Evolution of GPDs as a tool for their extraction

Hervé Dutrieux (William & Mary)

Some collaborators

William & Mary and JLab: K. Orginos, J. Karpie, C. Monahan, ...

PARTONS @ Saclay and Warsaw: H. Moutarde, C. Mezrag, V. Bertone, P. Sznajder, ...

Marseille: S. Zafeiropoulos

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Outlook

- **Generalized parton distributions**
- Phenomenology of GPDs from Deeply Virtual Compton Scattering
- Parton distributions on the lattice: challenges and ideas
- Conclusions

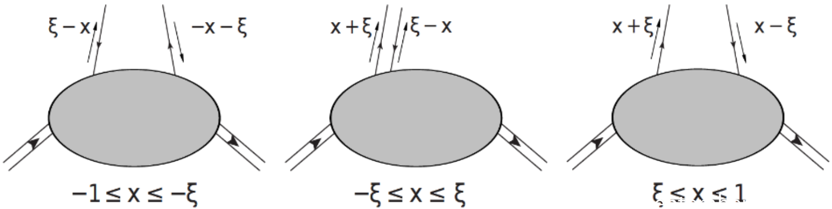
Generalized parton distributions

Spin-1/2 hadron, parton-helicity averaged quark GPDs H^q and E^q in the lightcone gauge [Müller et al, 1994], [Radyushkin, 1996], [Ji, 1997]

$$\frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi} e^{ixP^+z^-} \left\langle p_2 \middle| \bar{\psi}^q\left(-\frac{z}{2}\right)\gamma^+\psi^q\left(\frac{z}{2}\right) \middle| p_1 \right\rangle\Big|_{z_\perp=0,\ z^+=0}$$

$$= \frac{1}{2P^+}\left(H^q(x,\xi,t)\bar{u}(p_2)\gamma^+ u(p_1) + E^q(x,\xi,t)\bar{u}(p_2)\frac{i\sigma^{+\mu}\Delta_\mu}{2M}u(p_1)\right) \tag{1}$$

$$p_2 - p_1 = \Delta,\ \ t = \Delta^2,\ \ P = \frac{1}{2}(p_1+p_2),\ \ \xi = -\frac{\Delta^+}{2P^+}\,. \tag{2}$$



Generalized parton distributions

Forward limit

$$\begin{cases} H^q(x,\xi=0,t=0) & = q(x)\Theta(x) - \bar{q}(-x)\Theta(-x) \\ H^g(x,\xi=0,t=0) & = xg(x)\Theta(x) - xg(-x)\Theta(-x) \end{cases} \tag{3}$$

where $\Theta(x)$ is the Heaviside step function.

Elastic form factors

$$\int_{-1}^{1} \mathrm{d}x\, H^q(x,\xi,t) = F_1^q(t), \qquad \int_{-1}^{1} \mathrm{d}x\, E^q(x,\xi,t) = F_2^q(t) \tag{4}$$

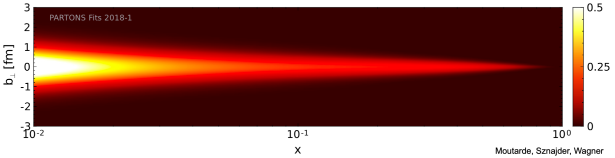
$\rightarrow$ **independent of ξ !**

Generalized parton distributions

Impact parameter distribution (IPD) [Burkardt, 2000]

$$I_a(x, \mathbf{b}_\perp, \mu^2) = \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp\cdot\Delta_\perp} F^a(x, 0, t = -\Delta_\perp^2, \mu^2) \quad (5)$$

is the density of partons with plus-momentum x and transverse position $\mathbf{b}_\perp$ from the center of plus momentum in a hadron $\rightarrow$ **hadron tomography**



Density of up quarks (valence GPD) in an unpolarized proton from a parametric fit to DVCS data in the PARTONS framework **[Moutarde et al, 2018]**.

Generalized parton distributions

Remarkably, GPDs allow access to gravitational form factors (GFFs) of the **energy-momentum tensor (EMT)** [Ji, 1997] defined for parton of type a

Gravitational form factors [Lorcé *et al*, 2017]

$$\langle p', s' | T_a^{\mu\nu} | p, s \rangle = \bar{u}(p', s') \Big\{ \frac{P^\mu P^\nu}{M} A_a(t) + \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu}\Delta^2}{M} C_a(t) + M\eta^{\mu\nu}\bar{C}_a(t) + \frac{P^{\{\mu} i\sigma^{\nu\}\rho}\Delta_\rho}{4M}\left[A_a(t) + B_a(t)\right] + \frac{P^{[\mu} i\sigma^{\nu]\rho}\Delta_\rho}{4M} D_a^{GFF}(t) \Big\} u(p, s) \quad (6)$$

Generalized parton distributions

- Link between **GFFs** and **GPDs** thanks to *e.g.* for quarks

$$\int_{-1}^{1} \mathrm{d}x\, x\, H^q(x,\xi,t,\mu^2) = A_q(t,\mu^2) + 4\xi^2 C_q(t,\mu^2) \tag{7}$$

$$\int_{-1}^{1} \mathrm{d}x\, x\, E^q(x,\xi,t,\mu^2) = B_q(t,\mu^2) - 4\xi^2 C_q(t,\mu^2) \tag{8}$$

- Ji's sum rule **[Ji, 1997]**

$$J^q = \frac{1}{2}\left(A_q(0) + B_q(0)\right) \tag{9}$$

- Radial distributions of hadron matter properties **[Polyakov, 2003]**: in the Breit frame ($\vec{P} = 0,\ t = -\vec{\Delta}^2$), radial pressure anisotropy profile

$$s_a(r) = -\frac{4M}{r^2}\int \frac{\mathrm{d}^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}}\, \frac{t^{-1/2}}{M^2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\left[t^{5/2}\, C_a(t)\right] \tag{10}$$

