GLUE AND SEA INSIDE PROTON: A LIGHT-FRONT HAMILTONIAN APPROACH

Chandan Mondal

Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou, China

BLFQ Collaboration

Siqi Xu, Hongyao Yu, Bolang Lin, Zimin Zhu, . . . , Xingbo Zhao (IMP) and James P. Vary (ISU)

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Overview

Introduction

Basis Light-Front Quantization (BLFQ) to

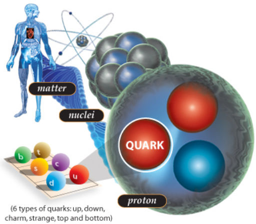
Proton : ($|qqq\rangle + |qqqg\rangle$)

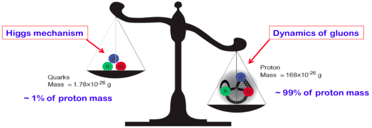
Proton : ($|qqq\rangle + |qqqg\rangle + |qqqq\bar{q}\rangle$)

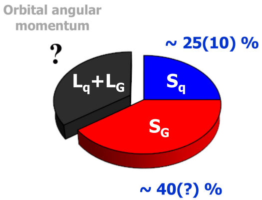
Conclusions

Fundamental Properties: Mass and Spin

- About 99% of the visible mass is contained within nuclei
- Nucleon: composite particles, built from nearly massless quarks ($\sim$ 1% of the nucleon mass) and gluons
- *How does 99% of the nucleon mass emerge?*
- Quantitative decomposition of *nucleon spin* in terms of quark and gluon degrees of freedom is not yet fully understood.
- *To address these fundamental issues $\rightarrow$ nature of the subatomic force between quarks and gluons, and the internal landscape of nucleons.*

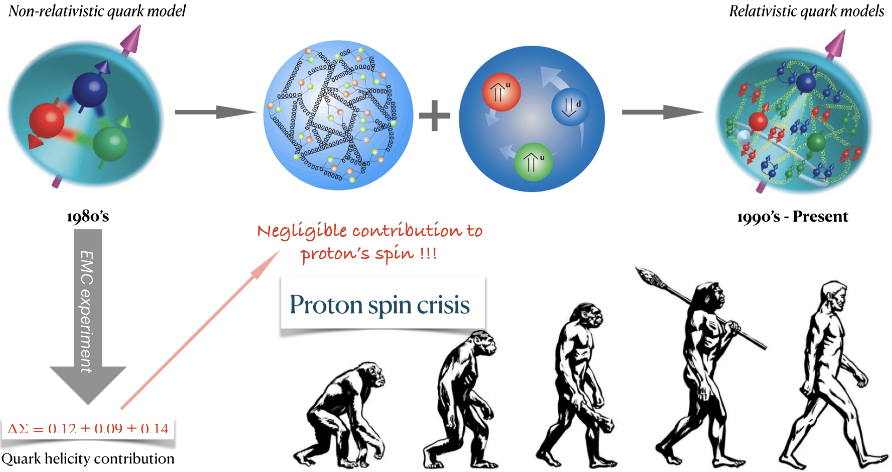




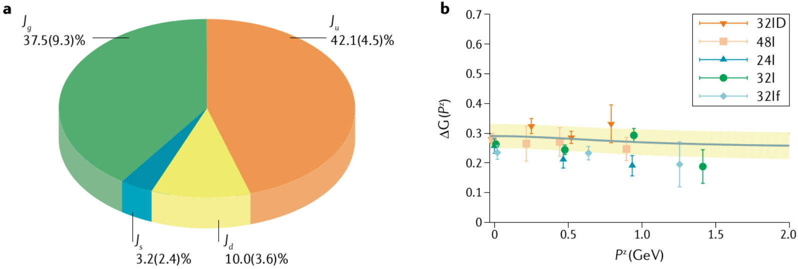


[1] Pictures (top to bottom) adopted from A. Signori, J. Qui, C. Lorce

Proton's Spin Evolution

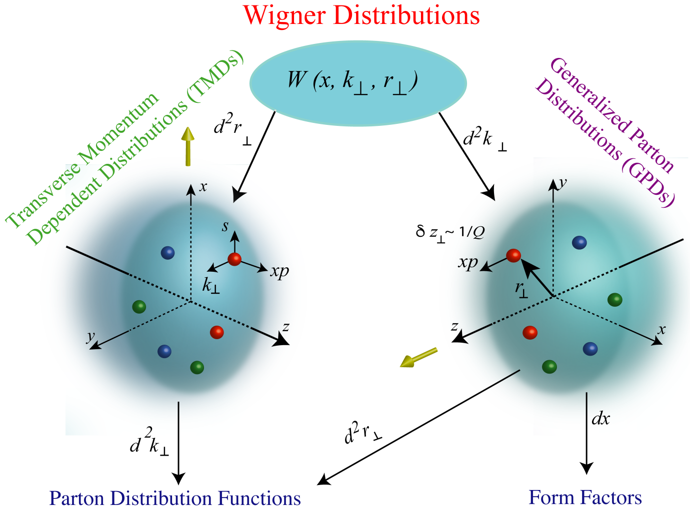


Spin sum rule	Formula	Terms	Characteristics
Frame independent (Ji)[30]	$\frac{1}{2}\Delta\Sigma + L_q^z + J_g = \frac{\hbar}{2}$	$\Delta\Sigma/2$ is the quark helicity L_q^z is the quark OAM J_g is the gluon contribution	The quark and gluon contributions, J_q and J_g, can be obtained from the GPD moments. A similar sum rule also works for the transverse angular momentum and has a simple parton interpretation
Infinite-momentum frame (Jaffe–Manohar)[31]	$\frac{1}{2}\Delta\Sigma + \Delta G + \ell_q + \ell_g = \frac{\hbar}{2}$	ΔG is the gluon helicity ℓ_q and ℓ_g are the quark and gluon canonical OAM, respectively	All terms have partonic interpretations; ℓ_q and ℓ_g are twist-three quantities. ΔG is measurable in experiments, including the RHIC spin and the EIC; ℓ_q and ℓ_g can be extracted from twist-three GPDs



[1] X. Ji, F. Yuan and Y. Zhao, Nature Reviews Physics 3, 65 (2021)
[2] Y.-B. Yang, R.S. Sufian, A. Alexandru et al., Phys. Rev. Lett. 118, 102001 (2017)

Hadron tomography



[1] Elke Aschenauer's, James P. Vary's talks: 18th Sept.

