中国科学院近代物理研究所
Institute of Modern Physics, Chinese Academy of Sciences

Towards a Hamiltonian First Principle Approach for Baryon

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With

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Light-Cone 2023: Hadrons and Symmetries

Sep 18 – 22, 2023 Rio de Janeiro, Brazil

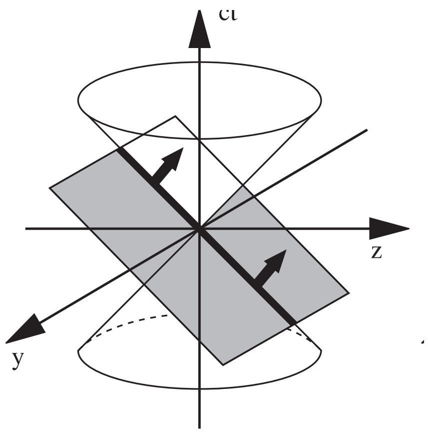
CBPF

Outline

- **B**asis **L**ight-**F**ront **Q**uantization (**BLFQ**)
- Our Current Study
- Conclusion
- Future

Light-Front Field Theory

➢ Light-Front Coordinates



Light-front variables:

$x^+ = x^0 + x^3$ (light-front time), $x^- = x^0 - x^3$, $x^\perp = x^{1,2}$

$P^- = P^0 - P^3$(light–front Hamiltonian), $P^+ = P^0 + P^3$, $P^\perp = P^{1,2}$

Dispersion relation: $P^- = \dfrac{m^2 + P_\perp^2}{P^+}$ Light-cone gauge: $A^+ = 0$

➢ Eigenvalue equation:

$$P^-|\beta\rangle = P_\beta^-|\beta\rangle$$

- $\boldsymbol{P^-}$: Light-Front Hamiltonian
- $|\boldsymbol{\beta}\rangle$: Eigenstates
- $\boldsymbol{P_\beta^-}$: Eigenvalues for eigenstates

Light-front wave functions → Baryon Structure: Form Factors (FFs),Parton distribution functions (PDFs) ...

- Frame-independent light-front wave functions
- Observables are defined on the light-front
- Light-front wave functions carry parton interpretation

Basis Light-Front Quantization

- Hamiltonian eigenvalue equation:

[Vary, et.al, Phys.Rev.C '10]

$$P^-|\beta\rangle = P^-_\beta|\beta\rangle$$

- P^-: Light-Front Hamiltonian
- $|\beta\rangle$: Eigenstates
- P^-_β: Eigenvalues for eigenstates

- Basis setup:

Fock sector expansion: $|\beta_{\text{nucleon}}\rangle = |qqq\rangle + |qqqg\rangle + |qqq\, q\bar{q}\rangle + \cdots\cdots$

Single particle basis $|\alpha\rangle = |n_1, m_1, n_2, m_2, n_3, m_3\rangle \otimes |k_1^+, k_2^+, k_3^+\rangle \otimes |\lambda_1, \lambda_2, \lambda_3, C\rangle$ in $|qqq\rangle$:

2-dimension harmonic oscillator	Discretized longitudinal momentum	Helicity and color
$\sum_i (2n_i + \lvert m_i \rvert + 1) \leq N_{\max}$	$\sum_i k_i^+ = K_{\max}$	$\Lambda = \sum_i (\lambda_i + m_i)$

- Advantages:

1. Rotational symmetry in transverse plane
2. Exact factorization between center-of-mass motion and intrinsic motion
3. Harmonic oscillator basis supplies correct infrared behavior for hadrons

Light-front Hamiltonian

➢ QCD light-front Hamiltonian can be derived from QCD Lagrangian:

$$\mathcal{L}_{QCD} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4} G^{\alpha}_{\mu\nu} G^{\mu\nu}_{\alpha}$$

$$P^-_{QCD} = H_K + H_I$$

$$H_K = \frac{1}{2}\int \mathrm{d}^3x\, \bar{\psi}\gamma^+ \frac{(\mathrm{i}\partial^\perp)^2 + m^2}{\mathrm{i}\partial^+}\psi - \frac{1}{2}\int \mathrm{d}^3x\, A^i_a (\mathrm{i}\partial^\perp)^2 A^i{}_a$$

$$\begin{aligned}
H_I = &+ g\int \mathrm{d}^3x\, \bar{\psi}\gamma_\mu A^\mu \psi \\
&+ \frac{1}{2}g^2 \int \mathrm{d}^3x\, \bar{\psi}\gamma_\mu A^\mu \frac{\gamma^+}{\mathrm{i}\partial^+}\gamma_\nu A^\nu \psi \\
&- \mathrm{i}g^2 \int \mathrm{d}^3x\, f^{abc}\bar{\psi}\gamma^+ T^c \psi \frac{1}{(\mathrm{i}\partial^+)^2}\left(\mathrm{i}\partial^+ A^\mu_a A_{\mu b}\right) \\
&+ \frac{1}{2}g^2 \int \mathrm{d}^3x\, \bar{\psi}\gamma^+ T^a \psi \frac{1}{(\mathrm{i}\partial^+)^2}\bar{\psi}\gamma^+ T^a \psi \\
&+ \mathrm{i}g \int \mathrm{d}^3x\, f^{abc}\mathrm{i}\partial^\mu A^{\nu a} A^b_\mu A^c_\nu \\
&- \frac{1}{2}g^2 \int \mathrm{d}^3x\, f^{abc} f^{ade}\mathrm{i}\partial^+ A^\mu_b A_{\mu c}\frac{1}{(\mathrm{i}\partial^+)^2}\left(\mathrm{i}\partial^+ A^\nu_d A_{\nu e}\right) \\
&+ \frac{1}{4}g^2 \int \mathrm{d}^3x\, f^{abc} f^{ade} A^\mu_b A^\nu_c A_{\mu d} A_{\nu e}.
\end{aligned}$$

- Interaction terms are simple to evaluate
- No more than three-body interaction
- Advantage to numerical approach

ψ: quark field operator
A^a_μ: gluon field operator

Publication

- **$|qqq\rangle$:**

Proton: [Mondal et al., Phys.Rev.D,102.016008] (2020)
[Xu et al., Phys.Rev.D,104.094036] (2021)
GPD: [Liu et al., Phys.Rev.D,105.094018] (2022)
[Kaur et al., arXiv:2307.09869 [hep-ph]] (2023)
TMD: [Hu et al., Phys.Lett.B,2022.137360] (2022)

Λ, Λ_c, **PDF : [Peng et al., Phys.Rev.D, 106.114040] (2022)**
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- $|\mathrm{qqq}\rangle + |\mathrm{qqqg}\rangle$:

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Talk: Chandan Mondal Sept. 19th 11:00-11:30
Sreeraj Nair Sept. 19th 16:00-16:30

- $|qqq\rangle + |qqqg\rangle + |qqqq\bar{q}\rangle$:

Talk: Glue and sea inside proton: A light-front Hamiltonian approach
– Chandan Mondal Sept. 19th 11:00-11:30

Light-Front Hamiltonian

$$P^- = H_{K.E.} + H_{trans} + H_{longi} + H_{OGE}$$

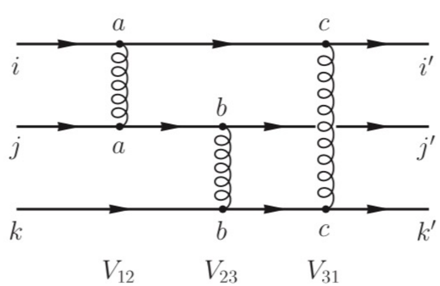
$$H_{K.E.} = \sum_i \frac{p_i^2 + m_q^2}{p_i^+}$$