Outlook

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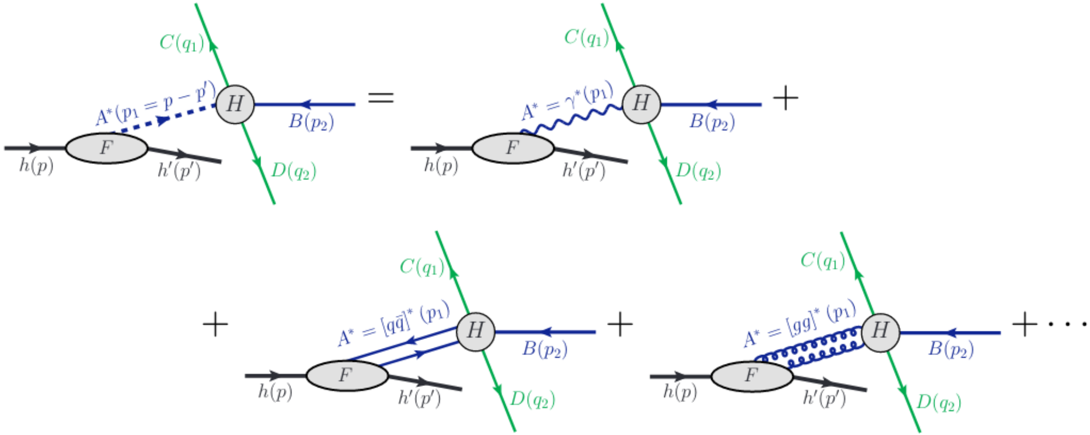
The experimental access to GPDs

Collinear factorisation of exclusive processes at small t/Q^2 in the Bjorken limit **[Qiu, Yu, 2022]**



Extraction of GFFs

DVCS observables parametrized in terms of **Compton form factors (CFFs)** which can be factorised at leading twist in terms of 4 chiral-even twist-2 GPDs owing to a perturbatively computable **coefficient function**:

CFF leading-twist convolution [Radyushkin, 1997], [Ji, Osborne, 1998], [Collins, Freund, 1999]

$$\mathcal{H}(\xi, t, Q^2) = \sum_a \int_{-1}^{1} \frac{\mathrm{d}x}{\xi} T^a\left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) \frac{H^a(x, \xi, t, \mu^2)}{|x|^{p_a}} \tag{11}$$

Plenty of challenges: ambiguities in defining ξ from experimental quantities up to order $\mathcal{O}(t/Q^2)$, related issue of kinematic power corrections and higher twists **[Braun et al, 2014]**, flavor decomposition **[Cuic, Kumericki, Schäfer, 2020]**, ...

$u, \bar{u}, d, \bar{d}, g \times$ 4 chiral-even GPDs = 20 GPDs $\times$ 3 dimensions = hundreds of parameters **[Guo et al, 2022]**

Extraction of GFFs

- From an analysis of DVCS based on **dispersion relations** (see *e.g.* [Anikin, Teryaev, 2007], [Diehl, Ivanov, 2007]), one extracts from DVCS data the **subtraction constant**

DVCS dispersion relation

$$\mathcal{C}_H(t, Q^2) = \mathrm{Re}\,\mathcal{H}(\xi, t, Q^2) - \frac{1}{\pi}\int_0^1 \mathrm{d}\xi'\,\mathrm{Im}\,\mathcal{H}(\xi', t, Q^2)\left(\frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'}\right) \tag{12}$$

$$\overset{LO}{=} 2\sum_q e_q^2 \int_{-1}^{1} \mathrm{d}z\, \frac{D^q(z, t, Q^2)}{1 - z} \tag{13}$$

- On the other hand, thanks to the **polynomiality property**, $C_q(t, \mu^2)$ depends only on the D-term via

Link GFF and D-term

$$\int_{-1}^{1} \mathrm{d}z\, z D^q(z, t, \mu^2) = 4\, C_q(t, \mu^2) \tag{14}$$

Extraction of GFFs

- How do we get from

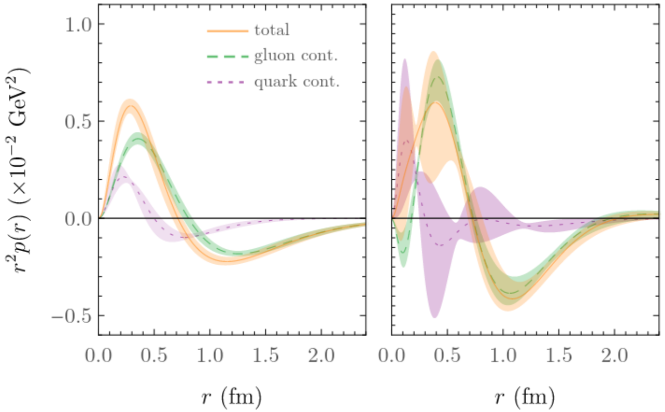
$$\int_{-1}^{1} \mathrm{d}z \, \frac{D^q(z,t,\mu^2)}{1-z} \quad \text{to} \quad \int_{-1}^{1} \mathrm{d}z \, z D^q(z,t,\mu^2) \; ? \tag{15}$$

- Each conformal moment of the D-term evolves at LO multiplicatively with a different anomalous dimension **[Lepage, Brodsky, 1979]**, **[Efremov, Radyushkin, 1979]**:

$$D^q(z,t,\mu^2) = (1-z^2) \sum_{\text{odd } n} d_n^q(t,\mu^2) \, C_n^{3/2}(z) \tag{16}$$