Basis Light-Front Quantization (BLFQ)

A computational framework for solving relativistic many-body bound state problems in quantum field theories

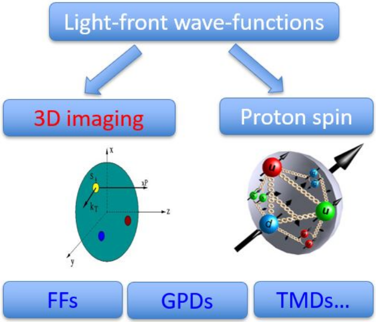
$$P^- P^+ |\Psi\rangle = M^2 |\Psi\rangle$$

- $P^- \equiv P^0 - P^3$: light-front Hamiltonian
- $P^+ \equiv P^0 + P^3$: longitudinal momentum
- $|\Psi\rangle$ mass eigenstate
- M^2 : mass squared eigenvalue for eigenstate $|\Psi\rangle$
- First-principle / effective Hamiltonian as input
- Evaluate observables

$$O \sim \langle \Psi | \hat{O} | \Psi \rangle$$

- direct access to light-front wavefunction of bound states

GOAL



[1] Vary, Honkanen, Li, Maris, Brodsky, Harindranath, *et. al.*, Phys. Rev. C 81, 035205 (2010).

- Fock expansion of baryonic bound states:

$$|\text{Baryon}\rangle = \psi_{(3q)}|qqq\rangle + \psi_{(3q+1g)}|qqqg\rangle + \psi_{(3q+q\bar{q})}|qqqq\bar{q}\rangle + \ldots ,$$

Solution proposed by BLFQ

Discrete basis and their direct product	Truncation
2D HO $\phi_{nm}(p^{\perp})$ in the transverse plane	$\sum_i (2n_i + \lvert m_i\rvert + 1) \leq N_{\max}$
Plane-wave in the longitudinal direction	$\sum_i k_i = K, \quad x_i = \frac{k_i}{K}$
Light-front helicity state for spin d.o.f.	$\sum_i (m_i + \lambda_i) = M_J$
$\alpha_i = (k_i, n_i, m_i, \lambda_i)$ $\lvert\alpha\rangle = \otimes_i \lvert\alpha_i\rangle$	Fock sector truncation

Large $N_{\max}$ and $K \to$ High UV cutoff & low IR cutoff

[1] Vary, Honkanen, Li, Maris, Brodsky, Harindranath, *et. al.*, Phys. Rev. C 81, 035205 (2010).

Nucleon within BLFQ

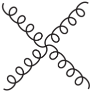
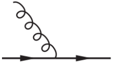
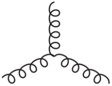
- The LF eigenvalue equation: $H_{\rm eff}|\Psi\rangle = M^2|\Psi\rangle$

$$H_{\rm eff} = \sum_a \frac{\vec{p}_{\perp a}^{\,2} + m_a^2}{x_a} + \frac{1}{2}\sum_{a\neq b}\kappa^4\left[x_a x_b(\vec{r}_{\perp a} - \vec{r}_{\perp b})^2 - \frac{\partial_{x_a}(x_a x_b \partial_{x_b})}{(m_a+m_b)^2}\right]$$
$$+ \frac{1}{2}\sum_{a\neq b}\frac{C_F 4\pi\alpha_s}{Q_{ab}^2}\bar{u}_{s'_a}(k'_a)\gamma^\mu u_{s_a}(k_a)\bar{u}_{s'_b}(k'_b)\gamma^\nu u_{s_b}(k_b) g_{\mu\nu}$$

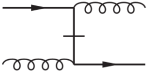
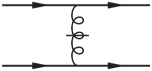
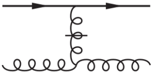
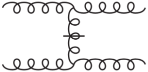
Publications:

- Mondal et al., Phys. Rev. D 102, 016008 (2020) : Form Factors, PDFs,...
- Xu et al., Phys. Rev. D 104, 094036 (2021) : Nucleon structure,...
- Liu et al., Phys. Rev. D 105, 094018 (2022) : Angular Momentum,...
- Kaur et al., arXiv:2307.09869 (2023) : Chiral-odd GPDs,...
- Hu et al., Phys. Lett. B 2022, 137360 (2022) : TMDs,...
- Peng et al., Phys. Rev. D 106, 114040 (2022) : Λ and Λ_c PDFs,...
- Zhu et al., Phys. Rev. D 108, 036009 (2023) : Λ and Λ_c TMDs,...
- Nair et al., under preparation : GFFs,...
- Peng et al., under preparation : Double parton correlations,...

Light-Front QCD with Light-Cone Gauge ($A^+ = 0$)

$$
\begin{aligned}
\hat{P}^-_{\mathrm{LFQCD}} = & \frac{1}{2}\int \mathrm{d}x^-\mathrm{d}^2x^\perp \;\; \overline{\psi}\gamma^+\frac{(i\partial^\perp)^2+m^2}{i\partial^+}\psi + A^{ia}(i\partial^\perp)^2A^{ia} \\
& + g_s\int \mathrm{d}x^-\mathrm{d}^2x^\perp \; \overline{\psi}\gamma_\mu A^{\mu a}T^a\psi \\
& + \frac{g_s^2}{2}\int \mathrm{d}x^-\mathrm{d}^2x^\perp \; \overline{\psi}\gamma_\mu A^{\mu a}T^a\frac{\gamma^+}{i\partial^+}\left(\gamma_\nu A^{\nu b}T^b\psi\right) \\
& + \frac{g_s^2}{2}\int \mathrm{d}x^-\mathrm{d}^2x^\perp \; \overline{\psi}\gamma^+T^a\psi\frac{1}{(i\partial^+)^2}\left(\overline{\psi}\gamma^+T^a\psi\right) \\
& - g_s^2\int \mathrm{d}x^-\mathrm{d}^2x^\perp \; if^{abc}\,\overline{\psi}\gamma^+T^c\psi\frac{1}{(i\partial^+)^2}\left(i\partial^+A^{\mu a}A^b_\mu\right) \\
& + g_s\int \mathrm{d}x^-\mathrm{d}^2x^\perp \; if^{abc}\,i\partial^\mu A^{\nu a}A^b_\mu A^c_\nu \\
& + \frac{g_s^2}{2}\int \mathrm{d}x^-\mathrm{d}^2x^\perp \; if^{abc}\,if^{ade}\,i\partial^+A^{\mu b}A^c_\mu\frac{1}{(i\partial^+)^2}\left(i\partial^+A^{\nu d}A^e_\nu\right) \\
& - \frac{g_s^2}{4}\int \mathrm{d}x^-\mathrm{d}^2x^\perp \; if^{abc}\,if^{ade}A^{\mu b}A^{\nu c}A^d_\mu A^e_\nu.
\end{aligned}
$$

[1] S.J. Brodsky, H.C. Pauli, S.S. Pinsky, Phys. Rep. 301, 299-486 (1998).