$H_{trans} \sim \kappa_T^4 r^2$ -- Brodsky, Teramond arXiv: 1203.4025

$H_{longi} \sim -\sum_{ij} \kappa_L^4 \partial_{x_i}\left(x_i x_j \partial_{x_j}\right)$ ---Y Li, X Zhao , P Maris , J Vary, PLB 758(2016)

$H_{OGE} = -\frac{C_F 4\pi\alpha_s}{Q^2} \sum_{i,j(i<j)} \bar{u}_{s_i'}(k_i')\gamma^\mu u_{s_i}(k_i)\bar{u}_{s_j'}(k_j')\gamma_\mu u_{s_j}(k_j)$ Color factor : $C_F = -\frac{2}{3}$

$$|P_{baryon}\rangle = |qqq\rangle$$



Three active-quark approach

Nucleon Radii and Axial Charges

- The magnetic moment of the proton and neutron

Quantity	BLFQ	Measurement[a]	Lattice
μ_p	2.443 ± 0.027	2.79	2.43(9)
μ_n	-1.405 ± 0.026	-1.91	$-1.54(6)$

- The radii of the proton and neutron

Quantity	BLFQ	Measurement	Lattice
r_E^p [fm]	$0.802^{+0.042}_{-0.040}$	0.833 ± 0.010	0.742(13)
r_M^p [fm]	$0.834^{+0.029}_{-0.029}$	0.851 ± 0.026	0.710(26)
$\langle r_E^2 \rangle^n$ [fm^2]	-0.033 ± 0.198	-0.1161 ± 0.0022	$-0.074(16)$
r_M^n [fm]	$0.861^{+0.021}_{-0.019}$	$0.864^{+0.009}_{-0.008}$	0.716(29)

- The axial charge and axial radius

Quantity	BLFQ	Extracted data	Lattice
g_A^u	1.16 ± 0.04	0.82 ± 0.07	0.830(26)
g_A^d	-0.248 ± 0.027	-0.45 ± 0.07	$-0.386(16)$
g_A^{u-d}	1.41 ± 0.06	1.2723 ± 0.0023	1.237(74)
$\sqrt{\langle r_A^2 \rangle}$ fm	$0.680^{+0.070}_{-0.073}$	0.667 ± 0.12	0.512(34)

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Light-Front Hamiltonian

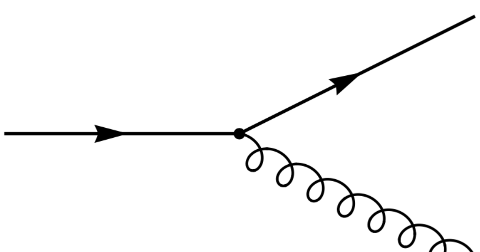
$$|P_{baryon}\rangle = \Psi_1|qqq\rangle + \Psi_2|qqqg\rangle$$

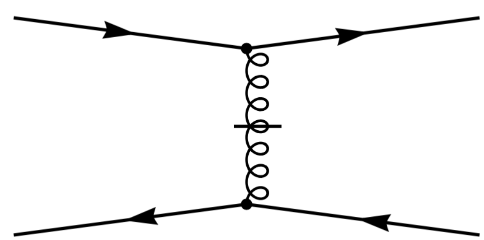
$$P^- = H_{K.E.} + H_{trans} + H_{longi} + H_{Interact}$$

$$H_{K.E.} = \sum_i \frac{p_i^2 + m_q^2}{p_i^+} \quad H_{trans} \sim \kappa_T^4 r^2 \quad H_{longi} \sim -\sum_{ij} \kappa_L^4 \partial_{x_i}\left(x_i x_j \partial_{x_j}\right)$$

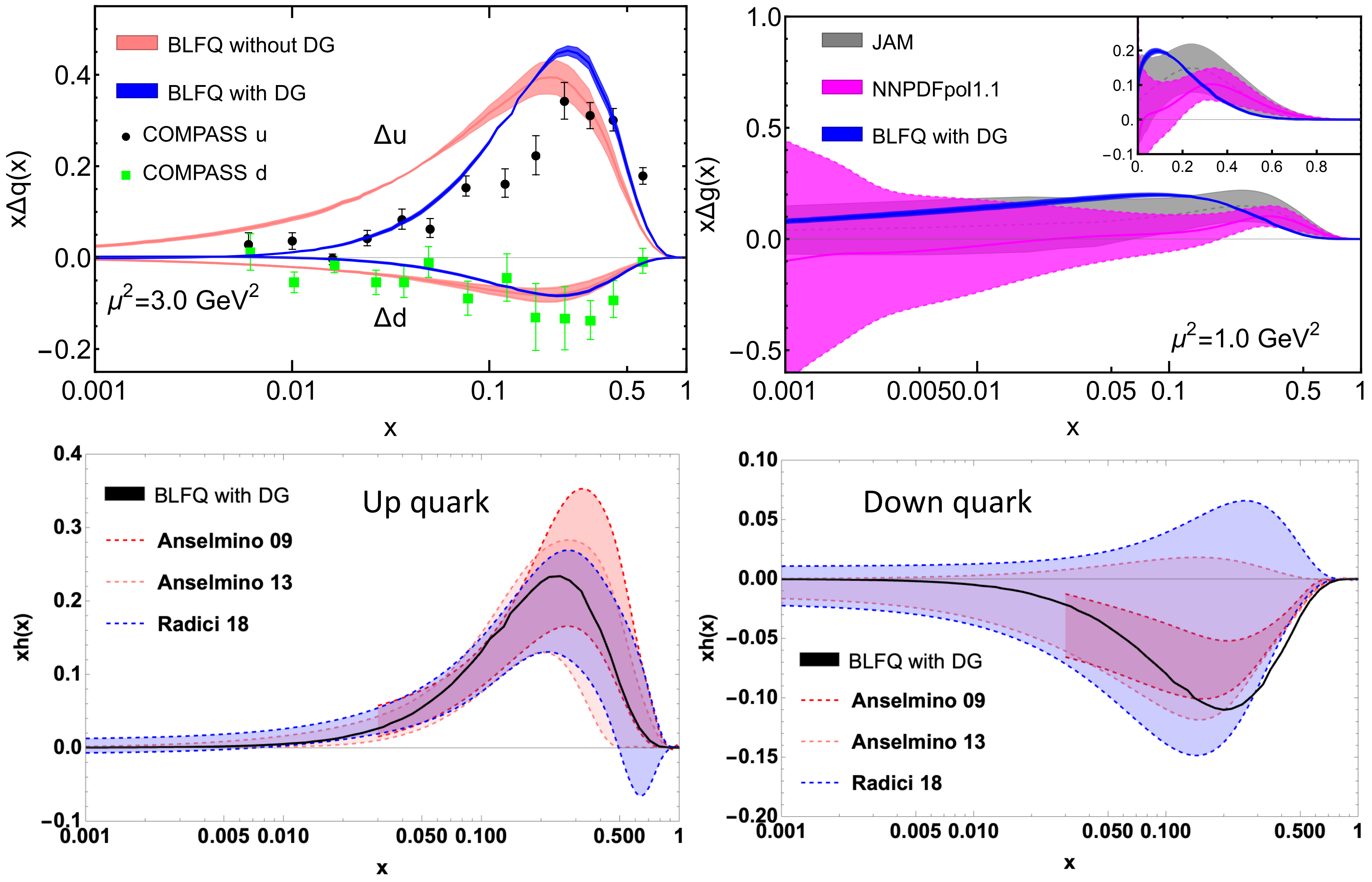
$$H_{OGE} = -\frac{C_F 4\pi\alpha_s}{Q^2} \sum_{i,j(i<j)} \bar{u}_{s_i'}(k_i')\gamma^\mu u_{s_i}(k_i)\bar{u}_{s_j'}(k_j')\gamma_\mu u_{s_j}(k_j)$$

$$H_{Interact} = H_{Vertex} + H_{inst} = g\bar{\psi}\,\gamma^\mu T^a\,\psi\,A_\mu^a + \frac{g^2 C_F}{2}\,j^+\frac{1}{(i\partial^+)^2}j^+$$