GFF C_a extraction

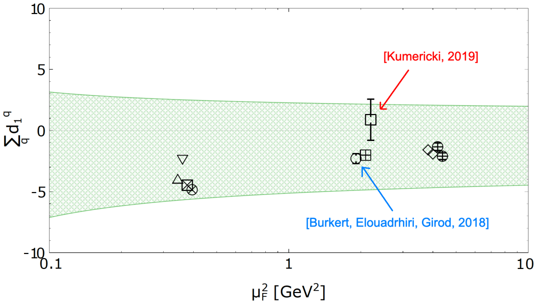
$$\int_{-1}^{1} \mathrm{d}z \, \frac{D^q(z,t,\mu^2)}{1-z} = 2 \sum_{\text{odd } n} d_n^q(t,\mu^2) \quad \text{and} \quad \int_{-1}^{1} \mathrm{d}z \, z D^q(z,t,\mu^2) = \frac{4}{5} \, d_1(t,\mu^2) \tag{17}$$



[Shanahan, Detmold, 2018]

Extraction of GFFs

- Published in **[Dutrieux *et al*, Eur.Phys.J.C 81 (2021) 4, 300]**. We perform a neural network fit of CFFs over world DVCS data, which gives a **subtraction constant compatible with 0** $\rightarrow$ also found in [Kumerički, 2019]. Then fixing the t-dependence with an Ansatz and assuming all d_n for $n > 1$ to be 0 gives



In green, 68% confidence interval found for $\sum_q d_1^q(t=0,\mu^2)$. Results obtained by the two other data-driven extractions highlighted. **But this is essentially a fit with one free parameter d_1^q whose uncertainty reflects the experimental uncertainty on the subtraction constant. What happens in case of a more flexible parametrization ?**

Extraction of GFFs

- To reduce bias, let us also allow d_3^q to be fitted jointly with d_1^q $(\mu_F^2 = 2\ \mathrm{GeV}^2)$

$d_1^{uds}(\mu_{\mathrm{F}}^2)$	-0.5 ± 1.2

$\longrightarrow$

$d_1^{uds}(\mu_{\mathrm{F}}^2)$	11 ± 25
$d_3^{uds}(\mu_{\mathrm{F}}^2)$	-11 ± 26

- Uncertainty explodes, and $d_1^q(\mu_F^2) \approx -d_3^q(\mu_F^2)$! What is going on ?

Extraction of GFFs

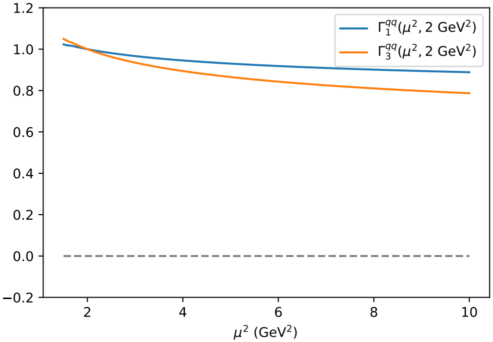
- The LO subtraction constant reads

$$\int_{-1}^{1} \mathrm{d}z \, \frac{D^q(z,t,\mu^2)}{1-z} = 2 \sum_{\text{odd } n} d_n^q(t,\mu^2) \tag{18}$$

  so at fixed scale μ_0^2, $d_1^q(\mu_0^2) = -d_3^q(\mu_0^2)$ does not bring any contribution on the subtraction constant. **Arbitrary large values of $d_1^q(\mu_0^2) = -d_3^q(\mu_0^2)$ are unconstrained by experimental data at fixed scale. We will call such an object a shadow D-term.**
- **Shadow distributions**
  Find a distribution with reasonable shape such that it gives no experimental contribution at one scale, and check how big its contribution becomes as you move from the initial scale $\rightarrow$ measures worst case uncertainty propagation from experiment to fit

Extraction of GFFs

in preparation



Simplified evolution in the qq sector

$$d_n^q(\mu^2) = \Gamma_n^{qq}(\mu^2, 2\ \text{GeV}^2) d_n^q(2\ \text{GeV}^2) \tag{19}$$

- current range of most DVCS data : [1.5, 4] GeV^2
- Over this range, Γ_1^{qq} and Γ_3^{qq} are numerically very close $\rightarrow$ little actual leverage in evolution to separate the two
- Estimate of the inflation on uncertainty when fitting jointly d_1 and d_3 compared to the sole d_1 :

$$\propto \left(1 - \frac{\Gamma_3^{qq}(Q^2_{\max}, Q^2_{\min})}{\Gamma_1^{qq}(Q^2_{\max}, Q^2_{\min})}\right)^{-1} \tag{20}$$

- **An increase thanks to EIC from [1.5, 4] GeV^2 to [1.5, 50] GeV^2 could yield a decrease by 3 times of the uncertainty on (d_1, d_3) due to the sole effect of increase in Q^2 range, without taking account a better experimental precision.**

Deconvoluting a Compton form factor

Position of the problem
Assuming a CFF has been extracted from experimental data with excellent precision – and the different gluon and flavor contributions have been separated –, we are left with the convolution:

$$\int_{-1}^{1} \frac{\mathrm{d}x}{\xi} T^q \left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2) \right) H^q(x, \xi, t, \mu^2) = T^q(Q^2, \mu^2) \otimes H^q(\mu^2) \tag{21}$$

where T^q is a coefficient function computed in pQCD. **Can we then "de-convolute" eq. (21) to recover $H^q(x, \xi, t, \mu^2)$ from $T^q(Q^2, \mu^2) \otimes H^q(\mu^2)$?**

Deconvoluting a Compton form factor

- Question was raised 20 years ago. Evolution was proposed as a crucial element in **[Freund, 1999]**, but the question has remained essentially open.
- **Plausible quantitative solution to this issue: study shadow GPDs** **[Bertone, HD, Mezrag, Moutarde, Sznajder, 2021]**.