Proton with One Dynamical Gluon

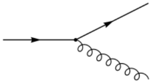
$$P^+P^-|\Psi\rangle = M^2|\Psi\rangle$$

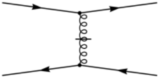
$$|\text{proton}\rangle = \psi_{uud}|uud\rangle + \psi_{uudg}|uudg\rangle$$

$$P^- = P^-_{\text{QCD}} + P^-_C$$

QCD Interaction:

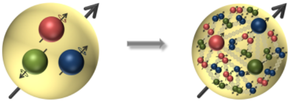
$$P^-_{\text{QCD}} = \int dx^- d^2x^\perp \Big\{ \frac{1}{2}\bar{\psi}\gamma^+ \frac{m_0^2 + (i\partial^\perp)^2}{i\partial^+}\psi - \frac{1}{2}A^i_a\left[m_g^2 + (i\partial^\perp)^2\right]A^i_a + g_s\bar{\psi}\gamma_\mu T^a A^\mu_a\psi + \frac{1}{2}g_s^2\bar{\psi}\gamma^+T^a\psi\frac{1}{(i\partial^+)^2}\bar{\psi}\gamma^+T^a\psi \Big\},$$





Confinement only in leading Fock:

$$P^-_C P^+ = \frac{\kappa^4}{2}\sum_{i\neq j}\left\{ \vec{r}^{\,2}_{ij\perp} - \frac{\partial_{x_i}(x_i x_j \partial_{x_j})}{(m_i+m_j)^2} \right\}$$



Parameters:

Truncation: Nmax=9, K=16.5
HO parameters: b=0.7GeV, b_{inst}=3GeV

m_u	m_d	m_g	κ	m_f	g
0.31GeV	0.25GeV	0.50GeV	0.54GeV	1.80GeV	2.40

[1] S. Xu, CM, X. Zhao, Y. Li, J. P. Vary, 2209.08584 [hep-ph].
[2] Brodsky, Teramond, Dosch and Erlich, Phys. Rep. 584, 1 (2015).
[3] Li, Maris, Zhao and Vary, Phys. Lett. B (2016).

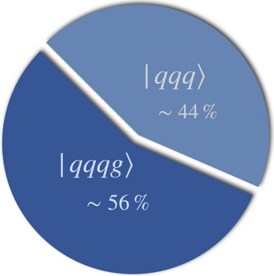
Fock Sector Decomposition

Fock expansion:

$$|\,\text{Proton}\rangle = a\,|\,uud\rangle + b\,|\,uudg\rangle + \ldots$$

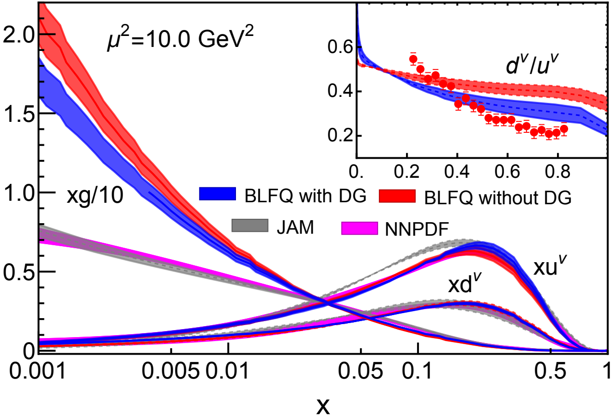
Light-front effective Hamiltonian :

$$H_{\text{eff}} = \sum_a \frac{\vec{p}_{\perp a}^{\,2} + m_a^2}{x_a} + H_{\text{confinement}} + H_{\text{vertex}} + H_{\text{inst}}$$



[1] Brodsky, Pauli, and Pinsky, Phys. Rep. 301, 299 (1998).
[2] Siqi Xu's talk on 20th 9:30 AM.

Unpolarized PDFs

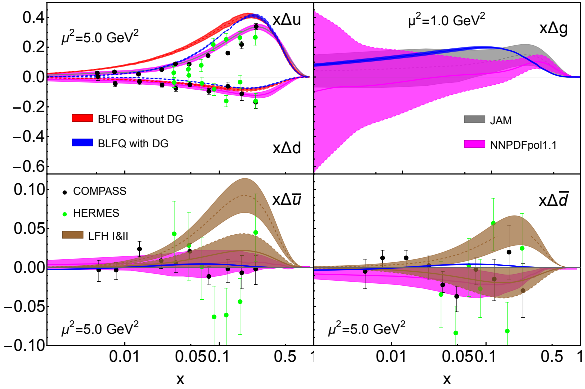


Including dynamical gluon (DG):

- Model scale : $\mu_0^2 = 0.195$ GeV^2 $\Rightarrow$ $\mu_0^2 = 0.23 - 0.25$ GeV^2
- Gluon distribution: closer to global fits.

[1] S. Xu, CM, X. Zhao, Y. Li, J. P. Vary, 2209.08584 [hep-ph].

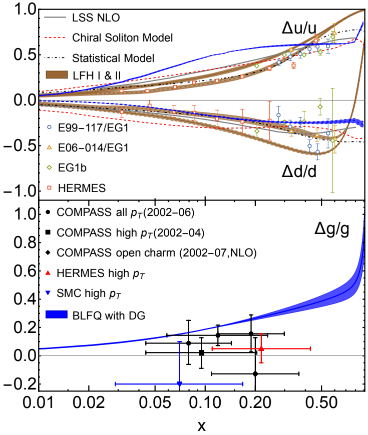
Helicity PDFs



- Quark spin: $\frac{1}{2}\Sigma_u = 0.438 \pm 0.004$, $\frac{1}{2}\Delta\Sigma_d = -0.080 \pm 0.002$.
- Gluon spin: $\Delta G = 0.131 \pm 0.003$, sizeable to the proton spin.
- PHENIX Collaboration: $\Delta G^{[0.02,0.3]} = 0.2 \pm 0.1$.
- Sea quarks: solely generated from the QCD evolution.

[1] BLFQ: 2209.08584 [hep-ph], LFH: 124 (2020), 082003; PHENIX: PRL 103 (2009) 012003].

Helicity Asymmetries



- Experimentally, the expected increase of $\Delta u/u$ is observed.
- For d quark: remains negative in the experimentally covered region.
- Global analyses favor negative values of $\Delta d/d$ at large-x.

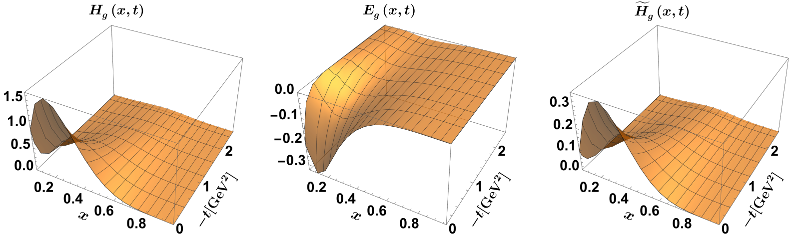
[1] S. Xu, CM, X. Zhao, Y. Li, J. P. Vary, 2209.08584 [hep-ph].