Parton distribution function



a lofty aspiration

From Now

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- $|qqq\rangle + |qqqg\rangle + |qqqq\bar{q}\rangle + |qqqgg\rangle$:

Light-Front QCD Hamiltonian

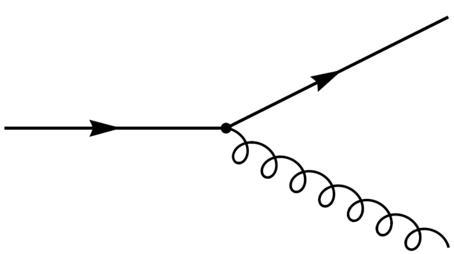
$$\left|P_{baryon}\right\rangle = \Psi_1\left|qqq\right\rangle + \Psi_2\left|qqqg\right\rangle + \Psi_{31}\left|qqq\,u\bar{u}\right\rangle + \Psi_{32}\left|qqq\,d\bar{d}\right\rangle + \Psi_{33}\left|qqq\,s\bar{s}\right\rangle$$

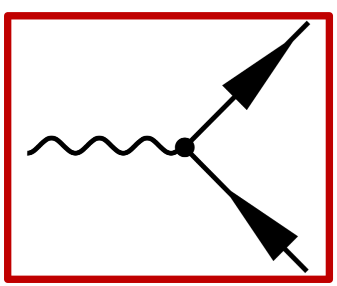
$$P^- = H_{K.E.} + \cancel{H_{trans} + H_{longi}} + H_{Interact}$$

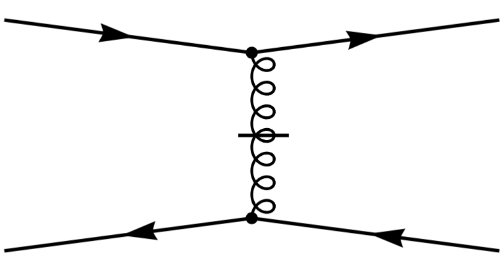
$$H_{K.E.} = \sum_i \frac{p_i^2 + m_q^2}{p_i^+}$$

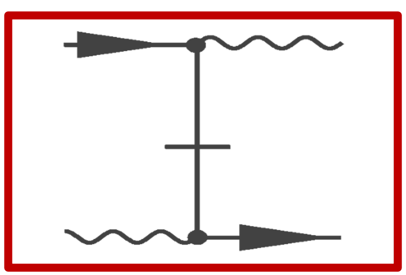
$$H_{Interact} = g\bar{\psi}\,\gamma^\mu T^a\,\psi\,A_\mu^a + \frac{g^2 C_F}{2}\,j^+\frac{1}{(i\partial^+)^2}j^+$$

$$H_{Interact} = g\bar{\psi}\,\gamma^\mu T^a\,\psi\,A_\mu^a + \frac{g^2 C_F}{2}\,j^+\frac{1}{(i\partial^+)^2}j^+ + \frac{g^2 C_F}{2}\,\bar{\psi}\gamma^\mu A_\mu \frac{\gamma^+}{i\partial^+} A_\nu \gamma^\nu \psi$$









Full BLFQ

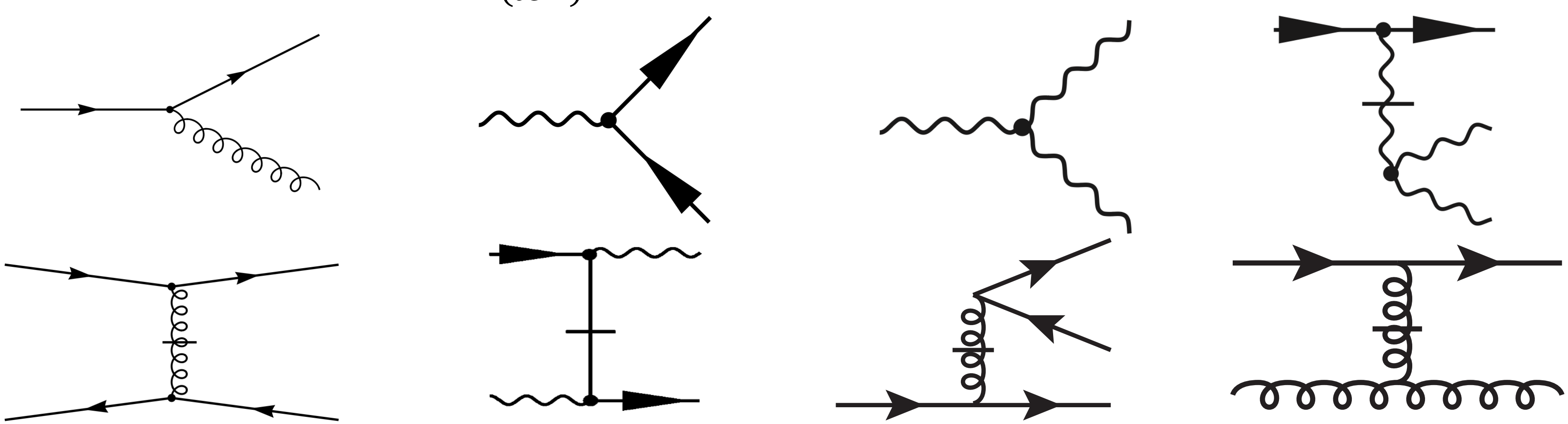
$$\left|P_{baryon}\right\rangle \rightarrow \left|qqq\right\rangle + \left|qqqg\right\rangle + \left|qqqu\bar{u}\right\rangle + \left|qqqd\bar{d}\right\rangle + \left|qqqs\bar{s}\right\rangle + \left|qqqgg\right\rangle$$

$$P^{-} = H_{K.E.} + H_{Interact}$$

$$H_{K.E.} = \sum_i \frac{p_i^2 + m_q^2}{p_i^+}$$

$$H_{Interact} = g\bar{\psi}\,\gamma^{\mu}T^{a}\,\psi\,A^{a}_{\mu} + \frac{g^2 C_F}{2}\,j^{+}\frac{1}{(i\partial^{+})^2}j^{+} + \frac{g^2 C_F}{2}\,\bar{\psi}\gamma^{\mu}A_{\mu}\frac{\gamma^{+}}{i\partial^{+}}A_{\nu}\gamma^{\nu}\psi$$

$$-g^2 C_F\bar{\psi}\gamma^{+}\psi\frac{1}{(i\partial^{+})^2}\,i\partial^{+}A^{a}_{\mu}A^{\mu}_{b} + igf^{abc}\,i\partial^{\mu}A^{\nu}_{a}A^{b}_{\mu}A^{c}_{\nu}$$



Fock Sector Decomposition

$$|P_{baryon}\rangle \rightarrow |qqq\rangle + |qqqg\rangle + |qqqu\bar{u}\rangle + |qqqd\bar{d}\rangle + |qqqs\bar{s}\rangle + |qqqgg\rangle$$

