_1)^2+M^2]}\\
&+\frac{(p_2)_\alpha(p_2+p_1)_\delta}{(p_2^2+M^2)[(p_2+p_1)^2+M^2][(p_2+p_1+p_4)^2+M^2]}\Big)\delta_{\gamma\beta}\\
&+\Big(\frac{(p_1)_\delta(p_1+p_4)_\gamma}{(p_1^2+M^2)[(p_1+p_4)^2+M^2][(p_1+p_4+p_3)^2+M^2]}\\
&+\frac{(p_2)_\gamma(p_2+p_3)_\delta}{(p_2^2+M^2)[(p_2+p_3)^2+M^2][(p_2+p_3+p_4)^2+M^2]}\Big)\delta_{\beta\alpha}\Big]\\
&+F^{acbd}\Big[\Big(\frac{(p_4)_\beta(p_4+p_2)_\gamma}{(p_4^2+M^2)[(p_4+p_2)^2+M^2][(p_4+p_2+p_3)^2+M^2]}\\
&+\frac{(p_1)_\gamma(p_1+p_3)_\beta}{(p_1^2+M^2)[(p_1+p_3)^2+M^2][(p_1+p_3+p_2)^2+M^2]}\Big)\delta_{\delta\alpha}\\
&+\Big(\frac{(p_4)_\alpha(p_4+p_1)_\gamma}{(p_4^2+M^2)[(p_4+p_1)^2+M^2][(p_4+p_1+p_2)^2+M^2]}\\
&+\frac{(p_2)_\gamma(p_2+p_3)_\alpha}{(p_2^2+M^2)[(p_2+p_3)^2+M^2][(p_2+p_3+p_1)^2+M^2]}\Big)\delta_{\delta\beta}\\
&+\Big(\frac{(p_1)_\delta(p_1+p_4)_\beta}{(p_1^2+M^2)[(p_1+p_4)^2+M^2][(p_1+p_4+p_2)^2+M^2]}\\
&+\frac{(p_3)_\beta(p_3+p_2)_\delta}{(p_3^2+M^2)[(p_3+p_2)^2+M^2][(p_3+p_2+p_4)^2+M^2]}\Big)\delta_{\gamma\alpha}\\
&+\Big(\frac{(p_2)_\delta(p_2+p_4)_\alpha}{(p_2^2+M^2)[(p_2+p_4)^2+M^2][(p_2+p_4+p_1)^2+M^2]}\\
&+\frac{(p_3)_\alpha(p_3+p_1)_\delta}{(p_3^2+M^2)[(p_3+p_1)^2+M^2][(p_3+p_1+p_4)^2+M^2]}\Big)\delta_{\gamma\beta}\Big]\\
&+F^{adbc}\Big[\Big(\frac{(p_4)_\gamma(p_4+p_3)_\alpha}{(p_4^2+M^2)[(p_4+p_3)^2+M^2][(p_4+p_3+p_1)^2+M^2]}\\
&+\frac{(p_2)_\alpha(p_2+p_1)_\gamma}{(p_2^2+M^2)[(p_2+p_1)^2+M^2][(p_2+p_1+p_3)^2+M^2]}\Big)\delta_{\delta\beta}\\
&+\Big(\frac{(p_4)_\beta(p_4+p_2)_\alpha}{(p_4^2+M^2)[(p_4+p_2)^2+M^2][(p_4+p_2+p_1)^2+M^2]}\\
&+\frac{(p_3)_\alpha(p_3+p_1)_\beta}{(p_3^2+M^2)[(p_3+p_1)^2+M^2][(p_3+p_1+p_2)^2+M^2]}\Big)\delta_{\delta\gamma}\\
&+\Big(\frac{(p_3)_\delta(p_3+p_4)_\beta}{(p_3^2+M^2)[(p_3+p_4)^2+M^2][(p_3+p_4+p_2)^2+M^2]}\\
&+\frac{(p_1)_\beta(p_1+p_2)_\delta}{(p_1^2+M^2)[(p_1+p_2)^2+M^2][(p_1+p_2+p_4)^2+M^2]}\Big)\delta_{\gamma\alpha}\\
&+\Big(\frac{(p_2)_\delta(p_2+p_4)_\gamma}{(p_2^2+M^2)[(p_2+p_4)^2+M^2][(p_2+p_4+p_3)^2+M^2]}\\
&+\frac{(p_1)_\gamma(p_1+p_3)_\delta}{(p_1^2+M^2)[(p_1+p_3)^2+M^2][(p_1+p_3+p_4)^2+M^2]}\Big)\delta_{\beta\alpha}\Big]\Big\}\,.
\end{aligned}
$$

(i.e., a very complicated momentum-dependent vertex)