Gluon GPDs

$$F^g(x,\Delta;\lambda,\lambda') = \frac{1}{2P^+}\,\bar{u}(p',\lambda')\left(\gamma^+ H^g(x,\xi,t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2M} E^g(x,\xi,t)\right)u(p,\lambda)\,,$$

$$\tilde{F}^g(x,\Delta;\lambda,\lambda') = \frac{1}{2P^+}\,\bar{u}(p',\lambda')\left(\gamma^+\gamma_5 \tilde{H}^g(x,\xi,t) + \frac{\Delta^+\gamma_5}{2M} \tilde{E}^g(x,\xi,t)\right)u(p,\lambda)\,.$$

Non-skewed GPDs



- Total Angular Momentum: $J = \frac{1}{2}\int \mathrm{d}x\, x[H(x,0) + E(x,0)]$; $J_g = 0.066$, 13.2% of the proton TAM.

[1] B. Lin, S. Nair, S.Xu, CM, X. Zhao, J. P. Vary, 2308.08275 [hep-ph].

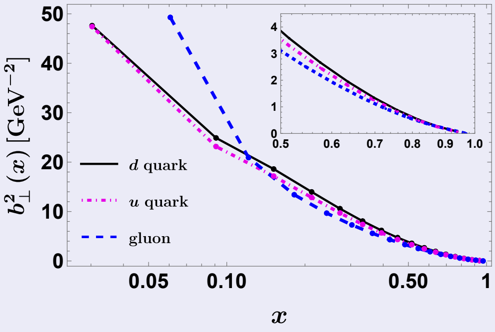
x-Dependent Squared Radius

$$\langle b_\perp^2 \rangle^i(x) = \frac{\int d^2\vec{b}_\perp b_\perp^2 H^i(x, b_\perp)}{\int d^2\vec{b}_\perp H^i(x, b_\perp)},$$

- Transverse squared radius:

$$\langle b_\perp^2 \rangle = \sum_i e_q \int_0^1 \mathrm{d}x f^i(x) \langle b_\perp^2 \rangle^i(x)$$

- BLFQ: $\langle b_\perp^2 \rangle = 0.47 \pm 0.04\ \mathrm{fm}^2$
- Experimental data [2]: $\langle b_\perp^2 \rangle_{\exp} = 0.43 \pm 0.01\ \mathrm{fm}^2$



[1] B. Lin, S. Nair, S.Xu, CM, X. Zhao, J. P. Vary, 2308.08275 [hep-ph].
[2] R. Dupre, M. Guidal and M. Vanderhaeghen, PRD 95, 011501 (2017).

BLFQ Predictions for Spin Decomposition

Quark and gluon helicities :

$$\Delta\Sigma_q = \int \mathrm{d}x\, \Delta q(x)$$

$$\Delta\Sigma_g = \int \mathrm{d}x\, \Delta G(x)$$

Total AM :

$$J_i = \int \mathrm{d}x\, x\, [H_i(x,0,0) + E_i(x,0,0)]$$

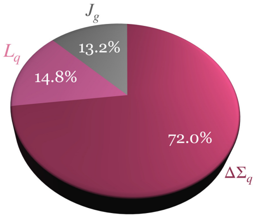
Kinetic OAM :

$$L_q = \int \mathrm{d}x\, [x\, \{H_q(x,0,0)+E_q(x,0,0)\} - \widetilde{H}_q(x,0,0)]$$

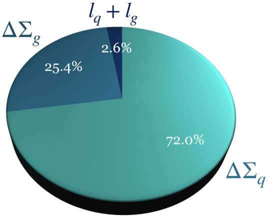
Canonical OAM :

$$l_i^z = -\int \mathrm{d}x\, \mathrm{d}^2\vec{p}_\perp\, \frac{\vec{p}_\perp^{\,2}}{M^2}\, F_{1,4}^i(x,0,\vec{p}_\perp^{\,2},0,0)$$

(a) Kinetic



(b) Canonical



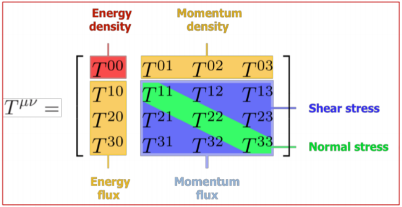
[1] S. Xu, CM, X. Zhao, Y. Li, J. P. Vary, 2209.08584 [hep-ph].

Nucleon Gravitational Form Factors

Nucleon scattering by the classical gravitational field is described by the gravitational (energy momentum tensor) form factors (GFFs).

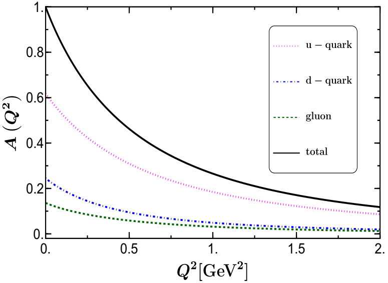
$$\langle P'|T_i^{\mu\nu}(0)|P\rangle = \bar{U}'\Big[-B_i(q^2)\frac{\bar{P}^\mu\bar{P}^\nu}{M} + (A_i(q^2)+B_i(q^2))\frac{1}{2}(\gamma^\mu\bar{P}^\nu + \gamma^\nu\bar{P}^\mu)$$
$$+ C_i(q^2)\frac{q^\mu q^\nu - q^2 g^{\mu\nu}}{M} + \bar{C}_i(q^2) M g^{\mu\nu}\Big]U$$

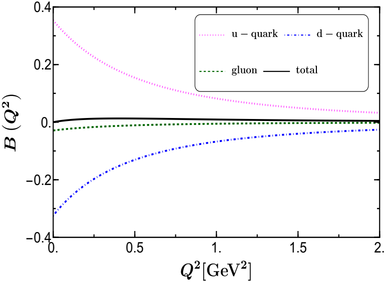
- Contain fundamental information about various mechanical properties.
- $A(Q^2)$ and $B(Q^2)$: T^{++} component
- $C(Q^2)$ and $\bar{C}(Q^2)$: T^{ij} components



[1] Sreeraj Nair talk today 4 PM

Preliminary Results : $A(Q^2)$ and $B(Q^2)$





- Spin sum rule: $J^i = \frac{1}{2}\left(A^i(0) + B^i(0)\right)$

$$\sum_i A^i(0) = 1 \text{ and } \sum_i B^i(0) = 0$$

$Q^2 = 0$	$i = u$	$i = d$	$i = g$	$u + d + g$
A_i	0.619	0.245	0.136	1.00
B_i	0.354	-0.325	-0.0289	~ 0

[1] S. Nair, CM, S. Xu, X. Zhao, J. P. Vary, in preparation.
[2] Sreeraj Nair talk today 4 PM

TMDs of Spin-1/2 Target

Gluon TMDs correlator :

$$\Phi^{g[ij]}(x, \vec{k}_\perp; S) = \frac{1}{xP^+}\int \frac{dz^-}{2\pi}\frac{d^2\vec{z}_\perp}{(2\pi)^2} e^{ikz}\langle P; S|F_a^{+j}(0)\mathcal{W}_{+\infty,ab}(0;z)F_b^{+i}(z)|P;S\rangle\,|_{z^+=0^+}$$