$|qqq\ q\bar{q}\rangle \sim$ 3 color singlet state

1 singlet ⊗ singlet

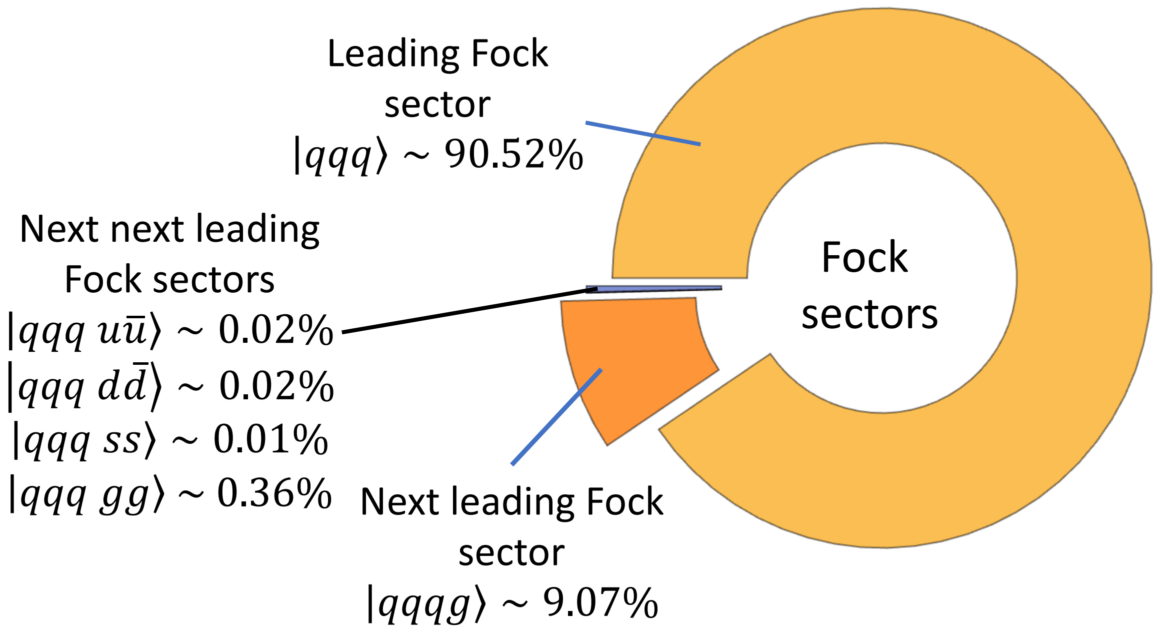
2 octet ⊗ octet

$|qqq\ gg\rangle \sim$ 6 color singlet state

1 singlet ⊗ singlet

4 octet ⊗ octet

1 decuplet ⊗ octet ⊗ octet



m_u	m_d	m_s	g	b
0.5 GeV	0.45 GeV	0.6 GeV	2.5	0.6 GeV

Truncation parameter: $N_{\max} = 7$ and $K_{\max} = 10$

Parton Distribution Function

- Parton distribution functions with five Fock sectors

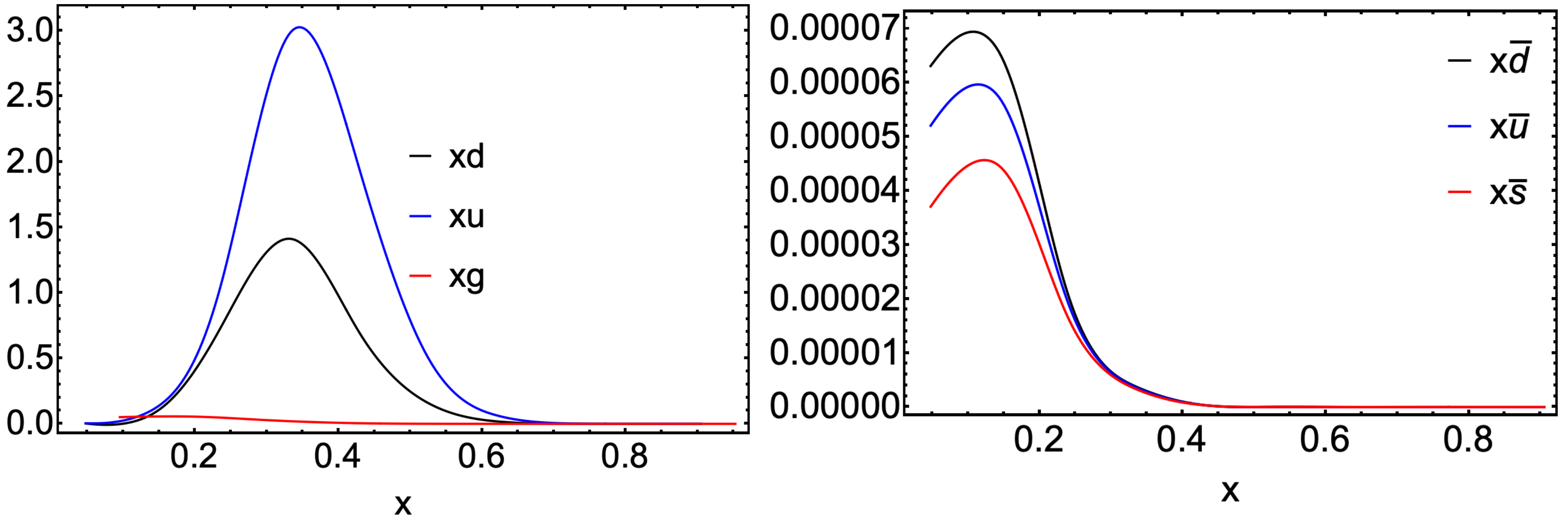
As we include $qqqgg$Fock sector, the endpoint behavior can be improved

Due to Fock sector truncation (no $qqq\ q\bar{q}\ g$, $qqq\ ggg$), our five-particle contribution too small