Definition of an NLO shadow GPD

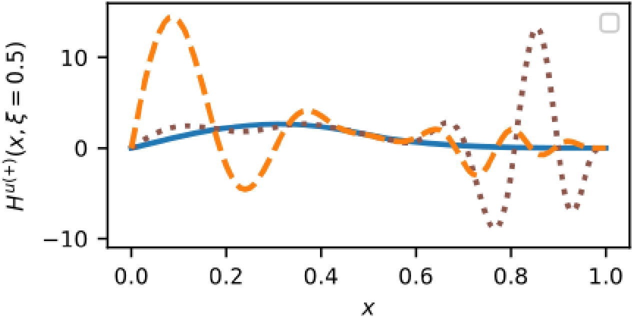
For a given scale μ_0^2,

$$\forall \xi, \forall t,\ T^q_{NLO}(Q^2, \mu_0^2) \otimes H^q(\mu_0^2) = 0 \quad \text{and} \quad H^q(x, \xi = 0, t = 0, \mu_0^2) = 0 \tag{22}$$

$$\text{so for } Q^2 \text{ and } \mu^2 \text{ close enough to } \mu_0^2,\ T^q_{NLO}(Q^2, \mu^2) \otimes H^q(\mu^2) = \mathcal{O}(\alpha_s^2(\mu^2)) \tag{23}$$

- Let H^q be an NLO shadow GPD, and G^q be any GPD. Then G^q and $G^q + H^q$ have the same forward limit, and the same NLO CFF up to a numerically small and theoretically subleading contribution.

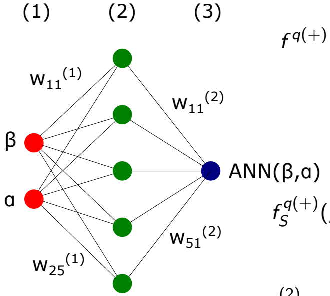
Deconvoluting a Compton form factor



- **Result**: the three models give CFFs that vary by $\approx 10^{-5}$ at moderate ξ over a range of $[1, 100]$ GeV^2 $\rightarrow$ **enormous inflation of uncertainty from experimental data at moderate** ξ
- **Limitation**: large fluctuations at large x unphysical, incompatible with positivity constraints. Very small contribution to uncertainty at small x and ξ [Moffat et al, 2023]

Deconvoluting a Compton form factor

To produce better physical models of GPDs, we have built a **neural network (NN) parametrization of DDs** in [HD et al, Eur.Phys.J.C 82 (2022) 3, 252], with emphasis on reproducing polynomiality, and shadow components.



Our neural network model for singlet DDs consists of three parts

$$f^{q(+)}(\beta,\alpha) = (1-x^2) f_C^{q(+)}(\beta,\alpha) + (x^2-\xi^2) f_S^{q(+)}(\beta,\alpha) + \xi f_D^{q(+)}(\beta,\alpha) \quad (24)$$

$$f_C^{q(+)} = \frac{q^{(+)}(\beta)}{1-\beta^2} \frac{\mathrm{ANN}_C(|\beta|,\alpha)}{\int_{|\beta|-1}^{1-|\beta|} \mathrm{ANN}_C(|\beta|,\alpha)} \quad (25)$$

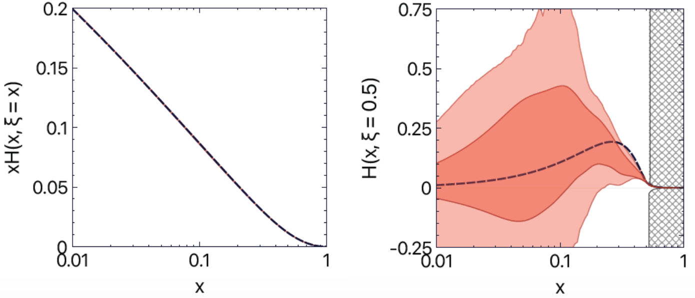
$$f_S^{q(+)}(\beta,\alpha) = q^{(+)}(\beta) N_S \left(\frac{\mathrm{ANN}_S^{(1)}(|\beta|,\alpha)}{\int_{|\beta|-1}^{1-|\beta|} \mathrm{ANN}_S^{(1)}(|\beta|,\alpha)} - \frac{\mathrm{ANN}_S^{(2)}(|\beta|,\alpha)}{\int_{|\beta|-1}^{1-|\beta|} \mathrm{ANN}_S^{(2)}(|\beta|,\alpha)} \right) \quad (26)$$

$$o_k^{(2)} = \varphi\left(b_k + w_{1,k}^{(1)}|\beta| + w_{2,k}^{(1)} \frac{\alpha}{1-|\beta|}\right) - \varphi\left(b_k + w_{1,k}^{(1)}|\beta| + w_{2,k}^{(1)}\right) + \left[w_{2,k}^{(1)} \to -w_{2,k}^{(1)}\right] \quad (27)$$

Deconvoluting a Compton form factor

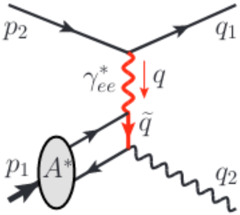
Introducing a simplified positivity constraint [Radyushkin, 1999], [Pire *et al*, 1999], [Diehl *et al*, 2001], [Pobylitsa, 2002]

$$|H^q(x,\xi)| \leq \sqrt{\frac{1}{1-\xi^2} f^q\left(\frac{x-\xi}{1-\xi}\right) f^q\left(\frac{x+\xi}{1+\xi}\right)} \tag{28}$$



Proof of concept: ANN model which reproduces exactly the LO CFF of a phenomenological model (GK), satisfies Lorentz covariance and positivity.

Perspectives

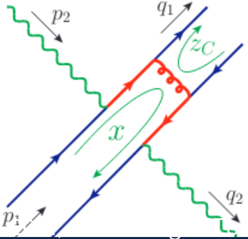


DVCS, TCS, DVMP: "moment-like" information on GPDs $\to x, \xi$ are not coupled directly to the hard scale [Qiu, Yu, 2022]

$$\tilde{q}^2 = \frac{Q^2 + q_2^2}{2\xi}\left[x - \xi\left(\frac{1 - q_2^2/Q^2}{1 + q_2^2/Q^2}\right)\right] + \mathcal{O}(t/Q^2) \qquad (29)$$

[Qiu, Yu, 2022]

Solution: entangle the flow of hard momentum with the x, ξ dependence: DDVCS [Guidal, Vanderhaeghen, 2003], [Belitsky, Müller, 2003], di-photon production [Pedrak et al, 2017], [Grocholski et al, 2020], photoproduction of photon-meson pair [Qiu, Yu, 2022] $\to$ avoids the single-photon channel!, ...