Parametrization

$$\Phi^g(x, \vec{k}_\perp; S) = \delta_\perp^{ij}\Phi^{g[ij]}(x, \vec{k}_\perp; S)$$
$$= f_1^g(x, \vec{k}_\perp^2) - \frac{\epsilon_\perp^{ij}k_\perp^i S_\perp^j}{M} f_{1T}^{\perp g}(x, \vec{k}_\perp^2)$$

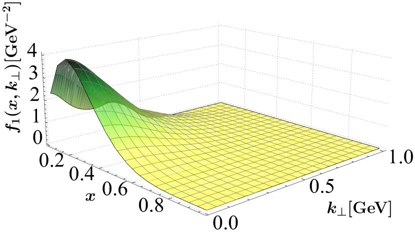
$$\tilde{\Phi}^g(x, \vec{k}_\perp; S) = i\epsilon_\perp^{ij}\Phi^{g[ij]}(x, \vec{k}_\perp; S)$$
$$= S^3 g_{1L}^g(x, \vec{k}_\perp^2) + \frac{\vec{k}_\perp\cdot\vec{S}_\perp}{M} g_{1T}^g(x, \vec{k}_\perp^2)$$

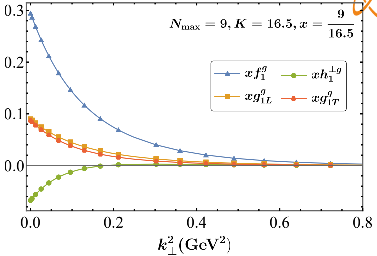
$$\Phi_T^{g,ij}(x, \vec{k}_\perp; S) = -\hat{S}\Phi^{g[ij]}(x, \vec{k}_\perp; S)$$
$$= -\frac{\hat{S}k_\perp^i k_\perp^j}{2M^2} h_1^{\perp g}(x, \vec{k}_\perp^2) + \frac{S^3\hat{S}k_\perp^i\epsilon_\perp^{jk}k_\perp^k}{2M^2} h_{1L}^{\perp g}(x, \vec{k}_\perp^2)$$
$$+ \frac{\hat{S}k_\perp^i\epsilon_\perp^{jk}S_\perp^k}{2M}\left(h_{1T}^g(x, \vec{k}_\perp^2) + \frac{\vec{k}_\perp^2}{2M^2} h_{1T}^{\perp g}(x, \vec{k}_\perp^2)\right)$$
$$+ \frac{\hat{S}k_\perp^i\epsilon_\perp^{jk}(2k_\perp^k\vec{k}_\perp\cdot\vec{S}_\perp - S_\perp^k\vec{k}_\perp^2)}{4M^3} h_{1T}^{\perp g}(x, \vec{k}_\perp^2),$$

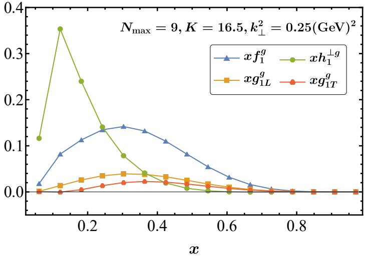
	PARTON SPIN		
GLUONS (TARGET SPIN)	$-g_T^{\alpha\beta}$	$\varepsilon_T^{\alpha\beta}$	$p_T^{\alpha\beta}, \ldots$
U	f_1^g		$h_1^{\perp g}$
L		g_1^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	h_1^g $h_{1T}^{\perp g}$

[1] A. Accardi *et al.*, Eur.Phys.J.A 52 (2016) 9, 268.
[2] Meißner, et. al. PRD D 76 (2007), 034002.

Gluon TMDs







- Positivity bounds

$$f_1^g(x, \boldsymbol{k}_\perp^2) > 0, \quad f_1^g(x, \boldsymbol{k}_\perp^2) \geq |g_{1L}^g(x, \boldsymbol{k}_\perp^2)|,$$

$$f_1^g(x, \boldsymbol{k}_\perp^2) \geq \frac{|\boldsymbol{k}_\perp|}{M} |g_{1T}^g(x, \boldsymbol{k}_\perp^2)|,$$

$$f_1^g(x, \boldsymbol{k}_\perp^2) \geq \frac{|\boldsymbol{k}_\perp|^2}{2M^2} |h_1^{\perp g}(x, \boldsymbol{k}_\perp^2)|$$

- Satisfies Mulders-Rodrigues relations

[1] Hongyao Yu, *et. al.* in preparation

Gluon TMDs

- Small-x limit

$$\lim_{x\to 0} \frac{\int d\boldsymbol{k}_\perp^2 |\boldsymbol{k}_\perp^2| h_1^{\perp g}(x,\boldsymbol{k}_\perp^2)}{2M^2 \int d\boldsymbol{k}_\perp^2 f_1^g(x,\boldsymbol{k}_\perp^2)} = 1.$$

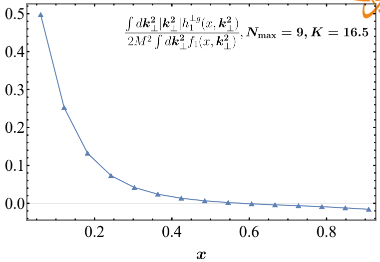
- Helicity asymmetry:

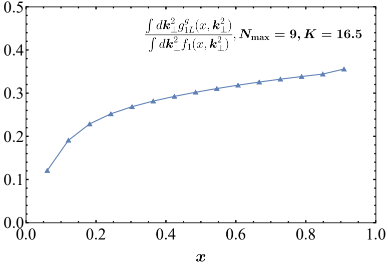
$$\lim_{x\to 0} \frac{\int d\boldsymbol{k}_\perp^2 g_{1L}^g(x,\boldsymbol{k}_\perp^2)}{\int d\boldsymbol{k}_\perp^2 f_1^g(x,\boldsymbol{k}_\perp^2)} = 0,$$

$$\lim_{x\to 1} \frac{\int d\boldsymbol{k}_\perp^2 g_{1L}^g(x,\boldsymbol{k}_\perp^2)}{\int d\boldsymbol{k}_\perp^2 f_1^g(x,\boldsymbol{k}_\perp^2)} = 1$$

.

- With larger truncation K, satisfies the limiting cases.