Our results qualitative agree with experimental results

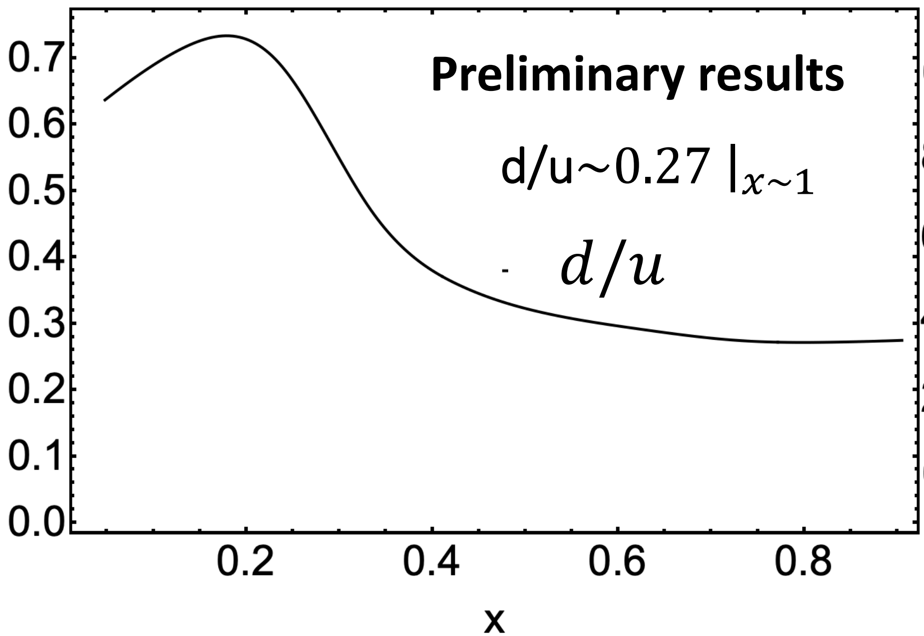
Preliminary results

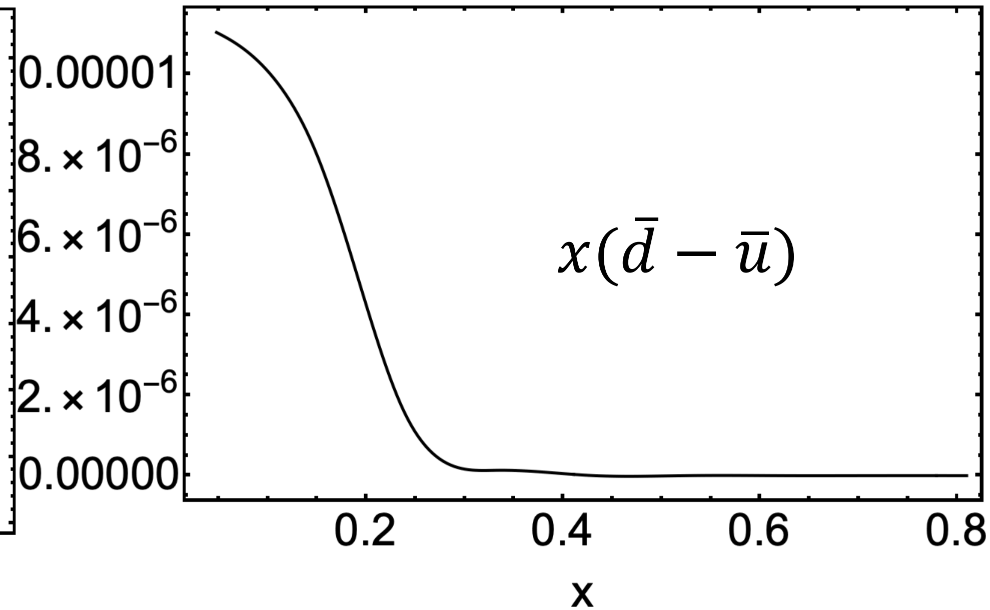
All results at the initial scale



Parton Distribution Function

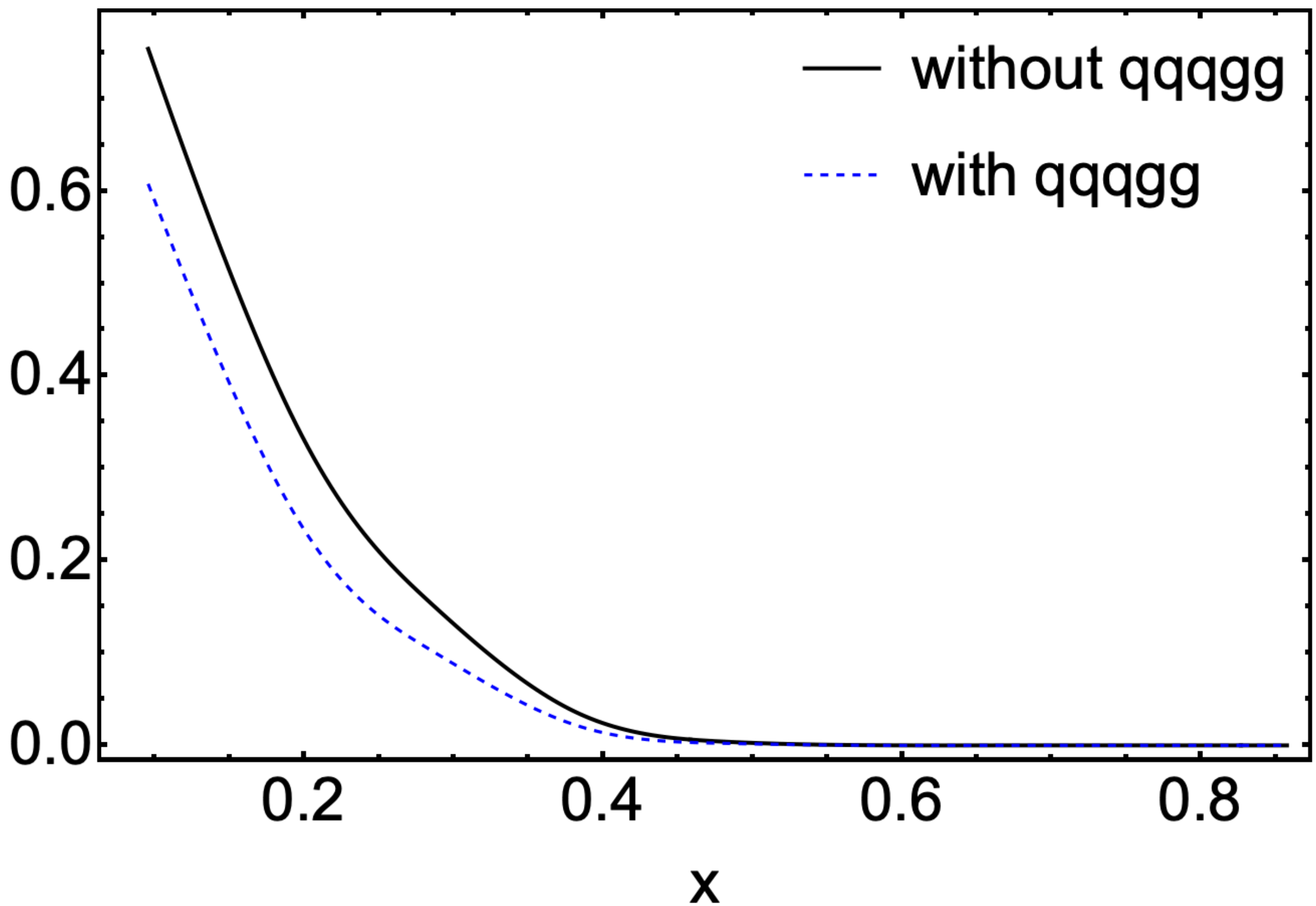
➢ Parton distribution functions with five Fock sectors





- One wave function → distributions of **valence quark**, **sea quark,** and **gluon**
- The **ratio of $\bar{d}$ to $\bar{u}$** → sea d is large than sea u
- BLFQ preliminary results qualitatively agree with Global fitting results

The Difference



As we include the $|qqq\ gg\rangle$ Fock sector, introduce the new interaction, the gluon distribution is suppressed.