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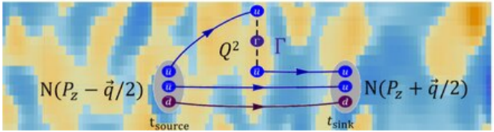
- **Parton distributions on the lattice: challenges and ideas**
- Conclusions

Parton distributions on the lattice

On a Euclidean lattice, equal-time spacelike separation $z^2 < 0$ [Ji, 2013]:

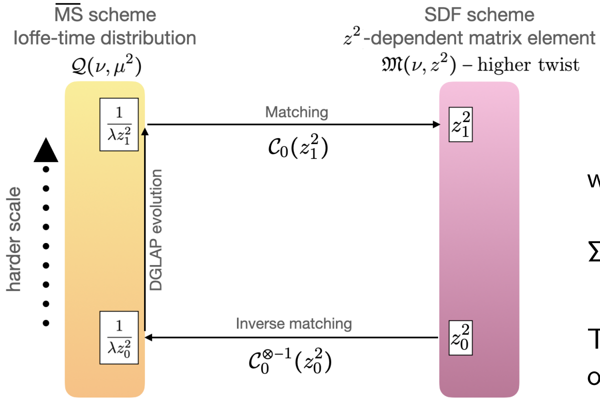
$$\left\langle p_2 \middle| \bar{\psi}^q(z)\,\gamma^\mu W(0,z)\psi^q(0) \middle| p_1 \right\rangle \tag{30}$$

- To compare to $\overline{MS}$ lightcone parton distributions, requires a matching computable in perturbation theory. [Radyushkin, 2017]
- The spacelike separation z^2 acts as a regulator of collinear divergence, playing a similar role to the factorisation scale. Evolution in z^2 provides a non-perturbative view of evolution!
- Warning: must have the physical z size small enough to have a partonic interpretation of the matrix element and an operational perturbative matching, typically $z \leq 0.2$ fm.



[Lin, Few-Body Systems 63:65, 2022]

Evolution in z^2 is derived from $\overline{MS}$ evolution through a back-and-forth matching procedure:



$$\mathcal{Q}(\nu, \mu^2) = \int_{-1}^{1} dx\, e^{ix\nu} q(x, \mu^2) \tag{31}$$

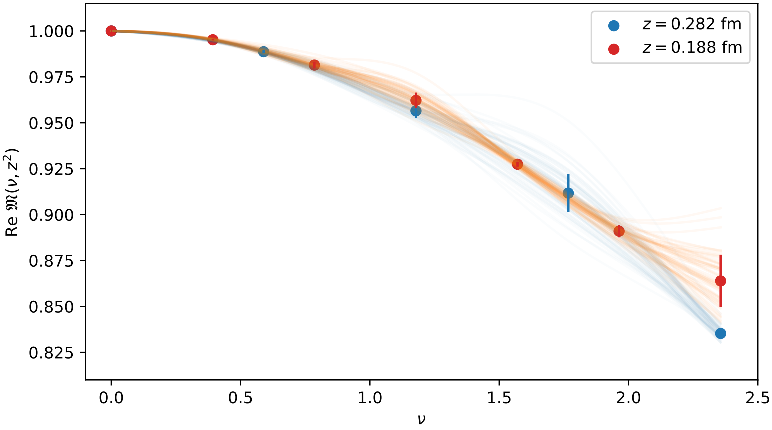
$$\mathfrak{M}(\nu, z_1^2) = \int_0^1 d\alpha\, \Sigma(\alpha; z_0^2, z_1^2) \mathfrak{M}(\alpha\nu; z_0^2)$$

where

$$\Sigma(z_0^2, z_1^2) = \mathcal{E}\left(\frac{1}{\lambda z_0^2}, \frac{1}{\lambda z_1^2}\right) \otimes \mathcal{C}_0(z_1^2) \otimes \mathcal{C}_0^{\otimes -1}(z_0^2)$$

The $\overline{MS}$ scale $1/(\lambda z^2)$ does not matter at all orders, but obviously does for a finite truncation. Uncertainties given by $\lambda \in [0.5, 2]$.

Parton distributions on the lattice



ν is the Fourier conjugate of x, so the resolution is $\Delta x \sim 1/\nu$
$m_\pi = 358$ MeV, $a = 0.094$ fm, $L = 3$ fm

Parton distributions on the lattice

We want to extract a z^2 evolution operator directly from the data by enforcing:

$$\mathfrak{M}(\nu, z_1^2) = \int_0^1 d\alpha \, \Sigma(\alpha; z_0^2, z_1^2) \mathfrak{M}(\alpha\nu; z_0^2)$$