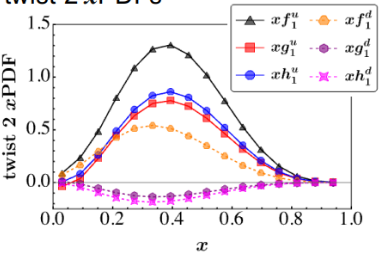




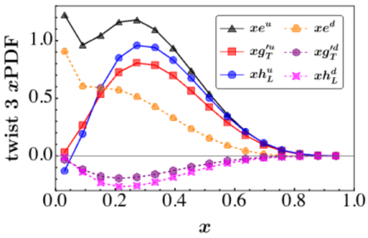
[1] Hongyao Yu, *et. al.* in preparation

xPDFs: Twist-2 vs Twist-3

twist-2 xPDFs

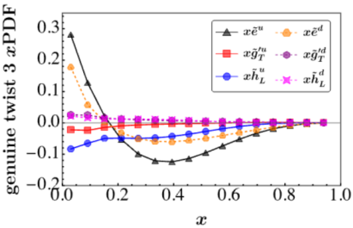


twist-3 xPDFs



$$\int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^2} f(x, k_\perp) = f(x)$$

genuine twist-3 xPDFs



Twist-3 PDFs:

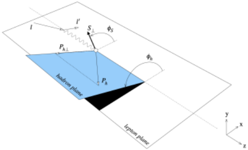
- more concentrating in small x
- similar magnitude to twist-2 PDFs

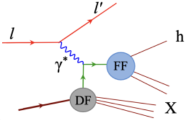
[1] Zhimin Zhu, *et. al.* in preparation

Semi-inclusive DIS

$$\frac{d\sigma}{dx\,dy\,dz\,dP_{hT}^2\,d\varphi_h\,d\psi} = \left[\frac{\alpha}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\right]\left(F_{UU,T}+\varepsilon F_{UU,L}\right)\times$$

$$\left\{\begin{aligned}
&1+\cos\phi_h\left(\sqrt{2\varepsilon(1+\varepsilon)}A_{UU}^{\cos\phi_h}\right)+\cos 2\phi_h\left(\varepsilon A_{UU}^{\cos 2\phi_h}\right)\\
&+\lambda\sin\phi_h\left(\sqrt{2\varepsilon(1-\varepsilon)}A_{LU}^{\sin\phi_h}\right)\\
&+S_L\left[\sin\phi_h\left(\sqrt{2\varepsilon(1+\varepsilon)}A_{UL}^{\sin\phi_h}\right)+\sin 2\phi_h\left(\varepsilon A_{UL}^{\sin 2\phi_h}\right)\right]\\
&+S_L\lambda\left[\sqrt{1-\varepsilon^2}A_{LL}+\cos\phi_h\left(\sqrt{2\varepsilon(1-\varepsilon)}A_{LL}^{\cos\phi_h}\right)\right]\\
&+S_T\left[\begin{aligned}&\sin(\phi_h-\phi_S)\left(A_{UT}^{\sin(\phi_h-\phi_S)}\right)\\&+\sin(\phi_h+\phi_S)\left(\varepsilon A_{UT}^{\sin(\phi_h+\phi_S)}\right)\\&+\sin(3\phi_h-\phi_S)\left(\varepsilon A_{UT}^{\sin(3\phi_h-\phi_S)}\right)\\&+\sin\phi_S\left(\sqrt{2\varepsilon(1+\varepsilon)}A_{UT}^{\sin\phi_S}\right)\\&+\sin(2\phi_h-\phi_S)\left(\sqrt{2\varepsilon(1+\varepsilon)}A_{UT}^{\sin(2\phi_h-\phi_S)}\right)\end{aligned}\right]\\
&+S_T\lambda\left[\begin{aligned}&\cos(\phi_h-\phi_S)\left(\sqrt{(1-\varepsilon^2)}A_{LT}^{\cos(\phi_h-\phi_S)}\right)\\&+\cos\phi_S\left(\sqrt{2\varepsilon(1-\varepsilon)}A_{LT}^{\cos\phi_S}\right)\\&+\cos(2\phi_h-\phi_S)\left(\sqrt{2\varepsilon(1-\varepsilon)}A_{LT}^{\cos(2\phi_h-\phi_S)}\right)\end{aligned}\right]
\end{aligned}\right\}$$





Factorization Theorem:

$A_{UT}^{\sin(\phi_h-\phi_S)} \propto f_{1T}^{\perp} \otimes D_1$ Twist-2

$A_{UT}^{\sin(\phi_h+\phi_S)} \propto h_1 \otimes H_1^{\perp}$

$A_{UT}^{\sin(3\phi_h-\phi_S)} \propto h_{1T}^{\perp} \otimes H_1^{\perp}$ Twist-3

$A_{UT}^{\sin(\phi_S)} \propto \frac{M}{Q}(f_T \otimes D_1 + h_1 \otimes H_1^{\perp} + \cdots)$

$A_{UT}^{\sin(2\phi_h-\phi_S)} \propto \frac{M}{Q}(h_T \otimes H_1^{\perp} + h_T^{\perp} H_1^{\perp} + \cdots)$

$A_{LT}^{\cos(\phi_h-\phi_S)} \propto g_{1T} \otimes D_1$

$A_{LT}^{\cos(\phi_S)} \propto \frac{M}{Q}(g_T \otimes D_1 + e_T \otimes H_1^{\perp} + \cdots)$

$A_{LT}^{\cos(2\phi_h-\phi_S)} \propto \frac{M}{Q}(e_T \otimes H_1^{\perp} + e_T^{\perp} \otimes H_1^{\perp} + \cdots)$

……

[1] Bacchetta, et al, JHEP 02 (2007) 093

[1] Zhimin Zhu, *et. al.* in preparation

Spin asymmetry in SIDIS process

twist-2 $F_{UU,T} = \mathcal{C}[f_1 D_1]$ $F_{UU,L} = 0$ $F_{LT}^{\cos(\phi_h-\phi_S)} = \mathcal{C}\Big[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M} g_{1T} D_1\Big]$ $\hat{\boldsymbol{h}} = \frac{\boldsymbol{P}_{h\perp}}{|\boldsymbol{P}_{h\perp}|}$

twist-3 $$F_{LT}^{\cos\phi_S} = \frac{2M}{Q}\mathcal{C}\Big\{-\Big(xg_T D_1 + \frac{M_h}{M}h_1\frac{\tilde{E}}{z}\Big) + \frac{\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\Big[\Big(xe_T H_1^\perp - \frac{M_h}{M}g_{1T}\frac{\tilde{D}^\perp}{z}\Big) + \Big(xe_T^\perp H_1^\perp + \frac{M_h}{M}f_{1T}^\perp\frac{\tilde{G}^\perp}{z}\Big)\Big]\Big\}$$

$$\sim -\frac{2M}{Q}\mathcal{C}[xg_T D_1]$$ supression factor

EOM relation: $$xg_T = x\tilde{g}_T - \frac{p_T^2}{2M^2}g_{1T} + \frac{m}{M}h_1$$