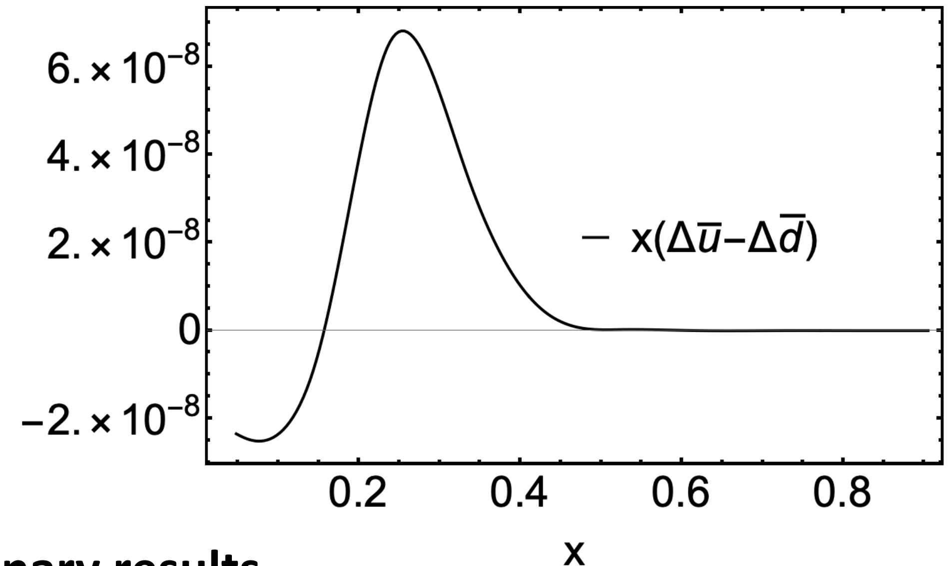
Helicity Parton Distribution Functions

- Helicity PDFs with five particle parton distribution

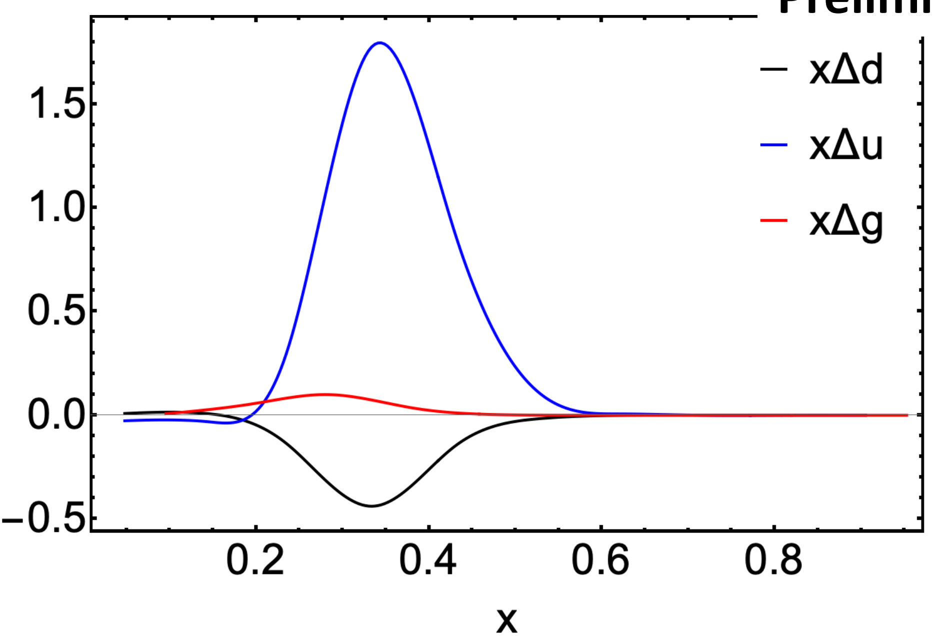
The behavior of sea asymmetry qualitatively agrees with JAM results

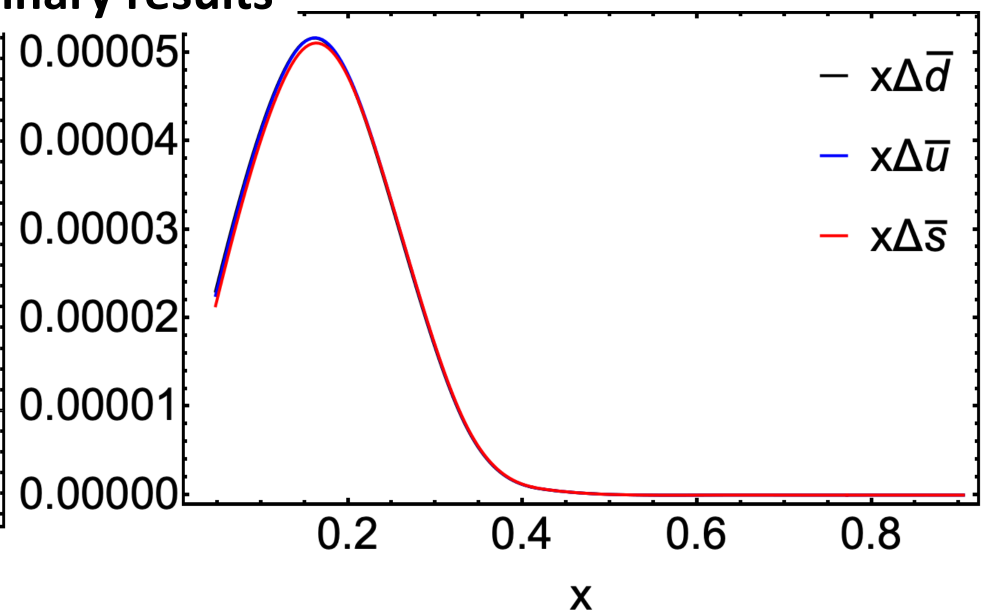
$\Delta\Sigma_u = 0.953$ $\Delta\Sigma = 0.743$

$\Delta\Sigma_d = -0.210$ $\Delta G = 0.081$



Preliminary results





Transversity Parton Distribution Functions

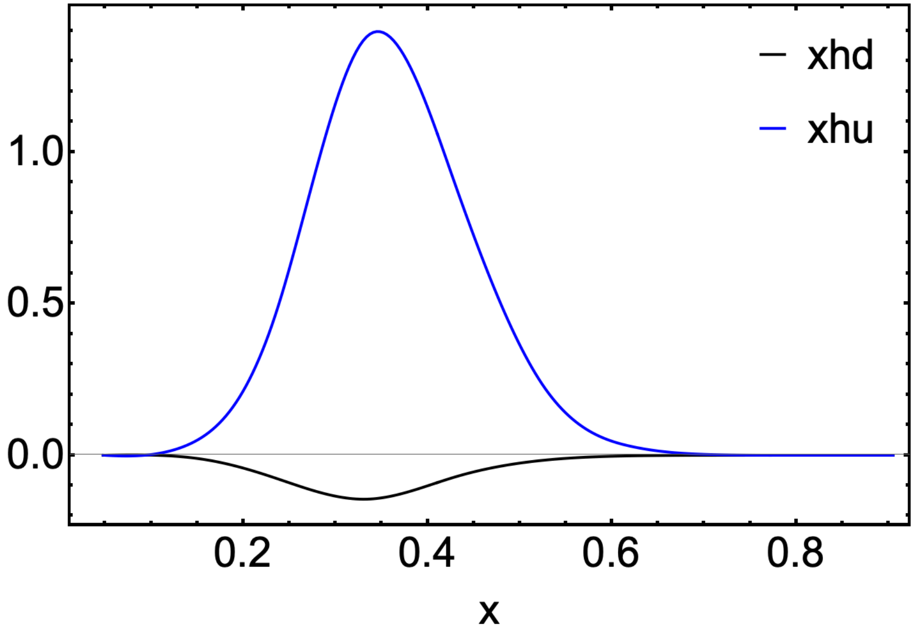
Transversity PDF of u and d has an opposite sign

According to current calculations, there is an asymmetry between $\bar{u}$, $\bar{d}$, and $\bar{s}$