Perturbative RGZ(2): 1-loop gluon propagator



- Four-gluon vertex modification (with Gribov's horizon)



$$= -g^4\gamma^4\delta(p_1+p_2+p_3+p_4)\Big\{F^{abcd}\Big[\Big(\frac{(p_4)_\gamma(p_4+p_3)_\beta}{(p_4^2+M^2)[(p_4+p_3)^2+M^2][(p_4+p_3+p_2)^2+M^2]}$$

$$+\frac{(p_1)_\beta(p_1+p_2)_\gamma}{(p_1^2+M^2)[(p_1+p_2)^2+M^2][(p_1+p_2+p_3)^2+M^2]}\Big)\delta_{\delta\alpha}$$

$$+\Big(\frac{(p_4)_\alpha(p_4+p_1)_\beta}{(p_4^2+M^2)[(p_4+p_1)^2+M^2][(p_4+p_2+p_1)^2+M^2]}$$

$$+\frac{(p_3)_\beta(p_3+p_2)_\alpha}{(p_3^2+M^2)[(p_3+p_2)^2+M^2][(p_3+p_2+p_1)^2+M^2]}\Big)\ldots$$

$$+\Big(\frac{(p_3)_\delta(p_3+p_4)_\alpha}{(p_3^2+M^2)[(p_3+p_4)^2+M^2][(p_3+p_4+p_1)^2+M^2]}$$

$$+\frac{(p_2)_\alpha(p_2+p_1)_\delta}{(p_2^2+M^2)[(p_2+p_1)^2+M^2][(p_2+p_1+p_4)^2+M^2]}\Big)\delta_{\gamma\beta}$$

$$+\Big(\frac{(p_1)_\delta(p_1+p_4)_\gamma}{(p_1^2+M^2)[(p_1+p_4)^2+M^2][(p_1+p_4+p_3)^2+M^2]}$$

$$+\frac{(p_2)_\gamma(p_2+p_3)_\delta}{(p_2^2+M^2)[(p_2+p_3)^2+M^2][(p_2+p_3+p_4)^2+M^2]}\Big)\delta_{\beta\alpha}\Big]$$

$$+F^{acbd}\Big[\Big(\frac{(p_4)_\beta(p_4+p_2)_\gamma}{(p_4^2+M^2)[(p_4+p_2)^2+M^2][(p_4+p_2+p_3)^2+M^2]}$$

$$+\frac{(p_1)_\gamma(p_1+p_2)_\beta}{(p_1^2+M^2)[(p_1+p_3)^2+M^2][(p_1+p_3+p_2)^2+M^2]}\Big)\delta_{\delta\alpha}$$

$$+\Big(\frac{(p_4)_\alpha(p_4+p_1)_\gamma}{(p_4^2+M^2)[(p_4+p_1)^2+M^2][(p_4+p_1+p_2)^2+M^2]}$$

$$+\frac{(p_2)_\gamma(p_2+p_3)_\alpha}{(p_2^2+M^2)[(p_2+p_3)^2+M^2][(p_2+p_3+p_1)^2+M^2]}\Big)\delta_{\delta\beta}$$

$$+\Big(\frac{(p_1)_\delta(p_1+p_4)_\beta}{(p_1^2+M^2)[(p_1+p_4)^2+M^2][(p_1+p_4+p_2)^2+M^2]}$$

$$+\frac{(p_3)_\beta(p_3+p_2)_\delta}{(p_2^2+M^2)[(p_3+p_2)^2+M^2][(p_3+p_2+p_4)^2+M^2]}\Big)\delta_{\gamma\alpha}$$

$$\ldots$$

$$+\frac{(p_2)_\alpha(p_2+p_1)_\gamma}{(p_2^2+M^2)[(p_2+p_1)^2+M^2][(p_2+p_1+p_3)^2+M^2]}\Big)\delta_{\delta\beta}$$

$$+\Big(\frac{(p_4)_\beta(p_4+p_2)_\alpha}{(p_4^2+M^2)[(p_4+p_2)^2+M^2][(p_4+p_2+p_1)^2+M^2]}$$

$$+\frac{(p_3)_\alpha(p_3+p_1)_\beta}{(p_3^2+M^2)[(p_3+p_1)^2+M^2][(p_3+p_1+p_2)^2+M^2]}\Big)\delta_{\delta\gamma}$$

$$\ldots$$

$$+\frac{(p_1)_\beta(p_1+p_2)_\delta}{(p_1^2+M^2)[(p_1+p_2)^2+M^2][(p_1+p_2+p_4)^2+M^2]}\Big)\delta_{\gamma\alpha}$$

$$\ldots$$

$$+\frac{(p_1)_\gamma(p_1+p_2)_\delta}{(p_1^2+M^2)[(p_1+p_2)^2+M^2][(p_1+p_2+p_4)^2+M^2]}\Big)\delta_{\beta\alpha}\Big]\Big\}\,.$$

(i.e., a very complicated momentum-dependent vertex)

$\langle AA\rangle$ nonlocal calculation is on the way

Gribov's problem...

- It's there! Why not take it seriously?
- Analytically and numerically (lattice) supported.
- Can be straightforwardly addressed (GZ/RGZ).
- Technically challenging!
- It may give insight into nonperturbative Physics (e.g. confinement).