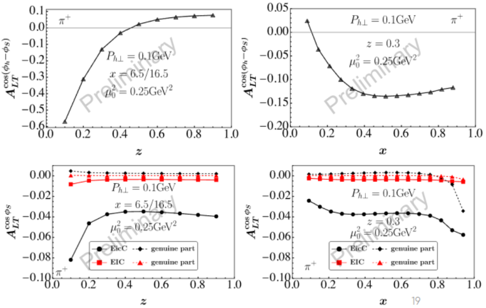
Kinematic parameters : M~1 GeV, $Q_{\rm EicC}$ ~ 10 GeV, $Q_{\rm EIC}$ ~ 100 GeV

Spin asymmetries :

twist-2 : $$A_{LT}^{\cos(\phi_h-\phi_S)} = \frac{F_{LT}^{\cos(\phi_h-\phi_S)}}{F_{UU,T}+\varepsilon F_{UU,L}}$$

twist-3 : $$A_{LT}^{\cos\phi_S} = \frac{F_{LT}^{\cos\phi_S}}{F_{UU,T}+\varepsilon F_{UU,L}}$$

- The twist-3 DSA, $A_{LT}^{\cos\phi_S}$, is smaller than the twist-2 DSA, $A_{LT}^{\cos(\phi_h-\phi_S)}$.
- Twist-3 spin asymmetries may be easier to measure in EicC than in EIC.



[1] Zhimin Zhu, *et. al.* in preparation

Effective Hamiltonian with Dynamical Gluon and Sea Quarks

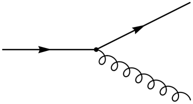
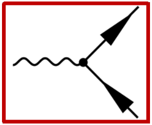
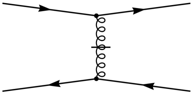
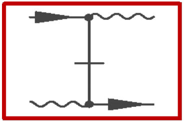
Fock expansion:

$$|\,\text{Proton}\rangle = a\,|\,uud\rangle + b\,|\,uudg\rangle + c_1\,|\,uudu\bar{u}\rangle + c_2\,|\,uudd\bar{d}\rangle + c_3\,|\,uuds\bar{s}\rangle + \ldots$$

Light-front QCD Hamiltonian :

$$H_{\text{eff}} = \sum_a \frac{\vec{p}^{\,2}_{\perp a} + m_a^2}{x_a} + \cancel{H_{\text{confinement}}} + H_{\text{vertex}} + H_{\text{inst}}$$

$$H_{\text{vertex}} + H_{\text{inst}} = g_s \bar{\psi}\gamma_\mu T^a A_a^\mu \psi + \frac{1}{2} g_s^2 \bar{\psi}\gamma^+ T^a \psi \frac{1}{(i\partial^+)^2} \bar{\psi}\gamma^+ T^a \psi$$
$$+ \frac{1}{2} g_s^2 \bar{\psi}\gamma^\mu A_\mu \frac{\gamma^+}{(i\partial^+)} A_\nu \gamma^\nu \psi$$

[1] Brodsky, Pauli, and Pinsky, Phys. Rep. 301, 299 (1998).

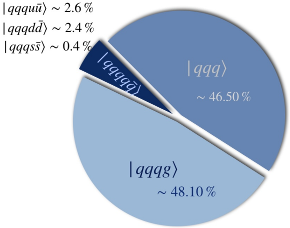
[2] Siqi Xu's talk on 20th 9:30 AM.

Fock Sector Decomposition

$$|\,\text{Proton}\rangle = a\,|\,uud\rangle + b\,|\,uudg\rangle + \ldots$$

$$\Downarrow$$

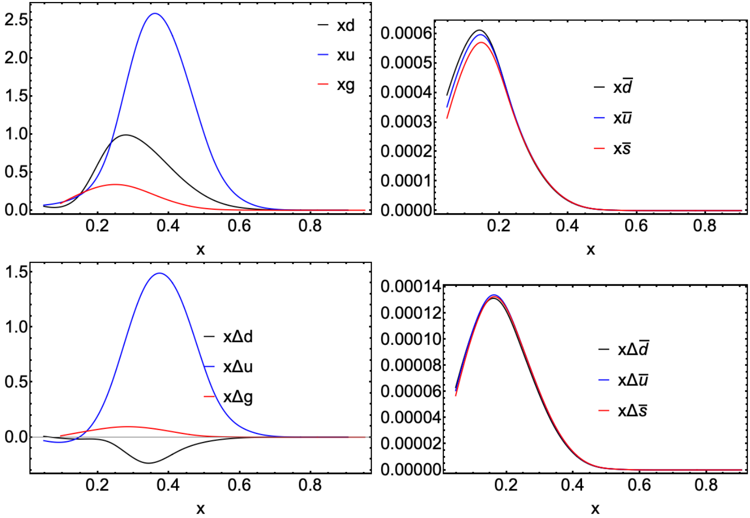
$$|\,\text{Proton}\rangle = a\,|\,uud\rangle + b\,|\,uudg\rangle$$
$$+\, c_1\,|\,uudu\bar{u}\rangle + c_2\,|\,uudd\bar{d}\rangle + c_3\,|\,uuds\bar{s}\rangle + \ldots$$



[1] S. Xu, CM, X. Zhao, Y. Li, J. P. Vary, 2209.08584 [hep-ph].
[0] Siqi Xu, *et. al.*, work in progress.

xPDFs

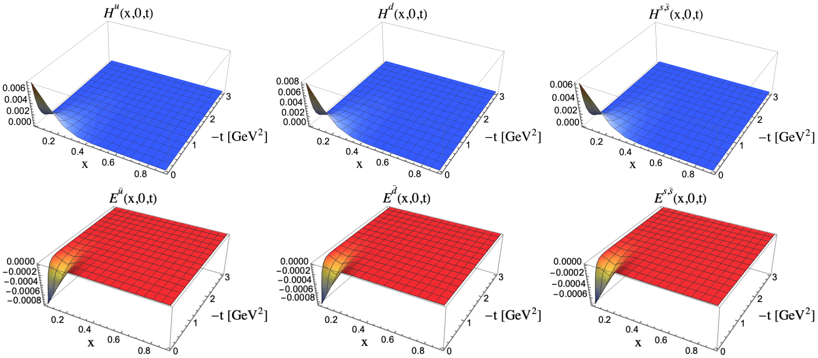
Preliminary results



0 Siqi Xu, CM, *et. al.*, in preparation

Sea Quark GPDs

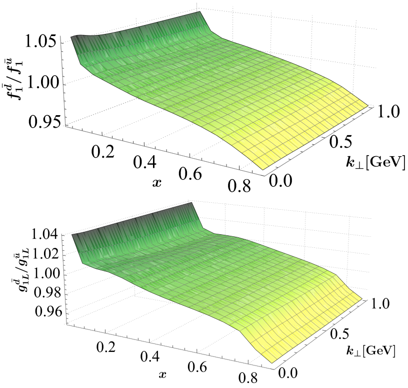
Preliminary results



[0] Siqi Xu, CM, *et. al.*, in preparation

Sea Quark TMDs Asymmetries

Preliminary results



[0] Hongyao Yu, *et. al.*, in preparation

Conclusions

- Basis Light-front Quantization : A non-perturbative approach based on light-front QCD Hamiltonian
- LF Hamiltonian ⇒ Wavefunctions ⇒ Observables.
- Explored gluon and sea quarks within proton based on $|qqq\rangle + |qqqg\rangle$ and $|qqq\rangle + |qqqg\rangle + |qqqq\bar{q}\rangle$, respectively.
- Provides good description of data/global fits for various observables.
- With one dynamical gluon, the quark spin contributes 70%; the gluon spin plays a substantial role (26%) in understanding the nucleon spin.