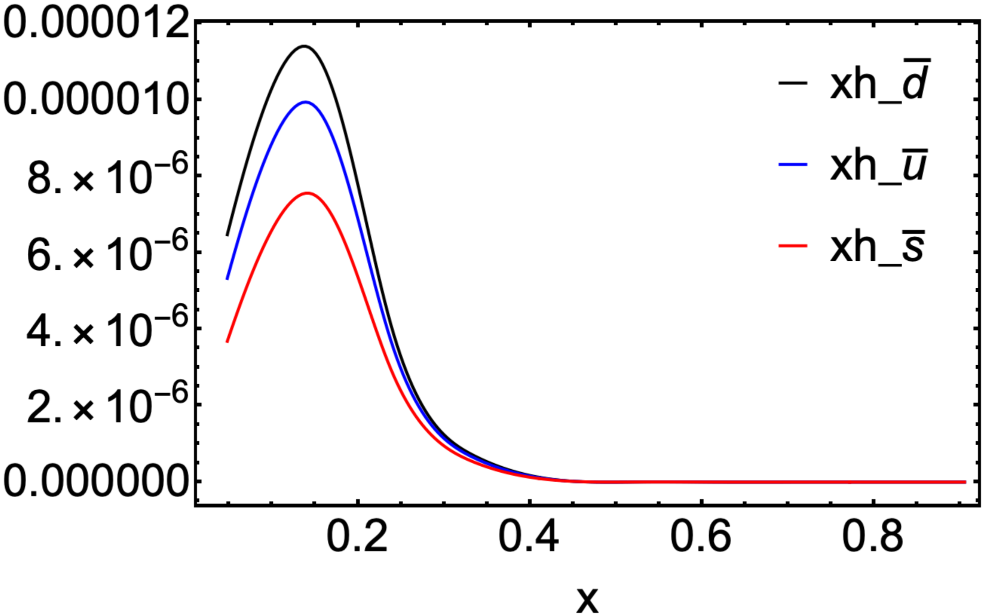
As we increase the truncation parameter, our results approach the experimental data.