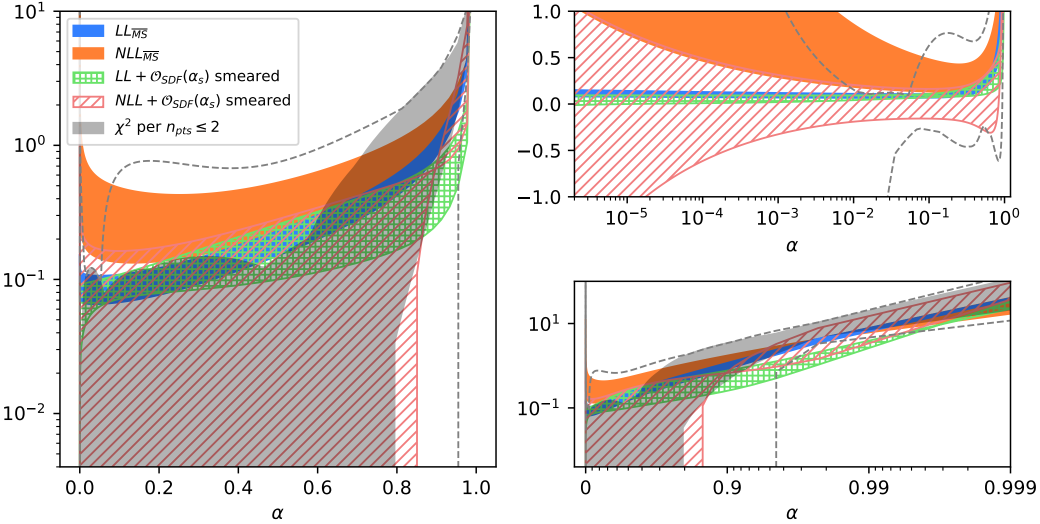
This is a fit of the type

$$y = f(x)$$

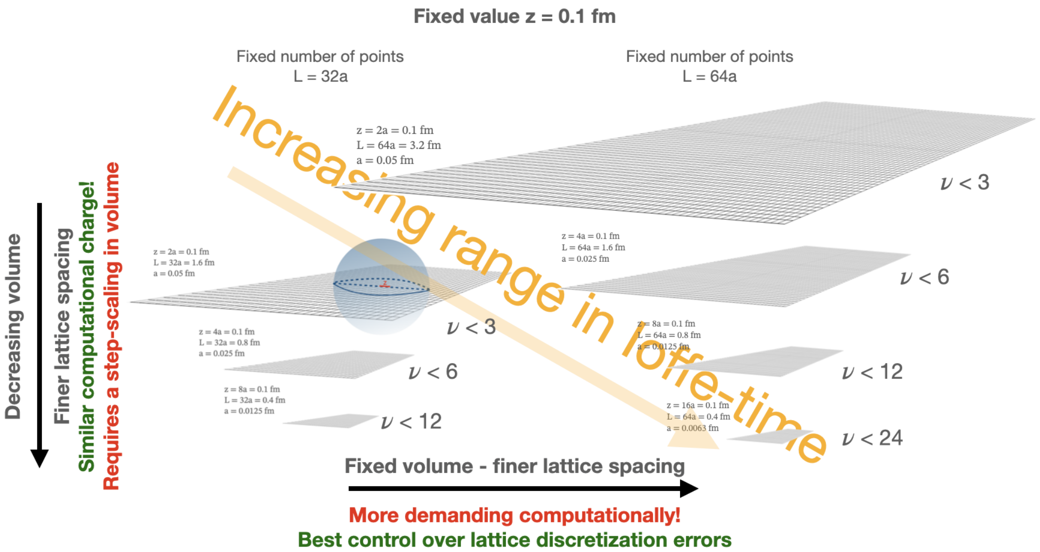
where x and y are noisy observations of true values x^* and y^*. The total least-squares method aims at minimizing

$$\begin{pmatrix} x - x^* \\ y - f(x^*) \end{pmatrix}^T \mathrm{Cov}[x, y] \begin{pmatrix} x - x^* \\ y - f(x^*) \end{pmatrix}$$

Parton distributions on the lattice

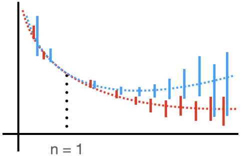


Large momentum, a dilemma

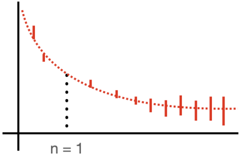


Large momentum, a dilemma

Many ways to introduce small volume computations in parton distributions, with various strategies to control finite volume effects. As an example,



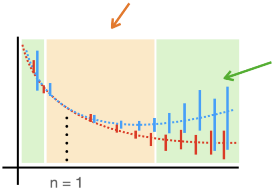
Larger volume = Smaller Ioffe time range
= Uncertainty of Mellin moments
degrades away from n = 1
+ different finite volume effects



Mellin moment of the matrix element

Small volume = Larger Ioffe time range
= Uncertainty of Mellin moments
small over larger range

Control region where uncertainty of Mellin moments is fairly similar to measure finite volume effects
Possible comparison with χPT



Region where we step-scale the small volume Mellin moments to take advantage of their better precision

Another cross-check: redo the computation at the larger volume with a finer lattice, to recover the Ioffe time range of the smaller volume, and check whether the predicted volume corrections were indeed correct!

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Conclusions

- Phenomenology of GPDs with lesser model dependence requires a global analysis program, over large kinematic range (EIC) and with many processes beyond the traditional DVCS, DVMP.
- One should be very careful when refering to “experimental” or lattice extractions of the modelling assumptions performed, and whether some features of interest arises from the data or from the modelling assumptions common to many studies.
- Lattice explorations are complementary, offer a very precious (largely) non-perturbative and first-principle view. They come however with statistical and systematic difficulties. A considerable improvement requires a much larger momentum, which demands imaginative solutions.

Thank you for your attention!

Polynomiality of Mellin moments: [Ji, 1998], [Radyushkin, 1999]
Translation of Lorentz covariance,

$$\int_{-1}^{1} \mathrm{d}x\, x^n H^q(x, \xi, t, \mu^2) = \sum_{k=0 \text{ even}}^{n+1} H^q_{n,k}(t, \mu^2)\, \xi^k \qquad (32)$$

This property implies that the GPD is the Radon transform of a **double distribution** F^q (DD) with an added ***D*-term** on the support $\Omega = \{(\beta, \alpha) \mid |\beta| + |\alpha| < 1\}$:

Double distribution formalism [Radyushkin, 1997], [Polyakov, Weiss, 1999]

$$H^q(x, \xi, t, \mu^2) = \int_\Omega \mathrm{d}\beta \mathrm{d}\alpha\, \delta(x - \beta - \alpha\xi) \left[F^q(\beta, \alpha, t, \mu^2) + \xi\delta(\beta) D^q(\alpha, t, \mu^2)\right] \qquad (33)$$