Outlook

- Include three-gluon and four-gluon interaction in the Hamiltonian.
- *This is not a complete picture . . . long way to go*.

Enormous amount of possibilities with future EICs . . . . . . Thank You

Overview of TMDs for Spin-1/2 Target

Quark correlator

$$\Phi_q^{[\Gamma]}\left(P, S; x = \frac{k^+}{P^+}, \vec{k}_\perp\right) = \frac{1}{2}\int \frac{dz^- dz^\perp}{2(2\pi)^3} e^{ik\cdot z} \langle P, S|\, \bar{\Psi}_q(0)\Gamma \mathcal{W}(0^\perp, z^\perp)\Psi_q(z)\, |P, S\rangle \Big|_{z^+=0},$$

Parameterization:

8 twist-2 TMDs:

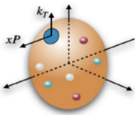
6 T-even terms

2 T-odd terms

$$\Phi^{[\gamma^+]} = f_1 - \frac{\epsilon_\perp^{ij} k_\perp^i S_\perp^j}{M} f_{1T}^\perp,$$

$$\Phi^{[\gamma^+\gamma^5]} = \Lambda g_{1L} + \frac{k_\perp \cdot \boldsymbol{S}_\perp}{M} g_{1T},$$

$$\Phi^{[i\sigma^{j+}\gamma^5]} = S_\perp^j h_1 + \Lambda \frac{k_\perp^j}{M} h_{1L}^\perp + S_\perp^i \frac{2k_\perp^i k_\perp^j - (k_\perp)^2 \delta^{ij}}{2M^2} h_{1T}^\perp + \frac{\epsilon_\perp^{ji} k_\perp^i}{M} h_1^\perp,$$



16 twist-3 TMDs:

8 T-even terms

8 T-odd terms

$$\Phi^{[1]} = \frac{M}{P^+}\left[e - \frac{\epsilon_T^{\rho\sigma} k_{\perp\rho} S_{T\sigma}}{M} e_T^\perp\right],$$

$$\Phi^{[i\gamma_5]} = \frac{M}{P^+}\left[S_L e_L - \frac{k_\perp \cdot S_T}{M} e_T\right],$$

$$\Phi^{[\gamma^\alpha]} = \frac{M}{P^+}\left[-\epsilon_T^{\alpha\rho} S_{T\rho} f_T - S_L \frac{\epsilon_T^{\alpha\rho} k_{\perp\rho}}{M} f_L^\perp - \frac{k_\perp^\alpha k_\perp^\rho - \frac{1}{2}k_\perp^2 g_T^{\alpha\rho}}{M^2} \epsilon_{T\rho\sigma} S_T^\sigma f_T^\perp + \frac{k_\perp^\alpha}{M} f^\perp\right],$$

$$\Phi^{[\gamma^\alpha\gamma_5]} = \frac{M}{P^+}\left[S_T^\alpha g_T + S_L \frac{k_\perp^\alpha}{M} g_L^\perp - \frac{k_\perp^\alpha k_\perp^\rho - \frac{1}{2}k_\perp^2 g_T^{\alpha\rho}}{M^2} S_{T\rho} g_T^\perp - \frac{\epsilon_T^{\alpha\rho} k_{\perp\rho}}{M} g^\perp\right],$$

$$\Phi^{[i\sigma^{\alpha\beta}\gamma_5]} = \frac{M}{P^+}\left[\frac{S_T^\alpha k_\perp^\beta - k_\perp^\alpha S_T^\beta}{M} h_T^\perp - \epsilon_T^{\alpha\beta} h\right],$$

$$\Phi^{[i\sigma^{+-}\gamma_5]} = \frac{M}{P^+}\left[S_L h_L - \frac{k_\perp \cdot S_T}{M} h_T\right].$$

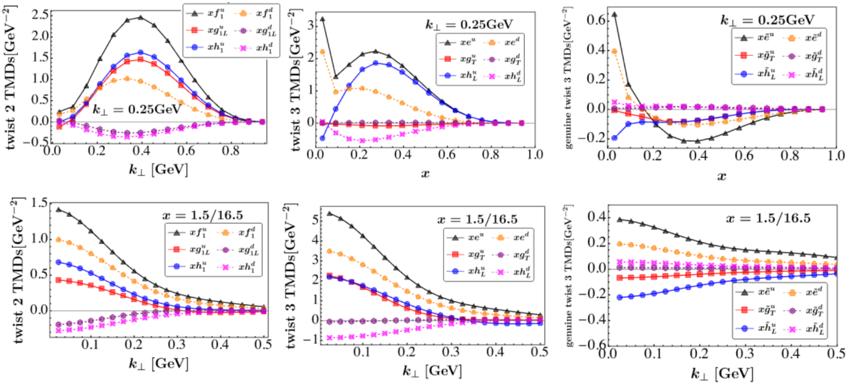
Jaffe-Ji notation:

f, e → unpolarized quarks
g → longitudinally polarized quarks
h → transversely polarized quarks

1 → the leading twist
L → longitudinally polarized hadron
T → transversely polarized hadron
$\perp$ → existing $k_\perp$ with a non-contracted index

[1] Meißner, et. al. JHEP08 (2009) 056.

Twist-2 vs Twist-3 Quark TMDs



Twist-3 TMDs:

- more concentrating in small x and k_T
- large than twist-2 TMDs

$$e\left(x,k_\perp^2\right) = \frac{m}{M}\frac{f_1\left(x,k_\perp^2\right)}{x} + \tilde{e}\left(x,k_\perp^2\right)$$

$$g_T(x,k_\perp^2) = \frac{m}{M}\frac{h_1(x,k_\perp^2)}{x} - \frac{k_\perp^2}{2M^2}\frac{g_{1T}(x,k_\perp^2)}{x} - \tilde{g}_T(x,k_\perp^2)$$

$$h_L\left(x,k_\perp^2\right) = \frac{m}{M}\frac{g_{1L}\left(x,k_\perp^2\right)}{x} - \frac{k_\perp^2}{M^2}\frac{h_{1L}^\perp\left(x,k_\perp^2\right)}{x} + \tilde{h}_L\left(x,k_\perp^2\right)$$

[1] Zhimin Zhu, *et. al.* in preparation