Tensor Charge: $\delta u = 0.91$, $\delta d = -0.10$ At initial scale

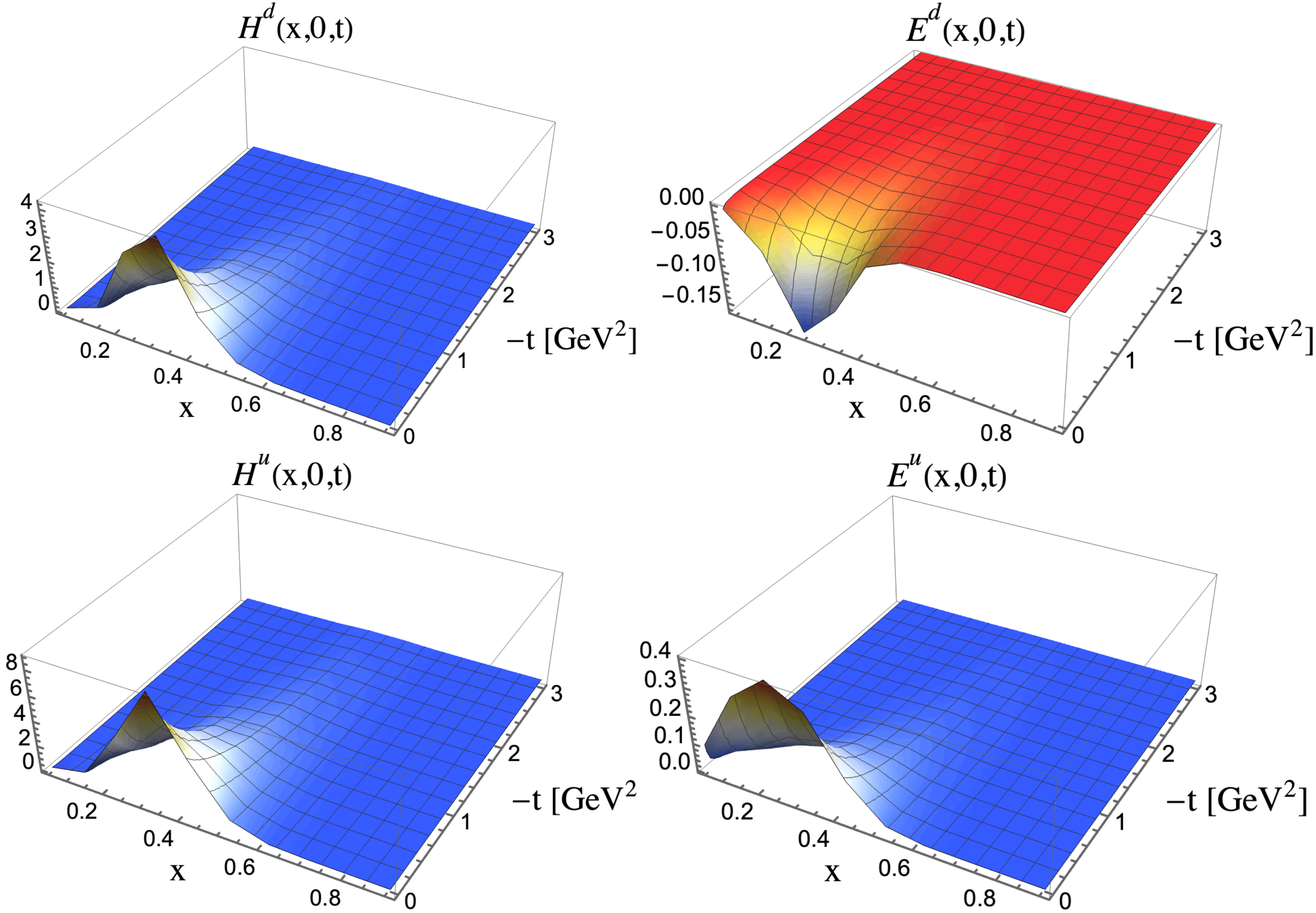
Preliminary results



Preliminary results



Generalized Parton Distribution Function



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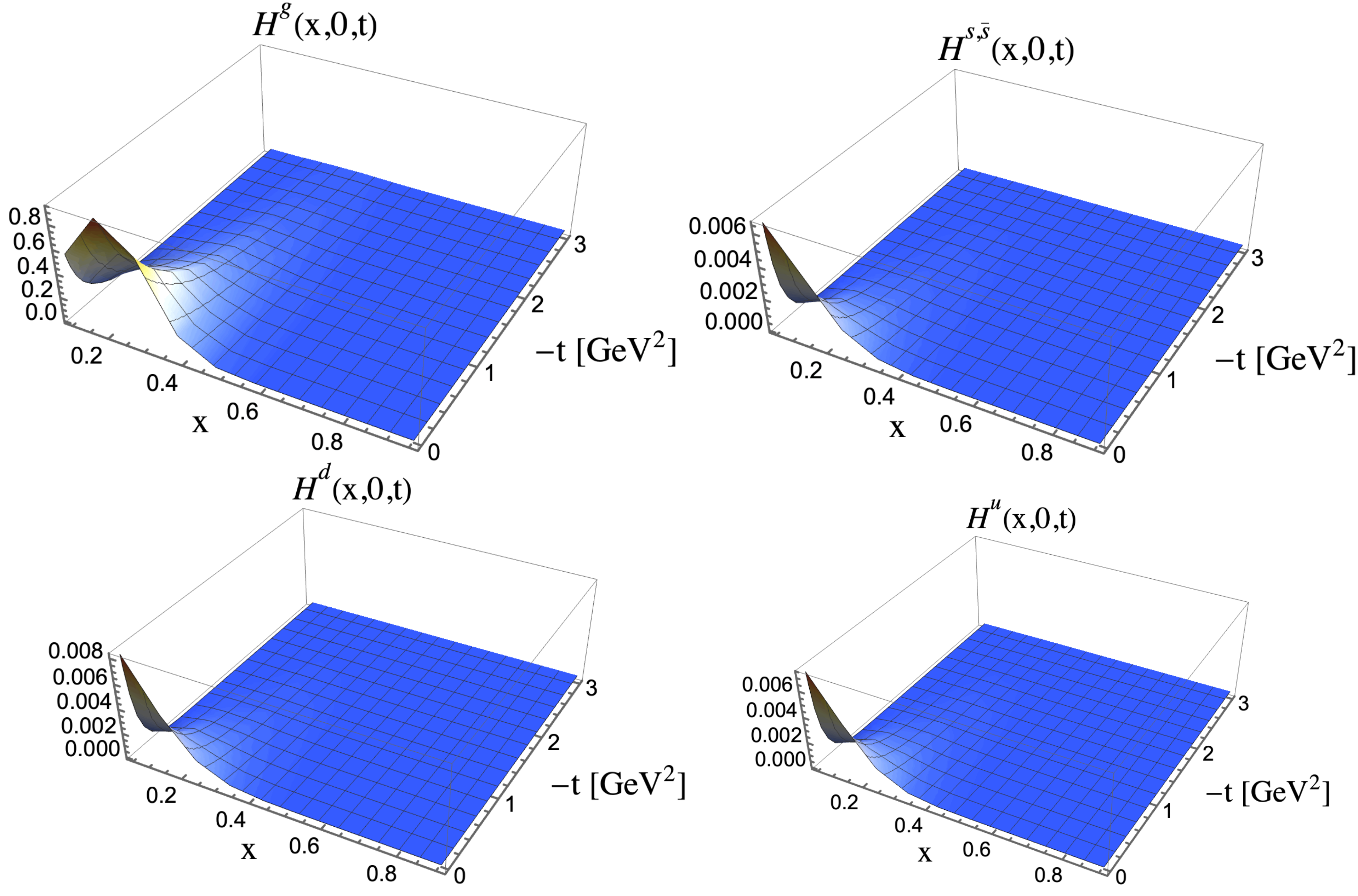


Conclusion

- BLFQ: a non-perturbative approach based on QCD Light-front Hamiltonian
- $|qqq\rangle + |qqqg\rangle + |qqq\ q\bar{q}\rangle + |qqq\ gg\rangle$ Fock sectors have been included
- Remove the effective interaction and Include the three-gluon vertex and relevant instantaneous interaction in QCD
- BLFQ results qualitatively agree with the global fitting
- Establish a platform to study the QCD from the first principle way in the light-front frame
- Welcome collaboration

Future

- Continue to expand the Fock sector: $|qqq\ q\bar{q}\ g\rangle$ and $|qqq\ ggg\rangle$……
- Include all QCD interaction
- Consider renormalization scheme
- Study the intrinsic charm, sea asymmetry
- Study the spin and mass decomposition
- Study the nucleus structure:

$$|deuterium\rangle = |qqq\ qqq\rangle + |qqq\ qqq\ g\rangle + |qqq\ qqq\ q\bar{q}\rangle + \ \ldots$$

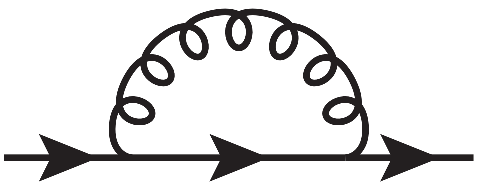
It’s when you follow the rain clouds that you find out where the rainbows are hiding.

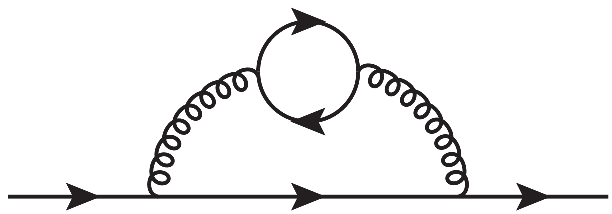
FSDR Scheme

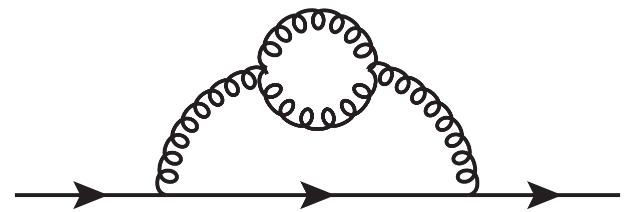
$$\left|P_{baryon}\right\rangle \rightarrow \left|qqq\right\rangle + \left|qqqg\right\rangle + \left|qqqu\bar{u}\right\rangle + \left|qqqd\bar{d}\right\rangle + \left|qqqs\bar{s}\right\rangle + \left|qqqgg\right\rangle$$

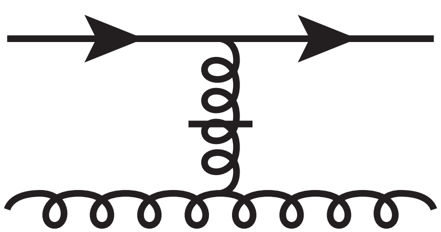
According to the Fock sector dependent renormalization:

- The mass counter term in $|qqq\rangle$:

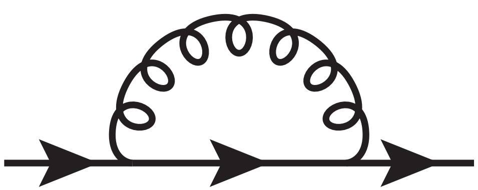


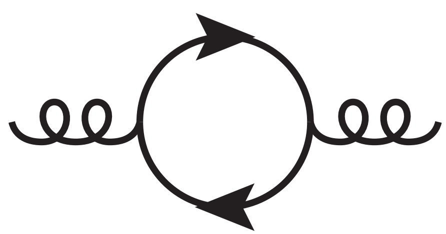


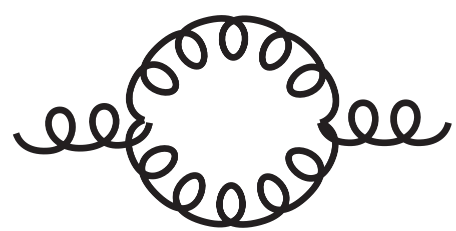




- The mass counter term in $|qqq\ g\rangle$:

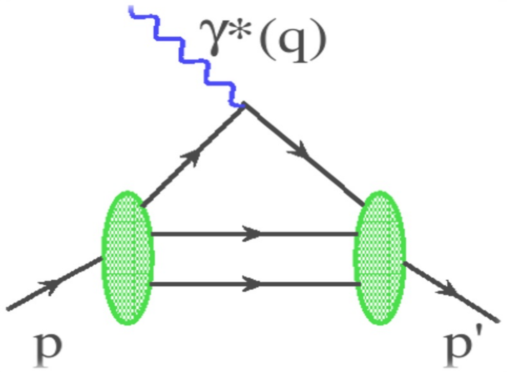






Nucleon Form Factor

$$\langle N(p')|J^{\mu}(0)|N(p)\rangle =$$
$$\bar{u}(p')\left[\gamma^{\mu}\underbrace{F_1(q^2)} + \frac{i\sigma^{\mu\nu}}{2m_N}q_{\nu}\underbrace{F_2(q^2)}\right]u(p)$$



As we include the FSDR:

the P-wave contribution → Significant Increase

Preliminary results