Shadow GPDs at leading order

- Published in [Bertone et al, Phys.Rev.D 103 (2021) 11, 114019]
- We search for our shadow GPDs as simple **double distributions (DD)** $F(\beta, \alpha, \mu^2)$ to respect polynomiality, with a zero D-term. Then, thanks to dispersion relations, we can restrict ourselves to the imaginary part only $\mathrm{Im}\ T^q(Q^2, \mu_0^2) \otimes H^q(\mu_0^2) = 0$.
- We search our DD as a polynomial of order N in (β, α), characterized by $\sim N^2$ coefficients c_{mn}:

$$F(\beta, \alpha, \mu_0^2) = \sum_{m+n \leq N} c_{mn}\, \alpha^m \beta^n \qquad (34)$$

- **Leading order** At LO, the imaginary part of the CFF gives

$$\mathrm{Im}\ T^q_{LO}(Q^2, \mu_0^2) \otimes H^q(\mu_0^2) \propto H^{q(+)}(\xi, \xi, \mu_0^2) \qquad (35)$$

and it is straightforward to build a system of $\sim N$ equations on the $\sim N^2$ coefficients c_{mn} of the polynomial DD and exhibit an infinite number of solutions cancelling the LO CFF.

Shadow GPDs at next-to-leading order

- **First study beyond leading order:** Apart from the **LO** part, the NLO CFF is composed of a **collinear part** (compensating the α_s^1 term resulting from the convolution of the LO coefficient function and the evoluted GPD) and a genuine **1-loop NLO** part.

$$\mathcal{H}^q(\xi, Q^2) = C_0^q \otimes H^{q(+)}(\mu_0^2) + \alpha_s(\mu^2)\, C_1^q \otimes H^{q(+)}(\mu_0^2) + \alpha_s(\mu^2)\, C_{coll}^q \otimes H^{q(+)}(\mu_0^2) \log\left(\frac{\mu^2}{Q^2}\right) \quad (36)$$

An explicit calculation of each term for our polynomial double distribution gives that

$$\mathrm{Im}\ T_{coll}^q(Q^2, \mu^2) \otimes H^q(\mu^2) \propto$$

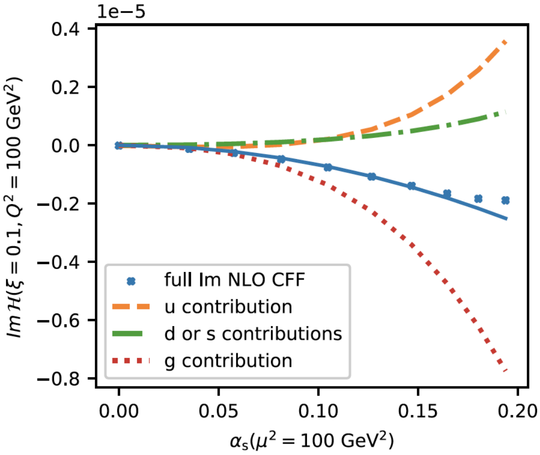
$$\alpha_s(\mu^2) \log\left(\frac{\mu^2}{Q^2}\right) \left[\left(\frac{3}{2} + \log\left(\frac{1-\xi}{2\xi}\right)\right) \mathrm{Im}\ T_{LO}^q \otimes H^q(\mu^2) + \sum_{w=1}^{N+1} \frac{k_w^{(coll)}}{(1+\xi)^w}\right] \quad (37)$$

and assuming $\mathrm{Im}\ T_{LO}^q \otimes H^q(\mu^2) = 0$,

$$\mathrm{Im}\ T_1^q(Q^2, \mu^2) \otimes H^q(\mu^2) \propto \alpha_s(\mu^2)\left[\log\left(\frac{1-\xi}{2\xi}\right) \mathrm{Im}\ T_{coll}^q \otimes H^q(\mu^2) + \sum_{w=1}^{N-1} \frac{k_w^{(1)}}{(1+\xi)^w}\right]$$

Shadow GPDs at next-to-leading order

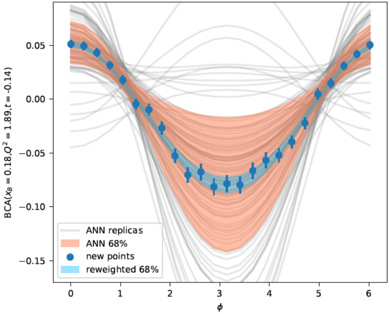
- Evolution of a shadow GPD of size $\mathcal{O}(1)$ on a lever-arm in Q^2 of $[1, 100]$ GeV^2 (typical collider kinematics) using APFEL++ code.

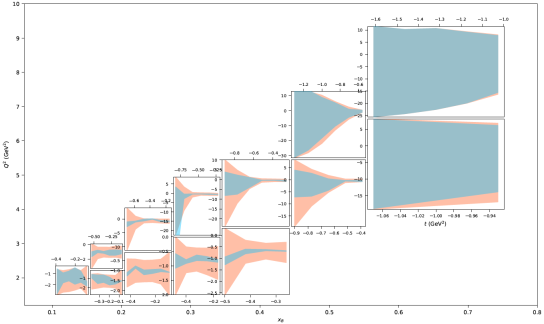


- Fit by $\alpha_s^2(\mu^2)$ is very good up to values of α_s of the order of its $\overline{MS}$ values. For larger values, large logs and higher orders slightly change the picture.
- The numerical effect of evolution remains very small. For a GPD of order 1, the NLO CFF is only of order 10^{-5}.

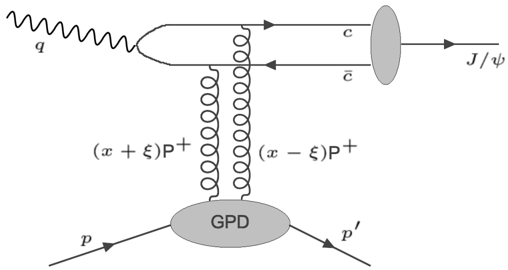
Perspectives

- Reducing uncertainties on CFFs itself is a very useful task. *e.g.* proton pressure anisotropy is compatible with 0 largely because of the uncertainty on $\mathrm{Re}\,\mathcal{H}$ in **[HD et al, Eur.Phys.J.C 81 (2021) 4, 300]**.
- The proposal to install a positron beam at JLab **[Afanasev et al, 2019]** can help on this task. We have performed in **[HD et al, Eur.Phys.J.A 57 (2021) 8, 250]** a reweighting of our neural network replicas of CFFs against simulated new experimental points.





GPDs at small x_B



LO depiction of J/ψ photoproduction

For moderate or small photon virtuality, description by **GPDs** and **non-relativistic matrix element** **[Ivanov et al, 2004]**:

$$\mathcal{F}(\xi,t) \propto \left(\frac{\langle O_1\rangle_V}{m_V^3}\right)^{1/2} \times \sum_{a=q,g} \int_{-1}^{1} \mathrm{d}x \, T^a(x,\xi) \, F^a(x,\xi,t) \quad (39)$$

where $\langle O_1\rangle_V^{1/2}$ is the NR QCD matrix element, T a hard-scattering kernel and $F(x,\xi,t)$ is the GPD. Hard scale provided by $m_V/2$ **[Jones et al, 2015]**.

$$\xi \approx \frac{x_B}{2} \sim 10^{-5} \quad (40)$$

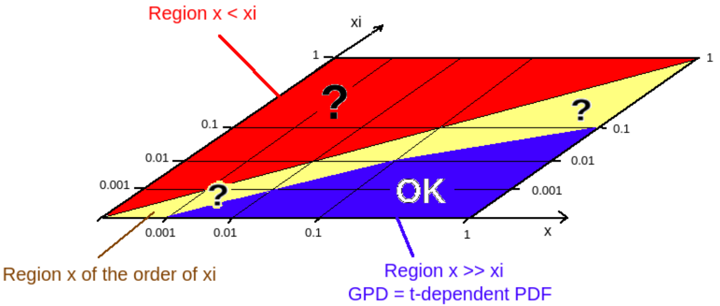
GPDs at small x_B

Why don't we just assume

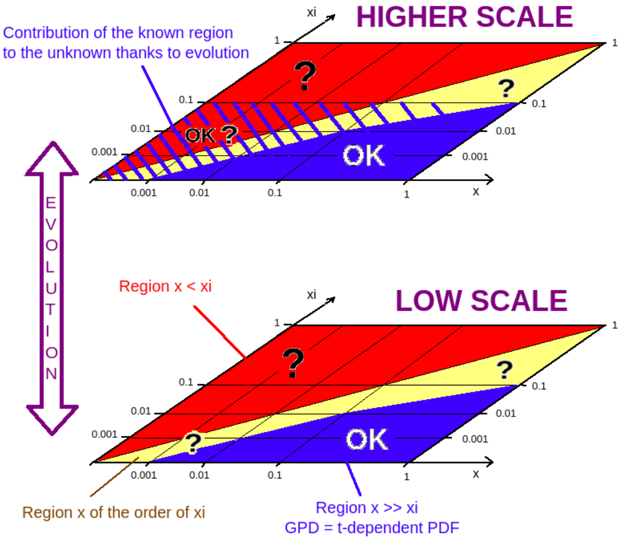
$$H(x, \xi, t, \mu^2) \approx H(x, 0, t, \mu^2) \quad \text{for } \xi \ll 1 \text{ even if } x \approx \xi? \tag{41}$$

Because significant asymmetry between incoming and outgoing $(x + \xi \gg x - \xi)$ parton momentum means very different dynamics, materialized *e.g.* by a very different behavior under evolution.

No reason for the ξ dependence to be negligible even at very small ξ. Skewness ratios $\frac{H(x,x)}{H(x,0)}$ as large as 1.6 have been advocated at small x. [Frankfurt et al, 1998] [Shuvaev et al, 1999]



GPDs at small x_B

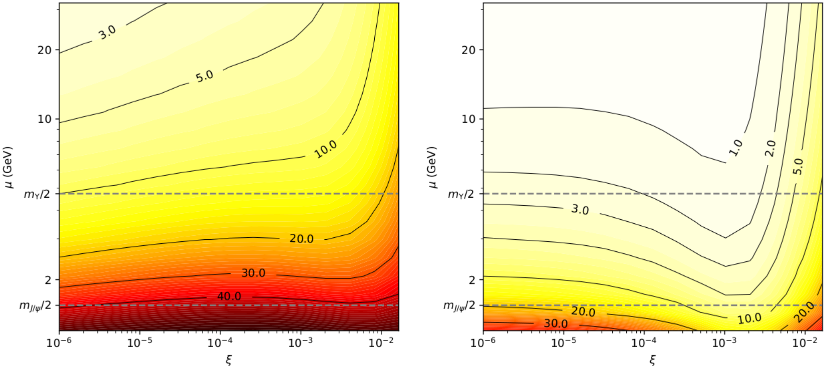


- Evolution displaces the GPD from the large x to the small x region
- Significant ξ dependence arises perturbatively in the small x and ξ region
- But how does it compare to the unknown ξ dependence at initial scale?

Obviously depends on the range of evolution, value of x and ξ, and profile of the known t-dependent PDF.

GPDs at small x_B

Example: working at $t = 0$, with the MMHT2014 PDF **[Harland-Lang et al, 2015]** at 1 GeV (**prior knowledge of t-dependent PDF**). We want to assess the dominance of the region $x \gg \xi$ at initial scale in the value of the GPD on the diagonal as scale increases.
Pessimistic assumption on unknown ξ dependence at $x = \xi$ for 1 GeV: 60%.



Uncertainty on the diagonal of the light sea quarks (left) and gluons (right) depending on $x = \xi$ and μ.
Stronger μ effect for gluons, divergence of PDFs at small x visible.

[HD et al, Phys.Rev.D 107 (2023) 11, 114019]